Focke, W. W.
Heat transfer analogies for plate heat exchangers,
SA. J. Chem, Eng.
7 (1) (1995) 1-15

HEAT TRANSFER ANALOGIES FOR PLATE HEAT EXCHANGERS

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ABSTRACT

Plate heat exchangers (PHE) are modular units composed of corrugated stainless steel plates. Flow is turbulent under typical operating conditions. It is shown that both the conventional transport analogies apply approximately for PHE's:

- Chilton-Colburn

$$f/j = \text{constant}$$

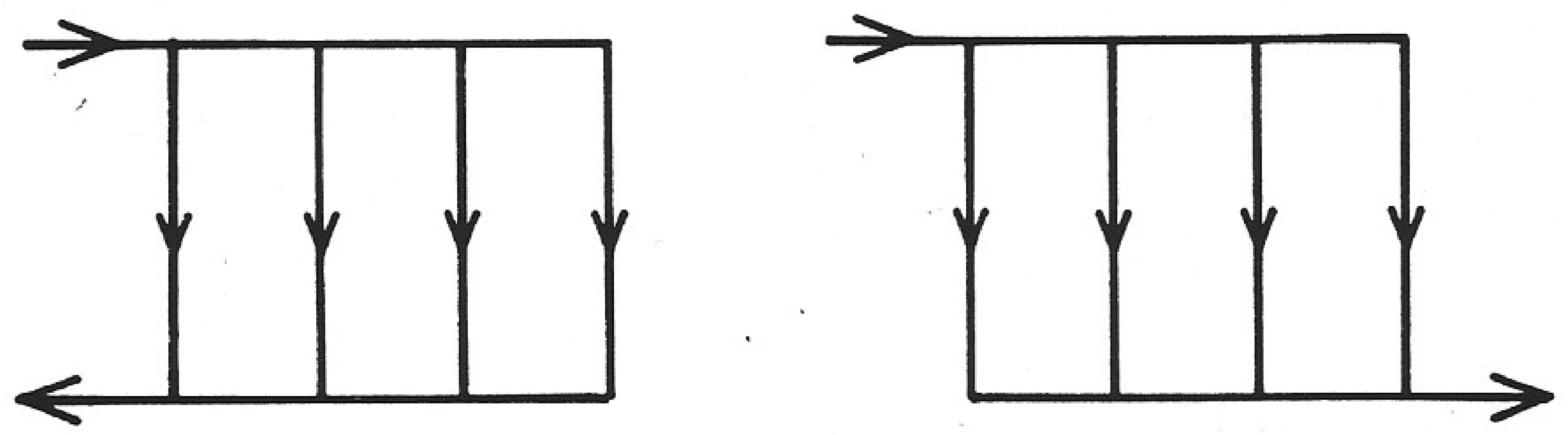
- Calderbank energy dissipation analogy

$$j\,Re = \text{constant}\ \ (fRe^3)^{1/4}$$

These expressions lead to a simple first-cut design procedure for plate heat exchangers. They also provide a theoretical framework for the concept of the θ value or "hardness" of a particular plate.

INTRODUCTION

The plate heat exchanger (PHE) is a modular unit based on a pack of corrugated plates which are clamped together in a frame. The flow channels are formed by the consecutive plates and peripheral elastomeric gaskets. Each plate has four corner ports and the gaskets are so arranged that the two process streams pass through alternate channels. Several different flow configurations are possible but the U-arrangement (Figure 1) is preferred. It allows all connections to be on one side thus obviating the need for disconnecting pipework for inspecting and cleaning purposes. It also features a more even flow distribution to the flow channels (1).



U-Arrangement Z-Arrangement

Figure 1. Plate heat exchanger flow configurations.

In a previous study a simplified model for a U-type PHE was developed (2). The model is based on the following idealization:

- Constant heat transfer coefficient
- No heat loss to surroundings
- Negligible fouling and wall resistances
- Equal flow distribution to channels
- True counter current flow
- Single pass on each side with equal number of channels
- Pressure drop in manifolds neglected

In terms of this simplified model suitable combinations of the base dimensionless numbers (Re, j, f) attain physical significance in terms of the exchanger dimensions of plate length, total surface area, cross sectional free flow area etc. (See Table 1 and Figure 2) This model is therefore a valuable conceptual tool allowing the design engineer to develop intuitive insight regarding the effect of exchanger size on performance. Presently this model will be used to generate a "short-cut" design procedure for U-type PHE's.

