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State controllability of a froth flotation cell

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Abstract: The control goals for a froth flotation cell are to maintain a stable pulp level and maximise the separation between valuable minerals to the concentrate and gangue to the tailings. Only a limited set of manipulated variables are available to achieve a target concentrate grade and tailings grade for a specific pulp level. The froth flotation cell can be represented by a non-linear phenomenological state-space model. The states of the model are the concentration of minerals in the froth and the pulp, and the volume of liquid in the froth and pulp. A state controllability analysis of the model indicates that the states are controllable using air flow-rate, tailings flow-rate, and feed-rate as manipulated variables. Therefore, given these three manipulated variables, a controller should be able to maximise the separation between minerals in the concentrate and tailings, while maintaining a pulp level within acceptable bounds.

Keywords: controllability, flotation, process control, process modelling

1. INTRODUCTION

The overall control strategy of a flotation plant can be divided into two levels. The primary control level consists of good control of local objectives such as pulp levels and flow-rates to stabilize the plant. The secondary level consists of modifying the operation of the circuit to obtain a specific concentrate grade, concentrate recovery, and tailings grade. The final concentrate grade and tailings grade are the main dynamic controlled variables for the flotation plant, whereas concentrate recovery is a steady-state controlled variable (Bergh and Yianatos, 1993, 2011).

Given the large number of variables which impact the performance of a flotation cell, it remains a challenge to control this non-linear process (Laurila et al., 2002). Apart from base level control, the number of successful industrial implementations of long-term model-based supervisory and optimisation control remains scarce (Shean and Cilliers, 2011; Bergh and Yianatos, 2011). Model predictive control is an attractive long-term non-linear control solution in the case where appropriate models are available to relate the target variables to the manipulated variables (Suichies et al., 2000; Jovanovic and Miljanovic, 2015).

The challenge for multi-component ore is to separate between strongly floatable, weakly floatable, and non-floatable minerals. Since each mineral has its own flotation characteristics, multiple flotation and re-grinding stages may be required to separate the valuable minerals from each other and from gangue (McIvor and Finch, 1991). For example, the industrial flotation cell evaluated by Jämsä-Jounela (1992) predominantly concentrates apatite, but to some degree calcite and dolomite as well. The main separation occurs between the apatite and gangue material, where apatite is produced as concentrate (along with some calcite and dolomite) and the gangue material as tailings. For good separation of minerals, a controller may aim to achieve a specific concentrate grade for one mineral and a specific tailing grade for another mineral. The question is if the available manipulated variables can drive the system to achieve both these aims without neglecting stabilizing control of the pulp level. This is a challenge as only a limited set of manipulated variables are available to control a cell (Laurila et al., 2002). The available manipulated variables include feed-rate, tailings flow-rate, air flow-rate, reagent addition, and froth wash water (Oosthuizen et al., 2017).

The aim of this paper is to evaluate the controllability of the dynamic target variables of a flotation cell – concentrate grades and tailings grades – based on the manipulated variables available to the primary control level: feed-rate, tailings flow-rate, and air flow-rate. As indicated by Oosthuizen et al. (2017), these are the most common variables available to plants for manipulation. Reagent addition and froth wash water is not considered here. The aim is to evaluate if the dynamic target variables can be controlled with the use of the three available manipulated variables. The controllability is evaluated against the simplified state-space model of a froth flotation cell of Bascur (1982). The data of Jämsä-Jounela (1992) is used as test scenario.

The paper is organized as follows: Section 2 describes the flotation process and provides an overview of the simplified model of Bascur (1982) of a single flotation cell, Section 3 provides the controllability theory, Section 4 describes and discusses the controllability analysis, and Section 5 gives a conclusion.

