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OPTIMAL POWER CONSUMPTION CONTROL OF SENSOR NODE BASED ON (*N, D*)-POLICY DISCRETE-TIME QUEUES

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Abstract. In this paper, we consider two types of power consumption control policies for the long lifetime of wireless sensor node based on the discrete-time Geo/G/1 queue. One is the max(*N, D*)-policy, which triggers transmission mode of radio server when the *N* and *D* policies are met simultaneously, and another is the min (*N, D*)-policy, which restarts transmission function of radio server when either of the *N* and *D* policies is first satisfied. Under two control policies, the steady-state queueing analysis of sensor node is mathematically carried out. The mean queueing measures of sensor node, such as the mean number of data packets, mean transmission time backlog, mean waiting time, mean busy period, mean busy cycle period, and so on, are derived. Two power consumption functions are constructed through the queueing measures obtained. Numerical experiments validate that two policies are feasible and efficient for power consumption control of sensor node. At a minimum power consumption, the superiority of the *N*-policy, *D*-policy, and two dyadic (*N, D*) policies is numerically compared. Some practical insights on the operation of two (*N, D*) polices in power consumption control of sensor node are obtained.

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1. Introduction

We introduce two different types of (*N, D*) policies to power consumption control of wireless sensor node(WSN) and numerically show that two policies are feasible and effective for power saving of a sensor node. By comparing the optimal *N, D*, and two (*N, D*) policies at a minimum power consumption, some practical insights on the power consumption control of two (*N, D*) policies are obtained.

Wireless sensor networks have a lot of potential applications in many fields, including, disaster managing, habit or environmental monitoring, security surveillance, medical healthcare monitoring, structural health monitoring, wildlife tracking, and so on [1–4]. For the long lifetime of a wireless sensor network, the power saving management of its wireless sensor nodes(WSNs) is very essential. However, the power consumption of sensor nodes comes from their limited-lifetime batteries. Moreover, because the sensor nodes are generally used in hostile and secret

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environments, and have small sizes, it is not easy to replace or recharge their batteries. Also, in many-to-one communication patterns, that is, several sensor nodes communicate with one sink (the information collector), the nodes near the sink node will swiftly exhaust their powers due to heavy transmission burden, but the nodes far from the sink possess very slow power consumption. This asymmetric energy consumption results in a phenomenon called an energy hole problem(EHP), which causes lots of energy waste and the end of network lifetime. In [5], the simulated experiment done by Lian *et al.* indicated that due to EHP about 90% of total power is not applied when the lifetime of network is finished, and Wadaa's analytical results [6] strengthened the above conclusion. Accordingly, it has been a hot attention problem for many network designers and engineers to control power consumption of sensor nodes in a wireless sensor network.

Considerable analytical techniques about the topic have been proposed. For detailed overviews of main method and results, the readers can see survey papers by Machado and Tekinay [7] and Anastasi *et al.* [8], and more later references [9–22]. Among them, Jiang *et al.* [20–22] modelled a sensor node in wireless sensor network by a continuous-time M/G/1 queue, and used the N, D and $\min(N, T)$ policies to optimize power expenditure of sensor node, respectively. In their works, if data packets(customers) arriving in sensor node(queueing system) reach N, then the N-policy triggers the transmission mode of radio server; If the sum of transmission times of waiting data packets exceeds D, then the D-policy turns the server on. Under the $\min(N, T)$-policy, if the first data packet arrives during the (mT)th $(m = 1, 2, \ldots)$ time units after the end of a transmission period(busy period), then the N-policy triggers the server when the Nth data packet arrives during the (mT)th $(m = 1, 2, \ldots)$ time units, or the T-policy triggers the server when the Nth data packet does not arrive during the (mT)th $(m = 1, 2, \ldots)$ time units and the (mT)th $(m = 1, 2, \ldots)$ time units terminate.

Inspired by the above researches, in this paper, we generalize the work of Jiang *et al.* [20, 21] in three aspects: discrete-time queue, different combinations of N and D polices, and power-saving comparison of several optimal policies. First, we consider discrete-time Geo/G/1 queues, while Jiang *et al.* [20, 21] considered continuous-time $M/M/1$ and $M/G/1$ queues. It is well known that the discrete-time queues are more suitable than their continuous-time counterparts in modelling many communication networks where time is divided into fixed-length time intervals. Hence, it is practical and realistic to model a sensor node by a discrete-time queue. Second, we consider the $\min(N, D)$ and $\max(N, D)$ policies. Under the N-policy, when the number of arrivals is much less than N, the first queued data packet may have rather long waiting time. When the value of D is very large, the D-policy may accumulate many data packets in the node buffer. The above two cases will undoubtedly incur high holding power consumption. So, the combination of N and D policies can counter the shortcomings stated above and has much greater flexibility for power consumption control of WSNs. Third, for the N, D, $\min(N, D)$ and $\max(N, D)$ policies, which policy can realize the minimum power consumption under the same condition? This motivates us to make an optimization comparison of the above four policies.

Our models can be applied to control the power consumption for some kind of wireless sensor network. For instance, we consider a wireless sensor network used in battlefield conditions for military purpose, which is responsible for the intelligence on enemy troop movements, damage, casualties, and so on. In such a network, the radio server of a wire sensor node(WSN) receives data packets and stores them in node buffer. For less setup power consumption and more intelligence, it is desirable that when there is enough packet number (N) or packet transmission backlog (D) to handle, the radio server begins its transmission mode. Here, a WSN can be modelled by the queueing models presented in this paper because its protocols are operated according to a unit of slot and the power consumption can be rationally controlled by the combination of N and D policies. It is interesting and important for the network designers in the military field to make optimal decisions regarding the start of sleep/wake-up mode of the radio server. In other words, how to trigger the sleep/wake-up mode of the radio server by means of the joint optimal values of N and D so that the power consumption of a WSN is minimized? In this application, data packets, transmission, sleep and wake-up modes of the radio server, correspond to customers, service, idle and busy states of the server, respectively, in the queueing terminology.

The contributions of this paper are threefold. Above all, the $\max(N, D)$ and $\min(N, D)$ policies are introduced into the power consumption analysis of a sensor node; Next, from the viewpoint of queueing theory we mathematically derive the mean queueing measures of a sensor node and avoid the complex derivation of probability generating function usually used in the discrete-time queueing theory. It is easy for the network practitioner to understand and use these mean queueing measures. Lastly, based on the same parameter setting, we numerically make a minimum power consumption comparison for the *N, D*, max(*N, D*), and min(*N, D*) policies so that the best energy-saving policy is provided for network practitioners.

The rest of this paper is organized as follows. Section 2 presents the queue assumptions to model a sensor node and some analytical preparations. In Sections 3 and 4, under two types of dyadic (*N, D*) policies we mathematically derive the steady-state queueing measures of a sensor node, including the mean number of data packets, the mean transmission time backlog, the mean waiting time, the mean busy period and the mean busy cycle period. Sections 5 and 6 set up two power consumption functions based on the mean queueing measures obtained. At a minimum energy consumption, the comparison of the optimal *N*-policy, *D*-policy and two dyadic (*N, D*) policies is numerically conducted. Conclusions are finally arrived at in Section 7.

2. Queue assumptions to simulate a sensor node and some preparations

In a wireless sensor network, we assume that the communication channel is error-free and communication pattern is many-to-one network. The time axis is marked by $t = 0, 1, 2, \ldots$, and is slotted into equal length intervals, called slots. The arrival and departure of data packets only happen at boundary epochs of time slots. Since the probability of an arrival and a departure occurring simultaneously is positive in the discrete-time setting, the order of the arrivals and departures must be stated. Suppose that a potential arrival only takes place within $(t^-, t),\ t = 0, 1, 2, \ldots$, and a potential departure only occurs within $(t,\ t^+),\ t = 1, 2, \ldots$, where t^- and t^+ represent $\lim_{\triangle t\to 0}(t - |\triangle t|)$ and $\lim_{\triangle t\to 0}(t + |\triangle t|)$, respectively. If a data packet arrives in the interval (t^-, t) and the radio server is idle at this moment, then its transmission (**service**) will be postponed to the time interval (t, t^+). That is, we consider a late arrival system with delayed access (**LAS-DA**). More details on the LAS-DA and related concepts can be referred to [25].

Inter-arrival times of data packets, $\tau_i, i = 1, 2, \ldots$, are independent and identically distributed (**i.i.d.**) with a common geometric distribution: $\Pr\{\tau_i = k\} = p(1 - p)^{k-1}$, where p is the probability that a data packet arrives in a time slot. The transmission times of data packets, $\{S_n, n \geq 1\}$, are i.i.d. with the probability distribution s_k =$\Pr\{S_n = k\}, k \geq 1(s_0 = 0)$. Let the mean transmission time $E(S)$ be finite and the second moment $E(S^2) < \infty$. The buffer capacity of a sensor node is enough large and the transmission order obeys the first-come-first-served (FCFS) discipline. In order to be efficient in power usage, when there are no data packets to process, the radio server goes to the sleep mode. The sleeping server will be wakened up to resume its transmission mode immediately according to two different types of dyadic (*N, D*) policies:

Max(*N, D*)-policy. the number of data packets arriving during the sleep period is greater than or equal to a predetermined positive integer N and the transmission time backlog(*i.e.*, the sum of transmission time) of data packets arriving during the sleep period exceeds a given non-negative integer D simultaneously;

Min(*N, D*)-policy. the number of data packets arriving during the sleep period reaches a predetermined positive integer N or the transmission time backlog of data packets arriving during the sleep period exceeds a given non-negative integer D, whichever comes first.

When no data packets present in the node, the transmission mode terminates and the sleep mode starts. We assume the arrival and transmission of data packets are mutually independent.

In this paper, our analysis is carried out under the steady-state condition, *i.e.*, the traffic intensity of system $\rho = pE(S) < 1$.

For ease of reference, some notations used in this paper are listed below.

WSN: the abbreviation of wireless sensor node,

$\bar{x}$: the complement of real number x, that is, $\bar{x} = 1 - x$,

$\sum_{k=i}^{j} = 0$, if $i > j$: the sum is equal to zero if the subscript i exceeds the superscript j,

C_k^j: $C_k^j = \frac{k!}{j!(k-j)!}, 0 \leq j \leq k$,

$\Pr\{A\}$: the probability of corresponding event A,

$s_n = \Pr\{S_k = n\}, n \geq 1$: the probability distribution of i.i.d.transmission times $S_k, k \geq 1(s_0 = 0)$,

$S^{(k)}(n) = \Pr\{S_1 + S_2 + \ldots + S_k \le n\}$: the distribution function of the k-fold convolution of transmission time with itself ($S^{(0)}(n) = 1, S^{(k)}(n) = 0, k > n \ge 1$),

$s^{(k)}(n) = \Pr\{S_1 + S_2 + \ldots + S_k = n\}$: the probability distribution of the k-fold convolution of transmission time with itself. Clearly, $s^{(k)}(n) = S^{(k)}(n) - S^{(k)}(n-1), n \ge k+1$, and $\{s^{(k)}(n), n \ge k\}$ is the k-fold convolution of $\{s_n, n \ge 1\}$, that is,

$$s^{(k)}(n) = \begin{cases} s_n, & k = 1, n \ge 1, \\ \sum_{j=1}^{n-k+1} s_j s^{(k-1)}(n-j), & 2 \le k \le n < \infty, \\ 0, & k > n \ge 1. \end{cases}$$

$E(X)$: the mean of corresponding random variable X,

$\rho = pE(S)$: the traffic intensity of the queueing systems presented in this paper,

$Q_{(N,\ D)}$: the number of data packets at the start of a busy period,

$\Phi_{(N,\ D)}$: the transmission time backlog at the start of a busy period, that is, the sum of transmission times of all data packets at the start of a busy period,

$B_{(N,\ D)}$: the busy period, that is, the time length that the radio server transmits the data packets,

$I_{(N,\ D)}$: the idle period, that is, the time length that the server is in sleep mode,

$C_{(N,\ D)}$: the busy cycle period, that is, the sum of the busy period and next idle period,

IP: the data packet that arrives during the idle period,

BP: the data packet that arrives during the busy period,

γ: the transmission end point of the last IP during the busy period,

Γ_γ: the number of BPs that arrive during the time length from the start epoch of a busy period to epoch γ, that is, the number of BPs that arrive during initial time length $\Phi_{N,D}$ of a busy period,

B_γ: the length of remaining busy period initiating with Γ_γ data packets,

$L_{(N,\ D)}$: in steady state, the number of data packets at any epoch t^+,

L_P: in steady state, the number of data packets left behind by a data packet at departure epoch,

L_{IP}: in steady state, the number of data packets left behind by an IP at departure epoch,

L_{BP}: in steady state, the number of data packets left behind by a BP at departure epoch,

$U^{\text{idle}}_{(N,\ D)}$: in steady state, the transmission time backlog at any epoch t^+ during the idle period,

$U^{\text{busy}}_{(N,\ D)}$: in steady state, the transmission time backlog at any epoch t^+ during the busy period,

$U_{(N,\ D)}$: in steady state, the transmission time backlog at any epoch t^+.

