INVENTORY MANAGEMENT FOR GROWING ITEMS IN MULTI-ECHELON SUPPLY CHAINS

by

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Abstract

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Abstract

Growing items have, in recent years, emerged as a distinct class of items within inventory modelling, similar to perishable or repairable items, for example. This class of items includes livestock, crops and fish, to name a few. The importance of inventory models developed specifically for growing items is due to the utility of these items to humanity and the financial implication that a poor inventory management system can have on a business. Most growing items are consumed as food products, albeit in a form that is suitable for human consumption. From a financial standpoint, inventory often accounts for the biggest portion of the current assets of a business' balance sheet and therefore, a poorly managed inventory system has the potential to financially cripple a business.

The objective of this research is to develop models for managing growing inventory items in multi-echelon supply chain settings. Food production operations are complex industrial systems that often involve multiple entities and processes. Food production systems often start with farming operations at the upstream end of the supply chain, where the live growing items are reared, and end with retail operations at the downstream end of the supply chain, where consumable food products are sold to end-users. The farming and retail ends of the supply chain are often connected by various forms of value-adding operations such as, in the case of livestock, de-feathering, stunning, slaughtering, processing and packaging. These value-adding activities transform the live items into a form that is safe for sale and consumption. Hence, a multi-echelon supply chain structure best represents these operations. This is a departure from most current literature whereby the models not only ignore the value-adding operations but also the multi-echelon nature of food production systems. By accounting for these shortcomings in the current literature, the models presented in this thesis are more realistic and are thus, useful for operations

and supply chain management practitioners who can use them when making ordering and shipment decisions in multi-echelon supply chains that involve growing items.

Furthermore, issues such as item mortality, quality control, pricing decisions, quantities of stock on shelves and expiration dates are also taken into account. These issues are important in food production systems and this further enhances the practical use of the models. The importance of these issues, along with that of collaboration between all supply chain members, is quantified through numerical experimentation. In certain instances, the profit generated across the supply chain can increase by as much as 15% if all members collaborate and integrate their ordering and shipment decisions. Prolonging the shelf life (expiration date) of food products by 40% can increase supply chain profits by as much as 21%. Furthermore, supply chain profits can be increased by as much as 10% and 21%, respectively, if survival rates of live inventory items and acceptable quality levels of the processed inventory are kept at 100%. While 100% survival rates and 100% acceptable quality levels might not be possible in reality, operations and supply chain management practitioners should strive to keep them as high as possible. Practitioners should also invest in preservation technologies that have the potential to improve the freshness of products. All these measures, along with increased collaboration between supply chain members, can be used by supply chain practitioners to increase profits across food production systems.

Research Outputs

The following research articles, all of which are derived from the work carried out for this thesis, have either been accepted for publication or are currently under review at accredited journals:

1. Sebatjane, M., Adetunji, O. A three-echelon supply chain for economic growing quantity model with price- and freshness-dependent demand: Pricing, ordering and shipment decisions. *Operations Research Perspectives* (Accepted for publication) `https://doi.org/10.1016/j.orp.2020.100153`

2. Sebatjane, M., Adetunji, O. Optimal inventory replenishment and shipment policies in a four-echelon supply chain for growing items with imperfect quality. *Production & Manufacturing Research* (Accepted for publication) `https://doi.org/10.1080/21693277.2020.1772148`

3. Sebatjane, M., Adetunji, O. Three-echelon supply chain inventory model for growing items. *Journal of Modelling in Management* (Accepted for publication) `https://doi.org/10.1108/JM2-05-2019-0110`

4. Sebatjane, M., Adetunji, O. Optimal lot-sizing and shipment decisions in a three-echelon supply chain for growing items with inventory level- and expiration date-dependent demand. *Applied Mathematical Modelling* (Revised manuscript resubmitted for second-round review)

5. Sebatjane, M., Adetunji, O. A four-echelon supply chain for growing items with imperfect quality and errors in quality inspection. *Annals of Operations Research* (Submitted for review)

6. Sebatjane, M., Adetunji, O. Optimal inventory replenishment and shipment policies in a three-echelon supply chain for growing items with expiration dates. *Opsearch* (Submitted for review)

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List of Acronyms

3PL Third Party Logistics.

DCF Discounted Cash Flow.

EGQ Economic Growing Quantity.

EOQ Economic Order Quantity.

EPQ Economic Production Quantity.

JELS Joint Economic Lot-Size.

LTL Less Than Truck Load.

SKU Stock Keeping Unit.

SSMD Single Setup-Multiple Deliveries.

SSSD Single Setup-Single Delivery.

TL Truck Load.

ZAR South African Rand.

Chapter 1

Introduction

1.1 Context

A vast majority of consumable food products are sourced primarily from either crops or livestock, both of which are living organisms. One of the defining characteristics of living organisms is their ability to grow. In the context of inventory modelling literature, crops and livestock constitute an important class of items, referred to as growing items. In recent years, growing items have emerged as an important research area within inventory modelling because of their role in food production systems.

End users seldom consume these growing items in their original form. There is usually some form of value-adding activity, through processing in most instances, that converts the growing items from their original form to a new form that is consumable and saleable. For example, live chickens which are growing items in their original form are never consumed in that form. Instead, the live chickens are slaughtered, processed into various forms (such as whole chicken portions, sausages, among other products) and packaged prior to being sold to end-users. For this reason, food production systems are complex industrial operations that involve multiple processes and multiple parties. In the context of inventory modelling literature, these processes and parties can be represented by supply chain echelons and supply chain members, respectively.

For the sake of simplicity, food supply chains can be envisioned as being comprised of three main supply chain members, namely, farmers, processors and retailers (Sebatjane and Adetunji, 2020c). The primary role of farmers is to rear the live growing items to maturity, while that of processors is to transform the mature live items into a saleable and consumable form through processing. Processing is an aggregated set of activities that encompasses individual activities such as de-feathering, stunning, slaughtering, chilling, cutting and packaging, in the case of livestock. The role of retailers is to sell the processed products to end-users. In the most basic form of this supply chain, the primary activities that occur at each of the three supply chain members' facilities are growing (or farming), processing and retailing. These activities represent echelons of the supply chain and thus, the most basic form of this supply chain setup has three echelons. However, in some instances, additional activities can occur at the supply chain members' facilities. For instance, a processor might add a quality control checkpoint to ensure that the products delivered to the retailer are of acceptable quality. This may represent an additional echelon to the supply chain setup.

Basic food products are a necessity for existence. This means that their availability and affordability to consumers are of utmost importance. The availability and affordability of food products at retail outlets, which represent the downstream echelon of the broader food supply chain, are dependent on factors from the upstream echelons of the supply chain such as the lot-sizes and growth rates (of the live newborn items) and the processing rates and holding costs (of the processed products), among other factors. The only way to avoid unavailability of food products is through proper inventory management techniques that are not only focused on the retail end of the supply chain but rather, encompass the entire food supply chain from the upstream farming echelon. Therefore, inventory management is a critical activity in food production systems. Inventory management in these settings is a complex activity not only because of the vast amounts of product varieties but also because of the number of supply chain members involved in the production of the products. Therefore, the availability and affordability of food products can be improved through better inventory management techniques driven by scientific principles. Accordingly, this thesis is aimed at developing models for managing growing inventory items in multi-echelon supply chains.

1.2 Relevance

1.2.1 The importance of growing items

Dani (2015) observed that most food supply chains are comprised of producers, processors, distributors, retailers and consumers. The producers are typically farmers who are involved in the production of fruits, vegetables, grains or livestock, among others. Processors transform the items from the producers into various saleable food items which meet consumer needs. Logistics providers ensure that the saleable food items reach consumers, who typically purchase these food items through retailers, in an acceptable condition. The business of food producers is typically the production (in the form of rearing, in the case of livestock) of growing items.

1.2.2 The importance of multi-echelon supply chain systems

Globalisation, outsourcing and rapidly evolving technology are some of the factors shaping today's competitive business landscape. In order to survive in this competitive landscape, businesses must offer improved products and services at reduced costs to customers. Consequently, businesses are forced to increase the efficiency of their operations as a way of reducing their costs and increasing their responsiveness. As a natural result, businesses are forced to look beyond the boundaries of their own operations in efforts to reduce their costs and increase their responsiveness further. Businesses have started to integrate and coordinate various decisions, such as inventory replenishment policies, not only within their own boundaries but with their suppliers and customers. Due to the growing focus on supply chain management in recent years, a lot of businesses have realised that inventories, which often cut company boundaries when moving from their point of origin to their destinations, can be managed more effectively through greater integration and collaboration among supply chain members.

1.3 Research gap identification

Currently, there aren't much known lot-sizing models in the literature for integrated multi-echelon supply chains where growing items are the primary source of the chain (i.e. growing items are at the upstream end of the supply chain). These types of models are tailored specifically for food production systems. In the current literature, most lot-sizing models that account for some features of food production systems, such as price-, inventory level- and freshness-dependent demand, quality control, and expiration dates, were developed from the perspective of a retailer and are thus, limited to the retail end of the supply chain. Consequently, these models did not account for the preceding stages in the supply chain. The production of food products often involves several stages. In the most simple supply chains, these stages are often the rearing of live inventory, the processing of the live inventory into a consumable form and the selling of the consumable (or processed) inventory to end consumers. Considering that supply chains are intricate networks with multiple echelons, it is important to study lot-sizing models in multi-echelon supply chains because they are more representative of real-life inventory systems.

1.4 Objectives

The main objective of this thesis is to develop inventory models for managing growing items in multi-echelon supply chains. Six inventory models are presented in this thesis and each of them represents a sub-objective of the thesis. Therefore, the sub-objectives of this thesis are to develop:

- A three-echelon supply chain inventory model for growing items.
- A three-echelon supply chain inventory model for growing items with price- and freshness-dependent demand.
- A four-echelon supply chain inventory model for growing items with imperfect quality.
- A three-echelon supply chain inventory model for growing items with expiration dates.
- A four-echelon supply chain inventory model for growing items with imperfect quality and errors in quality inspection.
- A three-echelon supply chain inventory model for growing items with inventory level- and freshness-dependent demand.

Brief descriptions of the six models are outlined as follows:

1.4.1 Sub-objective 1: A three-echelon supply chain inventory model for growing items

Food production serves a very important role in society. A simple food production system often starts with the rearing (growing) of live items, followed by the processing of the items into a consumable and saleable form and ends with the selling of the processed items to consumers. An integrated inventory model for a three-echelon supply chain, with a farmer, a processor and a retailer, is developed. The farmer grows newborn items and then delivers them to a processor once the items mature. At the processing plant, the items are slaughtered, cut, processed and packaged at a certain rate. In the context of the model, all activities that occur at the processing plant are collectively termed processing and they are assumed to occur at a finite rate. The processor then delivers a certain number of equally-sized batches of processed inventory to the retailer who satisfies customer demand. The proposed supply chain inventory system is formulated as a cost minimisation problem with the cycle time (and by extension the order quantity) and the number of shipments as the decision variables.

1.4.2 Sub-objective 2: A three-echelon supply chain inventory model for growing items with price- and freshness-dependent demand

The demand for perishable food products is often influenced by the selling price and the age of the items. This is because perishable food products, which include most grocery items, have become commodities from consumers' point of view, hence, there are very little differences between competing brands. Consequently, factors like price and freshness (or age) become important determinants of consumer demand. This fact has been used to develop numerous inventory control models for perishable products. However, all these models were developed from the perspective of a retailer. Today's increasingly competitive business environment has forced companies to collaborate with fellow supply chain members in an effort to improve profitability and operational efficiency. With this in mind, a model for inventory management, in a perishable food products supply chain that begins with farming operations where live inventory items are reared and ends with the consumption of processed inventory, is developed. The farming and consumption (retail) stages are connected by a processing stage during which live inventory is processed into a consumable form. Consumer demand at the retail stage is a function of the selling price and the freshness of the processed inventory. The farming, processing and retail stages are the three-echelons of the proposed supply chain aimed at maximising the joint supply chain profit.

1.4.3 Sub-objective 3: A four-echelon supply chain inventory model for growing items with imperfect quality

Quality control is an important consideration in food production systems which often start with farming and processing operations and finish with consumption. This study develops an integrated inventory control model for a four-echelon supply chain (with farming, processing, screening and retail operations). The farmer grows newborn items and then delivers them to a processor once the items are mature enough. At the processing plant, the items are slaughtered, processed, packaged and screened for quality. The processor then delivers a certain number of equally-sized batches of good quality processed inventory to the retailer who satisfies customer demand for good quality processed inventory. The processor sells the processed poorer quality inventory at a discounted price and as a single batch to secondary markets. The proposed supply chain inventory system is formulated as a profit maximisation problem with the number of batches of good quality processed inventory and the order quantity as the decision variables.

1.4.4 Sub-objective 4: A three-echelon supply chain inventory model for growing items with expiration dates

A vast majority of consumable food products have a specified shelf life or expiration date. The products are no longer suitable for consumption beyond their expiration dates. Furthermore, most of these products are derived from a variety of growing items such as crops or livestock and there is usually some form of processing performed on the live items in order to transform them to a consumable form. Using this logic, a model for managing growing inventory is developed for a three-echelon supply chain, with farming, processing and retailing operations. At the farming echelon, the items are reared but there is the possibility that some of them might die. The surviving items are then transferred to the processing echelon for slaughtering, processing and packaging. The final echelon is retail where the processed items, which have a specific shelf life or expiration date, are used to meet consumer demand.

1.4.5 Sub-objective 5: A four-echelon supply chain inventory model for growing items with imperfect quality and errors in quality inspection

In order to safeguard the livelihood of consumers, food producers are required, either by law or regulatory bodies, to inspect their products for quality before selling the products to consumers. This is because food processing, as is the case with most production systems, is not perfect and there is a possibility that some of the processed products might not meet the required quality standard. Likewise, the inspection process is seldom perfect meaning that it is subject to errors and thus, it is possible that some of the processed products might be incorrectly classified. In light of this, an inventory control model for a four-echelon food processing supply chain is developed. The supply chain has a farming echelon where live items are grown; a processing echelon where the live items are transformed into processed inventory; an inspection echelon where the processed inventory is classified into good and poorer quality classes under the assumption that the inspection process is subject to type I and type II errors; and a retail echelon where the processed inventory of good quality is sold to consumers. The supply chain is modelled as a profit maximisation problem.

1.4.6 Sub-objective 6: A three-echelon supply chain inventory model for growing items with inventory level- and freshness-dependent demand

The freshness condition of food products have a major effect on consumers' purchasing behaviour for those products. Furthermore, marketing theory has shown that increasing the levels of inventory on display may stimulate consumer demand. Moreover, the primary source of a vast majority of perishable food products is growing items such as livestock and crops. Before these growing items can be consumed, they are often processed into a form that is safe for human consumption. With all these factors in mind, an integrated model for inventory control in a three-echelon supply chain for growing items, with farming, processing and retail echelons, is formulated. Consumer demand at the retail end of the supply chain is assumed to be dependent on the inventory level and the freshness condition, a function of the expiration date, of the products. The effectiveness of a profit enhancement mechanism which relaxes the traditional zero-ending inventory policy at the retail end of the supply chain is investigated. In this situation, it is assumed that the retailer always keeps on-hand inventory and starts a new replenishment cycle once the inventory level drops to a certain minimum value. The retailer has a clearance sale at the end of the cycle to clear out the ending on-hand inventory.

1.5 Thesis organisation

Apart from the introductory chapter, this thesis has four additional chapters that are are organised as follows:

Chapter 2 presents a review of relevant inventory models in the literature. The review is limited to inventory models for growing items, items in multi-echelon inventory systems, items with price-dependent demand, items with imperfect quality, perishable items with expiration dates, items with imperfect quality and errors in quality inspection and items with inventory level-dependent demand.

Chapter 3 builds on the literature review by reviewing the mathematical theory behind a few of the reviewed models that form the foundation for the development of the novel models that are presented in Chapter 4.

The main objective of the thesis is realised in Chapter 4 through the development of six inventory models for growing items in multi-echelon supply chain settings. Each of the models is developed under certain conditions which represent situations that might arise in food production systems. The conditions include price-, inventory level- and freshness-dependent demand, quality control, quality control with errors in the quality inspection process and expiration dates.

The thesis is concluded in Chapter 5 through brief discussions of the summary of findings from the thesis; the contributions made by the thesis to the literature on inventory management; the benefits that operations and supply chain management practitioners in industry can draw from the results of the thesis; and possible areas for future research on inventory management for growing items in multi-echelon supply chains.

Chapter 2

Literature Review

2.1 Introduction

Inventory often accounts for the biggest portion of the current assets category of a business' balance sheet (Hillier and Lieberman, 2001). Consequently, issues in inventory management can have huge negative financial effects on an entire business. Beyond the financial implications, a poorly management inventory system can reduce customer satisfaction levels which inadvertently leads to even more financial strain because of the resulting lower customer retention rates. These negative effects not only affect the focal business but they ripple across the entire supply chain.

Inventory management is about optimising the quantity of on-hand stock. It is essentially a balancing act aimed at avoiding two undesirable extremes of a spectrum, with one being overstocking and the other understocking. When a business overstocks a product, not only does the business incur relatively high costs for keeping the product in storage (in addition to the cost of procuring the product), the business also loses out on opportunity costs that might have been gained as a result of the capital investment in inventory. On the other hand, if a product is understocked, the business has to turn away customers. This not only leads to lost sales but also poor customer satisfaction levels and lost potential repeat business in the future.

An effective inventory management system is imperative for business success. Consequently, decisions about the quantity of stock to order and the frequency of replenishing the stock are important management issues (Stevenson, 2018). Mathematical models are often used to aid management in reaching decisions about order quantities and frequencies. Harris (1913) developed the first model aimed at addressing these decisions. This model is often referred to as the economic order quantity (EOQ) model and it was aimed at determined the optimal lot size (i.e. order quantity) and replenishment frequency that minimised the total inventory management costs. In its most basic form, the EOQ model achieves its objective by balancing the holding costs (of storing the inventory) with the fixed costs (of placing an order).

2.2 Relevant inventory management models

Owing to the restrictive assumptions in Harris (1913)'s model which limited its practical applications to most real life inventory systems, various researchers have extended the model, either by relaxing the original assumptions (both implicit and explicit) or adding new ones, to suit different practical settings (Andriolo et al., 2014). These new extended lot-sizing models are often more useful to practitioners. Several of such lot-sizing models form the basis of the work presented in this thesis. These are models developed specifically for growing items, items in multi-echelon supply chain systems, items with imperfect quality, perishable items with expiration dates, items with imperfect quality and quality inspection errors, items with price-dependent demand and items with inventory-level dependent demand, which are attributed to Rezaei (2014), Goyal (1977), Salameh and Jaber (2000), Sarkar (2012), Khan et al. (2011), Whitin (1955) and Baker and Urban (1988), respectively.

2.2.1 Lot-sizing models for growing items

Through the development of an EOQ model for items that experience a weight increase during a replenishment cycle, Rezaei (2014) introduced a new class of items to inventory control modelling, namely growing items. These items are not suitable for consumption at the time they are procured, so before they are used to meet demand (i.e. consumed), they are fed and consequently, enabled to grow. Examples of items that fall under this class include seafood, livestock and grains, to name a few.

Given the foundational nature of Rezaei (2014)'s model, in terms of not accounting for shortages, quantity discounts and multiple-items, among other popular features of EOQ models, the model has been extended to account for some these shortcomings. For example, Khalilpourazari and Pasandideh (2019), Nobil et al. (2019) and Sebatjane and Adetunji (2019b) extended the model to scenarios with multiple items, shortages and quantity discounts, respectively. Khalilpourazari and Pasandideh (2019) solved the multi-item variant of Rezaei (2014)'s EOQ model through an exact solution methodology for small problem sizes and two semi-heuristic algorithms for medium and large problem sizes because of the proposed model's non-linearity and the presence of multiple local optimum solutions. Nobil et al. (2019) extended Rezaei (2014)'s work by assuming that shortages are permitted and are fully backordered during the consumption (or selling) period of the replenishment cycle. Sebatjane and Adetunji (2019b) incorporated quantity discounts, incremental quantity discounts to be exact, to the literature on lot sizing for growing items. Since most growing items are sold as various food items downstream in retail supply chains and these supply chains are often characterised by low profit margins, quantity discounts are a one of improving margins through increased increased purchase volumes. Besides the incorporation of these common EOQ extensions, other researchers have extended the model by accounting for specific characteristics of food production systems. For instance, noting that food products are often screened for quality before being put on sale, Sebatjane and Adetunji (2019a) considered a situation where a random percentage of the matured items is of inferior quality and as a result, it is removed from the lot and salvaged. Despite the presence of two revenue streams in this situation, one from the good quality inventory used to meet demand and the other from salvaging the inferior quality inventory, the overall impact of having higher percentages of inferior quality items was negative since more items have to be ordered to meet a given rate of demand. Another extension based on the characteristics of food production systems was presented by Malekitabar et al. (2019) who considered a case study for trout fish production and developed a model for inventory control when there is a revenue sharing contract between the party responsible for growing the fish and the one responsible for selling it. In addition, the authors compared the effectiveness of the revenue sharing contract with a revenue and cost sharing contract and found the latter to be more cost efficient. Sebatjane and Adetunji (2020c) developed an inventory control model for a three-level supply chain for growing items with separate farming, processing and retail levels. Through the consideration of probability functions for survival and mortality throughout the growth period of the replenishment cycle, Gharaei and Almehdawe (2020) incorporated item mortality to Rezaei (2014)'s model and consequently created a new type of EOQ model, referred to as the economic growing quantity (EGQ) model. Given the presence of various illnesses and predators in food production value chains, the EGQ is more representative of an actual inventory control system for growing items (which are living organisms) because living organisms are not immune to death.

2.2.2 Joint economic lot-sizing models

The idea of coordinating inventory decisions in a multi-echelon system, which is often attributed to Clark and Scarf (1960)'s work, predates the concept of supply chain management. Building upon Clark and Scarf (1960)'s model, Goyal (1977) studied a coordinated vendor-buyer inventory system, in which it was assumed that a vendor resells products to a buyer. The aim of the study was to determine the economic lot-size for both parties, i.e. the joint economic lot-size (JELS), with the aim of minimising the joint total costs of managing inventory. In its most basic form, the JELS problem considers a vendor and a buyer involved in the production and selling, respectively, of a single type of item with the aim of finding the optimal inventory replenishment policy for both parties. Goyal (1977)'s model was developed under the assumption of an infinite production rate and a lot-for-lot production policy at the vendor. The shipment policy between the vendor and the buyer in Goyal (1977)'s JELS model would later be referred to as the single setup-single delivery (SSSD) policy, whereby the vendor produces for only one cycle and then delivers the entire lot to the buyer.

Over the years, numerous extensions of Goyal (1977)'s model have been presented starting with Banerjee (1986) who relaxed the infinite production rate assumption and extended the model to a case where the vendor produces items at a given (i.e. finite) production rate on a lot-for-lot basis. In this instance, the vendor produces enough items for just the period of interest. Goyal (1988) introduced the (SSMD) variant of Goyal (1977)'s model formulated under the assumption that the vendor produces enough items to supply the buyer with an integer number of orders (i.e. for a SSMD policy, the vendor makes multiple deliveries to the buyer) for each single production setup). The SSMD policy resulted in lower total system costs because of the smaller lot sizes that attract relatively cheaper holding costs and are consumed much quicker meaning they spend less time in storage. Lu (1995) relaxed the single vendor-single buyer assumption and formulated an inventory model for a situation whereby the buyer is supplying products to more than one buyer. Hill (1999) investigated different replenishment policies in the integrated vendor-buyer inventory system. The aim was to investigate the benefits (or lack thereof) of adopting a different replenishment policy to the classic case where the vendor delivers equal sized shipments to the buyer. Producing and delivering unequal shipments (in particular, a different first shipment) led to slightly lower total costs.

Pan and Yang (2002) relaxed the zero lead-time assumption in Goyal (1977)'s model and formulated an inventory model for an integrated system under the assumption of a variable lead-time. Valentini and Zavanella (2003) incorporated the concept of consignment stock to the vendor-buyer inventory system by considering a case where the inventory is kept at the buyer's premises while it is legally owned by the vendor. When modelling the inventory system, the buyer's holding costs were divided into a financial component and a physical component, with the financial component being paid by the buyer and the physical component by the vendor. Ouyang et al. (2007) developed an extension that considered shortages, variable lead-time and quality improvement efforts. Chen and Kang (2007) presented a model that considered a case where the vendor permits the buyer to pay for the ordered items at a later stage following receipt of the items. Revenue sharing contacts are often put in place to enhance collaboration between supply chain members. Ho et al. (2008) studied a coordinated vendor-buyer inventory system where the vendor permits the buyer to delay payment and simultaneously offers the buyer discounts for paying in cash and at the same time, the buyer also permits their customer to delay payments. Giri and Bardhan (2012) incorporated revenue sharing to an integrated inventory system with a buyer and a vendor for an exponentially deteriorating item in a market with price-dependent demand. Sarkar (2013) investigated the effect of three different probabilistic deterioration functions, namely uniform, triangular and beta, on the inventory replenishment policy of a two-member supply chain. Geetha and Uthayakumar (2014) developed a model for jointly optimising pricing and replenishment policies in a two echelon (vendor-buyer) supply chain whereby the vendor not only allows the buyer to pay for the inventory at a later date through trade credit financing, but also grants the buyer freight discounts for transporting the inventory based on the weight of the order. Priyan and Manivannan (2017) studied a version of the JELS problem for a case where the vendor's production process produces some imperfect quality items that are screened out at the buyer's facility under the assumptions that the screening process is prone to errors and that the fraction of items that are of imperfect quality is a triangular fuzzy variable. Dye et al. (2018) presented an inventory model for a vendor-buyer system with deteriorating items and investments in preservation technologies in an effort to slow down the deterioration process. Most of the research published on the JELS problem seldom accounts for the cost of transporting goods from the vendor to the buyer, Wangsa and Wee (2018) developed a model which considered stochastic demand and compared two transportation modes, namely, less-than truck load (LTL) and truck load (TL) shipping. Saha et al. (2018) formulated an inventory control model for a two-echelon (manufacturer-retailer) supply chain aimed at optimising not only the retailer's order quantity, but also their selling price and delivery lead-time. Al-Khazraji et al. (2018) used particle swarm optimisation to solve a multi-objective model for inventory control in a supply chain with a single factory and a single retailer when the demand rate is assumed to be fluctuating. Jaipuria and Mahapatra (2019) investigated bullwhip effects on a supply chain under a novel forecasting technique. The technique was developed by combing an existing forecasting technique, namely discrete wavelet transformation, with a genetic algorithm previously used to forecast demand. Furthermore, the supply chain under study was assumed to have adopted a periodic review policy for managing its inventory. Taleizadeh et al. (2019) analysed two different inventory control policies in an imperfect manufacturing system with multiple items. The two policies stem from possible cases derived from the way in which imperfect quality items are dealt with, the first being that they are reworked and the second being that the items are sold at a discounted price. Jaggi et al. (2019) developed an integrated inventory control model for a two-member supply chain (with a buyer and vendor) dealing in deteriorating items under trade credit financing. Furthermore, the demand rate was assumed to be dependent on the amount of stock displayed on shelves and the vendor sold the items to the buyer on a short-term credit financing contract. Vats et al. (2019) investigated the effectiveness of a demand aggregation approach to inventory management in a supply chain with a multiple distributors and multiple retailers that makes of a reorder point ordering policy under stochastic demand conditions.

2.2.3 Lot-sizing models for items with imperfect quality

Item quality was incorporated to inventory management research by Salameh and Jaber (2000) through relaxing the implicit assumption made in most inventory models that all the items received in each order are of good quality. The authors proposed an inventory situation in which a specific percentage of the ordered items is of inferior quality. Before the items are sold, they are all subjected to a screening process in order to isolate the good quality items from those of poorer quality. A simpler method, with minimal effects on the expected total profit, for computing the EOQ in Salameh and Jaber (2000)'s model was proposed by Goyal and Cardenas-Barron (2002). Cardenas-Barron (2000) and Maddah and Jaber (2008) rectified computational mistakes made in Salameh and Jaber (2000)'s model pertaining to the EOQ and the expected total profit functions.

Imperfect quality was first considered in integrated inventory systems by Huang (2002) through extending Salameh and Jaber (2000)'s work to a supply chain with a vendor who produces items and a buyer who screens them for quality. Based upon Goyal and Cardenas-Barron (2002)'s correction of Salameh and Jaber (2000)'s model, Goyal et al. (2003) corrected the earlier integrated inventory control model for imperfect quality items proposed by Huang (2002). The models proposed by Huang (2002) and Goyal et al. (2003) were formulated under the assumption that the percentage of poorer quality items is random, Ouyang et al. (2006) deviated from this assumption and developed a model under the assumption that the poorer quality fraction is a triangular fuzzy number. Konstantaras et al. (2007) considered a version of the EOQ model for items with imperfect quality in a system where the items are inspected in-house at a secondary warehouse. Kreng and Tan (2011) relaxed the single buyer-single vendor assumption made in most models and developed one with a single buyer and multiple vendors. Kreng (2011) developed an extension which considered two-way trade-credit financing (i.e. the vendor offers the buyer a grace period to settle the bill and the buyer also offers a grace period to consumers). Hsu and Hsu (2013b) extended Goyal et al. (2003)'s model to a case with permissible (and fully backordered) shortages. Khan et al. (2014) studied an integrated inventory control system with imperfect quality items by taking the effects of two human factors, namely, learning in production (i.e. production efficiency improves from one production cycle to the next because of the experience the operators get) and errors in inspection (i.e. the inspectors can make mistakes during the screening process), into consideration. The recent emphasis on green supply chain management motivated Zanoni et al. (2014) to formulate a model for a two-echelon supply chain where the demand is a function of both the selling price and the environmental performance of the supply chain. Khan et al. (2016) developed a coordinated inventory control model for items with inferior quality for a vendor and a buyer in a supply chain where the vendor has the responsibility of managing the inventory at the buyer's facility. Jauhari and Saga (2017) developed an extension of the integrated vendor-buyer inventory control system with imperfect quality items by considering the demand rate to be stochastic, the ordering cost to be fuzzy and the (vendor's) production rate to be flexible. Castellano et al. (2017) studied an inventory control system with quality considerations, price discounts based on permitted shortages, stochastic demand and controllable lead time under a periodic review policy. Tiwari et al. (2018b) proposed an inventory control model for a vendor and a buyer producing and selling, respectively, deteriorating items with both environmental and quality considerations.

2.2.4 Lot-sizing models for perishable items with expiration dates

Perishable items are one of the most researched areas within inventory theory because of their applicability to various situations where inventoried items lose some of the utility or value over time. Following the publication of Ghare and Schrader (1963) model on perishable inventory control, numerous models which consider deterioration in different ways have been presented. Most notably, Covert and Phillip (1973) generalised Ghare and Schrader (1963)'s work by relaxing the constant deterioration rate assumption and considering a deterioration rate characterised by a Weibull distribution with two parameters. These two models have have spawned most of the literature on inventory management for deteriorating items. One of the most recent development in deteriorating inventory control is the incorporation of expiration dates, which in essence assumes that the items' deterioration rate is time dependent and consequently, the items have a maximum lifetime.

Recently, a number of studies which consider the deterioration to be a function of the inventory item's age have been formulated. These types of inventory systems are representative of retail stores selling products like fresh produce, meat and baked goods which have specific shelf lives or expiration dates. One of the earliest inventory control studies to explicitly consider expiration dates is by Hsu et al. (2007) who presented a model for deteriorating items with a maximum lifetime and a demand rate that varies seasonally. Sarkar (2012) proposed an EOQ model for a retailer selling items with an expiration date provided that the supplier permits the retailer to pay for the order at a later date (i.e. not at the time the order is delivered). Sarkar (2012) optimised an inventory system for expiring items under the assumption that the supplier of the items offers the firm selling the items trade credit financing (i.e. a grace period for settling debt for the ordered items). Wang et al. (2014) developed an extension of Sarkar (2012)'s model which deemed the demand rate to be a function of the duration of the credit period. Wu et al. (2014) formulated a model for optimising both the order quantity and the credit period in an inventory system with perishable items with an expiration date under the assumption that the retailer receives upstream trade credit from the supplier of the items and offers downstream trade credit to end consumers. Wu et al. (2016) used the items' expiration date to define a freshness index, stating in percentage terms how fresh an item is, and used it to develop a model for optimising the lot size provided that the demand rate for the items depends on both the inventory level and the freshness level. Teng et al. (2016) studied an inventory system for expiring items under an advance payment agreement between the supplier of the items and the seller whereby the supplier requires the seller to pay a certain portion of the procurement cost before the order can be delivered to the seller. Retailers selling products with expiration dates often discount the products as the expiration dates approach. Using this logic, Banerjee and Agrawal (2017) developed a model for optimising ordering, pricing and discounting policies for an inventory system consisting of expiring items with a demand rate that is dependent on the selling price. Feng et al. (2017) relaxed the assumption made in

Wu et al. (2016)'s model, that the inventory level at the end of a replenishment cycle is zero, and formulated a profit maximisation model (with the optimal cycle time, ending inventory and price as the decision variables) for an inventory system with expiration dates and freshness and price-dependent demand. Another extension to Wu et al. (2016)'s model was developed by Wu et al. (2018) through the consideration of a permissible delay in payment which is an incentive policy where by the supplier delivers the order to the seller and does not require payment at the moment of delivery but instead offers the seller a specific amount of time to settle the bill. Li and Teng (2019) incorporated trade credit financing to Wu et al. (2016)'s model and considered the duration of the credit period to be an additional decision variable. Khan et al. (2019) proposed an EOQ-type model for deteriorating products with expiration dates when shortages are permitted and end user demand depends on the products' selling prices.

2.2.5 Lot-sizing models for items with imperfect quality and inspection errors

Salameh and Jaber (2000) introduced the EOQ model for items with imperfect quality. This model was essentially an extension of classic EOQ model, formulated by Harris (1913), which relaxed the implicit assumption that the entire lot size is of perfect quality. Salameh and Jaber (2000) theorised an inventory situation where by the quality of a random fraction of the ordered items is of unacceptable to end users. The model assumed that the fraction of imperfect quality items is removed from the lot through a screening (or inspection) process and sold as a single batch at a discounted price when the inspection process ends. An implicit assumption of this particular model was that the inspection process is 100% effective at separating the good quality items from the poorer quality items. Khan et al. (2011) recognised that this assumption seldom holds for most production systems. Consequently, Khan et al. (2011) developed an extension to Salameh and Jaber (2000)'s work which accounted for an inspection process that is prone to errors. In their model, Khan et al. (2011) assumed that the inspection is subject to two types of errors, namely, type I and type II errors, representing situations where poorer quality items are classified as good quality items and where good quality items are classified as poorer quality items, respectively. Similar to the model by Salameh and Jaber (2000), it was assumed that the good quality items are sold throughout the replenishment cycle at a given price while the items of poorer quality are sold as a single batch once the inspection process ends at a discounted price. Furthermore, Khan et al. (2011) assumed that the incorrectly classified items of poorer quality (sold to consumer as good quality items) are returned throughout the cycle and incur penalty costs.

Given the relevance of the model to real life production system that have imperfect inspection processes, Khan et al. (2011)'s model has received considerable attention from various researchers. For instance, Hsu and Hsu (2013a) developed an extension of the model that accounted for shortages by assuming that the inventory system under study permits shortages which are fully backordered. Khan et al. (2014) extended the concept of inspection errors to a inventory system in a two-echelon supply chain with a single buyer and a single vendor. This particular model assumed that the vendor manufacturers inventory items that are used by the buyer (i.e. the seller or retailer) to meet consumer demand. However, some of the items manufactured by the vendor are of unacceptable quality. With this in mind, the buyer inspects the items prior to selling them to consumers but the inspection process is subject to type I and type II errors. The wrongfully classified items are returned by consumers. Chang et al. (2016) formulated an EOQ model for an inventory system in which a percentage of the items the seller receives is of imperfect quality and the seller has an imperfect inspection process which is not capable of correctly classifying all the items. Furthermore, the supplier of the items offers the seller trade credit financing whereby the seller is allowed a specified grace period to settle the bill for the items. Trade credit financing was the subject of another extension to Khan et al. (2011)'s model, this time by Zhou et al. (2016). Rout et al. (2019) extended the concept of inspection errors in an imperfect production process to an EPQ model for deteriorating items whose rate of decay is assumed to be a fuzzy random variable. Dey and Giri (2019) developed an inventory control model for a two-member supply chain, with a single vendor and a single buyer, where the vendor's production process is imperfect and consequently, a percentage of the items sent to the buyer, who inspect them for quality, is of unacceptable quality. Furthermore, the buyer's inspection process is assumed to be subject to both errors and learning. For the errors aspect, Dey and Giri (2019) assumed that the buyer;'s inspection process can make type I and type II errors and for the learning aspect, the authors assumed that the buyer's inspection process improves, in terms of correctly classifying the items, with every batch inspected.

2.2.6 Lot-sizing models for items with price-dependent demand

Lot sizing models for items with a demand rate that is influenced by the selling price have been studied since the publication of the seminal work by Whitin (1955). Price-dependent demand is still a popular topic in supply chain modelling as evidenced by recent works by Gan et al. (2018), Oliveira et al. (2018) and Raza and Govindaluri (2019), to name a few. In recent times, the demand rate's price dependency has been combined with various other factors. One of the more popular factors has been the freshness of the inventory items which is incorporated through the consideration of expiration dates.

The first inventory control model for perishable items with a demand rate that is influenced by the item's age and selling price is credited to Wu et al. (2016). In addition, the demand rate was assumed to also be a function of the item's inventory level. In developing the model, the authors also assumed a non-zero ending inventory policy whereby once the inventory reaches a certain point, it is salvaged so that it is not completely wasted after its expiration date. In addition, the capacity of the shelf space was assumed to be limited. The model was formulated as a profit maximisation problem with the cycle time, selling price and the ending inventory level as the decision variables.

Numerous researchers have built upon Wu et al. (2016) work. For example, Chen et al. (2016) formulated a model aimed at optimising not only the price, cycle time and ending inventory level, but also the available shelf space. Motivated by the fact that retailers often discount stocks of perishables when their expiration dates are approaching, Feng et al. (2017) developed an inventory control model for a retailer who has a closeout sale just before the items expire. Dobson et al. (2017) took a different approach to the assumption that the demand is a function of the age of the items and developed an EOQ model for a situation where customers gauge the freshness of the items before making a purchase and they can decide to either buy the item or not, regardless of its age. In addition to considering a demand rate that depends on the age, inventory level and selling price of a perishable item, Wu et al. (2017) incorporated a trapezoidal type demand pattern which is representative of most products' life cycles which are characterised by an increasing rate during the introduction phase, a flat rate at the maturity phase and a decreasing rate during the decline phase. Li et al. (2017) and Li and Teng (2018) incorporated advance payment schemes and reference selling prices, respectively, to Wu et al. (2016)'s model. In the advance payment model, the authors assumed that the supplier of the perishable items requires the retailer to pay a portion of the purchase price prior to receipt of the order. For the model that considers reference prices, the authors assume that the selling price has a certain threshold beyond which customers are not willing to purchase the items at all. Li and Teng (2019) included the length of the credit term as a third decision variable in Wu et al. (2016)'s model by extending it to a case where the supplier allows the retailer to purchase the items on credit and grants the retailer a certain amount of time to settle debt.

The aforementioned studies are all limited to the retail end of the supply chain. There have been a few studies dedicated to inventory management of fresh produce. Cai et al. (2010) formulated a model for optimising both the selling price and the replenishment policy in a fresh produce supply chain with a single producer, responsible for growing the produce, and a single distributor who is in charge of transporting the produce from the producer's facility to retail outlets. Cai et al. (2013) developed a model for maximising profit in a fresh produce supply chain with a producer, a third party logistics (3PL) provider and a distributor under the assumption that the demand rate for the produce is stochastic and sensitive to the selling price and the freshness condition of the produce. Ma et al. (2019) hypothesised a situation where the supply chain members do not have access to the same type of information such as order lead times, demand and delivery times, to name a few. This leads to a distortion in the amount of information and this is termed asymmetric information in economic theory. Ma et al. (2019) compared centralised and decentralised inventory replenishment policies in agricultural supply chains with as symmetric information provide that the demand for the agricultural products is price- and freshness-sensitive.

2.2.7 Lot-sizing models for items with inventory level-dependent demand rates

Motivated by the observation in Levin et al. (1972), Corstjens and Doyle (1981) and Silver and Peterson Silver and Peterson (1985), to name a few, that the presence of large piles of inventory induces consumer demand, Baker and Urban (1988) formulated an EOQ for items whose demand rate is a function of the on-hand inventory level. The authors used a power-form function to represent the demand rate's inventory level dependency and they assumed that a new replenishment cycle only starts when the inventory level in the current cycle reaches zero.

Mandal and Phaujdar (1989) presented an extension of Baker and Urban (1988)'s work that considered deteriorating items and made use of a linear function to represent the demand rate's inventory level dependency. Urban (1992) developed an extension that showed the benefits of adopting a non-zero ending inventory policy. While consumer demand has been shown to be affected by the inventory level, there are other factors that influence it too, for example, the selling price. With this in mind, Urban and Baker (1997) and Teng and Chang (2005) formulated inventory control models for items whose demand rate is a multivariate function of the inventory level and the selling price. Urban and Baker (1997)'s work also considered a non-zero ending inventory policy and a price reduction at the end of the replenishment cycle while Teng and Chang (2005)'s model considered deteriorating items. Hou and Lin (2006) used the DCF approach to study the effects of inflation and the time value of money on an inventory system with a demand rate that depends on both the selling price and the level of inventory of the items. Goyal and Chang (2009) developed an EOQ model for a system with an inventory level-dependent demand rate for a retailer who has a warehouse (used for storing inventory) and a display area where the inventory is sold with the aim of optimising the initial order quantity from the supplier and transfer quantity from the warehouse to the display area. Pando et al. (2012) presented a profit maximisation model for a scenario where the demand rate is an increasing function of the inventory level and the holding cost varies according to the amount of time the inventory has been in the system. Duan et al. (2012) developed two inventory control models, one with backordered shortages and the other without, for deteriorating items with a demand rate that depends on the inventory level. Krommyda et al. (2015) studied an inventory system with two substitutable items (i.e. a fraction of the demand rate for one item can be met with the second item in case of a stock out situation), each with a demand rate that is a function of the stock level of both items at a given time. Sargut and Isik (2017) developed a dynamic programming-based heuristic for optimising the inventory replenishment frequency and backorder quantity in a perishable inventory system with permitted shortages under production capacity constraints. Pando et al. (2019) studied an inventory system aimed at maximising the ratio of the profit generated to the total costs incurred under the assumption that the demand rate depends on the level of inventory and that the holding cost is a non-linear function of both time and the inventory level. Urban (2005) conducted an extensive review of the literature on inventory control models with inventory level-dependent demand rates.

2.3 Concluding remarks

The presented literature highlighted the contributions made by various researchers to the literature that forms the basis of the work presented in this thesis (namely, lot sizing models for growing items, multi-echelon inventory systems and items with imperfect quality, expiration dates, imperfect quality and inspections errors, price-dependent demand and inventory level-dependent demand). Despite its simplistic nature, the classic EOQ model, attributed to Harris (1913), is the groundwork behind all these models. While these models have made significant contributions to the literature, there are still gaps that might be filled, through the development of new models, to suit various practical situations in food production systems.

Two aspects of the reviewed literature, namely, lot-sizing models for growing items and those for items in multi-echelon inventory systems, stand out among the rest with ample opportunities for further development. In combination with one another, these two aspects of the reviewed literature are favourable for the development of new models that suited specifically for complex food production systems. For this reason, the common thread among all six novel models presented in this thesis is growing inventory items and multi-echelon inventory systems.

The other aspects of the reviewed literature (specifically, lot-sizing models for items with price-dependent demand, items with imperfect quality, perishable items with expiration dates, items with imperfect quality and errors in quality inspection and items with inventory level-dependent demand) also play a vital role in the development of the six novel models. These aspects are incorporated into each of the models to formulate models that are not only more realistic (and thus, practical) but are also suited specifically for food production systems. This is because aspects such as pricing decisions, quality inspection, expiration dates and stock levels play important roles in food production systems. For instance, quality control ensures that the consumption of food products, which move between different supply chain echelon for value-adding activities such as processing and packaging, is not compromised. In addition, it is imperative for managers in retail stores to keep track of the expiration dates of stocked food products so as to minimise wastage that results from having to dispose of expired food because it is unsuitable for sale. Both the selling price and the displayed inventory level of products have been shown to be important determinants of consumer demand. Generally, demand increases with decreasing selling prices and it increases with increasing levels of stock on shelves.