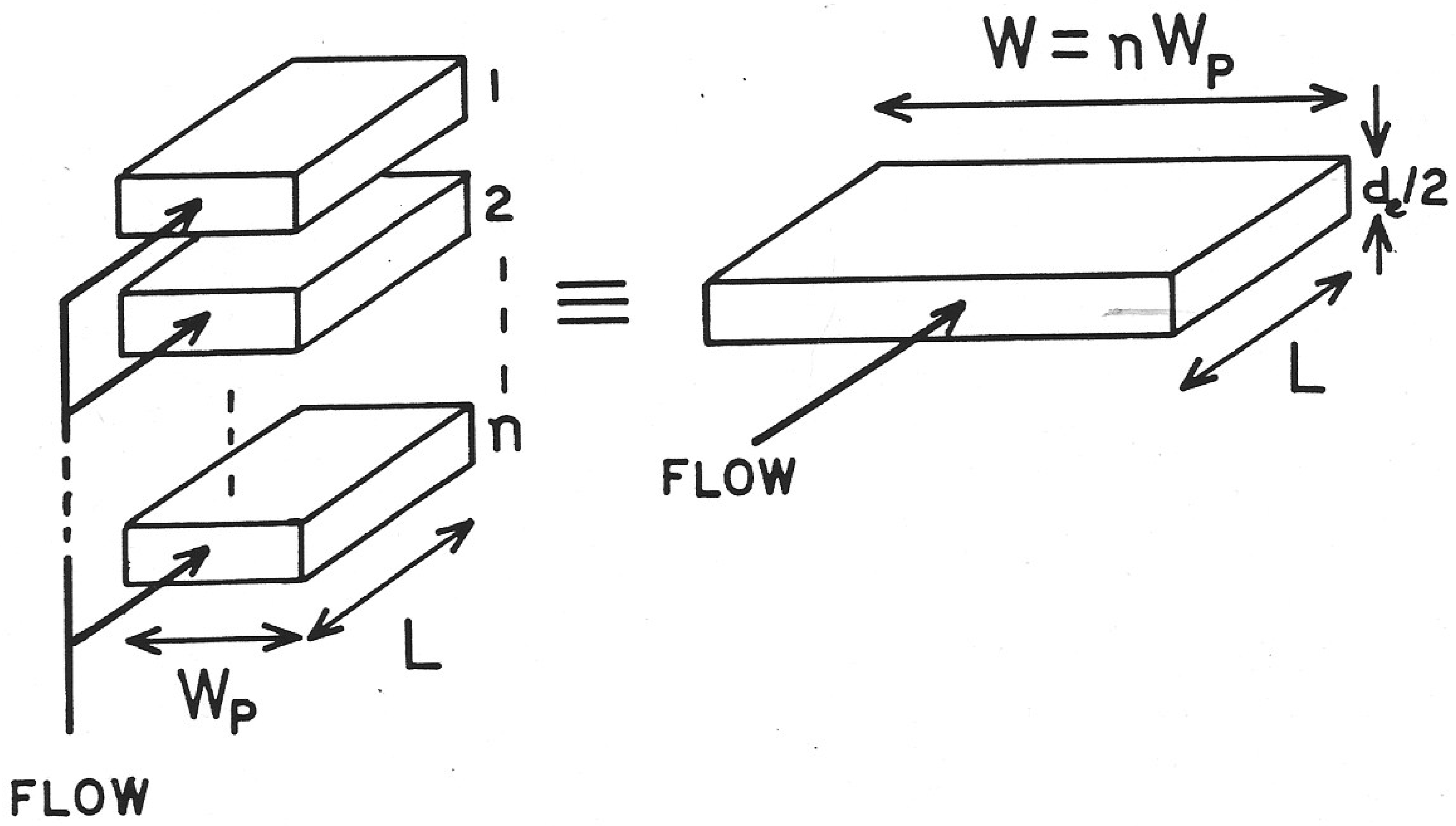


Figure 2. Simplified model of a Plate Heat Exchanger (2). A pack of channels per stream is equivalent to a single channel with width $W = nW_p$.

TABLE 1 - Physical significance of some dimensionless groups in the Idealized Plate Heat Exchanger (2).

