# UNDERSPECIFICATION OF THE EMPIRICAL RETURN-FACTOR MODEL AND A FACTOR ANALYTIC AUGMENTATION AS A SOLUTION TO FACTOR OMISSION

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## Abstract

This empirical paper comprehensively sets out the impact of underspecification on a key foundational concept in empirical finance, the linear factor model. It places emphasis on the extensive consequences of factor omission for model estimation and interpretation. Factor omission in time-series models that relate asset returns to pre-specified factor sets is a common problem. A proposed standard and widely-used solution is the inclusion of a residual market factor which is assumed to be a catch-all proxy for omitted factors. This study shows that a specification that incorporates a set of carefully selected macroeconomic factors will be underspecified. The inclusion of residual market factors will alleviate but not eliminate the consequences of underspecification. Although the early use of factor analytically derived factor scores in factor models has been criticized, augmenting a model comprising pre-specified factors with statistical factors derived from the residuals results in an accurately specified model for which the diagonality assumption holds. Consequently, this paper shows that a factor analytic augmentation is an effective and readily implementable solution to the factor omission problem.

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# 1 Introduction

The Arbitrage Pricing Theory (APT), introduced by Ross (1976), is the basis of multifactor representations of the return generating process relating asset returns to factor sets (Liow, 2004; Sadorksy, 2008). The underpinning time-series linear factor model is also a building block of asset pricing models and its correct specification is required if pricing models are to be estimated and tested (Elton, Gruber & Blake, 1995; Drakos, 2002; French, 2017; Elton & Gruber, 2018). Early multifactor literature relies on factor analytic and principal component techniques to determine the number of factors in returns and to estimate multifactor models using these factors as explanatory factors (see Chimanga & Kotze, 2009). However, the use of factor scores poses a challenge. Factor coefficients and the associated signs have no meaning or interpretation and the scaling of coefficients is arbitrary (Yli-Olli & Virtanen, 1992; Priestley, 1996). The limitations of statistical techniques in factor extraction and identification is what spurs the use of pre-specified macroeconomic factors in place of factor scores, beginning with the work of Chen, Roll and Ross (1986) (see also Chen & Jordan, 1993; Connor, 1995; Panetta, 2002).

However, macroeconomic linear factor models often suffer misspecification in the form of factor omission. Reasons for the purposeful or accidental omission of factors are numerous. The identification of macroeconomic factors that proxy for pervasive influences in returns is not straightforward (Connor, 1995; Panetta, 2002; Bilson, Brailsford & Hooper, 2001). Relevant and important factors may be omitted from analysis as a result of inaccurate data or data that is unavailable at the desired frequency (Van Rensburg, 1995; Clements & Galvão, 2008: 546). Relevant factors may be omitted if model parsimony and interpretability is desired and influences that are not fully reflected by macroeconomic factors, such as investor sentiment and behaviour, may need to be considered directly to arrive at a comprehensive specification (Hughes, 1984; Middleton & Satchell, 2001, Malkiel, 2003; Lin, Rahman & Yung, 2009; Driffill, 2011). The assumption of linearity may itself be restrictive, resulting in a failure to account for non-linearities, such as a differential impact according to economic state (Reinganum, 1981; Funke & Matsuda, 2006). Finally, macroeconomic factors are likely to be poor proxies for pervasive influences in stock returns, rendering purely macroeconomic factor models underspecified (Connor, 1995; Middleton & Satchel, 2001; Van Rensburg, 2000; Spyridis, Sevic & Theriou, 2012: 40).

Factor omission has consequences for model estimation and interpretation. Omitted factors result in inflated model intercepts and may impact inferences relating to performance. Factor omission, by inflating the residuals of a model, will reduce the power of statistical tests. This may lead to a misidentification of relevant factors in a model, which will appear to be erroneously insignificant (Chang, 1991; Van Rensburg, 2000; 2002). Factor omission will lead to endogeneity, resulting in biased and inconsistent parameter estimates that make reliable inference making almost impossible (Chenhall & Moers, 2007; Roberts & Whited, 2013; Studenmund, 2014). If omitted factors are not included in a specification, estimated coefficients will be unstable. Instability will be driven by the changing relationship between factors that are included and the omitted factors (Panetta, 2002). A well-specified return generating model should capture a significant portion of time-varying volatility in returns and factor omission may induce heteroscedasticity (also see Bera, Bubnys & Park, 1988; Koutoulas & Kryzanowski, 1994). In the presence of heteroscedasticity, inferences based upon models estimated using least squares will be unreliable as standard errors will be overestimated (Sadorsky & Henriques, 2001).

The key message in the literature is that factor omission adversely impacts time-series return-factor models in numerous ways. To resolve underspecification, a residual market factor derived from the residuals of a regression of returns on a broad market index onto a set of factors can be used. The residual market factor is assumed to be a catch-all proxy for unspecified omitted factors. The use of a residual market factor in such models is the default approach to addressing underspecification (Deetz, Poddig, Sidorovitch & Varmaz, 2009; Czaja, Scholz & Wilkens, 2010). However, as only the true market portfolio will reflect *all* return generating factors, and such a portfolio is impossible to identify or construct, a broad market index is unlikely to produce a residual market factor that fully reflects omitted influences (Born & Moser, 1988). The solution may therefore lie in combining macroeconomic factors with a residual market factor or factors *and* a statistical augmentation to arrive at an adequately specified model. This approach is suggested by the Northfield U.S. Macroeconomic Equity Risk model developed by Northfield Information Services (NIS) (2015). The factor analytic augmentation comprises statistical factor scores derived from the residuals which reflect pervasive influences that are either transient or not reflected by the pre-specified factors used to explain returns in a given specification.

The theoretical case for a factor analytic augmentation alongside pre-specified factors is made by Middleton and Satchell (2001) who argue that underspecification ought to be avoided in factor models that use derived statistical factors. Also, Van

Rensburg (1997) estimates a three-factor model comprising returns on the DJIA, and two macroeconomic factors to describe returns on a sample of well-traded stocks on the Johannesburg Stock Exchange (JSE). He goes on to extract two factors from the residuals of this specification which explain an additional 43,9% of variation in the residuals. This suggests that omitted factors are relegated to the residuals and that these factors can be appended to a factor model to account for omitted influences. Following Middleton and Satchell (2001) and Van Rensburg (1997), there is nothing that precludes the inclusion of such factors to arrive at an adequately specified model alongside a macroeconomic factor set and a residual market factor or factors.

This study provides a comprehensive overview of the consequences of underspecification and the effectiveness of residual market factors in mitigating the consequences of factor omission, within the context of return-factor models. It shows that residual market factors are not an effective proxy for omitted factors. Importantly, it demonstrates the effectiveness of a factor analytic augmentation in addressing underspecification. This study should be of value to researchers who work in asset pricing and those who wish to understand the interactions between asset prices and their hypothesized determinants using time-series factor models. Although South African data is used and the explanatory factor set is macroeconomic, inferences should be applicable across markets, in different contexts and to factor-return models in general. Linear factor models are a foundational concept in finance and the basis of numerous applications in finance. Asset pricing models, such as the CAPM, the APT and later prominent models, such as the Fama and French (1993) three-factor model, the Carhart (1997) four-factor model and the Fama and French (2015) five-factor model rely upon underpinning time-series factor models representative of the return generating process (see also Chimanga & Kotze, 2009; Lambert & Hübner, 2013; Zaremba, Czapkiewicz, Szczygielski & Kaganov, 2018). Factor models are also applied within an empirical context, one that is not concerned with the asset pricing equilibrium. Studies falling into this latter paradigm seek to relate asset returns to factors that are assumed to impact asset prices, whether statistical or macroeconomic. Examples of such predominantly non-equilibrium applications can be found in Berry, Burmeister and McElroy (1988), Van Rensburg (1995), Sadorsky (2008) and Szczygielski and Chipeta (2015). The accurate estimation and interpretation of such models becomes questionable if factors have been omitted.