Owing to their suitability for food production systems, the six models presented in this thesis not only enrich the reviewed literature but they also represent new research areas that involve growing items in multi-echelon supply chains. The results from the six models can help researchers and practitioners in food production system that involve growing items in multi-echelon settings to gain better a understanding of the inventory management practices.

Chapter 3

Review of Foundational Models

3.1 Introduction

Seven previously published lot-sizing models in the literature are the foundation of the six novel models presented in this thesis. In combination with one another, the lot-sizing model for growing items, credited to Rezaei (2014), and the lot-sizing models for items in multi-echelon supply chain settings, credited to Goyal (1977), form the basis of all six models presented in the thesis. The ideas behind these two foundational models (in combination with one another) are utilised in combination with the ideas behind the other five foundational models (namely, models for items with price-dependent demand, items with imperfect quality, perishable items with expiration dates, items with imperfect quality and inspections errors and items with inventory level-dependent demand) in the development of the six original models presented in this thesis. In this chapter, the mathematical theory governing these seven foundational models is briefly reviewed.

3.2 Notations

The following notations are used during the development of the models presented in this thesis:

a	The fraction of processed inventory that are of poorer quality
$b^{'}$	The weight of good quality (as classified by the inspection process) processed inventory received by the retailer per retail cycle
$b^{''}$	The weight of poorer quality processed inventory that was incorrectly classified (as good quality processed inventory by the inspection process) received by the retailer per retail cycle
c_f	The farmer's feeding cost per weight unit of live inventory per unit time
D	The demand rate, in weight units per unit time, for processed items of good quality
$F(t)$	Freshness index of the processed inventory at time t (a function of the expiration date)
G	The weight of the ending inventory (for the case with a non-zero ending inventory policy)
h_r	The retailer's holding cost per weight unit per unit time
h_p	The processor's processing facility holding cost per weight unit per unit time

h_s	The processor's inspection facility holding cost per weight unit per unit time
$I(t)$	The weight of the processed inventory at time t
K_f	The farmer's setup cost per cycle
K_p	The processor's processing facility setup cost per cycle
K_r	The retailer's ordering cost per cycle
K_s	The processor's cost of transferring a single batch of good quality processed inventory from the inspection facility to the retailer
L	The maximum lifetime (i.e. expiration date or shelf life) of the processed inventory
l_r	The cost (per weight unit) of accepting poorer quality processed inventory
l_s	The cost (per weight unit) of rejecting good quality processed inventory
m_f	The farmer's mortality cost per weight unit of mortal inventory per unit time
n	The number of shipments from the processor to the retailer per unit cycle of the processor
n_p	The number of batches of processed inventory sent by the processor from the processing facility to the inspection facility per unit cycle of the processing cycle
n_s	The number of batches of good quality processed inventory delivered to the retailer (by the processor from the inspection facility) during a single inspection run
p_f	The farmer's selling price per weight unit of live items
p_p	The processor's selling price per weight unit of (or good quality, in the case of imperfect quality models) inventory
p_q	The processor's selling price per weight unit of poorer quality inventory
p_r	The retailer's selling price per weight unit of (or good quality, in the case of imperfect quality models) processed inventory
p_s	Retailer's salvage price per weight unit of processed inventory
p_v	The purchasing cost per weight unit of live newborn item
Q_1	The weight of the items in the retailer's lot
R	The processing rate, in weight units per unit time
S	The capacity of the retailer's shelf space in weight units
$s^{'}$	The weight of each batch of good quality processed inventory delivered from the processor's inspection facility to the retailer per inspection cycle
$s^{''}$	The weight of poorer quality processed inventory allowed to accumulate at the processor's inspection facility per inspection cycle
T	The retailer's cycle time
T_f	The duration of the farmer's growth period
T_p	The processor's cycle time
T_s	The duration of time required to inspect the entire lot-size
u_1	The probability of a Type I error (i.e. classifying poorer quality processed inventory as good quality processed inventory)
u_2	The probability of a Type II error (i.e. classifying good quality processed inventory as poorer quality processed inventory)
v	The inspection cost per weight unit
w_0	The weight of each newborn item
w_1	The target (or maturity) weight of each item

$w(t)$	The weight of an item at anytime t
x	The fraction of the live items which survive throughout the growth period
y	The (equivalent) number of items in the retailer's lot per cycle
z	The inspection rate in weight units per unit time
α	The items' asymptotic weight
β	The integration constant
δ	Scale parameter of the demand rate (or asymptotic level of demand attainable when the inventory level is considered most favourable to consumers)
λ	The exponential growth rate of the items
ω	Price elasticity of the demand rate
ϕ	Maximum size of the market for processed inventory (or asymptotic level of demand attainable when the selling price is considered most favourable to customers)
ψ	Shape parameter of the demand rate (or the inventory level-elasticity of the demand)
τ	The duration of time between consecutive deliveries of good quality batches of processed inventory from the inspection facility to the retailer
$\theta(t)$	The age-dependent deterioration rate of the processed inventory at time t

3.3 Foundational models

3.3.1 Growing items

Item growth was first included in an inventory model by Rezaei (2014) through the development of an EOQ model for a new class of items termed growing items. This class of items is made up of living organisms with the capability to grow and includes livestock, fish and crops, to name a few. Rezaei (2014) considered a situation where a company purchases y live newborn items that are capable of growing. At the time of purchase, each newborn has an initial weight of w_0. This implies that the total weight of all the ordered newborn items is $Q_0 = yw_0$. The cost of purchasing each newborn item is p_v per weight unit. The company rears the items for a period of T_f time units and at the end of that period, the weight of each newborn item would have grown to w_1. Likewise, the total weight of all the ordered fully grown items would have increased to $Q_1 = yw_1$. At that point, the company slaughters all the fully grown items instantaneously and uses (i.e. sells) the slaughtered inventory items to meet consumer demand of D weight units per unit time. The inventory system profile for this situation is depicted by Figure 3.3.1.

The feeding and holding cost per weight unit per unit time are c_f and h_r, respectively, and the company incurs these costs for time periods T_f and $(T - T_f)$, respectively, which correspond to feeding and consumption periods, respectively. The fixed cost of setting up a new replenishment cycle is K_r and the company incurs this cost at the beginning of each cycle. The company sells the slaughtered inventory at a price of p_r per weight units. Given that the demand rate is D and the weight of the demanded (i.e. fully grown and slaughtered) inventory is replenishment Q_1, this means that the company's replenishment frequency (or cycle time), T, equals $Q_1/D = yw_1/D$. The company's total profit per unit time, TPU, is the company's revenue per unit time less the company's total costs (i.e.

the sum of the setup, purchasing, feeding and holding costs) per unit time, and hence,

$$TPU = p_r D - \frac{K_r D}{y w_1} - \frac{p_v w_0 D}{w_1} - \frac{c_f D}{w_1} \int_o^t w(t)dt - \frac{h_r y w_1}{2}. \tag{3.3.1}$$

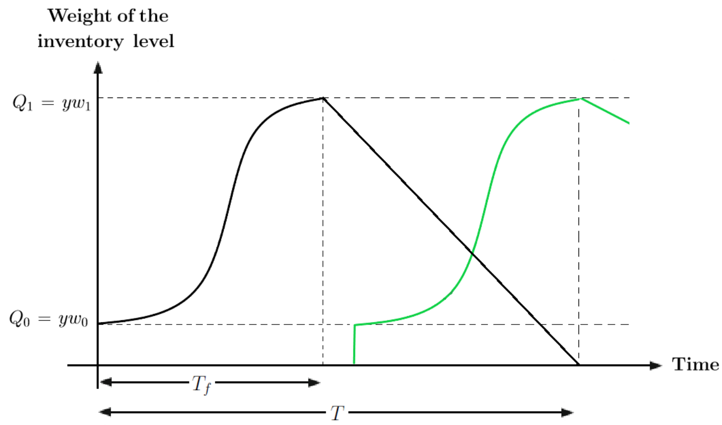


Figure 3.3.1: Inventory system profile for growing items

For the growth function, w_t, Rezaei (2014) used an approximation first developed by Richards (1959). The company's optimal lot-size (y^*), determined by setting the first derivative of TPU with respect to y to zero, is

$$y^* = \sqrt{\frac{2K_r D}{h_r w(t)^2}} \tag{3.3.2}$$

3.3.2 Multi-echelon inventory systems

Goyal (1977) developed the first model, often referred to as the JELS model, aimed at jointly optimising the inventory replenishment policy adopted in a two-echelon inventory system with a single vendor and a single buyer. The vendor is responsible for producing the demanded items and the buyer sells the demanded items to consumers. In essence, there is a production echelon (at the vendor's facility) and a consumption echelon (at the buyer's facility). The behaviour of the inventory system in this type of supply chain setup is depicted by Figure 3.3.2. When a new replenishment cycle starts, the vendor starts producing items at a rate of R per unit time. Once the quantity of items produced is enough to make up a batch of size Q, the vendor ships this batch to the buyer who meets end consumer demand of D items per unit time. The buyer receives a new order every T time units. Given that the buyer's lot-size is Q and buyer's the demand rate is D, the buyer's cycle cycle time $T = Q/D$. The production rate, R, is greater than the demand rate, D, and as a result, the vendor does not produce through the entirety of their replenishment cycle, of duration nT. This means that the vendor delivers n shipments, each with Q items, to the buyer throughout a single production run, with n being an integer number. The vendor incurs a fixed production setup cost of K_p at the start of each production cycle. Keeping a single item in stock costs the vendor a holding cost of h_p per unit time. On the other hand, the buyer's holding cost is h_r per unit time and the buyer incurs a fixed cost of K_r for placing an order.

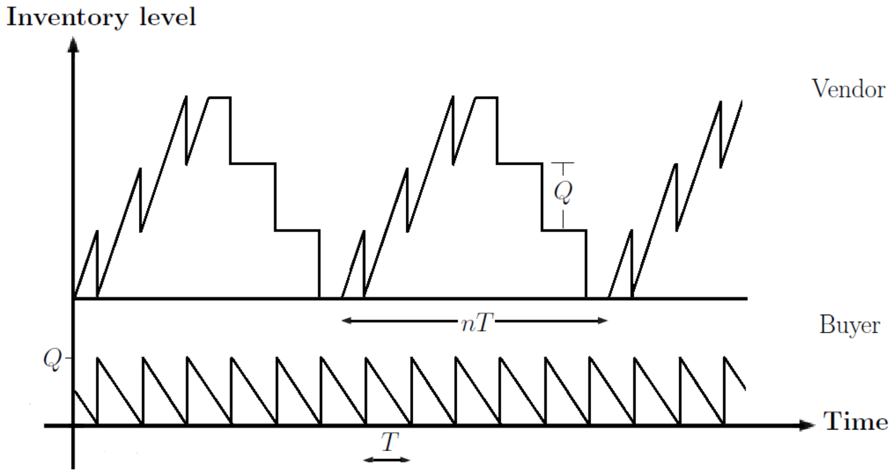


Figure 3.3.2: Inventory system profile for a vendor and a buyer in a two-echelon inventory system

Goyal (1977)'s model was aimed at minimising the total joint inventory management cost for both parties. Consequently, the joint total cost (i.e. for both echelons) per unit time, TPU_{sc}, is

$$TPU_{sc} = \frac{K_r D}{Q} + \frac{h_r Q}{2} + \frac{K_p D}{nQ} + \frac{h_p Q}{2}\left[(n-1)\left(1 - \frac{D}{R}\right) + \frac{D}{R}\right]. \tag{3.3.3}$$

The buyer's optimal lot-size, Q^*, is determined by setting the first derivative of Equation (3.3.3) with respect to Q to zero, resulting in

$$Q^* = \sqrt{\frac{2D\left(K_r + \frac{K_p}{n}\right)}{h_r + h_p\left[(n-1)\left(1 - \frac{D}{R}\right) + \frac{D}{R}\right]}}. \tag{3.3.4}$$

The optimal number of shipments delivered by the vendor to the buyer per production cycle, n^*, can be determined either by setting the first derivative of Equation (3.3.3) with respect to n to zero or through an iterative procedure.

3.3.3 Price-dependent demand

Price-dependent demand rates have been of interest to researchers studying the optimisation of inventory systems since the publication of Whitin (1955)'s work. In this type of inventory system, a company purchases Q items at the beginning of each inventory replenishment cycle at a cost of p_v per item. The company incurs a fixed ordering cost of K_r per cycle. The company uses the lot-size Q to meet a consumer demand rate of D. However, the demand rate is not a deterministic constant (unlike in most inventory models in the literature), instead it is affected by the selling price of the items. This is based on marketing theory which has shown that the demand for consumer goods generally increases with decreasing selling prices (Robinson and Lakhani, 1975). Numerous functions have been used to model this relationship. One such function is

$$D = \phi e^{-\omega p_r}, \tag{3.3.5}$$

first adopted by Robinson and Lakhani (1975) where ϕ represents the maximum number of potential customers, p_r is the selling price of the items and ω is the price elasticity of the demand rate. If the relationship between the demand and the selling price is given by Equation (3.3.5), the changes to the inventory level over time can be represented by Figure 3.3.3.

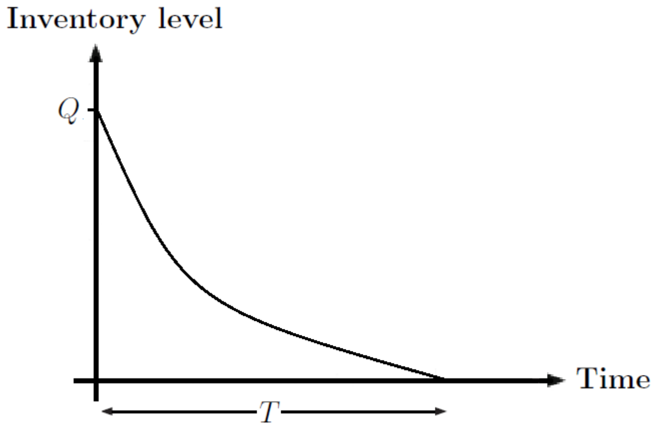


Figure 3.3.3: Inventory system profile for items with price-dependent demand rate

In addition to the fixed cost of placing an order, the company also incurs holding costs, with charge a of h_r per item per unit time. The company's total cost is the sum of the ordering, purchasing and ordering costs. The company's profit is computed by subtracting its revenue from its total costs. Consequently, the company's profit per unit time, TPU, is

$$TPU = p_r\phi e^{-\omega p_r} - \frac{K_r\phi e^{-\omega p_r}}{Q} - p_v\phi e^{-\omega p_r} - \frac{h_r Q}{2}. \tag{3.3.6}$$

The optimal selling price, p_r^*, is obtained through trial and errors methods and it is used to to compute the company's optimal lot-size, Q^*, as

$$Q^* = \sqrt{\frac{2K_r\phi e^{-\omega p_r^*}}{h_r}}. \tag{3.3.7}$$

3.3.4 Imperfect quality

Salameh and Jaber (2000) are credited with the EOQ model for items with imperfect quality. In this type of inventory system, not all of the ordered items are of good quality and there is an inspection process in place to separate good and poorer quality items. Figure 3.3.4 depicts the typical inventory profile for such an inventory system.

Salameh and Jaber (2000) studied an inventory system where, at the start of each replenishment cycle, a company places an order for Q items. The company is charged a fixed ordering cost of K_r for order placement at the beginning of each cycle. In addition, the company pays a purchasing cost of p_v for each item. However, not all the items in the company's order are of good quality. A certain fraction, a, of the order is of poorer quality and this can not be used to meet end consumer demand which is for good quality items. This means the quantity of poorer quality items in each lot, b', equals aQ. The holding cost is h_r per item per unit time. In order to ensure that only good quality items are sold to consumers, the company inspects the entire lot for quality before putting it on sale at a rate of z items per unit time for time period τ. The company sells the fraction of the lot that is of good quality, $(1-a)Q$, at a selling price of p_r per item throughout the cycle and salvages the fraction of the lot that is of poorer quality, $b' = aQ$, as a single batch at a discounted price of p_q per item at the end of the inspection process. This means that the company has two revenue streams, one from the sales of good quality units and the other from the poorer quality sales. On the cost front, in addition to the purchasing, ordering and holding costs, the company also incurs inspection costs for separating the lot into good and poorer quality classes, with the cost of inspecting each unit amounting to v. The poorer quality fraction, a, is assumed to be a random variable and consequently, the company's expected total profit per unit time, $E[TPU]$, is

$$E[TPU] = p_r D + \frac{p_q D E[a]}{\big(1 - E[a]\big)} - \frac{p_v D}{\big(1 - E[a]\big)} - \frac{K_r D}{Q\big(1 - E[a]\big)} - \frac{vD}{\big(1 - E[a]\big)} - h_r \left\{ \frac{Q\big(1 - E[a]\big)}{2} + \frac{QDE[a]}{z\big(1 - E[a]\big)} \right\}. \quad (3.3.8)$$

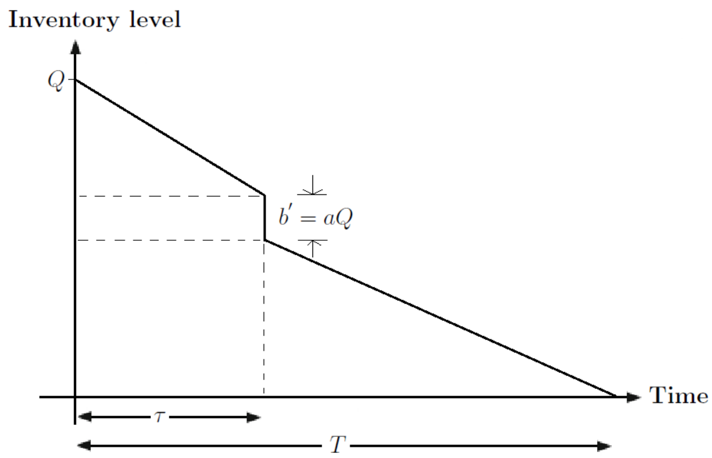


Figure 3.3.4: Inventory system profile for items with imperfect quality

The value of Q that maximises Equation (3.3.8), which corresponds to the optimal lot-size Q^*, is determined as

$$Q^* = \sqrt{\frac{2K_r D}{h_r \left\{ \big(1 - E[a]\big)^2 + \frac{2E[a]D}{z} \right\}}}. \quad (3.3.9)$$

3.3.5 Expiration date

The typical behaviour of the inventory level in an inventory system for perishable items with expiration is depicted by Figure 3.3.5. This type of inventory system is based on the classic perishable (or deteriorating) inventory system, first proposed by Ghare and Schrader (1963). In formulating their model, Ghare and Schrader (1963) used a fixed and deterministic quantity $\theta(t)$ to describe the rate of deterioration. Sarkar (2012) described the deterioration rate in terms of the expiration date (or shelf life) of demanded items as

$$\theta(t) = \frac{1}{1 + L - t}, \quad (3.3.10)$$

where L is the expiration date. Since the items can not be used to meet consumer demand (with a rate D) after the expiration date, the replenishment cycle time T is less than or equal to expiration date (i.e. $L \geq T$). Throughout the replenishment cycle, the inventory level is depleted as a result of both consumer demand and deterioration and consequently, the differential equation

$$\frac{dI(t)}{dt} = -D - \theta(t)I(t), \qquad 0 \leq t \leq T, \tag{3.3.11}$$

describes the changes to the inventory level. After solving Equation (3.3.11), the initial lot-size is determined as

$$Q = D(1+L)\ln\left(\frac{1+L}{1+L-T}\right). \tag{3.3.12}$$

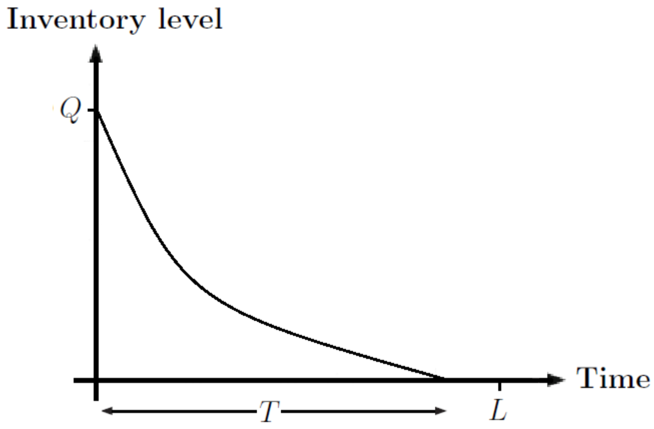


Figure 3.3.5: Inventory system profile for items with expiration dates

Inventory systems of this kind typically consider a company that places an order for deteriorating items at the beginning of a replenishment cycle. The company incurs a fixed cost of K_r per cycle associated with placing the order. The company also incurs holding costs, charged at h_r per item per unit time, for keeping the items in stock throughout the cycle. Therefore, the company's total cost per unit time, TCU, is

$$TCU = \frac{K_r}{T} + \frac{h_r D}{T}\left[\frac{(1+L)^2}{2}\ln\left(\frac{1+L}{1+L-T}\right) + \frac{T^2}{4} - \frac{(1+L)T}{2}\right]. \tag{3.3.13}$$

A closed form solution for the company's optimal cycle time, T^*, can not be determined from Equation (3.3.13). However, various search techniques can be used to solve for T^*. Substituting T^* into Equation (3.3.12) yields the optimal lot-size Q^*.

3.3.6 Imperfect quality and inspection errors

Khan et al. (2011) studied an inventory system for items with imperfect quality under the assumption that the inspection process used to separate the good and poorer quality items is prone to errors. Khan et al. (2011)'s model is essentially an extension of Salameh and Jaber (2000)'s work, described previously in Subsection 3.3.4, that considers two types of inspection errors, namely type I and type II errors. The behaviour of the inventory level over time for such an inventory system is depicted by Figure 3.3.6. A type I error is committed when a good quality item is incorrectly classified by the inspection process as being a poorer quality items whereas, a type II error describes a situation where a poorer quality item is incorrectly classified as a good quality unit. The probabilities of committing type I and type II errors are u_1 and u_2, respectively.

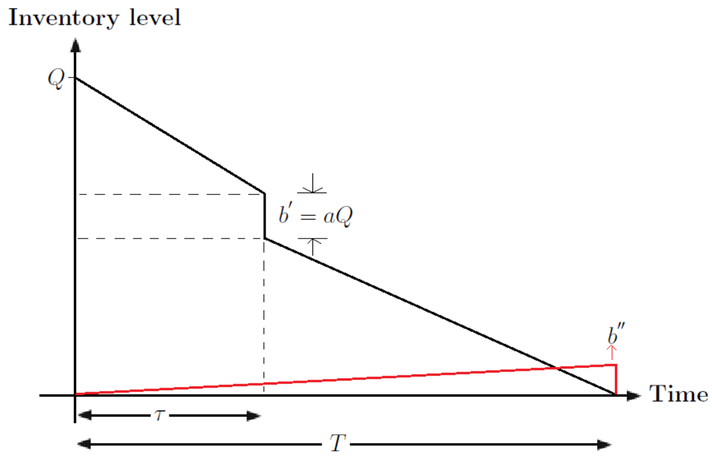


Figure 3.3.6: Inventory systems profile for items with imperfect quality and inspection errors

Since Khan et al. (2011)'s model is an extension of Salameh and Jaber (2000)'s, all the basic assumptions from Salameh and Jaber (2000)'s model hold with exception to the errors made by the inspection process. There are penalty costs associated with committing the inspection errors. If a poorer quality unit is accepted, the company incurs a cost of l_r per (incorrectly classified) item and if a good quality item is rejected, the company incurs a cost of l_s per (incorrectly classified) item. When a poorer quality unit is sold to a customer, customers can return the item for a replacement item at no additional cost. The quantity of poorer quality items that are sold to customers (as a result of inspection errors) is $b'' = aQu_2$ and this quantity is returned to the company and sold to secondary markets at a discounted price (of p_q per item) along with the poorer quality items. The fraction of items that are of poorer quality, a, is considered a random variable, as is the case in Salameh and Jaber (2000)'s model. Additionally, the probabilities of type I and type II errors, u_1 and u_2, respectively, are also considered random variables and consequently, the expected value of the total profit per unit time, $E[TPU]$ is

$$\begin{aligned} E[TPU] = p_r D &+ \frac{p_r D E[a] E[u_2]}{\big(1 - E[a]\big)\big(1 - E[u_1]\big)} + \frac{p_q D E[u_1]}{\big(1 - E[u_1]\big)} + \frac{p_q D E[a]}{\big(1 - E[a]\big)\big(1 - E[u_1]\big)} \\ &- \frac{p_v D}{\big(1 - E[a]\big)\big(1 - E[u_1]\big)} - \frac{K_r D}{Q\big(1 - E[a]\big)\big(1 - E[u_1]\big)} - \frac{vD}{\big(1 - E[a]\big)\big(1 - E[u_1]\big)} \\ &- \frac{l_s\big(1 - E[a]\big) D E[u_1]}{\big(1 - E[a]\big)\big(1 - E[u_1]\big)} - \frac{l_r D E[a] E[u_2]}{\big(1 - E[a]\big)\big(1 - E[u_1]\big)} \\ &- h_r \left\{ \frac{Q\big(1 - E[a]\big)\big(1 - E[u_1]\big)}{2} + \frac{Q D E[a]\Big[\big(1 - E[u_1]\big) + E[u_2]\Big]}{z\big(1 - E[a]\big)\big(1 - E[u_1]\big)} \right\}, \end{aligned} \quad (3.3.14)$$

and the associated optimal lot-size, Q^*, is

$$Q^* = \sqrt{\frac{2K_rD}{h_r\left\{\left(1-E[a]\right)^2\left(1-E[u_1]\right) + \frac{2E[a]D\left[\left(1-E[u_1]\right)+E[u_2]\right]}{z\left(1-E[u_1]\right)}\right\}}}. \tag{3.3.15}$$

3.3.7 Inventory level-dependent demand

Baker and Urban (1988) developed a model for a situation where the demand for a company's product is a function of the level of on-hand inventory of that particular product. Figure 3.3.7 depicts the typical inventory profile for such a situation. Based on marketing theory, consumer demand generally increases with an increasing level of stock on display (Levin et al., 1972). Baker and Urban (1988) used the function

$$D = \delta\left[I(t)\right]^{\psi}, \qquad 0 \leq t \leq T, \tag{3.3.16}$$

to describe the relationship between the demand rate, D, and the inventory level over time, $I(t)$, where δ represents the scaling parameter for the demand rate (i.e. the maximum attainable demand) and ψ represents the shape parameter (i.e. the elasticity of the demand with respect to the inventory level displayed on shelves). Therefore, the inventory level changes with time according to the differential equation

$$\frac{dI(t)}{dt} = -D = -\delta\left[I(t)\right]^{\psi}, \qquad 0 \leq t \leq T. \tag{3.3.17}$$

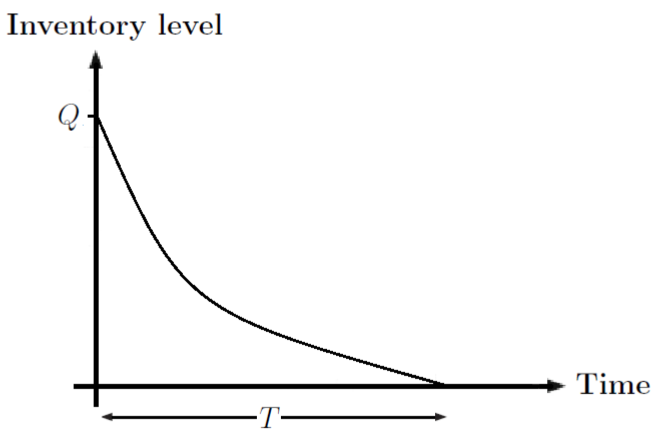


Figure 3.3.7: Inventory system profile for items with inventory level-dependent demand rate

Upon solving Equation (3.3.17), an expression for $I(t)$ (in terms of Q) is obtained and this is used to compute the company's holding costs per unit time. The company is charged a holding cost of h_r per unit time for keeping a single item in stock. Moreover, the company incurs a fixed ordering cost of K_r whenever an order is placed and the cost of purchasing each item is p_v. If the company sells the items at a selling price of p_r per item, then its total profit per unit time, TPU, is

$$TPU = p_r\delta(1-\psi)Q^{\psi} - \frac{K_r\delta(1-\psi)}{Q^{(1-\psi)}} - p_v\delta(1-\psi)Q^{\psi} - \frac{h_r(1-\psi)Q}{2-\psi}. \tag{3.3.18}$$

Owing to the difficulty associated with obtaining a closed form solution to Equation (3.3.18), various search techniques are used to determine the company's optimal lot-size Q^*.

3.4 Concluding remarks

In this chapter, the mathematical theory behind the seven foundational models that lay the groundwork for the development of the six original models that are presented in this thesis, is reviewed. The theory in these foundational models describes common situations that inventory managers in food production systems might face, for instance, quality control, perishability and pricing polices, to name a few. Two of the foundational models, specifically, the model for growing items and the JELS model, are the common thread among the six original models developed in the next chapter of the thesis. In combination with one another, these two specific models perfectly describe the inventory system of most food supply chains which start with the rearing of growing items such as livestock or crops and finish with retail sale of packaged food products. The upstream farming and the downstream retail ends of food supply chains are often connected by processing operations which add value to the grown items and transforms them into form that is suitable for human consumption. In most cases, the different stages of the supply chain are handled by different entities. With this in mind, the reviewed foundational models are used as a basis for developing the six original models for managing growing inventory items in multi-echelon supply settings that are reminiscent of actual food production systems.

Chapter 4

Presentation of Novel Models

4.1 A three-echelon supply chain inventory model for growing items*

4.1.1 Introduction

4.1.1.1 Context

As a result of heightened competition in today's business environment, many companies have been forced to look beyond the four walls of their own organisations when trying to improve efficiency or reduce costs in their operations. A lot of companies have realised that collaborating with their customers and suppliers can lead to sizeable improvements in responsiveness, efficiency and cost reductions (Ben-Daya and Al-Nassar, 2008). This collaboration entails integrating various decisions between all supply chain members. One such decision is the inventory replenishment policy adopted by the chain members.

Owing to the potential cost savings that can be realised as a result of coordinating inventory decisions with other supply chain members, numerous researchers have developed integrated inventory models which are aimed at minimising total inventory costs among all chain members.

4.1.1.2 Purpose

This section attempts to formulate an integrated inventory model for growing items in a supply chain with a farmer, a processor and a retailer. Growing items are classified as those which experience an increase in weight during the replenishment cycle (Rezaei, 2014). The items are able to gain weight because they are fed by the farmer until maturity. Once mature, the items are delivered to a processing plant plant for slaughtering, cutting and packaging (herein collectively called processing). For each processing run, the processor delivers a certain number of equal-sized batches of processed (ready for sale) items to the retailer who sells them to customers.

*A modified version of this section has been accepted for publication as Sebatjane and Adetunji (2020c) in the *Journal of Modelling in Management*.

4.1.1.3 Relevance

Growing items, which include livestock and crops, among others, are a vital part of food supply chains. Most growing items are consumed as saleable food items in various forms. Prior to reaching end consumers, these items usually undergo various stages and processes which ensure that they are ready for consumption (from a health and safety perspective). Growing items are received as newborn items by a farmer who rears (or grows) them until their weights are suitable for consumption. Following this, the items are processed (which includes slaughtering) and transformed into various saleable items. Consumer demand for saleable/processed items is met by a retailer who receives the items from the processor. These stages can represent the three echelons of a supply chain, with a farming operation, a processing plant and a retail store being the echelons.

From the literature review, it appears that no study (focusing on growing inventory items) has considered a three-echelon supply chain with farming, processing and retailing operations. This section is aimed at addressing this gap in the literature given that the proposed inventory system represents a typical situation in the food production supply chain. The results from this section can be used by purchasing managers in industries involved in food production when ordering growing items.

4.1.1.4 Organisation

In addition to the introductory subsection, this section has four other subsections. The proposed supply chain inventory system is briefly described and then modelled mathematically in Subsections 4.1.2 and 4.1.3, respectively. A numerical analysis is given in Subsection 4.1.4, from which important managerial insights are drawn through sensitivity analysis and a cost efficiency analysis of the proposed replenishment policy. The section is then concluded in Subsection 4.1.5.

4.1.2 Problem description

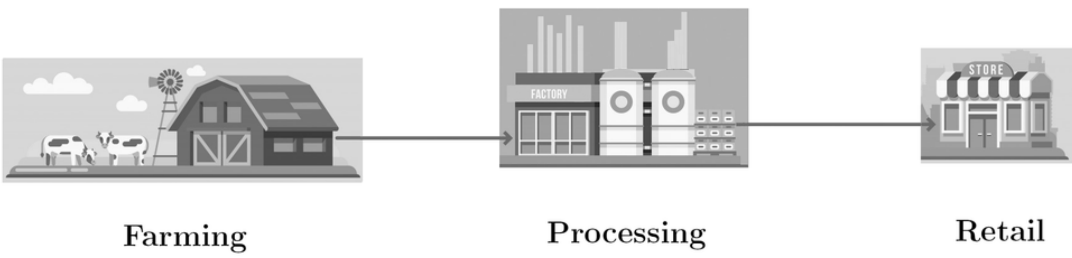


Figure 4.1.1: A three-echelon supply chain for growing items with farming, processing and retail operations

The problem considered is that of a growing items inventory system in a supply chain with farming, processing and retailing echelons of operations. This type of supply chain setup, depicted in Figure 4.1.1, is representative of most food production chains. At the first stage, a farmer purchases newborn items and rears them till they grow to a given target weight. As soon as the weight of the items has reached the maturity weight (i.e. the predefined target weight), the farmer delivers them to a processing plant for slaughtering, cutting and packaging. All the activities carried out by the processor are collectively called processing and they occur at a specified finite rate. The processor then delivers the ready-for-consumption items to a retailer who meets customer demand.

The behaviour of the inventory at each of the three supply chain echelons is depicted by Figure 4.1.2. The farmer's inventory level represents the weight of the live items which increases throughout the growth period as a result of feeding. On the other hand, the behaviour of the processor's and the retailer's inventory level represents the weight of the processed items. The farmer's growth period is synchronised with the processor's production cycle time so that when a growth period ends, the processor is ready to start production. Once the processor has produced enough saleable items to make up a batch, a shipment is delivered to the retailer. During a single processing run, the processor delivers multiple equal-sized shipments of saleable items to the retailer (i.e. a SSMD policy is adopted between the processor and the retailer).

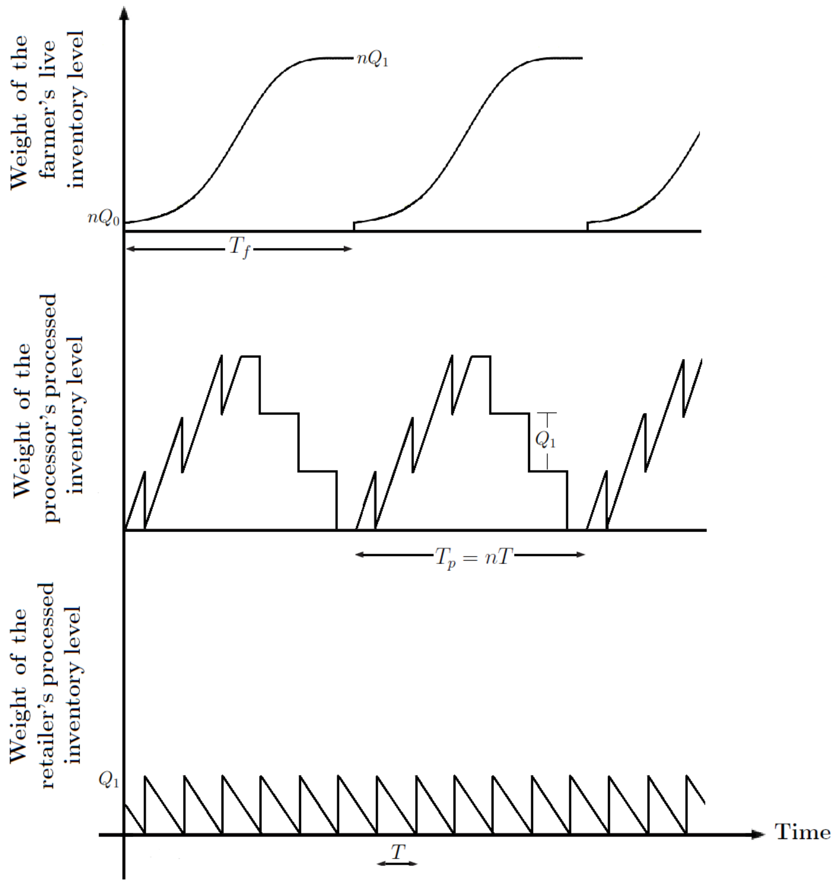


Figure 4.1.2: Inventory system profile for growing items in a supply chain with a single farmer, a single processor and a single retailer

The model is aimed at determining an inventory replenishment policy which minimises the total cost of managing inventory in the supply chain, made up of the total costs incurred by each of the three supply chain members. The decision variables of the model are the number of shipments made by the processor to the retailer in each production setup and the replenishment interval (and consequently the order quantity).

In order to model the inventory problem at hand, a number of assumptions were made to aid the model development process. These include:

- The supply chain comprises of a single farmer, a single processor processor and a single retailer, dealing in a single type of growing item.

- The (processor's) processing rate is greater than the (retailer's) demand rate, both of which are deterministic constants.
- The arrival of successive shipments of processed inventory from the processor to the retailer is scheduled to occur when the previous shipment has just been depleted.
- The processor delivers processed inventory to the retailer just at the moment the processed inventory is enough to make up a batch size.
- The retailer's replenishment interval is an integer multiple of the processor's replenishment interval.
- Holding costs are incurred only for the processed items.
- The retailer's holding costs are higher than those of the processor as a result of value adding as the items move downstream in the supply chain.

4.1.3 Model development

The start of a new growing cycle is marked by the farmer purchasing ny live newborn items, each weighing w_0 at the time of purchase. After reaching a specific weight, the items are instantaneously delivered to a processor, who delivers n shipments of y processed items to a retailer. This implies that in order for the retailer to meet customer demand for processed items, the farmer should also take into account the number of shipments made by the processor to the retailer during a single processing run when ordering newborn items. This means that the weight of all the newborn items ordered, nQ_0, is given by nyw_0. The farmer's growth period is of duration T_f and during this time, the items are fed so that they can grow. The maturity weight of each item (i.e. weight at the end of a growing cycle) is w_1 and once the items grow to this target weight, they are sent to a processing facility for slaughtering, cutting and packaging. Likewise, the weight of all the fully grown ordered items is given by $nQ_1 = nyw_1$.

While different items have different growth rates, the general pattern of growth is similar. Growth functions have a characteristic "S"-shape, and for this reason the logistic function is used to model the growth pattern. The logistic growth function makes use of three parameters to represent item growth over time. The items' growth function can be represented by

$$w_t = \frac{\alpha}{1 + \beta e^{-\lambda t}}, \tag{4.1.1}$$

where λ, β and α denote the exponential growth rate, the constant of integration and the items' asymptotic weight respectively.

When the growth period concludes at T_f, the items are instantaneously delivered to the processor who transforms them into a consumable form and at that point, the weight of each item would have increased to w_1. Using Equation (4.1.1), the farmer's growth period can be computed as

$$T_f = -\frac{\ln\left[\frac{1}{\beta}\left(\frac{\alpha}{w_1} - 1\right)\right]}{\lambda}. \tag{4.1.2}$$

The farmer and the processor's cycle times are synchronised in such a way that during the course of a single processing run, the farmer makes a single delivery of mature items to the processor when the growth period ends (i.e. the farmer and the processor operate on a SSSD policy) as shown in Figure 4.1.1. As a way of ensuring that the fully matured items are ready for processing at the right time (i.e. in the sense that their weight has reached the target weight), the growth period needs to be at most of equal duration to the processor's cycle time. Hence, the relationship between the farmer's growth period and the processor's cycle time is given by

$$T_f \leq T_p. \tag{4.1.3}$$

Since $R > D$, the processor does not produce for their entire cycle (i.e. T_p). This means that T_p has a processing portion, during which the processor transforms inputs from the farm into processed items ready for consumption and ships them to the retailer, and a non-processing portion where by the processor supplies the retailer with processed items from the accumulated stock . Stock accumulates because $R > D$. The processor and the retailer operate on a SSMD replenishment policy. This means that for each production setup, the processor delivers a certain number (in this case n) of equal-sized shipments of processed inventory to the retailer. This means that the weight of all the ordered inventory delivered by the farmer to the processor is nQ_1. The processor thus makes n deliveries of weight Q_1 to the retailer throughout the processor's replenishment cycle. Consequently, the relationship between the processor's cycle time, T_p, and the retailer's cycle time, T, is given by

$$T_p = nT. \tag{4.1.4}$$

4.1.3.1 Retailer's total cost components

The behaviour of the retailer's processed inventory level is depicted in Figure 4.1.3. The retailer's total cost function is the sum of the holding and the ordering costs.

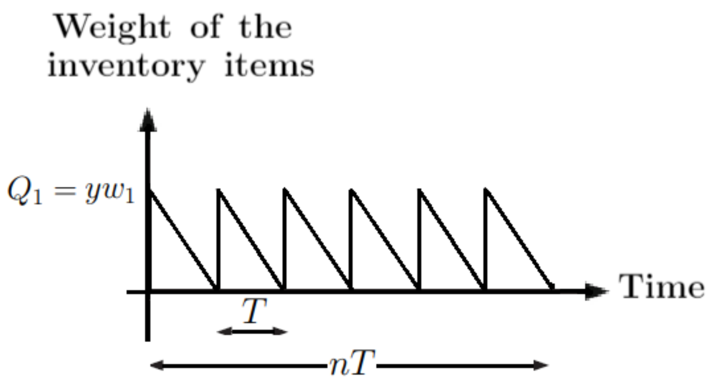


Figure 4.1.3: Inventory system profile for the retailer

4.1.3.1.1 Retailer's ordering cost per unit time

At the beginning of a retailer's cycle, the processor delivers a shipment of size Q_1 in order for the retailer to meet a demand rate of D. This implies that the retailer places D/Q_1 orders per unit time. Whenever the retailer places an order, a cost of K_r is incurred. Thus the ordering cost incurred by the retailer per unit time, OCU_r, is determined as

$$OCU_r = \frac{K_r D}{Q_1} = \frac{K_r D}{yw_1}. \tag{4.1.5}$$

4.1.3.1.2 Retailer's holding cost per unit time

The retailer's cost for holding inventory per unit time, HCU_r, is determined as the product of the average weight of the inventory level and the cost of holding a single weight unit of inventory for a single time unit (h_r). The average weight of the inventory items, as determined from Figure 4.1.3, is given by $Q_1/2$. The holding cost incurred by the retailer per unit time is thus

$$HCU_r = \frac{h_r Q_1}{2} = \frac{h_r y w_1}{2}. \tag{4.1.6}$$

4.1.3.1.3 Retailer's total cost per unit time

The total cost incurred by the retailer per unit time, TCU_r, is determined by adding Equations (4.1.5) and (4.1.6), resulting in

$$TCU_r = \frac{K_r D}{y w_1} + \frac{h_r y w_1}{2}. \tag{4.1.7}$$

The retailer places D/Q_1 orders per unit time in order to satisfy the demand rate. The retailer's cycle time is the reciprocal of the number of times an order is placed per unit time (i.e. $T = Q_1/D$). From this expression of the retailer's cycle time, their lot-size can be derived as

$$y = \frac{TD}{w_1}. \tag{4.1.8}$$

Equation (4.1.7) can be expressed in terms of T, by substituting y with the expression from Equation (4.1.8), as

$$TCU_r = \frac{K_r}{T} + \frac{h_r T D}{2}. \tag{4.1.9}$$

4.1.3.2 Processor's total cost components

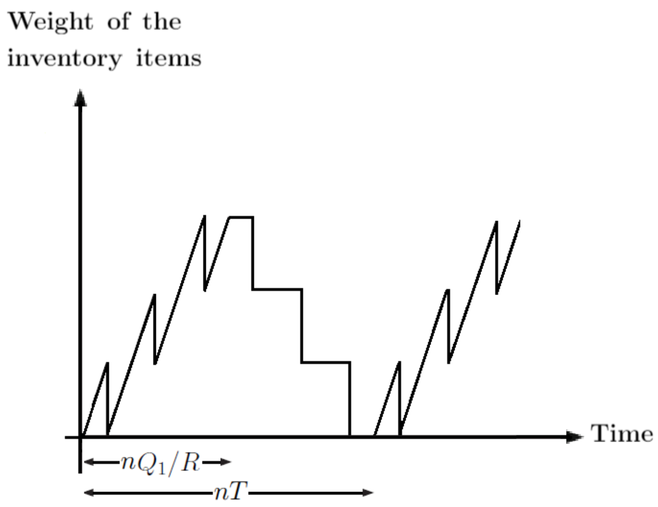


Figure 4.1.4: Inventory system profile for the processor

The processor's processed inventory system behaviour is depicted by Figure 4.1.4. The total cost of managing inventory at the processing facility is comprised of two components, namely the holding and setup costs.