1. BASE DIMENSIONLESS GROUPS

Geometry	A_c/A	$= d_e/4L$
Heat transfer	j	$= \dfrac{d_e \; ntu \; Pr^{2/3}}{4L}$
Friction factor	f	$= \dfrac{8\tau \Delta P A_c^{\;3}}{w^2 \; A}$
Reynolds Number	Re	$= \dfrac{4 \; w \; L}{\mu A}$

2. EXCHANGER DIMENSIONS

Surface Area (A)	$1/Re$	$= \dfrac{\mu A}{4 \; w \; L}$
Cross sectional free flow area (A_c)	$(f/j)^{\;1/2}$	$= A_c \left(\dfrac{8 \; \tau \; \Delta P}{w^2 \; ntu \; Pr^{\;2/3}} \right)^{1/2}$
Plate length	$(f \; Re^2/j^3)$	$= L \left(\dfrac{128 \; \tau \; \Delta P}{\mu^2 \; ntu^3 Pr^2} \right)^{1/2}$

3. PERFORMANCE GROUPS

Transfer units per unit surface area	$j \; Re$	$= \dfrac{w d_e \; ntu \; Pr^{\;2/3}}{\mu A}$
Pressure drop per unit surface area (Energy dissipation)	$f \; Re^3$	$= \dfrac{8w \; \tau \; d_e^3 \; \Delta P}{\mu^3 \; A}$
Pressure drop per unit exchanger length	$f \; Re^2$	$= \dfrac{2\tau \; \Delta P d_e^{\;3}}{\mu^2 \; L}$

TRANSPORT ANALOGIES

Transport analogies are rooted in the fact that the differential equations governing heat and momentum transfer in turbulent fluid flows are similar. Thus the Chilton-Colburn relationship

$$j = f/2 \tag{1}$$

is found to apply for turbulent flow in constant-area ducts. For normal operating conditions the flow in PHE's may be regarded as turbulent. The flow area goodness factor

$$K = (f/j)^{\frac{1}{2}} \tag{2}$$

is a dimensionless measure of the cross sectional free flow area of an exchanger. (See Table 1). The Chilton-Colburn analogy predicts a constant value K ~ 1,41. We have plotted K versus Re for a series of plate patterns tested at CSIR (3) in Figure 3. Over the flow ranges investigated K is approximately constant for each individual pattern. However K varies strongly with the corrugation inclination angle β.

As shown by Calderbank et al (4) and Banerjee (5) there is also an analogy between transfer and energy dissipation. They respectively used semi-empirical arguments and a surface renewal model of wall turbulence and found that

$$j\ Re = C\ (f\ Re^3)^{1/4} \tag{3}$$

where $j\ Re$ and $f\ Re^3$ are dimensionless representations of heat transfer and energy dissipation per unit exchanger surface area (See Table 1). To test this analogy we have also plotted $C = j\ (Re/f)^{1/4}$ in Figure 3. for our plate patterns. Again we find that C is remarkably constant and independent of geometry over the Re range investigated, i.e. for all the present plates $C \approx 0,23$.