This empirical study contributes to the literature on asset price and return modelling by providing a comprehensive and in-depth overview of the empirical consequences of underspecification in linear (return-)factor models. It suggests and outlines a readily implementable solution for factor omission in such models, that of a factor analytic augmentation. It shows that factor analysis *still* has a role to play in the estimation of models relating returns to factor sets and can be a useful tool for correctly specifying such models. It also contributes by investigating whether residual market factors adequately address factor omission and alleviate the consequences of underspecification. The use of these factors to correct for factor omission matters as this is often the standard and default approach to controlling for factor omission in factor models (Deetz, Poddig, Sidorovitch & Varmaz, 2009; Czaja, Scholz & Wilkens, 2010). However, the efficacy of using residual market factors alongside a specified factor set to account for omitted factors is not directly or widely explored in the literature. This paper addresses this gap. Furthermore, its novelty lies in that it considers the impact of underspecification on certain aspects that have been seldom considered, e.g. the conditional variance structure, residual correlation matrix structures. The consequences of underspecification are often considered in isolation, focusing on a limited number of aspects at a time and often indirectly. Examples of these isolated and limited aspects are the inflation of idiosyncratic factors (such as residual variance) used in tests of asset pricing models (Yli-Olli & Virtanen, 1992), the violation of the diagonality assumption and an upward bias in standard errors (Van Rensburg, 2000; 2002), coefficient bias (Dominguez, 1992), the misidentification of priced factors (Jorion, 1991) and pricing errors associated with the use of macroeconomic factors in the macroeconomic APT (Middleton & Satchell, 2001). This paper contributes by reviewing these consequences comprehensively and holistically and therefore acts as a reference for researchers, econometricians and modellers of asset prices. A further contribution lies in the econometric methodology used. Factor models are estimated using maximum likelihood (ML) estimation with ARCH/GARCH errors. This methodology is usually used to model volatility. However, using this methodology permits the conditional mean coefficients to reflect conditional variance structures, yielding more efficient and more accurate coefficient estimates while providing further insights into the impact of factor omission (Hamilton, 2010; Armitage & Brzeszczyński 2011).

A comparative approach is undertaken. A linear factor model comprising macroeconomic factors is compared against a model that combines macroeconomic factors and a single residual market factor derived from the domestic market aggregate, followed by a model that incorporates a second residual market factor, derived from a global market index. These models are then compared against a specification that incorporates a factor analytic augmentation. Results indicate that, as expected, factor omission results in biased coefficients, decreased efficiency, increased instances of residual serial correlation, increased prediction errors and a loss of predictive accuracy. Notable and novel findings are that factor omission impacts residual variance structures and that residual market factors, at least in the South African context, do not resolve factor omission. Importantly, a factor analytic augmentation is an effective solution to factor omission that ensures that the diagonality assumption holds. Although the use of statistical factors has been criticized in early literature, there is value in augmenting specifications comprising pre-specified factors with factors derived from the residuals of a linear factor model. A factor analytic augmentation eliminates systematic residual co-movement, which has the effect of invalidating a model.

This study proceeds as follows; Section 2 outlines the data and methodology applied, Section 3 reports the results of the analysis and outlines the implementation of a factor analytic augmentation. Section 4 concludes the study.

## 2 Data and methodology

### 2.1 Data

Monthly industrial data is obtained from the IRESS Expert database, comprising industrial sectors of the South African stock market, the Johannesburg Stock Exchange (JSE), spanning the period January 2001 to December 2016. Only industrial sectors with a full data history are included in the sample. This constitutes data for 26 industrial sectors out of a total of 33 sectors at the time of writing. Month-end data is used and continuously compounded excess returns are derived from total monthly returns using the closing yield on the R186 government bond (see Nel, 2011; PWC, 2015).

Table 1 lists the industrial sectors comprising the sample and the economic sectors to which they below, together with corresponding JSE index codes.[1]

[1] Descriptive statistics for the return series are reported in Table A1 and A2 in the Appendix. The Appendix is available from the authors upon request and includes unabridged and supplementary results that are referenced in this study.

**Table 1: List of industrial sectors**

| Economic Sector | Industrial Sector | Index Code |
|---|---|---|
| **Basic Materials** | Chemicals | J135 |
| | Forestry & Paper | J173 |
| | Ind. Metals & Mining | J175 |
| | Mining | J177 |
| **Industrials** | Constr. & Materials | J235 |
| | General Industrials | J272 |
| | Elec. & Elec. Equip. | J273 |
| | Indust. Engineering | J275 |
| | Indust. Transp. | J277 |
| | Support Services | J279 |
| **Consumer Goods** | Automobiles & Parts | J335 |
| | Beverages | J353 |
| | Food Producers | J357 |
| **Health Care** | Health Care Equip. & Services | J453 |
| | Pharm & Biotech. | J457 |
| **Consumer Services** | Food & Drug Retailers | J533 |
| | General Retailers | J537 |
| | Media | J555 |
| | Travel & Leisure | J575 |
| **Telecommunication** | Fixed Line Telecoms. | J653 |
| **Financials** | Banks | J835 |
| | Non-life Insurance | J853 |
| | Life Insurance | J857 |
| | General Financial | J877 |
| | Equity Investment Instruments | J898 |
| **Technology** | Software & Comp. Serv. | J953 |

## 2.2 Model specification and estimation

A comparative approach is undertaken in this study. Four specifications are estimated. Each specification is estimated for returns on each industrial sector and comparisons are undertaken on numerous aspects to quantify the impact of factor omission. The macroeconomic model comprises only macroeconomic factors identified (and interpreted) as proxies for systematic influences in South African stock returns in Szczygielski, Brümmer and Wolmarans (2020a) and Szczygielski, Brümmer, Wolmarans and Zaremba (2020b).

$$R_{it} = \alpha + b_{iBP}BP_{t-1} + b_{iLEAD}LEAD_{t-1} + b_{iBUS}BUS_t + b_{iUSD\varepsilon}USD\varepsilon_t + b_{iMET}MET_t + b_{iLTY}LTY_t + b_{iTLI}TLI\varepsilon_t + \varepsilon_{it} \quad (1)$$

where $R_{it}$ is the (excess) return on industrial sector index *i* at time *t*, the $b_i$'s are the sensitivities to innovations in the respective macroeconomic factors, namely the number of building plans passed ($BP_{t-1}$), the leading composite (domestic) business cycle indicator ($LEAD_{t-1}$), domestic business activity ($BUS_t$), the (orthogonalised) Rand Dollar exchange rate ($USD_{\varepsilon_t}$), world metal prices in US Dollars ($MET_t$), long-term interest rates ($LTY_t$) and the (orthogonalised) composite leading economic conditions indicator for South Africa's trading partners ($TLI_{\varepsilon_t}$). Szczygielski *et al.* (2020b) show that although these factors, chosen from an extensive set of factors, are proxies for statistically derived factors assumed to represent pervasive influences in returns, they are poor proxies for these influences. Likelihood ratio tests confirm that these factors produce an underspecified model (see Table A3 in the Appendix).

The second model, the macro-residual model, comprises the macroeconomic factors above and a residual market factor. The residual market factor is derived by regressing returns on the JSE All Share Index – the broad market aggregate – and treating the residuals of this auxiliary regression as the first (and conventional) residual market factor (Burmeister & Wall, 1986; Berry *et al.,* 1988; Van Rensburg, 1997; 2000):

$$R_{it} = \alpha + b_{iBP}BP_{t-1} + b_{iLEAD}LEAD_{t-1} + b_{iBUS}BUS_t + b_{iUSD\varepsilon}USD\varepsilon_t + b_{iMET}MET_t + b_{iLTY}LTY_t + b_{iTLI}TLI\varepsilon_t + b_{iM\varepsilon}M\varepsilon_t + \varepsilon_{it} \quad (2)$$

where all factors are as in equation (1), with the exception of the residual market factor, ($M_{\varepsilon_t}$). The residual market factor is derived by regressing returns on the JSE All Share Index onto the macroeconomic factor set and using the residuals of this regression in place of returns on the JSE All Share Index as a factor.[2] By taking this approach, we orthogonalize the market index, deriving a series that is uncorrelated with the remaining macroeconomic factors in equation (2).