4.1.3.2.1 Processor's setup cost per unit time

During the processor's replenishment cycle, of duration $T_p = nT$, the farmer delivers a single shipment of size nQ_1. So as to satisfy the demand rate D, the processor sets up for processing D/nQ_1 times in a single time unit. The processor's setup cost per unit time, SCU_p, is computed by multiplying the cost of setting up for a single processing run, K_p, by the number of times per unit time that the processor has to setup for processing. Thus,

$$SCU_p = \frac{K_p D}{nQ_1} = \frac{K_p D}{nyw_1}. \tag{4.1.10}$$

4.1.3.2.2 Processor's holding cost per unit time

The cost of holding inventory at the processing facility per unit time, HCU_p, is computed by multiplying the average inventory level at the processor ($\bar{I}_p$) by the holding cost per weight unit per unit time (h_p). The expression for the average inventory level in an integrated inventory system operating under a SSMD replenishment policy is difficult to compute given the irregular shape of Figure 4.1.4. To address this, various researchers, for instance Yang et al. (2007), have derived this expression by redrawing Figure 4.1.4 into a more regular shape as given in Figure 4.1.5.

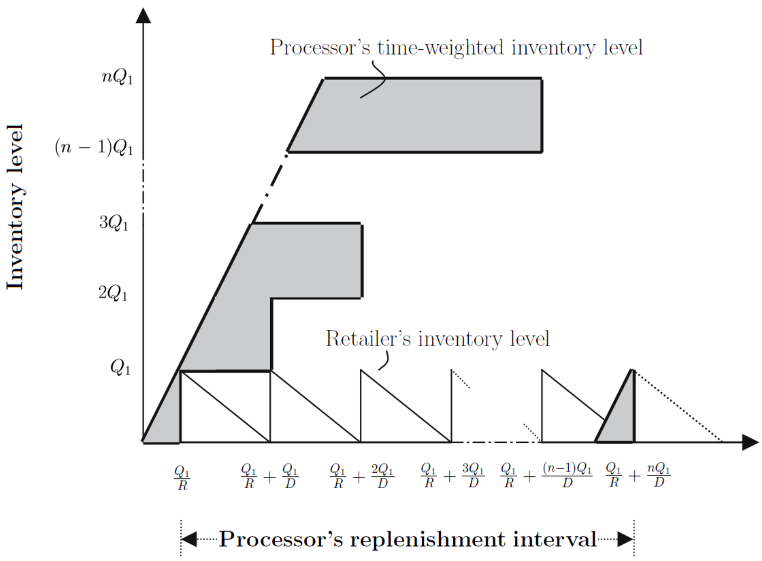


Figure 4.1.5: Inventory system profile the processor and the retailer [modified from Yang et al. (2007)]

The processor's average inventory level can be derived using Figure 4.1.5 as

$$\begin{aligned}
\bar{I}_p &= \frac{\text{Processor's time-weighted inventory}}{\text{Processor's replenishment interval}} \\
&= \frac{\frac{nQ_1^2}{2R} + Q_1^2\left(\frac{1}{D}-\frac{1}{R}\right) + 2Q_1^2\left(\frac{1}{D}-\frac{1}{R}\right) + \cdots + (n-1)Q_1^2\left(\frac{1}{D}-\frac{1}{R}\right)}{nQ_1/D} \\
&= \frac{D}{nQ_1}\left[\frac{nQ_1^2}{2R} + \frac{n(n-1)Q_1^2}{2}\left(\frac{1}{D}-\frac{1}{R}\right)\right] \\
&= \frac{Q_1}{2}\left[\Big(n-1\Big)\Big(1-\frac{D}{R}\Big) + \frac{D}{R}\right].
\end{aligned} \tag{4.1.11}$$

The processor's holding cost per unit time, HC_p, is computed as

$$HCU_p = h_p \bar{I}_p = \frac{h_p Q_1}{2}\left[\Big(n-1\Big)\Big(1-\frac{D}{R}\Big)+\frac{D}{R}\right] = \frac{h_p y w_1}{2}\left[\Big(n-1\Big)\Big(1-\frac{D}{R}\Big)+\frac{D}{R}\right]. \quad (4.1.12)$$

4.1.3.2.3 Processor's total cost per unit time

Adding Equations (4.1.10) and (4.1.12) results in the processor's total cost per unit time, TCU_p, which is

$$TCU_p = \frac{K_p D}{n y w_1} + \frac{h_p y w_1}{2}\left[\Big(n-1\Big)\Big(1-\frac{D}{R}\Big)+\frac{D}{R}\right]. \quad (4.1.13)$$

Equation (4.1.13) can be rewritten in terms of T by replacing y with Equation (4.1.8) and the result is

$$TCU_p = \frac{K_p}{nT} + \frac{h_p T D}{2}\left[\Big(n-1\Big)\Big(1-\frac{D}{R}\Big)+\frac{D}{R}\right]. \quad (4.1.14)$$

4.1.3.3 Farmer's total cost components

Figure 4.1.6 represents the farmer's (live) inventory system profile throughout the growth period. The farmer's total cost function is determined as the sum of their feeding and setup costs.

4.1.3.3.1 Farmer's setup cost

In order for the retailer to meet a demand rate of D, the farmer delivers a shipment of size nQ_1 to the processor (who supplies the retailer with n shipments, each of size Q_1). This means that the farmer places D/nQ_1 orders per unit time. Whenever the farmer places an order, a cost of K_f is incurred. This means that the farmer's setup cost per unit time, SCU_f, is

$$SCU_f = \frac{K_f D}{nQ_1} = \frac{K_f D}{n y w_1}. \quad (4.1.15)$$

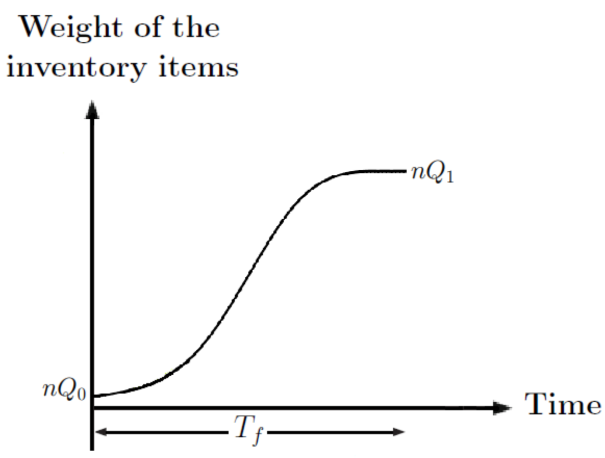


Figure 4.1.6: The farmer's inventory system profile

4.1.3.3.2 Farmer's feeding cost per unit time

The farmer's feeding cost component is dependent on the average weight of the inventory. Similar to the processor's average inventory, the farmer's average inventory, $\bar{I}_f$, is computed using Figure 4.1.6 as the product of the integral of the growth function (i.e. the area under the graph) and the number of ordered items divided by the replenishment interval. Thus,

$$\bar{I}_f = \frac{ny \int_0^{T_f} w_t \, dt}{nT} = \frac{y}{T}\Bigg(\alpha T_f + \frac{\alpha}{\lambda}\Big[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\Big]\Bigg). \tag{4.1.16}$$

The farmer's feeding cost per unit time, FCU_f, is determined by multiplying the feeding cost per weight unit per unit time (c_f) and the farmer's average inventory level ($\bar{I}_f$) and and it is thus

$$FCU_f = \frac{c_f y}{T}\Bigg(\alpha T_f + \frac{\alpha}{\lambda}\Big[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\Big]\Bigg). \tag{4.1.17}$$

4.1.3.3.3 Farmer's total cost per unit time

In order to determine the farmer's total cost per unit time, TCU_f, Equations (4.1.15) and (4.1.17) are summed and the result is

$$TCU_f = \frac{K_f}{nT} + \frac{c_f D}{w_1}\Bigg(\alpha T_f + \frac{\alpha}{\lambda}\Big[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\Big]\Bigg), \tag{4.1.18}$$

after substituting y with the expression from Equation (4.1.8).

4.1.3.4 Total supply chain cost per unit time

The total supply chain cost per unit time, TCU_{sc}, is the sum of Equations (4.1.9), (4.1.14) and (4.1.18), resulting in

$$\begin{aligned} TCU_{sc} = \frac{K_r}{T} + \frac{h_r T D}{2} + \frac{K_p}{nT} + \frac{h_p T D}{2}\left[\Big(n-1\Big)\Big(1-\frac{D}{R}\Big) + \frac{D}{R}\right] + \frac{K_f}{nT} \\ + \frac{c_f D}{w_1}\Bigg(\alpha T_f + \frac{\alpha}{\lambda}\Big[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\Big]\Bigg). \end{aligned} \tag{4.1.19}$$

The value of T which minimises the objective function is computed by equating the objective function's first derivative (with respect to T) to zero and the result is

$$\begin{aligned} \frac{\partial TCU_{sc}}{\partial T} &= -\frac{K_r}{T^2} + \frac{h_r D}{2} - \frac{K_p}{nT^2} + \frac{h_p D}{2}\left[\Big(n-1\Big)\Big(1-\frac{D}{R}\Big) + \frac{D}{R}\right] - \frac{K_f}{nT^2} = 0 \\ \implies T &= \sqrt{\frac{2\Big(K_r + \frac{K_p}{n} + \frac{K_f}{n}\Big)}{\Big\{h_r + h_p\Big[(n-1)\left(1-\frac{D}{R}\right) + \frac{D}{R}\Big]\Big\}D}}. \end{aligned} \tag{4.1.20}$$

The retailer's order quantity is determined by substituting Equation (4.1.20) into Equation (4.1.8), resulting in

$$y = \sqrt{\frac{2D\Big(K_r + \frac{K_p}{n} + \frac{K_f}{n}\Big)}{\Big\{h_r + h_p\Big[(n-1)\left(1-\frac{D}{R}\right) + \frac{D}{R}\Big]\Big\}w_1^2}}. \tag{4.1.21}$$

4.1.3.4.1 Model constraints

Two constraints are imposed on the proposed supply chain inventory model, the first constraint is to make the solution procedure tractable while the second ensures that the solution is feasible. The first constraint is that the number of shipments (of processed items) delivered by the processor to the retailer per unit cycle of the processor, n, is an integer. The second constraint, formulated by substituting Equation (4.1.4) into Equation (4.1.3), ensures feasibility by guaranteeing that mature items are ready in the sense that their weight has reached the target weight) for processing at the required time. These two constraints along with the objective function, given in Equation (4.1.19), are utilised to formulate the mathematical problem defining the proposed inventory system, which is

$$\text{Min.}\quad \Bigg\{ TCU_{sc} = \frac{K_r}{T} + \frac{h_r T D}{2} + \frac{K_p}{nT} + \frac{h_p T D}{2}\left[\Big(n-1\Big)\Big(1-\frac{D}{R}\Big) + \frac{D}{R}\right] + \frac{K_f}{nT} + \frac{c_f D}{w_1}\left(\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right)\Bigg\}$$

$$\text{s.t.}\quad n \in \mathbb{Z}, \quad T_f \leq nT. \quad (4.1.22)$$

4.1.3.4.2 Solution algorithm

The values of n and T which minimise the objective function are determined through an iterative procedure.

Step 1: Begin with $n = 1$.

Step 2: Compute T using Equation (4.1.20) and then use that to compute TCU_{sc} using Equation (4.1.19).

Step 3: Increase n by 1 and then calculate T and TCU_{sc} using Equations (4.1.20) and (4.1.19).

Step 4: If the value of TCU_{sc} decreases, then go to Step 3. Otherwise, the previously calculated value of TUC_{sc} (along with the corresponding T and n values) is the best solution.

Step 5: Check the best solution's feasibility. The solution is feasible provided that $T_f \leq nT$. T_f is calculated from Equation (4.1.2). If it is feasible, then the best solution is the optimal solution and proceed to Step 7. If it is not feasible, proceed to Step 6.

Step 6: If $T_f \geq nT$, set T_f to nT (i.e. T to T_f/n) and use it to compute TCU_{sc} using Equation (4.1.19) and then proceed to Step 7.

Step 7: End.

The farmer's order quantity is simply the product of the number of shipments that the processor delivers to the retailer in a single processing setup and the retailer's order quantity (i.e. ny).

4.1.3.4.3 Proof of convexity of the objective function

As a way of showing that a unique solution to the objective function exists, it must be proven that the objective function is convex because it seeks to minimise the total supply chain cost. The partial derivatives of TCU_{sc} with respect to T and n are

$$\frac{\partial TCU_{sc}}{\partial T} = -\frac{K_r}{T^2} + \frac{h_r D}{2} - \frac{K_p}{nT^2} + \frac{h_p D}{2}\left[\left(n-1\right)\left(1-\frac{D}{R}\right) + \frac{D}{R}\right] - \frac{K_f}{nT^2}, \tag{4.1.23}$$

$$\frac{\partial^2 TCU_{sc}}{\partial T^2} = \frac{K_r}{T^3} + \frac{K_p}{nT^3} + \frac{K_f}{nT^3}, \tag{4.1.24}$$

$$\frac{\partial TCU_{sc}}{\partial n} = -\frac{K_p}{n^2 T} + \frac{h_p T D}{2}\left(1-\frac{D}{R}\right) - \frac{K_f}{n^2 T}, \tag{4.1.25}$$

$$\frac{\partial^2 TCU_{sc}}{\partial n^2} = \frac{K_p}{n^3 T} + \frac{K_f}{n^3 T}, \tag{4.1.26}$$

$$\frac{\partial^2 TCU_{sc}}{\partial T \partial n} = \frac{\partial^2 TCU_{sc}}{\partial n \partial T} = -\frac{K_p}{n^2 T^2} + \frac{h_p D}{2}\left(1-\frac{D}{R}\right) - \frac{K_f}{n^2 T^2}. \tag{4.1.27}$$

The quadratic form of the Hessian matrix is given by

$$\begin{bmatrix} T & n \end{bmatrix} \begin{bmatrix} \frac{K_r}{T^3} + \frac{K_p}{nT^3} + \frac{K_f}{nT^3} & -\frac{K_p}{n^2T^2} + \frac{h_pD}{2}\left(1-\frac{D}{R}\right) - \frac{K_f}{n^2T^2} \\ -\frac{K_p}{n^2T^2} + \frac{h_pD}{2}\left(1-\frac{D}{R}\right) - \frac{K_f}{n^2T^2} & \frac{K_p}{n^3T} + \frac{K_f}{n^3T} \end{bmatrix} \begin{bmatrix} T \\ n \end{bmatrix}$$
$$= \frac{2}{T}\left(K_r + \frac{K_p}{n} + \frac{K_f}{n}\right) > 0. \tag{4.1.28}$$

From Equation (4.1.28), the objective is proven to be convex because it's quadratic from is shown to be positive.

4.1.4 Numerical results

4.1.4.1 Numerical example

As a way of showing the potential practical applications of the inventory problem discussed, a numerical example considering a supply chain with three members (i.e. a farmer, a processor and a retailer) involved at various stages of the food production chain is presented. The farmer grows newborn lambs and ships them to a processor for slaughtering, cutting and packaging. The processor then ships an integer number of batches of processed item to the retailer. The following parameters apply to the supply chain inventory problem at hand: D =10 000 kg/year, R =12 500kg/year, K_r =80 000 ZAR, h_r =20 ZAR/kg/year, K_p =60 000 ZAR, h_p =15 ZAR/kg/year, K_f =40 000 ZAR, c_f =10 ZAR/kg/year, α =51 kg, β =5, λ =6.2 /year and w_1=45 kg. The results from the example are presented in Table 4.1.1.

According to Table 4.1.1, the farmer should order ny = 640 newborn lambs at the beginning of a growing cycle and deliver them to to the processor after (T_f=) 0.5746 years when the weight of each sheep has reached the target weight of 45 kg . The processor should deliver (n=) four shipments of processed sheep in each processing setup to the retailer and each shipment should have $y = 160$ processed sheep. The weight of each shipment would be 7 200 kg ($Q_1 = yw_1$) and the retailer would replenish orders every $T = 0.7157$ years. By following this policy the total supply chain cost would amount to 329 214.74 ZAR/year. Figure 4.1.7 shows the response of the objective function to changes in the decision variables.

Table 4.1.1: Results from the numerical example

Number of shipments (n)	**Retailer's cycle time** (T)	**Retailer's order quantity** (y)	**Total supply chain cost** (TCU_{sc})
1	1.0607	236	375 197.98
2	0.8619	192	337 448.78
3	0.7723	172	329 271.54
4	**0.7157**	**160**	**329 214.74**
5	0.6742	150	332 434.67
6	0.6414	143	337 227.71
7	0.6141	137	342 846.52

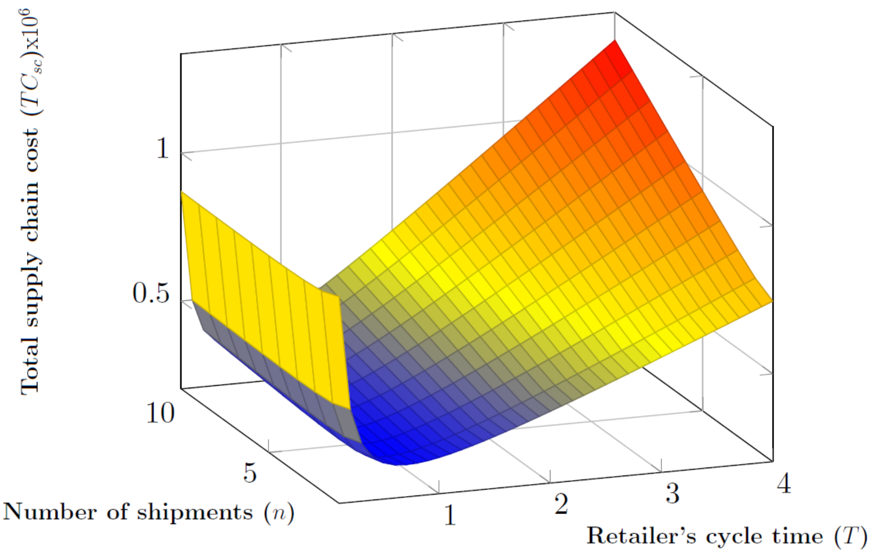


Figure 4.1.7: The total supply chain cost's response to changes in the number of shipments (made by the processor to the retailer) and the retailer's cycle time.

4.1.4.2 Sensitivity analysis

A sensitivity analysis was conducted on the proposed supply chain system's major input parameters and the results are presented in Table 4.1.2, from which the following observations are made:

- The retailer's ordering cost is the most sensitive input parameter to both the objective function (TCU_{sc}) and the decision variables (T and n). A 50% decrease in the retailer's ordering cost reduces their cycle time by 27.0% and consequently the number of shipments of ready-for-consumption (i.e. processed) items made by the processor to the retailer changes from four to five (representing a 25% increase). As a result, the cost of managing inventory in the supply chain decreases by 19.3%.

Table 4.1.2: Results from the sensitivity analysis

Parameters	% change	**Retailer's cycle time (T)** years	% change	**Number of shipments (n)** shipments	% change	**Total supply chain cost (TCU_{sc})** ZAR/year	% change
Base example		**0.7157**		**4**		**329 214.74**	
K_r	-50	0.5222	-27.0	5	+25	265 569.24	-19.3
	-25	0.6439	-10.0	4	0	299 794.31	-8.9
	+25	0.8377	+17.1	3	-25	354 115.70	+7.6
	+50	0.8983	+25.5	3	-25	377 156.52	+14.5
h_r	-50	0.8997	+25.7	3	-25	287 712.64	-12.6
	-25	0.8288	+15.8	3	-25	309 282.62	-6.1
	+25	0.6757	-5.6	4	0	346 592.13	+5.3
	+50	0.6086	-15.0	4	0	363 048.09	+10.3
K_p	-50	0.7375	+3.0	3	-25	316 024.72	-4.0
	-25	0.7551	+5.5	3	-25	322 724.59	-2.0
	+25	0.7283	+1.8	4	0	334 408.56	+1.6
	+50	0.7408	+3.5	4	0	339 513.58	+3.1
h_p	-50	0.7906	+10.5	5	+25	288 768.94	-12.3
	-25	0.7644	+17.0	4	0	309 784.90	-5.4
	+25	0.7303	+2.0	3	-25	346 162.85	+5.1
	+50	0.6944	-3.0	3	-25	362 181.17	+10.0
K_f	-50	0.6984	-2.4	4	0	322 143.15	-2.1
	-25	0.7071	-1.2	4	0	325 700.51	-1.1
	+25	0.7241	+1.2	4	0	332 687.39	+1.1
	+50	0.7325	+2.4	4	0	336 119.88	+2.1
c_f	-50	0.7157	0	4	0	311 321.38	-5.4
	-25	0.7157	0	4	0	320 268.06	-2.7
	+25	0.7157	0	4	0	338 161.43	+2.7
	+50	0.7157	0	4	0	347 108.11	+5.4

- Of all the input parameters tested in the sensitivity analysis, the farmer's feeding cost has the least effect on the objective function and the two decision variables.
- The total supply chain cost is affected by changes in any of the input parameters. However, the effects vary by parameter. In general, the effects caused by changes in the retailer and the processor's input parameters are greater than those of the farmer's parameters. Changes in the farmer's feeding and setup costs have the smallest effects, with 50% decreases in these two costs resulting in decreases of only 5.4% and 2.1% respectively in the total supply chain cost. On the other hand, 50% decreases in equivalent costs at the processor (i.e. holding costs and setup costs) resulted in decreases of 12.3% and 4.0% respectively and at the retailer (i.e. holding costs and ordering costs), the decreases in the total supply chain cost were 12.6% and 19.3% respectively.

- With the exception of the farmer's input parameters, changes to all the other input parameters affected the number of shipments made by the processor to the retailer during a single processing run. The shipments delivered by the processor increased as the retailer's holding cost and the processor's setup cost increased.

- In general, decisions of downstream supply chain members have greater effects on the decision variables than the upstream member, with the retailer's input parameters having the greatest effect followed by the processor's parameters. The effects of changes to the farmer's individual cost components on the number of shipments of processed inventory made by the processor, the retailer's cycle time and the total supply chain cost are less than the effects of changes to the processor's and the retailer's individual cost components. For example, the total chain cost increases by 14.5% when the setup cost increases by 50% while the same percentage increase in the retailer's ordering cost and the processor setup cost only increases the cost by 3.1% and 2.1% respectively.

- Given that the input parameters for the downstream supply chain members have a greater effect, it would be advisable for managers to prioritise most of their cost saving initiatives at the retail and processing operations as opposed to the farming operations.

4.1.4.3 Further cost reduction mechanisms in the system

Two properties of this inventory management system further help in reducing the overall cost. The first is that as soon as the processor has processed enough items to make a batch, a shipment is made to the retailer (i.e. the processor does not wait for the cycle to end in order to deliver processed items to the retailer) and consequently the processor makes a number of shipments of processed items to the retailer during a single processing setup. The second property is the assumption that inventory replenishment decisions are made for the benefit of all supply chain members as opposed to optimising the decisions of individual members. The effectiveness of these cost saving initiatives is tested by comparing them with more conventional replenishment policies.

4.1.4.3.1 Comparison of results with an equal-cycle time replenishment policy

Table 4.1.3: Efficiency of the proposed replenishment policy against an equal-cycle time policy

Variable	Base (Proposed replenishment policy)	Equal-cycle time replenishment policy	% difference
TCU^*_{sc}	329 214.74	375 197.98	+14.0
T^*	0.7157	1.0607	+48.2
n^*	4	1	-75

One of the assumptions made when formulating the proposed inventory system is that the processor delivers a certain (integer) number of shipments, of equal size, to the retailer during the course of a single processing cycle. This implies that the processor's cycle time is a product of the retailer's cycle time and the integer number of shipments and therefore their cycle times are not equal. To determine whether the proposed replenishment policy performs better than an equal-cycle time approach, where by the cycle times of all the parties involved are equal, the equal-cycle time counter-part of the inventory problem at hand is computed by setting the number of shipments from the processor to the retailer to one. When an equal-cycle time policy is adopted, the total supply chain cost increases from 329 214.74 ZAR/year to 375 197.98 ZAR/year (while the retailer's cycle time increases from 0.7157 years to 1.0607 years) as shown in Table 4.1.3. From a cost saving perspective, the proposed inventory system leads to a 14.0% decrease in the total supply chain cost when compared to an equal-cycle time replenishment policy. This indicates that the proposed policy is more cost efficient than the common-cycle time policy. In essence, opting for an integer-multiplier replenishment policy is more effective at reducing total supply chain costs than using an equal-cycle time approach.

4.1.4.3.2 Comparison of results with an independent replenishment policy

Table 4.1.4: Efficiency of the proposed replenishment policy against an independent policy

Variable	Base (Proposed replenishment policy)	Independent replenishment policy	% difference
TCU_r^*	183 349.91	178 885.44	-2.4
TCU_p^*	96 105.34	100 623.06	+4.7
TCU_f^*	49 759.49	58 147.41	+16.9
TCU_{sc}^*	329 214.74	337 655.91	+2.6

If a system approach is not adopted when managing inventory in the supply chain, then the chain members act independently and are thus only concerned with reducing their own costs as opposed to reducing the total system-wide costs. A comparison of the proposed inventory system, which is aimed at reducing the total supply chain costs, with its independent counter-part is given in Table 4.1.4. In order to determine the costs for the different chain members when an independent replenishment policy is adopted, it was assumed that the retailer, who faces customer demand for processed items, makes inventory replenishment decisions aimed at reducing their own costs and passes these down to the processor. This means that the retailer orders processed items from the processor based on their own EOQ and forces it to the upstream chain members. In order to fulfil the order, the processor places an order for grown items from the farmer. The processor then optimises the number of shipments made to the retailer, while adhering to the retailer's EOQ. As shown in Table 4.1.4, opting for an independent replenishment policy is not beneficial to all chain members and it also increased the cost of managing inventory across the entire chain (from 329 214.74 ZAR/year to 337 655.91 ZAR/year). While the retailer's cost was reduced by 2.4%, the processor's and the farmer's total costs increased by 4.7% and 16.9% respectively. The cost saving realised when all three members are collaborating, which amounts to 2.6%, highlights the importance of integrating inventory decisions with all supply chain members.

4.1.5 Concluding remarks

Growing items, which include crops and livestock, serve as a staple of most diets. Prior to being sold to consumers, they are usually grown at a farming operation, then processed in a manufacturing plant and finally sold to consumers through a retailer. Using this logic, a three-level supply chain (consisting of a farmer, a processor and a retailer) inventory system for growing items is proposed and a corresponding model is developed. An iterative solution algorithm for solving the model is proposed and applied to a numerical example. Through numerical experimentation, the proposed integrated inventory system is found to be 2.6% more cost efficient than its independent (i.e. non-coordinated) counterpart and 14.0% better than its equal-cycle time counterpart.

One of the limitations of the proposed model is that it only considers a serial supply chain with one member at each echelon. While this makes the mathematical formulation easier, it is not representative of most real-life supply chains. Secondly, mortality is an important issue when dealing with live items and this was not accounted for in this section. These two shortcomings of the proposed inventory system present opportunities for future research direction in addition to other popular EOQ extensions such as the incorporation of shortages, maintenance issues and permissible delay in payments. The three-echelon serial supply chain setup in the proposed model is a simplified version of an actual food production system, which is often a complex network, so another possible area of future exploration could be the inclusion of more echelons as well as the transportation nodes which connect those echelons.

The potential cost savings that can be realised in food production systems by adopting certain elements of the proposed inventory control model are important for both financial and social reasons. From an economic perspective, cost saving resulting from better inventory control can bolster profit margins and, in the process, improve both the financial performance of the supply chain and shareholder value. Taking a less capitalistic view, the resulting cost savings can be used for social reasons particularly because growing items are the primary source of most consumable food products. Supply chains can cushion consumers against some of the effects of rising food prices by absorbing a portion of the proposed price increases. By so doing, consumers will not be burdened as much because the price increases will not be as severe. Given that food products are an essential part of daily living, managers in food production systems should always be on the lookout for opportunities to save costs and pass the savings down to consumers. Reducing the costs of managing inventory is one way for managers to achieve this and the model presented in this section represents a piece of the puzzle aimed at reducing costs across food production chains.

4.2 A three-echelon supply chain inventory model for growing items with price- and freshness-dependent demand†

4.2.1 Introduction

4.2.1.1 Context

Lately, a number of inventory control models that are aimed at jointly optimising lot sizes and selling prices of perishable items have been developed, for example, Chen et al. (2016), Wu et al. (2016) and Feng et al. (2017). These three models, as well as other extensions based on them, were formulated specifically for perishable food items, and consequently, the demand rate used in these models had a few characteristics peculiar to perishable food products. The focus of this study is on two of those characteristics, which are the dependence of the demand rate on price and freshness of the items. These are two of the most important demand characteristics of perishable food products such as meat, seafood, fruits and vegetables. Two reasons may be adduced to the importance of these characteristics. Firstly, given the commoditised nature of groceries (and by extension food products), there is very little to differentiate between competing brands. As a result, the selling price is one of the most important factors that affect consumers' purchase decisions. Secondly, consumers prefer perishable food products when they are fresh implying that consumers are less likely to buy a particular item if it has been on shelves for longer periods of time because the longer it is on the shelves, the less fresh it becomes.

Although the aforementioned studies accurately depict the inventory behaviour of perishable food products, they are all focused on (and limited to) the retail end of the supply chain. Evidently, decisions affecting the price and length of stay of perishable food items on shelves are not limited to those taken at the retail end of the chain. In reality, the primary source of these products is living organisms such as crops and livestock which are reared at farms. Furthermore, products purchased at retail outlets are seldom in their original form. Most of the times, they have to be transformed into a different form that is suitable for human consumption and the transformation processes often take place in food processing plants.

4.2.1.2 Purpose

The aim of this study is to consider the implications of price and freshness (measured through the age of the item) on the inventory management policies of a multi-echelon supply chain of growing items. To this end, an integrated model for managing inventory in a three-echelon supply chain for growing items is proposed. The three echelons correspond to the farming, processing and consumption (retail) stages of a simplified value chain for perishable food products. At the farming echelon, live items are reared until such time as their weight reaches a given target. Following this, the items are transformed into a form that is safe for consumption. In the case of meat, the transformation process typically entails slaughtering, cutting and packaging. In the context of this study, all the tasks

† A modified version of this section has been accepted for publication as Sebatjane and Adetunji (2020b) in *Operations Research Perspectives*.

taking place at this echelon are collectively termed processing and they occur at a given finite rate. At the final echelon, consumer demand for the processed item is met through sales, and this demand is assumed to be a function of the product's selling price and freshness, measured through the age of the product from the time of processing.

4.2.1.3 Relevance

Perishable inventory control models such as those of Wu et al. (2016) and Feng et al. (2017) recognised that freshness and selling price, among other factors, are important determinants of demand for perishable food products. Nonetheless, these studies, along with their various extensions, were focused entirely on optimising purchasing decisions at the retail end of the supply chain. The current increasingly competitive business climate has forced businesses to seek external sources of cost and operational efficiencies in addition to intra-organisational optimisation. For this reason, a lot of businesses have been using supply chain integration as a tool for competitiveness.

This study extends the concept of supply chain integration to inventory control mechanisms used in perishable food products supply chains dealing with growing items such as livestock. In essence, the study considers an end-to-end supply chain for perishable food products, with the downstream end corresponding to consumption (of processed inventory) and the upstream end corresponding to rearing (of live inventory). Seeing that considerable cost and operational efficiencies can be achieved by integrating purchasing decisions among all supply chain members (Ben-Daya and Al-Nassar, 2008), the model presented in this study can serve as a guideline for production and operations managers in multi-echelon supply chains for growing items when making purchasing, shipment and pricing decisions.

4.2.1.4 Organisation

Other than the introductory section, this section has five more subsections. Prior to the model development phase which is presented in Subsection 4.2.3, the assumptions employed are stated in Subsection 4.2.2. Theoretical results which prove the model's optimality are given in Subsection 4.2.4 while numerical results which highlight potential practical applications of the model are given in Subsection 4.2.5. The section is wrapped up in Subsection 4.2.6 through the presentation of concluding remarks and suggestions for future research.

4.2.2 Assumptions

The supply chain under consideration has three echelons and there is a single member at each echelon. Figure 4.2.1 is a depiction of the proposed inventory control system. The inventory profile at the uppermost portion of the figure shows the changes to the weight of the ordered live items at the farming echelon. The middle portion of the figure depicts the weight of the processed inventory as the live items are slaughtered, prepared and packaged (i.e. processed). The lowermost portion of the figure also shows the processed inventory at the retail echelon.

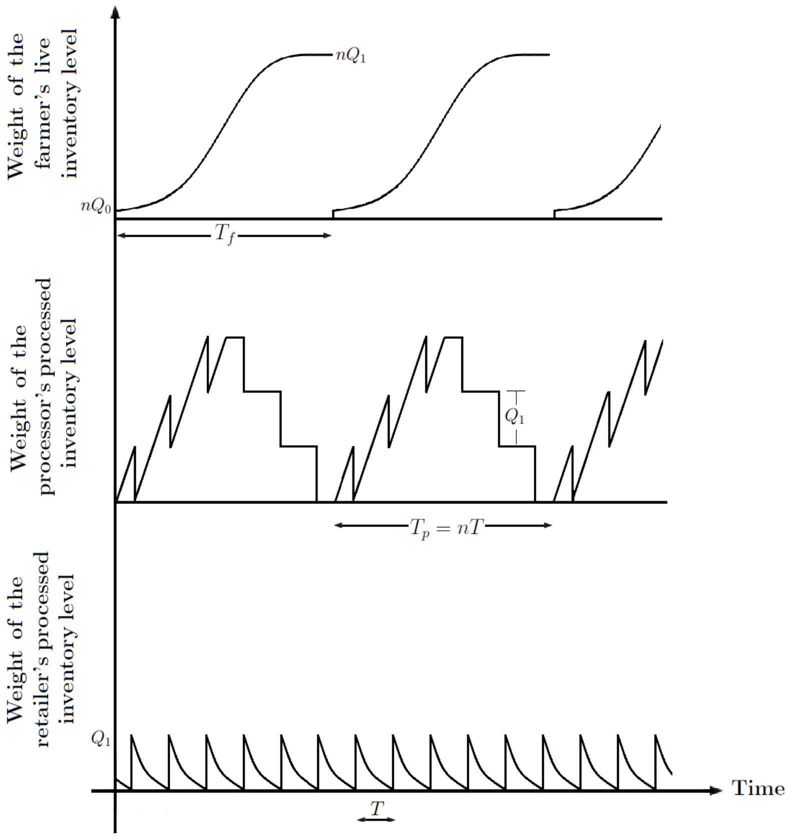


Figure 4.2.1: Behaviour of the weight of the live inventory at the growing facility, the weight of the processed inventory at the processing plant and the weight of the processed inventory at the retail outlet.

At the farming echelon, a farmer procures ny live newborn items and rears them. Given that the initial weight of each live item at the time the farmer receives the order is w_0, the weight of all the newborn items ordered, nQ_0, is therefore equal to nyw_0. The items' growth function is approximated by

$$w(t) = \frac{\alpha}{1 + \beta e^{-\lambda t}}, \tag{4.2.1}$$

which is the logistic function where α is the items' asymptotic weight, β is the integration constant and λ is the exponential rate of growth for the items. This function is chosen because of its distinctive "S"-shape which is reminiscent of the growth pattern of livestock. The farmer rears the live items for a period of T_f time units. This period ends when the weight of each item reaches the target maturity weight w_1. This implies that the duration of the growth period is

$$T_f = -\frac{\ln\left[\frac{1}{\beta}\left(\frac{\alpha}{w_1} - 1\right)\right]}{\lambda}, \tag{4.2.2}$$

as determined from substituting T_f and w_1 into Equation (4.2.1). The weight of all the live mature items is thus given by $nQ_1 = nyw_1$. This entire lot is then transferred to the processing plant. The live mature items are scheduled to arrive at the processing plant just as the processor starts a new processing cycle of duration T_p. For this reason, it is imperative for the live items to have grown to the target weight by the time the growing period ends after T_f time units. To ensure that this happens, the constraint

$$T_f \leq T_p, \tag{4.2.3}$$

is imposed on the inventory system under consideration.

At the processing echelon, the live inventory items are processed at a rate of R and they are transformed into processed inventory which is used to meet consumer demand at the supply chain's next echelon. The processing rate, R, is assumed to be a deterministic constant greater than the demand rate D. Consequently, processing does not take place for the entirety of the processor's cycle. This is because the weight of the processed inventory accumulates at a rate of $R - D$ and therefore, the demand can be met without having to continuously process the live items throughout the whole cycle. In essence, the processor's cycle can be divided into two portions: when there is processing and when there is no processing of items. During the processing time, the live inventory is processed and shipped to the retailer in equally-sized batches weighing $Q_1 = yw_1$. Both processing (of the live inventory) and shipping (of the processed inventory to the retailer) take place simultaneously during this time. This implies that processor starts shipping to the retailer once they have processed enough inventory to make up a batch (of weight Q_1). During the non-processing time, the processor continues to ship batches of processed inventory to the retailer without having to process because processed inventory would have accumulated during the processing time of the cycle since $R > D$. Granted that the processor receives a lot weighing nQ_1 from the farmer and ships it to the retailer (after processing it), in equally-weighted batches (each with a weight of Q_1) and at equally-spaced time intervals, T, the processor therefore makes n deliveries of processed inventory during the course of a single processing cycle T_p. This implies that the retailer's cycle time, T, is an integer multiple (in this instance the integer is n) of the processor's cycle time. Hence,

$$T_p = nT. \tag{4.2.4}$$

At the final echelon, the retailer receives orders of processed inventory from the processor at regular time intervals of duration T in order to meet the consumer demand rate (for processed inventory) of D. Each order of processed inventory that the processor ships to the retailer weighs Q_1. The demand rate is assumed to be affected by the items' selling price and freshness index. Classic economic and marketing theories affirm that the sales of an item are influenced by its selling price, among other factors. In essence, lower prices tend to spike the sales of an item and for this reason, the demand rate is assumed to be an exponentially decreasing function of the price. This is in accordance with studies by Feng et al. (2017), Wu et al. (2017) and Feng and Chan (2019), to name a few. Hence,

$$D \propto \phi e^{-\omega p_r}, \tag{4.2.5}$$

where ϕ represents the maximum size of the market for the processed inventory (asymptotic level of demand attainable when the selling price is considered most favourable to customers), ω is the price elasticity of the demand rate and p_r is the retailer's selling price per weight unit of the processed inventory. All three variables are positive numbers and thus, $\phi e^{-\omega p_r} > 0$.

Another aspect that has an effect on the demand for perishable food products is the freshness of the items. A vast majority of consumable food products have shelf lives that are often expressed as expiration or sell-by dates which essentially represent the maximum life times of those products. The printed expiration dates have an effect on consumers likelihood to make purchases. In essence, a consumer's likelihood of purchasing an item diminishes as the item ages (i.e. as it gets closer to its expiration date). Wu et al. (2016) (as well as subsequent models spun off from that particular model) used the Aarhenius equation to represent the freshness index of items. Therefore,

$$F(t) = \frac{L-t}{L}, \tag{4.2.6}$$

where L is the maximum shelf life or expiration date of the item. From Equation (4.2.6), the item is at its freshest (i.e. 100% freshness index) at $t = 0$ and it reaches its minimum freshness level of 0% at its expiration date L. The processed inventory is no longer suitable for consumption at its maximum shelf life meaning that the duration of the retailer's replenishment cycle cannot be greater than the shelf life (i.e. $L > T$).

In accordance with Chen et al. (2016), Wu et al. (2016) and Feng et al. (2017), Equations (4.2.5) and (4.2.6) are combined to formulate the demand as a multiplicative function of the selling price (in this case, per weight unit) and the freshness index of the inventory. Hence, the demand rate is

$$D = \left(\phi e^{-\omega p_r}\right)\left(\frac{L-t}{L}\right), \qquad 0 \leq t \leq T. \tag{4.2.7}$$

The proposed inventory control system is feasible when $R > D$. Since the demand rate varies with time, the only way to guarantee that this condition is met is by ensuring that the maximum possible demand rate does not exceed the processing rate. From Equation (4.2.7), the demand rate reaches its maximum value when the inventory is at its freshest (i.e. $t = 0$) and the retailer's selling price is zero (i.e. $p_r = 0$). This means that the maximum possible demand rate is ϕ and therefore, $R > D$ can be expressed as $R > \phi$.

4.2.3 Model formulation

The proposed inventory control model in the three-echelon supply chain system is formulated as a profit maximisation problem. All three members of the supply chain have a common goal of improving the supply chain's profit by reducing the costs associated with managing inventory across the chain. Each member's profit is calculated by subtracting the costs associated with managing inventory from the revenue generated from the sales of the inventory.

Consumer demand is for the processed inventory and this particular inventory, tracked at the processor's and the retailer's facilities, incurs purchasing, setup (or ordering, in the case of the retailer) and holding costs. On the other hand, the live inventory which is tracked at the farmer's facility incurs purchasing, setup and feeding costs, with the last cost being dependent on the weight of the item.

The model's objective function is the total supply chain profit and its decision variables are retailer's cycle time, the retailer' selling price and the number of batches of processed inventory shipped to the retailer per processing cycle.

4.2.3.1 The retail echelon

The start of the retailer's replenishment cycle is marked by the receipt of an order for processed inventory weighing Q_1. This inventory is displayed on shelves at the retail outlet and it can only be kept for a specified amount of time, known as the expiration date. Once this date has elapsed, the inventory can no longer be used to meet consumer demand. Figure 4.2.2 is a representation of the changes that occur to the weight of the retailer's inventory throughout the cycle.