2. FLOTATION

Flotation involves three phases: solids, liquid and gas. The aim of flotation is to separate valuable minerals from gangue minerals. The separation is based on differences in the surface properties of the minerals. In general, the

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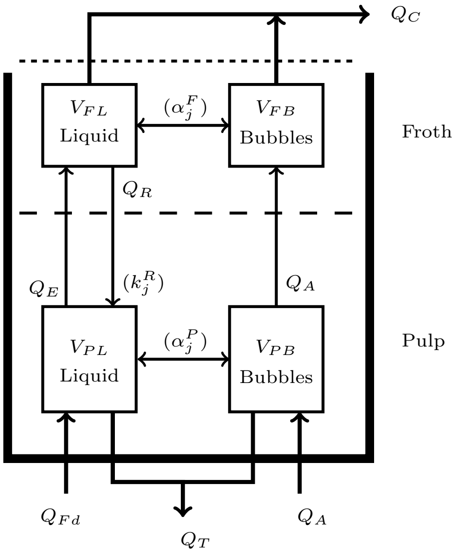

Fig. 1. The flotation process in a flotation cell.

surface of the valuable minerals should be hydrophobic, and the surface of gangue should be hydrophilic. When air is pumped through the pulp, the hydrophobic particles attach to the bubbles and float to the top, and the hydrophilic particles attach to the liquid and flow to the bottom. A more complete review of froth flotation can be found in: Dobby (2002), Laurila et al. (2002), Gupta and Yan (2006), and Wills (2006).

There are two approaches to model the set of complex physical and chemical reactions in a flotation cell: kinetic (first-principle) modelling and data-driven modelling. The kinetic modelling approach involves flotation rates, mass and flow conservation, and the probability of adhesion/detachment. The data-driven modelling approach involves grade vs. recovery predictions, soft sensing and evaluation of the relationship between flotation performance and froth features (Oosthuizen et al., 2017).

Bascur (1982) developed a detailed phenomenological kinetic model of a flotation cell. The model represents the behaviour of particles of different mineralogical composition and sizes under a wide range of steady-state and dynamic operating conditions. The model links the hydrodynamic characteristics of the flotation cell to the particle/bubble and water transport through the cell. Bascur (1982) simplified this model and validated the model against industrial data. The simplified model was used by Jämsä-Jounela (1992) to develop an adaptive controller for an industrial rougher flotation bank. The process of fitting the parameters of the simplified model to industrial sampling campaign data is briefly described by Sun et al. (2018).

2.1 Process Description

The flotation process for a single flotation cell is shown in Fig. 1. There are two distinguishable sections in the flotation cell: pulp at the bottom and froth at the top. The pulp and froth sections are subdivided into a liquid and an air (bubble) phase where:

- V_{FL} is the unaerated liquid volume in the froth,
- V_{PL} is the unaerated liquid volume in the pulp,
- V_{FB} is the volume of bubbles in the froth,
- V_{PB} is the volume of bubbles in the pulp,
- $V_P = V_{PB} + V_{PL}$ is the total pulp volume, and
- $V_F = V_{FB} + V_{FL}$ is the total froth volume.

Slurry containing valuable minerals and gangue (Q_{Fd}) and air (Q_A) is pumped from the bottom of the cell to form part of the pulp volume. In general, the aim is for valuable particles in the pulp to attach to bubbles and flow from the pulp volume to the froth volume, and for gangue material to remain in the liquid and leave the cell through the tailings stream (Q_T). The valuable particles in the froth volume generally remain attached to bubbles and leave the cell in the concentrate stream (Q_C). However, both valuable and gangue material in the pulp can become trapped between bubbles and enter from the pulp phase into the froth phase. This process is called entrainment (Q_E). In contrast, particles in the froth phase can detach from the bubbles and drain back to the pulp volume (Q_R). Thus, the aim is to concentrate the valuable material, and pass the gangue to the tailings. (In some cases the aim may be the opposite - for gangue to be concentrated and valuable material to pass to the tailings - but this is not considered here.)