The third model, the unrestricted model, incorporates a second residual market factor, derived by jointly regressing the macroeconomic factors and returns on the JSE All Share Index onto returns on the US dollar denominated MSCI World Market Index in a single regression. The resultant residuals are then used in place of returns on the

[2] All orthogonalizing models used to derive residual market factors are estimated by applying the least squares methodology.

MSCI World Market Index as an orthogonal factor. The orthogonalization process for the second residual market factor is therefore the same as that for the first residual market factor derived from returns on the JSE All Share Index, $M_{\varepsilon_t}$. This specific index is widely used in the literature to account for global influences on local stock markets that may not be fully reflected by a domestic market index (Harvey, 1995; Clare & Priestly, 1998; Brown, Hiraki, Arakawa & Ohno 2009).

$$R_{it} = \alpha + b_{iBP}BP_{t-1} + b_{iLEAD}LEAD_{t-1} + b_{iBUS}BUS_t + b_{iUSD\varepsilon}USD\varepsilon_t + b_{iMET}MET_t + b_{iLTY}LTY_t + b_{iTLI}TLI\varepsilon_t + b_{iIM\varepsilon}M\varepsilon_t + b_{iIM\varepsilon}IM\varepsilon_t + \varepsilon_{it} \qquad (3)$$

where all factors are as in equation (2), with the exception of the second residual market factor, $IM_{\varepsilon_t}$.

The fourth and final specification is the augmented model that incorporates a factor analytic augmentation:

$$R_{it} = \alpha + b_{iBP}BP_{t-1} + b_{iLEAD}LEAD_{t-1} + b_{iBUS}BUS_t + b_{iUSD\varepsilon}USD\varepsilon_t + b_{iMET}MET_t + b_{iLTY}LTY_t + b_{iTLI}TLI\varepsilon_t + b_{iIM\varepsilon}M\varepsilon_t + b_{iIM\varepsilon}IM\varepsilon_t + b_{i1}f_{1t} + b_{i2}f_{2t} + \varepsilon_{it}^* \qquad (4)$$

where all factors are as in equation (3), with the exception of two statistical factors, $f_1$ and $f_2$ which are factor scores derived from the residuals of equation (3). These factor scores, derived using the Bartlett (1937) method, comprise the factor analytic augmentation and represent influences not reflected by the macroeconomic and residual market factors (see also DiStefano, Zhu & Mîndrilă, 2009: 4-5). The (now) theoretically purely idiosyncratic component is represented by $\varepsilon_{it}^*$ (Burmeister & McElroy, 1991). Both statistical factors are derived from the residuals of equation (3) and are subjected to an orthogonal varimax rotation. Consequently, they are uncorrelated with all other factors and each other. The augmented model should outperform the remaining three models on all or most of the aspects considered below if the factor analytic augmentation resolves underspecification.

### 2.3 A note on implementing the factor analytic augmentation

A factor analytic augmentation can be implemented in two contexts. In the first instance, factor models are estimated to derive inputs for asset pricing relations, namely the betas. This requires that the same specification is estimated across a number of assets, be they individual stocks or portfolios (see for example Brown *et al.*, 2009; Spyridis *et al.*, 2012). This is the approach taken in this study; the same specification is estimated for a total of 26 industrial sectors comprising the South African stock market. In the second instance, factor models are estimated with pre-specified factors to study the behaviour of a single series (e.g. industry, market index) and the estimated factor sensitivities are not used for any other purposes than direct interpretation and inference making about the structure of the empirical-return generating process (see Sadorsky, 2001; Sadorksy & Henriques, 2001; Szczygielski & Chipeta, 2015).

In the first instance, a similar approach to that followed in this study can be applied. In the first step, a reduced-form model comprising the desired pre-specified factors is estimated. In this study, this is equation (3). In the second step, the resultant residual correlation matrix is factor analysed and extracted factors are appended to the initial specification to produce an expanded-form specification. In this study, this is equation (4). In the second instance, no restricted-form model is initially estimated. Instead, a broader set of return series from the same market as the single series of interest is factor analysed to extract factor scores representative of pervasive influences. In the second step, the extracted factor scores are orthogonalized against the pre-specified factor set that is used to represent the proposed model for the series of interest. This is the same approach followed in deriving the residual market factors as outlined above; factor scores are regressed against the factor set comprising the model and the residuals are used as orthogonal factors to augment the specification of interest. This produces the functional form of equation (4) but for a single series, with the factor analytic augmentation comprising the residuals of factor score regressions. The appeal of this approach is that while it requires return data beyond that of the series of interest, the model is still tailored and specific to the series of interest. The statistical factors now represent any common factors that have not been explicitly reflected by the pre-specified factor set.

### 2.4 Model estimation

As inference and efficiency are of interest, equations (1) to (4) are estimated using maximum likelihood (ML) estimation with ARCH/GARCH errors modelled as either an ARCH($p$) or GARCH ($p$,$q$) process (see Andersen, Bollerslev, Diebold & Vega, 2003):

$$h_{it} = \omega_i + \sum_{p\geq 1}^{p} \alpha_i \, \varepsilon_{it-p}^2 \tag{5}$$

$$h_{it} = \omega_i + \sum_{p\geq 1}^{p} \alpha_i \, \varepsilon_{it-p}^2 + \sum_{q\geq 1}^{q} \beta_i \, h_{it-q} \tag{6}$$

where $h_{it}$ is the conditional residual variance for return series $i$, $\omega_i$ is the intercept term, $\varepsilon_{it-p}^2$ are the squared residuals conditional on model specification and $h_{it-q}$ is the previous forecast of the conditional variance and its associated GARCH coefficient, $\beta_i$ (in equation 6) (Engle, 2004). Following Armitage and Brzeszczyński (2011), ARCH($p$) and GARCH($p$,$q$) specifications are fitted until there are no remaining ARCH effects, as established by applying the ARCH LM test at lower and higher orders (ARCH(1) and ARCH(5)), and the residuals do not exhibit the presence of non-linear dependence indicative of non-stationary variance, as established by applying $Q$-statistics at the first and fifth orders and are free of serial correlation (Akgiray, 1989). Residuals are assumed to be conditionally normally distributed and if this is not the case as indicated by the (post-estimation) Jarque-Bera (JB) test, quasi-maximum likelihood (QML) estimates with Bollerslev-Wooldridge robust standard errors and covariance are obtained (Mittelhammer, Judge & Miller, 2000; Fan, Qi & Xiu, 2014).

### 2.5 Comparative approach

The approach followed in this paper is comparative. Various aspects of each specification (outlined below) are compared across specifications. The macroeconomic model (equation (1)) together with the augmented specification (equation (4)) are viewed as baseline cases on opposite sides. The macroeconomic model is viewed as an inherently underspecified model whereas the augmented model is viewed as a fully (adequately) specified model. To confirm that this is the case, the structure of the residual correlation matrices derived from specifications (1) to (4) is investigated to establish whether there are any remaining factors in the respective residual correlation matrices. In other words, the validity of the diagonality assumption is investigated and the correlation structure is quantified (see Section 2.7 for a detailed outline of the methodology applied).

### 2.6 Comparative aspects

The comparative analysis begins with a consideration of a more direct (as opposed to subtle) impact of underspecification. Factor omission will bias residual variance upwards. Consequently, coefficient standard errors will be biased upwards and test statistics will be lower. This may result in an erroneous tendency to not reject the null hypothesis of a coefficient equalling zero (Type II error) and lead to a misidentification of the linear factor model (Chang, 1991; Van Rensburg, 2000; Brooks, 2008; Wooldridge, 2013). To determine the impact of inflated standard errors on the identification of significant factors, differences between the number of significant coefficients are first noted and this is followed by a comparison of the magnitudes of the mean standard errors.