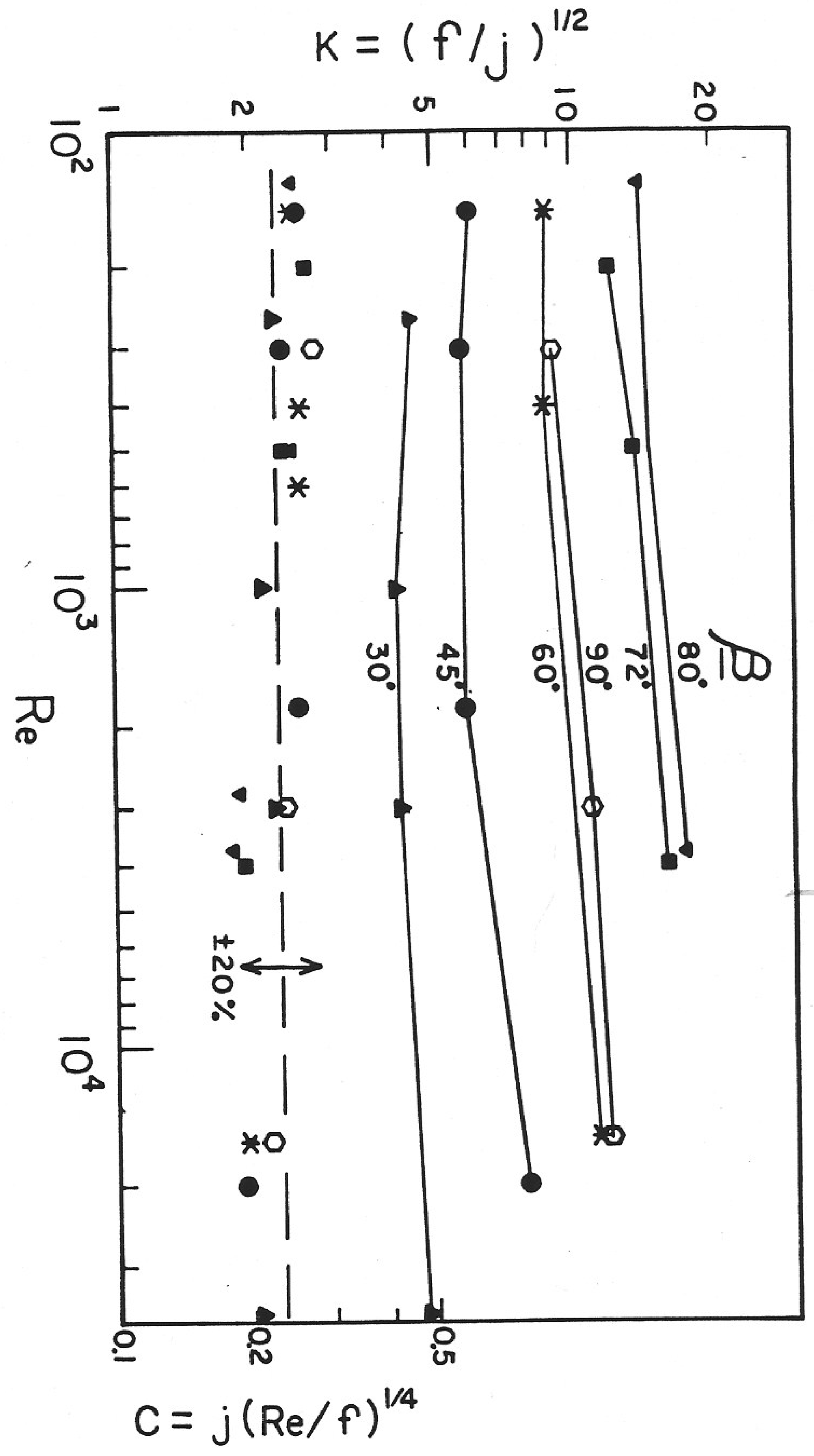


Figure 3. Test of transport analogy predictions for plate heat exchangers.