Remark 2.1. In discrete-time queues, we note that under the max(N, D)-policy, if $N > D + 1$, then the max(N, D) policy becomes the N-Policy; Under the min(N, D)-policy, if $N > D + 1$, then the min(N, D) policy coincides with the D-Policy. So, in this paper, we consider the values of N and D as $1 \le N \le D + 1$, $N = 1, 2, \ldots; D = 0, 1, 2, \ldots$ so that the N and D policies both take effect for energy consumption control.

Remark 2.2. In our study, the difficulty is to derive the mean number of data packets and mean transmission time backlog of data packets. The reason is that the BPs' transmission times are independent, while the IPs' transmission times $\{S_1, S_2, \ldots, S_{Q_{(N,\ D)}}\}$ are conditionally dependent, given $Q_{N,D}$, that is, the $\{S_1, S_2, \ldots, S_{Q_{(N,\ D)}} | Q_{(N,\ D)}\}$ are dependent. In reality, under the max(N, D)-policy, when $Q_{(N,\ D)} = k, k = N + 1, N + 2, \ldots, D + 1$, we have $\sum_{i=1}^{k-1} S_i \le D < \sum_{i=1}^{k} S_i$, and when $Q_{(N,\ D)} = N$, the $\sum_{i=1}^{N} S_i > D$ holds, These inequalities indicate the property of conditional dependence mentioned above. Likewise, under the min(N, D)-policy, if $Q_{(N,\ D)} = n, n = 1, 2, \ldots, N - 1$, then the D-policy triggers the transmission mode of radio server and the D is exceeded by the transmission time of the nth data packet for the first time, *i.e.*, $\sum_{i=1}^{n-1} S_i \le D < \sum_{i=1}^{n} S_i$; If $Q_{(N,\ D)} = N$, then the radio server is turned on by the N-policy. In this case, we get $\sum_{i=1}^{N-1} S_i \le D$, and $\sum_{i=1}^{N} S_i \le D + 1$(or $\sum_{i=1}^{N} S_i > D + 1$). So the IPs' transmission times are also conditionally dependent. Due to this dependence, which the N, T, and NT policies don't possess, the mean number of data packets can not be obtained by the decomposition property of queue length in queueing theory.

It can be seen that the BPs' transmission times are stochastically different from the IPs' transmission times. Since the above complex queueing behavior was overlooked and the decomposition property of queue length was applied, the mean number of data packets (mean queue length) obtained by Gu *et al.* [24] under the min(N, D)-policy (see Cor. 4.9 in [24]) is different from our result (see Eq. (4.18) at the end of Sect. 4).

Remark 2.3. In [23], Lee *et al.* analyzed the queueing measures of the continuous-time M/G/1 queue with the min(N, D)-policy. In this paper, Section 4 presents a performance measure analysis for the discrete-time Geo/G/1 queue with the min(N, D)-policy. It should be noted that there are some differences between our research on the min(N, D)-policy and the work of Lee *et al.* [23]. First, the values of N and D are different. For the values of N and D in our model, see Remark 1. In Lee *et al.* [23], N is a positive integer and D is a non-negative real number; Second, the analytical technique is different. We derive the mean queueing measures of a sensor node to construct power consumption functions only using the means of random variables, and avoid the analysis of abstract stochastic process and complex derivation of probability generating function in Lee *et al.* [23]. Specially, the derivations of mean waiting time and mean queue length are completely different. Our derivation and results are simple and easy for network practitioners to understand and utilize. Finally, in our study, the analysis on mean busy period is new, and applicable for the continuous-time $M/G/1$ and discrete-time Geo/G/1 queues with the min(N, D), max(N, D), min(D, T), and max(D, T) policies.

For the Geo/G/1 queue with the min(N, D)-policy, besides the differences from Gu *et al.* [24] and Lee *et al.* [23] presented in Remarks 2.2 and 2.3, our objective lies in investigating its effectiveness in power consumption control of sensor node and making a comparison with the N, D, and max(N, D) policies.

3. Performance measures of sensor node under the max(N, D)-policy

3.1. The number of data packets at the start of busy period

Let $Q_{(N,\ D)}$ denote the number of data packets at the start of a busy period. Note that under the max(N, D)-policy, the sleeping server resumes its transmission mode when the N-policy and the D-policy are satisfied simultaneously, therefore $N \leq Q_{(N,\ D)} \leq D+1$. When $Q_{(N,\ D)} = k,\ k = N+1, N+2, \ldots, D+1$, then the sum of transmission times of these k data packets exceeds D for the first time, that is, $\sum_{i=1}^{k-1} S_i \leq D < \sum_{i=1}^{k} S_i$; When $Q_{(N,\ D)} = N$, then the transmission time backlog of these N data packets is greater than D, that is, $\sum_{i=1}^{N} S_i > D$. Thus the distribution of $Q_{(N,\ D)}$ is

$$\Pr\{Q_{(N,\ D)} = k\} = \begin{cases} S^{(k-1)}(D) - S^{(k)}(D), & k = N+1, N+2, \ldots, D+1, \\ 1 - S^{(N)}(D), & k = N, \end{cases} \tag{3.1}$$

which leads to the first and second moments of $Q_{(N,\ D)}$ as

$$E(Q_{(N,\ D)}) = \sum_{k=N}^{D+1} k\Pr\left\{Q_{(N,\ D)} = k\right\} = N + \sum_{k=N}^{D} S^{(k)}(D), \tag{3.2}$$

$$\begin{aligned} E(Q_{(N,\ D)}(Q_{(N,\ D)} - 1)) &= \sum_{k=N}^{D+1} k(k-1)\Pr\left\{Q_{(N,\ D)} = k\right\} \\ &= N(N-1) + 2\sum_{k=N}^{D} kS^{(k)}(D). \end{aligned} \tag{3.3}$$

3.2. The transmission time backlog at the start of a busy period

Denote $\Phi_{(N,\ D)}$ as the transmission time backlog at the start of a busy period. Then $\Phi_{(N,\ D)} = x,\ x = D+1, D+2, \ldots$, comes under two cases: (1) N data packets accumulate at the start of a busy period and their transmission time backlog is x; (2) $k+1, N \leq k \leq D$, data packets accumulate at the start of a busy period, in which the transmission time backlog of the first k data packets is n, $n = k, k+1, \ldots, D$, and D is exceeded by the transmission time of the $(k+1)$th data packet. So, the probability distribution of $\Phi_{(N,\ D)}$ is

$$\Pr\{\Phi_{(N,\ D)} = x\} = s^{(N)}(x) + \sum_{k=N}^{D}\sum_{n=k}^{D} s^{(k)}(n)s_{x-n}, x = D+1, D+2, \ldots, \tag{3.4}$$

where $s_{x-n} = s^{(1)}(x-n) = \Pr(S_1 = x-n)$ (see the notations in Sect. 2).

From (3.4), we get the mean of $\Phi_{(N,\ D)}$ as

$$E(\Phi_{(N,\ D)}) = \sum_{x=D+1}^{\infty} xs^{(N)}(x) + \sum_{k=N}^{D}\sum_{x=D+1}^{\infty} x\sum_{n=k}^{D} s^{(k)}(n)s_{x-n}. \tag{3.5}$$

For the second term of right hand side in (3.5), we have

$$\sum_{k=N}^{D}\sum_{x=D+1}^{\infty} x\sum_{n=k}^{D} s^{(k)}(n)s_{x-n} = \sum_{x=D+1}^{\infty} xs^{(D+1)}(x) - \sum_{n=D+1}^{\infty} ns^{(N)}(n) - E(S)\sum_{k=N}^{D}\sum_{n=D+1}^{\infty} s^{(k)}(n), \tag{3.6}$$

where $E(S)$ is the mean transmission time (see queue assumptions in Sect. 2).

Using (3.6) in (3.5) and noting that $\sum\limits_{n=D+1}^{\infty} s^{(k)}(n) = 1 - S^{(k)}(D)$, $N \le k \le D$, we can derive

$$E(\Phi_{(N,\ D)}) = E(S)\left[N + \sum_{k=N}^{D} S^{(k)}(D)\right]. \tag{3.7}$$

Here, the detailed derivations of (3.6) and (3.7) can be found in Appendixes A and B, respectively.

By similar computation, we can get the second moments of $\Phi_{(N,\ D)}$ as

$$\begin{aligned} E\left(\Phi^2_{(N,\ D)}\right) &= \sum_{k=D+1}^{\infty} k^2 \Pr\{\Phi_{(N,\ D)} = k\} = E(S^2)\left[N + \sum_{k=N}^{D} S^{(k)}(D)\right] \\ &\quad + E(S)\left[D(D+1)E(S) - 2\sum_{k=N}^{D}\sum_{n=D+1}^{\infty} ns^{(k)}(n)\right]. \end{aligned} \tag{3.8}$$

3.3. The busy period and busy cycle period

To derive the mean busy period $E(B_{(N,\ D)})$, we recall the definitions of γ and Γ_γ in Section 2. Noting that at epoch γ the number of data packets in a sensor node, Γ_γ, is exactly equal to the number of those arriving during initial time length $\Phi_{(N,\ D)}$ of a busy period, so we get the probability distribution of Γ_γ as

$$\Pr\{\Gamma_\gamma = n\} = \sum_{i=n}^{\infty} C_i^n p^n \bar{p}^{i-n} \Pr\{\Phi_{(N,\ D)} = i\}, n = 0, 1, 2, \ldots, \tag{3.9}$$

which yields the first and second moments of Γ_γ as follows:

$$E(\Gamma_\gamma) = \sum_{i=0}^{\infty} \Pr\{\Phi_{(N,\ D)} = i\}\sum_{n=0}^{i} nC_i^n p^n \bar{p}^{i-n} = pE(\Phi_{(N,\ D)}), \tag{3.10}$$

$$E(\Gamma_\gamma(\Gamma_\gamma - 1)) = p^2 E(\Phi_{(N,\ D)}(\Phi_{(N,\ D)} - 1)). \tag{3.11}$$

Let B_γ be the remaining busy period initiating with Γ_γ data packets, then B_γ can exactly be viewed as the busy period initiating with Γ_γ customers in the ordinary Geo/G/1 queue. From the FCFS service discipline and the Galton-Watson branching process approach, $B_\gamma = B_1 + B_2 + \ldots + B_{\Gamma_\gamma}$, where $B_i, i \geq 1$ are i.i.d. and represent the busy period initiating with one customer in the ordinary Geo/G/1 queue, with mean $E(B) = \frac{E(S)}{1-\rho}$. Thus, using (3.10) yields

$$\begin{aligned} E(B_{(N,\ D)}) &= E(\Phi_{(N,\ D)} + B_1 + B_2 + \ldots + B_{\Gamma_\gamma}) \\ &= E(\Phi_{(N,\ D)}) + E(B)E(\Gamma_\gamma) \\ &= \frac{E(\Phi_{(N,\ D)})}{1-\rho}, \end{aligned} \tag{3.12}$$

where $E(\Phi_{(N,\ D)})$ is given by (3.7).

The mean idle period, $E(I_{(N,\ D)})$, is easily obtained from the following two relation equations: $pE(I_{(N,\ D)}) = E(Q_{(N,\ D)})$ and $E(S)E(Q_{(N,\ D)}) = E(\Phi_{(N,\ D)})$. Again by (3.12) and (3.7), we get the mean busy cycle period as

$$E(C_{(N,\ D)}) = E(I_{(N,\ D)}) + E(B_{(N,\ D)}) = \frac{1}{p(1-\rho)}\left[N + \sum_{k=N}^{D} S^{(k)}(D)\right]. \tag{3.13}$$

Remark 3.1. As far as we know, the above analysis on mean busy period is new. From the process of analysis, the approach is applicable for the continuous-time M/G/1 and discrete-time queues involving the *D*-policy, such as the Geo/G/1 queues with min(*N, D*)-policy, min(*D, T*)-policy, max(*D, T*)-policy, and so on.