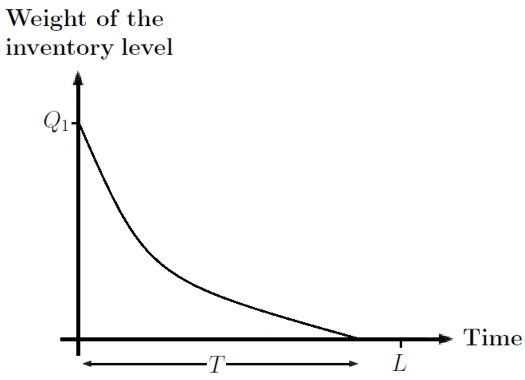


Figure 4.2.2: The retailer's processed inventory system behaviour

During the course of a replenishment cycle, the weight of the retailer's processed inventory is depleted due to consumer demand. As a result, the weight of the retailer's processed inventory is governed by the differential equation

$$\frac{dI(t)}{dt} = -D = -\left(\phi e^{-\omega p_r}\right)\left(\frac{L-t}{L}\right), \qquad 0 \leq t \leq T. \tag{4.2.8}$$

Equation (4.2.8) can be re-arranged into

$$dI(t) = \left(\phi e^{-\omega p_r}\right)\left(-1 + \frac{t}{L}\right)dt, \qquad 0 \leq t \leq T. \tag{4.2.9}$$

Integrating the left and the right hand sides of Equation (4.2.9) leads to

$$I(t) = \left(\phi e^{-\omega p_r}\right)\left(-t + \frac{t^2}{2L}\right) + C. \tag{4.2.10}$$

Since the weight of the processed inventory at the retailer reaches zero at time T, the boundary condition $I(T) = 0$ is binding. Through substitution, it follows that

$$C = -\left(\phi e^{-\omega p_r}\right)\left(-T + \frac{T^2}{2L}\right) = \left(\phi e^{-\omega p_r}\right)\left(T - \frac{T^2}{2L}\right). \tag{4.2.11}$$

By substituting Equation (4.2.11) into Equation (4.2.10) and re-arranging the terms, the weight of the retailer's processed inventory level is determined as

$$I(t) = \frac{\left(\phi e^{-\omega p_r}\right)}{2L}\left[t^2 + 2L(T-t) - T^2\right] \tag{4.2.12}$$

Given that the retailer receives an order weighing Q_1 at the start of each cycle (i.e. $t = 0$), the boundary condition $I(0) = Q_1$ is binding. Through substitution, it follows that

$$Q_1 = I(0) = \frac{\left(\phi e^{-\omega p_r}\right)\left(2LT - T^2\right)}{2L}. \tag{4.2.13}$$

Granted that $Q_1 = yw_1$, the equivalent number of items in the retailer's lot is thus

$$y = \frac{\left(\phi e^{-\omega p_r}\right)\left(2LT - T^2\right)}{2Lw_1}. \tag{4.2.14}$$

The retailer's cyclic holding cost (i.e. during the time period $[0, T]$) is determined using Equation (4.2.12) as

$$HC_r = h_r \int_0^T I(t)\,dt = h_r \left[\frac{\left(\phi e^{-\omega p_r}\right)\left(3LT^2 - 2T^3\right)}{6L} \right]. \tag{4.2.15}$$

The retailer's cyclic profit function is defined as the cyclic total revenue less the sum of the cyclic ordering, purchasing and holding costs. It follows that

$$TP_r = \frac{p_r\left(\phi e^{-\omega p_r}\right)\left(2LT - T^2\right)}{2L} - \frac{p_p\left(\phi e^{-\omega p_r}\right)\left(2LT - T^2\right)}{2L} - K_r - \frac{h_r\left(\phi e^{-\omega p_r}\right)\left(3LT^2 - 2T^3\right)}{6L}. \tag{4.2.16}$$

The first term in Equation (4.2.16) represents the cyclic revenue and it is the product of the selling price per weight unit charged to consumers (p_r) and the weight of processed items sold per cycle (Q_1). The second term is the cyclic purchasing cost and it is defined as the product of the weight of processed items purchased from the processor (Q_1) and the price that the processor charges for the inventory (p_p). The third term denotes the fixed cost associated with placing an order during each cycle while the last term is the cyclic holding cost from Equation (4.2.15).

The retailer's total profit per unit time is determined by dividing their cyclic profit by their cycle duration T and thus,

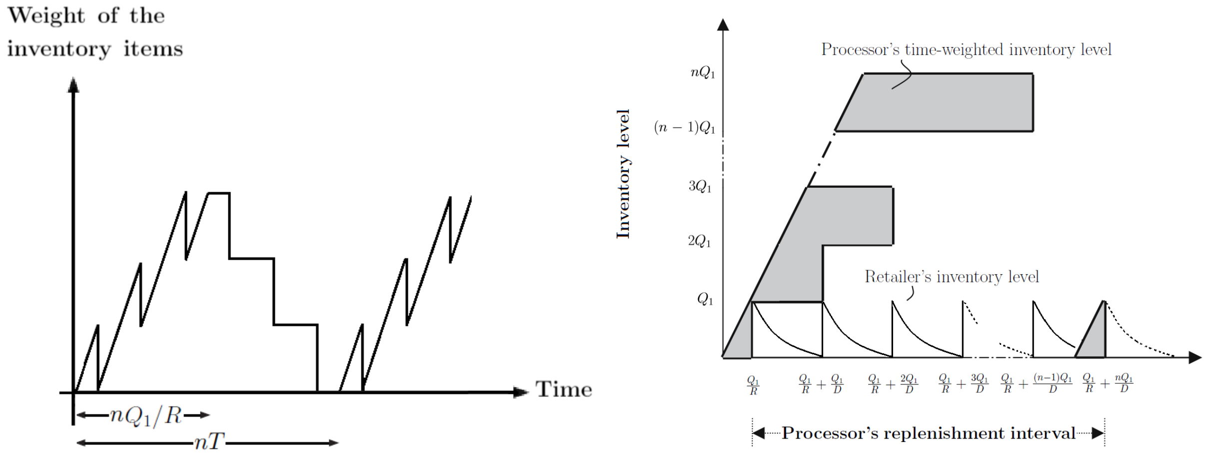
$$TPU_r = \frac{\left(\phi e^{-\omega p_r}\right)\left(2LT - T^2\right)\left(p_r - p_p\right)}{2LT} - \frac{K_r}{T} - \frac{h_r\left(\phi e^{-\omega p_r}\right)\left(3LT^2 - 2T^3\right)}{6LT}. \tag{4.2.17}$$

4.2.3.2 The processing echelon

The processor is responsible for transforming the live inventory into consumable processed inventory. When a new processing cycle starts, the processor receives an order of live items weighing nQ_1 from the farmer and processes the entire order at a rate of R. Throughout the cycle, the processor delivers n shipments of processed inventory to the retailer. The shipments are all of equal weight, meaning that they each weigh Q_1. The behaviour of the processor's processed inventory level is depicted in Figure 4.2.3a which is redrawn into Figure 4.2.3b for ease of computing the area under the graph. This method of redrawing the inventory system profile is adapted from a version of the JELS problem formulated by Yang et al. (2007).

The processor's cyclic holding cost is computed by multiplying the holding cost per weight unit by the area under the processor's inventory system which essentially shows the processor's time-weighted inventory level. The area under the graph in Figure 4.2.3b is thus

$$\begin{aligned} \text{Area}_p &= \text{Processor's time-weighted inventory} \\ &= \frac{nQ_1^2}{2R} + Q_1^2\left(\frac{1}{D(p)} - \frac{1}{R}\right) + 2Q_1^2\left(\frac{1}{D(p)} - \frac{1}{R}\right) + \cdots + (n-1)Q_1^2\left(\frac{1}{D(p)} - \frac{1}{R}\right) \\ &= \frac{nQ_1^2}{2R} + \frac{n(n-1)Q_1^2}{2}\left(\frac{1}{D(p)} - \frac{1}{R}\right). \end{aligned} \tag{4.2.18}$$



(a) Original. *(b) Redrawn (modified from Yang et al. (2007)).*

Figure 4.2.3: The processor's processed inventory system behaviour.

The demand rate in Equation (4.2.18) is a function of the retailer's selling price p_r. If p_r is held constant, then the demand rate in each cycle interval T is equal, and since T is used as the time basis for the analysis, all demands for all time intervals can be aggregated for ease of derivation. Hence, the processor's holding cost per cycle becomes

$$HC_p = h_p\left[\frac{nQ_1^2}{2R} + \frac{n(n-1)Q_1^2}{2}\left(\frac{T}{Q_1} - \frac{1}{R}\right)\right], \tag{4.2.19}$$

after replacing $D(p)$ in Equation (4.2.18) with Q_1/T so that all the terms are expressed in terms of T which is one of the model's decision variables. The expression for D as given in Equation (4.2.7) is not used because it varies with time and this becomes problematic when solving the model. Instead, a static approximation of D is used. Since the retailer receives orders of processed inventory weighing Q_1 at equally-spaced time intervals of duration T in order to meet a demand rate of D, the retailer places $\approx D/Q_1$ orders per unit time. This means that the retailer's cycle time $T \approx Q_1/D$. Likewise, $D \approx Q_1/T$.

The processor's profit per cycle is defined as the cyclic revenue minus the sum of the cyclic setup and holding costs. Thus,

$$TP_p = p_p nQ_1 - p_f nQ_1 - K_p - h_p\left[\frac{nQ_1^2}{2R} + \frac{n(n-1)Q_1^2}{2}\left(\frac{T}{Q_1} - \frac{1}{R}\right)\right]. \tag{4.2.20}$$

The first term in Equation (4.2.20) represents the processor's cyclic profit and it is determined as the product of the weight of the processed inventory sold to the retailer in a single processing run (nQ_1) and the price (per weight unit) that the processor charges the retailer for the processed inventory (p_p). The second term denotes the processor's procurement cost per cycle and it is computed by multiplying the weight of the mature live inventory that the processor procures from the farmer (nQ_1) and the price that the farmer charges for the inventory (p_f). The third term denotes the fixed cost of setting up the processing facility at the beginning of each processing cycle. The last term represents the the cyclic holding cost as determined in Equation (4.2.19).

Dividing Equation (4.2.20) by the processor's cycle time, $T_p = nT$, yields an expression for the processor's total profit per unit time. After substituting Q_1 with Equation (4.2.13), the expression becomes

$$TPU_p = \frac{\left(\phi e^{-\omega p_r}\right)\left(2LT - T^2\right)\left(p_p - p_f\right)}{2LT} - \frac{K_p}{nT} - \frac{h_p}{2TR}\left[\frac{\left(\phi e^{-\omega p_r}\right)\left(2LT - T^2\right)}{2L}\right]^2 - \frac{h_p(n-1)}{2T}\left[\frac{\left(\phi e^{-\omega p_r}\right)\left(2LT - T^2\right)}{2L}\right]^2\left[\frac{2LT}{\left(\phi e^{-\omega p_r}\right)\left(2LT - T^2\right)} - \frac{1}{R}\right]. \quad (4.2.21)$$

4.2.3.3 The farming echelon

Whenever the farmer's replenishment cycle begins, ny live day-old newly born items are procured and grown to maturity. The items are deemed mature when the weight of each item reaches w_1 after T_f time units. This means that the weight of all the items in the farmer's lot would be $nQ_1 = nyw_1$ by the time they are transferred to the processing plant. This is up from an initial purchase weight of $nQ_0 = nyw_0$ for all the ordered items. Figure 4.2.4 depicts the growth trajectory of the items at the farmer's growing facility.

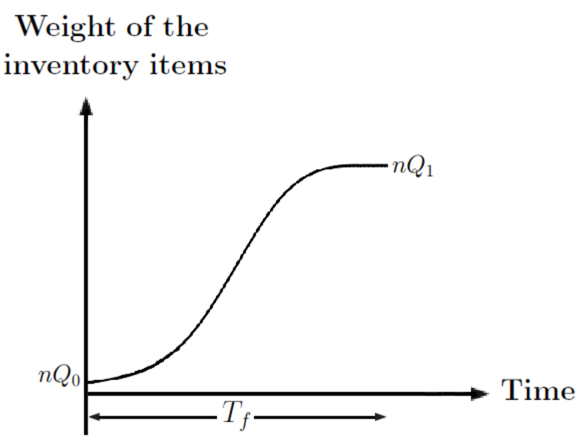


Figure 4.2.4: The farmer's live inventory system behaviour

The farmer's cyclic feeding cost (during the time period $[0, T_f]$) is defined as the product of the feeding cost per weight unit, c_f, and the area under the graph of the growth period as given in Figure 4.2.4. Therefore,

$$FC_f = c_f \int_0^{T_f} nyw(t)\, dt = c_f ny\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1 + \beta e^{-\lambda T_f}\right) - \ln\left(1 + \beta\right)\right]\right\}. \quad (4.2.22)$$

The farmer's profit per cycle is therefore

$$TP_f = p_f nQ_1 - p_v nQ_0 - K_f - c_f ny\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1 + \beta e^{-\lambda T_f}\right) - \ln\left(1 + \beta\right)\right]\right\}. \quad (4.2.23)$$

The first term in Equation (4.2.23) denotes the farmer's revenue per cycle and it is computed by multiplying the price (per weight unit) that the farmer charges to the processor for the live mature inventory (p_f) by the weight of the lot that the farmer sells to the processor (nQ_1). The second term is the cyclic procurement cost and it is computed as the product of the price (per weight unit) that the farmer is charged for the live newborn inventory (p_v) by their initial supplier and the weight of the lot that the farmer receives from their initial supplier (nQ_0). The third term is the fixed cost of setting up a new growing cycle while the last term is the cyclic feeding cost as determined from Equation (4.2.22).

The farmer's and the processor's cycles are synchronised to ensure that the processor is starts a new processing cycle at the instant that the items have reached the target maturity weight when the growth period ends at T_f time units. To achieve the synchronisation, the farmer and the processor share the same replenishment interval of $T_p = nT$ time units. In order to determine the farmer's profit per unit time, the profit per cycle, as given in Equation (4.2.23), is divided by the duration of the replenishment interval, nT, and the result becomes

$$TPU_f = \frac{p_f\left(\phi e^{-\omega p_r}\right)\left(2LT - T^2\right)}{2LT} - \frac{p_v w_0\left(\phi e^{-\omega p_r}\right)\left(2LT - T^2\right)}{2LTw_1} - \frac{K_f}{nT} - \frac{c_f\left(\phi e^{-\omega p_r}\right)\left(2LT - T^2\right)}{2LTw_1}\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1 + \beta e^{-\lambda T_f}\right) - \ln\left(1 + \beta\right)\right]\right\}, \quad (4.2.24)$$

after replacing Q_0 with yw_0 and substituting Q_1 and y with Equations (4.2.13) and (4.2.14), respectively.

4.2.3.4 The entire supply chain

4.2.3.4.1 Problem formulation

The total profit generated across the entire supply chain is the sum of the profits generated at each of the three echelons. Therefore, the total supply chain profit per unit time, TPU_{sc}, is the sum of Equations (4.2.17), (4.2.21) and (4.2.24). The mathematical formulation of the proposed inventory system is thus

$$\text{Maximise:} \quad TPU_{sc} = \frac{p_r\left(\phi e^{-\omega p_r}\right)\left(2LT - T^2\right)}{2LT} - \frac{K_r}{T} - \frac{h_r\left(\phi e^{-\omega p_r}\right)\left(3LT^2 - 2T^3\right)}{6LT} - \frac{K_p}{nT} - \frac{h_p}{2TR}\left[\frac{\left(\phi e^{-\omega p_r}\right)\left(2LT - T^2\right)}{2L}\right]^2 - \frac{h_p(n-1)}{2T}\left[\frac{\left(\phi e^{-\omega p_r}\right)\left(2LT - T^2\right)}{2L}\right]^2\left[\frac{2LT}{\left(\phi e^{-\omega p_r}\right)\left(2LT - T^2\right)} - \frac{1}{R}\right] - \frac{p_v w_0\left(\phi e^{-\omega p_r}\right)\left(2LT - T^2\right)}{2LTw_1} - \frac{K_f}{nT} - \frac{c_f\left(\phi e^{-\omega p_r}\right)\left(2LT - T^2\right)}{2LTw_1}\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1 + \beta e^{-\lambda T_f}\right) - \ln\left(1 + \beta\right)\right]\right\}$$

$$\text{subject to:} \quad T_f \leq nT, \quad n \in \mathbb{Z}. \quad (4.2.25)$$

The first constraint is from Equation (4.2.3) and it guarantees feasibility by ensuring that the live items are ready (in terms of having grown to the pre-defined target weight) for processing when they are transferred from the farming to the processing echelon. The second constraint is that the number of shipments of processed inventory delivered by the processor to the retailer is a positive whole number. This constraint makes the problem readily solvable because it is not possible for the processor to make non-integer deliveries to the retailer.

4.2.3.4.2 Solution procedure

The values of T, n and p_r that maximise TPU_{sc} are determined through the following iterative procedure:

Step 1 Set n to 1.

Step 2 Find the values of T and p_r that maximise Equation (4.2.25).

Step 3 Increase n by 1 and find the values of T and p_r that maximise Equation (4.2.25). Carry on to Step 4.

Step 4 If the latest value of TPU_{sc} increases, go back to Step 3. If the value of TPU_{sc} decreases, the previously calculated value of TPU_{sc} (along with the corresponding T, n and p_r values) is the best solution and if this case, carry on to Step 5.

Step 5 Verify the solution's feasibility with regard to the constraint $T_f \leq nT$. T_f is calculated from Equation (4.2.2). If the solution is feasible, those values of T, n and p_r are optimal and if this is the case, carry on to Step 7. If the solution is not feasible, carry on to Step 6.

Step 6 If the constraint is violated, set T to T_f/n and use that T value to calculate TPU_{sc} using Equation (4.2.25) and then carry on to Step 7.

Step 7 End.

4.2.4 Theoretical results

The concavity of the total supply chain profit (TPU_{sc}) with respect to the model's three decision variables, namely, the retailer's cycle time (T) and selling price (p_r) and the number of shipments of processed inventory delivered to the retailer per processing cycle (n), is investigated in two ways. Firstly, the concavity of TPU_{sc} in T for fixed values of p_r and n is proven. Secondly, the concavity of TPU_{sc} in p_r and n for a fixed T value is also proven. Together, these two results show that the model's objective function (i.e. TPU_{sc}) is concave and that there are unique T, p_r and n values that maximise this objective function.

Theorem 4.2.1. *For all $p_r > 0$ and $n > 0$, TPU_{sc} is a concave function of T. Therefore, a unique value of T that maximises TPU_{sc} exists.*

Proof. For settled values of p_r and n, the first and second derivatives of TPU_{sc}, as given in Equation (4.2.25), with respect to T are

$$\frac{\partial TPU_{sc}}{\partial T} = -\frac{p_r\left(\phi e^{-\omega p_r}\right)}{2L} + \frac{K_r}{T^2} - \frac{h_r\left(\phi e^{-\omega p_r}\right)(3L-4T)}{6L} + \frac{K_p}{nT^2} - \frac{h_p\left(\phi e^{-\omega p_r}\right)^2(T-2L)(3T-2L)}{8L^2R} + \frac{p_v w_0\left(\phi e^{-\omega p_r}\right)}{2Lw_1} + \frac{K_f}{nT^2} + \frac{c_f\left(\phi e^{-\omega p_r}\right)}{2Lw_1}\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right\} \quad (4.2.26)$$

$$\frac{\partial^2 TPU_{sc}}{\partial T^2} = -\frac{K_r}{T^3} - \frac{2h_r\left(\phi e^{-\omega p_r}\right)}{3L} - \frac{K_p}{nT^3} - \frac{h_p\left(\phi e^{-\omega p_r}\right)^2(3T-4L)}{4L^2R} - \frac{K_f}{nT^3} < 0 \quad (4.2.27)$$

Given that the second derivative of TPU_{sc} with respect to T is negative, as shown in Equation (4.2.27), it is apparent that TPU_{sc} is a concave function of T for any settled values of $p_r > 0$ and $n > 0$. This means that there is a unique T value that maximises TPU_{sc}. □

Theorem 4.2.2. *For all $T > 0$, TPU_{sc} is a concave function of both p_r and n. Therefore, unique values of p_r and n that maximise TPU_{sc} exist.*

Proof. For compactness, Equation (4.2.25) can be written in terms of Q_1 as

$$TPU_{sc} = \frac{p_r Q_1}{T} - \frac{K_r}{T} - \frac{h_r\left(\phi e^{-\omega p_r}\right)\left(3LT^2 - 2T^3\right)}{6LT} - \frac{K_p}{nT} - \frac{h_p Q_1^2}{2TR} - \frac{h_p(n-1)Q_1^2}{2T}\left(\frac{T}{Q_1} - \frac{1}{R}\right)$$
$$- \frac{p_v w_0 Q_1}{Tw_1} - \frac{K_f}{nT} - \frac{c_f Q_1}{Tw_1}\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1 + \beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right\}. \quad (4.2.28)$$

The fact that Q_1 is a function of p_r does not have an impact on the concavity of TPU_{sc} with respect to p_r and n because Q_1 is always positive since it is not possible for the retailer to receive an order of processed inventory with a negative weight. Recall, from Equation (4.2.13), that $Q_1 = \left(\phi e^{-\omega p_r}\right)\left(2LT - T^2\right)/2L$. Given that ϕ, ω and p_r are all > 0, $\phi e^{-\omega p_r}$ will always be > 0. Furthermore, it is not possible to have negative time duration and thus, L and T are > 0. Since the retailer can not sell the processed past its expiration date, $L > T$, and thus, $2LT - T^2$ will always be > 0. Therefore, Q_1 will always be positive.

For a settled value of T, the first and second derivatives of TPU_{sc}, as given in Equation (4.2.28), with respect to n and p_r are

$$\frac{\partial TPU_{sc}}{\partial n} = \frac{K_p}{n^2 T} - \frac{h_p Q_1^2}{2T}\left(\frac{T}{Q_1} - \frac{1}{R}\right) + \frac{K_f}{n^2 T} \quad (4.2.29)$$

$$\frac{\partial^2 TPU_{sc}}{\partial n^2} = -\frac{K_p}{n^3 T} - \frac{K_f}{n^3 T} \quad (4.2.30)$$

$$\frac{\partial TPU_{sc}}{\partial p_r} = \frac{Q_1}{T} + \frac{h_r \phi \omega e^{-\omega p_r}(3LT^2 - 2T^3)}{6LT} \quad (4.2.31)$$

$$\frac{\partial^2 TPU_{sc}}{\partial p^2} = -\frac{h_r \phi \omega^2 e^{-\omega p_r}(3LT^2 - 2T^3)}{6LT} \quad (4.2.32)$$

$$\frac{\partial^2 TPU_{sc}}{\partial n \partial p_r} = 0 \quad (4.2.33)$$

The quadratic form of the Hessian matrix of TPU_{sc} as given in Equation (4.2.28) is therefore

$$\begin{bmatrix} n & p_r \end{bmatrix} \begin{bmatrix} -\frac{K_p}{n^3 T} - \frac{K_f}{n^3 T} & 0 \\ 0 & -\frac{h_r \phi \omega^2 e^{-\omega p_r}(3LT^2 - 2T^3)}{6LT} \end{bmatrix} \begin{bmatrix} n \\ p_r \end{bmatrix}$$
$$= -\frac{K_p}{nT} - \frac{K_f}{nT} - \frac{h_r \phi \omega^2 {p_r}^2 e^{-\omega p_r}(3LT^2 - 2T^3)}{6LT} < 0. \quad (4.2.34)$$

Since the quadratic form of the Hessian matrix is negative, TPU_{sc} is a concave function of $n > 0$ and $p_r > 0$ for any given value of T. This means that TPU_{sc} is a concave function of n and p_r for a settled value of T and therefore, unique values of n and p_r that maximise TPU_{sc} exist. □

4.2.5 Numerical results

A numerical example that considers a chicken production system in a three-echelon supply chain is used to solve and analyse the proposed inventory control model. The example makes use of the following parameters: $L = 4$ days; R=320 kg/day; K_f=7 500 ZAR; c_f=1 ZAR/kg/day; K_p=5 000 ZAR; h_p=0.5 ZAR/kg/day; K_r=1 000 ZAR; h_r=1 ZAR/kg/day; p_v = 10 ZAR/kg; p_f = 15 ZAR/kg; p_p = 30 ZAR/kg; w_0= 0.06kg; w_1= 2kg; a = 300 kg/day; b = 0.03 kg/ZAR; α=6.87 kg; β=120; λ=0.12 /day.

Decision variables and objective function	Quantity
T^*	1.86 days
n^*	18 shipments
p_r^*	49.59 ZAR/kg
TPU_{sc}^*	759.98 ZAR/day

Table 4.2.1: Results from the example

The example is solved using the Solver function in Microsoft Excel and the results are presented in Table 4.2.1. The decision variables are used to determine the ordering and shipment policies to be followed by all three supply chain members. When a new cycle starts, the farmer should order ($ny \approx$) 871 newborn items with a total weight of ($nQ_0 =$) 55.7 kg. After ($T_f =$) 32.5 days, the items would have reached the targeted maturity weight and the total weight of the live inventory would be ($nQ_1 =$) 1 741.7 kg. The farmer should then send the live inventory to the next echelon where it is transformed into processed inventory. During the processing cycle, the processor should deliver ($n =$) 18 shipments of processed inventory to the retailer, with each shipment weighing ($Q_1 =$) 96.8 kg, at regularly spaced time intervals of ($T =$) 1.86 days. The retailer should sell the processed inventory at ($p_r =$) 49.59 ZAR/kg. The farmer and the processor should start new cycles every ($nT =$) 33.5 days. If this policy is followed, the supply chain should make a profit of about 759.98 ZAR/day.

4.2.5.1 Sensitivity analysis

The relative importance, in terms of impact on the objective function and the three decision variables, of some of the model's input parameters is investigated by means of a sensitivity analysis. The results from the analysis are summarised in Table 4.2.2 from which the following note-worthy observations are drawn:

- The parameters that affect the demand rate, namely ω, ϕ and L, have the greatest impact on the objective function and the three decision variables.
- As ω increases, TPU_{sc} decreases. ω is the price elasticity of the demand rate which represents consumer's sensitivity to the selling price. Higher values of ω imply that consumers are more price-conscious. Therefore, when ω increases, the model responds by lowering the retailer's selling price (in an effort to increase demand) and ordering frequency (in an effort to reduce fixed costs). Lower selling prices lead to reduced revenue and less frequent ordering means that the processed inventory is kept in stock for much longer which reduces its freshness and by extension it's demand. Management can take advantage of this observation by targeting consumers who are less price-conscious in their marketing activities.

Table 4.2.2: Sensitivity analysis of various input parameters

	% change	**Retailer's cycle time** (T^*) days	% change	**Number of shipments** (n^*) shipments	% change	**Retailer's selling price** (p_r^*) ZAR/kg	% change	**Total supply chain profit** (TPU_{sc}^*) ZAR/day	% change
Base		**1.86**		**18**		**49.59**		**759.98**	
h_r	-40	1.87	+0.4	18	0	49.25	-0.7	777.49	+2.3
	-20	1.86	+0.2	18	0	49.42	-0.3	768.71	+1.1
	+20	1.86	-0.2	18	0	49.75	+0.3	751.31	-1.1
	+40	1.82	-2.4	19	+5.6	50.09	+1.0	742.77	-2.3
K_r	-40	1.41	-24.1	23	+27.8	49.06	-1.1	1 005.78	+32.3
	-20	1.65	-11.2	20	+11.1	49.35	-0.5	874.70	+15.1
	+20	2.03	+9.2	17	-5.6	49.91	+0.6	657.01	-13.5
	+40	2.20	+18.3	16	-11.1	50.16	+1.1	562.77	-25.9
h_p	-40	1.79	-3.7	24	+33.3	48.23	-2.7	921.67	+21.3
	-20	1.81	-2.8	21	+16.7	49.03	-1.1	835.04	+9.9
	+20	1.91	+2.7	17	-5.6	50.60	+2.0	692.50	-8.9
	+40	1.91	+2.7	17	-5.6	51.79	+4.4	628.18	-17.3
K_p	-40	1.80	-3.0	18	0	49.36	-0.5	820.85	+8.0
	-20	1.82	-2.0	18	0	49.44	-0.3	790.14	+4.0
	+20	1.86	-0.2	19	+5.6	49.91	+0.7	731.17	-3.8
	+40	1.85	-0.4	20	+11.1	50.24	+1.3	703.39	-7.4
c_f	-40	1.71	-8.1	19	+5.6	45.12	-9.0	976.81	+28.5
	-20	1.80	-3.0	18	0	47.29	-4.6	864.07	+13.7
	+20	1.93	+3.6	18	0	51.93	+4.7	663.94	-12.6
	+40	2.00	+7.3	18	0	54.28	+9.5	575.41	-24.3
K_f	-40	1.80	-3.0	18	0	49.36	-0.5	851.64	+12.1
	-20	1.80	-3.0	18	0	49.36	-0.5	805.45	+6.0
	+20	1.87	+0.7	19	+5.6	49.99	+0.8	717.06	-5.6
	+40	1.89	+1.4	20	+11.1	50.38	+1.6	676.64	-11.0
ϕ	-40	2.59	+39.3	18	0	53.80	+8.5	134.78	-82.3
	-20	2.14	+14.9	18	0	51.27	+3.4	428.38	-43.6
	+20	1.62	-12.7	20	+11.1	48.68	-1.8	1 118.33	+47.2
	+40	1.48	-20.6	22	+22.2	47.95	-3.3	1 494.49	+96.6
ω	-40	1.30	-30.2	25	+38.9	70.60	+42.4	2 646.83	+248.3
	-20	1.55	-16.9	21	+16.7	57.23	+15.4	1 435.69	+88.9
	+20	2.16	+16.2	17	-5.6	45.12	-9.0	348.56	-54.1
	+40	2.53	+36.1	16	-11.1	42.39	-14.5	86.36	-88.6
L	-40	1.46	-21.4	24	+33.3	49.95	+0.7	495.05	-39.6
	-20	1.66	-10.9	21	+16.7	49.83	+0.5	638.24	-16.0
	+20	1.99	+7.1	17	-5.6	49.64	+0.1	849.36	+11.8
	+40	2.17	+16.4	15	-16.7	49.42	-0.3	918.29	+20.8

- As ϕ increases, TPU_{sc} increases. In order to understand this response, it is important to recall that ϕ is the asymptotic level of demand attainable when the cost is considered most favourable to customers. In essence, ϕ is the maximum size of the market for the processed inventory. As the size of the market increases, the retailer has to replenish the processed inventory more frequently (i.e. reduce the cycle time) because of the increased potential customer base. By so doing, the processed inventory is kept much fresher than it would have been if it was replenished less frequently which spikes consumer demand further. When consumer demand is increased and the market is large, the retailer can charge higher prices which increases revenue. While increasing the selling price negatively affects consumer demand, the negative effect is cushioned by the positive effects brought by the larger market size and the more frequent replenishment cycles which ensures that the inventory does not get close to its expiration date. To take advantage of this observation, management should increase their marketing (or advertising) spend which will increase their potential customer base. In the short term, this will increase costs but the long term benefits of having a larger potential customer base will outweigh the initial marketing spend.

- As L increases, TPU_{sc} increases. In addition to maximising profit, the model aims to ensure that the processed inventory does not expire and so when the inventory can last for longer periods of time (because of increased L values), the model prompts the retailer to order less frequently (i.e. increase the cycle time) because the risk of expiration is reduced. By so doing, the retailer would receive fewer shipments (of relatively larger sizes). While this reduces the fixed costs, it reduces demand because of reduced freshness since the inventory will be kept in stock for a relatively longer period of time because of less frequent ordering. However, this negative effect is outweighed by the positive effect of the reduced fixed costs. In order to take advantage of this observation, management should invest in preservation technologies such as (more advanced) refrigeration which has the potential to prolong the shelf life of the processed inventory. Once again, the initial investment will be large in the short term, but the long term benefits will outweigh this initial investment.

- When any of the fixed costs (i.e. K_r, K_p and K_f) increase, TPU_{sc} decreases. In an effort to reduce the fixed costs, the model's response is to reduce the replenishment frequency (i.e. increase the cycle time) by placing larger orders. This leads to increased holding costs because the processed inventory will spend more time in stock. This inadvertently reduces consumer demand because if the inventory is kept in stock for longer periods of time, its freshness levels decrease which has a negative effect on demand.

4.2.5.2 Investigating the benefits of supply chain integration

The proposed inventory control model advocates for the integration of ordering and shipment decisions among all supply chain members. This is because organisations have realised that significant cost savings can be achieved through collaboration and integration of certain decisions, such as inventory replenishment policies, with all the supply chain members (Ben-Daya and Al-Nassar, 2008). In order to investigate the benefits of (or lack thereof) integrating inventory decisions with all parties in the supply chain, the proposed supply chain system (which calls for the integration of inventory replenishment policies) is compared with two alternative scenarios which do not encourage supply chain integration, in varying degrees. The results from this analysis are presented in Table 4.2.3.

The base scenario corresponds to the proposed inventory system. The two alternative scenarios include one with partial integration of replenishment decisions where by some of the supply chain members collaborate and coordinate replenishment decisions (i.e. Scenario 1) and one with no integration at all where each of the members act independently (i.e. Scenario 2). For illustrative purposes, Scenario 1 considers a case where the processor and the retailer coordinate their replenishment policy with the aim of optimising (i.e. finding T, n and p_r values that maximise) the joint profit generated among them. On the other hand, Scenario 2 considers a case where the retailer acts independently with the aim of optimising (i.e. finding T, n and p_r values that maximise) their individual profit.

Table 4.2.3: Quantifying the importance of collaboration

Variables	Base Scenario (Full collaboration)	Scenario 1 (S1) (Partial collaboration)		Scenario 2 (S2) (No collaboration)	
	Quantity	Quantity	% difference	Quantity	% difference
Collaborating parties	Retailer; Processor; Farmer	Processor; Retailer		Retailer	
T^* (days)	1.86	2.03	+9.1	2.28	+22.6
n^* (shipments)	18	16	-11.1	18	0
p_r^* (ZAR/kg)	49.59	54.93	+10.8	64.32	+29.7
TPU_{cp}^* (ZAR/day)	720.32 (S1) or 437.80 (S2)	748.31	+3.9	599.57	+37.0
TPU_{sc}^* (ZAR/day)	759.98	735.77	-3.2	645.79	-15.0

From the the results, it is clear that supply chain integration will all members is beneficial to the entire supply chain. The two alternative scenarios succeeded in optimising the profit of the collaborating members but at the expense of the non-collaborating members and most importantly, the supply chain. The partial collaboration scenario resulted in a 3.2% reduction in the supply chain profit (but a 3.9% increase in the profit of the collaborating partners, symbolised by TC_{cp}). The scenario with no collaborating members performed even worse, in terms of supply chain profit maximisation, with a 15.0% decrease in the supply chain profit (along with a 37.0% increase in the retailer's individual profit).

The results show that the proposed inventory coordination mechanism, which encourages collaboration among all supply chain members, is better at maximising supply chain profit than alternative replenishment mechanisms which call for either partial collaboration or no collaboration at all.

4.2.6 Concluding remarks

Perishable food products constitute a significant portion of grocery retail sales. Considering the commoditised nature of grocery items and the fact that retailers often carry various brands of the same type of perishable food product, pricing and freshness become important catalysts for consumer demand. A number of studies in the literature have proposed inventory control models for perishable perishable products whose demand rate depends on the selling price and freshness or expiration date of the products. The common denominator among all these previous studies which considered the demand rate's price and freshness dependency has been a focus on the retail end of the supply chain. In reality, retailers do not exist in isolation, they have suppliers and their suppliers might also have suppliers (i.e. retailers are part of a supply chain network).

This section presents a model for managing inventory in a three-echelon supply chain for growing items. The echelons include a farming operation where items are reared, a processing plant where the items are processed so as to get them into a form that is suitable for human consumption and a retail outlet where consumer demand is met. The demand rate is affected by the selling price and the expiration date of the processed inventory. The significance of the proposed model lies in the fact that it in more representative, when compared to previous studies in the literature, of an actual perishable food supply chain. This is because it not only accounts for pricing policies and expiration dates at the retail stage, but also the preceding farming and processing stages (i.e. it considers an end-to-end supply chain for perishable products). The most important characteristics of the proposed model are the integration of replenishment and shipment policies among all supply chain members and the demand rate's dependency on the selling price and the expiration date. The importance of these characteristics are quantified through numerical experimentation.

Despite being more representative of an actual end-to-end supply chain for growing items, the model presented in this section still makes use of a few assumptions that have the potential to restrict its applicability to actual food supply chains. For instance, incentive strategies like quantity discounts, revenue sharing contracts, pre-payment agreements and trade credit financing, among others, are not taken into account. These incentive policies are often used by supply chain members in food production systems as a way of boosting profits because of the relatively low margins generated in food retail outlets. Therefore, the current literature can be enriched by incorporating some of these incentive policies to the model presented in this section.

4.3 A four-echelon supply chain inventory model for growing items with imperfect quality[‡]

4.3.1 Introduction

4.3.1.1 Context

Building upon Rezaei (2014)'s EOQ model for growing items, Sebatjane and Adetunji (2019a) developed an extension which incorporated imperfect quality. This extension captures some aspects of realistic food production systems, particularly item growth and the presence of items that are of inferior quality. Nonetheless, the model falls short of being an accurate representation of a real life inventory system because of some of the assumptions made when it was being developed, three of which are addressed in this section. The first assumption is that immediately after the items have grow to the target weight, they are slaughtered and sold to market instantly and the second one is that all the newborn items survive throughout the growing cycle. The third assumption is that one entity is responsible for all the activities, this is not always the case, especially in today's business environment where the benefits of supply chain management are documented. Consequently, multiple entities are involved in getting the slaughtered product to market, albeit at different stages. While these assumptions simplify the modelling process, they are unrealistic because there is usually some form of processing prior to selling the items to end consumers and growing items, which are living organisms, are not immune to mortality.

To overcome some of the shortcomings in Sebatjane and Adetunji (2019a)'s model, an integrated multi-echelon supply chain is suggested. The proposed supply chain consists of three members (i.e. a farmer, a processor and a retailer) and four echelons (i.e. farming, processing, inspection and selling). The farmer, who receives the items as newborns at the beginning of a replenishment cycle, is responsible for growing the items provided that some of the items do not survive to the end of the growing period due to mortality. The processor is involved in the transformation of the live fully grown items into saleable items through slaughtering, preparation and packaging (collectively termed processing), while the retailer sells the processed items to end-users. Prior to sending the processed items to the retailer, the processor inspects them for quality to ensure that consumers get them in the correct form (from a consumer health and safety perspective).

4.3.1.2 Objectives

The main objective of this section is to formulate a coordinated model for managing inventory in a supply chain with growing items while taking into account item mortality and quality control initiatives. In addition to the main objective, the section has a few sub-objectives that are aimed at investigating, through numerical experimentation, the effects of item mortality during the farming stage, the presence of imperfect quality inventory after processing and the adopted shipment policy between the processor and the retailer on the supply chain's profit and ordering policy.

[‡]A modified version of this section has been accepted for publication as Sebatjane and Adetunji (2020a) in *Production & Manufacturing Research*.

4.3.1.3 Relevance

The presence of illnesses, pests (in the case of crops) and predators (in the case of livestock) makes item mortality an important factor to consider when studying inventory systems with growing items. In reality, not all the newborn items ordered when a replenishment cycle starts make it to the processing stage due to mortality.

The primary source of most consumable food products is growing items. However, these items are rarely consumed in their original form. The items go through a number of stages before they are in a form suitable for consumption. These stages represent supply chain echelons and in the context of this section, four are identified, namely, farming, processing, inspection and selling. This section uses this fact to propose a model for inventory control in a four-echelon supply chain for growing items when there is a possibility of having inferior quality items. For this reason, government regulations require food items to be checked for quality before they are sold to consumers in order to ensure that the health and safety of consumers is not compromised.

Given that growing items were only incorporated in inventory theory recently (Rezaei, 2014), it appears that no study has considered an integrated inventory control system for growing items in a four-echelon supply chain with item processing considering mortality (in the case live items) and quality (in the case of processed items). This study is aimed at filling this void in the literature. The model and results from the numerical analysis can be used as a guideline for managers in charge of making purchasing decisions in industries involved in the broader food chain.

4.3.1.4 Organisation

Besides the introductory subsection, this section has four additional subsections. In addition to providing a description of the proposed inventory control system, Subsection 4.3.2 also lists the assumptions used when formulating the model. The model is then developed in Subsection 4.3.3. Following this, important managerial insights are drawn from a numerical example presented in Subsection 4.3.4 and then concluding remarks are given in Subsection 4.3.5.

4.3.2 Problem description

The inventory problem at hand considers a four-echelon supply chain for growing items with the farming, processing, inspection and retail operations as the echelons. The farmer is responsible for growing the items which are received as newborns. The fully grown items are then instantaneously shipped to the processor for transformation into a saleable form. The items are deemed fully-grown or mature once they grow to a specified weight. The processing stage is not perfect and it thus produces some items of poorer quality, in addition to those of good quality. As a way of ensuring that the health and safety of end users is not compromised, the processor is also responsible for quality control (in addition to processing the items) and for this reason the processor has two facilities, one for processing and the other one for quality inspection. Throughout the inspection process, the processor delivers an integer number of batches of good quality processed inventory to the retailer who meets consumer demand for good quality processed inventory. The processed poorer quality inventory is accumulated throughout the inspection process and are then sold as a single batch to secondary markets when the inspection process ends.

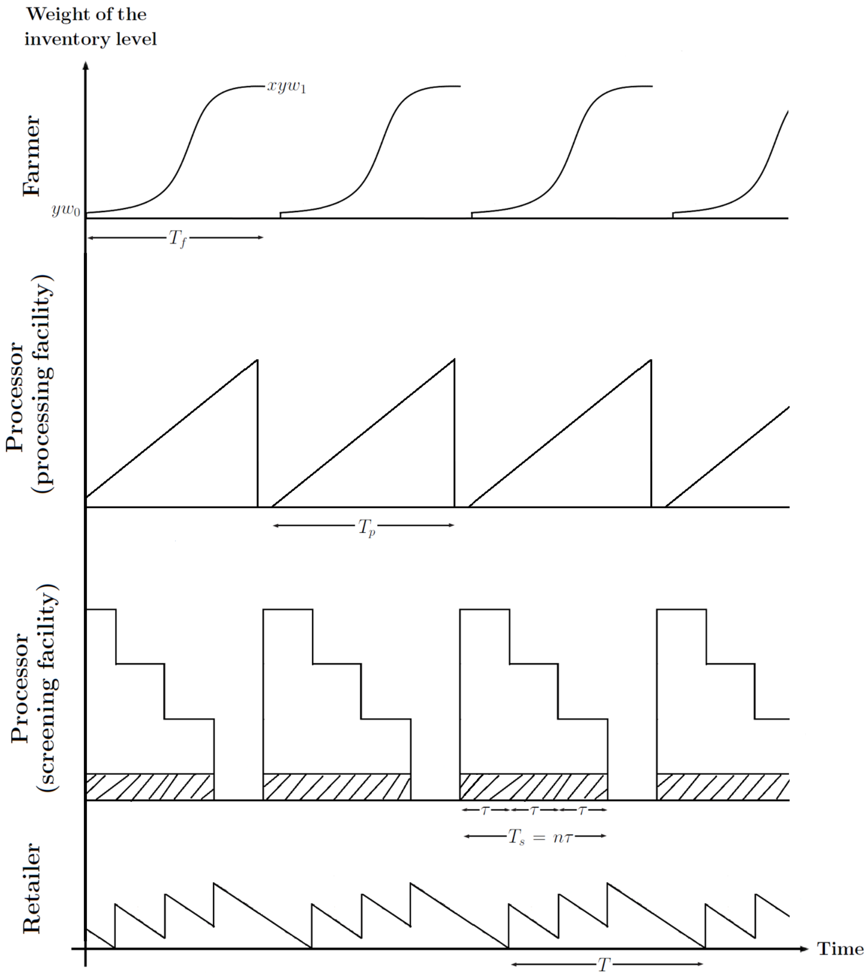


Figure 4.3.1: Inventory system profile for a farmer, a processor and a retailer in a supply chain for growing items with imperfect processing (with $n_s = 3$ for illustrative purposes)

The inventory system profile for the problem at hand is depicted by Figure 4.3.1. For the farmer's inventory profile, the weight depicted in the graph is that of the live items as they grow throughout the cycle. In the case of the processor's and the retailer's inventory profiles, the graph shows the changes that occur to the processed inventory. At the processing facility, live items are transformed into saleable processed items and the weight of the processed inventory increases gradually at a certain rate. At the inspection warehouse, the processed inventory is inspected for quality and batches of good quality are transferred to the retailer while the poorer quality processed inventory (shown as the shaded portion in the figure) accumulates at the warehouse. The processed poorer quality inventory is then sold off as a single batch when inspection ends.

A number of assumption are made in order to model the proposed supply chain inventory system. These include:

- There is only one farmer, one processor and one retailer in the supply chain involved, respectively, in the growing, the processing and the inspection and the selling of a single type of item.
- A fraction of the live inventory items does not survive until the end of the growth period.

- The processor's processing rate is greater than the retailer's demand rate and both quantities are deterministic constants.
- A fraction of the processed inventory does not meet the required quality standard.
- As soon as processing is complete, the entire processed lot is transferred to an inspection warehouse where a 100% inspection process takes place in which the items are classified as either being of good quality or poorer quality.
- The inspection rate is greater than the demand rate.
- During the inspection process, equally-sized batches of good quality processed inventory are sent to the retailer.
- The retailer uses the good quality processed inventory to meet consumer demand as soon as the first batch is delivered.
- The poorer quality processed inventory are allowed to accumulate at the processor's inspection warehouse from where they are sold when a single inspection run ends.
- Poorer quality processed items can not be reworked.
- Holding costs are incurred only for the ready-to-consume (i.e. processed) items.
- The retailer's holding costs are higher than those of the processor due to value adding as the items move downstream in the supply chain.
- Order lead time is zero and shortages are not permitted.

