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Fixed Point Results for Multivalued Prešić Type Weakly Contractive Mappings

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Abstract: We present fixed points results of multivalued Prešić type k-step iterative mappings satisfying generalized weakly contraction conditions in metric spaces. An example is presented to support the main result proved herein. The stability of fixed point sets of multivalued Prešić type weakly contractive mappings are also established. Global attractivity result for the class of matrix difference equations is derived as application of the result presented herein. These results generalize and extend various comparable results in the existing literature.

Keywords: fixed point; multivalued mapping; Prešić type weakly contraction; stability of fixed point set; global attractivity

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1. Introduction and Preliminaries

A Banach contraction principle [1] is the simplest and useful tool having a variety of scientific applications such as proving the existence of the solution of linear, nonlinear, differential, integral, and difference equations. Several extensions and generalizations of Banach contraction principle are present in the current literature. Kannan [2] established fixed point theorem on certain type of contraction mappings that are independent of the Banach contraction principle. Boyd and Wong [3] extended the Banach contraction and derived a ϕ-generalized contraction mapping in metric spaces. Rhoades [4] obtained fixed point results for weakly contractive maps. Dutta and Choudhury [5] established (ψ, ϕ)-contraction mappings and proved fixed point results of these mappings. Choudhury et al. [6] proved fixed points of multivalued α- admissible mappings and stability of fixed point sets in metric spaces. Latif and Beg obtined [7] the geometric fixed points for single and multivalued mappings.

We begin with Banach contraction principle, its extensions, some definitions and results that are used in the sequel.

Theorem 1. *Let (X, d) be a complete metric space and $f : X \to X$. If for any $x, y \in X$, the following condition holds:*

$$d(fx, fy) \leq \alpha\, d(x, y),$$

where a constant $\alpha \in [0, 1)$. Then, there exists at most one point $x^* \in X$ having $x^* = f(x^*)$. In addition, for any $x_0 \in X$, given iterative sequence $x_{n+1} = f(x_n)$ converges to x^*.

Definition 1. *In a given metric space (X, d), self-mapping $f : X \to X$ is called weakly contractive if for any $x, y \in X$, we have*

$$d(fx, fy) \leq d(x, y) - \varphi(d(x, y)),$$

where a continuous and non-decreasing function $\varphi : [0, \infty) \to [0, \infty)$ is satisfying $\varphi(0) = 0$ and $\lim_{t\to\infty} \varphi(t) = \infty$.

For a positive integer $k \geq 1$, consider a mapping $f : X^k \to X$. An element $x^* \in X$ is called a fixed point of f if $x^* = f(x^*, x^*, \ldots, x^*)$.

The kth order nonlinear difference equation is given by

$$z_{n+k} = f(z_n, x_{n+1}, \ldots, z_{n+k-1}), \; n = 0, 1, 2, \ldots \tag{1}$$

with the initial values $z_1, z_2, \ldots, z_k \in X$.

The problem related to Equation (1) becomes the problem of fixed point theory in the sense that an element x^* in X solves Equation (1) if and only if x^* is the fixed point of mapping $S : X \to X$ defined as

$$S(x) = f(x, x, \ldots, x), \text{ for all } x \in X.$$

Prešić [8] obtained very useful result in this direction given as follows:

Theorem 2 ([8])**.** *Let (X, d) be a complete metric space. If a mapping $f : X^k \to X$, for a positive integer k satisfies that*

$$d(f(z_1, z_2, \ldots, z_k), f(z_2, \ldots, z_k, z_{k+1})) \leq r_1 d(z_1, z_2) + r_2 d(z_2, z_3) + \ldots + r_k d(z_k, z_{k+1}),$$

for all $z_1, z_2, \ldots, z_{k+1} \in X$, where constants $r_1, r_2, \ldots, r_k \geq 0$ such that $r_1 + r_2 + \ldots + r_k < 1$. Then, there exists at most one point $z^* \in X$ such that $f(z^*, z^*, \ldots, z^*) = z^*$. Moreover, for any arbitrary points $z_1, z_2, \ldots, z_k \in X$, the sequence $\{z_{n+k}\}$ given in Equation (1) converges to z^*.

Observe that, by taking $k = 1$, Theorem 2 becomes the Banach contraction principle.

Ćirić and Prešić [9] extended Theorem 2 in the following way.

Theorem 3 ([9])**.** *Let (X, d) be a complete metric space. If a mapping $f : X^k \to X$, for a positive integer k satisfies that*

$$d(f(z_1, z_2, \ldots, z_k), f(z_2, \ldots, z_k, z_{k+1})) \leq r \max\{d(z_1, z_2), d(z_2, z_3), \ldots, d(z_k, z_{k+1})\},$$

for all $z_1, z_2, \ldots, z_{k+1} \in X$, where $0 \leq r < 1$. Then, there exists a point $z^* \in X$ such that $f(z^*, z^*, \ldots, z^*) = z^*$. Moreover, for any arbitrary points $z_1, z_2, \ldots, z_k \in X$, the sequence $\{z_{n+k}\}$ given in Equation (1) converges to z^* and

$$\lim_{n\to\infty} z_n = f(\lim_{n\to\infty} z_n, \lim_{n\to\infty} z_n, \ldots, \lim_{n\to\infty} z_n).$$

If, in addition,

$$d(f(z_1, z_1, \ldots, z_1), f(z_2, z_2 \ldots, z_2)) < d(z_1, z_2)$$

holds for all $z_1, z_2 \in X$, with $z_1 \neq z_2$, then z^* is the unique point in X with $f(z^*, z^*, \ldots, z^*) = z^*$.

The significance of Theorems 2 and 3 lies in the study of global asymptotic stability of the equilibrium problem for the nonlinear difference Equation (1) were obtained in [10,11].