3.4. The transmission time backlog at any epoch t^+

3.4.1. The transmission time backlog at any epoch t^+ in the idle period

Note that at epoch t^+ in the idle period, the transmission time backlog is zero, with probability $\frac{\frac{1}{p}}{E(I_{N,D})}$, and the transmission time backlog of k data packets is n, with probability $\frac{\frac{1}{p}s^{(k)}(n)}{E(I_{N,D})}$, where $k = 1, 2, \ldots, N-1,\ n = k, k+1, \ldots, D$; or $k = 1, 2, \ldots, N-1,\ n = D+1, D+2, \ldots$; or $k = N, N+1, \ldots, D,\ n = k, k+1, \ldots, D$. Here, the ranges for k and n are determined by the max(*N, D*)-policy. In fact, under the max(*N, D*)-policy, the number k of data packets and their transmission time backlog n during an idle period meet: (1) if $1 \leq k \leq N-1$, then we have $k \leq n \leq D$ or $n \geq D+1$; (2) if $N \leq k \leq D$, then we have $k \leq n \leq D$. From Remark 2.1, we know that the values of N and D satisfy $1 \leq N \leq D+1, N = 1, 2, \ldots; D = 0, 1, 2, \ldots$. If $k \geq D+1$, then we have $k \geq D+1 \geq N$. In this case, the busy period will begin. Therefore, in an idle period, the inequality $k \geq D+1$ does not hold. According to the definition of mean value, the steady-state mean transmission time backlog at epoch t^+ in the idle period, is obtained as

$$\begin{aligned} E(U^{\text{idle}}_{(N,\ D)}) &= \sum_{k=1}^{N-1}\sum_{n=k}^{D} n\frac{\frac{1}{p}s^{(k)}(n)}{E(I_{N,D})} + \sum_{k=N}^{D}\sum_{n=k}^{D} n\frac{\frac{1}{p}s^{(k)}(n)}{E(I_{N,D})} + \sum_{k=1}^{N-1}\sum_{n=D+1}^{\infty} n\frac{\frac{1}{p}s^{(k)}(n)}{E(I_{N,D})} \\ &= \frac{\sum_{k=1}^{D}\sum_{n=k}^{D} ns^{(k)}(n) + \sum_{k=1}^{N-1}\sum_{n=D+1}^{\infty} ns^{(k)}(n)}{N + \sum_{k=N}^{D} S^{(k)}(D)}, \end{aligned} \tag{3.14}$$

where the last equation is derived by means of the relation $pE(I_{(N,\ D)}) = E(Q_{(N,\ D)})$, and (3.2).

3.4.2. The transmission time backlog at any epoch t^+ in the busy period

We note that the busy period can be considered as a delay cycle incurred by the BPs, with initial delay $\Phi_{N,D}$, in the ordinary Geo/G/1 queue. Thus, according to the decomposition theory of the Geo/G/1 queue with generalized vacations (see Takagi [26]), we get the steady-state mean transmission time backlog at epoch t^+ in the busy period as

$$\begin{aligned} E(U^{\text{busy}}_{(N,\ D)}) &= E\left(U_{\text{Geo}/G/1}\right) + E\left(\Phi^-_{(N,\ D)}\right) \\ &= E\left(U_{\text{Geo}/G/1}\right) + \frac{E\left(\Phi_{(N,\ D)}(\Phi_{(N,\ D)} - 1)\right)}{2E(\Phi_{(N,\ D)})} \\ &= \frac{pE(S(S-1))}{2\rho(1-\rho)} + \frac{\frac{1}{2}D(D+1)E(S) - \sum\limits_{k=N}^{D}\sum\limits_{n=D+1}^{\infty} ns^{(k)}(n)}{N + \sum\limits_{k=N}^{D} S^{(k)}(D)}, \end{aligned} \tag{3.15}$$

where $E(U_{Geo/G/1}) = \frac{pE(S(S-1))}{2(1-\rho)}$ is the steady-state mean service time backlog at epoch t^+ in the ordinary Geo/G/1 queue, and $E(\Phi^-_{(N,\ D)}) = \frac{E(\Phi_{(N,\ D)}(\Phi_{(N,\ D)}-1))}{2E(\Phi_{(N,\ D)})}$ is the mean forward recurrence time of the discrete-time renewal process generated by i.i.d. $\Phi_{(N,\ D)}$'s. Also, by (3.7) and (3.8), the last equation of (3.15) is obtained.

3.4.3. The steady-state transmission time backlog at an arbitrary time t^+

Let p_{IP} denote the probability that an arbitrary data packet is an IP, p_{BP} the probability that an arbitrary data packet is a BP, and $E(\Lambda_{cycle})$ the mean number of data packets arriving in a busy cycle period, then $p_{IP} = \frac{E(Q_{(N,\ D)})}{E(\Lambda_{\text{cycle}})} = \frac{E(Q_{(N,\ D)})}{pE(C_{(N,\ D)})} = 1-\rho$, and $p_{BP} = 1 - p_{IP} = \rho$.

Conditioning on the type of data packets, utilizing (3.14) and (3.15), and noting that

$$\begin{aligned} \sum_{k=1}^{D}\sum_{n=k}^{D} ns^{(k)}(n) + \sum_{k=1}^{N-1}\sum_{n=D+1}^{\infty} ns^{(k)}(n) &= \sum_{k=1}^{D}\left[kE(S) - \sum_{n=D+1}^{\infty} ns^{(k)}(n)\right] + \sum_{k=1}^{N-1}\sum_{n=D+1}^{\infty} ns^{(k)}(n) \\ &= \frac{1}{2}D(D+1)E(S) - \sum_{k=N}^{D}\sum_{n=D+1}^{\infty} ns^{(k)}(n), \end{aligned} \tag{3.16}$$

we easily get the steady-state mean transmission time backlog at any epoch t^+ as

$$\begin{aligned} E(U_{(N,\ D)}) &= p_{IP}E\left(U^{\text{idle}}_{(N,\ D)}\right) + p_{BP}E\left(U^{\text{busy}}_{(N,\ D)}\right) \\ &= \frac{pE(S(S-1))}{2(1-\rho)} + \frac{\sum\limits_{k=1}^{D}\sum\limits_{n=k}^{D} ns^{(k)}(n) + \sum\limits_{k=1}^{N-1}\sum\limits_{n=D+1}^{\infty} ns^{(k)}(n)}{N + \sum\limits_{k=N}^{D} S^{(k)}(D)}. \end{aligned} \tag{3.17}$$

3.5. The mean waiting time of an arbitrary data packet

3.5.1. The mean waiting time of an arbitrary IP

According to queue assumptions with LAS-DA, if an arbitrary IP arrives within $((t+1)^-, (t+1)), t = 0, 1, 2, \ldots$ during an idle period, then its transmission waiting time, $W^{IP}_{(N,\ D)}$, is the sum of two parts. One is the transmission time backlog it sees upon arrival, which is equal to the transmission time backlog $U^{\text{idle}}_{(N,\ D)}$ at epoch t^+ during the idle period since no arrival occurs within $(t^+, (t+1)^-)$. Another is the remaining idle period after its arrival, denoted by $I_{\text{remaining}}$, that is, a time length from the arrival of this IP to the start of subsequent busy period.

To derive $E(I_{\text{remaining}})$, we assume that this IP under consideration is the kth arrival in an idle period (with probability $\frac{1}{E(Q_{(N,\ D)})}$), and just after its arrival, the total transmission time backlog including itself is x (with probability $s^{(k)}(x)$), where $k = 1, 2, \ldots, N-1,\ x = k, k+1, \ldots, D$; or $k = 1, 2, \ldots, N-1,\ x = D+1, D+2, \ldots$;

or $k = N, N+1, \ldots, D,\ x = k, k+1, \ldots, D$. Then since the mean number of data packets at the start of a busy period is equal to the product of arrival rate p and mean idle period, we have

$$\begin{aligned} pE(I_{\text{remaining}}) &= \sum_{k=1}^{N-1}\sum_{x=k}^{D}\frac{s^{(k)}(x)}{E(Q_{(N,\ D)})}E(Q_{(N-k,D-x)}) + \sum_{k=N}^{D}\sum_{x=k}^{D}\frac{s^{(k)}(x)}{E(Q_{(N,\ D)})} \\ &\quad \times E(Q_{(1,D-x)}) + \sum_{k=1}^{N-1}\sum_{x=D+1}^{\infty}\frac{s^{(k)}(x)}{E(Q_{(N,\ D)})}E(Q_{(N-k,N-k-1)}) \\ &= \sum_{k=1}^{N-1}\sum_{x=k}^{D}\frac{s^{(k)}(x)}{E(Q_{N,D})}\left[N-k+\sum_{n=N-k}^{D-x}S^{(n)}(D-x)\right] \\ &\quad + \sum_{k=1}^{N-1}\sum_{x=D+1}^{\infty}(N-k)\frac{s^{(k)}(x)}{E(Q_{N,D})} \\ &\quad + \sum_{k=N}^{D}\sum_{x=k}^{D}\frac{s^{(k)}(x)}{E(Q_{N,D})}\left[1+\sum_{n=1}^{D-x}S^{(n)}(D-x)\right], \end{aligned} \tag{3.18}$$

where $Q_{(u,v)}$ denotes the number of data packets at the start of a busy period under the $\max(u,v)$ policy, and $E(Q_{(u,v)})$ is determined by equation (3.2).

From (3.18) and (3.2), we can derive

$$E(I_{\text{remaining}}) = \frac{\frac{N(N-1)}{2} + \sum_{k=N}^{D} kS^{(k)}(D)}{p\left[N + \sum_{k=N}^{D} S^{(k)}(D)\right]}, \tag{3.19}$$

where

$$\frac{N(N-1)}{2} = \sum_{k=1}^{N-1}\sum_{x=k}^{D} s^{(k)}(x)(N-k) + \sum_{k=1}^{N-1}\sum_{x=D+1}^{\infty}(N-k)s^{(k)}(x), \tag{3.20}$$

$$\begin{aligned} \sum_{k=N}^{D} kS^{(k)}(D) &= \sum_{k=N}^{D}\sum_{x=k}^{D} s^{(k)}(x) + \sum_{k=1}^{N-1}\sum_{x=k}^{D} s^{(k)}(x)\sum_{n=N-k}^{D-x}S^{(n)}(D-x) \\ &\quad + \sum_{k=N}^{D}\sum_{x=k}^{D} s^{(k)}(x)\sum_{n=1}^{D-x}S^{(n)}(D-x). \end{aligned} \tag{3.21}$$

For the detailed derivations of (3.20) and (3.21), see the Appendix C.

Thus, we get the mean waiting time of an arbitrary IP as

$$\begin{aligned} E\left(W^{IP}_{(N,\ D)}\right) &= E\left(U^{\text{idle}}_{(N,\ D)}\right) + E(I_{\text{remaining}}) \\ &= \frac{\frac{N(N-1)}{2} + \sum_{k=N}^{D} kS^{(k)}(D)}{p\left[N + \sum_{k=N}^{D} S^{(k)}(D))\right]} + \frac{\sum_{k=1}^{D}\sum_{n=k}^{D} ns^{(k)}(n) + \sum_{k=1}^{N-1}\sum_{n=D+1}^{\infty} ns^{(k)}(n)}{N + \sum_{k=N}^{D} S^{(k)}(D)}. \end{aligned} \tag{3.22}$$

3.5.2. The mean waiting time of an arbitrary BP

Assume that an arbitrary BP arrives within $((t+1)^-, (t+1)), t = 0, 1, 2, \ldots$ during the busy period, then its transmission waiting time, $W^{BP}_{(N,\ D)}$, is the transmission time backlog it sees upon arrival, which exactly equals the transmission time backlog $U^{\text{busy}}_{(N,\ D)}$ at epoch t^{+} during the busy period since no arrival and departure occur within $(t^{+},(t+1)^{-})$. Therefore, we get $E(W^{BP}_{(N,\ D)}) = E(U^{\text{busy}}_{(N,\ D)})$, that is,

$$E(W^{BP}_{(N,\ D)}) = \frac{pE(S(S-1))}{2\rho(1-\rho)} + \frac{\frac{1}{2}D(D+1)E(S) - \sum_{k=N}^{D}\sum_{n=D+1}^{\infty} ns^{(k)}(n)}{N + \sum_{k=N}^{D} S^{(k)}(D)}. \tag{3.23}$$

3.5.3. The mean waiting time of an arbitrary data packet

Conditioning on the type of data packets, and using (3.22), (3.23), and (3.16), we obtain the mean waiting time of an arbitrary data packet as

$$\begin{aligned} E\left(W_{(N,D)}\right) &= p_{IP}E\left(W^{IP}_{(N,D)}\right) + p_{BP}E\left(W^{BP}_{(N,D)}\right) \\ &= \frac{\sum_{k=1}^{D}\sum_{n=k}^{D} ns^{(k)}(n) + \sum_{k=1}^{N-1}\sum_{n=D+1}^{\infty} ns^{(k)}(n)}{N + \sum_{k=N}^{D} S^{(k)}(D)} \\ &\quad + \frac{(1-\rho)\left[\frac{N(N-1)}{2} + \sum_{k=N}^{D} kS^{(k)}(D)\right]}{p\left[N + \sum_{k=N}^{D} S^{(k)}(D)\right]} + \frac{pE(S(S-1))}{2(1-\rho)}. \end{aligned} \tag{3.24}$$

3.6. The mean number of data packets in sensor node

From the Little's formula and (3.24), we get the mean number of data packets in sensor node as

$$\begin{aligned} E(L_{(N,\ D)}) &= \rho + pE(W_{(N,\ D)}) = \rho + \frac{p^{2}E(S(S-1))}{2(1-\rho)} \\ &\quad + \frac{(1-\rho)\left[\frac{N(N-1)}{2} + \sum_{k=N}^{D} kS^{(k)}(D)\right]}{N + \sum_{k=N}^{D} S^{(k)}(D)} \\ &\quad + \frac{p\left[\sum_{k=1}^{D}\sum_{n=k}^{D} ns^{(k)}(n) + \sum_{k=1}^{N-1}\sum_{n=D+1}^{\infty} ns^{(k)}(n)\right]}{N + \sum_{k=N}^{D} S^{(k)}(D)}. \end{aligned} \tag{3.25}$$

4. Performance measures of sensor node under the min(N, D)-policy

In this section, we consider the min(N, D)-policy for a sensor node. Under this policy, the sleeping radio server is wakened up to restart its transmission mode as soon as there are N data packets accumulated in the node buffer or the transmission time backlog of data packets arriving during the idle period is greater than D. Similar to the analysis in the previous section, the performance measures of sensor node can be derived.