The following types of chemicals are generally used to influence the flotation process: collectors and frothers. Collectors are chemicals that cause the surface of a mineral to be hydrophobic so that the mineral can be 'collected'. Frothers are chemicals that reduce the surface tension of water to stabilize the bubbles at the top of the flotation cell into a froth layer to ease concentrate removal.

2.2 Industrial Circuit Measurements

In order to build a successful controller at an industrial plant, it is necessary that the manipulated variables have adequate actuators, and that the manipulated and controlled variables are measurable.

In terms of the manipulated variables, a magnetic flow metre is commonly used to measure Q_T. An outlet-valve is used as actuator for Q_T. Plants generally use simple Proportional-Integral (PI) controllers to manipulate Q_T to control V_P. Feed-forward control can be used to account for flow-rate variations upstream. An alternative to PI control of V_P is to design a controller which considers all the levels in the circuit simultaneously, such as the FloatStar level stabiliser developed at Mintek (Schubert et al., 1995). Such an approach has been shown to be beneficial operationally and economically (Craig and Henning, 2000).

Since cells are generally grouped in flotation banks such that the concentrate from one cell flows directly into the next cell, there is little control of Q_{Fd} into cells. In the case where the slurry is introduced into the flotation bank by means of a surge tank with a pump, Q_{Fd} into the first cell in the flotation bank can be manipulated. A magnetic flow metre is generally used to measure Q_{Fd}.

The typical methods to measure Q_A on industrial circuits are:

- Thermal gas mass flow sensors,
- Differential pressure metre with venturi tube, and

- Differential pressure transmitter with Pitot or annubar tube.

According to Wills (2006), the flotation process reacts quicker to changes in Q_A than changes in V_P. Manipulation of Q_A is also significantly cheaper than reagent addition. It is therefore a very effective manipulated variable to control the process.

In terms of the controlled variables, the typical methods to measure V_P are:

- Float with a target plate and ultrasonic transmitter,
- Float with angle arms and a capacitive angle transmitter, and
- Reflex radar.

The mineral concentration of species j at the concentrate (C_j^F) and at the tailings (C_j^P) can be measured by means of on-line X-ray fluorescence analysers. When multiple cells are present in a flotation circuit, only the grade of the final concentrate stream is measured as this stream is the final product of the circuit. In certain cases the feed grade, tailings grade, and the intermediate concentrate grade between flotation banks are also measured, but this scenario is not common (Shean and Cilliers, 2011).

2.3 Process modelling

A brief overview of the simplified phenomenological model developed by Bascur (1982) for a single flotation cell is presented below. The data of the industrial sampling campaign by Jämsä-Jounela (1992) of a flotation cell is used as example where four mineral species are considered: apatite ($j = 1$), calcite ($j = 2$), dolomite ($j = 3$), and other ($j = 4$). The model nomenclature is shown in Table 1.

Table 1. Nomenclature

Parm	Unit	Description
α_j^F	-	Froth attachment and detachment rate constant for species j
α_j^P	-	Pulp attachment and detachment rate constant for species j
γ_j	-	Grade of species j
υ_j	-	Recovery of species j
ρ_j	t/m^3	Density of species j
C_j^{Fd}	t/m^3	Concentration of species j in feed
C_j^F	t/m^3	Concentration of species j in froth
C_j^P	t/m^3	Concentration of species j in pulp
k_j^R	-	Drainage rate constant
Q_A	m^3/h	Aeration-rate
Q_C	m^3/h	Concentrate flow-rate
Q_E	m^3/h	Entrainment flow-rate
Q_{Fd}	m^3/h	Pulp feed flow-rate
Q_R	m^3/h	Drainage flow-rate
Q_T	m^3/h	Tailings flow-rate
V_{FB}	m^3	Volume of froth bubbles
V_{FL}	m^3	Volume of froth liquid
V_{PB}	m^3	Volume of pulp bubbles
V_{PL}	m^3	Volume of pulp liquid
V_{Tot}	m^3	Total cell volume