4.3.3 Model formulation

4.3.3.1 Farmer's profit

Figure 4.3.2 depicts the weight of the farmer's live inventory throughout the growth period. As the items are fed, their weight increases gradually from an initial newborn weight of w_0 to a target maturity weight of w_1 at the end of the growing period (of duration T_f).

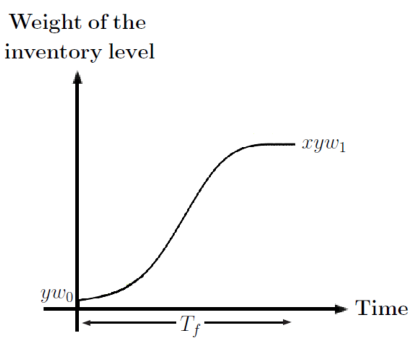


Figure 4.3.2: The farmer's inventory system profile

The farmer's replenishment cycle starts with the placement of an order for y live newborn items. The farmer procures live newborn items from a vendor who charges p_v

per weight unit of the newborn items. Given that at the time of purchase the weight of each item is w_0, the farmer's purchase cost per cycle, PC_f, is therefore

$$PC_f = p_v y w_0. \tag{4.3.1}$$

There is a fixed cost of K_f associated with setting up for a new growing cycle. The farmer's setup cost per cycle, KC_f, is therefore

$$KC_f = K_f. \tag{4.3.2}$$

The items are allowed to grow until their weight reaches a certain target (i.e. the maturity weight w_1). At that point, they are delivered to the processor for processing and inspection. Due to its "S"-shaped curve, the logistic function is used to model the items' growth function, given by

$$w(t) = \frac{\alpha}{1+\beta e^{-\lambda t}}. \tag{4.3.3}$$

When the farmer's growth period ends, the weight of the surviving items would have reached the target weight of w_1. From Equation (4.3.3), the length of the growth period, T_f, is computed using the target maturity weight as

$$T_f = -\frac{\ln\left[\frac{1}{\beta}\left(\frac{\alpha}{w_1}-1\right)\right]}{\lambda}. \tag{4.3.4}$$

The farmer's cyclic feeding cost, FC_f, computed by multiplying the feeding cost per weight unit (c_f), the fraction of items which survive (x), and the area under the graph showing the items' growth trajectory (as given in Figure 4.3.2), is given by

$$FC_f = c_f x \int_0^{T_f} y w(t)\, dt = c_f x y \left\{ \alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right\}. \tag{4.3.5}$$

The farmer incurs a cost associated with disposing the fraction of newborn items which do not survive until the end of the growing cycle. The farmer's mortality cost cycle, MC_f, is determined as the product of the farmer's average inventory level (i.e the area under the graph of the farmer's inventory system profile), the fraction of items which do not not survive $(1-x)$ and the mortality cost per weight unit per unit time (m_f). Hence,

$$MC_f = m_f(1-x)\int_0^{T_f} y w(t)\, dt = m_f(1-x)y\left\{ \alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right\}. \tag{4.3.6}$$

Since the items have a survival rate of x, when the farmer's replenishment cycle ends, the weight of all the surviving items would be given by xyw_1. This lot is then transferred to the processor for further processing and inspection. The weight xyw_1 represents the processor's lot size. The farmer charges the processor p_f for each weight unit of the surviving live inventory. The farmer's revenue per cycle, TR_f, is therefore

$$TR_f = p_f x y w_1. \tag{4.3.7}$$

The farmer's total profit per cycle, TP_f, is the farmer's cyclic revenue [i.e. Equation (4.3.7)] less the farmer's cyclic total costs [i.e the sum of Equations (4.3.1), (4.3.2), (4.3.5) and (4.3.6)] and it is given by

$$TP_f = p_f x y w_1 - p_v y w_0 - K_f - y\left\{ \alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right\}\left[c_f x + m_f(1-x)\right]. \tag{4.3.8}$$

4.3.3.2 Processor's profit

4.3.3.2.1 Processor's processing facility

The inventory system profile for processed inventory in the processing facility, depicted in Figure 4.3.3, is used to determine various components of the costs incurred in the processing facility. The weight of the processed inventory increases at a processing rate of R as the processor transforms the live items (received from the farmer) into saleable processed items.

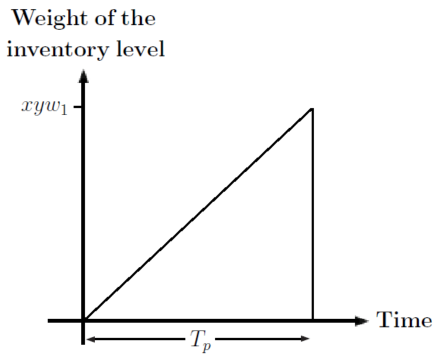


Figure 4.3.3: Inventory system profile for the processor's processing facility

The entire lot size received from the farmer (i.e. xyw_1) is processed at a rate R. This means that the duration of the processing period is

$$T_p = \frac{xyw_1}{R}. \tag{4.3.9}$$

The processor incurs a fixed cost of K_p at the start of each processing run and therefore the setup cost per cycle, KC_p, is

$$KC_p = K_p. \tag{4.3.10}$$

The processor procures live fully grown items from the farmer for processing. Given that the farmer charges p_f per weight unit of inventory, the processor's procurement cost per cycle, PC_p, is

$$PC_p = p_f xyw_1. \tag{4.3.11}$$

The holding cost per cycle at the processing facility, HC_p, is computed by multiplying the holding cost per weight unit in the facility, h_p, by the area under the inventory system graph, as given in Figure 4.3.3, and it is given by

$$HC_p = h_p\left(\frac{x^2y^2w_1^2}{2R}\right). \tag{4.3.12}$$

The total cost (per cycle) at the processing facility, TC_p, is determined by adding Equations (4.3.10), (4.3.11) and (4.3.12), resulting in

$$TC_p = K_p + p_f xyw_1 + h_p\left(\frac{x^2y^2w_1^2}{2R}\right). \tag{4.3.13}$$

4.3.3.2.2 Processor's inspection facility

Once the entire lot is processed, it is transferred to an inspection warehouse for quality control. In the inspection warehouse, the processor incurs holding, inspection and transfer costs. Figure 4.3.4 depicts the changes to the weight of the processed inventory in the inspection warehouse. A fraction , a, of the processed inventory does not meet the required quality standard (i.e these inventory items are of poorer quality). This means that the weight of good quality processed inventory is $xyw_1(1-a)$, and from this the processor ships n_s batches (each weighing $s^{'}$) to the retailer at equally-spaced time intervals.

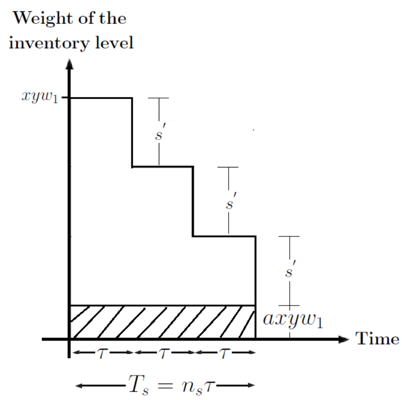


Figure 4.3.4: Inventory system profile for the processor's inspection facility (with $n_s = 3$ for illustrative purposes)

Since the processor inspects the entire lot they received from the farmer (i.e. xyw_1) for quality at a rate of z, the duration of the inspection period, T_s, is thus

$$T_s = \frac{xyw_1}{z}. \tag{4.3.14}$$

At equally-spaced time intervals of τ, the processor delivers an integer number of shipments (n_s in this case) of good quality processed inventory to the retailer. Therefore the duration of the time interval between successive deliveries of good quality items is

$$\tau = \frac{xyw_1}{n_s z}. \tag{4.3.15}$$

In each inspection run, the weight of items of good quality that the processor has is $xyw_1(1-a)$. From this, the processor delivers n_s batches of good quality processed inventory, each batch being of size $s^{'}$, to the retailer. The weight of each batch of good quality processed inventory is thus

$$s^{'} = \frac{xyw_1(1-a)}{n_s}. \tag{4.3.16}$$

The processor also incurs a fixed cost for sending a batch of good quality processed inventory to the retailer. Since the processor sends n_s batches to the retailer during a single inspection run, the transfer cost per cycle, KC_s, is thus

$$KC_s = n_s K_s. \tag{4.3.17}$$

The entire lot is subjected to quality inspection process in order to separate the items. The processor incurs a cost of v per weight unit of the processed inventory inspected. Therefore, the processor's inspection cost per cycle, VC_s, is

$$VC_s = vxyw_1. \tag{4.3.18}$$

The holding cost per cycle at the inspection warehouse, HC_s, is computed by multiplying the holding cost per weight unit by the area under the graph depicting the processed inventory system profile as given in Figure 4.3.4. It follows that

$$HC_s = h_s\left[\frac{x^2y^2w_1^2}{z} - \frac{(n_s-1)x^2y^2w_1^2(1-a)}{2n_sz}\right]. \tag{4.3.19}$$

The cyclic total cost incurred by the processor at the inspection warehouse (TC_s), computed by summing Equations (4.3.17), (4.3.18) and (4.3.19), is therefore

$$TC_s = n_sK_s + vxyw_1 + h_s\left[\frac{x^2y^2w_1^2}{z} - \frac{(n_s-1)x^2y^2w_1^2(1-a)}{2n_sz}\right]. \tag{4.3.20}$$

The processor delivers n_s batches of good quality items to the retailer with each batch weighing $s^{'}$. Noting that all the batches delivered during a single processing cycle have a combined weight of $xyw_1(1-a)$ and that the processor charges the retailer p_p per weight unit of good quality processed inventory, the processor's cyclic revenue from good quality processed inventory, TR_p, is thus

$$TR_p = p_pxyw_1(1-a). \tag{4.3.21}$$

After the inspection process, the processor sells the poorer quality processed inventory at a cost of p_q per weight unit to secondary markets in a single batch. This means that the processor's cyclic revenue from sales of poorer quality processed inventory, TR_q, is

$$TR_q = p_qxyw_1a. \tag{4.3.22}$$

The processor's cyclic profit, TP_p, is computed as the revenue from both good and poorer quality inventory [i.e the sum of Equations (4.3.21) and (4.3.22)] less the total cost [i.e the sum of Equations (4.3.13) and (4.3.20)] and is thus

$$\begin{aligned} TP_p = p_pxyw_1(1-a) + p_qxyw_1a - K_p - p_fxyw_1 - h_p\left(\frac{x^2y^2w_1^2}{2R}\right) - n_sK_s - vxyw_1 \\ - h_s\left[\frac{x^2y^2w_1^2}{z} - \frac{(n_s-1)x^2y^2w_1^2(1-a)}{2n_sz}\right]. \end{aligned} \tag{4.3.23}$$

4.3.3.3 Retailer's profit

Figure 4.3.5 shows the behaviour of the retailer's processed inventory level over time. The retailer faces a demand rate (for good quality processed inventory) of D. To meet this demand, the retailer receives n_s batches of size $s^{'}$ from the processor after the items have been inspected for quality. Altogether, these batches add up to $xyw_1(1-a)$ between successive order cycles (i.e. when the retailer's inventory level reaches zero). The duration of time between successive order cycles is therefore

$$T = \frac{xyw_1(1-a)}{D}. \tag{4.3.24}$$

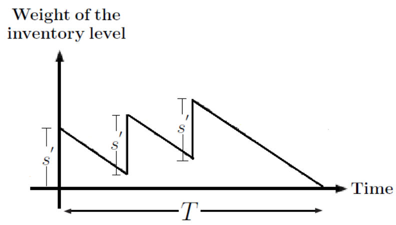


Figure 4.3.5: The retailer's inventory system profile (with $n_s = 3$ for illustrative purposes)

A fixed ordering cost cost of K_r is incurred whenever the retailer places an order of $xyw_1(1-a)$ good quality items (received in n_s batches of $s^{'}$). The retailer's cyclic ordering cost, KC_r, is thus

$$KC_r = K_r \tag{4.3.25}$$

The retailer procures processed good quality inventory from the processor at a cost of p_p per weight unit. Given that the retailer procures good quality inventory weighing $xyw_1(1-a)$ per cycle, their (the retailer's) procurement cost per cycle, PC_r, is

$$PC_r = p_p xyw_1(1-a). \tag{4.3.26}$$

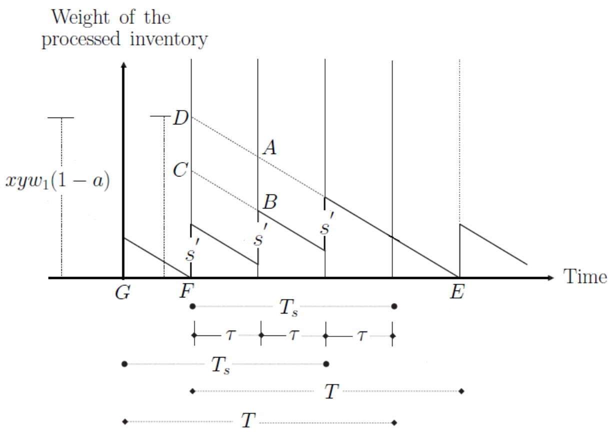


Figure 4.3.6: Redrawn version of the retailer's inventory system profile (with $n_s = 3$ for illustrative purposes)[modified from Konstantaras et al. (2007)]

Figure 4.3.6, a redrawn version of Figure 4.3.5, is utilised to determine the area under the graph of the retailer's processed inventory level (in weight units). The redrawn version makes the computation of the area easier and the method is adopted from Konstantaras et al. (2007). This area is computed by subtracting the area of $n_s(n_s-1)/2$ parallelograms of type ABCD from the area of triangle DEF. It follows that

$$\begin{aligned}
\text{Area}_r &= \left[\text{Area of triangle DEF}\right] - \left[\frac{n_s(n_s-1)}{2}(\text{Area of parallelogram ABCD})\right] \\
&= \left[\frac{xyw_1(1-a)T}{2}\right] - \left[\frac{n_s(n_s-1)s^{'}\tau}{2}\right] \\
&= \left[\frac{xyw_1(1-a)T}{2} - \frac{(n_s-1)x^2y^2w_1^2(1-a)}{2n_s z}\right].
\end{aligned} \tag{4.3.27}$$

This area is multiplied with the holding cost per weight unit to determine the holding cost incurred by the retailer in each cycle, HC_r, as

$$HC_r = h_r\left[\frac{xyw_1(1-a)T}{2} - \frac{(n_s-1)x^2y^2w_1^2(1-a)}{2n_sz}\right]. \tag{4.3.28}$$

The retailer sells the good quality processed inventory to consumers at a selling price of p_r per weight unit. This implies that the retailer's cyclic revenue, TR_r, is

$$TR_r = p_rxyw_1(1-a). \tag{4.3.29}$$

The retailer's total profit per cycle, TP_r, is the revenue generated from the sales of good quality processed inventory minus the sum of the ordering and holding costs. It follows that

$$TP_r = p_rxyw_1(1-a) - K_r - h_r\left[\frac{xyw_1(1-a)T}{2} - \frac{(n_s-1)x^2y^2w_1^2(1-a)}{2n_sz}\right] \tag{4.3.30}$$

4.3.3.4 Supply chain profit

The supply chain's profit profit is computed as the sum of the profits generated at each supply chain echelon, as given in Equations (4.3.8), (4.3.23) and (4.3.30). Therefore the cyclic supply chain profit, TP_{sc}, is

$$\begin{aligned}TP_{sc} = {} & p_rxyw_1(1-a) + p_qxyw_1a - K_r - h_r\left[\frac{xyw_1(1-a)T}{2} - \frac{(n_s-1)x^2y^2w_1^2(1-a)}{2n_sz}\right] \\ & - K_p - h_p\left(\frac{x^2y^2w_1^2}{2R}\right) - n_sK_s - vxyw_1 - h_s\left[\frac{x^2y^2w_1^2}{z} - \frac{(n_s-1)x^2y^2w_1^2(1-a)}{2n_sz}\right] - p_vyw_0 \\ & - K_f - y\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right\}\left[c_fx + m_f(1-x)\right]. \end{aligned} \tag{4.3.31}$$

To ensure that the retailer never runs out of processed inventory, the farmer and the processor start new replenishment cycles every T time units which is the amount of time it takes for the processed inventory at the retailer's facility to reach zero. Therefore, the total supply chain profit per unit time, TPU_{sc}, is determined by dividing Equation (4.3.31) by the replenishment cycle time, T, as given in Equation (4.3.24) and the result is

$$\begin{aligned}TPU_{sc} = {} & p_rD + \frac{p_qaD}{(1-a)} - \frac{K_rD}{xyw_1(1-a)} - h_r\left[\frac{xyw_1(1-a)}{2} - \frac{(n_s-1)xyw_1D}{2n_sz}\right] \\ & - \frac{K_pD}{xyw_1(1-a)} - h_p\left[\frac{xyw_1D}{2R(1-a)}\right] - \frac{n_sK_sD}{xyw_1(1-a)} - \frac{vD}{(1-a)} \\ & - h_s\left[\frac{xyw_1D}{s(1-a)} - \frac{(n_s-1)xyw_1D}{2n_sz}\right] - \frac{p_vw_0D}{xw_1(1-a)} - \frac{K_fD}{xyw_1(1-a)} \\ & - \frac{D}{xw_1(1-a)}\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right\}\left[c_fx + m_f(1-x)\right]. \end{aligned} \tag{4.3.32}$$

Both the live items' survival rate, x, and the fraction of processed inventory that is of poorer quality, a, are considered as random variables with known probability density functions given by $f(x)$ and $f(a)$ respectively. Therefore, the expected value of Equation (4.3.32) is

$$\begin{aligned} E[TPU_{sc}] = p_r D + \frac{p_q E[a] D}{(1-E[a])} - \frac{K_r D}{E[x] y w_1 (1-E[a])} \\ - h_r \left[\frac{E[x] y w_1 (1-E[a])}{2} - \frac{(n_s - 1) E[x] y w_1 D}{2 n_s z} \right] - \frac{K_p D}{E[x] y w_1 (1-E[a])} \\ - h_p \left[\frac{E[x] y w_1 D}{2R(1-E[a])} \right] - \frac{n_s K_s D}{E[x] y w_1 (1-E[a])} - \frac{v D}{(1-E[a])} \\ - h_s \left[\frac{E[x] y w_1 D}{s(1-E[a])} - \frac{(n_s - 1) E[x] y w_1 D}{2 n_s z} \right] - \frac{p_v w_0 D}{E[x] w_1 (1-E[a])} - \frac{K_f D}{E[x] y w_1 (1-E[a])} \\ - \frac{D}{E[x] w_1 (1-E[a])} \left\{ \alpha T_f + \frac{\alpha}{\lambda} \left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln(1+\beta) \right] \right\} \left\{ c_f E[x] + m_f (1-E[x]) \right\}. \end{aligned} \tag{4.3.33}$$

The order quantity which maximises the total profit generated by the supply chain is determined by setting the first derivative of Equation (4.3.33) with respect to y to zero. The result is

$$y = \sqrt{\frac{2D\left(K_r + K_p + n_s K_s + K_f\right)}{\left\{ h_r \left[E\left[(1-a)^2\right] - \frac{D(n_s-1)E[1-a]}{n_s z} \right] + h_p \left(\frac{D}{R}\right) + h_s \left[\frac{2D}{z} - \frac{D(n_s-1)E[1-a]}{n_s z} \right] \right\} E[x^2] w_1^2}}. \tag{4.3.34}$$

4.3.3.4.1 Constraints governing the proposed inventory system

The feasibility and tractability of the proposed inventory control model is dependent on the imposition of three constraints.

The first constraint ensures that shortages do not occur in the processor's inspection facility during the inspection process. By defining $E[W]$ as the expected weight of good quality processed inventory less the weight of poor quality processed inventory in each cycle, the equation

$$E[W] = E[xyw_1] - E[axyw_1] = E[(1-a)xyw_1], \tag{4.3.35}$$

is formulated. One of the assumptions made when developing the model is that shortages are not permitted. As a way of ensuring that shortages are avoided, the expected weight of good quality processed inventory should be greater than or equal to the demand during the inspection period T_s. Thus,

$$E[W] \geq DT_s. \tag{4.3.36}$$

By substituting Equation (4.3.35) and T_s [as given in Equation (4.3.14)] into Equation (4.3.36), the first constraint is formulated as

$$E[a] \leq 1 - \frac{D}{z}. \tag{4.3.37}$$

The second constraint is that the number of batches of good quality processed inventory delivered to the retailer during a single inspection run (n_s) should be an integer.

This constraint not only ensures that the proposed solution procedure is tractable, it also makes the problem practical. The second constraint is thus

$$n_s \in \mathbb{Z}. \tag{4.3.38}$$

The third constraint relates to the common replenishment cycle time (T) for all echelons. The constraint guarantees that the solution is feasible. Since a new processing cycle is set up every T time units, the farmer's growth period (T_f) must at most be equal to the common replenishment cycle time (T) so that the weight of the live items has reached the target maturity weight at the start of a new processing run. Therefore, the third constraint is

$$T_f \leq T. \tag{4.3.39}$$

In addition, the cycle time T also places a restriction on the processor's processing duration T_p in a similar manner (i.e. $T_p \leq T$). But since the processing rate (R) is assumed to be greater that the demand rate (D), this constraints will not be violated and therefore it's not necessary to explicitly state it.

4.3.3.4.2 Solution procedure

An iterative solution algorithm is used to determine the model's optimal solution. The procedure is as follows:

Step 1: Start with $n_s = 1$.

Step 2:

- Step 2a: Compute y using Equation (4.3.34).
- Step 2b: Check the model's feasibility with regards to the third constraint as given in Equation (4.3.39). To accomplish this, the values of T_f and T are first calculated using Equations (4.3.4) and (4.3.24). If the model is feasible, keep the calculated y value and continue to Step 2d. If not, continue to Step 2c.
- Step 2c: If $T_f \geq T$, equate T to T_f and use the new T value to determine a new y value from Equation (4.3.24). Continue to Step 2d.
- Step 2d: Check the model's feasibility with regards to the first constraint as given in Equation (4.3.37). If it is feasible, continue to Step 2e. If not, the problem is infeasible and in this case continue to Step 4.
- Step 2e: Compute $E[TPU]$ using Equation (4.3.33).

Step 3: Increase n_s by 1 and then repeat Step 2. If the value of $E[TPU_{sc}]$ increases, then go to Step 3. If not, the previously calculated value of $E[TPU_{sc}]$ along with corresponding y and n_s values represent the best solution.

Step 4: End.

4.3.3.5 Proof of the supply chain profit function's concavity

To show that a unique solution to the presented model exists, it must be proven that the expected total supply chain profit function ($E[TPU_{sc}]$), as specified in Equation (4.3.33), is concave in both the farmer's order order quantity (y) and the number of shipments made by the processor from the inspection warehouse to the retailer (n_s).

The partial derivatives of ($E[TPU_{sc}]$) with respect to y and n_s are

$$\frac{\partial E[TPU_{sc}]}{\partial y} = \frac{K_r D}{E[x]y^2 w_1(1-E[a])} - h_r\left[\frac{E[x]w_1(1-E[a])}{2} - \frac{(n_s-1)E[x]w_1 D}{2n_s z}\right] + \frac{K_p D}{E[x]y^2 w_1(1-E[a])} - h_s\left[\frac{E[x]w_1 D}{s(1-E[a])} - \frac{(n_s-1)E[x]w_1 D}{2n_s z}\right] - h_p\left[\frac{E[x]w_1 D}{2R(1-E[a])}\right] + \frac{n_s K_s D}{E[x]y^2 w_1(1-E[a])} + \frac{K_f D}{E[x]y^2 w_1(1-E[a])}, \quad (4.3.40)$$

$$\frac{\partial^2 E[TPU_{sc}]}{\partial y^2} = -\frac{D(K_r+K_p+n_s K_s+K_f)}{E[x]y^3 w_1(1-E[a])} \quad (4.3.41)$$

$$\frac{\partial E[TPU_{sc}]}{\partial n} = \frac{E[x]yw_1 D(h_r+h_s)}{2n^2 s} - \frac{K_s D}{E[x]yw_1(1-E[a])} \quad (4.3.42)$$

$$\frac{\partial^2 E[TPU_{sc}]}{\partial n^2} = -\frac{E[x]yw_1 D(h_r+h_s)}{2n^3 s} \quad (4.3.43)$$

$$\frac{\partial^2 E[TPU_{sc}]}{\partial y \partial n} = -\frac{E[x]w_1 D(h_r+h_s)}{2n^2 s} + \frac{K_s D}{E[x]y^2 w_1(1-E[a])}. \quad (4.3.44)$$

Based on those partial derivatives, the quadratic form of the Hessian matrix is therefore

$$\begin{bmatrix} y & n_s \end{bmatrix} \begin{bmatrix} -\frac{D(K_r+K_p+n_sK_s+K_f)}{E[x]y^3w_1(1-E[a])} & -\frac{E[x]w_1D(h_r+h_s)}{2n^2s} + \frac{K_sD}{E[x]y^2w_1(1-E[a])} \\ -\frac{E[x]w_1D(h_r+h_s)}{2n^2s} + \frac{K_sD}{E[x]y^2w_1(1-E[a])} & -\frac{E[x]yw_1D(h_r+h_s)}{2n^3s} \end{bmatrix} \begin{bmatrix} y \\ n_s \end{bmatrix} = -\frac{D(K_r+K_p+K_f)}{E[x]yw_1(1-E[a])} - \frac{E[x]yw_1D(h_r+h_s)}{2n_s z} < 0. \quad (4.3.45)$$

The quadratic form of the Hessian matrix, as determined in Equation (4.3.45), proves that the expected total profit function is concave because the quadratic form of the Hessian is shown to be negative.

4.3.4 Numerical results

A numerical example which considers a mutton production system with a farmer, a processor (who also inspects the items for quality after processing) and a retailer is used to solve and analyse the proposed inventory control model. The example makes use of the following parameters: D=250 kg/week; R=300 kg/week; w_0= 8.5 kg; w_1=30 kg; K_r=2 500 ZAR; h_r=1 ZAR/kg/week; p_r=50 ZAR/kg; K_p=25 000 ZAR; h_p=0.5 ZAR/kg/week; p_p=30 ZAR/kg; K_s=200 ZAR; h_s=0.5 ZAR/kg/week; p_q=20 ZAR/kg; v=0.5 ZAR/kg; z=1 000 kg/week; K_f=30 000 ZAR; p_f=15 ZAR/kg; c_f=1 ZAR/kg/week; m_f=2 ZAR/kg/week; p_v=10 ZAR/kg; α=51 kg; β=5; λ=0.12 /week. x and a are assumed to be random variables uniformly distributed over [0.8, 1] and [0, 0.05], respectively. Their probability density functions are given by

$$f(x) = \begin{cases} 5, & 0.8 \le x \le 1 \\ 1, & \text{otherwise.} \end{cases}$$

$$f(a) = \begin{cases} 32, & 0 \leq a \leq 0.05 \\ 0, & \text{otherwise.} \end{cases}$$

This implies that

$$E[x] = \int_{0.8}^{1} 5x \quad dx = 5\left[\frac{(1^2 - 0.8^2)}{2}\right] = 0.9$$

$$E[a] = \int_{0}^{0.05} 32a \quad da = 32\left[\frac{(0.05^2 - 0^2)}{2}\right] = 0.04$$

Table 4.3.1 presents the results from the numerical example which was solved using the proposed iterative solution procedure. The table also shows the profit function's concavity. The optimal values of the decision variables were found to be n_s^*=9 shipments (of processed inventory delivered by the processor to the retailer during a single processing cycle) and y^*=179 newborn lambs (ordered by the farmer at the beginning of a growing cycle). The model's objective function, $E[TPU^*]$, amounted to 2 191.76 ZAR/week.

Table 4.3.1: Results from the numerical example showing the objective function's concavity

Number of shipments (n_s)	**Farmer's order quantity** (y)	**Retailer's cycle time** (T)	**Total supply chain profit** ($E[TPU_{sc}]$)
1	158	16.38	1 528.28
2	168	17.41	1 928.35
3	172	17.82	2 060.35
4	174	18.06	2 122.20
5	176	18.21	2 155.45
6	177	18.33	2 174.24
7	178	18.42	2 184.72
8	178	18.50	2 189.98
9	**179**	**18.57**	**2 191.76**
10	180	18.63	2 191.09
11	180	18.68	2 188.65
12	181	18.73	2 184.88
13	181	18.78	2 180.09
14	182	18.83	2 174.50

The optimal inventory shipment and replenishment policies for all chain members are determined using those two decision variables. The farmer should order (y=) 179 live newborn items when a growing cycle commences. When the farmer receives the order, the weight of all the ordered newborn items (yw_0) should be approximately 1 522 kg. Since (x=) 90% of the initially ordered newborn items survive and reach the target weight at the end of the growth period, the weight of the surviving items delivered to the processor's processing facility (xyw_1) should be about 4 833 kg. After processing the entire lot, the processor transfers it to an inspection warehouse where the processed inventory is inspected for quality. Throughout the inspection process, the processor should deliver (n_s=) 9 batches of good quality inventory, each weighing (s'=) 516 kg, which has been separated from the poorer quality inventory. This implies that the weight of good quality processed inventory $[xyw_1(1-a)]$ amounts to 4 640 kg per processing cycle. The poorer quality inventory (xyw_1a) in each processing cycle weighs 193 kg and it is sold as a single batch to secondary markets when inspection is complete. The processor should deliver good quality processed inventory to the retailer every τ= 0.54 weeks once enough inventory to make a batch (s') has been inspected. The retailer's inventory level will reach zero every (T=) 18.57 weeks, which also represents the time between successive farming and processing cycles at the farmer's growing facility and the processor's processing facility, respectively.

4.3.4.1 Sensitivity analysis

In order to investigate the relative importance (in terms of effects on the model's objective function and decision variables) of the input parameters, a sensitivity analysis was conducted and its results are presented in Table 4.3.2. The following observations from the sensitivity analysis are notable:

- The expected supply chain profit is most sensitive to changes in the selling prices of the items at different stages in the supply chain (i.e. p_v, p_f, p_p and p_r). The higher these input parameters are, the higher the profit. This is not a surprising result given that as the items move further downstream along the supply chain, the selling prices increase because of value-adding. Furthermore,none of these parameters have an effect on the model's two decision variables. However, this does not understate the importance of these parameter because the impact they have on the profit is disproportionately large.

- The survival rate of the items has the second highest impact on the expected profit. Unlike the selling prices, this parameter has an effect (a significant one at that) on the farmer's EOQ. As more items survive during the farmer's growth period, less items are required to satisfy a given demand rate. While this implies that the farmer would have to feed more items and the processor would have to process and hold more inventory, the net effect on the supply chain profit is positive most likely because the model introduced a mortality cost, which is sort of like a penalty cost for items dying. Having fewer items die during the farmer's growth period means that the effect of this penalty cost is less severe.

- The fraction of processed items which are of inferior quality also affected both the profit and the order quantity, but the effect was very small. This is probably due to the relatively low value of 0.04 in the base example. As this fraction increases, the total profit decreases despite the fact that the poorer quality are sold a sold when the inspection process ends. But since they are sold at a lower price than the one being charged for the good quality inventory, this extra source of revenue does not increase the supply chain profit.

- Changes to all the different fixed costs in the system have the same general effect on the supply chain profit and the order quantity. When the fixed costs are increased the profit across the supply chain decreases and the optimal order quantity increases. Nonetheless, the severity of the effect varies between the fixed costs. The fixed costs at the farmer's and the processor's setup costs have the greatest effects while the fixed costs at the downstream echelons, namely the retailer's ordering cost and the transfer cost from the inspection facility, have significantly lower effects.

Table 4.3.2: Sensitivity analysis of various input parameters

Parameters	% change	**Farmer's order quantity (y)** items	% change	**Number of shipments (n_s)** shipments	% change	**Supply chain profit ($E[TPU_{sc}]$)** ZAR/week	% change
Base example		**179**		**9**		**2 177.29**	
K_r	-50	177	-1.1	9	0	2 259.44	+3.1
	-25	178	-0.5	9	0	2 225.51	+1.5
	+25	180	+0.5	9	0	2 158.18	-1.5
	+50	181	+1.1	9	0	2 124.78	-3.1
h_r	-50	211	+17.8	9	0	3 156.41	+44.0
	-25	193	+7.8	9	0	2 654.41	+21.1
	+25	168	-6.3	9	0	1 760.42	-19.7
	+50	158	-11.6	9	0	1 354.78	-38.2
K_p	-50	158	-11.5	8	-11.1	2 905.79	+32.6
	-25	169	-5.4	9	0	2 537.75	+15.8
	+25	189	+5.5	10	+11.1	1 863.94	-15.0
	+50	198	+10.4	10	+11.1	1 552.01	-29.2
h_p	-50	197	+9.8	10	+11.1	2 741.13	+25.1
	-25	188	+4.7	10	+11.1	2 459.93	+12.2
	+25	172	-3.9	9	0	1 934.61	-11.7
	+50	166	-7.3	9	0	1 687.05	-23.0
K_s	-50	179	+0.1	13	+44.4	2 249.68	+2.6
	-25	179	+0.2	11	+22.2	2 218.16	+1.2
	+25	179	0	8	-11.1	2 168.39	-1.1
	+50	180	+0.3	8	-11.1	2 146.88	-2.0
h_s	-50	184	+3.0	9	0	2 374.86	+8.4
	-25	182	+1.4	9	0	2 282.64	+4.1
	+25	177	-1.4	9	0	2 102.15	-4.1
	+50	175	-2.4	10	+11.1	2 014.16	-8.1
K_f	-50	154	-13.9	8	-11.1	3 060.07	+39.6
	-25	167	-6.5	9	0	2 609.35	+19.1
	+25	191	+6.5	10	+11.1	1 800.41	-17.9
	+50	201	+12.3	10	+11.1	1 431.05	-34.7
c_f	-50	179	0	9	0	3 492.18	+59.3
	-25	179	0	9	0	2 841.97	+29.7
	+25	179	0	9	0	1 541.55	-29.7
	+50	179	0	9	0	891.34	-59.3
m_f	-50	179	0	9	0	2 480.74	+13.2
	-25	179	0	9	0	2 336.25	+6.6
	+25	179	0	9	0	2 047.27	-6.6
	+50	179	0	9	0	1 902.78	-13.2
$p_v; p_f; p_p; p_r$	-50	179	0	9	0	-3 752.49	-271.2
	-25	179	0	9	0	-780.37	-135.6
	+25	179	0	9	0	5 163.88	+136.5
	+50	179	0	9	0	8 136.01	+271.2
$E[x]$	-50	179	0	9	0	-4 407.73	-301.1
	-25	179	0	9	0	-8.07	-100.4
	+25	143	-20.0	9	0	3 511.65	+60.2
	+50	119	-33.3	9	0	4 391.59	+100.4
$E[a]$	-50	177	-1.2	9	0	2 221.32	+1.3
	-25	178	-0.6	9	0	2 206.81	+0.7
	+25	180	+0.6	9	0	2 176.13	-0.7
	+50	181	+1.3	9	0	2 159.91	-1.5

- The effect of changes to all the holding costs (i.e. those incurred at the retail store and those incurred by the processor's at both the inspection facility and the processing plant) on the order quantity, number of deliveries and profit is as expected whereby increases in the holding costs cause the EOQ and the profit to decrease. When cost of holding processed inventory increases, the model tries to mitigate this by ordering less newborn items per cycle. This means that less live items need to be fed and the processed inventory spends less time in holding. While this reduces some of the variable inventory management costs, it has an adverse effect on the fixed costs because more order need to be placed in order to meet a specified demand rate. Given that the model has quite a few fixed costs at all four echelons (namely, the setup costs at the farming and processing facilities, the ordering cost at the retail facility and the transfer cost at the inspection facility), the net effect effect on the profit is negative.

4.3.4.2 Comparison with alternative scenarios

The proposed model for managing growing inventory items in a supply chain with farming, processing, inspection and retail stages incorporates a number of concepts to the literature on inventory control for growing items. These concepts include the delivery of multiple batches during a single processing run , the possibility of item mortality and the possibility of having inferior quality inventory in a lot. To quantify the importance of these concepts, three alternative scenarios, coinciding with each of the three concepts, are considered. Table 4.3.3 summarises the results from this analysis.

Table 4.3.3: Comparison of the proposed inventory control system with various alternative cases

Variables	**Base case**	**Case I (Single shipment)**		**Case II (Perfect survival)**		**Case III (Perfect quality)**	
	Quantity	Quantity	% difference	Quantity	% difference	Quantity	% difference
y^* (items)	179	158	-11.9	161	-10.0	175	-2.5
n_s^* (shipments)	9	1	-88.9	9	0	9	0
T^* (weeks)	18.57	16.38	-11.9	18.57	0	18.86	+1.6
$E[TPU_{sc}]^*$ (ZAR/week)	2 191.76	1 528.28	-30.3	2 851.71	+31.0	2 248.75	+2.6

In the first case, it is assumed that the processor delivers a single batch of processed inventory to the retailer when the inspection process ends. In this case, the profit reduces by 30.5%. While this leads reduces the cost of sending processed inventory to the retailer, the overall effect on the supply chain chain is negative because of the increased holding costs at the processing and inspection facilities. Furthermore, the processor has to ship each batch to the retailer at more frequent intervals to meet the demand rate and this increases the fixed costs because new growing, processing and retailing cycle start more frequently. Therefore, having the processor deliver multiple batches of processed inventory to the retailer per processing cycle is beneficial from a cost perspective.

For the second case, the survival rate of the live items during the growing period is assumed to be 100%. The profit generated across the supply chain increases by 31.0%. There are two factors which contribute to this, the first one (and the most critical) is that the farmer can meet the demand rate from a smaller lot size because all the items survive. This reduces the procurement cost and the holding cost at the growing facility. The second contributing factor is that the farmer incurs zero penalty costs for item mortality (i.e. the mortality cost) if none of the live items die. This demonstrates the importance of taking measures to keep item mortality rates during the growing period as low as possible.

In the third case, all the processed inventory is assumed to be of good quality and in this instance, the supply chain cost increases by 2.6%. This is significant given the low base value of the imperfect fraction of the items used in the numerical (of 0.04). The increase is also important because when all the items are of good the supply chain looses the revenue stream from inferior quality items sold to secondary markets and despite this, the supply chain profit still increases. This observation shows the importance of keeping items are that are rejected for quality reasons as low as possible.

4.3.5 Concluding remarks

In this section, an inventory control model for an integrated four-echelon supply chain for growing items is developed. When compared to similar models from previous studies, the proposed model sets itself apart not only via the four-echelon supply chain setup with discrete farming, processing, inspection and retail activities but also through the explicit consideration of the possibility that some of the live inventory items might die during the growing period and that some of the processed inventory might be of inferior quality.

Through numerical experimentation, it was shown that the total profit generated by the supply chain is affected by the survival rates of live items, the percentage of processed items that do not meet the quality standard and the shipment policy between the processor and the retailer. Consequently, production and operations managers in food production chains with quality inspection operations should take these three factors into consideration when making procurement decisions.

Despite being representative of an actual food production system, the model presented in this section still makes a few assumptions that might limit its practical applications. For instance, item deterioration, pricing decisions and incentive strategies like quantity discounts, revenue sharing contracts and trade credit financing, to name a few, are all not taken into consideration. These are critical issues in food production supply chains which are often characterised by short product life cycles and relatively low profit margins. It might be conducive for future research to focus on incorporating some of these issues to the proposed model.

4.4 A three-echelon supply chain inventory model for growing items with expiration dates[††]

4.4.1 Introduction

4.4.1.1 Context

One of the most fundamental changes to business management in recent years has been that businesses compete within supply chains, as opposed to competing as individual entities Lambert (2008). Businesses executives have realised that competitive advantages such as customer service, responsiveness and cost efficiency, among others, can be improved through collaboration with suppliers and customers. One form of collaboration is integrating inventory replenishment decisions with other supply chain members.

A multitude of researchers have used this collaboration mechanism to develop various integrated inventory models in systems with more than one party. Nonetheless, most of these models have been developed specifically for either conventional or deteriorating items. While these two groups of items are important, there are other groups of items which are also important for different reasons. For instance, growing items are the primary source of most food items. Given that these items are rarely consumed in their original form (i.e. most of them are processed before being put on sale) and that there are usually multiple parties involved in the food production chain, growing items (in the context of inventory control modelling) are the perfect candidate for an extension of the multi-echelon inventory model.

4.4.1.2 Purpose

In this section, a coordinated model for inventory control in a three-echelon supply chain for growing items is developed. The farming, processing and retail stages of the supply chain represent the three echelons of the supply chain. At the farming echelon, newborn items are procured and grown to maturity. It is assumed that some of the items do not survive at the farming operation due to factors like scavengers and illnesses. The mature items are then transferred to a processing plant for slaughtering, processing and packaging. Following this, the processor delivers processed inventory, in an integer number of shipments per processing run, to the retailer who meets consumer demand. At the the retail store, the processed inventory is displayed on shelves and consequently, the inventory has an associated shelf life or expiration date.

4.4.1.3 Relevance

The proposed inventory model accounts for a number of important issues in food production chains, namely the possibility of mortality (with reference to the live growing items), the integration of inventory replenishment decisions among multiple supply chain members and deterioration (of shelved stock items). Growing items are, like most living organisms, not immune to illnesses and various other health issues which might result in mortality. This makes item mortality an important consideration in the upstream portion of most food production chains. At the other end of the chain (downstream), shelf

[††]A modified version of this section has been submitted to *Opsearch* for review.

life becomes very critical because of government health and safety regulations regarding consumable food products.

4.4.1.4 Organisation

Apart from the introduction, this section has five other subsections. The proposed inventory system is briefly outlined in Subsection 4.4.2 which also includes the assumptions utilised during the model development phase. The inventory system under consideration is then modelled as a cost minimisation problem in Subsection 4.4.3. This is followed by a derivation of a special case of the model and a proof of the model's optimality in Subsection 4.4.4. Managerial insights are drawn from a numerical example presented in Subsection 4.4.5 which also shows the potential practical applications of the model. Concluding remarks and suggestions for future research are presented in Subsection 4.4.6.

4.4.2 Problem description

The proposed supply chain inventory model consists of three echelons representing different stages of a typical food production chain, namely farming, processing and retail operations. At the farming echelon, newborn items are procured and reared until maturity. The items are declared mature once their weight reaches a pre-defined target. Following this, the live items are instantaneously transferred to the next echelon which is processing. The live items are slaughtered, prepared and packaged in preparation for consumption (or sale) at the processing plant. For convenience, all activities carried out at the processing plant are collectively called processing and they are carried out at a given processing rate. The processed inventory is delivered to the last echelon (i.e. retail) in a number of equally-sized shipments per processing run. At the retail outlet, the processed inventory is placed on shelves in order to meet consumer demand. However, the processed inventory can only be displayed on the shelves for a given amount of time. The inventory continuously loses some of its utility over time and at the end of the shelf life, often specified as an expiration date, it is no longer suitable for consumption.

The inventory system profile for the problem at hand is depicted by Figure 4.4.1, which shows the behaviour (over time) of the farmer's live inventory, the processor's processed inventory and the retailer's processed and deteriorating inventory. The processor's cycle time is coordinated with the cycle times of the other two supply chain members on the basis of the behaviour of the inventory at that particular member's operations. For instance, the processor's cycle time is an integer multiple of the retailer's cycle time because the processed inventory is replenished frequently in an effort to keep it as fresh as possible due to its expiration dates. On the other hand, growing the newborn items requires a relatively longer period of time and a result the growing cycle is setup up such that when it ends a new processing cycle commences (as shown in Figure 4.4.1). In a nut shell, the farmer and the processor operate on a SSSD inventory replenishment policy while the processor and the retailer operate on a SSMD policy. The difference between these two policies lies in the number of shipments delivered by upstream supply chain member to the downstream member per cycle of the upstream member. For the SSSD replenishment policy, the farmer delivers one shipment of mature items to the processor during a single growing cycle while in the case of the SSMD policy, the processor delivers multiple shipments (an integer number) of processed inventory to the retailer during a single processing cycle.