4.1. The number of arriving data packets at the start of a busy period

Under the min(N, D)-policy, the number of data packets $Q_{(N,\ D)}$ at the start of a busy period depends on the fact that either the D- or N-policy triggers the busy period. If the server starts its busy period in the light of the D-policy, then $Q_{(N,\ D)} = k, k = 1, 2, \ldots, N-1$, with probability $\Pr\{\sum_{i=1}^{k-1} S_i \le D < \sum_{i=1}^{k} S_i\}$. If a busy period is triggered by the N-policy, then $Q_{(N,\ D)} = N$, with probability $\Pr\{\sum_{i=1}^{N-1} S_i \le D\}$. So the probability distribution, the first and second moments of $Q_{(N,\ D)}$ are given respectively, by

$$\Pr\{Q_{(N,\ D)} = k\} = \begin{cases} S^{(k-1)}(D) - S^{(k)}(D), & k = 1, 2, \ldots, N-1, \\ S^{(N-1)}(D), & k = N, \end{cases} \tag{4.1}$$

$$E(Q_{(N,\ D)}) = \sum_{k=1}^{N} k\Pr\{Q_{(N,\ D)} = k\} = 1 + \sum_{k=1}^{N-1} S^{(k)}(D), \tag{4.2}$$

$$E(Q_{(N,\ D)}(Q_{(N,\ D)} - 1)) = \sum_{k=1}^{N} k(k-1)\Pr\{Q_{(N,\ D)} = k\} = 2\sum_{k=1}^{N-1} kS^{(k)}(D). \tag{4.3}$$

4.2. The transmission time backlog at the start of a busy period

Let $\Phi_{(N,\ D)}$ denote the transmission time backlog at the start of a busy period. When $\Phi_{(N,\ D)} = x,\ x = D+1, D+2, \ldots$, two cases may happen:(a) there is only a data packet at the start of a busy period but its transmission time x is greater than D; (b) the transmission time backlog of the first $k(1 \le k \le N-1)$ data packets is $n,\ n = k, k+1, \ldots, D$, and D is exceeded by the transmission time of the $(k+1)$th data packet. When $\Phi_{(N,\ D)} = x,\ x = N, N+1, \ldots, D$, then there are N waiting data packets at the start of a busy period and their transmission time backlog is equal to x. So the distribution of $\Phi_{(N,\ D)}$ is

$$\Pr\{\Phi_{(N,\ D)} = x\} = \begin{cases} s^{(N)}(x), & x = N, N+1, \ldots, D, \\ s_x + \sum\limits_{k=1}^{N-1} \sum\limits_{n=k}^{D} s^{(k)}(n) s_{x-n}, & x = D+1, D+2, \ldots. \end{cases} \tag{4.4}$$

From (4.4), the mean of $\Phi_{N,D}$ is

$$E(\Phi_{(N,\ D)}) = \sum_{x=N}^{D} xs^{(N)}(x) + \sum_{x=D+1}^{\infty} xs_x + \sum_{k=1}^{N-1} \sum_{x=D+1}^{\infty} x \sum_{n=k}^{D} s^{(k)}(n) s_{x-n}, \tag{4.5}$$

where the third term on the right hand side of (4.5) becomes

$$\begin{aligned} \sum_{k=1}^{N-1} \sum_{x=D+1}^{\infty} x \sum_{n=k}^{D} s^{(k)}(n) s_{x-n} &= \sum_{k=1}^{N-1} \sum_{x=D+1}^{\infty} x \left[\sum_{n=k}^{x} s^{(k)}(n) s_{x-n} - \sum_{n=D+1}^{x} s^{(k)}(n) s_{x-n} \right] \\ &= \sum_{k=1}^{N-1} \sum_{x=D+1}^{\infty} xs^{(k+1)}(x) - \sum_{k=1}^{N-1} \sum_{n=D+1}^{\infty} s^{(k)}(n) \sum_{x=n}^{\infty} [ns_{x-n} + (x-n)s_{x-n}] \\ &= \sum_{k=1}^{N-1} \sum_{x=D+1}^{\infty} xs^{(k+1)}(x) - \sum_{k=1}^{N-1} \sum_{n=D+1}^{\infty} ns^{(k)}(n) - E(S) \sum_{k=1}^{N-1} \sum_{n=D+1}^{\infty} s^{(k)}(n) \\ &= \sum_{x=D+1}^{\infty} xs^{(N)}(x) - \sum_{n=D+1}^{\infty} xs_x - E(S) \sum_{k=1}^{N-1} \left[1 - S^{(k)}(D)\right]. \end{aligned} \tag{4.6}$$

Substituting (4.6) into (4.5) gives rise to

$$E(\Phi_{(N,\ D)}) = E(S) \left[1 + \sum_{k=1}^{N-1} S^{(k)}(D)\right]. \tag{4.7}$$

Similar computation yields

$$E(\Phi_{N,D}(\Phi_{N,D}-1)) = E(S(S-1))\sum_{k=0}^{N-1} S^{(k)}(D) + 2E(S)\sum_{k=1}^{N-1}\sum_{n=k}^{D} ns^{(k)}(n). \tag{4.8}$$

4.3. The busy period and busy cycle period

From Remark 3.1, we know that the busy period formula (3.12) is still available for the Geo/G/1 queue with the min($N,\ D$) policy, hence, after using (32), we obtain the mean busy period as

$$E(B_{(N,\ D)}) = \frac{E(\Phi_{(N,\ D)})}{1-\rho} = \frac{E(S)}{1-\rho}\left[1+\sum_{k=1}^{N-1} S^{(k)}(D)\right]. \tag{4.9}$$

Noting that $pE(I_{(N,\ D)}) = E(Q_{(N,\ D)})$, and utilizing (4.2), we get the mean busy cycle period as

$$E(C_{(N,\ D)}) = E(I_{(N,\ D)}) + E(B_{(N,\ D)}) = \frac{1}{p(1-\rho)}\left[1+\sum_{k=1}^{N-1} S^{(k)}(D)\right]. \tag{4.10}$$

4.4. The transmission time backlog at any epoch t^+

4.4.1. The transmission time backlog at any epoch t^+ in the idle period

In steady state, at any epoch t^+ during the idle period, the transmission time backlog of k data packets is n with probability $\frac{\frac{1}{p}s^{(k)}(n)}{E(I_{(N,\ D)})}$, $k=1,2,\ldots,N-1,\ n=k,k+1,\ldots,D$, and the transmission time backlog is zero with probability $\frac{\frac{1}{p}}{E(I_{(N,\ D)})}$. Thus, at any epoch t^+ in the idle period, the mean transmission time backlog is

$$E(U^{\text{idle}}_{(N,\ D)}) = \sum_{k=1}^{N-1}\sum_{n=k}^{D} n\frac{\frac{1}{p}s^{(k)}(n)}{E(I_{(N,\ D)})} = \frac{\sum\limits_{k=1}^{N-1}\sum\limits_{n=k}^{D} ns^{(k)}(n)}{1+\sum\limits_{k=1}^{N-1} S^{(k)}(D)}. \tag{4.11}$$

4.4.2. The transmission time backlog at any epoch t^+ in the busy period

By same analysis in Section 3.4.2 and the decomposition theory of the Geo/G/1 queue with generalized vacations (see Takagi [26]), we obtain the steady-state mean transmission time backlog at epoch t^+ in the busy period as

$$\begin{aligned} E(U^{\text{busy}}_{(N,\ D)}) &= \frac{pE(S(S-1))}{2(1-\rho)} + \frac{E(\Phi_{(N,\ D)}(\Phi_{(N,\ D)}-1))}{2E(\Phi_{(N,\ D)})} \\ &= \frac{pE(S(S-1))}{2\rho(1-\rho)} + \frac{\sum\limits_{k=1}^{N-1}\sum\limits_{n=k}^{D} ns^{(k)}(n)}{1+\sum\limits_{k=1}^{N-1} S^{(k)}(D)}, \end{aligned} \tag{4.12}$$

where $E(\Phi_{(N,\ D)}(\Phi_{(N,\ D)}-1))$ and $E(\Phi_{(N,\ D)})$ are given in (4.8) and (4.7), respectively.

4.4.3. The steady-state transmission time backlog at an arbitrary time t^+

Let p_{IP} denote the probability that an arbitrary data packet is an IP, p_{BP} the probability that an arbitrary data packet is a BP, and $E(\Lambda_{cycle})$ the mean number of data packets arriving in a busy cycle period, then $p_{IP} = \frac{E(Q_{(N,\ D)})}{E(\Lambda_{\text{cycle}})} = \frac{E(Q_{(N,\ D)})}{pE(C_{(N,\ D)})} = 1-\rho$, and $p_{BP} = 1-p_{IP} = \rho$.

Conditioning on the type of data packets, and using (4.11) and (4.12), we easily get the steady-state mean transmission time backlog at any epoch t^+ as

$$E(U_{(N,\ D)}) = p_{IP}E\left(U^{\text{idle}}_{(N,\ D)}\right) + p_{BP}E\left(U^{\text{busy}}_{(N,\ D)}\right) = \frac{pE(S(S-1))}{2(1-\rho)} + \frac{\sum_{k=1}^{N-1}\sum_{n=k}^{D} ns^{(k)}(n)}{1+\sum_{k=1}^{N-1} S^{(k)}(D)}. \tag{4.13}$$

4.5. The mean waiting time of an arbitrary data packet

4.5.1. The mean waiting time of an arbitrary IP

According to queue assumptions with LAS-DA, if an arbitrary IP arrives within $((t+1)^-, (t+1)), t = 0, 1, 2, \ldots$ during an idle period, then its transmission waiting time, $W^{IP}_{(N,\ D)}$, is the sum of two parts. One is the transmission time backlog it sees upon arrival, which is equal to the transmission time backlog $U^{\text{idle}}_{(N,\ D)}$ at epoch t^+ during an idle period since no arrival occurs within $(t^+, (t+1)^-)$. Another is the remaining idle period after its arrival, denoted by $I_{\text{remaining}}$, that is, a time length from the arrival of this IP to the start of subsequent busy period.

To derive $E(I_{\text{remaining}})$, we assume that this IP under consideration is the kth $(1 \le k \le N-1)$ arrival in an idle period (with probability $\frac{1}{E(Q_{(N,\ D)})}$), and just after its arrival, the total transmission time backlog including itself is $x, k \le x \le D$, (with probability $s^{(k)}(x)$). In this case, the mean arrival number during $I_{\text{remaining}}$ is exactly equal to the mean number of data packets at the start of a busy period under the $\min(N-k, D-x)$ policy. Then we have

$$\begin{aligned} pE(I_{\text{remaining}}) &= \sum_{k=1}^{N-1}\sum_{x=k}^{D} \frac{s^{(k)}(x)}{E(Q_{(N,\ D)})} E(Q_{(N-k,D-x)}) \\ &= \frac{1}{E(Q_{(N,\ D)})} \sum_{k=1}^{N-1}\sum_{n=0}^{N-k-1}\sum_{x=k}^{D} s^{(k)}(x)S^{(n)}(D-x) \\ &= \frac{1}{E(Q_{(N,\ D)})} \sum_{k=1}^{N-1}\sum_{n=0}^{N-k-1} S^{(n+k)}(D) \\ &= \frac{\sum_{k=1}^{N-1} kS^{(k)}(D)}{1+\sum_{k=1}^{N-1} S^{(k)}(D)}, \end{aligned} \tag{4.14}$$

where $Q_{(N-k,D-x)}$ denotes the number of data packets at the start of a busy period under the $\min(N-k, D-x)$ policy, and $E(Q_{(N-k,D-x)})$ is determined by equation (4.2).