Păcurar [12] established a result for Prešić–Kannan operators in the following way.

Theorem 4 ([12]). *Let (X, d) be a complete metric space. If a mapping $f : X^k \to X$, for a positive integer k satisfies that*

$$d(f(z_1, z_2, \ldots, z_k), f(z_2, z_3, \ldots, z_{k+1})) \leq a \sum_{i=1}^{k+1} d(z_i, f(z_i, z_i, \ldots, z_i)),$$

holds for all $(z_1, z_2, \ldots, z_{k+1}) \in X^{k+1}$, where a constant $a \in \mathbb{R}$ with $0 < ak(k+1) < 1$, then:

(i) f has at most one fixed point $z^* \in X$.
(ii) For arbitrary points $z_1, z_2, \ldots, z_k \in X$, the sequence $\{z_{n+k}\}$ defined by Equation (1) converges to z^*.

Recently, Abbas et al. [10] obtained the iterative approximation of fixed points of generalized weak Prešić type operators.

Theorem 5 ([10]). *Let (X, d) be a complete metric space. If a mapping $f : X^k \to X$, for a positive integer k satisfies that*

$$\begin{aligned} d(f(z_1, z_2, \ldots, z_k), f(z_2, z_3, \ldots, z_{k+1})) \quad &\leq \quad \max\{d(z_i, z_{i+1}) : 1 \leq i \leq k\} \\ &\quad -\phi(\max\{d(z_i, z_{i+1}) : 1 \leq i \leq k\}), \end{aligned}$$

for all $(z_1, z_2, \ldots, z_{k+1}) \in X^{k+1}$, where $\phi : [0, \infty) \to [0, \infty)$ is a lower semi-continuous function with $\phi(t) = 0$ if and only if $t = 0$. Then, for any arbitrary points $z_1, z_2, \ldots, z_k \in X$, the sequence $\{z_{n+k}\}$ defined by Equation (1) converges to $u \in X$ such that $u = f(u, u \ldots, u)$. Moreover, if

$$d(f(x_1, x_1, \ldots, x_1), f(x_2, x_2 \ldots, x_2)) \leq d(x_1, x_2) - \phi(d(x_1, x_2)),$$

holds for all $x_1, x_2 \in X$ with $x_1 \neq x_2$, then fixed point of f is unique.

Various other useful results of generalized Prešić type operators and its applications in various spaces are presented in [11,13–20]. Recently, Alecsa [21] introduced Prešić convex contraction and obtained the unique fixed point in the setup of metric spaces. Ali et al. [22] obtained the best proximity results of nonself operators in the metric space structure. Babu et al. [23] proved the fixed point results of Prešić type mapping in b-Dislocated metric spaces. Common fixed point of Prešić type mappings were established in [24].

For a metric space (X, d), we set:

$$\begin{aligned} N(X) &= \{A : A \text{ is a non-empty subset of } X\}, \\ B(X) &= \{A : A \text{ is a non-empty bounded subset of } X\}, \\ CB(X) &= \{A : A \text{ is a non-empty closed and bounded subset of } X\} \text{ and} \\ C(X) &= \{A : A \text{ is a non-empty compact subset of } X\}. \end{aligned}$$

For $A \in N(X)$ and $x \in X$, the distance $d(x, A$ is given as

$$d(x, A) = \inf\{d(x, z) : z \in A\}.$$

For $A, B \in N(X)$, define

$$\delta(A, B) = \sup\{d(x, B) : x \in A\}, \text{ and}$$

$$H(A, B) = \max\{\delta(A, B), \delta(B, A)\}.$$

Then, H is called Pompeiu–Hausdorff metric on $CB(X)$. Furthermore, $(CB(X), H)$ is complete metric space if (X, d) is compete metric space.

Nadler [25] extended the Banach contraction mapping principle to multivalued functions and established the following result.

Theorem 6. *Let (X,d) be a complete metric space and $T : X \to CB(X)$. If for any $x_1, x_2 \in X$, the following holds:*

$$H(Tx_1, Tx_2) \leq \lambda d(x_1, x_2),$$

where, $0 \leq \lambda < 1$. Then, there exists u in X such that $u \in T(u)$, that is, T has a fixed point.

We state the following Lemma given by Nadler in [25].

Lemma 1. *Let (X,d) be a metric space. If $A, B \in C(X)$ and $h \geq 1$. Then, for each $x_1 \in A$, there exists $x_2 \in B$ such that*

$$d(x_1, x_2) \leq hH(A, B).$$

From Lemma 1, we can obtain the following result.

Lemma 2. *Let (X,d) be a metric space. If $A, B \in C(X)$, $T : X \to C(X)$ and $h \geq 1$. Then, for any $a, b \in A$ and $x_1 \in T(a)$, there exists $x_2 \in T(b)$ such that*

$$d(x_1, x_2) \leq hH(T(a), T(b)).$$

Several useful fixed point results for multivalued mappings were established after the work of Nadler.

Recently, Shulka et al. [20] introduced the notion of set-valued Prešić type contraction mapping in product spaces.

Definition 2. *Let (X,d) be a metric space. A mapping $T : X^k \to CB(X)$ is known as a set-valued Prešić type contraction if it satisfies*

$$H(T(x_1, x_2, ..., x_k), T(x_2, x_3, ..., x_{k+1})) \leq \sum_{i=1}^{k} \alpha_i \, d(x_i, x_{i+1}),$$

for all $(x_1, x_2, ..., x_{k+1}) \in X^{k+1}$, where nonnegative constants α_i satisfies $\sum\limits_{i=1}^{k} \alpha_i < 1$.

For the mapping $T : X^k \to N(X)$, a point $u \in X$ is called a fixed point of T if $u \in T(u, u, ..., u)$. By $Fix(T)$, we denote the collection of all fixed points of mapping T.

