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# Distributed resource allocation for multi-zone commercial buildings

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## Abstract

Advanced building structures are extremely complicated with a large number of dynamically interacting components and widely equipped with multi-zone air conditioning (ME A/C) systems. In this paper, multi-zone commercial building systems are built as second order Resistance-Capacitance (RC) models with individual zones communicating to the whole network through neighbors. The presented thermal dynamics are different from that in our previous work [8], which does not consider the wall's thermal dynamics. The second order RC models are more accurate in capturing the thermal dynamics of a building in comparison with the first order model employed in [8]. We first propose a hierarchical distributed method using ME A/C systems in commercial building for demand response. The idea of the upper layer strategy is based on the balance of building's energy demand and users' comfort level to generate reference signals. The lower layer is designed as distributed model predictive controllers (DMPC) to track the reference signals whereas energy cost and demand are reduced. Simulation results are provided to verify the effectiveness of the proposed control.

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*Keywords:* ME A/C system, distributed model predictive control, energy efficiency, multi-zone commercial buildings;

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## 1. Introduction

Commercial buildings consumed  $19 \times 10^{18}$  J of primary energy in 2009, which represents 46% of total building energy consumption in primary energy. These values suggest that commercial buildings have become one of the main targets for control approaches designed to lower energy usage or improve energy efficiency and comfort levels. In recent years, most commercial buildings commonly use simple ON/OFF and PID controllers for controlling their air conditioning (A/C) systems. In [1], the results demonstrated that model predictive control (MPC) is better than

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other control methods. An MPC is capable of shifting peak demands from peak to off-peak hours [2]. Other advantages of an MPC algorithm for building HVAC systems include robustness, tunability and flexibility [3].

Despite the abundance of study in DX A/C system control, economic and optimal control system have not widely been addressed in industry. An obstacle to the successful use of MPC is the dimension of the building A/C systems. Commercial buildings including multiple connected zones and rooms are widely equipped with a large number of A/C systems. A centralized control approach is impractical and undesirable.

Distributed MPC can be used so that each zone controller optimizes its own and neighbor variables to achieve control objective. Nowadays, most research has focused on distributed optimization problems such as distributed flexibility characterization [4], distributed economic MPC [5]. In [4], although the authors built second order resistance-capacitance (RC) models which can be used to control multi-zones temperature, the authors treated the temperatures of its neighbors as exogenous variables. In [5], despite considering the interaction between rooms, the authors only considered building thermal dynamics as first order RC models, which cannot fully reflect the building dynamics.

Motivated by the above issues, the multi-zone building's thermal dynamics can be built by second order RC models and new hierarchical distributed dynamic controller designed to control multiple zones' thermal comfort and indoor air quality (IAQ) at comfort levels. Energy efficiency and energy cost savings are also achieved in this paper.