The analysis turns next to a direct consideration of residual variance across specifications. The mean of the residual variances for each model is compared across specifications by applying a paired-sample *t*-test.[3] The Brown-Forsythe test for the equality of variance is applied to individual residual series generated by the specifications for each sector (see Brown & Forsythe, 1974). If factor omission significantly inflates residual variance, because variance reflects the dispersion associated with omitted factors, then differences will be statistically significant (Lehmann, 1990; Dominguez, 1992). The Brown-Forsythe test will indicate widespread rejections of the null hypothesis of homogenous residual variance across individual sector residuals.

The estimation of equations (1) to (4) using ML estimation with ARCH/GARCH errors permits the quantification of two potential consequences of underspecification. The residuals of specifications that are free of underspecification or are better specified are likely to exhibit simpler and lower order ARCH($p$) and GARCH($p,q$) processes as the inclusion of additional factors in equations (2) to (4) should reduce or eliminate impure heteroscedasticity (Bucevska, 2011). Furthermore, within this econometric framework, the level of conditional heteroscedasticity will impact coefficient estimates. Bera *et al.* (1988) show that the greater the level of conditional heteroscedasticity in the residuals, attributable to factor omission, the greater the deviation from least squares coefficients. ML estimation with ARCH/GARCH errors permits the volatility structure to enter coefficient estimates in the conditional mean (also see Engle, 2001; Hamilton, 2010; Armitage & Brzeszczyński, 2011). To quantify bias induced by factor omission, differences between the ML and least squares coefficient estimates are reported, with greater differences attributed to information associated with omitted factors reflected in the residual variance which enters the loglikelihood function used in parameter estimation under ML estimation (Singh, Kumar & Pandey, 2010). Macroeconomic factor coefficients are not affected if least squares estimation is applied, as the residual market factors and the statistical factors are orthogonal to the macroeconomic factor set.

Commonly considered regression diagnostics and robustness measures are considered next. Primary comparisons focus on differences across specifications at the individual sector level. Secondary form analysis considers the magnitude of the

[3] As there may be ambiguity regarding the distribution of the values for each series, the non-parametric Wilcoxon matched-pairs signed-rank test is applied following each *t*-test.

mean test statistics across specifications to determine effect size (see Sullivan & Feinn, 2012 for a discussion of effect size analysis). The first test applied is Wald's test of linear restrictions to confirm whether a specification has explanatory power in its entirety (Sadorsky & Henriques, 2001). The JB test is applied to the residuals to determine whether underspecification impacts the distribution of the residuals. Outliers may be associated with omitted factors, inducing non-normality in the residuals (Downing & Clark, 2010). Consequently, the number of departures from normality should be lowest for the augmented specification. As with heteroscedasticity, serial correlation may be impure and attributable to omitted factors and may bias standard errors, impacting conventional significance tests (Granger & Newbold, 1974; Mutsune, 2008; Wooldridge, 2013). This also has the potential to result in a misidentification of significant relationships. To test for serial correlation in the residuals, $Q$-statistics are estimated for first order serial correlation ($Q(1)$), and for the first five serial correlation coefficients ($Q(5)$). A better specified model will generally be free of impure residual serial correlation and any remaining significant residual serial correlation will be pure in nature, unless the functional form is incorrect (Adams & Coe, 1990; Claar, 2006).[4] $Q$-statistics are estimated for *squared* residuals for the first serial correlation coefficient, $Q^2(1)$, and for five orders of serial correlation, $Q^2(5)$ to test for non-linear dependence in the residuals. The ARCH LM test is also applied to test for first and fifth order ARCH effects (ARCH(1) and ARCH(5)). Residual series should be free of non-linear dependence and ARCH effects as ARCH($p$) and GARCH($p,q$) specifications have been fitted to ensure that this is the case.

A model should yield predictions that resemble realised (actual) returns. Prediction errors are quantified by the magnitude of the residuals and the mean of residuals[5] should be zero if prediction errors are negligible. Aggregate residual series derived from each specification are reported and tested against a null hypothesis of the mean equalling zero using the $t$-test. As an additional method of comparing specifications, comparisons are also made using Theil's inequality coefficient (Chang, 1991; Chen & Jordan, 1993). Related decompositions are considered,[6] namely the bias proportion ($U_{\text{BIAS}}$), which indicates the discrepancy that arises between the mean values of the predictions and actual observations, the variance proportion, ($U_{\text{VAR}}$), which indicates whether the variability of the actual series is greater (lower) than that predicted by a

[4] The RESET test is applied to exclude functional form misspecification. Unreported results indicate that the specifications are free of functional form misspecification. Any reduction is serial correlation can therefore be attributed to reductions in impure serial correlation.
[5] To conduct this part of the analysis, a mean is calculated from the means of individual residual series.
[6] Fair (1984) states that Theil's $U$ statistic may also be used to evaluate *ex-post* forecasts and for comparative purposes across models.

specific model, and the covariance proportion, ($U_{COV}$), which measures the extent to which prediction errors are attributable to unsystematic errors or residual components (Elkhafif, 1996; Watson & Teelucksingh, 2002; Brooks & Tsolacos, 2010; Kacapyr, 2014).

### 2.7 Analysis of residual correlation structures

The final part of the analysis considers the structure of the residual correlation matrices derived from specifications (1) to (4). The minimum average partial (MAP) test, which seeks to extract the number of factors so that the residual correlation matrix most closely resembles an identity matrix (off-diagonal elements are zero) is applied to the residual correlation matrices to determine whether any common factors have been relegated to the residuals (Meyers, 1973; Zwick & Velicer, 1986; Van Rensburg, 1995; Ledesma & Valero-Mora, 2007). If there are no omitted factors, then no common factors will be extracted or if factors are extracted, their communality will be low (Yong & Pearce, 2013; Elton, Gruber, Brown & Goetzmann, 2014). To investigate the strength of any remaining correlation in the residual matrices without extracting factors indicative of co-movement attributable to omitted common factors, the Kaiser-Meyer-Olkin (KMO) index, which indicates the proportion of shared variance in the series, is estimated. Values between 0,8 and 1 are indicative of desirable sampling adequacy - high levels of interdependence. For a well-specified model that captures most pervasive influences, the KMO index should be ideally below 0,5 indicating low levels of interdependence that are unlikely to yield factor scores reflective of unspecified omitted factors (Kaiser, 1974; Madaree, 2018). The analysis of the residual structure constitutes an investigation into the validity of the diagonality assumption for the specifications considered. The diagonality assumption should hold for a model that is free of factor omission – an adequately specified model.

## 3 Results

### 3.1 Results of comparisons across specifications

Table 2 summarizes the results of comparisons across specifications on the aspects outlined in Section 2.6 Factor omission is associated with inflated regression standard errors attributable to inflated residual variance. This has the effect of lowering the power of statistical tests, potentially leading to an erroneous tendency to not reject the null hypothesis of no relationship (Lehmann, 1990; Dominguez, 1992; Van Rensburg, 2002). The result is an understatement of the importance of factors (row

(3) in Table 2). The augmented model (equation (4)), the baseline case for a fully specified model, is the specification with the most statistically significant coefficients; 119 of 182 (65,38%) coefficients for the seven macroeconomic factors are statistically significant. Fewer coefficients are statistically significant with the omission of factors (row (1)). For the macroeconomic specification (equation (1)) 93 of 182 (51,099%) coefficients are statistically significant, 26 fewer relative to the augmented model. For the macro-residual model (equation (2)), 105 of 182 (57,692%) estimated coefficients are statistically significant and for the unrestricted model (equation (3)), 109 of 182 (59,890%) coefficients are statistically significant. As may be expected, explanatory power, as measured by the $\bar{R}^2$ decreases with the omission of factors (row (2a)). Similarly, higher AIC and BIC values indicate a deterioration in the ability to replicate returns and a greater deviation from the true return generating process (row (2b)&(2c)) (Spiegelhalter, Best, Carlin & Van der Linde, 2014).