The associate operator τ of mapping T is defined as $\tau : X \to N(X)$ by $\tau(z) = T(z, z, ..., z)$ for all $z \in X$.

The aim of this paper is to introduce the concept of Prešić type weakly contractive multivalued mappings and then to study the fixed point result for such mappings in metric spaces. We also give an example to support the results presented herein. Furthermore, the stability of fixed point sets of multivalued Prešić type weakly contractive mappings is also obtained.

2. Main Results

In this section, several fixed point results for multivalued Prešić type mappings are established. First, we prove the following main result.

Theorem 7. *Let (X,d) be a complete metric space, k a positive integer and $T : X^k \to C(X)$. Suppose that*

$$\begin{aligned} H(T(x_1,x_2,\ldots,x_k),T(x_2,x_3,\ldots,x_{k+1})) &\le \max\{d(x_i,x_{i+1}) : 1 \le i \le k\} \\ &\quad -\phi(\max\{d(x_i,x_{i+1}) : 1 \le i \le k\}) \end{aligned} \tag{2}$$

holds for all $(x_1,\ldots,x_{k+1}) \in X^{k+1}$, where $\phi : [0,\infty) \to [0,\infty)$ is lower semi-continuous and $\phi(\alpha) = 0$ if and only if $\alpha = 0$. Then, for any arbitrary points $x_1,\ldots,x_k \in X$, the sequence $\{x_{n+k}\}$ defined by

$$x_{n+k} \in T(x_n, x_{n+1}, ..., x_{n+k-1}),\ n = 1, 2, \ldots$$

converges to $u \in X$ and $u \in Fix(T)$. Moreover, if

$$H(T(x,x,...,x),T(y,y,...,y)) \le d(x,y) - \phi(d(x,y)), \tag{3}$$

satisfies for all $x, y \in X$ with $x \ne y$, then $Fix(T) = \{u\}$.

Proof. Let $x_1, ..., x_k$ be arbitrary k elements in X. Define the sequence $\{x_n\}$ in X by

$$x_{n+k} \in T(x_n, x_{n+1}, ..., x_{n+k-1}),\ n = 1, 2, \ldots \text{ with } n \ge k.$$

If $x_i = x_{i+1}$ for all $i = n, n+1, ..., n+k-1$, then $x_i \in T(x_i, x_i, ..., x_i)$, that is, x_i is the fixed point of T. Thus, we assume that $x_i \ne x_{i+1}$ for some $i = n, n+1, ..., n+k-1$. Let $n \le k$. Using Equation (2) and Lemma 1, we have

$$\begin{aligned} d(x_{k+n},x_{k+n+1}) &\le H(T(x_n,\ldots,x_{k+n-1}),T(x_{n+1},\ldots,x_{k+n})) \\ &\le \max\{d(x_i,x_{i+1}) : n \le i \le k+n-1\} \\ &\quad -\phi(\max\{d(x_i,x_{i+1}) : n \le i \le k+n-1\}) \\ &< \max\{d(x_i,x_{i+1}) : n \le i \le k+n-1\}, \end{aligned}$$

$$\begin{aligned} d(x_{k+1},x_{k+2}) &\le H(T(x_1,\ldots,x_k),T(x_2,\ldots,x_{k+1})) \\ &\le \max\{d(x_i,x_{i+1}) : 1 \le i \le k\} - \phi(\max\{d(x_i,x_{i+1}) : 1 \le i \le k\}) \\ &< \max\{d(x_i,x_{i+1}) : 1 \le i \le k\}, \end{aligned}$$

$$\begin{aligned} d(x_k,x_{k+1}) &\le H(T(x_0,\ldots,x_{k-1}),T(x_1,\ldots,x_k)) \\ &\le \max\{d(x_i,x_{i+1}) : 0 \le i \le k-1\} \\ &\quad -\phi(\max\{d(x_i,x_{i+1}) : 0 \le i \le k-1\}) \\ &< \max\{d(x_i,x_{i+1}) : 0 \le i \le k-1\}, \\ &\quad \vdots \end{aligned}$$

Thus, we conclude that $\{d(x_{n+k-1},x_{n+k})\}$ is monotone non-increasing and bounded below. Hence, a real number $c \ge 0$ exists such that

$$\lim_{n\to\infty} d(x_{n+k-1},x_{n+k}) = \lim_{n\to\infty} \max\{d(x_{n+i},x_{n+i+1}) : 0 \le i \le k-1\} = c.$$

We now claim that $c = 0$. It follows that, on taking upper limits as $n \to \infty$ on both sided of the given inequality,

$$\begin{aligned} d(x_{k+n}, x_{k+n+1}) &\leq H(T(x_n, \ldots, x_{k+n-1}), T(x_{n+1}, \ldots, x_{k+n})) \\ &\leq \max\{d(x_i, x_{i+1}) : n \leq i \leq k+n-1\} \\ &\quad -\phi\left(\max\{d(x_i, x_{i+1}) : n \leq i \leq k+n-1\}\right), \end{aligned}$$

we obtain that

$$c \leq c - \phi(c),$$

which gives, $\phi(c) \leq 0$ and so $\phi(c) = 0$ by the property of ϕ. Hence,

$$\lim_{n\to\infty} d(x_{n+k-1}, x_{n+k}) = 0. \tag{4}$$

Next, we prove that $\{x_n\}$ is Cauchy sequence. Suppose that $n, m \in \mathbb{N}$ with $m > n$. From Equation (2), it follows that