Nomenclature			
$A_1^i$	heat transfer area in the wet-cooling region of the DX evaporator $i$ , $m^2$	$T_d^i$	air temperature leaving the dry-cooling region on air side of the DX evaporator $i$ , $^{\circ}C$
$A_2^i$	heat transfer area in the wet-cooling region of the DX evaporator $i$ , $m^2$	$T_{mix}$	mixing air temperature between the outside air and return air, $^{\circ}C$
$A_{win}$	represents the total window area, $m^2$	$T_z^i$	zone $i$ 's indoor air temperature, $^{\circ}C$
$C^i$	zone $i$ 's thermal capacitance, $J / ^{\circ}C$	$T^{(i,j)}$	inside temperature of the wall that separates zone $i$ and $j$ , $^{\circ}C$
$C^{(i,j)}$	thermal capacitance of the wall that separates zone $i$ and zone $j$ , $^{\circ}C / kW$	$T_w^i$	air temperature of the DX evaporator wall, $^{\circ}C$
$C_a$	specific heat of air, $kJ kg^{-1} ^{\circ}C^{-1}$	$T^0$	outside air temperature, $^{\circ}C$
$C_c^i$	$CO_2$ concentration in the conditioning room $i$ , ppm	$T_s^i$	zone $i$ 's supply air temperature, $^{\circ}C$
$C_l^i$	pollutant load of room $i$ , $m^3 / s$ ,	$V^i$	volume of the conditioned room $i$ , $m^3$
$C_s$	$CO_2$ concentration of air supply, ppm	$V_{h1}^i$	air side volume in the dry-cooling region on air side of the DX evaporator $i$ , $m^3$
$G_i$	amount of $CO_2$ emission rate by people of room $i$ , $m^3 / s$	$V_{h2}^i$	air side volume in the wet-cooling region on air side of the DX evaporator $i$ , $m^3$
$h_{fg}$	latent heat of vaporization of water, $kJ / kg$	$v_f^i$	zone $i$ 's supply air flow, $m^3 / s$
$h_{r1}^i$	enthalpy of refrigerant at inlet of evaporator $i$ , $kJ / kg$	$W_{mix}$	mixing moisture content of outside air and return air, $kg / kg$
$h_{r2}^i$	enthalpy of refrigerant at outlet of evaporator $i$ , $kJ / kg$	$W_s^i$	moisture content of air leaving the DX evaporator $i$ , $kg / kg$
$k_p, k_I$	proportional and integral coefficients,	$W_z^i$	moisture content of air conditioned room $i$ , $kg / kg$
$M_l^i$	moisture load in the conditioned room $i$ , $kg/s$	$W_0$	moisture content of the outdoor air, $kg / kg$

$Occp_i$	number of occupants at room $i$ ,	$\alpha_{dc}^i$	heat transfer coefficient between air and the evaporator wall in the dry-cooling region $i$ , $\text{kW m}^{-2}\text{°C}^{-1}$
$Q_l^i$	sensible heat load in the conditioned room $i$ , kW	$\alpha_{wc}^i$	heat transfer coefficient between air and the evaporator wall in the wet-cooling region $i$ , $\text{kW m}^{-2}\text{°C}^{-1}$
$R^i$	thermal resistance of the wall that separates zone $i$ and outside, $\text{°C / kW}$	$\rho$	density of moist air, $\text{kg / m}^3$
$R^{(i,j)}$	thermal resistance of the wall that separates zone $i$ and zone $j$ , $\text{°C / kW}$		

## 2. Multi-zone commercial building model

### 2.1. Description of a building ME A/C system

The schematic of a typical ME A/C system is illustrated in Fig. 1, which makes up Indoor Air Unit (IAU), Variable Air Volume (VAV) boxes. The IAU is equipped with dampers, the evaporator cooling/heating coils, and a Variable Frequency Drive (VFD) fan: the dampers mix the return air from the building with the outside air; the cooling/heating coil cools down or heat up the mixed air; the VFD regulates its own speed based on the total air flow rate which can be controlled by VAV boxes. Each VAV box has a damper to control the air flow rate supplied to the zone.

### 2.2. Dynamic model of the building ME A/C system

The thermal dynamic model of a  $m$  zone can be built by a connected network of second order Resistance-Capacitance models of individual zones [1]. The thermal dynamics of the  $i^{th}$  zone ( $i = 1, 2, \dots, m$ ) can be expressed in [8] and the energy balance of the wall  $i$  is described by:

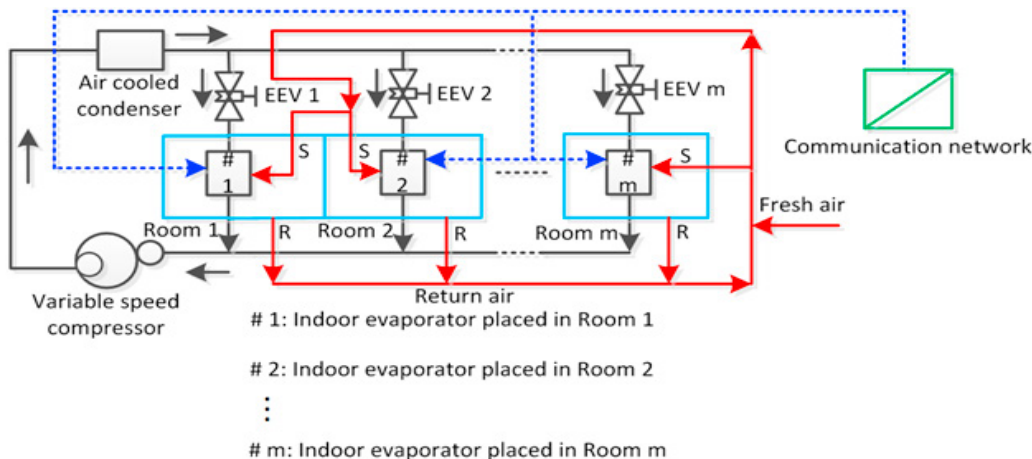


Figure 1. Multi-zone building ME A/C system

$$C^{(i,j)} \frac{dT_w^{(i,j)}}{dt} = \frac{T_z^i - T_w^{(i,j)}}{R^{(i,j)}} + \frac{T_z^j - T_w^{(i,j)}}{R^{(i,j)}}. \tag{1}$$

The details of the physical equations are addressed in [7], [9].

To make the ME A/C system cooperatively controlling multi-zones' thermal comfort and air quality, we suppose that the ME A/C system is equipped with a communication network based on wireless sensors. In this network, they can share information (e.g.,  $T_z^i$ ,  $W_z^i$  and  $C_c^i$ ) with each other, as shown in Fig. 1. Model the information flow between them as a network graph  $G=(V, \mathcal{G}, A)$ , where  $V = \{1, 2, \dots, m\}$  is the index set of different rooms and zones of the ME A/C system,  $\mathcal{G} \subset V \times V$  is the edge set of ordered pairs of the ME A/C system, and  $A = [a_{ij}] \in \mathbb{R}^{m \times m}$  is the adjacency matrix with entries  $a_{ij} = 1$  or  $a_{ij} = 0$ .

The system dynamic equations can be written as equations of the following:

$$\dot{x}^i = f_i(x^i, x^{-i}, u^i, w^i), \quad i = 1, 2, \dots, m, \tag{2}$$

where  $x^i \in [h_s^i, T_z^i, T_d^i, T_w^i, W_z^i, C_c^i, T_w^{(i,j)}]^T$  denote is the state of the subsystem  $S_i$ ;  $u^i = [v_f^i, m_r^i]^T$  are the constrained control signals;  $w^i \in [Q_l^i, M_l^i, C_c^i]^T$  represent the load variables of room  $i$ ;  $x^{-i}$  concatenates the states of all subsystems  $S_j$  ( $j \in V$ ) of the subsystem  $S_i$ , i.e.,  $x^{-i} = (x^1, \dots, x^{i-1}, x^{i+1}, \dots, x^m)$ .

In this paper, the energy consumption of the multi-zone commercial building ME A/C system includes the compressor, the condenser and the supply fans. These components of energy models have been built in [9].

### 3. Hierarchical control design for commercial buildings

This section designs two distributed controllers to achieve energy efficiency improvement, demand reduction, as well as comfort levels.

#### 3.1. High level controller

The high level controller is used an open loop optimal controller to generate tradeoff reference signals for low level controllers by minimizing the demand and energy costs of ME A/C system under a TOU rate structure while meeting the requirements of each room's thermal comfort and IAQ. Due to space limitations, the centralized controller is omitted here. Before studying the distributed open loop controller, we make the following assumption for the system model.

Under steady state, the total heat gain from the wall is sometimes less dominant compared with the heat gain from the outside plus the indoor heat gain in every zone. Therefore, we can ignore the interacting term

$\sum_{j=1}^m \frac{T_w^{(i,j)} - T_z^i}{R^{(i,j)}}$  in the equation. Then, a simplified optimization problem is derived for one zone  $i$  as follows:

$$x_i^r(t_k^u) = \arg \min_{x_i^r(t_k^u), u_i^r(t_k^u)} \left( \underbrace{\sum_{k=1}^n w_1 (E_c(t_k^u) P_{tot,i}(t_k^u) \delta^u)}_{\text{energy cost}} + \underbrace{w_2 (D_c(t_k^u) \max_{1 \leq k \leq n} \{P_{tot,i}(t_k^u)\})}_{\text{demand cost}} \right), \tag{3a}$$