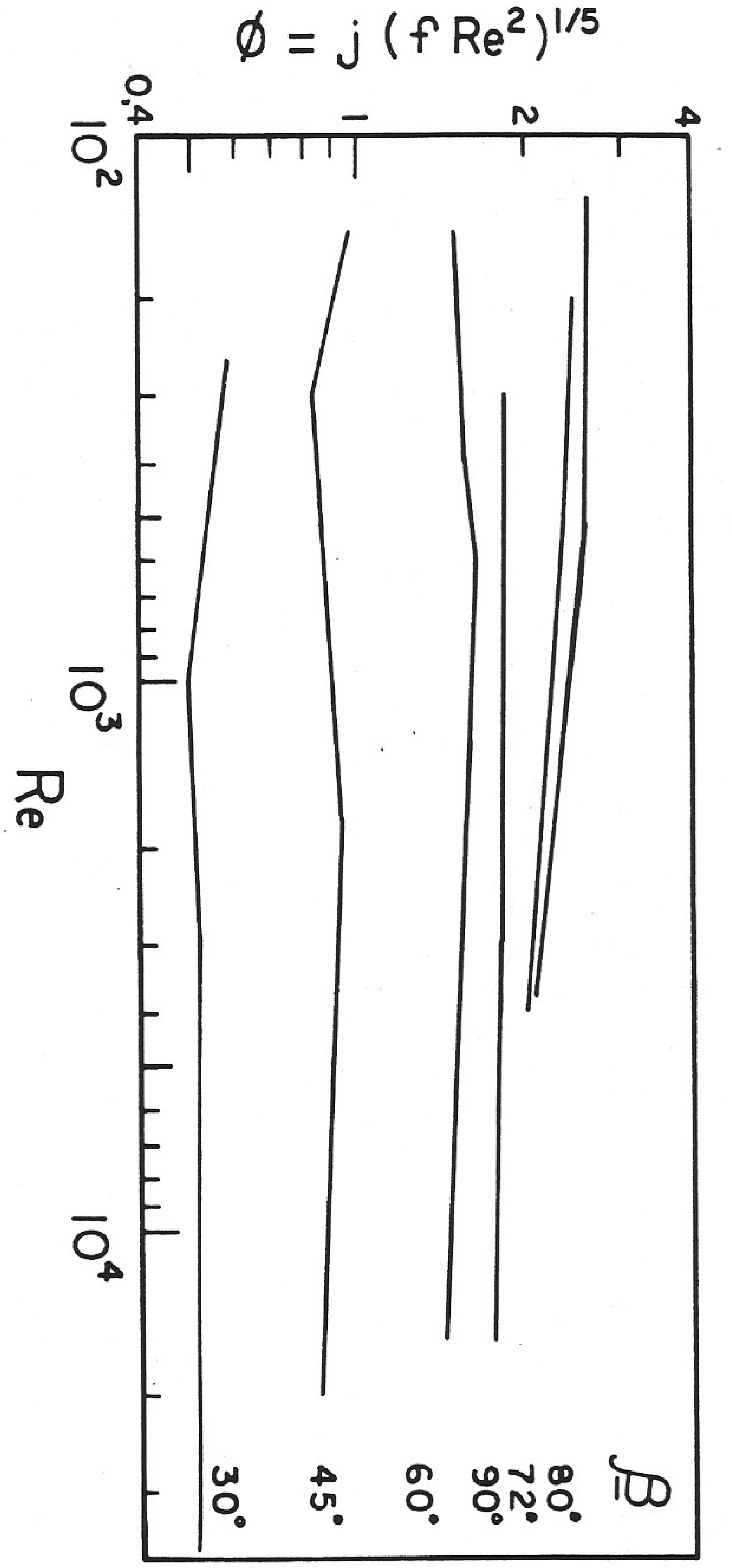


Figure 4. Plot of Φ (equation (4)) versus Re for CSIR plate patterns.

We conclude that both the transport analogies hold approximately for the plate patterns tested.
From Table 1 we note that the characteristic group for pressure drop is f Re^2 and for heat transfer it is j. By combining the two heat transfer analogies we find a relationship between these two variables:

$$j\ (f\ Re^2)^{1/5} = (K^6\ C^8)^{1/5} = \Phi \tag{4}$$

A plot of j $(f\ Re^2)^{1/5}$ for the present plate patterns is shown in Figure 4. Interestingly one finds that the variation in this factor for a given plate is significantly less than either C or K alone. This may imply that Φ is a more characteristic constant for plate heat exchangers than either C or K.

APPROXIMATE DESIGN OF PLATE HEAT EXCHANGERS

For a specific application we usually require that a minimum amount of heat be transferred from one stream to another. Usually the inlet temperatures of both streams are known while the minimum outlet temperature of at least one stream is specified. From a heat balance

$$q = C_c\ (t_o - t_i) = C_h\ (T_i - T_o) \tag{5}$$

the required exchanger effectiveness can be obtained via

$$\epsilon = q/[C_{min}\ (T_i - t_i)] \tag{6}$$

Because ϵ is a monotomically increasing function of NTU and since we idealize for true countercurrent flow, the minimum number of transfer units required are:

$$NTU_{min} = \ln\ [(1 - R_c\epsilon)/(1 - \epsilon)]/[1 - R_c] \tag{7}$$

In general pressure drop restrictions are also imposed. We can therefore write the design constraints as:

$$NTU \geq NTU_{min} \tag{8}$$

$$\Delta P_i \leq \Delta P_{i,max} \qquad (i = 1,2) \tag{9}$$

The cross-sectional free flow area (A_c) and the heat transfer surface area (A) are identical for both fluid streams. From the heat transfer analogies (equations (2) and (3)) we then obtain for a pack of identical plates

$$\frac{\Delta P_1}{\Delta P_2} = \left(\frac{\tau_2}{\tau_1}\right)\left(\frac{\mu_2}{\mu_1}\right)^{4/3}\left(\frac{w_1}{w_2}\right)^{5/3}$$

$$= F_p \tag{10}$$

$$\frac{ntu_1}{ntu_2} = \left(\frac{w_2}{w_1}\right)^{2}\left(\frac{Pr_2}{Pr_1}\right)^{2/3}\left(\frac{\tau_1\,\Delta P_1}{\tau_2\,\Delta P_2}\right)$$