$$\begin{aligned} d(x_{k+n}, x_{k+m}) &\leq H(T(x_n, \ldots, x_{k+n-1}), T(x_m, \ldots, x_{k+m-1})) \\ &\leq H(T(x_n, \ldots, x_{k+n-1}), T(x_{n+1}, \ldots, x_{k+n})) \\ &\quad +H(T(x_{n+1}, \ldots, x_{k+n}), T(x_{n+2}, \ldots, x_{k+n+1})) \\ &\quad +\ldots+H(T(x_{m-1}, \ldots, x_{k+m-2}), T(x_m, \ldots, x_{k+m-1})) \\ &\leq \max\{d(x_i, x_{i+1}) : n \leq i \leq k+n-1\} \\ &\quad -\phi\left(\max\{d(x_i, x_{i+1}) : n \leq i \leq k+n-1\}\right) \\ &\quad +\max\{d(x_i, x_{i+1}) : n+1 \leq i \leq k+n\} \\ &\quad -\phi\left(\max\{d(x_i, x_{i+1}) : n+1 \leq i \leq k+n\}\right) \\ &\quad +\ldots+\max\{d(x_i, x_{i+1}) : m-1 \leq i \leq k+m-2\} \\ &\quad -\phi\left(\max\{d(x_i, x_{i+1}) : m-1 \leq i \leq k+m-2\}\right). \end{aligned}$$

On upper limiting as $n, m \to \infty$, we have

$$\lim_{n\to\infty} d(x_{k+n}, x_{k+m}) = 0.$$

It follows that $\{x_n\}$ is a Cauchy sequence in complete metric space (X, d). Thus, an element u in X exists such that

$$\lim_{n,m\to\infty} d(x_n, x_m) = \lim_{n\to\infty} d(x_n, u) = 0. \tag{5}$$

Now, for any $n \in \mathbb{N}$, we obtain

$$\begin{aligned} d(u, T(u, u, \ldots, u)) &\leq d(u, x_{n+k}) + d(x_{n+k}, T(u, u, \ldots, u)) \\ &\leq d(u, x_{n+k}) + H(T(x_n, x_{n+1}, \ldots, x_{n+k-1}), T(u, u, \ldots, u)) \\ &\leq d(u, x_{n+k}) + H(T(u, u, \ldots, u), T(u, u, \ldots, x_n)) \\ &\quad +H(T(u, u, \ldots, x_n), T(u, \ldots, x_n, x_{n+1})) \\ &\quad +\ldots+H(T(u, x_n, x_{n+1}, \ldots, x_{n+k-2}), T(x_n, x_{n+1}, \ldots, x_{n+k-1})) \\ &\leq d(u, x_{n+k}) + d(u, x_n) - \phi\left(d(u, x_n)\right) \\ &\quad +\max\{d(u, x_n), d(x_n, x_{n+1})\} - \phi\left(\max\{d(u, x_n), d(x_n, x_{n+1})\}\right) \\ &\quad +\ldots+\max\{d(u, x_n), d(x_n, x_{n+1}), \ldots, d(x_{n+k-2}, x_{n+k-1})\} \\ &\quad -\phi\left(\max\{d(u, x_n), d(x_n, x_{n+1}), \ldots, d(x_{n+k-2}, x_{n+k-1})\}\right). \end{aligned}$$

On upper limiting as $n \to \infty$ on both sides of the above inequality and employing Equation (5), we get

$$d(u, T(u, u, \ldots, u)) \leq 0,$$

and so $u \in T(u, u, \ldots, u)$, that is, u is a fixed point of T.

Now, we prove that T has a unique fixed point. Contrary suppose that another element $v \in X$ exists with $v \neq u$, satisfying $v \in T(v, v, \ldots, v)$. From Equation (3), we obtain

$$\begin{aligned} d(u,v) &\leq H(T(u,u,\ldots,u), T(v,v,\ldots,v)) \\ &\leq d(u,v) - \phi\left(d(u,v)\right) \\ &< d(u,v), \end{aligned}$$

a contradiction. Hence, fixed point of T is unique, that is, a unique point u in X exists that is satisfying $u \in T(u, u, \ldots, u)$. □

Example 1. *Let $X = [0,2]$ and d be a usual metric on X. For $k \geq 1$ a positive integer, define the mapping $T : X^k \to C(X)$ by*

$$T(x_1, x_2, ..., x_k) = [0, \frac{\max\{x_1, x_2, ..., x_k\}}{4k^2}] \text{ for all } x_1, ..., x_k \in X.$$

Define $\phi : [0, \infty) \to [0, \infty)$ as:

$$\phi(\alpha) = \begin{cases} \frac{\alpha}{5}, & \text{if } \alpha \in [0, \frac{5}{2}), \\ \frac{2^{2n}(2^{n+1}\alpha - 3)}{2^{2n+1} - 1}, & \text{if } \alpha \in [\frac{2^{2n}+1}{2^n}, \frac{2^{2(n+1)}+1}{2^{n+1}}],\ n \in \mathbb{N} \end{cases}$$

Note that ϕ is lower semi-continuous on $[0, \infty)$ and $\phi(\alpha) = 0$ if and only if $\alpha = 0$.
For all $x_1, x_2, ..., x_{k+1} \in X$, we have

$$\begin{aligned} & H(T(x_1, x_2, ..., x_k), T(x_2, x_3, ..., x_{k+1})) \\ \leq\ & \frac{1}{4k} |x_1 - x_{k+1}| \\ \leq\ & \frac{1}{4} \max\{|x_i - x_{i+1}| : 1 \leq i \leq k\} \\ \leq\ & \frac{4}{5} \max\{d(x_i, x_{i+1}) : 1 \leq i \leq k\} \\ =\ & \max\{d(x_i, x_{i+1}) : 1 \leq i \leq k\} - \phi(\max\{d(x_i, x_{i+1}) : 1 \leq i \leq k\}). \end{aligned}$$

In addition, for all $x_1, x_2 \in X$, we have

$$\begin{aligned} H(T(x_1, x_1, ..., x_1), T(x_2, x_2, ..., x_2)) &\leq \frac{1}{8} |x_1 - x_2| \\ &\leq \frac{4}{5} d(x_1, x_2) \\ &= d(x_1, x_2) - \phi(d(x_1, x_2)). \end{aligned}$$

Hence, T satisfies Equations (2) and (3). All the conditions of Theorem 7 are satisfied. In addition, for any arbitrary points $x_1, x_2, ..., x_k \in X$, the sequence $\{x_n\}$ defined by $x_{n+k} \in T(x_n, x_{n+1}, ..., x_{n+k-1})$, $n = 1, 2, \ldots$ converges to $u = 0$, a unique fixed point of T.