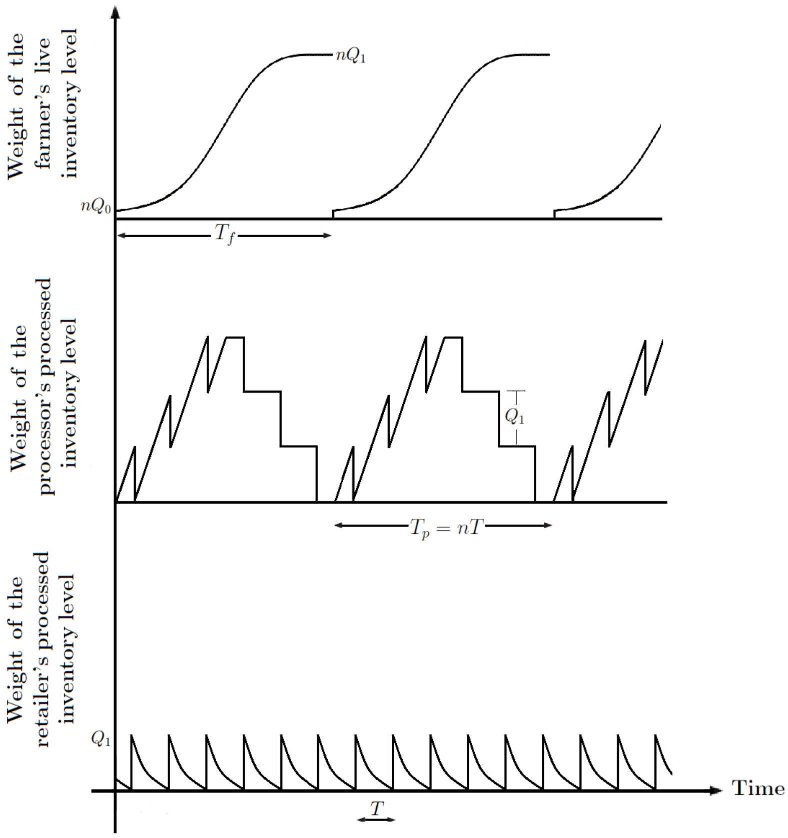


Figure 4.4.1: Inventory system profile showing the weight of the live inventory at the farmer's growing facility and the weight of the processed inventory at the processor's and the retailer's facilities.

The proposed inventory control problem is formulated as a cost minimisation problem aimed at determining the optimal number of newborn items that the farmer should order when a growing cycle commences (and by extension the processor and the retailer's order quantities and cycle times) and the optimal number of shipments processor should deliver to the retailer during a single processing run.

The model representing the proposed inventory system is developed under the following assumptions:

- There is only one farmer, one processor and one retailer in the supply chain dealing in one type of growing item.
- A fraction of the ordered items dies before reaching maturity weight.
- The (processor's) processing rate is greater than the (retailer's) demand rate, both of which are deterministic constants.
- The arrival of successive shipments of processed inventory from the processor to the retailer is scheduled to occur when the previous shipment has just been depleted.
- The processor delivers processed inventory to the retailer just at the moment the processed inventory is enough to make up a batch size.
- The retailer's replenishment interval is an integer multiple of the processor's replenishment interval.

- The live inventory incurs feeding costs (during the growth period) while the processed inventory incurs holding costs (during the processing and selling periods).
- Once the processed inventory reaches the retailer's shelves, it has a specified shelf life (or maximum lifetime) indicated by an expiration date. Beyond this point, the inventory has lost all utility and it cannot be used to meet consumer demand.

4.4.3 Model development

The procurement of ny newborn items marks the start of the farmer's replenishment cycle. At the time they are procured, each of the newborn items weighs w_0. Multiplying the number of items procured by the weight of the items yields the weight of all the ordered items at the time of procurement (i.e. $nQ_0 = nyw_0$). The farmer feeds the live items throughout the growth cycle, of duration T_f, and stops only when the weight of each item increases to the target maturity weight of w_1. The live items have a survival rate of x [i.e during the growth period, $(1-x)$ of the initially ordered newborn items die]. This implies that the weight of all the surviving ordered mature items (nQ_1) is therefore

$$nQ_1 = nxyw_1. \tag{4.4.1}$$

The logistic function, given by

$$w(t) = \frac{\alpha}{1 + \beta e^{-\lambda t}}, \tag{4.4.2}$$

is used to represent the items' growth function. It is chosen because of its distinctive "S"-shaped curve which is representative of the pattern of growth in most living organisms. The function describes the changes to the weight of items during the growth period and it makes use of three parameters, namely the items' asymptotic weight, the integration constant and the growth rate (represented by the symbols α, β and λ respectively). When the growth period is complete (i.e. when the weight of each has reached the target weight w_1 at time T_f), the items are delivered to the processor for slaughtering, preparation and packaging (i.e processing). Equation (4.4.2) can be rewritten in terms of the target weight and growth cycle duration as

$$T_f = -\frac{\ln\left[\frac{1}{\beta}\left(\frac{\alpha}{w_1} - 1\right)\right]}{\lambda}. \tag{4.4.3}$$

Since the processor and the retailer operate on a SSMD replenishment policy, for each processing run, the processor delivers an integer number (n) of equally-sized shipments of processed inventory to the retailer. The implications of this are that the retailer places n orders, of total weight Q_1, per processing setup. Likewise, the retailer's cycle time, T, and the processor's cycle time, T_p, are linked by the relation

$$T_p = nT. \tag{4.4.4}$$

This indicates that the processor should process live items of total weight nQ_1 per processing run. And since the farmer and the processor operate on a SSSD policy, the farmer should grow the same weight units of live inventory during each processing setup.

The total cost of managing inventory in the proposed three-echelon supply chain is made up of the individual inventory management costs incurred at each of the three echelons.

4.4.3.1 Retail operations

The inventory system profile for the retailer's processed and deteriorating inventory is illustrated by Figure 4.4.2. When a replenishment cycle commences, the retailer receives an order weighing Q_1 from the processor in order to meet consumer demand, with a rate D, for processed inventory. The retailer keeps the processed inventory on shelves and it deteriorates as a result. For health reasons, the processed inventory has a specified shelf life, L, beyond which it is no longer suitable for consumption.

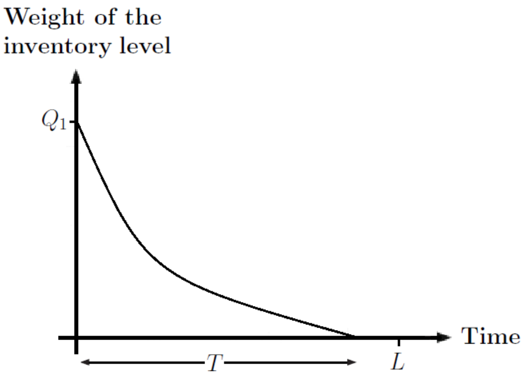


Figure 4.4.2: Inventory system profile the retailer

The deterioration experienced by the retailer's processed inventory is age-dependent in the sense that the longer the items are on the shelf, the greater the deterioration. The rate of deterioration peaks (at 100%) at the expiration date. Beyond this point, the inventory is no longer useful in the sense that it can no longer be used to fulfil consumer demand. This type of deterioration, associated with items with a maximum lifetime, has a rate

$$\theta(t) = \frac{1}{1 + L - t}, \tag{4.4.5}$$

for $0 \leq t \leq T$ and it is adopted from works such as Sarkar (2012) and Wang et al. (2014), to name a few. Since the items' deterioration rate cannot exceeds 100% (i.e. $\theta(t) \leq 1$), Equation (4.4.5) implies that the retailer's replenishment cycle (T) is less than or equal to the shelf life/ maximum lifetime of the items (L).

Throughout retailer's replenishment cycle, the weight of their inventory decreases due to consumer demand and deterioration. Accordingly, the changes to the weight of the retailer's processed inventory can be represented by the differential equation

$$\frac{dI(t)}{dt} = -D - \theta(t)I(t), \qquad 0 \leq t \leq T. \tag{4.4.6}$$

Since the weight of the processed inventory is completely depleted at time T, Equation (4.4.6) has the boundary condition $I(T) = 0$. Using the boundary condition to solve Equation (4.4.6) results in

$$I(t) = D(1 + L - t)\ln\left(\frac{1 + L - t}{1 + L - T}\right), \qquad 0 \leq t \leq T. \tag{4.4.7}$$

The weight of the retailer's order quantity, computed by subsisting $t = 0$ in Equation (4.4.7), is therefore

$$Q_1 = I(0) = D(1 + L)\ln\left(\frac{1 + L}{1 + L - T}\right). \tag{4.4.8}$$

Consequently, the corresponding number of items (or the retailer's lot size) is

$$y = \frac{1}{xw_1}\left[D(1+L)\ln\left(\frac{1+L}{1+L-T}\right)\right]. \tag{4.4.9}$$

The retailer incurs a cost of h_r for holding a single weight unit of the processed inventory per unit time. This cost is multiplied by the average weight of the processed inventory (i.e. the area under the graph in Figure 4.4.2 divided by the replenishment interval) in order to determine the holding cost per unit time as

$$HCU_r = h_r\frac{\int_0^T I(t)\,dt}{T} = \frac{h_r D}{T}\left[\frac{(1+L)^2}{2}\ln\left(\frac{1+L}{1+L-T}\right) + \frac{T^2}{4} - \frac{(1+L)T}{2}\right]. \tag{4.4.10}$$

Furthermore, the retailer incurs a fixed ordering cost of K_r whenever a new order for processed inventory is placed. This means that the ordering cost per unit time is

$$KCU_r = \frac{K_r}{T}. \tag{4.4.11}$$

The total cost (per unit time) of managing inventory at the retailer, computed by summing Equations (4.4.10) and (4.4.11), is thus

$$TCU_r = \frac{K_r}{T} + \frac{h_r D}{T}\left[\frac{(1+L)^2}{2}\ln\left(\frac{1+L}{1+L-T}\right) + \frac{T^2}{4} - \frac{(1+L)T}{2}\right]. \tag{4.4.12}$$

4.4.3.2 Processing operations

The total cost of managing inventory at the processing plant is made up of the holding and setup costs. The profile of the processor's inventory (in weight units) is represented by Figure 4.4.3a. The processor receives one delivery from the farmer at the beginning of each replenishment cycle with a duration of $T_p = nT$. The weight of each shipment received is $nQ_1 = nxyw_1$. When a new replenishment cycle starts, the processor incurs a fixed cost of K_p for preparing the processing facility for slaughtering, preparation and packaging (all are collectively termed processing and they occur at a rate of R). This implies that the setup cost per unit time is

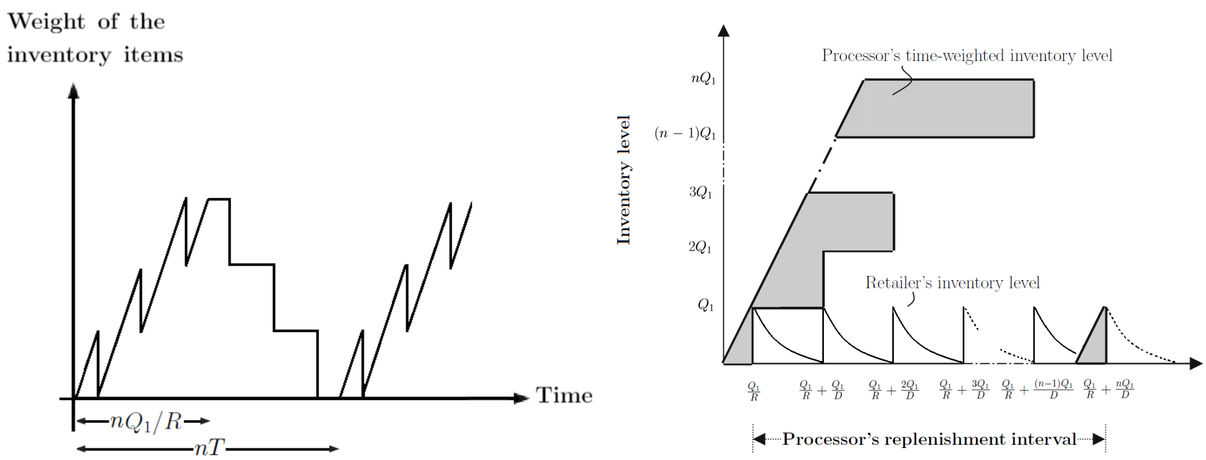
$$KCU_p = \frac{K_p}{nT}. \tag{4.4.13}$$

In order to determine the processor's holding costs per unit time, the average weight of the processed inventory (in weight units) is multiplied by the holding cost (in weight units per unit time, i.e. h_p). It follows that

$$HCU_p = \frac{h_p TD}{2}\left[\left(n-1\right)\left(1-\frac{D}{R}\right) + \frac{D}{R}\right]. \tag{4.4.14}$$

The average weight of the processed inventory is determined by dividing the area under the processed inventory level, as shown in Figure 4.4.3a, by the duration of the replenishment interval. So as to easily determine the area under Figure 4.4.3a, the figure is redrawn into Figure 4.4.3b. This approach is adapted from Yang et al. (2007)'s JELS model and the resulting expression for the average inventory level is

$$\begin{aligned}
\text{Average inventory}_p &= \frac{\text{Processor's time-weighted inventory}}{\text{Processor's replenishment interval}} \\
&= \frac{\frac{nQ_1^2}{2P} + Q_1^2\left(\frac{1}{D} - \frac{1}{R}\right) + 2Q_1^2\left(\frac{1}{R} - \frac{1}{R}\right) + \cdots + (n-1)Q_1^2\left(\frac{1}{D} - \frac{1}{R}\right)}{nQ_1/D} \\
&= \frac{D}{nQ_1}\left[\frac{nQ_1^2}{2R} + \frac{n(n-1)Q_1^2}{2}\left(\frac{1}{D} - \frac{1}{R}\right)\right] \\
&= \frac{Q_1}{2}\left[\left(n-1\right)\left(1 - \frac{D}{R}\right) + \frac{D}{R}\right].
\end{aligned} \tag{4.4.15}$$



(a) Original. *(b) Redrawn (modified from Yang et al. (2007)).*

Figure 4.4.3: The processor's processed inventory system behaviour.

The cost of managing the processor's inventory (per unit time) is therefore

$$TCU_p = \frac{K_p}{nT} + \frac{h_pTD}{2}\left[\left(n-1\right)\left(1 - \frac{D}{R}\right) + \frac{D}{R}\right]. \tag{4.4.16}$$

4.4.3.3 Farming operations

The farmer, whose inventory system profile is given by Figure 4.4.4, is responsible for rearing the live newborn items to maturity The items are deemed mature once they have grown to a pre-defined target weight. The farmer's total inventory management cost is comprised of the setup, feeding and mortality costs. In order for the retailer to meet a demand rate (for processed inventory) of D, the farmer delivers a shipment of processed inventory weighing nQ_1 to the processor, who in turn supplies the retailer with n shipments of processed inventory each weighing Q_1. Since the farmer and the processor operate on a SSSD replenishment policy, for each of the processor's processing setups the farmer starts a single replenishment cycle. Given that the farmer pays a fixed setup cost of K_f when a new replenishment cycle begins, the farmer's setup cost per unit time is therefore

$$KCU_f = \frac{K_f}{nT}. \tag{4.4.17}$$

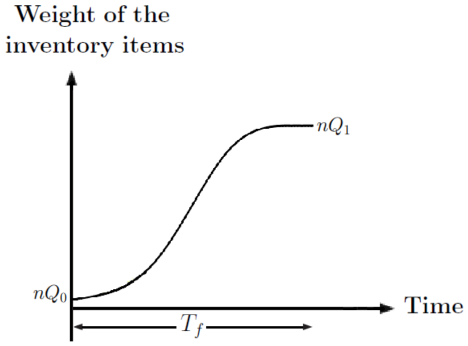


Figure 4.4.4: Inventory system profile the farmer

With exception to the setup cost, all of the farmer's other cost components are dependent on the average weight of the farmer's live inventory. The average weight is determined by dividing the area under the inventory system graph, depicted by Figure 4.4.4, by the duration of the replenishment cycle and it is given by

$$\begin{aligned}\text{Average inventory}_f &= \frac{\text{Farmer's time-weighted inventory}}{\text{Farmer's replenishment interval}}\\ &= \frac{\int_0^{T_f} nyw(t)\,dt}{nT}\\ &= \frac{D(1+L)\ln\frac{1+L}{1+L-T}}{Txw_1}\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right\}.\end{aligned} \tag{4.4.18}$$

The farmer incurs a cost associated with disposing the fraction of newborn items which do not survive until the end of the growing cycle. The farmer's mortality cost per unit time is computed as the product of the farmer's average inventory level, the fraction of items which do not not survive $(1-x)$ and the mortality cost per weight unit per unit time (m_f). The mortality cost per unit time is therefore

$$MCU_f = \frac{m_f(1-x)D(1+L)\ln\frac{1+L}{1+L-T}}{Txw_1}\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right\}. \tag{4.4.19}$$

Similarly, the farmer's feeding cost per unit time is determined as the product of the farmer's average inventory level, the fraction of items which survive (x) and the feeding cost per weight unit per unit time (c_f). It follows that

$$FCU_f = \frac{c_f x D(1+L)\ln\frac{1+L}{1+L-T}}{Txw_1}\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right\}. \tag{4.4.20}$$

The farmer's total cost per unit time is the sum of Equations (4.4.17), (4.4.19) and (4.4.20) and it is given by

$$TCU_f = \frac{K_f}{nT} + \frac{c_f x + m_f(1-x)}{Txw_1}\left(\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right)D(1+L)\ln\left(\frac{1+L}{1+L-T}\right). \tag{4.4.21}$$

4.4.3.4 Whole supply chain

4.4.3.4.1 Constraints governing the proposed inventory system

The feasibility and tractability for the proposed inventory control model is dependent on the imposition of two constraints. Firstly, the number of shipments of processed inventory delivered to the retailer per processing setup (n) should be an integer. This makes the solution procedure tractable. Secondly, the duration of the farmer's growth period (T_f) should be less than or equal to the duration of the processor's cycle time ($T_p = nT$). This ensures that the solution to the problem is feasible by assuring that the weight of the live items has reached maturity at the start of the processing run.

4.4.3.4.2 Total inventory management cost across the whole supply chain

The total supply chain (inventory management) cost per unit time is determined by adding Equations (4.4.12), (4.4.16) and (4.4.21). Furthermore, the fraction of items items which survive throughout the farmer's replenishment cycle, x, is considered a random variable with a given probability density function $f(x)$. The two constraints and the expected value of the total supply chain cost per unit time are used to formulate the inventory problem at hand as

$$\text{Min.} \quad \Bigg\{ E[TCU_{sc}] = \frac{K_r}{T} + \frac{h_r D}{T}\left[\frac{(1+L)^2}{2}\ln\left(\frac{1+L}{1+L-T}\right) + \frac{T^2}{4} - \frac{(1+L)T}{2}\right] + \frac{K_p}{nT} + \frac{h_p T D}{2}\left[\left(n-1\right)\left(1-\frac{D}{R}\right)+\frac{D}{R}\right] + \frac{K_f}{nT} + \left[\frac{c_f E[x] + m_f E[1-x]}{TE[x]w_1}\right]\left[\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right]\left[D(1+L)\ln\left(\frac{1+L}{1+L-T}\right)\right]\Bigg\}$$

$$\text{s.t.} \quad n \in \mathbb{Z}, \quad T_f \leq nT. \quad (4.4.22)$$

4.4.3.4.3 Solution procedure

The following iterative procedure is followed when calculating the optimal values of n and T:

Step 1 Set n to 1.

Step 2 Find the value of T which minimises Equation (4.4.22).

Step 3 Increase n by 1 and find the value of T which minimises Equation (4.4.22). Carry on to Step 4.

Step 4 If the latest value of $E[TC_{sc}]$ decreases, go back to Step 3. If the value of $E[TCU_{sc}]$ increases, the previously calculated value of $E[TCU_{sc}$ (along with corresponding T and n values) is the best solution and in this case carry on to Step 5.

Step 5 Verify the solution's feasibility with regard to the constraint $T_f \leq nT$. T_f is calculated from Equation (4.4.2). If the solution is feasible, those values of n and T are optimal and if this is the case, carry on to Step 7. If the solution is not feasible, carry on to Step 6.

Step 6 If the constraint is violated, set T to T_f/n and use that T value to calculate $E[TCU_{sc}]$ using Equation (4.4.22) and then carry on to Step 7.

Step 7 End.

4.4.4 Special case and theoretical results

4.4.4.1 Special case with no mortality nor deterioration

A special case of the proposed inventory model is derived by disregarding the possibility of some of the live items dying throughout the growing cycle and the fact that the processed inventory has a specified shelf life at the retail store. Before deriving this result, two scenarios (which aid in the derivation) are briefly discussed, namely one with no deterioration at the retail echelon and one with no mortality at the farming echelon.

4.4.4.1.1 Scenario I: Infinite shelf life (i.e. no deterioration)

Using the result

$$\lim_{L\to\infty}\left(\frac{1+L}{T}\right)\ln\left(\frac{1+L}{1+L-T}\right)=1, \tag{4.4.23}$$

from Wang et al. (2014), the weight of the retailer's order for processed inventory, as given in Equation (4.4.8), can be rewritten in terms of the result in Equation (4.4.23) as

$$Q_1 = DT\left(\frac{1+L}{T}\right)\ln\left(\frac{1+L}{1+L-T}\right). \tag{4.4.24}$$

This means that when the processed inventory is assumed to have an infinite shelf life (i.e. in the absence of deterioration), the retailer's order for processed inventory weighs

$$Q_1 = DT, \tag{4.4.25}$$

as $L \to \infty$.

Likewise, the result

$$\lim_{L\to\infty}\left[\frac{(1+L)^2}{2}\ln\left(\frac{1+L}{1+L-T}\right)-\frac{(1+L)T}{2}\right]=\frac{T^2}{4}, \tag{4.4.26}$$

from Wang et al. (2014) is used to evaluate the retailer's holding cost per unit time as given in Equation (4.4.10). This holding cost can be rewritten in terms of the result in Equation (4.4.26) as

$$HCU_r = \frac{h_rD}{T}\left[\frac{(1+L)^2}{2}\ln\left(\frac{1+L}{1+L-T}\right)-\frac{(1+L)T}{2}+\frac{T^2}{4}\right]. \tag{4.4.27}$$

Consequently, when the processed inventory is assumed to have an infinite shelf life, the retailer's holding cost per unit time becomes

$$HCU_r = \frac{h_rD}{T}\left(\frac{T^2}{4}+\frac{T^2}{4}\right)=\frac{h_rDT}{2}, \tag{4.4.28}$$

as $L \to \infty$.

Therefore, the retailer's total cost associated with managing the processed inventory per unit time becomes

$$TCU_r = \frac{K_r}{T} + \frac{h_r DT}{2}. \tag{4.4.29}$$

Similarly, as $L \to \infty$, the farmer's total cost associated with managing the processed inventory per unit time becomes

$$TCU_f = \frac{K_f}{nT} + \frac{\left[c_f x + m_f(1-x)\right]D}{xw_1}\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right\}. \tag{4.4.30}$$

Since the processor's total cost function is not a function of L, it remains the same as the one given in Equation (4.4.16) even under this scenario.

4.4.4.1.2 Scenario II: No mortality

When all the live items are assumed to survive throughout the growing period (i.e. 100% survival rate or simply, $x = 1$), the farmer's total cost associated with managing the live inventory per unit time becomes

$$TCU_f = \frac{K_f}{nT} + \frac{c_f}{Tw_1}\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right\}D(1+L)\ln\left(\frac{1+L}{1+L-T}\right). \tag{4.4.31}$$

Under this scenario, the retailer and the processor's total cost function remain the same as those in Equations (4.4.12) and (4.4.16), respectively.

4.4.4.1.3 Special case

A special case of the proposed inventory model is derived by assuming that all the live ordered items survive (at the farming echelon) and that the processed inventory has an infinite shelf life (at the retail echelon). In essence, this special case is derived by letting $x = 1$ and $L \to \infty$ (simultaneously, as opposed to doing it separately as was the case in the two aforementioned scenarios). Since the processor's total cost function is not affected by neither x nor L, it remains the same as the one given in Equation (4.4.16). The retailer's total cost function is only affected by L and it becomes the same as the one given in Equation (4.4.29) which corresponds to a situation where $L \to \infty$. Since, the farmer's total cost function is affected by both x and L, the farmer's new total cost for the special case is determined by evaluating Equation (4.4.31) for $L \to \infty$. Equation (4.4.31) can be rewritten in terms of the result in Equation (4.4.23) as

$$\begin{aligned} TCU_f &= \frac{K_f}{nT} + \frac{c_f}{Tw_1}\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right\}DT\left(\frac{1+L}{T}\right)\ln\left(\frac{1+L}{1+L-T}\right) \\ &= \frac{K_f}{nT} + \frac{c_f D}{w_1}\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right\}, \end{aligned} \tag{4.4.32}$$

as $L \to \infty$.

Consequently, the total inventory management cost across the supply chain becomes

$$\begin{aligned} TCU_{sc} = \frac{K_r}{T} + \frac{h_r DT}{2} + \frac{K_p}{nT} + \frac{h_p TD}{2}\left[\left(n-1\right)\left(1-\frac{D}{R}\right) + \frac{D}{R}\right] + \frac{K_f}{nT} \\ + \frac{c_f D}{w_1}\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right\}. \end{aligned} \tag{4.4.33}$$

The result in Equation (4.4.33) corresponds to the one in Sebatjane and Adetunji (2020c) who developed an inventory control model for a three-echelon supply chain for growing items without considering the possibility of mortality at the farming echelon and the shelf life of the processed inventory at the retail echelon.

4.4.4.2 Theoretical results

It is necessary to show that the objective function of the proposed inventory control model has a unique solution which actually minimises the function. This is achieved by proving that the function is convex.

Theorem 4.4.1. *For a certain $n > 0$, $E[TCU_{sc}]$ is a convex function of T and thus there exists a unique value of T which minimises $E[TCU_{sc}]$.*

Proof. The following auxiliary functions are derived by rewriting the objective function to be of the form $E[TCU_{sc}] = \frac{g(T)}{h(T)}$

$$\begin{aligned} g(T) = K_r + h_r D\left[\frac{(1+L)^2}{2}\ln\left(\frac{1+L}{1+L-T}\right) + \frac{T^2}{4} - \frac{(1+L)T}{2}\right] + \frac{K_p}{n} \\ + \frac{h_p T^2 D}{2}\left[\left(n-1\right)\left(1-\frac{D}{R}\right) + \frac{D}{R}\right] + \frac{K_f}{n} \\ + \left[\frac{c_f E[x] + m_f E[1-x]}{E[x]w_1}\right]\left[\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right]\left[D(1+L)\ln\left(\frac{1+L}{1+L-T}\right)\right], \end{aligned} \tag{4.4.34}$$

and

$$h(T) = T. \tag{4.4.35}$$

Taking the first and the second derivatives of $g(T)$ with respect to T for any specified n results in

$$\begin{aligned} g'(T) = h_r D\left[\frac{(1+L)^2}{2(1+L-T)} + \frac{T}{2} - \frac{1+L}{2}\right] + \frac{h_p T D}{2}\left[\left(n-1\right)\left(1-\frac{D}{R}\right) + \frac{D}{R}\right] \\ + \left[\frac{c_f E[x] + m_f E[1-x]}{E[x]w_1}\right]\left[\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right]\left[\frac{D(1+L)}{(1+L-T)}\right]. \end{aligned} \tag{4.4.36}$$

and

$$\begin{aligned} g''(T) = h_r D\left[\frac{(1+L)^2}{2(1+L-T)^2} + \frac{1}{2}\right] + \frac{h_p D}{2}\left[\left(n-1\right)\left(1-\frac{D}{R}\right) + \frac{D}{R}\right] \\ + \left[\frac{c_f E[x] + m_f E[1-x]}{E[x]w_1}\right]\left[\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right]\left[\frac{D(1+L)}{(1+L-T)^2}\right]. \end{aligned} \tag{4.4.37}$$

To show that $g(T)$ is strictly convex, it should be guaranteed that $\frac{(1+L)^2}{2(1+L-T)^2}$ and $\frac{D(1+L)}{(1+L-T)^2}$ are always positive. This achieved using Lemma 4.4.1. Consequently, $g''(T) > 0$ for all

$T > 0$ and therefore $g(T)$ is a differentiable and positive convex function. Given that $h(T)$ is also differentiable and positive convex function, $E[TCU_{sc}]$ is a convex function of T for a given value of n and hence, there exists a unique optimal value of T. □

Lemma 4.4.1. *$\frac{(1+L)^2}{2(1+L-T)^2}$ and $\frac{D(1+L)}{(1+L-T)^2}$ are always positive for all $T > 0$.*

Proof. Let

$$\Delta_1(T) = \frac{(1+L)^2}{2(1+L-T)^2}, \tag{4.4.38}$$

$$\Delta_2(T) = \frac{D(1+L)}{(1+L-T)^2}. \tag{4.4.39}$$

Taking the first derivatives of $\Delta_1(T)$ and $\Delta_2(T)$ with respect to T results in

$$\Delta_1'(T) = \frac{(1+L)^2}{(1+L-T)^3}, \tag{4.4.40}$$

$$\Delta_2'(T) = \frac{2D(1+L)}{(1+L-T)^3}. \tag{4.4.41}$$

Since $\Delta_1'(T) > 0$ and $\Delta_2'(T) > 0$, $\Delta_1(T)$ and $\Delta_2(T)$ are increasing functions of T. Therefore, for all $T > 0$, $\frac{(1+L)^2}{2(1+L-T)^2}$ and $\frac{D(1+L)}{(1+L-T)^2}$ are always positive. □

Theorem 4.4.2. *For all $T > 0$ values, $E[TCU_{sc}]$ is a convex function of n and consequently, there exists a unique value of n which minimises $E[TCU_{sc}]$.*

Proof. Taking the first and the second derivatives of $E[TCU_{sc}]$ with respect to n for any specified T results in

$$\frac{\partial E[TCU_{sc}]}{\partial n} = -\frac{K_p}{n^2 T} + \frac{h_p T D}{2} - \frac{K_f}{n^2 T}, \tag{4.4.42}$$

$$\frac{\partial^2 E[TCU_{sc}]}{\partial n^2} = \frac{K_p}{n^3 T} + \frac{K_f}{n^3 T}. \tag{4.4.43}$$

$E[TCU_{sc}]$ is a convex function of n because $\frac{\partial^2 E[TCU_{sc}]}{\partial n^2} > 0$. This indicates that a unique value of n which minimises $E[TCU_{sc}]$ exists. □

4.4.5 Numerical results

4.4.5.1 Base example

As a way of demonstrating the potential practical applications of the proposed inventory system, an example which considers a farmer, a processor and a retailer involved in the chicken production supply chain (at different stages) is considered. The retailer meets end consumer demand for processed chicken at a store and can only keep the chicken on shelves for a maximum of 4 days. The example considers the following input parameters: L=4 days; w_0= 0.06kg; w_1= 2kg; D=100 kg/day; R=150 kg/day; K_f=7 500 ZAR; c_f=1 ZAR/kg/day; m_f=2 ZAR/kg/day; K_p=5 000 ZAR; h_p=0.5 ZAR/kg/day; K_r=1 000 ZAR; h_r=1 ZAR/kg/day; α=6.87 kg; β=120; λ=0.11 /day. The fraction of items which survive throughout the farmer's growth period, x, is assumed to be a random variable that is uniformly distributed over [0.8, 1] with a probability density function given by

$$f(x) = \begin{cases} 5, & 0.8 \leq x \leq 1 \\ 1, & \text{otherwise.} \end{cases}$$

This means that

$$E[x] = \int_{0.8}^{1} 5x \quad dx = 5\left[\frac{(1^2 - 0.8^2)}{2}\right] = 0.9$$

The example is solved using Solver, a Microsoft Excel add-in, and the results are presented in Table 4.4.1.

Decision variables and objective function	**Quantity**
T^*	1.79 days
n^*	22 shipments
$E[TCU_{sc}]^*$	2 909.78 ZAR/day

Table 4.4.1: Optimal number of shipments per processing run, cycle time and expected total profit

From those results, the optimal inventory replenishment and shipment policies for all three supply chain members are determined. The farmer should place an order for ($ny =$) 2 706 live newborn (day old) items. The weight of the all the ordered newborn items (nQ_0) would amount to 162 kg. When the growth period ends, ($E[x] =$) 90% of the initially ordered items would have survived. This implies that the weight of the surviving mature items (nQ_1) would be 5 412 kg. The farmer should then transfer the entire lot to the processing plant. Throughout the processing cycle, the processor should deliver the items (now in a consumable form) to the retailer in ($n =$) 22 equally sized batches. Each batch that the retailer receives will weigh about ($Q_1 =$) 246 kg. The retailer should replenish their inventory every ($T =$) 1.79 days so that the processed items don't expire (after $L = 4$ days). The farmer and the processor should start new growing and processing cycles every (nT=) 39.6 days. By following this replenishment and shipment policy, the total costs of managing inventory in the supply will be minimised at 2 909.78 ZAR/day.

4.4.5.2 Sensitivity analysis

A sensitivity analysis was performed on the major input parameters in the base example in order to observe the effect of changes (increases and decreases of 25% and 50%) to those parameters on the objective function ($E[TCU_{sc}]$) and the two decision variables (T and n). The results are given in Table 4.4.2 and the following observations are note-worthy:

- While the shelf life (or expiration date) of the processed items affected the total inventory management cost across the supply chain, the effect was minimal when compared to those it had on the number of shipments and the retailer's cycle time. Case in point, a 50% reduction in the shelf life increased the cost by about 9%, increased the number of shipments by roughly 32% and reduced the retailer's cycle time by roughly 25%. The effects on the shipment and replenishment policy are not surprising considering that the retailer does not want to keep the products beyond their expiration dates. Consequently, when the shelf life of the products is reduced, the model's optimal solution recommends placing orders for relatively smaller lot sizes, but more frequently.

Table 4.4.2: Sensitivity analysis of various input parameters

Parameters	% change	Retailer's cycle time (T^*) days	% change	Number of shipments (n^*) shipments	% change	Supply chain cost ($E[TCU_{sc}^*]$) ZAR/day	% change
Base example		**1.79**		**22**		**2 909.78**	
L	-50	1.34	-24.9	29	+31.8	3 183.07	+9.4
	-25	1.60	-10.6	24	+9.1	3 015.86	+3.6
	+25	1.96	+9.8	20	-9.1	2 835.93	-2.5
	+50	2.14	+19.4	18	-18.2	2 781.36	-4.4
K_r	-50	1.38	-23.1	28	+27.3	2 595.94	-10.8
	-25	1.61	-9.8	24	+9.1	2 763.34	-5.0
	+25	1.95	+9.1	20	-9.1	3 042.64	+4.6
	+50	2.08	+16.1	19	-13.6	3 166.25	+8.8
h_r	-50	1.87	+4.6	21	-4.5	2 855.84	-1.9
	-25	1.84	+3.0	21	-4.5	2 883.07	-0.9
	+25	1.76	-1.4	22	0	2 935.59	+0.9
	+50	1.74	-2.8	22	0	2 960.96	+1.8
K_p	-50	1.78	-0.4	20	-9.1	2 841.84	-2.3
	-25	1.81	+1.3	20	-9.1	2 876.62	-1.1
	+25	1.79	+0.1	23	+4.5	2 941.26	+1.1
	+50	1.79	+0.1	24	+9.1	2 971.36	+2.1
h_p	-50	1.81	+1.4	30	+36.4	2 713.09	-6.8
	-25	1.80	+0.7	25	+13.6	2 819.45	-3.1
	+25	1.78	-0.8	20	-9.1	2 989.93	+2.8
	+50	1.77	-1.0	20	-9.1	3 067.44	+5.4
K_f	-50	1.77	-1.0	20	-9.1	2 806.59	-3.5
	-25	1.80	+0.4	20	-9.1	2 859.30	-1.7
	+25	1.80	+0.8	23	+4.5	2 956.36	+1.6
	+50	1.79	0	25	+13.6	3 000.24	+3.1
c_f	-50	2.06	+14.9	19	-13.6	2 247.90	-22.7
	-25	1.92	+7.5	20	-9.1	2 582.00	-11.3
	+25	1.70	-5.0	23	+4.5	3 232.47	+11.1
	+50	1.62	-9.3	24	+9.1	3 551.25	+22.0
m_f	-50	1.85	+3.3	21	-4.5	2 764.71	-5.0
	-25	1.83	+2.4	21	-4.5	2 837.34	-2.5
	+25	1.77	-0.9	22	0	2 981.80	+2.5
	+50	1.76	-1.7	22	0	3 053.66	+4.9
$E[x]$	-50	1.29	-28.0	30	+36.4	5 680.03	+95.2
	-25	1.56	-12.9	25	+13.6	3 855.18	+32.5
	+25	2.03	+13.5	19	-13.6	2 322.78	-20.2
	+50	2.26	+26.2	17	-22.7	1 917.81	-34.1

- The effects of changes to the retailer's ordering cost were also significant, but highly anticipated as well. When the cost of placing an order increases, the model's most obvious response is to let the retailer place orders less frequently (by increasing the lot size). However, this can have negative effects on the total cost, particularly due to the increased holding costs as a result of the larger lot size. For example, increasing the ordering cost by 50% increases the cycle time (and lot size) by about 17%. While this counters the effect of the increased ordering cost, it also increases the total cost by approximately 9%.

- When the retailer's holding costs are increased, the model's optimal solution responds by prompting the retailer to order less processed items. The main benefit of ordering smaller lot sizes is that they attract relatively lower holding costs, firstly, because there are fewer items that need to be kept in storage and secondly, because when the lot size is smaller, the retailer's cycle time is reduced which means that the processed inventory spends less time in storage and consequently, the holding costs are reduced. However, meeting a given demand rate from smaller lots leads to an increase in the number of shipments delivered to the retailer. This response is emulated by the results of the sensitivity analysis where, for example, a 50% increases in the retailer's holding costs resulted in a 3% reduction in the cycle time and a 2% increase in the shipments.

- Changes to the processor's setup and holding costs followed an identical response pattern to the retailer's holding and ordering costs. The major difference being that the response in the upstream members is not as sever as those shown by the downstream members. For example, when the processor and the retailer's fixed costs (i.e. setup and ordering costs, respectively) are reduced by 50%, the cycle time and total cost decreased by roughly 23% and 11% respectively in case of the retailer, while for the processor, the changes were about 1% and 2% respectively.

- A reduction in the farmer's feeding and mortality costs prompts the farmer to order more live newborn items. As a result, the processor and the retailer will receive relatively larger lots of mature items for processing and selling respectively. Consequently, the cycle time will increase and the number of shipments will decrease. The increased cycle time means that the processed inventory spends more time in holding and consequently, the total cost increase.

- As the fraction of live items which survive during the growing cycle increases, the number of shipments and the total cost decrease while the cycle time increases. The model's optimal solution responds this way because the a given demand rate for processed items can be met from a smaller lot size of newborn items since the survival rate has improved. While this increases the holding costs at the processor (since there are more mature surviving items that need to be processed), the reduced mortality costs at the farmer cushions against this and consequently, the total supply chain cost decreases despite the increased processor's costs.

4.4.5.3 Comparisons with alternative scenarios

The proposed inventory replenishment and shipment policy is compared with three alternative scenarios in order to investigate the benefits (or lack thereof) that might be reaped if those alternative scenarios occurred. The first of these alternative scenarios considers the shelf life of the processed items to be infinite (i.e. the items do not expire). The second alternative scenario considers a situation where the survival rate of the live items during the farmer's growth period is 100%, while the last scenario considers an independent replenishment policy where the replenishment decisions are made for the benefit of individual chain members as opposed to optimising costs for the whole supply chain. The results from the comparison are presented in Table 4.4.3.

For the first scenario, the processed items were assumed to have no expiration dates. When the items have an infinite shelf life at the retail store, the retailer's optimal cycle time increases significantly and the number of shipments delivered by the processor to the retailer decreases notably. This is achieved by ordering larger lot sizes. This scenario is actually beneficial to the whole supply chain because of the decreased fixed costs since fewer setups are required if the processed inventory can spends longer time periods in stock without expiring. In the example studied, the supply chain cost decreased by 21.6% under this scenario. While it might not be practical for management to infinitely increase the shelf life of the inventory, this result should motivate management to invest in better preservation technologies which have the potential to prolong the life time of the inventory.

Table 4.4.3: Performance of the proposed inventory control mechanism against various alternative scenarios

Variables	Base Scenario (Proposed system)	Scenario 1 (Infinite shelf life)		Scenario 2 (No mortality)		Scenario 3 (Independent)	
	Quantity	Quantity	% difference	Quantity	% difference	Quantity	% difference
T^* (days)	1.709	4.23	+136.3	1.91	+16.7	3.04	+70.0
n^* (shipments)	22	9	-59.1	20	-9.1	22	0
y^* (items)	123	235	+91.1	121	-1.6	260	+111.6
TCU_r^* (ZAR/day)	663.18	447.89	-32.5	635.61	-4.2	540.02	-18.6
TCU_p^* (ZAR/day)	469.90	483.98	+3.0	465.49	-0.9	657.54	+39.9
TCU_f^* (ZAR/day)	1 777.70	1 350.25	-24.0	1 517.64	-14.6	2 086.33	+17.4
TCU_{sc}^* (ZAR/day)	2 909.78	2 282.12	-21.6	2 618.74	-10.0	3 283.90	+12.9

For the second scenario, it was simply assumed that all (100%) of the initially ordered newborn items survive throughout the farmer's growth cycle. The increased inventory management resulting from mortality also show that item mortality is an important consideration in food production systems which derive most of their products primarily from growing items. Since growing items are living organisms, it is possible for the to die. The financial benefits of having no mortality are not only reaped by the farmer, they also trickle (albeit lower) downstream across the supply chain. The total cost of inventory control across the supply chain decreased by 10% when the survival rate was 100% (compared to 90% in the base case). This result should motivate management to take measures aimed at increasing the survival rate of the items such as immunisations and inoculations.

For the scenario with an independent replenishment policy (i.e. third scenario), it was assumed that the retailer (who faces consumer demand for processed items) optimises their own inventory replenishment and shipment decisions (while ensuring that the lot does not expire) and these are passed down to the upstream chain members. This resulted in a sizeable reduction (of 18.6%) in the retailer's inventory management costs, however, the total costs of managing inventory across the supply chain increased (by 12.9%). The benefits of coordinating replenishment and shipment decisions with all supply chain members outweigh those achieved through individual optimisation. This emphasises the importance of one of the main objectives of supply chain management which collaborating will all chain members towards a common goal (for the benefit of all parties involved).

The cost differences between the proposed inventory system and the three alternative scenarios highlight the importance of the three major concepts incorporated in the proposed model, namely, item mortality, expiration dates and the integration of shipment and replenishment decisions with all supply chain members. It would, therefore, be advisable for procurement managers in food production systems with an inventory control setup similar to the proposed one to pay close attention to those three issues as they have sizable effects on the financial and operational performance of the supply chain.

4.4.6 Concluding remarks

Operations managers and inventory control specialists at various stages of food production systems are faced with a number of issues. For instance, at the down stream end of the supply chain, retailers are confronted with short product life cycles and their aim is to sell the inventory as fast as possible in order to avoid expiration and to reduce holding costs. One way of achieving this goal is to keep stock levels low but doing so puts the retailers at a higher risk of losing sales due to stock-outs. At the upstream end of the chain, item mortality is a threat to the livelihood of growing items. Another issue facing all supply chain members is deciding whether to make inventory replenishment decisions individually or jointly with other chain members.

This section take all these issues into consideration and develops a coordinated model for inventory control in a supply chain with distinct farming, processing and retail echelons. In addition to determining the optimal replenishment policies to be followed at each echelon, the model demonstrates the benefits (through cost savings) of supply chain integration as well as the drawbacks of item mortality which are not only detrimental at the farming echelon, but are also amplified across the entire supply chain.