$$= \left(\frac{w_2}{w_1}\right)^{1/3}\left(\frac{Pr_2}{Pr_1}\right)^{2/3}\left(\frac{\mu_1}{\mu_2}\right)^{1/3}$$

$$= F_n \tag{11}$$

Note that both F_p and F_n are constants that depend on mass flow rates and fluid physical properties only! Also

$$NTU = (1/ntu_1 + 1/ntu_2)^{-1}$$

$$= ntu_1 \; (1 + F_n)^{-1} \tag{12}$$

For the discussion to follow it is now convenient to assume (without loss of generality) that stream 1 is the pressure drop limited stream i.e: provided that the pressure drop constraint on stream 1 is met, we will also have:

$$\Delta P_2 = \Delta P_1 / F_p$$

$$\leq \Delta P_{2,\ max} \qquad (13)$$

This means that the heat exchanger design constraints have now been reduced to:

$$ntu_1 \geq ntu_{1,\ min}$$

$$= NTU_{min} \left[1 + F_n \right] \qquad (14)$$

$$\Delta P_1 \leq \Delta P_{1,\ max} \qquad (15)$$

The surface area of the exchanger can be determined from the heat transfer-energy dissipation analogy:

$$A = w_1 \left(\frac{1}{2C^{4/3}}\right) \left(\frac{Pr_1^{\ 8/9}}{\mu_1^{1/3}\ \tau^{1/3}}\right) \left(\frac{ntu_1^{\ 4/3}}{\Delta P_1^{\ 1/3}}\right) d_e^{\ 1/3} \qquad (16)$$

This shows that:

- Surface area scales linearly with mass flow rate as expected.
- Surface area scales with the cube root of the equivalent diameter.
- Surface area requirements decrease with increasing pressure drop and decreasing ntu.

This implies that the optimum surface area will be the same for all our plates (recall C = constant and same for all patterns) and given by:

$$A_{min} = 3{,}5\ w_1 \left(\frac{Pr_1^{\ 8/9}}{\mu_1^{1/3}\ \tau^{1/3}}\right) \left(\frac{ntu_{1,min}^{\ 4/3}}{\Delta P_{1,max}^{\ 1/3}}\right) d_e^{\ 1/3} \qquad (17)$$

Implied in this equation is the design freedom to vary plate length and width continuously in order that both the ntu and ΔP constraints can be matched. The required plate length will obviously vary with corrugation angle β for instance.

However in practice plates are only manufactured in a limited number of sizes. It is therefore necessary to consider the effect of a plate length constraint.

Substituting the definitions for j and f Re^2 from Table 1 into equation (4) and rearranging leads to:

$$ntu_1 \, Pr_1^{2/3} \, (\tau_1 \Delta P_1 / \mu_1^2)^{1/5} = 3{,}48 \, \Phi \, (L^6/d_e^8)^{1/5} \qquad (18)$$

Depends only on performance parameters (design constraints P_1, ntu_1) and fluid physical properties	Depends on thermo-hydraulic performance characteristic of plate pattern ($\Phi = \Phi(\beta)$), plate length and equivalent diameter

In the first instance equation (18) may be used to calculate the required plate length for the optimum surface area design. More important however we can use this equation for short cut plate selection purposes. Define:

$$\Theta_s = ntu_{1,min} \, Pr_1^{2/3} \, (\tau_1 \Delta P_{1,max} / \mu_1^2)^{1/5} \qquad (19)$$

$$\Theta_p = 3{,}48 \, \Phi \, (L^6/d_e^8)^{1/5} \qquad (20)$$

= "plate hardness" and is a constant for any given plate.

The optimum plate will match the Θ_s value of the specific application. In order to achieve design flexibility, a set of plates should be available with Θ_p values evenly spanning the typical range of Θ_s values found in industrial applications. The arguments presented here can also be used to develop detailed design procedures for PHE's (6).