Thus, we get the mean waiting time of an arbitrary IP as

$$E(W^{IP}_{(N,\ D)}) = E(U^{\text{idle}}_{(N,\ D)}) + E(I_{\text{remaining}}) = \frac{\sum_{k=1}^{N-1}\sum_{n=k}^{D} ns^{(k)}(n) + \frac{1}{p}\sum_{k=1}^{N-1} kS^{(k)}(D)}{1+\sum_{k=1}^{N-1} S^{(k)}(D)}. \tag{4.15}$$

4.5.2. The mean waiting time of an arbitrary BP

Assume that an arbitrary BP arrives within $((t+1)^-, (t+1)), t = 0, 1, 2, \ldots$ during a busy period, then its transmission waiting time, $W^{BP}_{(N,\ D)}$, is the transmission time backlog it sees upon arrival, which exactly equals the transmission time backlog $U^{\text{busy}}_{(N,\ D)}$ at epoch t^+ during a busy period since no arrival and departure occur within $(t^+, (t+1)^-)$. Therefore, we get

$$E(W^{BP}_{(N,\ D)}) = E(U^{busy}_{(N,\ D)}) = \frac{pE(S(S-1))}{2\rho(1-\rho)} + \frac{\sum_{k=1}^{N-1}\sum_{n=k}^{D} ns^{(k)}(n)}{1+\sum_{k=1}^{N-1} S^{(k)}(D)}. \tag{4.16}$$

4.5.3. The mean waiting time of an arbitrary data packet

Conditioning on the type of data packets, and using (4.15) and (4.16), we obtain the mean transmission waiting time of an arbitrary queued data packet as

$$\begin{aligned} E(W_{(N,\ D)}) &= p_{IP}E(W^{IP}_{(N,\ D)}) + p_{BP}E(W^{BP}_{(N,\ D)}) \\ &= \frac{pE(S(S-1))}{2(1-\rho)} + \frac{\sum_{k=1}^{N-1}\sum_{n=k}^{D} ns^{(k)}(n)}{1+\sum_{k=1}^{N-1} S^{(k)}(D)} + \frac{(1-\rho)\sum_{k=1}^{N-1} kS^{(k)}(D)}{p\left[1+\sum_{k=1}^{N-1} S^{(k)}(D)\right]}. \end{aligned} \tag{4.17}$$

4.6. The mean number of data packets in sensor node

From the Little's formula and (4.17), we get the mean number of data packets in sensor node as

$$\begin{aligned} E(L_{(N,\ D)}) = \rho + pE(W_{(N,\ D)}) = \rho + \frac{p^2E(S(S-1))}{2(1-\rho)} \\ + \frac{(1-\rho)\sum_{k=1}^{N-1} kS^{(k)}(D) + p\sum_{k=1}^{N-1}\sum_{n=k}^{D} ns^{(k)}(n)}{1+\sum_{k=1}^{N-1} S^{(k)}(D)}. \end{aligned} \tag{4.18}$$

5. Special cases

Case 1.

(1) Under the max(N, D) policy in Section 3, when $D = N-1$ and $\sum_{k=i}^{j} = 0, i > j$, note that $E(Q_{(N,\ D)}) = N$ and

$$\sum_{k=1}^{D}\sum_{n=k}^{D} ns^{(k)}(n) + \sum_{k=1}^{N-1}\sum_{n=D+1}^{\infty} ns^{(k)}(n) = E(S)\frac{N(N-1)}{2},$$

we get the steady-state measures of sensor node under the N-policy. For instance, the mean number of data packets, mean busy period, mean busy cycle period, and mean transmission time backlog, are given respectively, by

$$E(L_N) = \rho + \frac{p^2E(S(S-1))}{2(1-\rho)} + \frac{N-1}{2}, \quad E(B_N) = \frac{NE(S)}{1-\rho}, \tag{5.1}$$

$$E(C_N) = \frac{N}{p(1-\rho)}, \quad E(U_N) = E(S)\frac{N-1}{2} + \frac{pE(S(S-1))}{2(1-\rho)}, \tag{5.2}$$

where $E(L_N)$ agrees with that in Luo *et al.* [27], and $E(U_N)$ is a new result.

(2) Under the min($N,\ D$) policy in Section 4, when $D \to \infty$, note that

$$S^{(k)}(D) \to 1, k \geq 1, \qquad E(Q_{(N,\ D)}) \to N,$$
$$\sum_{k=1}^{N-1} kS^{(k)}(D) \to \frac{N(N-1)}{2}, \quad \sum_{k=1}^{N-1}\sum_{n=k}^{D} ns^{(k)}(n) \to E(S)\frac{N(N-1)}{2},$$

we can still obtain the performance measures of sensor node under the N-policy, such as (5.1) and (5.2).

Case 2.

(1) Under the max($N,\ D$) policy in Section 3, when $N = 1$ and $\sum_{k=i}^{j} = 0, i > j$, we easily derive the corresponding measures of sensor node under the D-policy. For example, the mean number of data packets, mean busy period, mean busy cycle period, and mean transmission time backlog, are given respectively, by

$$E(L_D) = \rho + \frac{p^2E(S(S-1))}{2(1-\rho)} + \frac{(1-\rho)\sum_{k=1}^{D} kS^{(k)}(D) + p\sum_{k=1}^{D}\sum_{n=k}^{D} ns^{(k)}(n)}{1+\sum_{k=1}^{D} S^{(k)}(D)}, \tag{5.3}$$

$$E(B_D) = \frac{E(S)}{1-\rho}\left[1+\sum_{k=1}^{D} S^{(k)}(D)\right], E(C_D) = \frac{1}{p(1-\rho)}\left[1+\sum_{k=1}^{D} S^{(k)}(D)\right], \tag{5.4}$$

$$E(U_D) = \frac{pE(S(S-1))}{2(1-\rho)} + \frac{\sum_{k=1}^{D}\sum_{n=k}^{D} ns^{(k)}(n)}{1+\sum_{k=1}^{D} S^{(k)}(D)}\cdot \tag{5.5}$$

(2) Under the min($N,\ D$) policy in Section 4, when $N = D + 1$, we still derive the corresponding performance measures of sensor node under the D-policy, such as (5.3), (5.4), and (5.5).

6. Optimal power-saving comparison and numerical illustration

In this section, we apply the performance measures obtained in the previous sections to construct two power consumption functions of sensor node. One is based on the mean number of data packets, and another is based on the mean transmission time backlog of data packets. Our aim is to examine the power-saving effectiveness of two ($N,\ D$) policies and compare minimum power consumption of the $N,\ D$ and two ($N,\ D$) policies. Let us define the power consumption elements as follows:

$C_s \equiv$ set-up power consumption per the busy cycle period, that is, the power consumption incurred by switching from idle state (sleep mode) to busy state (transmission mode) and vice versa in each busy cycle period,

$C_h \equiv$ holding power consumption for each data packet in a sensor node,

$C_i \equiv$ power consumption when the radio server is idle (in sleep mode),

$C_b \equiv$ power consumption when the radio server is busy (in transmission mode),

$C_u \equiv$ holding power consumption for per unit of transmission time backlog in the sensor node.

6.1. Power consumption function based on mean number of data packets

Following the definition presented by Jiang *et al.* [18], the power consumption function of sensor node based on mean number of data packets, is given by

$$F(N,\ D) = \frac{C_s}{E(C_{(N,\ D)})} + C_hE(L_{(N,\ D)}) + C_b\frac{E(B_{(N,\ D)})}{E(C_{(N,\ D)})} + C_i\frac{E(I_{(N,\ D)})}{E(C_{(N,\ D)})}\cdot \tag{6.1}$$

Let $F_1(N,\ D)$, $F_2(N,\ D)$, $F_N(N)$, and $F_D(D)$ denote the power consumption functions under the max(N, D), min(N, D), N, and D policies, respectively, then by putting the performance measures in Sections 3, 4 and 5 into (6.1), we get

$$F_1(N,\ D) = \frac{C_s p(1-\rho) + C_h(1-\rho)\left[\frac{N(N-1)}{2} + \sum\limits_{k=N}^{D} kS^{(k)}(D)\right]}{N + \sum\limits_{k=N}^{D} S^{(k)}(D)} + \frac{C_h p\left[\sum\limits_{k=1}^{D}\sum\limits_{n=k}^{D} ns^{(k)}(n) + \sum\limits_{k=1}^{N-1}\sum\limits_{n=D+1}^{\infty} ns^{(k)}(n)\right]}{N + \sum\limits_{k=N}^{D} S^{(k)}(D)} + A, \tag{6.2}$$

$$F_2(N,\ D) = \frac{C_s p(1-\rho) + C_h\left[p\sum\limits_{k=1}^{N-1}\sum\limits_{n=k}^{D} ns^{(k)}(n) + (1-\rho)\sum\limits_{k=1}^{N-1} kS^{(k)}(D)\right]}{1 + \sum\limits_{k=1}^{N-1} S^{(k)}(D)} + A, \tag{6.3}$$

$$F_N(N) = F_1(N, N-1) = F_2(N, \infty), \tag{6.4}$$

$$F_D(D) = F_1(1, D) = F_2(D+1, D), \tag{6.5}$$

where $A = C_b\rho + C_i(1-\rho) + C_h\left[\rho + \frac{p^2 E(S(S-1))}{2(1-\rho)}\right]$. Equations (6.4) and (6.5) are derived from Cases 1 and 2 in Section 5.

To examine the power-saving effectiveness and improvement degree, we introduce the power-saving factors for the N, D, max(N, D), and min(N, D) policies, denoted by $P_N(N)$, $P_D(D)$, and $P_i(N,\ D), i = 1, 2$, respectively. Their definitions are as follows:

$$P_i(N,\ D) = \frac{F_{\text{Geo}/G/1} - F_i(N,\ D)}{F_{\text{Geo}/G/1}}, \quad i = 1, 2, \tag{6.6}$$

$$P_N(N) = P_1(N, N-1) = P_2(N, \infty), \tag{6.7}$$

$$P_D(D) = P_1(1, D) = P_2(D+1, D), \tag{6.8}$$

where $F_{\text{Geo}/G/1} = F_1(1, 0)$ is the power consumption of sensor node without any threshold policy.

From the above definition, we know that if the power-saving factor of a threshold policy exceeds zero, then this policy is effective for power consumption control, and large power-saving factor means good power consumption control.

6.2. Power consumption function based on mean transmission time backlog of data packets

To investigate the power consumption control of four different policies from the viewpoint of mean transmission time backlog, we define the second kind of power consumption function as follows

$$f(N,\ D) = \frac{C_s}{E(C_{(N,\ D)})} + C_u E(U_{(N,\ D)}) + C_b\frac{E(B_{(N,\ D)})}{E(C_{(N,\ D)})} + C_i\frac{E(I_{(N,\ D)})}{E(C_{(N,\ D)})}. \tag{6.9}$$

Let $f_1(N,\ D)$, $f_2(N,\ D)$, $f_N(N)$, and $f_D(D)$ denote the second kind of power consumption functions under the max(N, D), min(N, D), N, and D policies, respectively, then by substituting the measures in Sections 3, 4 and 5 into (6.9), we get

$$f_1(N,\ D) = \frac{C_s p(1-\rho) + C_u\left[\sum_{k=1}^{D}\sum_{n=k}^{D} ns^{(k)}(n) + \sum_{k=1}^{N-1}\sum_{n=D+1}^{\infty} ns^{(k)}(n)\right]}{N + \sum_{k=N}^{D} S^{(k)}(D)} + B, \tag{6.10}$$

$$f_2(N,\ D) = \frac{C_s p(1-\rho) + C_u \sum_{k=1}^{N-1}\sum_{n=k}^{D} ns^{(k)}(n)}{1 + \sum_{k=1}^{N-1} S^{(k)}(D)} + B, \tag{6.11}$$

$$f_N(N) = f_1(N, N-1) = f_2(N, \infty), \tag{6.12}$$

$$f_D(D) = f_1(1, D) = f_2(D+1, D), \tag{6.13}$$

where $B = C_b\rho + C_i(1-\rho) + C_u\frac{pE(S(S-1))}{2(1-\rho)}$. Equations (6.12) and (6.13) are derived from Cases 1 and 2 in Section 5.

The corresponding power-saving factors for four policies, are denoted by $p_N(N)$, $p_D(D)$, and $p_i(N,\ D), i = 1, 2$, respectively. Their definitions are as follows:

$$p_i(N,\ D) = \frac{f_{\mathrm{Geo}/G/1} - f_i(N,\ D)}{f_{\mathrm{Geo}/G/1}}, \quad i = 1, 2, \tag{6.14}$$

$$p_N(N) = p_1(N, N-1) = p_2(N, \infty), \tag{6.15}$$

$$p_D(D) = p_1(1, D) = p_2(D+1, D), \tag{6.16}$$

where $f_{\mathrm{Geo}/G/1} = f_1(1, 0)$ is the power consumption of sensor node without any threshold policy.

6.3. Optimal power-saving policy

Since the power consumption functions in Sections 6.1 and 6.2 are highly non-linear and complicated, it is not easy to analytically find the optimal energy-saving policies and compare their minimum energy consumptions. But from the optimization theory of dynamic programme [28], the optimal power-saving policy that minimizes $F_N(N)$ in (6.4), denoted by N_N^*, can be obtained as follows:

$$N_N^* = \min\{N \geq 1 \mid F_1(N+1, N) - F_1(N, N-1) > 0\}. \tag{6.17}$$

That is to say, N_N^* is the first positive integer N that makes the inequality $F_1(N+1, N) - F_1(N, N-1) > 0$ hold.

Likewise, the optimal power-saving policy that minimizes $F_D(D)$ in (6.5), denoted by D_D^*, is

$$D_D^* = \min\{D \geq 0 \mid F_1(1, D+1) - F_1(1, D) > 0\}. \tag{6.18}$$

So, an algorithm to obtain N_N^* and D_D^* is easy.