Mineral Concentrations The mass balance for mineralogical species j in the flotation cell can be written as:

$$\begin{aligned} V_{PL}\frac{dC_j^P}{dt} =& Q_{Fd}C_j^{Fd} - \frac{Q_E + Q_A\alpha_j^P\frac{V_{PL}}{V_{PB}}}{\left(1+\alpha_j^P\right)}C_j^P \qquad (1a)\\ &+ \frac{Q_R k_j^R C_j^F}{\left(1+\alpha_j^F\right)} - \frac{Q_T C_j^P}{\left(1+\alpha_j^P\right)} \end{aligned}$$

$$\begin{aligned} V_{FL}\frac{dC_j^F}{dt} =& \frac{Q_E + Q_A\alpha_j^P\frac{V_{PL}}{V_{PB}}}{\left(1+\alpha_j^P\right)}C_j^P - Q_C C_j^F \qquad (1b)\\ &- \frac{Q_R k_j^R C_j^F}{\left(1+\alpha_j^F\right)} \end{aligned}$$

where C_j^{Fd}, C_j^F, and C_j^P (t/m^3) represent the concentration of species j in the feed, froth, and pulp respectively; α_j^P and α_j^F are the attachment/detachment rate constants of the species in the pulp and froth respectively; k_j^R is the drainage rate constant of species j. The model above assumes ideal conditions such that the bubbles exiting through the tailing stream is zero and the flow-rate of bubbles in the concentrate is equal to Q_A.

Volume balance The liquid volume balance at the pulp and froth interface is given by:

$$\frac{dV_{PL}}{dt} = Q_{Fd} - Q_T - Q_E + Q_R \qquad (2a)$$

$$\frac{dV_{FL}}{dt} = Q_E - Q_R - Q_C. \qquad (2b)$$

The physical volume of the cell V_{Tot} remains fixed, such that the following equation should always be satisfied:

$$V_{Tot} = V_{PB} + V_{PL} + V_{FB} + V_{FL}. \qquad (3)$$

Entrainment and Drainage According to Bascur (1982), the entrainment flow-rate can be calculated from the number of bubbles rising per unit time:

$$Q_E = \frac{6\delta Q_A}{d_{BP}} = r_E Q_A \qquad (4)$$

where δ is the film thickness around the bubble, d_{BP} is the mean bubble diameter in the pulp, and $r_E = \frac{6\delta}{d_{BP}}$. Similarly, the drainage flow rate can be expressed as:

$$Q_R = Q_A\frac{1-\phi_f}{\phi_f A_1} = r_R Q_A \qquad (5)$$

where $\left(1-\phi_f\right)/\phi_f$ is the average liquid to air fraction, A_1 is the cross sectional area in the centre of the froth volume, and $r_R = \frac{1-\phi_f}{\phi_f A_1}$.

It is assumed under ideal conditions that $V_{PB} = V_{FB}$. Also, it is assumed the ratio of V_{FL} to V_{FB} is equal to the ratio of Q_C to Q_A, i.e. $\frac{V_{FL}}{V_{FB}} = \frac{Q_C}{Q_A}$. Therefore, given these assumptions and ratios, Q_C can be expressed in terms of Q_A, V_{PL} and V_{FL} as:

$$Q_C = \frac{2V_{FL}Q_A}{V_{Tot} - V_{PL} - V_{FL}}. \qquad (6)$$

Performance measures The performance of the flotation process is measured according to the grade (γ_j) and recovery (υ_j) of species j in the concentrate:

$$\gamma_j = C_j^F \qquad (7a)$$

$$\upsilon_j = \frac{Q_C C_j^F}{Q_{Fd}C_j^{Fd}}. \qquad (7b)$$

Since grade and recovery are inversely proportional (Bauer and Craig, 2008), a trade-off exists between these variables. The control challenge is to achieve the optimal relation between γ and υ.