If we take $\phi(t) = (1 - \lambda)t$ for all $t \in [0, \infty)$ in Theorem 7, then we obtain the following immediate consequence of Theorem 7.

Corollary 1. *Let (X,d) be a complete metric space, k a positive integer and $T : X^k \to C(X)$. If there exists $\lambda \in [0,1)$ such that*

$$H(T(x_1,x_2,\ldots,x_k),T(x_2,x_3,\ldots,x_{k+1})) \leq \lambda \max\{d(x_i,x_{i+1}) : 1 \leq i \leq k\}, \tag{6}$$

for all $(x_1,x_2,\ldots,x_{k+1}) \in X^{k+1}$. Then, for any arbitrary points $x_1,\ldots,x_k \in X$, the sequence $\{x_{n+k}\}$ defined by

$$x_{n+k} \in T(x_n,x_{n+1},...,x_{n+k-1}),\ n = 1,2,\ldots$$

converges to $u \in X$ and $u \in Fix(T)$. Moreover, if

$$H(T(x,...,x),T(y,...,y)) \leq \lambda d(x,y), \tag{7}$$

holds for all $x,y \in X$ with $x \neq y$, then $Fix(T) = \{u\}$.

Corollary 2. *Let (X,d) be a complete metric space, k a positive integer and $T : X^k \to CB(X)$. Suppose that there exists $\lambda_1,\lambda_2,...,\lambda_k$ non-negative constants with $\lambda_1+\lambda_2+...+\lambda_k < 1$ such that*

$$H(T(x_1,\ldots,x_k),T(x_2,\ldots,x_{k+1})) \leq \lambda_1 d(x_1,x_2)+\lambda_2 d(x_2,x_3)+...+\lambda_k d(x_k,x_{k+1}), \tag{8}$$

for all $(x_1,\ldots,x_{k+1}) \in X^{k+1}$. Then, for any arbitrary points $x_1,\ldots,x_k \in X$, the sequence $\{x_{n+k}\}$ defined by

$$x_{n+k} \in T(x_n,x_{n+1},...,x_{n+k-1}),\ n = 1,2,\ldots$$

converges to $u \in X$ and $u \in Fix(T)$. Moreover, if

$$H(T(x,...,x),T(y,...,y)) \leq (\lambda_1+\lambda_2+...+\lambda_k)\, d(x,y), \tag{9}$$

holds for all $x,y \in X$ with $x \neq y$, then $Fix(T) = \{u\}$.

Proof. Clearly, the condition in Equation (6) implies the condition in Equation (8) with $\lambda = \lambda_1+\lambda_2+\ldots+\lambda_k$. Now, let $x_1,x_2 \in X$ with $x_1 \neq x_2$. From Equation (9), we have

$$\begin{aligned} H(T(x_1,x_1,\ldots,x_1),T(x_2,x_2,\ldots,x_2)) &\leq H(T(x_1,x_1,\ldots,x_1),T(x_1,\ldots,x_1,x_2)) \\ &\quad +H(T(x_1,\ldots,x_1,x_2),T(x_1,\ldots,x_1,x_2,x_2)) \\ &\quad +\ldots+H(T(x_1,x_2,\ldots,x_2),T(x_2,x_2,\ldots,x_2)) \\ &\leq (\lambda_k+\lambda_{k-1}+\ldots+\lambda_1)d(x_1,x_2) = \lambda d(x_1,x_2), \end{aligned}$$

where $\lambda = \lambda_k+\lambda_{k-1}+\ldots+\lambda_1 \in [0,1)$. Thus, the conditions of Corollary 1 are satisfied and the result follows. □

From Theorem 7, we get the following fixed point result for single-valued mapping.

Theorem 8. *Let (X,d) be a complete metric space, k a positive integer and $T : X^k \to X$. If there exists $\phi : [0,\infty) \to [0,\infty)$, a lower semi-continuous function with $\phi(\alpha) = 0$ if and only if $\alpha = 0$ such that*

$$\begin{aligned} d(T(x_1,x_2,\ldots,x_k),T(x_2,x_3,\ldots,x_{k+1})) &\leq \max\{d(x_i,x_{i+1}) : 1 \leq i \leq k\} \\ -\phi(\max\{d(x_i,x_{i+1}) &: 1 \leq i \leq k\}) \end{aligned} \tag{10}$$

holds for all $(x_1,\ldots,x_{k+1}) \in X^{k+1}$. Then, for any arbitrary points $x_1,\ldots,x_k \in X$, the sequence $\{x_{n+k}\}$ defined by

$$x_{n+k} = T(x_n,x_{n+1},...,x_{n+k-1}),\ n = 1,2,\ldots$$

converges to $u \in X$ and u is the fixed point of T. Moreover, if

$$d(T(x_1, x_1, ..., x_1), T(x_2, x_2, ..., x_2)) \leq d(x_1, x_2) - \phi(d(x_1, x_2)), \tag{11}$$

holds for all $x_1, x_2 \in X$ with $x_1 \neq x_2$, then $Fix(T) = \{u\}$.