## Table 2: Summary of the different aspects of specifications estimated

| | (A) Macroeconomic (eq (1)) | | (B) Macro-residual (eq (2)) | | (C) Unrestricted (eq (3)) | | (D) Augmented (eq (4)) | |
|---|---|---|---|---|---|---|---|---|
| (1) Factor(s) omitted | $M\varepsilon_t, IM\varepsilon_t, f_{1t}, f_{2t}$ | | $IM\varepsilon_t, f_{1t}, f_{2t}$ | | $f_{1t}, f_{2t}$ | | - | |
| (2) Model fit/relative fit | | | | | | | | |
| (2a) Mean $\bar{R}^2$ | 0.142$^{\blacktriangledown *}$ | | 0.310$^{\blacktriangledown * \blacktriangle *}$ | | 0.322$^{\blacktriangledown * \blacktriangle *}$ | | 0.504 | |
| (2b) Mean AIC | -2.736$^{\blacktriangle *}$ | | -2.985$^{\blacktriangle * \blacktriangledown *}$ | | -3.001$^{\blacktriangle * \blacktriangledown *}$ | | -3.348 | |
| (2c) Mean BIC | -2.556$^{\blacktriangle *}$ | | -2.789$^{\blacktriangle * \blacktriangledown *}$ | | -2.789$^{\blacktriangle * \blacktriangledown *}$ | | -3.114 | |
| (3) No. of sig macroeconomic coefficients (out of 182) | 93 (-26) | | 105 (-14) | | 109 (-10) | | 119 (-) | |
| (4) Intercept & factor significance over/under-statement relative to augmented model | Intercept [+3], $BP_{t-1}$ [-7], $LEAD_{t-1}$ [-5], $BUS_t$ [-5], $USD\varepsilon_t$ [-6], $MET_t$ [-1], $TLI\varepsilon_t$ [-2] | | Intercept [+3], $BP_{t-1}$ [-3], $LEAD_{t-1}$ [-4], $BUS_t$ [-3], $USD\varepsilon_t$ [-3], $MET_t$ [+1], $TLI\varepsilon_t$ [-2] | | Intercept [+2], $BP_{t-1}$ [-1], $LEAD_{t-1}$ [-4], $BUS_t$ [-3], $USD\varepsilon_t$ [-2], $MET_t$ [+2], $TLI\varepsilon_t$ [-2] | | - | |
| (5) Standard errors | Highest | | Lower | | Low | | Lowest | |
| (6) Z-scores magnitudes | Lowest | | Low | | Higher | | Highest | |
| (7) Mean residual variance | 0.004307$^{\blacktriangle ***}$ | | 0.003484$^{\blacktriangle *** \blacktriangledown ***}$ | | 0.003411$^{\blacktriangle *** \blacktriangledown ***}$ | | 0.002483 | |
| (8) Variance great than augmented | 22/26 | | 18/26$^{\blacktriangledown}$ | | 13/26$^{\blacktriangledown}$ | | - | |
| (9) Variance structures | ARCH(1)[10]$^{\blacktriangledown}$, ARCH(2)[1]$^{\blacktriangle}$, GARCH(1,1)[15]$^{\blacktriangle}$ | | ARCH(1)[13]$^{\blacktriangledown \blacktriangle}$, GARCH(1,1)[12]$^{\blacktriangle \blacktriangledown}$, GARCH(2,1)[1]$^{\blacktriangle \blacktriangledown}$ | | ARCH(1)[15]$^{\blacktriangledown \blacktriangle}$, GARCH(1,1)[10]$^{\blacktriangle \blacktriangledown}$, GARCH(2,1)[1]$^{\blacktriangle \blacktriangledown}$ | | ARCH(1)[18], GARCH(1,1)[8] | |
| (10) Coefficient bias (ML/LS) | | | | | | | | |
| (10a) Significant ML LS differences within individual specifications | Intercept$^{*** \blacktriangle}$, $BP_{t-1}{}^{*** \blacktriangle}$, $MET_t{}^{w \blacktriangledown}$, $LTY_t{}^{w \blacktriangle}$ | | Intercept$^{** \blacktriangle}$ | | Intercept$^{** \blacktriangle}$ | | $USD\varepsilon_t{}^{w}$, $MET_t{}^{*w}$, $TLI\varepsilon_t{}^{*}$ | |
| (10b) ML LS differences magnitude | Highest | | Lower | | Low | | Lowest | |
| (11) Coefficient differences | Intercept$^{\blacktriangle ***}$,$BP_{t-1}{}^{\blacktriangledown ***}$, $TLI\varepsilon_t{}^{\blacktriangle **}$ | | $BP_{t-1}{}^{\blacktriangledown \blacktriangle **}$, $LEAD_{t-1}{}^{\blacktriangle \blacktriangle **}$, $BUS_t{}^{\blacktriangledown * \blacktriangledown *}$, $LTY_t{}^{\blacktriangle \blacktriangle **}$,$TLI\varepsilon_t{}^{\blacktriangle ** \blacktriangle}$ | | $BP_{t-1}{}^{\blacktriangledown \blacktriangle ***}$, $LEAD_{t-1}{}^{\blacktriangle \blacktriangle **}$, $BUS_t{}^{\blacktriangledown * \blacktriangledown w}$, $LTY_t{}^{\blacktriangle \blacktriangle **}$, $TLI\varepsilon_t{}^{\blacktriangle ** \blacktriangle}$ | | - | |
| (12) Intercepts | | | | | | | | |
| (12a) Intercept magnitude | 0.008$^{\blacktriangle ***}$ | | 0.007$^{\blacktriangle ** \blacktriangledown ***}$ | | 0.007$^{\blacktriangle * \blacktriangledown **}$ | | 0.006 | |
| (12b) Number significant | 16/26 | | 16/26 | | 15/26 | | 13/26 | |
| **(13) Diagnostics** | **Mean Value** | **Total Sig.** | **Mean Value** | **Total Sig.** | **Mean Value** | **Total Sig.** | **Mean Value** | **Total Sig.** |
| (13a) *F*-Test | 6.692$^{\blacktriangledown}$ | 26/26 | 16.143$^{\blacktriangledown \blacktriangle}$ | 26/26 | 15.530$^{\blacktriangledown \blacktriangle}$ | 26/26 | 36.528 | 26/26 |
| (13b) JB Test | 15.868$^{\blacktriangle}$ | 14/26$^{\blacktriangledown}$ | 17.862$^{\blacktriangle \blacktriangle}$ | 12/26$^{\blacktriangledown \blacktriangledown}$ | 15.229$^{\blacktriangle \blacktriangledown}$ | 14/26$^{\blacktriangledown -}$ | 11.511 | 15/26 |
| (13c) $Q(1)$ | 1.919$^{\blacktriangle}$ | 8/26$^{\blacktriangle}$ | 1.365$^{\blacktriangledown \blacktriangledown}$ | 4/26$^{\blacktriangledown \blacktriangledown}$ | 1.322$^{\blacktriangledown \blacktriangledown}$ | 3/26$^{\blacktriangledown \blacktriangledown}$ | 1.744 | 5/26 |
| (13d) $Q(5)$ | 7.107$^{\blacktriangle}$ | 8/26$^{\blacktriangle}$ | 6.412$^{\blacktriangledown \blacktriangledown}$ | 3/26$^{\blacktriangledown \blacktriangledown}$ | 6.614$^{\blacktriangledown \blacktriangledown}$ | 5/26$^{\blacktriangle \blacktriangledown}$ | 6.586 | 4/26 |
| (13e) $Q^2$ (1) | 0.524$^{\blacktriangle}$ | 0/26 | 0.478$^{\blacktriangle \blacktriangledown}$ | 0/26 | 0.406$^{\blacktriangle \blacktriangledown}$ | 0/26 | 0.236 | 0/26 |
| (13f) $Q^2$ (1) | 3.638$^{\blacktriangledown}$ | 0/26 | 3.517$^{\blacktriangledown \blacktriangledown}$ | 0/26 | 3.686$^{\blacktriangledown \blacktriangle}$ | 0/26 | 3.812 | 0/26 |
| (13g) ARCH(1) | 0.454$^{\blacktriangle}$ | 0/26 | 0.469$^{\blacktriangle \blacktriangle}$ | 0/26 | 0.398$^{\blacktriangle \blacktriangledown}$ | 0/26 | 0.235 | 0/26 |
| (13h) ARCH(5) | 0.724$^{\blacktriangledown}$ | 0/26 | 0.653$^{\blacktriangledown \blacktriangledown}$ | 0/26 | 0.712$^{\blacktriangledown \blacktriangledown}$ | 0/26 | 0.786 | 0/26 |
| (14) Predictive ability | | | | | | | | |
| (14a) Mean errors | -0.0019261$^{*** \blacktriangledown ***}$ | | -0.0012502$^{** \blacktriangledown ** \blacktriangle **}$ | | -0.0010557$^{** \blacktriangledown ** \blacktriangle **}$ | | -0.0005692 | |
| (14b) Theil $U$ | 0.633$^{\blacktriangle ***}$ | | 0.511$^{\blacktriangle * \blacktriangledown *}$ | | 0.503$^{\blacktriangle * \blacktriangledown *}$ | | 0.395 | |
| (14c) Bias ($U_{BIAS}$) | 0.002355$^{\blacktriangle **}$ | | 0.002002$^{\blacktriangle W \blacktriangledown}$ | | 0.001408$^{\blacktriangle \blacktriangledown *}$ | | 0.001036 | |
| (14d) Variance ($U_{VAR}$) | 0.412575$^{\blacktriangle ***}$ | | 0.269569$^{\blacktriangle * \blacktriangledown *}$ | | 0.265393$^{\blacktriangle * \blacktriangledown *}$ | | 0.178215 | |
| (14e) Covariance ($U_{COV}$) | 0.585071$^{\blacktriangledown ***}$ | | 0.727644$^{\blacktriangledown * \blacktriangle *}$ | | 0.733199$^{\blacktriangledown * \blacktriangle *}$ | | 0.820820 | |
| (15) Residual matrix | | | | | | | | |
| (15a) No. factors extracted | 3 | | 3 | | 2 | | 1 | |
| (15b) Mean communality | 0.399$^{\blacktriangle}$ | | 0.300$^{\blacktriangle \blacktriangledown}$ | | 0.248$^{\blacktriangle \blacktriangledown}$ | | 0.066 | |
| (15c) KMO Index | 0.888$^{\blacktriangle}$ | | 0.790$^{\blacktriangle \blacktriangledown}$ | | 0.780$^{\blacktriangle \blacktriangledown}$ | | 0.052 | |