Several assumptions, which have the potential to limit the practical applications of the model, were made during the model development process. These include, but are not limited to, deterministic demand and processing rates, one type of growing item in the inventory system and the absence of incentive policies between the supply chain members. Food production systems are not isolated from macroeconomic conditions and are therefore characterised by uncertainty and more often than not, retailers often stock multiple food products derived from growing items. Furthermore, incentive policies like quantity discounts, pre-payments and delayed payments are not uncommon in food production chains where margins are relatively low. Any of these factors, along with various popular EOQ extensions, can be used (solely or in combination with one another) to extend the proposed model.

4.5 A four-echelon supply chain inventory model for growing items with imperfect quality and errors in quality inspection**

4.5.1 Introduction

4.5.1.1 Context

Commercial food processing operations are complex industrial production systems often involving multiple processes and entities. One of the most critical processes, from a consumer health perspective, is the inspection of the food products for quality control. The purpose of inspection is to separate the products into two groups, those of good quality and those of poorer quality. Inspection processes are not error-free and therefore, it is possible for food products to be incorrectly classified at the inspection stage. In other words, poorer quality processed inventory might be mistakenly classified as being of good quality (i.e. a type I inspection error might occur). Conversely, a type II error might occur whereby good quality processed inventory is classified as being of poorer quality.

4.5.1.2 Purpose

This section is aimed at formulating a model for inventory management in a multi-echelon supply chain for growing items. The proposed supply chain has three members (namely, a single farmer, a single processor and a single retailer) and four echelons (namely, the farmer's growing facility, the processor's processing facility, the processor's inspection facility and the retailer's selling facility).

At the farming echelon, the farmer rears live newborn items until maturity provided that some of the items might die throughout the growing cycle as a result of predators and illnesses. At the end of each growing period, the farmer sends a batch of mature items to the processor. The processor operates two facilities, corresponding to two echelons (i.e. processing and inspection). At the processing echelon, the live inventory received from the farmer is transformed into a form that is consumable (and saleable). A given fraction of the processed inventory is of poorer quality. During a single processing cycle, the processor ships an integer number of processed inventory from the processing facility to the inspection facility. At the inspection echelon, the processed inventory is classified into one of two classes, namely, good and poorer quality processed inventory, provide that the inspection is subject to type I and type II errors that result in some items being misclassified. During the course of a single inspection cycle, the processor ships an integer number of good quality processed inventory, as classified according to the inspection process, from the inspection facility to the retailer's selling facility. The processor sells the poorer quality processed inventory (as classified by the inspection process) to secondary markets at a discounted price. At the retail echelon, good quality processed inventory (as per the inspection process) is used to meet consumer demand. Some of the inventory is incorrectly classified, a fraction of the poorer quality processed will be used to meet consumer demand. When consumers mistakenly receive poorer quality processed inventory as a result of errors in inspection, they return it to the retailer who sells it at a discounted

**A modified version of this section has been submitted to *Annals of Operations Research* for review.

price to secondary markets. There are penalty costs associated with committing any of the two types of errors.

4.5.1.3 Relevance

This section accounts for a number of pertinent issues related to inventory management in food processing systems. These issues include the possibility of mortality at the farming echelon, the shipments policies adopted among the echelons, the quality inspection at the inspection echelon and the possibility of committing type I and type II errors at the inspection echelon. Because real life food processing systems are faced with all these issues, the results from this section can be used by production and operations managers as a guideline when making procurement and shipment decisions at different stages (or echelons) of the broader food processing value chain.

4.5.1.4 Organisation

Other than the introduction, this section has five additional subsections. The introduction is followed, in Subsection 4.5.2, by an overview of the assumptions employed in the section. In Subsection 4.5.3, a model representing the proposed inventory system is developed. Theoretical results that demonstrate the proposed model's optimality are presented in Subsection 4.5.4. Prior to concluding the section in Subsection 4.5.6, numerical results showing the potential practical applications of the model are provided in Subsection 4.5.5.

4.5.2 Problem description

Figure 4.5.1 depicts the proposed four-echelon supply chain system. At the farming echelon, the farmer procures live newborn items (and thus have the capability to grow). The farmer rears the items until such a time that their weight has reached a specified target, at which point the items are deemed mature. The farmer then delivers the mature items to the next echelon, which is the processing facility where the live inventory is transformed into processed inventory. Following processing operations, the now processed inventory is delivered to the next echelon, which is the inspection facility where the quality control measures are in place. The processor sends an integer number of shipments of processed inventory during from the processing facility to the inspection facility during a single processing cycle. This is different from the shipment policy between the farming facility and the processing facility whereby the farmer delivers a single shipment of mature live inventory to the processing facility per farming cycle. This is because the process of growing live items takes a relatively longer period of time. At the inspection facility, the processed inventory is classified into two groups, namely, good and poorer quality processed inventory. However, the inspection process is subject errors and therefore, some of the processed inventory is incorrectly classified. The processed inventory that is classified as being of poorer quality (including the incorrectly classified good quality inventory) is sold as a single batch to secondary markets at a discounted price. The processed inventory that is classified as being of good quality (including the incorrectly classified poorer quality inventory) is shipped to the final echelon which is the retail echelon where consumer demand for good quality processed inventory is met. Since some of the inventory that is sold to consumers would have been incorrectly classified, it is returned to the retailer (shown as the red area of the lowermost portion of Figure 4.5.1) and replaced with good quality processed inventory. The returned inventory is then sold to secondary markets at a discounted price.

The proposed supply chain system is studied as a profit maximisation problem. The objective function of the problem is the total profit generated across the supply chain being the objective function and problem's decision variables are the order quantity for live inventory items, the number of shipments of processed inventory delivered from the processing facility to the inspection facility per processing cycle and the number of shipments of processed inventory delivered from the inspection facility to the retail facility per inspection cycle.

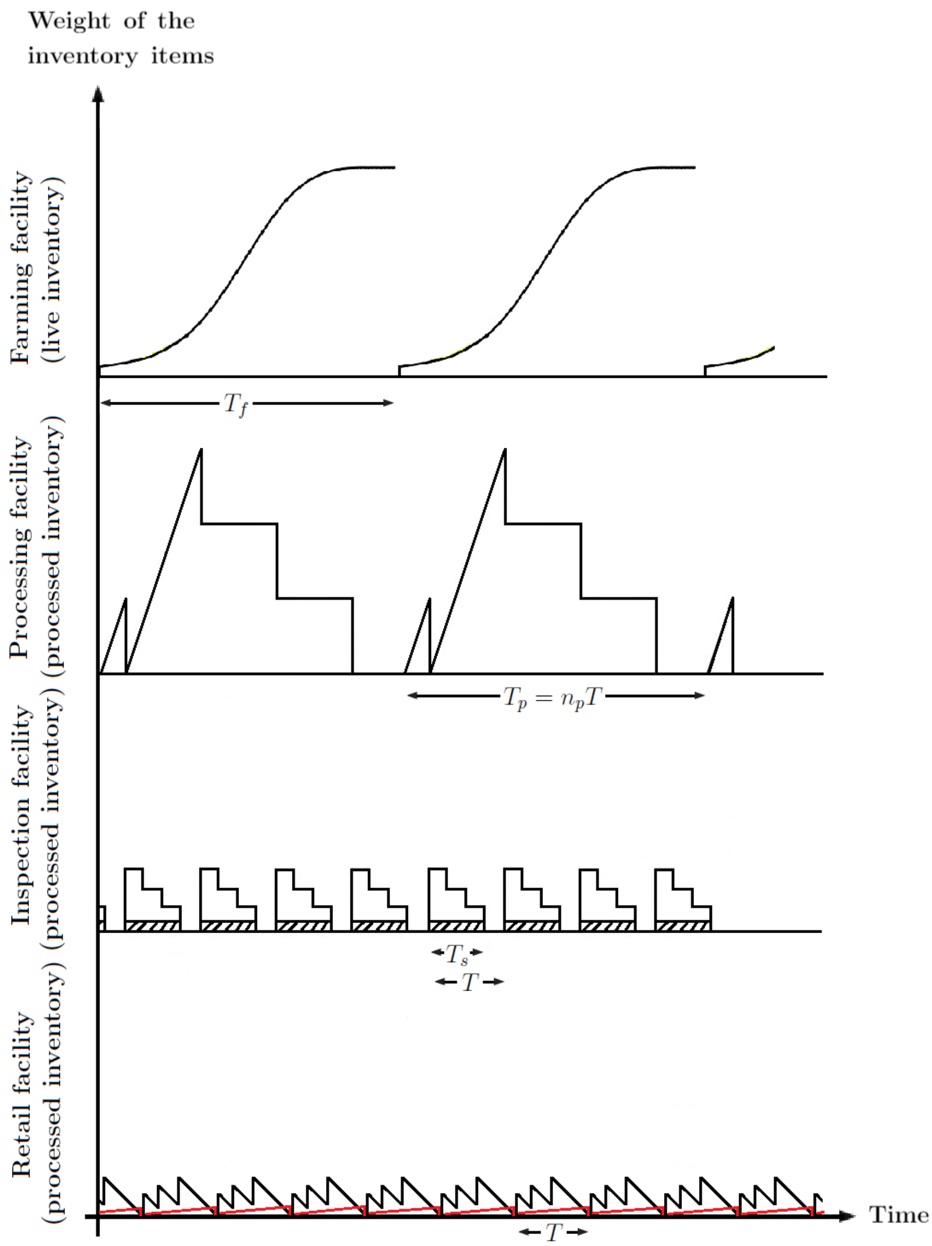


Figure 4.5.1: Inventory system profile for a farmer, a processor and a retailer in a supply chain for growing items with imperfect quality and errors in the inspection process

The following assumption are employed when modelling the proposed four-echelon supply chain inventory system:

- The supply chain is comprised of a single farmer, a single processor and a single retailer involved in the growing, processing and retailing of a single type of growing item.
- A fraction of the ordered live items dies at the farmer's growing facility before reaching the maturity weight.
- The processing rate at the processor's processing facility is greater than the demand rate at the retailer's retail facility and both rates are deterministic constants.
- A fraction of the processed inventory is of poorer quality (i.e. it does not meet the required quality standard).
- Throughout the course of a single processing cycle, the processor transfers an integer number of shipments of processed inventory to the inspection facility where a 100% inspection process takes place in which the processed inventory is classified into two groups, namely, good quality and poorer quality processed inventory.
- The inspection process is not perfect and is thus prone to errors.
- Two types of errors can occur, namely, type I (whereby poorer quality processed inventory is incorrectly classified as being of good quality) and type II errors (whereby good quality processed inventory is incorrectly classified as being of poorer quality).
- During the inspection process, the processor transfers equally-sized shipments of good quality processed inventory (as classified by the inspection process which is prone to errors) from the inspection facility to the retailer's selling facility.
- Throughout the inspection process, the processor allows the poorer quality processed inventory (as classified by the inspection process which is prone to errors) to accumulate at inspection facility.
- At the end of each inspection run, the poorer quality processed inventory (as classified by the inspection process which is prone to errors) is sold as a single batch to secondary markets at a discounted price.
- Since the inspection process is prone to errors, some of the poorer quality processed inventory sold to secondary markets is in fact good quality processed inventory (that was incorrectly classified by the inspection process). The processor incurs a penalty cost for selling incorrectly classified processed inventory.
- The retailer uses the good quality processed inventory (as classified by the inspection process which is prone to errors) to meet consumer demand as soon as the first shipment is delivered by the processor (from the inspection facility).
- Since the inspection process is prone to errors, some of the good quality processed inventory that the retailer sells to end consumers is in fact poorer quality processed inventory (that was incorrectly classified by the inspection process). The retailer incurs a penalty cost for selling incorrectly classified processed inventory.

- Consumers who received poorer quality processed inventory that was incorrectly classified as good quality inventory can return the inventory to the retailer who will replace their order with a new order for good quality processed inventory.
- The live inventory incurs feeding costs (during the growth cycle) while the processed inventory incurs holding costs (during the processing, inspection and selling cycles).
- The probabilities of survival, type I errors and type II errors and the fraction of poorer quality processed inventory are assumed to be random variables with known probability density functions.

4.5.3 Model development

4.5.3.1 Profit generated by the farmer

A replenishment cycle starts with the purchase of $n_p y$ newborn items. The weight of the each newborn item is w_0. This implies that the weight of all the ordered newborn item, $n_p Q_0$, is given by $n_p y w_0$. The weight of each item increases throughout the replenishment cycle until it reaches a target of w_1. All the ordered newborn items grow at the same rate and when the weight of each of them reaches the target weight, of w_1, the items are transferred to the processing echelon. However, not all the items survive to the end of the growth period due to illnesses and scavengers. Based on a survival rate of x, the total weight of all the ordered surviving items at the end of the growing period, $n_p Q_1$, is given by $x n_p y w_1$. This quantity is transferred to the processor for processing and quality control.

The general pattern of growth is similar for different items despite the fact that different items have different growth rates. Growth functions have a characteristic "S"-shaped curve, and for this reason the logistic function is used to model the growth pattern. The items' growth function can be represented by

$$w_t = \frac{\alpha}{1 + \beta e^{-\lambda t}}. \tag{4.5.1}$$

The logistic growth function makes use of three parameters to represent item growth over time. These are λ, β and α which denote the exponential growth rate, the constant of integration and the items' asymptotic weight respectively.

When the growth period concludes at T_f, the farmer instantaneously delivers the surviving mature items to the processor. At this point, each item would be fully grown and its weight would have reached the target weight of w_1. The duration of the growth period can be determined from Equation (4.5.1) by replacing w_t with the specified target maturity weight (w_1) and T_f with t and the result is

$$T_f = -\frac{\ln \left[\frac{1}{\beta} \left(\frac{\alpha}{w_1} - 1 \right) \right]}{\lambda}. \tag{4.5.2}$$

Figure 4.5.2, a diagrammatic representation of the farmer's live inventory profile over time, is used to evaluate the area under the farmer's inventory system profile. This area represents the farmer's average inventory level (in weight units). The area is evaluated as

$$\text{Area}_f = \int_0^{T_f} n_p y w(t)\, dt = n_p y \left\{ \alpha T_f + \frac{\alpha}{\lambda} \left[\ln \left(1 + \beta e^{-\lambda T_f} \right) - \ln (1 + \beta) \right] \right\}. \tag{4.5.3}$$

There is a cost associated with feeding the items (i.e the feeding cost) that the farmer incurs. The cyclic feeding cost incurred by the farmer, FC_f, is computed as the product of the farmer's average inventory level as determined in Equation (4.5.3), the fraction of surviving items (x) and the farmer's feeding cost per weight unit per unit time (c_f). Thus,

$$FC_f = c_f x n_p y \left\{ \alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right\}. \tag{4.5.4}$$

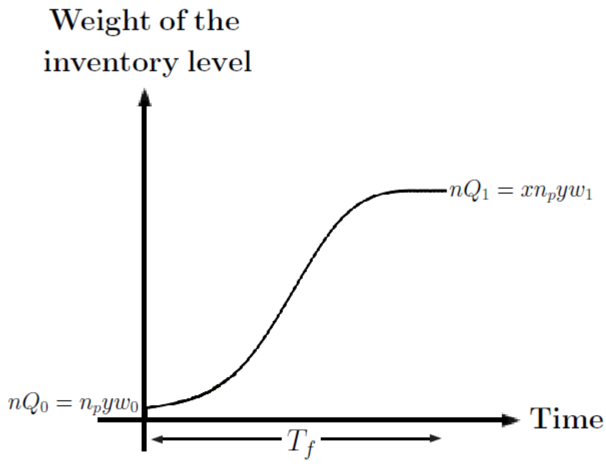


Figure 4.5.2: The farmer's inventory system profile at the growing facility

In addition, the farmer incurs a cost associated with disposing the fraction of newborn items which do not survive until the end of the growing cycle. The farmer's mortality cost per cycle, MC, is computed as the product of the farmer's average inventory level, the fraction of items which do not not survive $(1-x)$ and the mortality cost per weight unit per unit time (m_f). Thus,

$$MC_f = m_f(1-x) n_p y \left\{ \alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right\}. \tag{4.5.5}$$

The profit generated by the farmer per cycle, TP_f, is calculated as the difference between their revenue and their total costs. This profit is given by

$$\begin{aligned} TP_f &= p_f n_p x y w_1 - p_v n_p y w_0 - K_f \\ &\quad - n_p y \left\{ \alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right\}\left[c_f x + m_f(1-x)\right]. \end{aligned} \tag{4.5.6}$$

The first term in Equation (4.5.6) represents the revenue generated by the farmer per cycle and it is computed as the product of the price that the farmer charges the processor for mature live inventory (p_f) and the quantity, in weight units, of live inventory that the farmer sells to the processor per cycle ($n_p x y w_1$). The next term is the farmer's procurement cost per cycle and it is calculated by multiplying the procurement cost that the farmer gets charged for the newborn inventory (p_v) and the total weight of newborn inventory that the farmer procures per cycle ($n_p y w_0$). The third term is the fixed setup cost incurred at the beginning of each cycle for preparing the growing facility (K_f) and the last term is the sum of Equations (4.5.5) and (4.5.6).

The duration of the farmer's growing period is dependent on the specified maturity weight of the items. Since the farmer delivers a single shipment of live items to the processing facility per processing cycle, it is imperative that the two cycles (i.e. the farming and processing cycles) are synchronised for better planning. In order to ensure the proposed synchronisation, it is assumed that the farmer's replenishment frequency is equal to that of the processor. Therefore, the farmer' total profit per unit time, TPU_p, is computed by dividing the farmer's total profit per cycle, as given in Equation (4.5.6), by the duration of the processor's replenishment cycle (i.e. n_pT) and hence,

$$\begin{aligned} TPU_f = {} & \frac{p_f xyw_1}{T} - \frac{p_v yw_0}{T} - \frac{K_f}{n_pT} \\ & - \frac{y}{n_pT}\left\{\alpha T_f + \frac{\alpha}{\lambda}\Big[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\Big]\right\}\big[c_f x + m_f(1-x)\big]. \end{aligned} \tag{4.5.7}$$

4.5.3.2 Profit generated by the processor

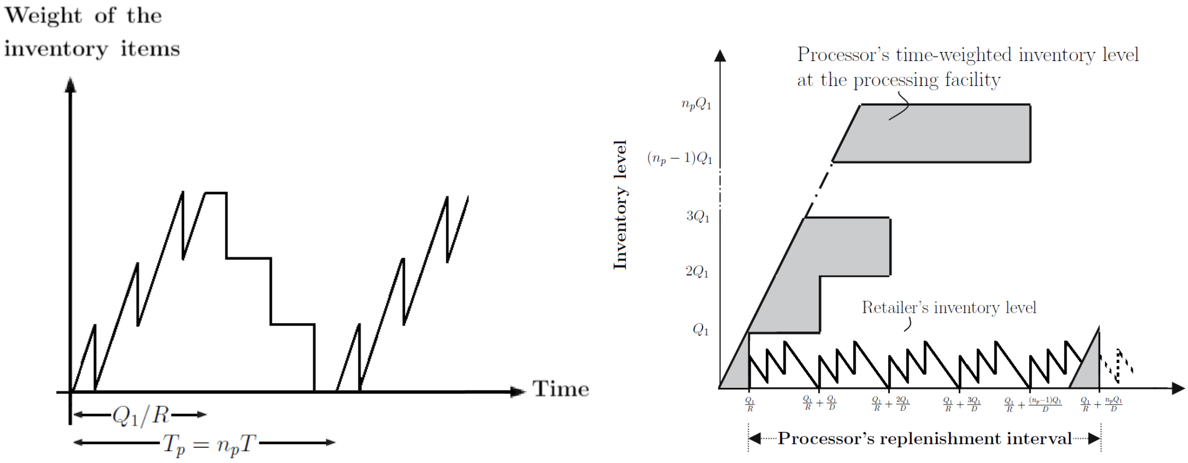
The processor operates two facilities, namely, a processing facility and a inspection facility. The processing facility is used to transform the live mature inventory items (received from the farmer) into processed inventory suitable for consumption. Prior to delivering the processed inventory to the retailer who meets end consumer demand, the processor transfers the processed inventory from the processing plant to a inspection house where quality control takes place. In the inspection facility, good quality processed inventory is separated from processed inventory of poorer quality. However, some of the good quality processed inventory is mistakenly classified as poorer quality processed inventory and vice versa.

4.5.3.2.1 Costs incurred at the processing facility

The processor receives live inventory from the farmer weighing $n_pQ_1 = n_pxyw_1$ at the beginning of each cycle. The live inventory is converted into processed inventory at a rate of R weight units per unit time. The processor sends n_p equally-sized batches of processed inventory to the inspection facility during the course of a single processing cycle (of duration T_p). Given that the batches are of equal size, this means that each batch sent to the inspection facility weighs xyw_1. A profile of the processor's processed inventory level (in the processing facility) over time is depicted in Figure 4.5.3a.

The average inventory level in the processing facility is determined by evaluating the area under the inventory profile as depicted in Figure 4.5.3a. Given the irregular shape of Figure 4.5.3a, it suffices to redraw that figure into Figure 4.5.3b which makes the computation of the area much easier. This method is adapted from Yang et al. (2007). The area under the inventory profile as depicted in Figure 4.5.3b is thus evaluated as

$$\begin{aligned} \text{Area}_p &= \frac{n_pQ_1^2}{2R} + Q_1^2\left(\frac{1}{D}-\frac{1}{R}\right) + 2Q_1^2\left(\frac{1}{D}-\frac{1}{R}\right) + \cdots + (n_p-1)Q_1^2\left(\frac{1}{D}-\frac{1}{R}\right) \\ &= \frac{\left(n_pQ_1\right)^2}{2R} + \frac{n_p(n_p-1)\left(n_pQ_1\right)^2}{2}\left(\frac{1}{D}-\frac{1}{R}\right) \\ &= \frac{\left(n_pxyw_1\right)^2}{2R} + \frac{n_p(n_p-1)\left(n_pxyw_1\right)^2}{2}\left(\frac{1}{D}-\frac{1}{R}\right). \end{aligned} \tag{4.5.8}$$



(a) Original. *(b) Redrawn for ease of computation.*

Figure 4.5.3: The inventory system for processor's processing plant.

The total cost incurred by the processor at the processing facility per cycle is the sum of the purchasing, setup and holding costs per cycle, given by

$$TC_p = p_f n_p xyw_1 + K_p + h_p \left[\frac{\left(n_p xyw_1\right)^2}{2R} + \frac{n_p(n_p-1)\left(n_p xyw_1\right)^2}{2} \left(\frac{1}{D} - \frac{1}{R} \right) \right]. \quad (4.5.9)$$

The first term in Equation (4.5.9) is the farmer's purchasing cost per cycle and it is calculated by multiplying the price that the processor gets charged (by the farmer) for mature live inventory (p_f) and the quantity, in weight units, of live inventory that the processor procures (from the farmer) per cycle ($n_p xyw_1$). The second term in Equation (4.5.9) is the fixed setup cost incurred by the processor for preparing the processing facility (K_p) at the start of each cycle. The last term in Equation (4.5.9) represents the holding cost incurred at the processing facility per cycle and this cost is calculated as the product of the average processed inventory level, i.e. Equation (4.5.8), and the cost of holding a single weight unit of processed inventory at the processing facility per unit time (h_p).

4.5.3.2.2 Costs incurred at the inspection facility

The processor sends n_p batches of processed inventory from the processing facility to the inspection warehouse during the course of a single processing run. This implies that the processor has n_p inspection runs per processing cycle. Each batch has a weight of $Q_1 = xyw_1$. The inventory system profile for the processed inventory at the inspection warehouse is depicted by Figure 4.5.4. It is assumed that a certain fraction (a) of the processed inventory received from the processing plant is of poorer quality. This means that for every batch received from the processing facility (with a weight of xyw_1), the weight of poorer quality processed inventory is xyw_1a and the good quality processed inventory weighs $xyw_1(1-a)$. For every inspection cycle, the entire lot received from the processing facility (i.e. xyw_1) is inspected in order to separate the processed inventory of good quality from that of poorer quality at a rate of z weight units per unit time. This implies that the duration of inspection period (i.e the time required to inspect the entire lot received) is

$$T_s = \frac{xyw_1}{z}. \quad (4.5.10)$$

During the course of a single inspection run, the processor delivers an integer number (n_s) of batches of good quality processed inventory to the retailer at regular intervals of duration τ. Given that the duration of a single inspection run is T_s, the duration of τ is computed by dividing T_s by n_s. Therefore,

$$\tau = \frac{xyw_1}{n_s z}. \tag{4.5.11}$$

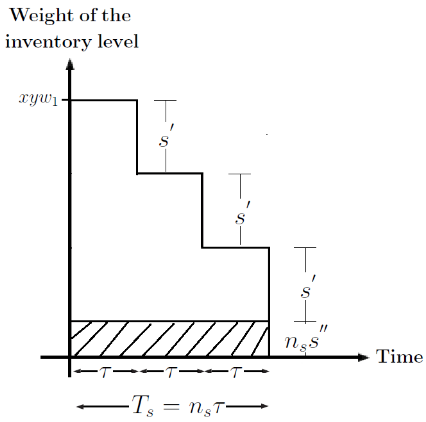


Figure 4.5.4: The processor's inventory system profile at the inspection facility

In an idealistic situation with a perfect inspection process (i.e. all good quality processed inventory is separated from poorer quality processed inventory), the weight of the good and poorer quality processed inventory per inspection run would be given by $xyw_1(1-a)$ and xyw_1a, respectively. In most production systems, inspection processes are seldom perfect. To account for this, probabilities of erroneously classifying good and poorer quality items are Incorporated. A type I error is committed when poorer quality processed inventory is classified as good quality processed inventory following the inspection process, while a type II error is committed if, at the end of inspection, good quality processed inventory is classified as poorer quality processed. The probabilities of committing type I and type II errors are given by u_1 and u_2, respectively. This means that the probabilities of correctly classifying good and poorer quality processed inventory are $(1-u_1)$ and $(1-u_2)$, respectively. Hence, the inspection process can lead to four possible outcomes or scenarios. These are: Scenario 1 - good quality processed inventory is classified as good quality processed inventory; Scenario 2 - good quality processed inventory is classified as poorer quality processed inventory; Scenario 3 - poorer quality processed inventory is classified as good quality processed inventory; and Scenario 4 - poorer quality processed inventory is classified as poorer quality processed inventory. Figure 4.5.5 depicts the four probabilities associated with the four different scenarios.

The weights of the processed inventory for each of the four scenarios are

$$\text{Scenario 1:} \quad xyw_1(1-a)(1-u_1)$$

$$\text{Scenario 2:} \quad xyw_1(1-a)u_1$$

$$\text{Scenario 3:} \quad xyw_1au_2$$

$$\text{Scenario 4:} \quad xyw_1a(1-u_2)$$

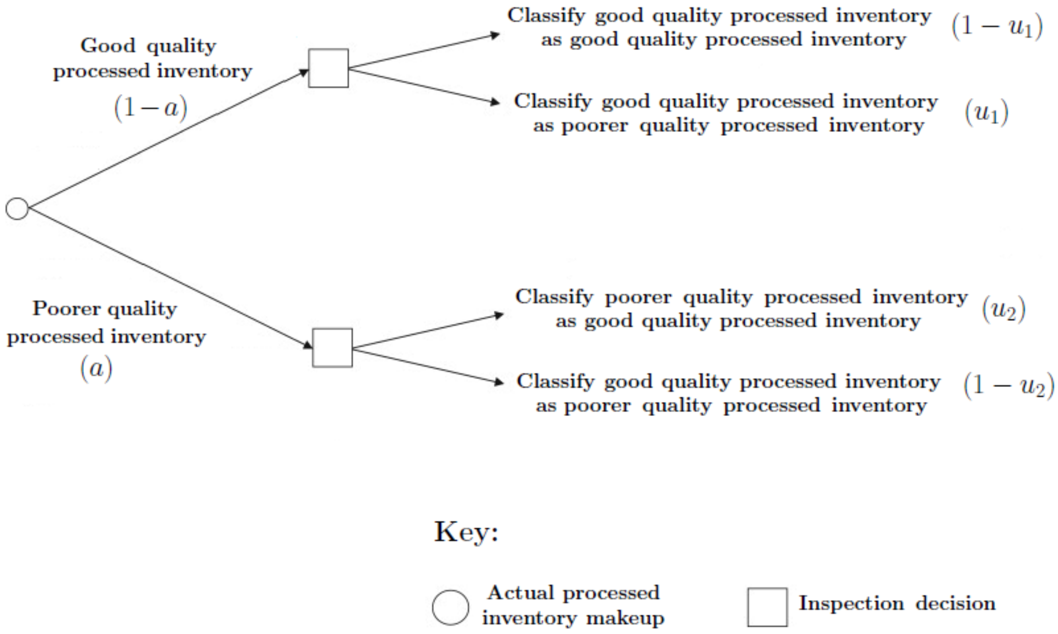


Figure 4.5.5: The probabilities correlated with each of the four possible inspection scenarios

The sum of the weights in Scenarios 1 and 3 represents the weight of the processed inventory that is classified as being of good quality in each inspection run, with Scenario 1 being correctly classified and Scenario 3 being misclassified. From this sum, the processor delivers n_s shipments to the retailer at equally-spaced time intervals of τ. This means that the weight of each batch delivered to the retailer after τ time units, is

$$s^{'} = \frac{xyw_1(1-a)(1-u_1) + xyw_1au_2}{n_s}. \tag{4.5.12}$$

Likewise, the sum of the weights in Scenarios 2 and 4 represents the weight of the processed inventory that is classified as being of poor quality in each inspection run, with Scenario 2 being misclassified and Scenario 4 being correctly classified. The processor lets this inventory accumulate throughout the inspection run and sells it to secondary markets (at a discounted price) at the end of each inspection run. This means that every τ time units, the processor allows

$$s^{''} = \frac{xyw_1(1-a)u_1 + xyw_1a(1-u_2)}{n_s}, \tag{4.5.13}$$

weight units of processed inventory classified as being of poorer quality to accumulate. At the end of each inspection run, the processor would have let n_s batches to accumulate and these are sold simultaneously as a single batch at a discounted price to secondary markets. This means that the weight of each batch sold by the processor to secondary markets at the end of each inspection run is $xyw_1(1-a)u_1 + xyw_1a(1-u_2)$.

The average inventory level of processed inventory in the inspection facility is used to compute the holding costs. The average inventory level is determined by evaluating the area under the processor's inventory system profile at the screening facility, as depicted in Figure 4.5.4. The area under Figure 4.5.4 is given by

$$\text{Area}_s = \frac{x^2y^2w_1^2}{z} - \frac{\left[(n_s-1)x^2y^2w_1^2\right]\left[(1-a)(1-u_1)+au_2\right]}{2n_sz}. \tag{4.5.14}$$

The total cost incurred by the processor in each inspection run is made up of the cost of transferring a batch to the retailer every τ time units, the inspection cost, the cost of and the holding cost.

$$TC_s = n_s K_s + vxyw_1 + l_s xyw_1(1-a)u_1 + h_s\left\{\frac{x^2y^2w_1^2}{z} - \frac{\left[(n_s-1)x^2y^2w_1^2\right]\left[(1-a)(1-u_1)+au_2\right]}{2n_s z}\right\}. \quad (4.5.15)$$

The first term in Equation (4.5.15) represents the transfer cost. This cost is included because it is assumed that the processor sends equally-sized batches of processed inventory classified as being of good quality to the retailer at time intervals of τ during a single inspection run, while the retailer's cycle time (defined at successive time intervals at which the retailer's processed inventory level reaches zero) is T. The processor does not wait for the retailer's inventory to reach zero before sending a batch of good quality processed inventory. The processor sends n_s batches during the retailer's cycle time of T. Hence, the processor incurs the cost of sending batches before the retailer's inventory level reaches zero. The transfer cost is computed by multiplying the number of batched sent to the retailer during the retailer's cycle time (n_s) and the fixed cost of sending a single batch (K_s). The second term in Equation (4.5.15) represents the inspection cost per inspection run and it is computed as the product of the cost of inspecting a single weight unit of inventory (v) and the weight of the processed inventory inspected per inspection run (xyw_1). The third term in Equation (4.5.15) is the cost of rejecting good quality processed inventory per inspection run. This cost is determined as the product of the cost of rejecting a single weight unit (l_s) and the weight of good quality processed inventory classified as poorer quality inventory [$xyw_1(1-a)u_1$]. The last term in Equation (4.5.15) is simply the holding cost per inspection run and it is the product of the holding cost per weight unit in the inspection facility (h_s) and the average inventory level in the inspection facility as given in Equation (4.5.14).

It should be noted that the total cost in Equation (4.5.15) is the cost incurred per inspection run. Since the processor sends n_p batches of processed inventory from the processing plant to the inspection warehouse in each processing run, the total cost incurred at the inspection facility per processing run is thus

$$TC_s = n_p n_s K_s + n_p vxyw_1 + n_p l_s xyw_1(1-a)u_1 + n_p h_s\left\{\frac{x^2y^2w_1^2}{z} - \frac{\left[(n_s-1)x^2y^2w_1^2\right]\left[(1-a)(1-u_1)+au_2\right]}{2n_s z}\right\}. \quad (4.5.16)$$

4.5.3.2.3 Profit generated by the processor

During each processing setup, the processor generates revenue from the sale of both good and poorer quality processed inventory. The revenue is given by

$$TR_p = n_p p_p xyw_1(1-a)(1-u_1) + n_p p_p xyw_1 au_2 + n_p p_q xyw_1(1-a)u_1 + n_p p_q xyw_1 a(1-u_2). \quad (4.5.17)$$

The first two terms in Equation (4.5.17) represent the revenue from sales of processed inventory classified as good quality following the inspection process. The weights of the processed inventory for these two terms correspond to Scenarios 1 and 3. Multiplying these weights [$xyw_1(1-a)(1-u_1)$ and xyw_1au_2, respectively] by the cost that the processor charges the retailer for each weight unit of processed inventory classified as good quality (p_p) yields the revenue from good quality inventory per inspection run. Since the are n_p inspection runs in each processing run, these terms are multiplied by the number of inspection runs (or batches of processed inventory sent from the processing facility to the inspection facility). The third and the fourth terms in Equation (4.5.17) represent the revenue from sales of processed inventory classified as being of poorer quality following the inspection process. The revenue per processing run is determined using the weights processed inventory in Scenarios 2 and 4 and the cost that the processor charges the secondary markets for each weight unit of processed inventory classified as poorer quality (p_q) inventory.

The processor's total profit per cycle (i.e. TP_p) is equal to the processor's total revenue per cycle, as given in Equation (4.5.17), less the processor's total costs per cycle, which is the sum of Equations (4.5.9) and (4.5.16), and thus,

$$TP_p = n_p p_p xyw_1(1-a)(1-u_1) + n_p p_p xyw_1 au_2 + n_p p_q xyw_1(1-a)u_1 + n_p p_q xyw_1 a(1-u_2)$$
$$p_f n_p xyw_1 - K_p - h_p \left[\frac{\left(n_p xyw_1\right)^2}{2R} - \frac{n_p(n_p-1)\left(n_p xyw_1\right)^2}{2} \left(\frac{1}{D} - \frac{1}{R} \right) \right] - n_p n_s K_s - n_p vxyw_1$$
$$- n_p l_s xyw_1(1-a)u_1 - n_p h_s \left\{ \frac{x^2y^2w_1^2}{z} - \frac{\left[(n_s-1)x^2y^2w_1^2\right]\left[(1-a)(1-u_1)+au_2\right]}{2n_s z} \right\}. \tag{4.5.18}$$

The processor' total profit per unit time, TPU_p, is computed by dividing the processor's total profit per cycle, as given in Equation (4.5.18), by the duration of the processor's replenishment cycle (i.e. n_pT) and hence,

$$TPU_p = \frac{p_p xyw_1(1-a)(1-u_1)}{T} + \frac{p_p xyw_1 au_2}{T} + \frac{p_q xyw_1(1-a)u_1}{T} + \frac{p_q xyw_1 a(1-u_2)}{T}$$
$$- \frac{p_f xyw_1}{T} - \frac{K_p}{n_pT} - \frac{h_p}{n_pT} \left[\frac{\left(n_p xyw_1\right)^2}{2R} + \frac{n_p(n_p-1)\left(n_p xyw_1\right)^2}{2} \left(\frac{1}{D} - \frac{1}{R} \right) \right] - \frac{n_s K_s}{T} - \frac{vxyw_1}{T}$$
$$- \frac{l_s xyw_1(1-a)u_1}{T} - \frac{h_s}{T} \left\{ \frac{x^2y^2w_1^2}{z} - \frac{\left[(n_s-1)x^2y^2w_1^2\right]\left[(1-a)(1-u_1)+au_2\right]}{2n_s z} \right\}. \tag{4.5.19}$$

4.5.3.3 Profit generated by the retailer

The retailer receives batches of processed inventory classified as good quality inventory from the processor's inspection facility every τ time units, with the weight of each of batch received given in Equation (4.5.12). The profile for the retailer's processed inventory is depicted by Figure 4.5.6. The retailer receives n_s batches of processed inventory during the course of a single replenishment cycle, defined at successive time intervals at which the retailer's processed inventory level reaches zero, of duration T. This means that the weight of processed inventory received during the retailer's replenishment cycle, b', is determined by multiplying Equation (4.5.12) by n_s. Hence,

$$b' = xyw_1(1-a)(1-u_1) + xyw_1 au_2. \tag{4.5.20}$$

All the processed inventory in Equation (4.5.20) was classified as being of good quality through the inspection process. Because it is assumed that the processor's inspection process is prone to errors, some of the processed inventory gets misclassified. As a result, some of the processed inventory used to fulfil end consumer demand throughout the retailer's cycle T would be of poorer quality. It is implicitly assumed that consumers are capable of judging the quality of the processed inventory and if they receive poorer quality processed inventory, they return it to the retailer who replaces them with good quality processed inventory. The poorer quality processed inventory sold to consumers that is later returned to the retailer is shown as b'' in Figure 4.5.6. b'' is essentially the portion of processed in Equation (4.5.20) that is incorrectly classified, and thus,

$$b'' = xyw_1au_2. \tag{4.5.21}$$

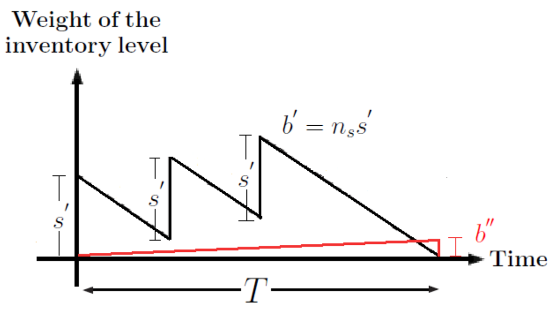


Figure 4.5.6: The retailer's inventory system profile at the retail/consumption facility

The incorrectly classified processed inventory is returned throughout T at a rate of xyw_1au_2/T. In order to avoid shortages, it is assumed that the weight of the processed inventory received by the retailer is at least equal to the adjusted demand for good quality processed inventory. The adjusted demand is the sum of the weight of the actual demand for good quality processed inventory (i.e. DT) and the weight of used to replace the poorer quality inventory that is returned from the market over the internal T (i.e. xyw_1au_2). Therefore,

$$xyw_1(1-a)(1-u_1) + xyw_1au_2 \geq DT + xyw_1au_2$$
$$xyw_1(1-a)(1-u_1) \geq DT$$

Hence, for the limiting case, the duration of the retailer's cycle time can be computed as

$$T = \frac{xyw_1(1-a)(1-u_1)}{D} \tag{4.5.22}$$

In order to determine the average inventory level at the retailer's facility, which is used to compute the retailer's holding cost, the area under the retailer's inventory system profile as given in Figure 4.5.6 is evaluated. Figure 4.5.6 is redrawn into Figure 4.5.7 (without the returned poorer quality processed inventory) for ease of computation, a method first used by Konstantaras et al. (2007), and it follows that

$$\begin{aligned}\text{Area}_r &= \left[\text{Area of triangle DEF}\right] - \left[\frac{n_s(n_s-1)}{2}(\text{Area of parallelogram ABCD})\right]\\ &= \left[\frac{xyw_1(1-a)(1-u_1)T}{2}\right] - \left[\frac{n_s(n_s-1)\tau s'}{2}\right]\\ &= \left[\frac{xyw_1(1-a)(1-u_1)T}{2} - \frac{(n_s-1)x^2y^2w_1^2(1-a)(1-u_1)}{2n_sz} - \frac{(n_s-1)x^2y^2w_1^2au_2}{2n_sz}\right].\end{aligned} \tag{4.5.23}$$

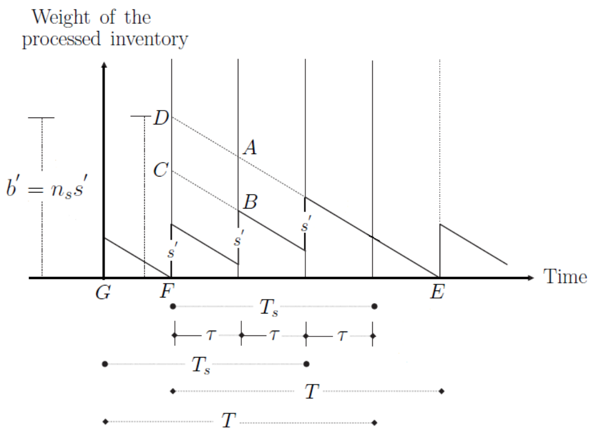


Figure 4.5.7: A redrawn version, adapted from Konstantaras et al. (2007) for ease of computation, of the retailer's inventory system profile

The retailer has two revenue streams in each replenishment cycle. The first one is from the sales of good quality processed inventory that was correctly classified during the inspection process. The second revenue stream is from the sales of the poorer quality processed inventory that is returned from the market because it was incorrectly classified during the inspection process. It is assumed that the retailer offers customers a full refund if they are sold incorrectly classified processed inventory and hence, the retailer does not generate any revenue for selling the misclassified inventory. The retailer's profit per cycle is thus

$$TR_r = p_r xyw_1(1-a)(1-u_1) + p_q xyw_1 au_2. \tag{4.5.24}$$

The first term in Equation (4.5.24) is the revenue from sales of correctly classified processed inventory which is sold at a price of p_r while the second term represents revenue from the sales of returned poorer quality processed inventory which is sold to secondary markets as a single batch and at a discounted price of p_q.

The retailer's profit per cycle is the revenue from the two streams less the total costs (made up of the purchasing cost, the fixed ordering cost, the cost of accepting poorer quality inventory as good quality inventory and the holding cost), and hence,

$$TP_r = p_r xyw_1(1-a)(1-u_1) + p_q xyw_1 au_2 - p_p xyw_1(1-a)(1-u_1) - p_p xyw_1 au_2 - K_r - l_r xyw_1 au_2 \\ - h_r \left\{ \frac{xyw_1(1-a)(1-u_1)T}{2} - \frac{\left[(n_s-1)x^2y^2w_1^2\right]\left[(1-a)(1-u_1)+au_2\right]}{2n_s z} \right\} \tag{4.5.25}$$

The first two terms in Equation (4.5.25) represent the revenue from the two streams as computed in Equation (4.5.24). The third and the fourth terms in Equation (4.5.25) represent the purchasing cost which is determined by multiplying the weights of the processed inventory received from the processor (i.e. both the correctly classified inventory and the incorrectly classified inventory which are $xyw_1(1-a)(1-u_1)$ and xyw_1au_2, respectively) and the purchasing cost charged by the processor (p_p). The fifth term in Equation (4.5.25) is simply the fixed cost of placing an order at the beginning of each replenishment cycle (K_r). The sixth term in Equation (4.5.25) is the cost of accepting poorer quality processed inventory per cycle. This cost is determined as the product of the cost of accepting a single weight unit of poorer quality inventory (l_r) and the weight of poorer quality processed inventory classified as good quality inventory [xyw_1au_2]. The last term in Equation (4.5.25) is simply the holding cost per cycle and it is the product of the holding cost per weight unit in the retailer's facility (h_r) and the average inventory level in the retail outlet as determined in Equation (4.5.23).