3. Stability of Prešić Type Multivalued Fixed Point Problems

We study the stability of fixed point sets of Prešić type weakly contractive multivalued mappings. First, we obtain the following result.

Theorem 9. *Let (X, d) be a complete metric space and $T_i : X^k \to C(X)$ for $i = 1, 2$ be two multivalued mappings. Suppose that there exists a lower semi-continuous function $\phi : [0, \infty) \to [0, \infty)$ with $\phi(\alpha) = 0$ if and only if $\alpha = 0$, such that*

$$\begin{aligned} H(T_i(x_1, \ldots, x_k), T_i(x_2, \ldots, x_{k+1})) \quad &\leq \quad \max\{d(x_i, x_{i+1}) : 1 \leq i \leq k\} \\ &\quad -\phi(\max\{d(x_i, x_{i+1}) : 1 \leq i \leq k\}), \end{aligned} \tag{12}$$

for all $(x_1, \ldots, x_{k+1}) \in X^{k+1}$ and

$$H(T_i(x_1, \ldots, x_1), T_i(x_2, \ldots, x_2)) \leq d(x_1, x_2) - \phi(d(x_1, x_2)) \tag{13}$$

for all $x_1, x_2 \in X$ hold. Then, $\phi(H(Fix(T_1), Fix(T_2))) \leq K$, where

$$K = \sup_{x \in X} H(T_1\,(x, ..., x)\,, T_2\,(x, ..., x)).$$

Proof. From Theorem 7, the set of fixed points of T_i is non-empty, that is, $Fix(T_i) \neq \emptyset$ for $i = 1, 2$. Let $y_0 \in T_1\,(y_0, ..., y_0)$. Then, by Lemma 1, there exists a $y_1 \in T_2\,(y_0, ..., y_0)$ such that

$$d(y_0, y_1) \leq H(T_1(y_0, ..., y_0), T_2(y_0, ..., y_0)). \tag{14}$$

Since $y_1 \in T_2\,(y_0, ..., y_0)$, again by Lemma 1, there exists a $y_2 \in T_2\,(y_1, ..., y_1)$ such that

$$d(y_1, y_2) \leq H(T_2(y_0, ..., y_0), T_2(y_1, ..., y_1)).$$

We construct a sequence $\{y_n\}$ such that, for all $n \geq 0$ with $y_{n+1} \in T_2(y_n, ..., y_n)$, we have

$$\begin{aligned} d(y_{k+1}, y_{k+2}) \quad &\leq \quad H(T_2(y_k, \ldots, y_k), T_2(y_{k+1}, \ldots, y_{k+1})) \\ &\leq \quad d(y_k, y_{k+1}) - \phi\left(d(y_k, y_{k+1})\right). \end{aligned}$$

In addition,

$$\begin{aligned} d(y_k, y_{k+1}) \quad &\leq \quad H(T(y_{k-1}, \ldots, y_{k-1}), T(y_k, \ldots, y_k)) \\ &\leq \quad d(y_{k-1}, y_k) - \phi\left(d(y_{k-1}, y_k)\right), \\ &\quad\; \vdots \end{aligned}$$

Thus, for $n \leq k$, we have

$$\begin{aligned} d(y_{k-n}, y_{k-n+1}) \quad &\leq \quad H(T(y_{k-n-1}, \ldots, y_{k-n-1}), T(y_{k-n}, \ldots, y_{k-n})) \\ &\leq \quad d(y_{k-n-1}, y_{k-n}) - \phi\left(d(y_{k-n-1}, y_{k-n})\right). \end{aligned}$$

Following similar arguments to those given in the proof of Theorem 7, we obtain that $\{y_n\}$ is a Cauchy sequence X and there exists a $u \in X$ such that $y_n \to u$ as $n \to \infty$. Let u be the fixed point of T_2, that is, $u \in T_2(u, ..., u)$. It follows that

$$\begin{aligned} d(y_0, y_1) &\leq H(T_1(y_0, ..., y_0), T_2\,(y_0, ..., y_0)) \\ &\leq \sup_{x \in X} H(T_1\,(x, ..., x)\,, T_2\,(x, ..., x)) = K\,(\text{say}). \end{aligned}$$

Note that

$$\begin{aligned} d(y_0, u) &\leq d(y_0, y_1) + d(y_1, u) \\ &\leq K + H\,(T_2\,(y_0, ..., y_0)\,, T_2\,(u, ..., u)) \\ &\leq K + d\,(y_0, u) - \phi\,(d\,(y_0, u))\,, \end{aligned}$$

that is,

$$\phi\,(d\,(y_0, u)) \leq K.$$

Thus, given an arbitrary $y_0 \in Fix(T_1)$, we can find a $u \in Fix(T_2)$ for which

$$\phi\,(d(y_0, u)) \leq K.$$

Similarly, it follows that, for arbitrary $z_0 \in Fix(T_2)$, an element $w \in Fix(T_1)$ exists such that

$$\phi\,(d(z_0, w)) \leq K.$$

Thus, we obtain

$$\phi\,(H(Fix(T_1), Fix(T_2))) \leq K.$$

□

Lemma 3. *Let (X, d) be a complete metric space and $\{T_n : X^k \to C(X) : n \in \mathbb{N}\}$ a sequence of multivalued mappings, uniformly convergent to $T : X^k \to C(X)$. If T_n satisfies Equations (12) and (13) for each $n \in \mathbb{N}$, then T also satisfies Equations (12) and (13), where the lower semi-continuous function $\phi : [0, \infty) \to [0, \infty)$ is satisfying $\phi(\alpha) = 0$ if and only if $\alpha = 0$.*