**Notes:** 1. Where stated, the first ▲ or ▼ symbol indicates that a value or the number of instances recorded is greater or lower than that observed for the augmented model (eq (4)). The second ▲ or ▼ symbol indicates that a value or the number of instances recorded in greater or lower than that observed for the macroeconomic model (eq (1)). 2. The asterisks, ***, ** or *, indicate statistical significance at the 1%, 5% and 10% levels of significance, respectively. Where reported in this table, factors are defined as follows: $BP_{t-1}$ - number of building plans passed, $LEAD_{t-1}$ - domestic composite cyclical leading indicator, $BUS_t$ - business activity, $USD_{\varepsilon_t}$ - (orthogonalized) fluctuations in the Rand-Dollar exchange rate, $MET_t$ - world metal prices, $LTY_t$ - long-term government bond yields, $TLI_{\varepsilon_t}$ - (orthogonalized) leading indicator for South Africa's trading partners, $M_{\varepsilon_t}$ - the residual market factor orthogonal to the macroeconomic factor set, derived from returns on the JSE All Share Index, $IM_{\varepsilon_t}$ - a second residual market factor orthogonal to the macroeconomic factor set and $M_{\varepsilon_t}$.

Some factors now appear to be pseudo-factors with a limited impact on less than half of the series in the sample, this constituting an example of a misidentified return generating process (Ferson & Harvey 1994). For example, in the macroeconomic model, these are $BP_{t-1}$ (seven [-7] fewer coefficients are now statistically significant relative to the augmented model), $LEAD_{t-1}$ [-5] and $BUS_t$ [-5]. Including $M_{\varepsilon_t}$ and subsequently $IM_{\varepsilon_t}$ in the macro-residual and unrestricted models goes some way to reducing the understatement of the significance of macroeconomic factors. $LEAD_{t-1}$ and $USD_{\varepsilon_t}$ now appear to have a pervasive although still understated impact relative to the augmented model (see row (4)).

The understatement of factors in the macroeconomic, macro-residual and unrestricted models relative to the augmented model can be explained by inflated standard errors. For example, the mean standard error for $LEAD_{t-1}$ in the macroeconomic model is 0,537 and 0,425 in the augmented model, 0,049 and 0,038 for $BUS_t$ and 1,611 and 1,124 for $LTY_t$, respectively. Associated with the increases in the standard errors are decreases in related *z*-scores (see for example the mean *z*-scores for $LEAD_{t-1}$ , $BUS_t$ and $LTY_t$ in the macroeconomic model relative to the augmented model in Table A7 in the Appendix). This is evidence of a loss of parameter efficiency arising from factor omission, which in turn, impacts significance tests (Chang, 1991; Van Rensburg, 2002). The inclusion of the residual market factors improves efficiency but the factor analytic augmentation results in the lowest standard errors (row (5)&(6)).

Inflated standard errors arise from inflated residual variance which reflects dispersion associated with omitted factors. The mean residual variance for the macroeconomic model (0,004307) is significantly higher than that of the macro-residual (0,003484), unrestricted (0,003411) and augmented (0,002483) models, respectively (row (7). The progressive inclusion of $M_{\varepsilon_t}$, $IM_{\varepsilon_t}$ and the factor analytic augmentation reduces residual variances with greatest gains achieved by incorporating $M_{\varepsilon_t}$. Furthermore, residual variance is lowest for the augmented model on an individual sector level. For the macroeconomic model, residual variance is greater than that of the augmented model for 22 industrial sectors. This declines to 18 of 26 sectors with the inclusion of $M_{\varepsilon_t}$ and to 13 sectors with the inclusion of $IM_{\varepsilon_t}$ (row (8). The upward bias in individual sector residual variances can explain the larger mean standard errors for the macroeconomic, macro-residual and unrestricted models and the understated impact of the macroeconomic factors (see Table A7 and A8 in the Appendix).

Inflated residual variance suggests that variance structures differ across specifications. The macroeconomic specification is associated with the most complex residual variance structures. The more complex GARCH(1,1) model describes residual variance for 15 industrial sectors (row (9). While the ARCH(*p*) specification is a short-memory model, the GARCH(*p*,*q)* model is a long-memory model that incorporates an adaptive learning mechanism ($h_{it-q}$ in equation (6)) and captures more complex volatility dynamics (Bollerslev, 1986; Koutoulas & Kryzanowski, 1994; Elyasiani & Mansur, 1998). Incorporating residual market factors simplifies conditional variance structures. Results point towards a shift to short-memory structures (row (9). The conditional variance structures of the macro-residual model still resemble those of the macroeconomic model. However, 13 series are now described by the short-memory ARCH(1) model and the long-memory GARCH(1,1) model is applied to 12 sectors. For the unrestricted model, the ARCH(1) process characterises the conditional variance of 15 sectors. The augmented model, favours short-memory structures; 18 sectors are described by ARCH(1) specifications. Greater complexity in underspecified models can be attributed to induced impure heteroscedasticity, which requires the fitting of more complex GARCH(*p*,*q*) specifications to ensure that residuals are free from non-linear dependence and ARCH effects (Koutoulas & Kryzanowski, 1994; Armitage & Brzeszczyński, 2011). The impact of factor omission on conditional variance structures is not explored in the literature, a gap addressed in this study. Nevertheless, in the presence of impure heteroscedasticity, conventional standard errors will substantially depart from appropriate values rendering inferences relating to the significance of coefficients unreliable (Greene, 2012).