2.4 State-space representation

The model of the flotation cell can be represented as a multi-input-multi-output non-linear state-space model:

$$\begin{aligned} \dot{x} &= g\left(x\right)u \\ y &= h\left(x,u\right). \end{aligned} \tag{8}$$

Since there are four mineral species in the data of Jämsä-Jounela (1992), the variable sets are:

$$x = [C_1^P, C_2^P, C_3^P, C_4^P, C_1^F, C_2^F, C_3^F, C_4^F, V_{PL}, V_{FL}]^T \tag{9a}$$

$$u = [Q_{Fd},\ Q_A,\ Q_T]^T, \tag{9b}$$

where x is the process states, and u is the manipulated variables. A constant collector and frother addition rate is assumed and is not included in the set of manipulated variables. In general, there is a linear relationship between the adhesion/detachment rates and chemical additions, which can be modelled if sufficient sampling campaign data is available.

With the use of (3)-(6), the differential equations in (1) and (2) can be expressed in terms of the states and manipulated variables in (9) for $j = 1, \ldots, 4$ as:

$$\begin{bmatrix} \frac{dC_j^P}{dt} \\ \frac{dC_j^F}{dt} \\ \frac{dV_{PL}}{dt} \\ \frac{dV_{FL}}{dt} \end{bmatrix} = \begin{bmatrix} \frac{C_j^{Fd}}{V_{PL}} & A_{12_j} & \frac{-C_j^P}{V_{PL}\left(1+\alpha_j^P\right)} \\ 0 & A_{22_j} & 0 \\ 1 & r_R - r_E & -1 \\ 0 & r_E - r_R - \frac{2V_{FL}}{V_X} & 0 \end{bmatrix} \begin{bmatrix} Q_{Fd} \\ Q_A \\ Q_T \end{bmatrix} \tag{10}$$

where

$$\begin{aligned} A_{12_j} &= \frac{r_R k_j^R C_j^F}{\left(1+\alpha_j^F\right)V_{PL}} - \frac{r_E C_j^P}{\left(1+\alpha_j^P\right)V_{PL}} - \frac{2\alpha_j^P C_j^P}{\left(1+\alpha_j^P\right)V_X} \\ A_{22_j} &= \frac{r_E C_j^P}{\left(1+\alpha_j^P\right)V_{FL}} + \frac{2\alpha_j^P V_{PL} C_j^P}{\left(1+\alpha_j^P\right)V_X V_{FL}} - \frac{r_R k_j^R C_j^F}{\left(1+\alpha_j^F\right)V_{FL}} - \frac{2C_j^F}{V_X} \\ V_X &= V_{Tot} - V_{PL} - V_{FL}. \end{aligned}$$

3. CONTROLLABILITY THEORY

This section introduces the controllability theory for the analyses in Section 4.

A multi-input-multi-output control-affine non-linear state-space model with $\dim(x) = n$ and $\dim(u) = m$ can be written as

$$\dot{x} = f\left(x\right) + \sum_{i=1}^{m} g_i\left(x\right)u_i. \tag{11}$$

The system in (11) is said to be *weakly* controllable at x_0 if there exists an input $u(t)$ such that any initial state $x(t_0)$ in a neighbourhood X_0 of x_0 can be driven to any other state $x(t_1) \in X_0$ in finite time $t_1 > t_0$. The system is said to be *locally* weakly controllable at x_0 if it is weakly controllable and there exists a neighbourhood $X_1 \subset X_0$ such that $x(t) \in X_1$ for $t_0 < t < t_1$ (Doyle III and Henson, 1997). If the system in (11) satisfies the so called controllability rank condition:

$$\dim\left(\mathcal{C}\right) = n \tag{12}$$

the system is locally (weakly) controllable (Hermann and Krener, 1977). The distribution $\mathcal{C}$ is given by:

$$\begin{aligned} \mathcal{C} = \text{span}\{\ & g_1, \ldots, g_m, \\ & \left[g_1, g_2\right], \left[g_1, g_3\right], \ldots, \\ & \left[g_1, \left[g_2, g_3\right]\right], \left[g_2, \left[g_1, g_3\right]\right] \ldots, \\ & \left[f, g_i\right], \left[f, \left[f, g_i\right]\right], \ldots\}. \end{aligned} \tag{13}$$