Proof. As T_n satisfies Equations (12) and (13) for every $n \in \mathbb{N}$, so that for $x_1, x_2, \ldots, x_{k+1} \in X$, we have

$$\begin{aligned} H(T_n(x_1, \ldots, x_k), T_n(x_2, \ldots, x_{k+1})) &\leq \max\{d(x_i, x_{i+1}) : 1 \leq i \leq k\} \\ -\phi(\max\{d(x_i, x_{i+1}) &: 1 \leq i \leq k\}) \end{aligned}$$

and

$$H(T_n(x_1, \ldots, x_1), T_n(x_2, \ldots, x_2)) \leq d(x_1, x_2) - \phi(d(x_1, x_2)).$$

As the sequence $\{T_n\}$ is uniformly convergent to T and ϕ is lower semi-continuous, taking the upper limit as $n \to \infty$ on both sides of the above two inequalities, we obtain that

$$\begin{aligned} H(T(x_1, x_2, \ldots, x_k), T(x_2, x_3, \ldots, x_{k+1})) &\leq \max\{d(x_i, x_{i+1}) : 1 \leq i \leq k\} \\ -\phi(\max\{d(x_i, x_{i+1}) &: 1 \leq i \leq k\}) \end{aligned}$$

and

$$H(T(x_1, \ldots, x_1), T(x_2, \ldots, x_2)) \leq d(x_1, x_2) - \phi(d(x_1, x_2)).$$

This shows that T satisfies Equations (12) and (13). □

Now, we present the following stability result.

Theorem 10. *Let (X, d) be a complete metric space, and $\{T_n : X^k \to C(X) : n \in \mathbb{N}\}$ a sequence of multivalued mappings, uniformly convergent to $T : X^k \to C(X)$. Suppose that T_n satisfies Equations (12) and (13) for each $n \in \mathbb{N}$, where the mapping ϕ is the same as in the statement of Theorem 9. Then*

$$\lim_{n\to\infty} H(Fix(T_n), Fix(T)) = 0;$$

that is, the fixed point sets of T_n are stable.

Proof. By Lemma 3, T satisfies Equations (12) and (13). Let $K_n = \sup_{x\in X} H(T_n x, Tx)$. Since the sequence $\{T_n\}$ is uniformly convergent to T on X, we have

$$\lim_{n\to\infty} K_n = \lim_{n\to\infty} \sup_{x\in X} H(T_n x, Tx) = 0.$$

Using Theorem 9, we obtain that

$$\phi\left(H(Fix(T_n), Fix(T))\right) \leq K_n \text{ for every } n \in \mathbb{N}.$$

Since ϕ is lower semi-continuous, we have

$$\liminf_{n\to\infty} \phi\left(H(Fix(T_n), Fix(T))\right) \leq \liminf_{n\to\infty} K_n = 0;$$

that is,

$$\lim_{n\to\infty} H(Fix(T_n), Fix(T)) = 0.$$

Hence, the proof is complete. □

4. Global Attractivity Results

Let $P(n)$ denote an open convex cone of all $n \times n$ Hermitian positive definite matrices. We investigate the weak asymptotic stability and global attractivity of the nonlinear matrix recursive sequence $\{U_n\}$ in $P(n)$ defined by

$$U_{n+k} = Q + \frac{1}{k}\sum_{i=0}^{k-1} A^*\eta(U_{n+i})A, \quad n = 1, 2, \ldots, \tag{15}$$

where k is the positive integer, Q is the $n \times n$ Hermitian positive semidefinite matrix, A the $n \times n$ nonsingular matrix, and A^* is a conjugate transpose matrix of A and the mapping $\eta : P(n) \to P(n)$.

Definition 3. *Let U be a non-empty set, k be a positive integer and $T : U^k \to U$. For given $u_1, u_2, \ldots, u_k \in U$, consider the recursive sequence $\{u_n\}$ in U defined by*

$$u_{n+k} = T(u_n, u_{n+1}, \ldots, u_{n+k-1}), \quad n = 1, 2, \ldots. \tag{16}$$

The solutions of Equation (16) are functions $\omega : \mathbb{N} \to U$, that is, for every $n \in \mathbb{N}$, $\omega(n+k) = \omega_{n+k} = T(\omega_n, \omega_{n+1}, ..., \omega_{n+k-1})$.

A point $\overline{u}$ in U is called equilibrium point of Equation (16) if it satisfies

$$\overline{u} = T(\overline{u}, \ldots, \overline{u}).$$

In addition, the equilibrium point $\overline{u}$ of Equation (16) is known as the global attractor if for arbitrary $u_1, u_2, \ldots, u_k \in U$, we have $d(u_n, \overline{u}) \to 0$ as $n \to \infty$.

The point $\overline{u}$ of Equation (16) is called weakly stable if, given $\varepsilon > 0$, there exists $\delta > 0$ such that for at least one solution of Equation (16) with initial values $u_1, u_2, ..., u_k$ and $\|u_1 - \overline{u}\| + \|u_2 - \overline{u}\| + ... + \|u_k - \overline{u}\| < \delta$ implies that $\|u_n - \overline{u}\| < \varepsilon$ for all $n \in \mathbb{N}$. In addition, the weakly stable point $\overline{u}$ of Equation (16) is called weakly asymptotically stable if, in addition $\lim_{n\to\infty} u_n = \overline{u}$.

We say that $\eta : U \to U$ is ϕ-contraction, if there exists a lower semi-continuous function $\phi : [0, \infty) \to [0, \infty)$ with $\phi(\alpha) = 0$ if and only if $\alpha = 0$ such that

$$d(\eta(u), \eta(v)) \leq d(u, v) - \phi\left(d(u, v)\right),$$

for all $u, v \in U$.

We endow $P(n)$ with the Thompson metric [26,27] by property of inverse matrix and congruence transformations, that is,

$$d(U, V) = d(U^{-1}, V^{-1}) = d(W^*UW, W^*VW), \tag{17}$$

where W is any nonsingular matrix.