As variance structures enter coefficient estimates, coefficients reflect bias associated with factor omission. The comparison of the differences between the ML estimates and least squares estimates for the macro-residual and unrestricted models produces encouraging results, which favour the residual market factors (see Table A7 in the Appendix). Overall, coefficient estimates for these models more closely approximate least squares coefficient estimates which are theoretically unbiased (assuming no factors are omitted) (Bera *et al.*, 1988; Lee & Lemieux, 2010: 289). Nevertheless, a closer examination suggests that differences between the ML and least squares coefficients for these two specifications are generally larger, although not significantly, relative to those for the augmented specification (row (10a), (10b)). For example, factors for which differences are larger are $BP_{t-1}, LEAD_{t-1}$, $BUS_t$ and $USD_{\varepsilon_t}$ for both specifications. This bias can be explained by omitted factors reflected in the structure of heteroscedasticity, a proxy for information.

Relatedly, ML coefficient magnitudes differ across specifications, reflecting differences in variance structures (row (11). Differences are statistically significant between the coefficient series of $BP_{t-1}$, and $TLI_{\varepsilon_t}$ for the macroeconomic and augmented model. This is expected; coefficients either underestimate or overestimate the impact of a factor if a model is underspecified. The inclusion of the residual market factors leads to somewhat ambiguous' conclusions. For both specifications, some factors are now associated with significantly lower and higher coefficients (e.g. $BUS_t$ and $TLI_{\varepsilon_t}$, respectively) than those of the augmented model whereas others are comparable to that of the augmented model (e.g. $BP_{t-1}$ and $LEAD_{t-1}$). The inclusion of $M_{\varepsilon_t}$ in the macro-residual model results in a more accurate approximation of the coefficients for $BP_{t-1}$ relative to the macroeconomic model.

Like coefficients, intercepts are also impacted by factor omission (row (12)). The means of the intercepts of the macroeconomic (0,008), the macro-residual and unrestricted models (both 0,007) are inflated and differ significantly to those of the augmented model. In the presence of underspecification, inferences drawn from the magnitude of the alphas of a model, whether it be macroeconomic or fundamental in nature, may be misleading as intercepts reflect omitted factors (see Lehmann & Modest, 1987; Dominguez, 1992; Van Rensburg, 2002: 92; Hübner, 2007). Also, the augmented model exhibits the lowest number of statistically significant intercepts (13 out of 26) (row (12b)).

Underspecification impacts diagnostic tests (row (13)). The macroeconomic model produces the lowest mean *F*-statistics (6,692) indicative of overall factor set significance whereas the augmented model (36,528) produces the highest (row (13a)). This is because a lower proportion of return variation is explained by the macroeconomic model (Blackwell, 2008; Kluve, Schneider, Uhlendorff & Zhao, 2012: 600). $F$-statistics increase with the inclusion of $M_{\varepsilon_t}$ (16,143) and $IM_{\varepsilon_t}$ (15,530) but remain lower than that of the augmented specification (see Sullivan & Feinn, 2012: 279 for a discussion of effect size that can be applied to these results).

The exclusion of factors does not have much of an impact on conditional normality on an individual sector basis (row (13b). The number of rejections of the assumption of residual normality does not differ much across specifications. However, the augmented model has the lowest mean JB statistic (11,511), estimated from individual JB statistics across sectors. The mean JB statistics for the macro-residual model (17,862) and the unrestricted model (15,229) are comparable to that of the macroeconomic model (15,868). Lower JB statistics for the augmented model can be attributed to the ability of the statistical factor scores to capture omitted factors that are associated with outliers which contribute to departures from normality. Therefore, it appears that the residual market factors do not capture all omitted factors (Downing & Clark, 2010).

Mean *Q*(1) and *Q*(5) statistics are not revealing (row (13c), (13d)). On an individual sector basis, the inclusion of residual market factors reduces the number of (joint) instances of significant serial correlation. Six and seven sectors modelled using the macro-residual and unrestricted models exhibit significant serial correlation respectively, as evident from *either* or *both* statistically significant *Q*(1) and *Q*(5) statistics (see Table A3 to A6 and Table A10 in the Appendix). In contrast, the residuals of the macroeconomic specification are significantly serially correlated for 12 of the industrial sectors. The contribution of the factor analytic augmentation to reducing serial residual correlation is ambiguous. A total of six sectors exhibit serial correlation at either lower or higher orders. The conclusion is that respecification with $M_{\varepsilon_t}$ included in the macro-residual model sufficiently reduces impure serial correlation (Adams & Coe, 1990; Claar, 2006).

Across specifications, $Q^2(1)$ and $Q^2(5)$ statistics indicate that residuals do not exhibit non-linear serial correlation (row (13e), (13f)). ARCH(1) and ARCH(5) LM tests confirm that residuals are free of ARCH effects (row (13g), (13h)). This is to be expected as the approach employed relies on fitting increasingly complex ARCH(*p*) and GARCH(*p*,*q*) specifications to ensure that residual series are free of non-linear dependence and heteroscedasticity.

A deterioration in the predictive ability of underspecified specifications can be attributed to the omission of relevant information (primary) and associated coefficient bias (secondary). The only specification for which the residuals, $\varepsilon_{it}$, do not differ significantly from zero is the augmented specification (row (14a). Mean residuals decrease (in absolute terms) with the inclusion of $M_{\varepsilon_t}$ (from 0,0019261 to 0,0012502) and decrease further with the inclusion of $IM_{\varepsilon_t}$ (from 0,0012502 to 0,0010557) but remain significantly different from zero and larger than those of the augmented model (0,0005692). Chang (1991) reports that the inclusion of a residual market factor in a linear factor model, together with macroeconomic factors, renders mean errors statistically insignificant. Here, it is the factor analytic augmentation that eliminates prediction errors.

*U* statistics can be used to determine the superiority of competing specifications. The macroeconomic, macro-residual and unrestricted models severely underperform the augmented model (14b). The *U* statistic is highest for the macroeconomic model (0,633) and lowest for the augmented model (0,395). Mean *U* statistics are lower for the macro-residual (0,511) and for the unrestricted (0,503) models, implying that the inclusion of $M_{\varepsilon_t}$ improves predictive accuracy although the contribution of $IM_{\varepsilon_t}$ is minor. The augmented model, again, outperforms all other specifications, notably the macro-residual and unrestricted models.

The exclusion of factors impacts the ability of a model to replicate series means although the impact is not severe enough to warrant concern, with the bias proportion, $U_{BIAS}$, remaining under below 0,1 (see Brooks & Tsolacos, 2010). The macroeconomic model is associated with the highest mean bias proportion (0,002355) suggesting that this model is prone to the greatest level of systematic over- or underprediction (row (14c). The inclusion of $M_{\varepsilon_t}$ yields ambiguous and potentially non-existent improvements. The mean bias proportion for the macro-residual model (0,002002) remains comparable to that of the macroeconomic model but differs from that of the augmented model (0,001036). The inclusion of $IM_{\varepsilon_t}$ in the unrestricted model (0,001408) reduces the mean bias proportion to an extent that is comparable to that of the augmented specification.

The macroeconomic model underperforms all specifications in replicating the variability of the series and its turning points, reporting the highest variance proportion component (0,412575), $U_{VAR}$, more than double that of the augmented model (0,178215) (row (14d)) (see Elkhafif, 1996; Kacapyr, 2014: 162). Including the residual market factors significantly decreases the variance proportions of the macro-residual (0,269569) and unrestricted models (0,265393). While these reductions are desirable, both residual market factors do not encompass information that is required to optimally replicate the variance of the actual series.

A greater proportion of prediction error in the macroeconomic, macro-residual and unrestricted modes is attributable to the structure of the respective specifications rather than inherent randomness in the data, relative to the augmented model for which the covariance proportion, $U_{COV}$, is closest to one (0,807705) (row (14e)

(Fildes & Kingsman, 2011). The two residual market factors fail to reduce the proportion of prediction errors to a level that is comparable to that of the augmented model. The covariance proportions for the macro-residual (0,727644) and unrestricted (0,733199) models are significantly lower than that of the augmented model but not as low as that of the macroeconomic model (0,412575). If accurate predictions are desired, a factor analytic augmentation should be included alongside pre-specified factors.