subject to the following constraints:

$$\tilde{f}_i(x^i(t_k^u), u^i(t_k^u), w^i(t_k^u)) = 0, \tag{3b}$$

$$|PMV_i(t_k^u)| \leq \alpha, \tag{3c}$$

$$x^i(t_k^u) \in X_i, \quad u^i(t_k^u) \in U_i, \quad h_1^i(x^i(t_k^u), u^i(t_k^u)) \leq 0, \quad h_2^i(x^i(t_k^u)) \leq 0, \quad t_k^u \in [0, K^u], \tag{3d}$$

where  $x_i^r(t_k^u)$  is a local optimal solution at time  $t_k^u$ ,  $\tilde{f}_i(x_i, u_i, w_i)$  is described by removing the interacting term.

### 3.2. Low level controller

The low level controller is designed as the closed-loop distributed controllers to steer ME A/C system to maintain each room's air temperature, humidity and CO2 concentration at their references calculated by the high level.

The high level transmits the reference signals to the low level controllers. Then the low level controllers are designed as closed-loop distributed MPC controllers to steer the ME A/C system reaching the reference signals. The linearized dynamic subsystem  $S_i$  for the nonlinear systems (2) at sampling time instant  $t_{c(k,q)}^l$  can be written as follows:

$$\begin{cases} \delta \dot{z}_1^i(s) = A_{ii}(t_{c(k,q)}^l) \delta z_1^i(s) + B_i(t_{c(k,q)}^l) \delta u_i(s), \\ y_1^i(s) = C_{ii} \delta z_1^i(s; t_{c(k,q)}^l) + y_i^0(s), \\ \dot{z}_2^i(s) = z_3^i(s), \\ \dot{z}_3^i(s) = w_i(s), i = 1, 2, \dots, m, \end{cases} \quad (4)$$

where the state variables  $z_1^i = [h_s^i, T_d^i, T_w^i, W_z^i, C_c^i]^T$ ,  $z_2^i = [T_z^i]$ ,  $z_3^i = [T_z^i]$ ,  $w_i$  is the virtual control variable and  $y_1^i = [W_z^i, C_c^i]^T$ ,  $y_2^i(s) = z_2^i(s)$  are the system outputs. The control variables can be designed as follows:

$$\begin{cases} z_4^i(s) = v_f^i(s), i = 1, 2, \dots, m, \\ z_4^i(s) = G(x^i(s), x^j(s), z_4^i(s), w_i(s)), s \in [t_{c(k,q)}^l, t_{c(k,q)}^l + T^l], j \in V_i, \end{cases} \quad (5)$$

where the dynamic feedback controllers in (5) so obtained is distributed. Therefore, to implement distributed control, the original nonlinear system can be converted to linearized systems (4). In this paper, we are using our previous results in [9] in the MPC design, then based on (4), the proposed optimization objective is given by:

$$\begin{aligned} \min_{u^i} \bar{J}_{i1}^l = & \int_{t_{c(k,q)}^l}^{t_{c(k,q)}^l + T^l} (\|\hat{y}_1^i(s; t_{c(k,q)}^l) - y_{1,r}^i(s; t_k^u)\|_{Q_{1i}}^2 + \|\delta \hat{u}^i(s; t_{c(k,q)}^l)\|_{R_{1i}}^2) ds \\ & + \|\hat{y}_1^i(t_{c(k,q)}^l + T^l; t_{c(k,q)}^l) - y_{1,r}^i(t_{c(k,q)}^l + T^l)\|_{P_{1i}}^2, i = 1, 2, \dots, m, \end{aligned} \quad (6)$$

and based on (4) and (5), the distributed MPC controllers with the designed objective function is given by:

$$\begin{aligned} \min_{z_4^i} \bar{J}_{i2}^l = & \int_{t_{c(k,q)}^l}^{t_{c(k,q)}^l + T^l} (\|\hat{y}_2^i(s; t_{c(k,q)}^l) - y_{2,r}^i(s; t_k^u)\|_{Q_{2i}}^2 + \|\hat{z}_4^i(s; t_{c(k,q)}^l)\|_{R_{2i}}^2) ds \\ & + \|\hat{y}_2^i(t_{c(k,q)}^l + T^l; t_{c(k,q)}^l) - y_{2,r}^i(t_{c(k,q)}^l + T^l)\|_{P_{2i}}^2, i = 1, 2, \dots, m, j \in V_i, \end{aligned} \quad (7)$$

where  $\delta \hat{u}^i(s; t_{c(k,q)}^l)$  is the deviation of the predicted input variable of the system (4) at time  $t_{c(k,q)}^l$ ,  $\hat{w}_i(s; t_{c(k,q)}^l)$  is the predicted control variable of the systems (5) at time  $t_{c(k,q)}^l$ ,  $\hat{y}_1^i(s; t_{c(k,q)}^l)$  and  $\hat{y}_2^i(s; t_{c(k,q)}^l)$  are the predicted output variable;  $y_{1,r}^i(s; t_k^u)$  and  $y_{2,r}^i(s; t_k^u)$  are the output references. Due to space limitations, the proposed hierarchical distributed control algorithm is omitted here.

Table 1. Comparisons of different strategies.

Strategy	Energy (kWh)	Energy cost (\$)
Proposed method	82.54	6.13
Baseline	84.32	7.24

## 4. Simulation study

A simulation study was performed for a six-zone commercial building. The weather and electricity pricing data, the system parameters and the coefficients of the energy models in this simulation are taken from [9]. In our previous work [9], the simulation results demonstrated that the high-level steady state distributed controller is effective. Figure 2 shows the tracking results of the low-level distributed controller. We observe that each room's air temperature, relative humidity and CO<sub>2</sub> concentration, by using the proposed control strategy, are tracking and maintaining at their desired reference points. It implies that the designed dynamic feedback controllers are effective to control the commercial building. It can be seen from Figure 2 that the reference points of air temperature, relative humidity and CO<sub>2</sub> concentration of each room with the proposed controllers are tallish during standard and peak hours. The reason is that during standard and peak hours, the proposed controllers automatically adjust their reference points upward according to the energy price policy and DR action respectively, such that the energy cost and energy consumption are minimized while both thermal comfort and IAQ are still maintained at comfortable ranges.

In addition to showing the performance of the proposed control strategy in terms of energy efficiency, the proposed energy minimization strategy compared with a baseline strategy is outlined in this section. The baseline strategy is designed as below. In the upper level, the peak demand charge is not included in the distributed optimization objective function. The lower level is designed as the closed-loop distributed controllers with the same optimization objectives as in (6) and (7). Table 1 summarizes the energy consumption and cost for both control strategies. It can be observed that using the proposed control strategy the commercial building can save more energy consumption and cost compared to the baseline strategy. Therefore, the performance of the proposed control strategy is superior as compared to the baseline method.

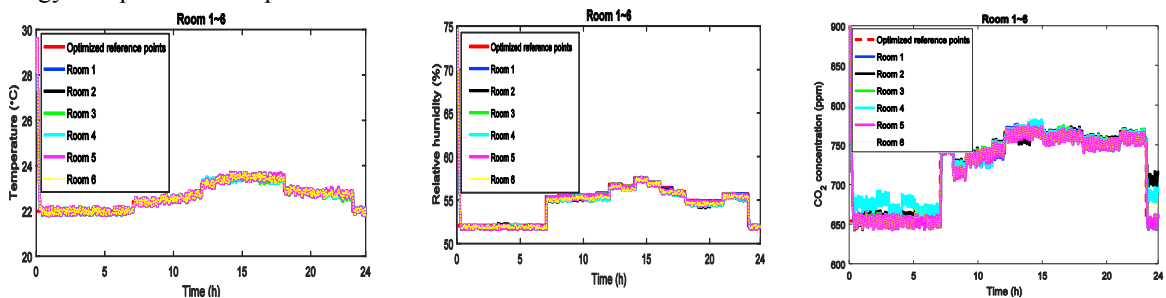


Figure 2. Tracking response of zone's air temperature, humidity and CO<sub>2</sub> concentration.

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