Note, $[f, g] = \frac{\partial g}{\partial x} f - \frac{\partial f}{\partial x} g$ is the Lie bracket. In the case where the non-linear system is linearised at a specific steady-state operating point (x_0, u_0), the controllability distribution corresponds to the controllability matrix $\mathcal{C} = \left[B, AB, \ldots A^{n-1}B\right]$ where $B = \frac{\partial}{\partial u} g(x)u|_{x_0,u_0}$ and $A = \frac{\partial}{\partial x}\left(f(x) + g(x)u\right)|_{x_0,u_0}$. The non-linear system in (11) is regarded as drift free if $f(x) \equiv 0$. The linearised case of a drift free system is not controllable as $A = 0$ for all x. However, the non-linear system may still satisfy the controllability rank condition such that the drift free system is controllable.

4. ANALYSIS AND RESULTS

The controllability of a single flotation cell is evaluated against the industrial sampling campaign data by Jämsä-Jounela (1992). The four mineral species present in the flotation cell are apatite ($j = 1$), calcite ($j = 2$), dolomite ($j = 3$), and other ($j = 4$). The cell primarily concentrates apatite, but calcite and dolomite are also present in the concentrate stream. The cell aims to improve the separation between apatite ($j = 1$) and gangue material ($j = 4$) while maintaining stable operation. In other words, the control aim is to increase C_1^F and C_4^P while V_{PL} remains within acceptable boundaries. Therefore, the aim of the analysis is to determine if the states C_1^F, C_4^P and V_{PL} in (10) are controllable using the manipulated variables in (9b).

4.1 Controllability of all process states

There are ten process states and three inputs in (9a), i.e. $n = 10$ and $m = 3$ for the state-space in (11). Therefore, the distribution $\mathcal{C}$ in (13) for the flotation system is given by:

$$\begin{aligned} \mathcal{C}_{10} = [\ & g_1, g_2, g_3, \left[g_1, g_2\right], \left[g_1, g_3\right] \left[g_2, g_3\right], \\ & \left[g_1, \left[g_2, g_3\right]\right], \left[g_2, \left[g_2, g_3\right]\right], \\ & \left[g_3, \left[g_2, g_3\right]\right], \left[g_2, \left[g_1, g_2\right]\right]\ ] \end{aligned} \tag{14}$$

where the functions g_i for $i = 1, 2, 3$ are given by the three columns of the system represented in (10).

The distribution $\mathcal{C}_{10}$ shown in (14) was evaluated with the Symbolic Math Toolbox of MATLAB. The full distribution is not shown here because of space restrictions. The result is $\dim\left(\mathcal{C}_{10}\right) = 10$, which implies all ten states are controllable using the three available manipulated variables. (Note, there are various combinations of repeated Lie brackets which can be used to form the distribution $\mathcal{C}_{10}$ in (14) such that $\dim\left(\mathcal{C}\right) = 10$. Only one such combination is shown in (14). It is only in the case when $[g_i, [g_1, g_3]]$ is used to form $\mathcal{C}_{10}$ that the dimension of the distribution reduces to 9.)

Since all ten states are controllable, the analysis indicates that the volume of pulp liquid and the mineral concentrations in the concentrate and tailings can be driven from one condition (x_0) to the next (x_1) using only 3 manipulated variables. Although the system is under-actuated and not functionally controllable, all states remain controllable. (A system is functionally controllable if there are at least as many manipulated variables as controlled variables, i.e. independent control of the controlled variables is possible (Skogestad and Postlethwaite, 2005).) A disadvantage of the analysis is that it does not indicate the magnitude, direction or time period of the control actions required to move from x_0 to x_1, simply that it is possible to move from x_0 to x_1.