Now, we present the following global attractivity result.

Theorem 11. *Let A be $n \times n$ nonsingular matrix and Q be an $n \times n$ Hermitian positive semidefinite matrix. Suppose that $\eta : P(n) \to P(n)$ is a ϕ-contraction with respect to the Thompson metric d, then for a positive integer k and for given $U_1, U_2, \ldots, U_k \in P(n)$, the sequence $\{U_n\}$ in $P(n)$ defined by Equation (15) has a unique equilibrium point $\overline{U}$ in $P(n)$. Moreover, $\overline{U}$ is weakly asymptotically stable and a global attractor.*

Proof. Define the mapping $T : P(n)^k \to P(n)$ by

$$T(U_1, U_2, \ldots, U_k) = Q + \frac{1}{k}[A^*\eta(U_1)A + A^*\eta(U_2)A + \ldots + A^*\eta(U_k)A],$$

for all $U_1, U_2, \ldots, U_k \in P(n)$. Then, we have

$$\begin{aligned}
&d(T(U_1, U_2, \ldots, U_k), T(U_2, U_3, \ldots, U_{k+1})) \\
&= d\left(Q + \frac{1}{k}\sum_{i=1}^{k} A^*\eta(U_i)A, Q + \frac{1}{k}\sum_{j=2}^{k+1} A^*\eta(U_j)A\right) \\
&\leq d\left(\frac{1}{k}\sum_{i=1}^{k} A^*\eta(U_i)A, \frac{1}{k}\sum_{j=2}^{k+1} A^*\eta(U_j)A\right) \\
&= d\left(\sum_{i=1}^{k}(\frac{1}{\sqrt{k}}A)^*\eta(U_i)(\frac{1}{\sqrt{k}}A), \sum_{j=2}^{k+1}(\frac{1}{\sqrt{k}}A)^*\eta(U_j)(\frac{1}{\sqrt{k}}A)\right).
\end{aligned}$$

Denote $V = \dfrac{1}{\sqrt{k}}A$. Then, we obtain

$$\begin{aligned}
&d(T(U_1, U_2, \ldots, U_k), T(U_2, U_3, \ldots, U_{k+1})) \\
&\leq d\left(\sum_{i=1}^{k} V^*\eta(U_i)V, \sum_{j=2}^{k+1} V^*\eta(U_j)V\right) \\
&= d\left(V^*\eta(U_1)V + \ldots + V^*\eta(U_k)V, V^*\eta(U_2)V + \ldots + V^*\eta(U_{k+1})V\right) \\
&\leq \max\left\{d(V^*\eta(U_1)V, V^*\eta(U_2)V), \ldots, d(V^*\eta(U_k)V, V^*\eta(U_{k+1})V)\right\} \\
&= \max\left\{d(V^*\eta(U_i)V, V^*\eta(U_{i+1})V) :\ i = 1, 2, \ldots, k\right\}.
\end{aligned}$$

The nonsingularity of matrix A implies that V is a nonsingular matrix. By using Equation (17), for all $i = 1, 2, \ldots, k$,

$$d(V^*\eta(U_i)V, V^*\eta(U_{i+1})V) = d(\eta(U_i), \eta(U_{i+1})).$$

Since η is ϕ-contraction, so that for all $i = 1, 2, \ldots, k$, we have

$$\begin{aligned} & d(T(U_1, U_2, \ldots, U_k), T(U_2, U_3, \ldots, U_{k+1}) \\ \leq \ & \max\{d(\eta(U_i), \eta(U_{i+1})) : i = 1, 2, ..., k\} \\ = \ & \max\left\{d(U_i, U_{i+1}) : i = 1, 2, \ldots, k\right\} - \phi\left(\max\left\{d(U_i, U_{i+1}) : i = 1, 2, \ldots, k\right\}\right) \end{aligned}$$

for all $U_1, U_2, \ldots, U_{k+1} \in P(n)$.

By employing Corollary 1, it follows that a global attractor equilibrium point $\overline{U} \in P(n)$ exists. Further, if we assume that, for $\overline{U}, \overline{V} \in P(n)$ with $\overline{U} \neq \overline{V}$, we have

$$\begin{aligned} d(T(\overline{U}, \overline{U}, \ldots, \overline{U}), T(\overline{V}, \overline{V}, \ldots, \overline{V})) &= d(Q + A^*\eta(\overline{U})A, Q + A^*\eta(\overline{V})A) \\ &\leq d(A^*\eta(\overline{U})A, A^*\eta(\overline{V})A) \\ &= d(\eta(\overline{U}), \eta(\overline{V})) \\ &\leq d(\overline{U}, \overline{V}) - \phi\left(d(\overline{U}, \overline{V})\right). \end{aligned}$$

Again, by employing Corollary 1, we obtain that the global attractor equilibrium point in $P(n)$ is unique. □

5. Results and Discussion

The convergence of multivalued Prešić type k-step iterative process for operators $T : X^k \to C(X)$ that are satisfying Prešić type generalized weakly contractive conditions is studied in the setup of metric spaces. An example is presented in support of our main theorem. We also establish the stability of fixed point sets of multivalued Prešić type weakly contractive mappings. Furthermore, the results of global attractivity for a collection of matrix difference equations are established. For further work in this direction, various new results of multivalued Prešić type contractions can be obtained such as fixed point results for multivalued Prešić type F-contractions and multivalued Prešić type convex contractions. These results extend and generalize the results presented by Abbas et al. [28] and Alecsa [21], respectively. Moreover, the multivalued Prešić type k-step iterative process can be derived for flour beetle population model, generalized Beddington–Holt stock recruitment model, and the delay model of a perennial grass [29]. In addition, the variety of problems related to dynamical systems, fixed points and equilibrium points dealing with multivalued mappings can be solved.

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