### 3.2 Results of an analysis of the residual correlation structure

The final part of the analysis (row (15) shows and confirms that a factor analytic augmentation eliminates underspecification by directly accounting for omitted sources of variation (Section 2.7; Van Rensburg, 2000). Three factors are extracted from the residuals of the macroeconomic model (row (15a). The macroeconomic factor set fails to account for almost 40% of the common variation reflected in the residuals, confirming that the model is underspecified (row (15a), (15b). The KMO index is 0,888. Kaiser (1974: 35) views KMO indices over 0,8 as "meritorious" and those above 0,9 as "marvellous" for factor extraction. There is no doubt that the residual correlation matrix of the macroeconomic model is characterised by extensive pairwise residual correlation attributable to omitted common factors (Elton *et al.*, 2014; Madaree, 2018). The inclusion of $M_{\varepsilon_t}$ brings some, albeit limited, improvement. Omitted factors now explain 30% of variation in returns but the KMO index is 0,79, indicating that correlation is still substantial. The inclusion of the second residual market factor yields almost negligible improvements in both the communality and the KMO index. Both the macro-residual and unrestricted models remain underspecified given the violation of the diagonality assumption.

The factor analytic augmentation resolves the factor omission problem. A single factor is extracted with a mean communality of 0,066. The KMO index value is 0,052 indicating a level of correlation too low for the meaningful extraction of factors (Kaiser, 1974).[7] Any remaining residual correlation appears to be negligible, implying that the augmented model is accurately specified. The residual correlation matrix does not show evidence of systematic co-movement, *approximating* the diagonality assumption (see Meyers, 1973).

[7] A closer examination of the sample reveals that only five sectors have communalities greater than 0,15. The sector with the highest loading on the single extracted factor is the general retailers' sector, with a loading of -0,492 and a corresponding communality of 0,242. With this sector excluded, the MAP test fails to identify a single factor implying that the extracted factor may be the result of strong interdependence between a limited number of industrial sectors.

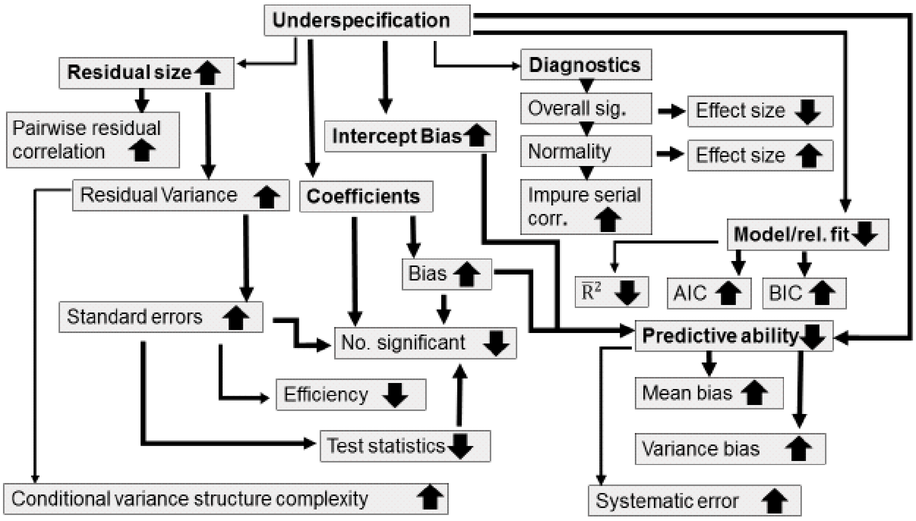
To confirm that statistical factors account for additional influences in returns not accounted for by the two-residual market factors, the equality of the residual correlation matrices derived from the macro-residual and unrestricted models is tested against that of the augmented model. The test produces highly significant Jennrich $\chi^2$ statistics of 680,222 and 663,927 respectively (d.f. 325), confirming that the factor analytic augmentation accounts for additional return co-movement (see McElroy & Burmeister, 1988). The violation of the diagonality assumption in the macroeconomic, macro-residual and unrestricted models challenges the validity of these models, but is resolved with the inclusion of a factor analytic augmentation.

The results of the preceding analysis show that, by and large, the diagonality assumption continues to be violated with the conventional inclusion of residual market factors. This matters for researchers and econometric modellers. The residual market factor is considered to be a default and effective solution to factor omission. The present analysis shows that this is not the case.

Although these findings may be limited to the South African stock market, researchers and econometricians should be mindful that the use of residual market factors will not fully eliminate biases associated with factor omission in empirical return-factor models. An explanation relates to the nature of the market proxy. The true market portfolio is an aggregation process that reflects the influences of all-pervasive factors but is unobservable (Brown & Brown, 1987; Born & Moser, 1988). Therefore, any market proxy will fail to reflect all relevant influences as it is not the true market portfolio. Residual market factors will never be an adequate proxy for all omitted factors. However, a factor analytic augmentation is an easily and readily implementable solution that mitigates factor omission.

### 3.3 A summary

Figure 1 summarizes the consequences of underspecification which are either reduced or resolved, with the inclusion of a factor analytic augmentation but not always with the conventional approach of using residual market factors. What is evident from this summary is that factor omission impacts numerous aspects of a specification and the consequences are often interrelated.




**Notes:**
1. ↑ ↓ indicate increases/decreases in magnitude/instances.
2. For diagnostics, effect size reported only if noted as meaningful.

**Figure 1: Summary of the impact of underspecification on factor models**

Source: Author's own

## 4 Conclusion

This study comprehensively investigates the econometric impact of underspecification. While the consequences of underspecification are frequently discussed in econometric textbooks, this study considers factor omission within a specific context. Linear factor models that relate asset return behaviour to a factor set are a key foundational concept in empirical finance and constitute a building block of numerous asset pricing models and other applications. As such, these findings and the suggested approach to addressing factor omission should be generalizable to other time-series models that relate returns to factors, be they macroeconomic, fundamental or characteristic-based.

The deterioration in the explanatory power of a model, the loss of efficiency, coefficient and intercept bias, as noted in this study, are widely-known consequences of underspecification. In this study, the impact of these consequences is comprehensively investigated within the context of the linear factor model. However, the ability of residual market factors or market indices to address underspecification has not been investigated directly or extensively. This study suggests that using residual market factors as a default approach to factor omission is not adequate. Also, the impact of factor omission on conditional variance structures while recognized, has not been widely studied. This study addresses this gap, finding that factor omission results in more complex variance structures, implying that a greater amount of information is now reflected in the second moment of the residuals. If permitted to impact coefficient estimates, this is a source of bias. Finally, this study provides insight into the structure of residual correlation matrices, treating these as a measure of the extent of factor omission.

An avenue of research related to underspecification is the identification and quantification of the threshold of the level of factor omission at which model interpretation and estimation are impacted (or not impacted). Factor models relating returns to factor sets are likely to always be underspecified. However, it may be that underspecification becomes problematic only beyond a certain point. One way of quantifying underspecification and the point at which it is problematic is to consider measures of sampling adequacy such as the KMO index used in this study (Section 2.7) to quantify residual co-movements related to factor omission and to relate co-movement levels to noticeable model underspecification. A suggested direction of research is the adaptation of such techniques into a robustness test for model underspecification.

Another avenue for further research relates to the impact of underspecification on the structure of the conditional variance and its impact on coefficient estimates. This can be investigated further by using a broader set of ARCH/GARCH-type models. There are relatively few studies that consider the impact of the conditional variance structure on coefficient estimates in the conditional mean (see Bera *et al.*, 1988; Hamilton, 2010; Brzeszczyński, Gajdka & Schabek, 2011; Armitage & Brzeszczyński, 2011). These studies propose that there is value in permitting the structure of conditional variance, which represents information in the second moment, to be reflected in coefficient estimates. A dedicated study of the impact of conditional variance structures on beta estimates in the conditional mean and a comparison of the benefits of such estimates with those derived using least squares may be of value.

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