4.2 Controllability of a reduced set of process states

The control goal mentioned previously considers two mineral species: apatite and gangue. If the analysis is limited to consider only these two mineral species, $j = 1$ and $j = 4$, there are only six process states to consider. In other words, $n = 6$ and $m = 3$ for the state-space in (11). This is a more tractable problem. In this case the distribution $\mathcal{C}$ in (13) for the flotation system is given by:

$$\mathcal{C}_6 = [\ g_1, g_2, g_3, [g_1, g_2], [g_1, g_3] [g_2, g_3]\ ]. \tag{15}$$

The result is $\dim(\mathcal{C}_6) = 6$, which implies all six states are controllable from the three available manipulated variables.

As mentioned in Section 2.2, the variable Q_{Fd} is not always readily available as a manipulated variable. This variable can be regarded as a disturbance such that only Q_A and Q_T are used as manipulated variables. Therefore, for $n = 6$ and $m = 2$ for the state-space in (11), the distribution can be represented as:

$$\begin{aligned}\mathcal{C}_{6'} = [\ \ & g_2, g_3, [g_2, g_3] [g_2, [g_2, g_3]], [g_3, [g_2, g_3]], \\ & [g_2, [g_3, [g_2, g_3]]]\ ].\end{aligned} \tag{16}$$

The result is $\dim(\mathcal{C}_{6'}) = 6$, which implies all six states are controllable from only two available manipulated variables: Q_A and Q_T.

As observed in (1), there is strong coupling between C_j^P and C_j^F. If for example the requirement is to increase C_1^F, it will naturally cause a reduction of C_1^P, and vice-versa. Therefore, control of C_1^F will imply a degree of control of C_1^P. Similarly, a strong coupling exists between V_{PL} and V_{FL} as seen in (2).

Therefore, based on the state controllability results, it is reasonable to expect a controller to achieve the control aim to maximise the separation between C_1^F in the concentrate and C_4^P in the tailings while maintaining V_{PL} within acceptable limits. This can be achieved with the use of only two manipulated variables, Q_A and Q_T, but the inclusion of Q_{Fd} may ease the task.

Although state controllability is neither a sufficient nor necessary condition for to design feedback controllers with acceptable performance, it affirms the possibility of moving the system from one condition to the next with the available manipulated variables.

5. CONCLUSION

The aim of this study was to investigate the controllability of a flotation cell represented by the simplified model of Bascur (1982). Since the function of a flotation cell is to separate valuable minerals from gangue, a controller should aim to maximise this separation by drawing as much valuable mineral to the concentrate and as much gangue to the tailings. However, the controller should not neglect control of the pulp volume in the cell.

The controllability study indicates that if the feed-rate of slurry to the cell (Q_{Fd}), the air flow-rate into the cell (Q_A), and the tailings flow-rate out of the cell (Q_T) can be manipulated, it is possible to achieve a specific pulp liquid volume (V_{PL}), a specific concentration of the valuable mineral in the concentrate (C_1^F), and a specific concentration of the gangue mineral in the tailings (C_4^P). Although these variables cannot be controlled independently, it is possible to design a controller to maximise the separation between C_1^F and C_4^P for a specific V_{PL} with the use of only three manipulated variables.

The results of this study indicate that the manipulated variables, Q_A, Q_{Fd}, and Q_T, which generally form part of the primary control level of the flotation cell, are sufficient to drive the metallurgical objectives of the cell. Although all the states are controllable in the case where four different mineral species are represented by the model, the control moves to drive the system from one condition to the next may be unrealistic. The problem becomes more tractable in the case where only two mineral species are considered. For two mineral species only Q_A and Q_T is sufficient to control the process.

Future research involves developing a non-linear controller capable of stabilizing the plant and maximising the metallurgical objectives of the plant with the use of only three manipulated variables as mentioned above. Further studies can also include a controllability analysis of multiple flotation cells connected in series such as in Jämsä-Jounela et al. (2003).

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