Furthermore, for a given positive integer N, the optimal value $D^*(N)$ of D so as to minimize $F_i(N, D), i = 1, 2$, is given by

$$D^*(N) = \min\{D \geq N-1 \mid F_i(N, D+1) - F_i(N,\ D) > 0\}. \tag{6.19}$$

Thus, we can get an algorithm to find the optimal power-saving policy (N^*, D^*) that minimizes $F_i(N,\ D), i = 1, 2$. This algorithm is presented as follows:

Step 1. Set $N = 1$. Calculate $D^*(N)$ and $F_i(N, D^*(N))$ through equations (6.19) and (6.2) or (6.3), respectively.
Step 2. Calculate $D^*(N+1)$ and $F_i(N+1, D^*(N+1))$ through equations (6.19) and (6.2) or (6.3), respectively.

Step 3. If $F_i(N+1, D^*(N+1)) > F_i(N, D^*(N))$, then the algorithm stops. The optimal power-saving policy that minimizes $F_i(N,\ D)$ is $(N^*, D^*) = (N, D^*(N))$. Otherwise, set $N = N+1$, and go to Step 2.

In the above three steps, we utilize (6.2) to find the optimal power-saving policy (N^*, D^*) that minimizes $F_1(N,\ D)$, and apply (6.3) to get the optimal power-saving policy (N^*, D^*) that minimizes $F_2(N,\ D)$.

Similarly, the optimal power-saving policies $N_N^*,\ D_D^*$ and (N^*, D^*) so as to minimize $f_N(N),\ f_D(D)$ and $f_i(N,\ D), i = 1, 2$ can be found from proper replacement in (6.17)–(6.19) and Steps 1–3. For example, replacing $F_i(\cdot,\cdot)$ and (6.2) by $f_1(\cdot,\cdot)$ and (6.10), respectively, in (6.19) and Steps 1–3, we can obtain the optimal power-saving policy (N^*, D^*) that minimizes $f_1(N,\ D)$.

6.4. Numerical illustration

In this subsection, we illustrate the power-saving effectiveness of two $(N,\ D)$ policies and compare minimum power consumptions of the $N,\ D$ and two $(N,\ D)$ policies. Note that $S^{(k)}(D) = \sum_{n=k}^{D} s^{(k)}(n)$, so it is difficult to compute $S^{(k)}(D), k = 1, 2, \ldots, D$ for an arbitrarily distributed transmission time due to the complexity of convolution computation. For the convenience of analysis, we assume that

- data packets arrive according to a Bernoulli process at a rate p,
- transmission time of data packet is geometrically distributed(in this case, since the transmission rate $\frac{p}{\rho}$ is the probability that a transmission is finished at a slot, we get $0 < p < \rho$),
- $C_s = 120,\ C_h = 1,\ C_u = 1,\ C_b = 210,\ C_i = 5$.

After the algorithm in Section 6.3 is performed, the numerical results are displayed in Tables 1–6. Tables 1–3 summarize the optimal power consumptions based on the mean number of data packets under the *N*,*D*, max($N,\ D$) and min($N,\ D$) policies. Since all power-saving factors in Tables 1–3 are greater than zero, the four policies are effective and feasible in power saving optimization of sensor node. Further, for a same p, under each policy, smaller the difference of ρ and p is, larger corresponding power-saving factor is, which means a better power consumption control. Figures 1–2 confirm some computational results in Table 1.

TABLE 1. Optimal power-saving comparison of four policies based on mean number of data packets ($p = 0.25$).

ρ	N-policy	D-policy	max($N,\ D$)-policy	min($N,\ D$)-policy
	$N_N^* = 6$	$D_D^* = 7$	$(N^*, D^*) = (6,\ \ 6)$	$(N^*, D^*) = (7,\ \ 8)$
0.3	$F_N(N_N^*) = 72.8214$	$F_D(D_D^*) = 72.8397$	$F_1(N^*, D^*) = 72.8056$	$F_2(N^*, D^*) = 72.8114$
	$P_N(N_N^*) = 0.1708$	$P_D(D_D^*) = 0.1706$	$P_1(N^*, D^*) = 0.1710$	$P_2(N^*, D^*) = 0.1709$
	$N_N^* = 6$	$D_D^* = 8$	$(N^*, D^*) = (6,\ \ 7)$	$(N^*, D^*) = (6,\ 10)$
0.4	$F_N(N_N^*) = 93.0000$	$F_D(D_D^*) = 93.0313$	$F_1(N^*, D^*) = 92.9956$	$F_2(N^*, D^*) = 92.9900$
	$P_N(N_N^*) = 0.1185$	$P_D(D_D^*) = 0.1182$	$P_1(N^*, D^*) = 0.1185$	$P_2(N^*, D^*) = 0.1186$
	$N_N^* = 5$	$D_D^* = 9$	$(N^*, D^*) = (5,\ \ 8)$	$(N^*, D^*) = (6,\ 11)$
0.5	$F_N(N_N^*) = 113.2500$	$F_D(D_D^*) = 113.2273$	$F_1(N^*, D^*) = 113.2007$	$F_2(N^*, D^*) = 113.1926$
	$P_N(N_N^*) = 0.0811$	$P_D(D_D^*) = 0.0813$	$P_1(N^*, D^*) = 0.0815$	$P_2(N^*, D^*) = 0.0816$
	$N_N^* = 5$	$D_D^* = 9$	$(N^*, D^*) = (4,\ \ 9)$	$(N^*, D^*) = (6,\ 10)$
0.6	$F_N(N_N^*) = 133.5250$	$F_D(D_D^*) = 133.4803$	$F_1(N^*, D^*) = 133.4669$	$F_2(N^*, D^*) = 133.4570$
	$P_N(N_N^*) = 0.0539$	$P_D(D_D^*) = 0.0542$	$P_1(N^*, D^*) = 0.0543$	$P_2(N^*, D^*) = 0.0543$
	$N_N^* = 4$	$D_D^* = 9$	$(N^*, D^*) = (2,\ \ 9)$	$(N^*, D^*) = (6,\ 10)$
0.7	$F_N(N_N^*) = 154.0000$	$F_D(D_D^*) = 153.8947$	$F_1(N^*, D^*) = 153.8929$	$F_2(N^*, D^*) = 153.8876$
	$P_N(N_N^*) = 0.0330$	$P_D(D_D^*) = 0.0336$	$P_1(N^*, D^*) = 0.0336$	$P_2(N^*, D^*) = 0.0337$
	$N_N^* = 3$	$D_D^* = 8$	$(N^*, D^*) = (1,\ \ 8)$	$(N^*, D^*) = (6,\ \ 8)$
0.8	$F_N(N_N^*) = 175.0000$	$F_D(D_D^*) = 174.8170$	$F_1(N^*, D^*) = 174.8170$	$F_2(N^*, D^*) = 174.8155$
	$P_N(N_N^*) = 0.0169$	$P_D(D_D^*) = 0.0179$	$P_1(N^*, D^*) = 0.0179$	$P_2(N^*, D^*) = 0.0179$
	$N_N^* = 2$	$D_D^* = 6$	$(N^*, D^*) = (1,\ \ 6)$	$(N^*, D^*) = (5,\ \ 6)$
0.9	$F_N(N_N^*) = 198.2500$	$F_D(D_D^*) = 198.0278$	$F_1(N^*, D^*) = 198.0278$	$F_2(N^*, D^*) = 198.0272$
	$P_N(N_N^*) = 0.0050$	$P_D(D_D^*) = 0.0061$	$P_1(N^*, D^*) = 0.0061$	$P_2(N^*, D^*) = 0.0061$

In Tables 1–3, it is seen that for $0 < p < \rho < 1$, the max(N, D) and min(N, D) policies are better than the N and D policies in the sense of minimum power consumption. None of the max(N, D) and min(N, D) policies is superior over the other, which depends on the arrival rate and traffic intensity. The same conclusion also holds for the superiority of the N and D policies. In addition, when the traffic intensity approaches 1 (*e.g.* $\rho = 0.9$), the max(N, D) policy and the D-policy have an identical minimum power consumption.

Tables 4–6 report the optimal power consumption based on mean transmission time backlog under four policies. It is observed that for $0 < p < \rho < 1$, four optimal policies reduce power consumption when their corresponding power-saving factors are all positive number. Also, for given p and same ρ, the optimal N-policy is the least favourable, and the best is two optimal (N, D) policies or the optimal D-policy because the max(N, D), min(N, D) and D polices have an identical minimum power consumption, which is smaller than that incurred by the N policy. Figures 3−4 show several optimal results in Table 4 for the case of $p = 0.25$.

TABLE 2. Optimal power-saving comparison of four policies based on mean number of data packets ($p = 0.5$).

ρ	N-policy	D-policy	max(N, D)-policy	min(N, D)-policy
	$N_N^* = 7$	$D_D^* = 7$	$(N^*, D^*) = (6,\ 7)$	$(N^*, D^*) = (7,\ 8)$
0.6	$F_N(N_N^*) = 135.1786$	$F_D(D_D^*) = 135.1646$	$F_1(N^*, D^*) = 135.1600$	$F_2(N^*, D^*) = 135.1631$
	$P_N(N_N^*) = 0.1150$	$P_D(D_D^*) = 0.1151$	$P_1(N^*, D^*) = 0.1152$	$P_2(N^*, D^*) = 0.1151$
	$N_N^* = 6$	$D_D^* = 7$	$(N^*, D^*) = (4,\ 7)$	$(N^*, D^*) = (7,\ 7)$
0.7	$F_N(N_N^*) = 155.1667$	$F_D(D_D^*) = 155.1190$	$F_1(N^*, D^*) = 155.1177$	$F_2(N^*, D^*) = 155.1167$
	$P_N(N_N^*) = 0.0746$	$P_D(D_D^*) = 0.0748$	$P_1(N^*, D^*) = 0.0748$	$P_2(N^*, D^*) = 0.0749$
	$N_N^* = 5$	$D_D^* = 6$	$(N^*, D^*) = (3,\ 6)$	$(N^*, D^*) = (7,\ 6)$
0.8	$F_N(N_N^*) = 175.4000$	$F_D(D_D^*) = 175.3125$	$F_1(N^*, D^*) = 175.3121$	$F_2(N^*, D^*) = 175.3125$
	$P_N(N_N^*) = 0.0415$	$P_D(D_D^*) = 0.0420$	$P_1(N^*, D^*) = 0.0420$	$P_2(N^*, D^*) = 0.0420$
	$N_N^* = 3$	$D_D^* = 5$	$(N^*, D^*) = (1,\ 5)$	$(N^*, D^*) = (5,\ 5)$
0.9	$F_N(N_N^*) = 197.0000$	$F_D(D_D^*) = 196.8464$	$F_1(N^*, D^*) = 196.8464$	$F_2(N^*, D^*) = 196.8442$
	$P_N(N_N^*) = 0.0150$	$P_D(D_D^*) = 0.0158$	$P_1(N^*, D^*) = 0.0158$	$P_2(N^*, D^*) = 0.0158$

TABLE 3. Optimal power-saving comparison of four policies based on mean number of data packets ($p = 0.75$).

ρ	N-policy	D-policy	max(N, D)-policy	min(N, D)-policy
	$N_N^* = 6$	$D_D^* = 5$	$(N^*, D^*) = (5,\ 5)$	$(N^*, D^*) = (6,\ 6)$
0.8	$F_N(N_N^*) = 175.5000$	$F_D(D_D^*) = 175.4931$	$F_1(N^*, D^*) = 175.4925$	$F_2(N^*, D^*) = 175.4923$
	$P_N(N_N^*) = 0.0665$	$P_D(D_D^*) = 0.0665$	$P_1(N^*, D^*) = 0.0665$	$P_2(N^*, D^*) = 0.0665$
	$N_N^* = 4$	$D_D^* = 4$	$(N^*, D^*) = (1,\ 4)$	$(N^*, D^*) = (5,\ 4)$
0.9	$F_N(N_N^*) = 195.5000$	$F_D(D_D^*) = 195.4423$	$F_1(N^*, D^*) = 195.4423$	$F_2(N^*, D^*) = 195.4423$
	$P_N(N_N^*) = 0.0262$	$P_D(D_D^*) = 0.0264$	$P_1(N^*, D^*) = 0.0264$	$P_2(N^*, D^*) = 0.0264$

When $C_h = C_u$ and other conditions are same, we know from Tables 1–6 that under each policy, the minimum power consumption based on the mean number of data packets is smaller than that based on the mean transmission time backlog. For example, in Tables 1 and 4, for $p = 0.25, \rho = 0.6$, we have $F_N(5) < f_N(3)$, $F_D(9) < f_D(5)$, $F_1(4, 9) < f_1(1, 5)$, and $F_2(6, 10) < f_2(6, 5)$. In Tables 2 and 5 or Tables 3 and 6, the same conclusion holds. Hence, under the same condition, we should select the power consumption function based on the mean number of data packets.