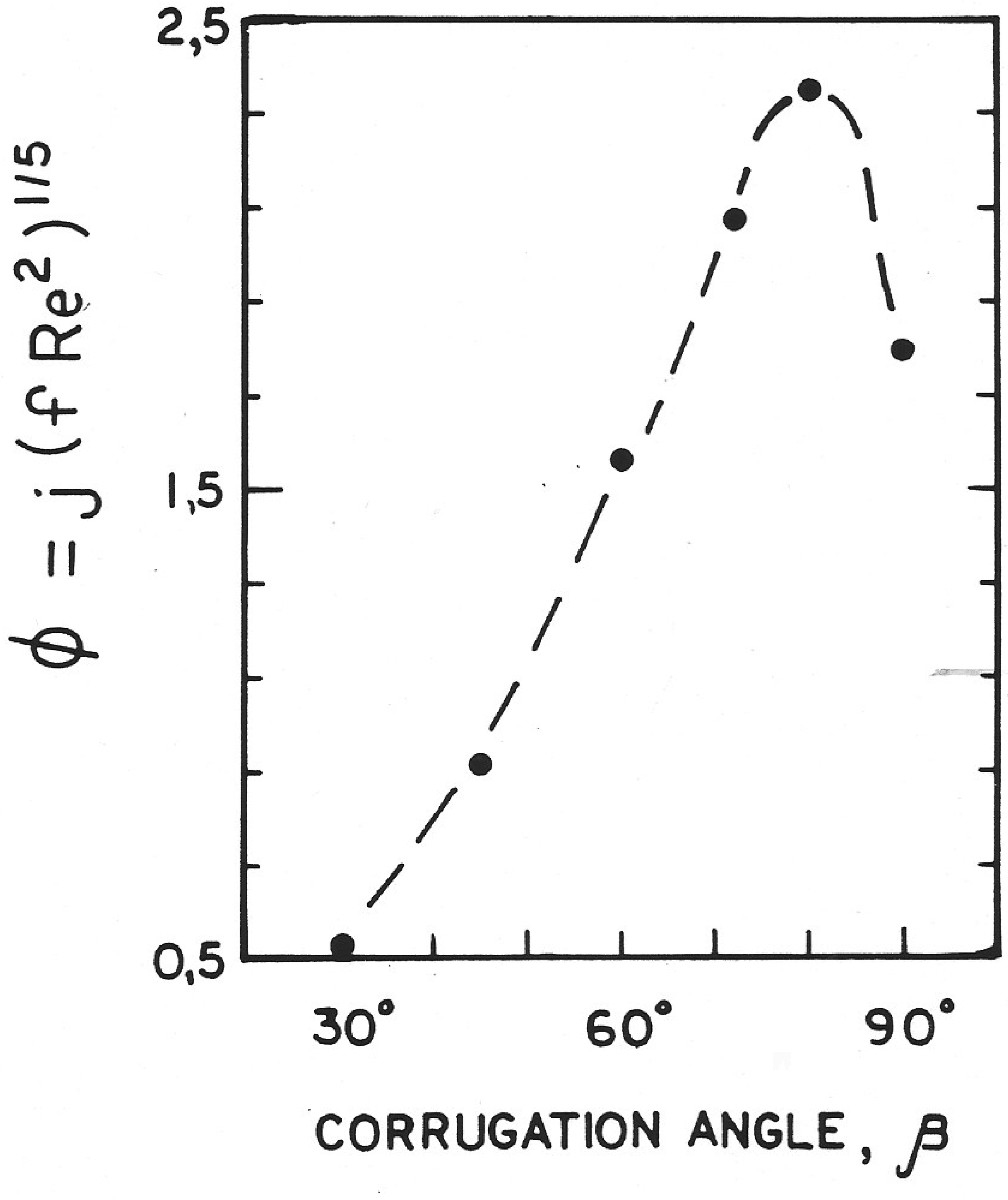


Figure 4. The effect of corrugation inclination angle on the characteristic pattern constant Φ.

The Φ values for the present plate patterns are plotted versus the corrugation inclination angle in Figure 4. Φ increases almost linearly with β between $\beta = 30^\circ$ and $\beta = 80^\circ$. Previously (8) we have shown that thermohydraulic performance of mixed plates channels are intermediate in performance to those of the parent plates. Thus in principle a mixture of 30° and 60° plates also provides for 45° plate behaviour.

Finally it is important to note that, according to equation (20), "plate hardness" is a stronger function of equivalent diameter than plate length.

The discussion presented above was based on a single set of experimental data. Unfortunately thermohydraulic performance data for commercial PHE's remain confidential. However Marriott (7) claims that equation (4) correlates the characteristics of commercial plates to within about 20%. It can therefore safely be assumed that the present disposition applies to commercial plates as well.