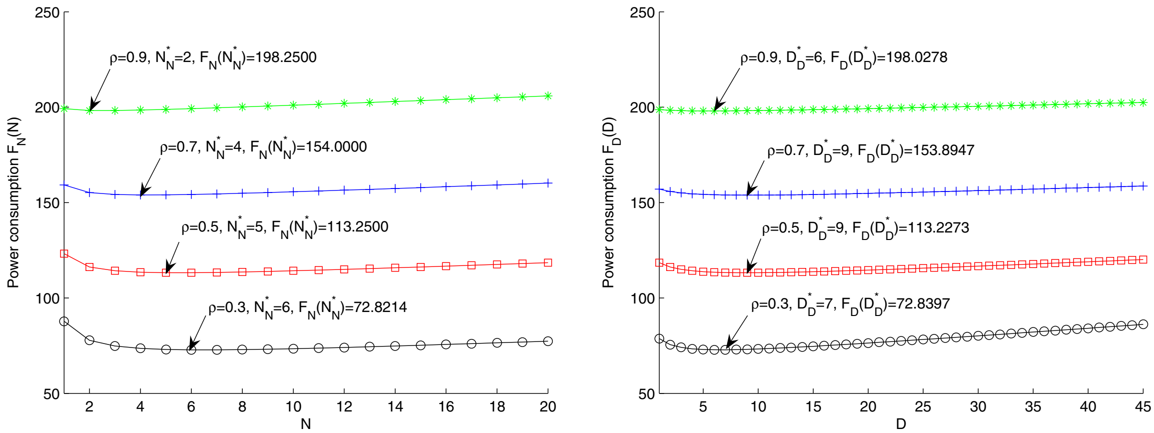

FIGURE 1. Power consumption based on mean number of data packets for $\rho = 0.3, 0.5, 0.7, 0.9$ and $p = 0.25$ under the N and D policies.

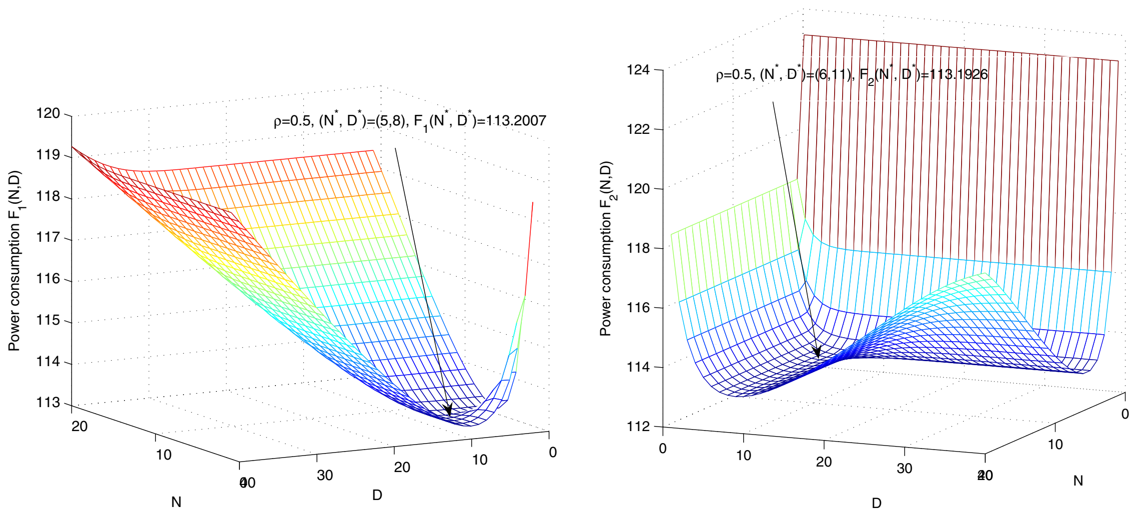

FIGURE 2. Power consumption based on mean number of data packets for $\rho = 0.5$, $p = 0.25$ under the $\max(N,\ D)$ and $\min(N,\ D)$ policies.

Remark 6.1. In numerical illustration and comparison, due to computational difficulties in convolution summation of transmission times, we only considered the case of geometrically distributed transmission time. In the case of generally distributed transmission time, we may need numerical approximations.

Remark 6.2. This paper mainly focuses on the discrete-time queueing analysis of the sensor node under the $\max(N,\ D)$ and $\min(N,\ D)$ policies. As a practical application, the power consumption optimization of a sensor node is numerically illustrated and compared. For power consumption optimization of the whole sensor network, we can first compute the mean arrival rate of each sensor node in each shell, and then use the formulas (6.2)–(6.3) (or (6.10)–(6.11)) to optimize the power consumption of each node. Finally, by the energy improvement in sensor nodes of the innermost shell, the power consumption optimization of the whole sensor network is realized. In this way, the lifetime of network is lengthened. For details one can refer to the work of Jiang *et al.* [18].

TABLE 4. Optimal power-saving comparison of four policies based on mean transmission time backlog of data packets ($p = 0.25$).

ρ	N-policy	D-policy	max(N, D)-policy	min(N, D)-policy
0.3	$N_N^* = 6$ $f_N(N_N^*) = 73.0857$ $p_N(N_N^*) = 0.1656$	$D_D^* = 6$ $f_D(D_D^*) = 73.0024$ $p_D(D_D^*) = 0.1665$	$(N^*, D^*) = (1,\ 6)$ $f_1(N^*, D^*) = 73.0024$ $p_1(N^*, D^*) = 0.1665$	$(N^*, D^*) = (7,\ 6)$ $f_2(N^*, D^*) = 73.0024$ $p_2(N^*, D^*) = 0.1665$
0.4	$N_N^* = 5$ $f_N(N_N^*) = 94.2000$ $p_N(N_N^*) = 0.1063$	$D_D^* = 6$ $f_D(D_D^*) = 93.9526$ $p_D(D_D^*) = 0.1086$	$(N^*, D^*) = (1,\ 6)$ $f_1(N^*, D^*) = 93.9526$ $p_1(N^*, D^*) = 0.1086$	$(N^*, D^*) = (7,\ 6)$ $f_2(N^*, D^*) = 93.9526$ $p_2(N^*, D^*) = 0.1086$
0.5	$N_N^* = 4$ $f_N(N_N^*) = 115.2500$ $p_N(N_N^*) = 0.0668$	$D_D^* = 6$ $f_D(D_D^*) = 114.8750$ $p_D(D_D^*) = 0.0698$	$(N^*, D^*) = (1,\ 6)$ $f_1(N^*, D^*) = 114.8750$ $p_1(N^*, D^*) = 0.0698$	$(N^*, D^*) = (7,\ 6)$ $f_2(N^*, D^*) = 114.8750$ $p_2(N^*, D^*) = 0.0698$
0.6	$N_N^* = 3$ $f_N(N_N^*) = 136.5000$ $p_N(N_N^*) = 0.0394$	$D_D^* = 5$ $f_D(D_D^*) = 136.0189$ $p_D(D_D^*) = 0.0428$	$(N^*, D^*) = (1,\ 5)$ $f_1(N^*, D^*) = 136.0189$ $p_1(N^*, D^*) = 0.0428$	$(N^*, D^*) = (6,\ 5)$ $f_2(N^*, D^*) = 136.0189$ $p_2(N^*, D^*) = 0.0428$
0.7	$N_N^* = 3$ $f_N(N_N^*) = 158.5000$ $p_N(N_N^*) = 0.0198$	$D_D^* = 5$ $f_D(D_D^*) = 157.8538$ $p_D(D_D^*) = 0.0238$	$(N^*, D^*) = (1,\ 5)$ $f_1(N^*, D^*) = 157.8538$ $p_1(N^*, D^*) = 0.0238$	$(N^*, D^*) = (6,\ 5)$ $f_2(N^*, D^*) = 157.8538$ $p_2(N^*, D^*) = 0.0238$
0.8	$N_N^* = 2$ $f_N(N_N^*) = 182.4000$ $p_N(N_N^*) = 0.0076$	$D_D^* = 4$ $f_D(D_D^*) = 181.8556$ $p_D(D_D^*) = 0.0106$	$(N^*, D^*) = (1,\ 4)$ $f_1(N^*, D^*) = 181.8556$ $p_1(N^*, D^*) = 0.0106$	$(N^*, D^*) = (5,\ 4)$ $f_2(N^*, D^*) = 181.8556$ $p_2(N^*, D^*) = 0.0106$
0.9	$N_N^* = 1$ $f_N(N_N^*) = 215.9000$ $p_N(N_N^*) = 0.0000$	$D_D^* = 2$ $f_D(D_D^*) = 215.3643$ $p_D(D_D^*) = 0.0025$	$(N^*, D^*) = (1,\ 2)$ $f_1(N^*, D^*) = 215.3643$ $p_1(N^*, D^*) = 0.0025$	$(N^*, D^*) = (3,\ 2)$ $f_2(N^*, D^*) = 215.3643$ $p_2(N^*, D^*) = 0.0025$

TABLE 5. Optimal power-saving comparison of four policies based on mean transmission time backlog of data packets ($p = 0.5$).

ρ	N-policy	D-policy	max(N, D)-policy	min(N, D)-policy
0.6	$N_N^* = 6$ $f_N(N_N^*) = 135.3000$ $p_N(N_N^*) = 0.1116$	$D_D^* = 6$ $f_D(D_D^*) = 135.2167$ $p_D(D_D^*) = 0.1122$	$(N^*, D^*) = (1,\ 6)$ $f_1(N^*, D^*) = 135.2167$ $p_1(N^*, D^*) = 0.1122$	$(N^*, D^*) = (7,\ 6)$ $f_2(N^*, D^*) = 135.2167$ $p_2(N^*, D^*) = 0.1122$
0.7	$N_N^* = 5$ $f_N(N_N^*) = 155.8333$ $p_N(N_N^*) = 0.0693$	$D_D^* = 6$ $f_D(D_D^*) = 155.6766$ $p_D(D_D^*) = 0.0702$	$(N^*, D^*) = (1,\ 6)$ $f_1(N^*, D^*) = 155.6766$ $p_1(N^*, D^*) = 0.0702$	$(N^*, D^*) = (7,\ 6)$ $f_2(N^*, D^*) = 155.6766$ $p_2(N^*, D^*) = 0.0702$
0.8	$N_N^* = 4$ $f_N(N_N^*) = 176.8000$ $p_N(N_N^*) = 0.0360$	$D_D^* = 5$ $f_D(D_D^*) = 176.5818$ $p_D(D_D^*) = 0.0372$	$(N^*, D^*) = (1,\ 5)$ $f_1(N^*, D^*) = 176.5818$ $p_1(N^*, D^*) = 0.0372$	$(N^*, D^*) = (6,\ 5)$ $f_2(N^*, D^*) = 176.5818$ $p_2(N^*, D^*) = 0.0372$
0.9	$N_N^* = 3$ $f_N(N_N^*) = 200.5000$ $p_N(N_N^*) = 0.0109$	$D_D^* = 3$ $f_D(D_D^*) = 200.2000$ $p_D(D_D^*) = 0.0123$	$(N^*, D^*) = (1,\ 3)$ $f_1(N^*, D^*) = 200.2000$ $p_1(N^*, D^*) = 0.0123$	$(N^*, D^*) = (4,\ 3)$ $f_2(N^*, D^*) = 200.2000$ $p_2(N^*, D^*) = 0.0123$

7. CONCLUSION

This paper deals with two power-saving policies of sensor node based on a Geo/G/1 queue. We obtain main queueing performance measures of sensor node, such as the mean number of data packets, the mean transmission time backlog, the mean waiting time, the mean busy cycle period, and so on. Two power consumption functions are constructed to show the power-saving effectiveness of two (N, D) policies. Various numerical experiments were provided to compare the superiority of the N, D, and two (N, D) policies at a minimum power consumption. Our numerical results indicate that: (1) if the power consumption function based on the mean number of data

TABLE 6. Optimal power-saving comparison of four policies based on mean transmission time backlog of data packets ($p = 0.75$).

ρ	N-policy	D-policy	max$(N,\ D)$-policy	min$(N,\ D)$-policy
	$N_N^* = 6$	$D_D^* = 5$	$(N^*, D^*) = (1,\ 5)$	$(N^*, D^*) = (6,\ 5)$
0.8	$f_N(N_N^*) = 174.9333$	$f_D(D_D^*) = 174.9040$	$f_1(N^*, D^*) = 174.9040$	$f_2(N^*, D^*) = 174.9040$
	$p_N(N_N^*) = 0.0659$	$p_D(D_D^*) = 0.0660$	$p_1(N^*, D^*) = 0.0660$	$p_2(N^*, D^*) = 0.0660$
	$N_N^* = 4$	$D_D^* = 4$	$(N^*, D^*) = (1,\ 4)$	$(N^*, D^*) = (5,\ 4)$
0.9	$f_N(N_N^*) = 195.3500$	$f_D(D_D^*) = 195.3000$	$f_1(N^*, D^*) = 195.3000$	$f_2(N^*, D^*) = 195.3000$
	$p_N(N_N^*) = 0.0247$	$p_D(D_D^*) = 0.0250$	$p_1(N^*, D^*) = 0.0250$	$p_2(N^*, D^*) = 0.0250$

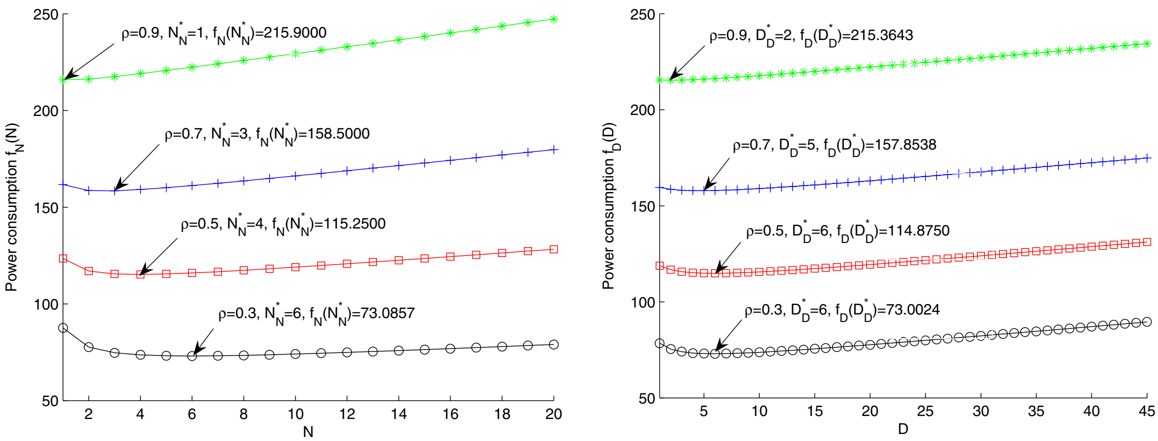

FIGURE 3. Power consumption based on mean transmission time backlog of data packets for $\rho = 0.3, 0.5, 0.7, 0.9$ and $p = 0.25$ under the N and D policies.

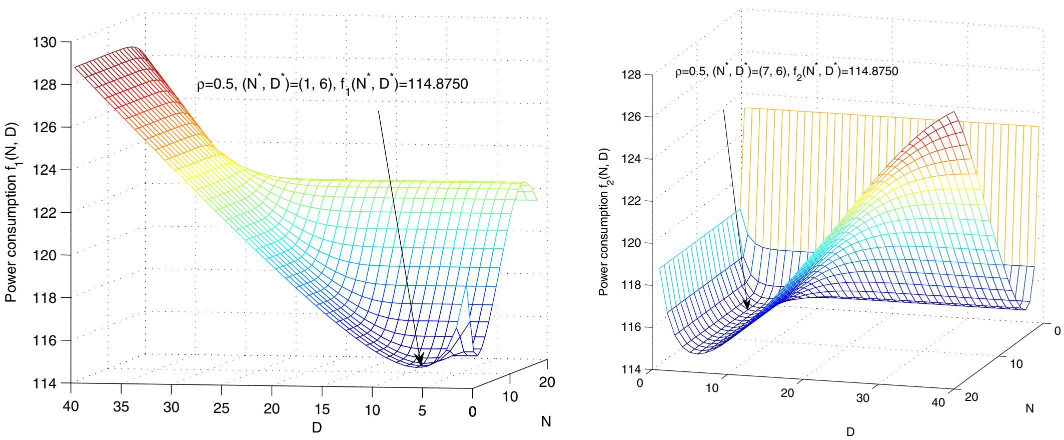

FIGURE 4. Power consumption based on mean transmission time backlog of data packets for $\rho = 0.5$ and $p = 0.25$ under the max$(N,\ D)$ and min$(N,\ D)$ policies.

packets is utilized, then two $(N,\ D)$ policies are better than the N and D policies. None of two $(N,\ D)$ policies is superior over the other, which depends on the arrival rate and traffic intensity; (2) if the power consumption function based on the mean transmission time backlog is used, then two optimal (N, D) policies and optimal D-policy have a same minimum power consumption; (3) under the same condition, the power consumption function based on the mean number of data packets is better than that based on the mean transmission time backlog. We expect that these results are helpful for the network engineers to make reasonable decisions in power-saving control. As a future work, the numerical approximation on convolution summation of generally distributed transmission time, and the power consumption optimization from single sensor node to whole sensor network, will be interesting and important.

Appendix A. Derivation of equation (3.6)

Above all, in the derivation of equation (3.6), the following relation equation will be used:

$$s^{(n+k)}(m) = \sum_{x=k}^{m-n} s^{(k)}(x)s^{(n)}(m-x), m \geq n+k; n, k = 1, 2, \dots,$$

whose proof is easy.

In fact, by total probability formula, we have

$$\begin{aligned} s^{(n+k)}(m) = \Pr\{S_1 + S_2 + \ldots + S_{n+k} = m\} &= \sum_{x=k}^{m-n} \Pr\left\{\sum_{i=1}^{k} S_i = x\right\} \Pr\left\{\sum_{i=1}^{n+k} S_i = m \mid \sum_{i=1}^{k} S_i = x\right\} \\ &= \sum_{x=k}^{m-n} \Pr\left\{\sum_{i=1}^{k} S_i = x\right\} \Pr\left\{\sum_{i=k+1}^{k+n} S_i = m-x\right\} \\ &= \sum_{x=k}^{m-n} s^{(k)}(x)s^{(n)}(m-x). \end{aligned}$$

Now, employing the above relation equation and noting that $s^{(1)}(x-n) = s_{x-n}$ and $\sum_{n=D+1}^{D} = 0$ (see the notations in p. 3), we have

$$\begin{aligned} \sum_{k=N}^{D} \sum_{x=D+1}^{\infty} x \sum_{n=k}^{D} s^{(k)}(n)s_{x-n} &= \sum_{k=N}^{D} \sum_{x=D+1}^{\infty} x \left[\sum_{n=k}^{x-1} s^{(k)}(n)s_{x-n} - \sum_{n=D+1}^{x-1} s^{(k)}(n)s_{x-n}\right] \\ &= \sum_{k=N}^{D} \sum_{x=D+1}^{\infty} xs^{(k+1)}(x) - \sum_{k=N}^{D} \sum_{n=D+1}^{\infty} s^{(k)}(n) \sum_{x=n+1}^{\infty} [ns_{x-n} + (x-n)s_{x-n}] \\ &= \sum_{k=N}^{D} \sum_{x=D+1}^{\infty} xs^{(k+1)}(x) - \sum_{k=N}^{D} \sum_{n=D+1}^{\infty} ns^{(k)}(n) - E(S) \sum_{k=N}^{D} \sum_{n=D+1}^{\infty} s^{(k)}(n) \\ &= \sum_{k=N+1}^{D+1} \sum_{x=D+1}^{\infty} xs^{(k)}(x) - \sum_{k=N}^{D} \sum_{n=D+1}^{\infty} ns^{(k)}(n) - E(S) \sum_{k=N}^{D} \sum_{n=D+1}^{\infty} s^{(k)}(n) \\ &= \sum_{x=D+1}^{\infty} xs^{(D+1)}(x) - \sum_{n=D+1}^{\infty} ns^{(N)}(n) - E(S) \sum_{k=N}^{D} \sum_{n=D+1}^{\infty} s^{(k)}(n), \end{aligned}$$

where $E(S)$ is the mean transmission time(see queue assumptions in Sect. 2).

Appendix B. Derivation of equation (3.7)

From the relation equation between $s^{(k)}(n)$ and $S^{(k)}(n)$: $s^{(k)}(n) = S^{(k)}(n) - S^{(k)}(n-1), n \geq k+1$, for $N \leq k \leq D$, we have

$$\begin{aligned}
\sum_{n=D+1}^{\infty} s^{(k)}(n) &= \sum_{n=D+1}^{\infty} \left[S^{(k)}(n) - S^{(k)}(n-1)\right] \\
&= \lim_{m\to\infty} \sum_{n=D+1}^{m} \left[S^{(k)}(n) - S^{(k)}(n-1)\right] \\
&= \lim_{m\to\infty} \left[\sum_{n=D+1}^{m} S^{(k)}(n) - \sum_{n=D}^{m-1} S^{(k)}(n)\right] \\
&= \lim_{m\to\infty} \left[S^{(k)}(m) - S^{(k)}(D)\right] \\
&= 1 - S^{(k)}(D),
\end{aligned}$$

Using (3.6) in (3.5) and noting that $\sum_{n=D+1}^{\infty} s^{(k)}(n) = 1 - S^{(k)}(D)$, $N \leq k \leq D$, we get

$$\begin{aligned}
E(\Phi_{(N,\ D)}) &= \sum_{x=D+1}^{\infty} x s^{(D+1)}(x) - E(S) \sum_{k=N}^{D} \sum_{n=D+1}^{\infty} s^{(k)}(n) \\
&= \sum_{x=D+1}^{\infty} x \Pr\{S_1 + S_2 + \ldots + S_{D+1} = x\} - E(S) \sum_{k=N}^{D} \left[1 - S^{(k)}(D)\right] \\
&= (D+1)E(S) - E(S) \left[D - N + 1 - \sum_{k=N}^{D} S^{(k)}(D)\right] \\
&= E(S) \left[N + \sum_{k=N}^{D} S^{(k)}(D)\right].
\end{aligned}$$

Appendix C. Derivations of equations (3.20) and (3.21)

By the following two relation equations:

$$\sum_{x=k}^{D} s^{(k)}(x) = S^{(k)}(D), 1 \leq k \leq N-1, \sum_{x=D+1}^{\infty} s^{(k)}(x) = 1 - S^{(k)}(D), 1 \leq k \leq N-1,$$

we have

$$\begin{aligned}
&\sum_{k=1}^{N-1} \sum_{x=k}^{D} s^{(k)}(x)(N-k) + \sum_{k=1}^{N-1} \sum_{x=D+1}^{\infty} (N-k) s^{(k)}(x) \\
&= \sum_{k=1}^{N-1} (N-k) S^{(k)}(D) + \sum_{k=1}^{N-1} (N-k)[1 - S^{(k)}(D)] \\
&= \frac{N(N-1)}{2},
\end{aligned}$$

which gives equation (3.20). In order to derive equation (3.21), we need to use the following relation equation:

$$S^{(n+k)}(D) = \sum_{x=k}^{D-n} s^{(k)}(x) S^{(n)}(D-x),$$

whose proof is easy.

In fact, from the relation equation

$$s^{(n+k)}(m) = \sum_{x=k}^{m-n} s^{(k)}(x) s^{(n)}(m-x), m \geq n+k; n, k = 1, 2, \ldots,$$

which is proved in the derivation of (3.6) in Appendix A, we get

$$\begin{aligned}
S^{(n+k)}(D) &= \sum_{m=n+k}^{D} s^{(n+k)}(m) = \sum_{m=n+k}^{D} \sum_{x=k}^{m-n} s^{(k)}(x) s^{(n)}(m-x) \\
&= \sum_{x=k}^{D-n} s^{(k)}(x) \sum_{m=n+x}^{D} s^{(n)}(m-x) = \sum_{x=k}^{D-n} s^{(k)}(x) \sum_{i=n}^{D-x} s^{(n)}(i) \\
&= \sum_{x=k}^{D-n} s^{(k)}(x) S^{(n)}(D-x).
\end{aligned}$$

Now, utilizing the above equation and noting that $\sum_{n=1}^{0} = 0$ (see the notations in p. 3), we obtain

$$\begin{aligned}
&\sum_{k=1}^{N-1} \sum_{x=k}^{D} s^{(k)}(x) \sum_{n=N-k}^{D-x} S^{(n)}(D-x) + \sum_{k=N}^{D} \sum_{x=k}^{D} s^{(k)}(x) \sum_{n=1}^{D-x} S^{(n)}(D-x) + \sum_{k=N}^{D} \sum_{x=k}^{D} s^{(k)}(x) \\
&= \sum_{k=1}^{N-1} \sum_{n=N-k}^{D-k} \sum_{x=k}^{D-n} s^{(k)}(x) S^{(n)}(D-x) + \sum_{k=N}^{D} \sum_{n=1}^{D-k} \sum_{x=k}^{D-n} s^{(k)}(x) S^{(n)}(D-x) + \sum_{k=N}^{D} S^{(k)}(D) \\
&= \sum_{k=1}^{N-1} \sum_{n=N-k}^{D-k} S^{(n+k)}(D) + \sum_{k=N}^{D} \sum_{n=1}^{D-k} S^{(n+k)}(D) + \sum_{k=N}^{D} S^{(k)}(D) \\
&= \sum_{k=1}^{N-1} \left[S^{(N)}(D) + S^{(N+1)}(D) + S^{(N+2)}(D) + \ldots + S^{(D)}(D) \right] \\
&\quad + \sum_{k=N}^{D-1} \left[S^{(k+1)}(D) + S^{(k+2)}(D) + \ldots + S^{(D)}(D) \right] + \sum_{k=N}^{D} S^{(k)}(D) \\
&= (N-1) \left[S^{(N)}(D) + S^{(N+1)}(D) + S^{(N+2)}(D) + \ldots + S^{(D)}(D) \right] \\
&\quad + \left[S^{(N+1)}(D) + 2S^{(N+2)}(D) + 3S^{(N+3)}(D) + \ldots + (D-N) S^{(D)}(D) \right] + \sum_{k=N}^{D} S^{(k)}(D) \\
&= \sum_{k=N}^{D} (k-1) S^{(k)}(D) + \sum_{k=N}^{D} S^{(k)}(D) = \sum_{k=N}^{D} k S^{(k)}(D),
\end{aligned}$$

which yields equation (3.21).

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