CONCLUSIONS

Both the Chilton-Colburn analogy and the heat transfer-power dissipation analogies hold approximately for plate heat exchangers. However it appears that a correlation of the type

$$j = \Phi \ (f \, Re^2)^{-1/5} \qquad (4)$$

provides the best fit with experimental data. Here Φ is a characteristic constant for a given plate pattern. It varies strongly with parameters such as corrugation inclination angle β.

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provides the best fit with experimental data. Here Φ is a characteristic constant for a given plate pattern. It varies strongly with parameters such as corrugation inclination angle β.

The optimum plate pattern can be determined by matching "plate hardness"

$$\Theta_p = 3{,}48\ \Phi\ (L^6/d_e^{\,8})^{1/5}$$

with the Θ_s value of the application:

$$\Theta_s = ntu_{1,min}\ Pr_1^{\,2/3}\ (\tau_1 \Delta P_{1,max} / \mu_1^{\,2})^{1/5}$$

where the subscript 1 refers to property values for the pressure drop limiting stream.

REFERENCES

1. NUIJENS, PGJM., FOCKE, WW., and OLIVIER, I. (1991) Flow distribution in multi-compartment equipment with linear supply and discharge manifolds. Submitted for Publication in S A I Ch E J

2. FOCKE, WW. (1986) Selecting optimum plate heat exchanger surface patterns. J Heat Transfer 108 153 - 160.

3. FOCKE, WW., ZACHARIADES, J., and OLIVIER, I. (1985) The effect of the corrugation inclination angle on the thermohydraulic performance of plate heat exhangers. Int J Heat Mass Transfer 28 1469 - 1479.

4. CALDERBANK, PH., and MOO-YOUNG, MB., (1961) The continuous heat and mass transfer properties of dispersions. Chem Eng Sci 16 39 - 54.

5. BANERJEE, S. (1971) A note on turbulent mass transfer at high Schmidt numbers. Chem Eng Sci 26 989 - 990.

6. SHAH, RK., and FOCKE WW. Plate heat exchangers and their design theory; in SHAH, RK., SUBBARNO, EC., MASHELKAR., RA., (1986) Heat transfer equipment design, Hemisphere, New York.

7. MARRIOTT, J., Alfa Laval, Sweden. Private Communication with FOCKE, WW.

8. FOCKE, WW., and LOMBARD, M., (1984). Thermohydraulic performance of plate heat exchanger configurations consisting of mixed plates. CSIR Report CENG 533.

NOMENCLATURE

A	heat exchanger surface area	[m^2]
A_c	cross-sectional free flow area	[m^2]
C	constant equation (3)	[-]
C_c	cold stream heat capacity rate	[W/°C]
C_h	hot stream heat capacity rate	[W/°C]
C_p	heat capacity	[kJ/kg°C]
d_e	equivalent diameter, twice the spacing between plates	[m]
F_n, F_p	constants equations (10) and (11)	[-]
f	friction factor (2 $Pd_e/\tau u^2 L$)	[-]
h	heat transfer coefficient	[W/m^2°C]
j	Colburn j factor (h $Pr^{2/3}$)/($\tau u C_p$)	[-]
K	flow area goodness factor, equation (2)	[-]
k	fluid thermal conductivity	[W/m°C]
L	channel length	[m]
n	number of channels per fluid	[-]
NTU	number of transfer units	[-]
ntu	stream number of transfer units (hA/C_i)	[-]
ΔP	pressure drop	[Pa]
Pr	Prandtl number	[-]
q	heat duty	[W]
Re	Reynolds number ($\tau u\, d_e$ /)	[-]
R_c	capacity ratio C_{min}/C_{max}	[-]
T	hot stream temperature	[°C]
t	cold stream temperature	[°C]
U	overall heat transfer coefficient	[W/m^2°C]
u	fluid velocity	[m/s]
W_p, W	plate width, width of idealized PHE channel	[m]
w	mass flow rate	[kg/s]

GREEK SYMBOLS

β	corrugation inclination angle to main flow direction	[rad]
ϵ	thermal effectiveness $q/C_{min}(T_i - t_i)$	[-]
μ	fluid viscosity	[Pa.s]
τ	fluid density	[kg/m^3]
Φ	constant, equation (4)	[-]
Θ_s	stream "hardness" factor, equation (19)	[$m^{-2/5}$]
Θ_p	plate "hardness" factor, equation (20)	[$m^{-2/5}$]

SUBSCRIPTS

i	in	1	pressure drop limited stream
o	out	2	other fluid stream
max	maximum		
min	minimum		