The retailer' total profit per unit time, TPU_r, is computed by dividing the retailer's total profit per cycle, as given in Equation (4.5.25), by the duration of the retailer's replenishment cycle (i.e. T) and thus,

$$TPU_r = p_r D + \frac{p_q a u_2 D}{(1-a)(1-u_1)} - p_p D - \frac{p_p a u_2 D}{(1-a)(1-u_1)} - \frac{K_r D}{xyw_1(1-a)(1-u_1)} - \frac{l_r a u_2}{(1-a)(1-u_1)} - h_r \left\{ \frac{xyw_1(1-a)(1-u_1)}{2} - \frac{\left[(n_s-1)xyw_1 D\right]\left[(1-a)(1-u_1)+au_2\right]}{2n_s z(1-a)(1-u_1)} \right\} \quad (4.5.26)$$

4.5.3.4 Profit generated across the supply chain

The total supply chain profit generated across the supply chain per unit time, TPU_{sc}, is determined by summing Equations (4.5.26), (4.5.19) and (4.5.7). Since it is assumed that the probabilities of survival (x), type I errors (u_1) and type II errors (u_2) and the fraction of poorer quality processed inventory (a) are random variables with known probability density functions, given by $f(x)$, $f(u_1)$, $f(u_2)$ and $f(a)$, respectively, the expected value of the supply chain profit, $E[TPU_{sc}]$, is thus defined as

$$\begin{aligned}
E[TPU_{sc}] &= p_r D + \frac{p_q E[a]E[u_2]D}{E[1-a]E[1-u_1]} - \frac{K_r D}{E[x]yw_1 E[1-a]E[1-u_1]} - \frac{l_r E[a]E[u_2]}{E[1-a]E[1-u_1]} \\
&- \frac{h_r D}{E[1-a]E[1-u_1]} \left\{ \frac{E[x]yw_1}{2} - \frac{\left\{(n_s-1)E[x]yw_1 D\right\}\left\{E[1-a]E[1-u_1]+E[a]E[u_2]\right\}}{2n_s z} \right\} \\
&+ \frac{p_q E[u_1]D}{E[1-u_1]} + \frac{p_q E[a]E[1-u_2]D}{E[1-a]E[1-u_1]} - \frac{K_p D}{n_p E[x]yw_1 E[1-a]E[1-u_1]} - \frac{n_s K_s D}{E[x]yw_1 E[1-a]E[1-u_1]} \\
&- \frac{h_p D}{E[1-a]E[1-u_1]} \left\{ \frac{n_p E[x]yw_1}{2R} + \frac{n_p(n_p-1)n_p E[x]yw_1}{2}\left(\frac{1}{D}-\frac{1}{R}\right) \right\} \\
&- \frac{h_s D}{E[1-a]E[1-u_1]} \left\{ \frac{E[x]yw_1}{z} - \frac{\left\{(n_s-1)E[x]yw_1 D\right\}\left\{E[1-a]E[1-u_1]+E[a]E[u_2]\right\}}{2n_s z E[1-a]E[1-u_1]} \right\} \\
&- \frac{l_s E[u_1]D}{E[1-u_1]} - \frac{vD}{E[1-a]E[1-u_1]} - \frac{K_f D}{n_p E[x]yw_1 E[1-a]E[1-u_1]} \\
&- \frac{\left\{c_f E[x] + m_f E[1-x]\right\}D}{n_p E[x]w_1 E[1-a]E[1-u_1]} \left\{ \alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln(1+\beta)\right] \right\} \\
&- \frac{p_v w_0 D}{E[x]w_1 E[1-a]E[1-u_1]}
\end{aligned} \quad (4.5.27)$$

The retailer's optimal lot size, y, is determined by setting the first derivative of $E[TPU_{sc}]$, as given in Equation (4.5.27), with respect to y to zero and solving for y. The first derivative of $E[TPU_{sc}]$ with respect to y is given in Equation (4.5.29). The result is

$$y = \sqrt{\frac{2D\left(K_r + \frac{K_p}{n_p} + n_s K_s + \frac{K_f}{n_p}\right)}{\left[h_r(\gamma_1) + h_p(\gamma_2) + h_s(\gamma_3)\right]E[x^2]w_1^2}}, \quad (4.5.28)$$

where

$$\gamma_1 = E[1-a]^2 E[1-u_1]^2 - \frac{\left[(n_s-1)D\right]\left\{E[1-a]E[1-u_1]+E[a]E[u_2]\right\}}{n_s z},$$

$$\gamma_2 = n_p \left[\frac{D}{R} + (n_p - 1) \left(\frac{1}{D} - \frac{1}{R} \right) \right],$$

and

$$\gamma_3 = \frac{2D}{z} - \frac{\left[(n_s - 1)D \right] \left\{ E[1-a]E[1-u_1] + E[a]E[u_2] \right\}}{n_s z}.$$

The tractability of the proposed inventory system is dependent on the imposition of two constraints while the feasibility of the system is dependent on the imposition of an additional constraint. The two constraints that ensure the tractability are that the number of shipments of processed inventory transferred from the processing facility to the inspection facility per processing cycle and the number of shipments of good quality processed inventory (as classified by the inspection process which is prone to errors) delivered from the inspection facility to the retail facility per inspection cycle should be integers (i.e. $n_p, n_s \in \mathbb{Z}$). These two constraints make the solution procedure tractable. The third constraint, which ensures the feasibility of the solution obtained, is that the duration of the farmer's growth period should be less than or equal to the duration of the processor's cycle time (i.e. $T_f \leq n_p T$). This ensures that the solution to the problem is feasible by assuring that the weight of the live items has reached the maturity weight at the start of the processing cycle which means that the items will be ready for processing at that time.

4.5.3.5 Solution procedure

An iterative procedure is used to compute the optimal values of y, n_p and n_s. The procedure is made up of two sub-processes, namely Steps 1 and 2. Step 1 is aimed at finding the optimal value of n_p while Step 2 is aimed at optimising y and n_s. The procedure is as follows:

Step 1 Set n_s to 1.

Step 1.1 Set n_p to 1.

Step 1.2 Compute the values of y and $E[TPU_{sc}]$ using Equations (4.5.28) and (4.5.27), respectively.

Step 1.3 Increase n_p by 1 and values of y and $E[TPU_{sc}]$ using Equations (4.5.28) and (4.5.27), respectively. Carry on to Step 1.4.

Step 1.4 If the latest value of $E[TPU_{sc}]$ increases, go back to Step 1.3. If the value of $E[TPU_{sc}]$ decreases, the previously calculated value of $E[TPU_{sc}$ (along with corresponding y and n_p values) is the best solution so far and if this is the case, carry on to Step 2. The corresponding n_p value is the optimal value.

Step 2 Set $n_s$to 2.

Step 2.1 Compute the values of y and $E[TPU_{sc}]$ using Equations (4.5.28) and (4.5.27), respectively.

Step 2.2 Increase n_s by 1 and calculate the values of y and $E[TPU_{sc}]$ using Equations (4.5.28) and (4.5.27), respectively. Carry on to Step 2.3.

Step 2.3 If the latest value of $E[TPU_{sc}]$ increases, go back to Step 2.2. If the value of $E[TPU_{sc}]$ decreases, the previously calculated value of $E[TPU_{sc}$ (along with corresponding y and n_s values) is the best solution so far and if this is the case, carry on to Step 2.4.

Step 2.4 Verify the solution's feasibility with regard to the feasibility constraint $T_f \leq n_p T$. If the solution is feasible, those values of y and n_s are optimal and if this is the case, carry on to Step 2.6. If the solution is not feasible, carry on to Step 6.

Step 2.5 If the constraint is violated, set T to T_f/n_p and use that T value to calculate new y and $E[TCU_{sc}]$ values using Equations (4.5.22) and (4.5.27), respectively, and then carry on to Step 2.6.

Step 2.6 End.

4.5.4 Theoretical results

In order to show that there are unique values of the model's three decision variables, namely, y, n_p and n_s, that maximise the model's objective function, $E[TPU_{sc}]$, it is necessary to demonstrate that $E[TPU_{sc}]$ is concave. The concavity of $E[TPU_{sc}]$ is explored in two ways. In the first way, it is proven that $E[TPU_{sc}]$ is a concave function of y for fixed values of n_p and n_s. Secondly, it is proven that $E[TPU_{sc}]$ is a concave function of n_p and n_s for a fixed value of y. Together, these pair of results not only demonstrate that $E[TPU_{sc}]$ is concave, but they also prove the existence of unique y, n_p and n_s values that maximise $E[TPU_{sc}]$.

Theorem 4.5.1. *$E[TPU_{sc}]$ is a concave function of y for all $n_p > 0$ and $n_s > 0$. Therefore, there exists a unique value of y that maximises $E[TPU_{sc}]$.*

Proof. For fixed values of n_p and n_s, the first and second derivatives of $E[TPU_{sc}]$, as given in Equation (4.5.27), with respect to y are

$$\begin{aligned}\frac{\partial E[TPU_{sc}]}{\partial y} &= \frac{K_r D}{E[x]y^2 w_1 E[1-a]E[1-u_1]} + \frac{K_p D}{n_p E[x]y^2 w_1 E[1-a]E[1-u_1]} \\ &\quad + \frac{n_s K_s D}{E[x]y^2 w_1 E[1-a]E[1-u_1]} + \frac{K_f D}{n_p E[x]y^2 w_1 E[1-a]E[1-u_1]} \\ &- \frac{h_r D}{E[1-a]E[1-u_1]}\left\{\frac{E[x]w_1}{2} - \frac{\{(n_s-1)E[x]w_1 D\}\{E[1-a]E[1-u_1]+E[a]E[u_2]\}}{2n_s z}\right\} \\ &- \frac{h_s D}{E[1-a]E[1-u_1]}\left\{\frac{E[x]w_1}{z} - \frac{\{(n_s-1)E[x]w_1 D\}\{E[1-a]E[1-u_1]+E[a]E[u_2]\}}{2n_s z E[1-a]E[1-u_1]}\right\} \\ &\quad - \frac{h_p D}{E[1-a]E[1-u_1]}\left\{\frac{n_p E[x]w_1}{2R} + \frac{n_p(n_p-1)n_p E[x]w_1}{2}\left(\frac{1}{D}-\frac{1}{R}\right)\right\} \qquad (4.5.29)\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 E[TPU_{sc}]}{\partial y^2} &= -\frac{K_r D}{E[x]y^3 w_1 E[1-a]E[1-u_1]} - \frac{K_p D}{n_p E[x]y^3 w_1 E[1-a]E[1-u_1]} \\ &\quad - \frac{n_s K_s D}{E[x]y^3 w_1 E[1-a]E[1-u_1]} - \frac{K_f D}{n_p E[x]y^3 w_1 E[1-a]E[1-u_1]} < 0 \qquad (4.5.30)\end{aligned}$$

From Equation (4.5.30), it is obvious that $E[TPU_{sc}]$ is a concave function of y for any fixed values of $n_p > 0$ and $n_s > 0$ because the second derivative of $E[TPU_{sc}]$ with respect to y is negative. This implies that a unique y value that maximises $E[TPU_{sc}]$ exists. □

Theorem 4.5.2. *For all $y > 0$, $E[TPU_{sc}]$ is a concave function of both n_p and n_s. Therefore, there exists unique values of n_p and n_s that maximise $E[TPU_{sc}]$.*

Proof. For a fixed value of y, the first and second derivatives of $E[TPU_{sc}]$, as given in Equation (4.5.27), with respect to n_p and n_s are

$$\frac{\partial E[TPU_{sc}]}{\partial n_p} = \frac{K_p D}{{n_p}^2 E[x]yw_1 E[1-a]E[1-u_1]} + \frac{K_f D}{{n_p}^2 E[x]yw_1 E[1-a]E[1-u_1]} + \frac{h_p D}{E[1-a]E[1-u_1]}\left\{\frac{E[x]yw_1}{2R} + \frac{(n_p-1)n_p E[x]yw_1}{2}\left(\frac{1}{D}-\frac{1}{R}\right)\right\} \quad (4.5.31)$$

$$\frac{\partial^2 E[TPU_{sc}]}{\partial {n_p}^2} = -\frac{K_p D}{{n_p}^3 E[x]yw_1 E[1-a]E[1-u_1]} - \frac{K_f D}{{n_p}^3 E[x]yw_1 E[1-a]E[1-u_1]} - \frac{h_p D}{E[1-a]E[1-u_1]}\left\{\frac{(n_p-1)E[x]yw_1}{2}\left(\frac{1}{D}-\frac{1}{R}\right)\right\} \quad (4.5.32)$$

$$\frac{\partial E[TPU_{sc}]}{\partial n_s} = -\frac{K_s D}{E[x]yw_1 E[1-a]E[1-u_1]} + \frac{h_s D}{E[1-a]E[1-u_1]}\left\{\frac{E[x]yw_1 D\{E[1-a]E[1-u_1]+E[a]E[u_2]\}}{2{n_s}^2 zE[1-a]E[1-u_1]}\right\} \quad (4.5.33)$$

$$\frac{\partial^2 E[TPU_{sc}]}{\partial {n_s}^2} = -\frac{h_s D}{E[1-a]E[1-u_1]}\left\{\frac{E[x]yw_1 D\{E[1-a]E[1-u_1]+E[a]E[u_2]\}}{2{n_s}^3 zE[1-a]E[1-u_1]}\right\} \quad (4.5.34)$$

$$\frac{\partial^2 E[TPU_{sc}]}{\partial n_p \partial n_s} = 0 \quad (4.5.35)$$

The quadratic form of the Hessian matrix of $E[TPU_{sc}]$ is defined as

$$\begin{bmatrix} n_p & n_s \end{bmatrix} \begin{bmatrix} \frac{\partial^2 E[TPU_{sc}]}{\partial {n_p}^2} & \frac{\partial^2 E[TPU_{sc}]}{\partial n_p \partial n_s} \\ \frac{\partial^2 E[TPU_{sc}]}{\partial n_p \partial n_s} & \frac{\partial^2 E[TPU_{sc}]}{\partial {n_s}^2} \end{bmatrix} \begin{bmatrix} n_p \\ n_s \end{bmatrix}, \quad (4.5.36)$$

which upon further expansion results in

$$-\frac{K_p D}{n_p E[x]yw_1 E[1-a]E[1-u_1]} - \frac{K_f D}{n_p E[x]yw_1 E[1-a]E[1-u_1]} - \frac{h_s D}{E[1-a]E[1-u_1]}\left\{\frac{E[x]yw_1 D\{E[1-a]E[1-u_1]+E[a]E[u_2]\}}{2{n_s}^3 zE[1-a]E[1-u_1]}\right\} < 0. \quad (4.5.37)$$

From Equation (4.5.37), it is obvious that $E[TPU_{sc}]$ is a concave function of $n_p > 0$ and $n_s > 0$ for any fixed value of y because the quadratic form of the Hessian matrix of $E[TPU_{sc}]$ is negative. This implies that unique n_p and n_s values that maximise $E[TPU_{sc}]$ exist. □

4.5.5 Numerical results

4.5.5.1 Numerical example

As a way of demonstrating the potential practical applications of the proposed inventory management policy, an example which considers the entire chicken production value chain with a farming facility, a processing facility, an inspection facility and a retail facility is solved. The retailer meets end consumer demand for good quality processed chicken. The example considers the following input parameters: D=250 kg/week; R=300 kg/week; w_0= 8.5 kg; w_1=30 kg; K_r=2 500 ZAR; h_r=1 ZAR/kg/week; p_r=50 ZAR/kg; l_r=500 ZAR/kg; K_p=25 000 ZAR; h_p=0.5 ZAR/kg/week; p_p=30 ZAR/kg; K_s=200 ZAR; h_s=0.5 ZAR/kg/week; p_q=20 ZAR/kg; v=0.5 ZAR/kg; z=1 000 kg/week; l_s=200 ZAR/kg; K_f=30 000 ZAR; p_f=15 ZAR/kg; c_f=1 ZAR/kg/week; m_f=2 ZAR/kg/week; p_v=10 ZAR/kg; α=51 kg; β=5; λ=0.12 /week. x, u_1, u_2 and a are assumed to be random variables uniformly distributed over [0.8, 1], [0, 0.05], [0, 0.05] and [0, 0.05], respectively. Their probability density functions are given by

$$f(x) = \begin{cases} 5, & 0.8 \leq x \leq 1 \\ 1, & \text{otherwise.} \end{cases}$$

$$f(a) = \begin{cases} 32, & 0 \leq a \leq 0.05 \\ 0, & \text{otherwise.} \end{cases}$$

$$f(u_1) = \begin{cases} 32, & 0 \leq u_1 \leq 0.05 \\ 0, & \text{otherwise.} \end{cases}$$

$$f(u_2) = \begin{cases} 32, & 0 \leq u_2 \leq 0.05 \\ 0, & \text{otherwise.} \end{cases}$$

This implies that

$$E[x] = \int_{0.8}^{1} 5x \quad dx = 5\left[\frac{(1^2 - 0.8^2)}{2}\right] = 0.9$$

Likewise,

$$E[a] = E[u_1] = E[u_2] = 32\left[\frac{(0.05^2 - 0^2)}{2}\right] = 0.04$$

The results from the example, solved using the Microsoft Excel add-in Solver , are presented in Table 4.5.1. From the results, in order to maximise the expected total supply chain profit, the farmer should place an order for ($n_p y \approx$) 227 live newborn chicks at the beginning of each growing cycle. The total weight of the live newborn inventory items ($n_p Q_0 = n_p y w_0$) would be roughly 1 926 kg at the time of purchase. Once items mature (i.e. the weight of each item reaches $w_1 = 30$ kg), the farmer should ship them to the processor for processing and inspection. About ($E[x] =$) 90% of the initially ordered orders survive until the end of the growth cycle, this means that by the time the farmer ships the items to the processing facility, the total weight of the live inventory ($n_p Q_1 = n_p x y w_1$) would be roughly 6 118 kg. After the entire lot is processed, the processor transfers it to the inspection facility, in (n_p^*=) 2 batches per processing cycle, where the processed inventory is inspected for quality and separated into good and poorer quality classes. Throughout the inspection process, the processor should deliver (n_s=) 5 batches of good quality inventory from the inspection facility to the retail facility, each weighing ($s^{'}$=) 565 kg. The poorer quality processed inventory in each processing cycle ($n_s s^{''}$) amounts to 235 kg, with $s^{''} \approx$ 47 kg being accumulated in each inspection cycle. The processor should deliver good quality processed inventory from the inspection facility to the retailer every τ= 0.61 weeks. The retailer's inventory level will reach zero every (T=) 11.28 weeks. If this order replenishment and shipment policy is followed, the supply chain should expect to make a profit of about ($E[TPU_{sc}]$ =) 1 836.52 ZAR/week.

Objective function and decision variables	Quantity
$E[TPU^*_{sc}]$	1 836.52 ZAR/week
n^*_p	2 shipments
n^*_s	5 shipments
y^*	113 items
T^*	11.28 weeks

Table 4.5.1: Optimal results from the numerical example

4.5.5.2 Sensitivity analysis

The impact of changes to some of the model's input parameters were tested through a sensitivity analysis, whose results are presented in Table 4.5.2. The following observations from the analysis were notable:

- The retailer's optimal lot size (y^*) is most sensitive to $E[x]$. A 40% decrease in $E[x]$ result in a 66.7% increase in y^*. If an increasing number of items do not survive during the farmer's growth period, obviously more items would need to be ordered in order to meet the specified demand. Changes to all the other input parameters apart from $E[u_2]$, c_f and m_f had minimal effects on y^*. All the changes to $E[u_2]$, c_f and m_f values tested had no effect on y^*.

- Changes to all the input parameters had no effect on the optimal number of shipment transferred from the processing facility to the inspection facility (n^*_p). The number of shipments remained at 2 regardless of the percentage change or input parameter.

- The optimal number of shipments of good quality processed inventory delivered from the inspection facility to the retail facility (n^*_s) remained the same the same as well for most input parameters. It only changed when K_s, K_p, h_p and K_f changed. All changes were quite minimal and limited to 20% with the only exception being a 40% increase in n^*_s as a result of a 40% increase in the value of K_s.

- The expected value of the optimal total supply chain profit $E[TPU^*_{sc}]$ was affected by changes to all the input parameters. However, the severity of the impact different across the parameters. $E[x]$ had the greatest effect while $E[u_2]$ had the least. A 40% increase in $E[x]$ and $E[u_2]$ resulted in a 257.1% increase in $E[TPU^*_{sc}]$ and a 4.8% decrease in $E[TPU^*_{sc}]$, respectively.

Table 4.5.2: Sensitivity analysis of various input parameters

	% change	Items in retailer's lot size (y^*)		Shipments from processing facility (n_p^*)		Shipments from screening facility (n_s^*)		Expected total supply chain profit ($E[TPU_{sc}^*]$)	
		items	% change	shipments	% change	shipments	% change	ZAR/week	% change
Base example		**113**		**2**		**5**		**1 836.52**	
K_r	-40	111	-1.6	2	0	5	0	1 934.90	+5.4
	-20	112	-0.8	2	0	5	0	1 885.51	+2.7
	+20	114	+0.8	2	0	5	0	1 787.92	-2.7
	+40	115	+1.6	2	0	5	0	1 739.71	-5.3
h_r	-40	124	+9.1	2	0	5	0	2 246.60	+22.5
	-20	118	+4.3	2	0	5	0	2 038.18	+11.1
	+20	109	-3.8	2	0	5	0	1 641.05	-10.7
	+40	105	-7.2	2	0	5	0	1 451.27	-21.2
K_s	-40	114	+0.3	2	0	7	+40.0	1 881.68	+2.5
	-20	114	+0.3	2	0	6	+20.0	1 857.66	+1.2
	+20	114	+0.3	2	0	5	0	1 817.03	-1.1
	+40	113	-0.3	2	0	4	-20.0	1 798.98	-2.1
h_s	-40	116	+2.0	2	0	5	0	1 931.39	+5.2
	-20	114	+1.0	2	0	5	0	1 857.66	+2.6
	+20	112	-0.9	2	0	5	0	1 817.03	-2.6
	+40	111	-1.9	2	0	5	0	1 798.98	-5.1
K_p	-40	104	-8.4	2	0	5	0	2 345.87	+28.0
	-20	109	-4.1	2	0	5	0	2 085.60	+13.7
	+20	118	+4.0	2	0	5	0	1 597.29	-13.1
	+40	123	+8.4	2	0	6	+20.0	1 367.40	-25.8
h_p	-40	128	+12.8	2	0	6	+20.0	2 605.58	+42.2
	-20	120	+6.2	2	0	6	+20.0	2 207.22	+20.4
	+20	108	-4.7	2	0	5	0	1 486.94	-19.2
	+40	103	-8.8	2	0	5	0	1 155.16	-37.4
K_f	-40	102	-10.2	2	0	5	0	2 453.47	+33.9
	-20	108	-5.0	2	0	5	0	2 136.71	+16.5
	+20	119	+4.7	2	0	5	0	1 550.53	-15.7
	+40	125	+9.9	2	0	6	+20.0	1 277.96	-30.7
c_f	-40	113	0	2	0	5	0	2 920.20	+59.5
	-20	113	0	2	0	5	0	2 378.36	+29.7
	+20	113	0	2	0	5	0	1 294.68	-29.7
	+40	113	0	2	0	5	0	752.23	-59.5
m_f	-40	113	0	2	0	5	0	2 077.34	+13.2
	-20	113	0	2	0	5	0	1 956.93	+6.6
	+20	113	0	2	0	5	0	1 716.11	-6.6
	+40	113	0	2	0	5	0	1 595.70	-13.2
$E[x]$	-40	189	+66.7	2	0	5	0	-2 746.46	-249.5
	-20	142	+25.0	2	0	5	0	117.91	-93.6
	+20	94	-6.7	2	0	5	0	2 981.81	+62.4
	+40	81	-28.6	2	0	5	0	3 800.65	+106.9
$E[u_1]$	-40	112	-0.7	2	0	5	0	2 099.22	+14.4
	-20	113	-0.4	2	0	5	0	1 969.02	+7.3
	+20	114	+0.4	2	0	5	0	1 701.67	-7.4
	+40	114	+0.7	2	0	5	0	1 564.40	-15.0
$E[u_2]$	-40	113	0	2	0	5	0	1 923.04	+4.8
	-20	113	0	2	0	5	0	1 879.78	+2.4
	+20	113	0	2	0	5	0	1 793.26	-2.4
	+40	113	0	2	0	5	0	1 750.00	-4.8

4.5.6 Concluding remarks

Food production operations are complex industrial systems that involve multiple entities and processes. The entities involved range from farmers, at the upstream end of the supply chain, who are responsible for rearing live items, to processors, who not only process the live items into a form that is suitable for consumption but also inspect the items for quality, and finally to retailers at the downstream end of the supply chain, who are responsible for selling the consumable food products to end consumers. Quality inspection is one of the most important processes in food production systems because consumer health is at stake. However, inspection processes are not perfect and are prone to errors. If these errors are not minimised, the repercussions are not only costly, in terms of liability and lost business, but they also place consumers' health at risk.

This section formulated an inventory model for a multi-echelon supply chain for growing items with imperfect quality and the possibility of committing errors while inspecting the items for quality. The financial impact of committing inspection errors is quantified through numerical results. Even when the probability of committing such errors is small (i.e. for instance, 4% as is the case in the numerical example, the impact on the supply chain profit is quite sizeable. Furthermore, small changes to these probabilities also lead to significant shifts in the supply chain profit. Production managers should aim to keep these errors to a minimum not only for the sake of maximising profits but also for protecting the health of consumers.

The proposed model can be developed further in several ways. For instance, the effects of learning on both processing and inspection operations can be incorporated. Additionally, the effectiveness of adopting different shipment policies, such as power-of-two policies instead of the integer shipment policy, can be explored. The deterministic demand assumption is not realistic and thus, stochastic demand patterns represent one potential area for further exploration.

4.6 A three-echelon supply chain inventory model for growing items with inventory level- and freshness-dependent demand[‡‡]

4.6.1 Introduction

4.6.1.1 Context

Perishables such as meat, fish and fresh produce, account for a significant portion of retail grocery sales (Agi and Soni, 2020). The primary source of perishable food products is growing items such as livestock, grains and crops, to name a few. In most instances, these growing items require some form of transformation process in order to get them into a consumable and saleable form. For example, livestock is slaughtered, processed and packaged prior to being sold while grains are usually harvested, processed and packaged. One of the defining features of most modern business enterprises is a focus on core competencies. Consequently, most activities that are classified as non-core competencies are outsourced. In the context of perishable food production systems, the implication is that more often than not, there are multiple parties involved in different stages of the value chain such as rearing the live items, processing the items and eventually selling them to end users.

Consumer demand for perishable food products is often met by retailers who display these products on shelves. As a result, the demand rate for a perishable product is often affected by its availability on shelves and its shelf life. The product's availability is essentially its inventory level displayed at the retail store and its shelf life is its expiration date which affects its freshness condition.

4.6.1.2 Purpose

The main objective of this section is to develop a model for inventory control in a perishable food production system. For simplicity, it is assumed that there are three distinct stages involved in the system, namely, rearing (or growing), processing and selling carried out by a farmer, a processor and a retailer, respectively.

The first stage of the proposed inventory system is concerned with the growing of live newly born items which are not immune to mortality and therefore the possibility of the items dying during the course of the growing cycle is considered. The mature items are then processed into a form that is safe for consumption. The term processing encompasses all the activities that take place at this stage which might include slaughtering, cleaning, processing and packaging. To keep the model tractable, all these activities are collectively called processing and it is assumed that they collectively take place at a finite rate. The final stage is when the processed (and thus, safe for consumption) inventory is sold to end consumers. The demand rate at this stage is a function of the inventory level and the expiration date of the perishables. The three stages correspond to supply chain echelons.

A secondary objective of this section is to develop an extension of the aforementioned model that relaxes the traditional zero-ending inventory policy at the retail end of the supply chain. Numerous previously published EOQ models that consider an inventory

[‡‡]A modified version of this section has been submitted to *Applied Mathematical Modelling* for review. A revision was requested and the revised manuscript was resubmitted.

level-dependent demand rate have shown that replenishing the inventory once it reaches a certain minimum level, as opposed to doing so when it reaches zero in most traditional EOQ models, can improve profitability. For this reason, the extension assumes that the retailer holds a clearance sale at the end of the cycle when the inventory reaches a specific minimum level.

4.6.1.3 Relevance

This section accounts for a number of important issues, from an inventory control perspective, in perishable food production systems. The first of these issues is that at the retail echelon, final consumer demand is a function of both the level of inventory and the expiration date of the perishables. This is important for two reasons, firstly, expiration dates are important determinants of product freshness which itself affects consumers' purchasing behaviour. Secondly, while higher inventory levels have been shown to encourage consumers' purchasing behaviour, it is important to keep the right amount of inventory because it incurs relatively expensive holding costs. The second issue is the possibility of item mortality at the growing echelon. This is an important issue because the inventory items of interest are living organisms and are therefore not immortal. The third issue pertains to investigating the benefits (or lack thereof) of the retailer adopting a non-zero ending inventory policy as a way of improving profitability. This is particularly important for perishable food products because they can not be sold once their expiration dates have elapsed so it might be beneficial to hold a clearance sale at the end of the replenishment cycle.

The proposed inventory system presents a realistic inventory control mechanism in perishable food supply chains which are often comprised of multiple echelons with various issues at each of those echelons. By accounting for all these factors, the results from this section have the potential to guide production managers when making procurement and shipment decisions in perishable food supply chains.

4.6.1.4 Organisation

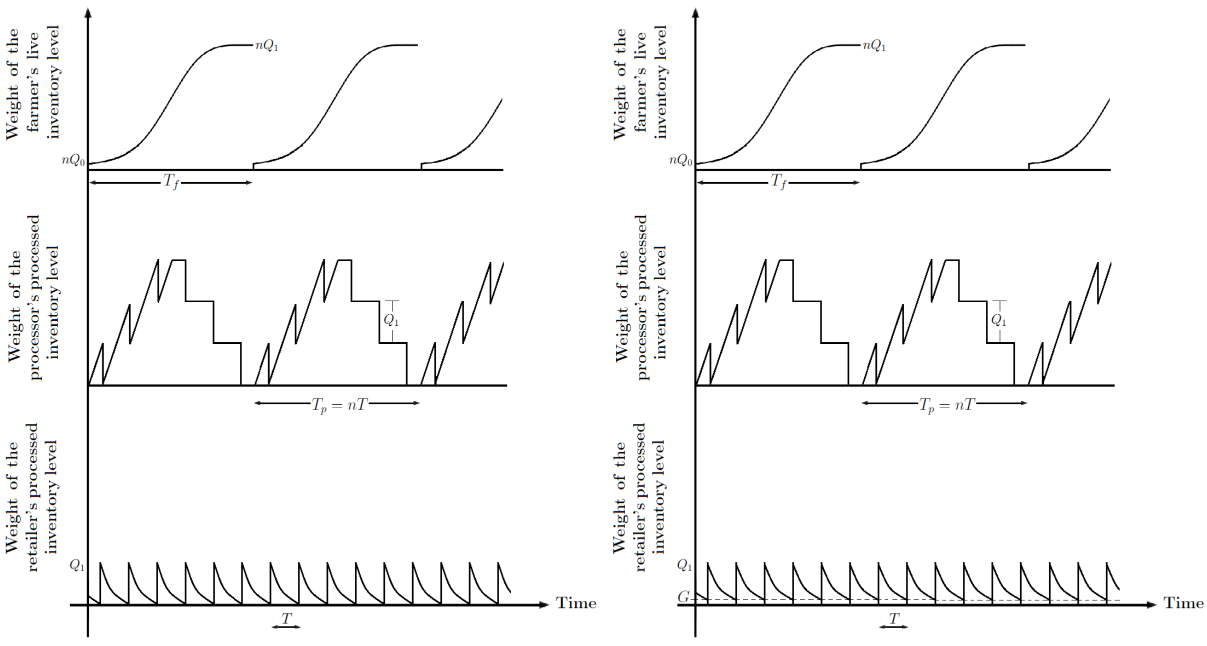
Besides the introduction, this section has five more subsections. The introduction is followed by the assumptions used to develop the model, which are presented in Subsection 4.6.2. Subsequently, the model development phase follows in Subsection 4.6.3. The zero-ending inventory assumption (at the retail echelon) made in the model is relaxed in Subsection 4.6.4 as a way of improving profitability. The results from two numerical examples, from which important managerial insights are drawn, are presented in Subsection 4.6.5. The section is then wrapped up, through the provision of concluding remarks and suggestions for further development of the proposed model, in Subsection 4.6.6.

4.6.2 Problem description

The proposed inventory control systems, depicted in Figures 4.6.1a and 4.6.1b, considers one farmer, one processor and one retailer involved in the rearing (i.e. growing) of live items, the transformation of the live items into consumable processed items and the selling of the processed items, respectively. The farmer tracks the live inventory while both the processor and the retailer track the processed inventory, with the latter tracking it at the retail outlet where it is used to meet consumer demand and the former tracking it at the processing plant as the live inventory is slaughtered, prepared and packaged (i.e. processed) into processed inventory.

The main objective of this section is to develop a model for inventory control in a three-echelon supply chain for growing items with a demand rate that is dependent on the inventory level and the expiration date. A secondary objective of this section is to extend the model to a case where a non-zero ending inventory policy is adopted as a way of improving profitability. Figure 4.6.1a shows the proposed inventory system with the traditional zero-ending inventory policy at the retail echelon. This addresses the main objective of this section. Figure 4.6.1b, showing the inventory profile when a non-zero ending inventory policy is adopted at the retail echelon, is an an extension of the model represented in Figure 4.6.1a and it is used to investigate the profit enhancement mechanism.

From Figures 4.6.1a and 4.6.1b, it should be noted that the delivery policy adopted by the farmer and the processor are different due to the nature of the inventory they track and deliver. The farmer, who delivers live inventory to the processor, makes use of a SSSD policy whereby the farmer delivers a single shipment of mature live items to the processor for each farming cycle. This is because live items often require considerable amount of time to grow. On the other hand, the processor, who delivers processed inventory to the retailer, adopts a SSMD policy. The reason being that the processed inventory is assumed to have a specified maximum shelf life (expiration date) and in order to avoid the inventory going bad and losing all utility (i.e. expiring), it is imperative for the retailer to replenish the inventory as frequently as possible in a reasonable manner while minimising the total inventory management costs.



(a) Zero ending inventory policy. *(b) Non-zero ending inventory policy.*

Figure 4.6.1: Changes to the weight of the live inventory at the farmer's growing facility, the weight of the processed inventory at the processor's processing facility and the weight of the processed inventory at the retailer's selling facility.

When a new replenishment cycle begins, the farmer places an order for live newborn items. The number of live items in each order is ny and each item has a newborn weight of w_0. The weight of all the ordered newborn items, nQ_0, is therefore the product of the number of items in the order and the weight of each item in the order (i.e. $nQ_0 = nyw_0$). The farmer feeds the items and this facilitates growth. The weight of the items changes with time according to the equation

$$w(t) = \frac{\alpha}{1 + \beta e^{-\lambda t}}. \tag{4.6.1}$$

The expression in Equation (4.6.1) is the logistic function and it is used to approximate the items' weight because of its characteristic "S" shape which is representative of actual item growth. At the beginning of the growth period, there is a slow rate of increase in weight and this is followed by a faster rate of increase (in weight) in the intermediate region. Following this is a slowing rate of weight increase which asymptotes towards the maximum possible weight the items can grow to. This pattern results in an "S"-shaped curve.

The farmer grows the items until they reach maturity, defined by a pre-defined target weight w_1. All the items in the order are assumed to grow at the same rate and thus, they reach maturity at the same time. The duration of the farmer's growth period (i.e.the duration of time required to grow the items to the target weight of w_1) can be determined from Equation (4.6.1) as

$$T_f = -\frac{\ln\left[\frac{1}{\beta}\left(\frac{\alpha}{w_1} - 1\right)\right]}{\lambda}. \tag{4.6.2}$$

Growing items are living organisms and can therefore contract various illnesses. This along with presence of predators means that not all the items survive to the end of the growth period. Based on a survival rate of x, the number of live grown items at the end of the growing cycle equals xny. The total weight of all the ordered surviving items at the end of the growing period, nQ_1, is determined by multiplying the number of surviving items and the maturity weight of each item (i.e. $nQ_1 = xnyw_1$).

Since the delivery policy between the farmer and the processor is a SSSD type policy, at the end of each growth period (after T_f time units), the farmer delivers the entire lot of mature surviving items (weighing nQ_1 weight units) in a single shipment. From Equation (4.6.2), it is clear that the duration of the farmer's growth period, T_f, is dependent on the target maturity weight and given that the farmer and the processor operate on a SSSD policy, it is imperative that the processor's cycle time, T_p, is at least equal to farmer's growth duration (i.e. T_f) and thus,

$$T_f \leq T_p. \tag{4.6.3}$$

The constraint in Equation (4.6.3) is imperative because it guarantees that live inventory delivered by the farmer to the processor would have grown to the target maturity weight by the time the growth period ends.

The processor receives an order of mature surviving inventory, with a weight nQ_1 weight units, whenever the farmer's growth period ends. At the processing plant, the mature surviving inventory is transformed into processed inventory at a processing rate of R. The delivery policy between the processor and the retailer is a SSMD and consequently, the processor starts delivering equally-sized batches of processed inventory to the retailer once the amount of surviving inventory that has been transformed into processed inventory is enough to make a batch. The processor delivers n batches of processed inventory to the retailer during a single processing cycle. Since the processor delivers equally-sized batches of processed inventory, this implies that each batch of processed inventory that the retailer receives has a weight of Q_1. The retailer utilises these batches to meet a consumer demand rate (for processed inventory) of D. In order to guarantee the feasibility of the proposed inventory system it is assumed that the rate at which the surviving inventory is transformed into processed inventory at the processing echelon is greater than the rate at which the processed inventory is consumed at the retail echelon (i.e. $R > D$). The processor delivers the batches at equally-spaced intervals of T time units. Consequently, these equally-spaced time intervals (i.e. the retailer's cycle time) and the processor's cycle time, T_p, are linked by

$$T_p = nT. \tag{4.6.4}$$

On receipt of an order, the retailer places the processed inventory on shelves. The retailer has a maximum shelf space for stocking S weight units of processed inventory which implies that the lot size has to be at most equal to the maximum shelf space (i.e. $Q_1 \leq S$). Once placed on shelves, the processed inventory has an associated maximum lifetime (or shelf life) which ensures that the inventory is safe for consumption from consumer health perspective. The shelf life of the processed inventory is indicated by its expiration date often imprinted on the packaging. End consumer demand for the processed inventory is assumed to be a function of both the inventory level displayed on shelves and the expiration date.

Levin et al. (1972) noted that an item's inventory level on display is an important determinant of demand. Higher inventory levels on display tend to entice consumers to buy more. In the context of inventory control modelling, a variety of functions have been used to represent this observation. One of the most widely used functions is the power function, first used by Baker and Urban (1988), which expresses the demand rate as a power function of the inventory level. Therefore,

$$D \propto \delta\big[I(t)\big]^{\psi}, \tag{4.6.5}$$

where $\delta > 0$ and $0 \leq \psi < 1$. Moreover, δ is the scaling parameter for the demand rate (or asymptotic level of demand attainable when the inventory level is considered most favourable to consumers) and ψ is the shape parameter representing the elasticity of the demand rate with respect to the inventory level displayed on the shelves. The relationship in Equation (4.6.5) suggests that the demand rate for processed inventory increases as the level of processed inventory at the retail echelon increases. Likewise, as the inventory level decreases so does the demand rate. The power form relationship also means that at the beginning of a replenishment cycle, whereby the inventory level is at its maximum, the rate at which the processed inventory depletes is higher and as time goes on, the rate at which the inventory is depleted slows down.

The demand rate's expiration date dependency is incorporated through the use of a freshness index. Most growing items are processed into fresh meat, produce or fish products which have relatively short shelf lives. This means that consumers' likeliness to purchase these items is dependent on the age of these items. In other words, when food products have expiration dates, consumers are more likely to purchase items whose expiration date are further away than those whose expiration dates are much closer. Wu et al. (2016) used the expiration date of an item to define the item's freshness index as

$$F(t) = \frac{L - t}{L}, \tag{4.6.6}$$

where L is the expiration date of the item. With the passage of time, the item becomes less fresh. Likewise, Equation (4.6.6) is used to quantify the freshness condition of the processed inventory. The processed inventory is at its freshest after it was just delivered to the retail store with $F(0) = 1$ and it is at its least freshest when it has expired (i.e. reached its maximum shelf life) with $F(L) = 0$. For health reasons, the processed inventory can no longer be sold to consumers once it has expired. This means that the retailer's cycle time, T, can not exceed the expiration date ($L > T$). For this reason, it is important for the retailer to replenish processed inventory frequently so that it does not expire. Hence, the adoption of a SSMD delivery policy between the processor and the retailer is very appropriate for this situation.

Equations (4.6.5) and (4.6.6) are combined to define the demand rate for the processed inventory as a multiplicative function of the inventory level on shelves and freshness condition. It follows that

$$D = \delta\big[I(t)\big]^{\psi}\left(\frac{L-t}{L}\right). \tag{4.6.7}$$

One of the feasibility conditions for the proposed inventory system is that $R > D$. Given that the demand rate varies with time, it is necessary to redefine this condition so that feasibility is guaranteed at all times. The demand rate peaks when the two factors which affect it, namely the inventory level and the freshness index, are at their maximum possible values. The demand rate's inventory level dependency reaches its maximum value when the shelf space is stocked to capacity (i.e $I(t) = S$) while the demand rate's freshness dependency reaches its maximum when the inventory is at its freshest (i.e $t = 0$). This means that the condition $R > D$ has to only be met when the demand rate reaches its maximum to ensure feasibility at all time. The maximum possible demand rate is at the beginning of the retailer's cycle when the shelf space is fully stocked and the processed inventory is furthest from its expiration date. Accordingly, the condition $R > D$ can be rewritten as $R > \delta S^{\psi}$.

4.6.3 Model development

The aforementioned inventory system is modelled as a profit maximisation problem with the retailer's cycle time and the number of shipments delivered by the processor to the retailer as the decision variables.

Consumer demand is for the processed inventory and this particular inventory, tracked at the processor's and the retailer's facilities, incurs procurement, setup (or ordering, in the case of the retail facility) and holding costs. On the other hand, the live inventory, tracked at the farming facility, incurs procurement, setup and feeding costs.

4.6.3.1 Retail echelon profit

Whenever a new replenishment cycle commences, the retailer receives an order for processed inventory (from the processor) weighing $Q_1 = xyw_1$. The inventory is displayed on shelves so as to induce consumer demand. The available shelf space has a maximum capacity of stocking S weight units of processed inventory with a shelf life of L time units. Figure 4.6.2 is a depiction of the retailer's processed inventory system profile.

Throughout a replenishment cycle, the weight of the processed inventory at the retail echelon is depleted due to demand which itself is a function of the processed inventory level and the freshness index of the inventory. Hence, the weight of the processed inventory is controlled by the differential equation

$$\frac{dI(t)}{dt} = -D = -\delta\big[I(t)\big]^{\psi}\left(\frac{L-t}{L}\right), \qquad 0 \le t \le T. \tag{4.6.8}$$

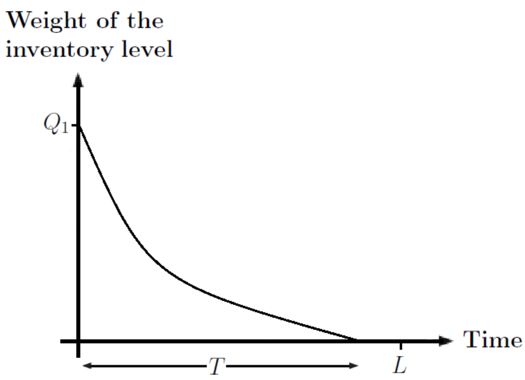


Figure 4.6.2: Changes to the weight of the retailer's processed inventory level

Through rearrangement of terms, Equation (4.6.8) can be rewritten as

$$dI(t) = -\delta\big[I(t)\big]^{\psi}\left(-1+\frac{t}{L}\right)dt, \qquad 0 \le t \le T. \tag{4.6.9}$$

Integrating both sides of Equation (4.6.9) results in

$$\frac{1}{1-\psi}\big[I(t)\big]^{1-\psi} = \delta\left(t-\frac{t^2}{2L}\right)+C. \tag{4.6.10}$$

The processed inventory is completely depleted at the end of each replenishment cycle (i.e. $I=0$ at $t=T$). Therefore, the boundary condition $I(T)=0$ is used to solve for C in Equation (4.6.10) as

$$C = \delta\left(T-\frac{T^2}{2L}\right). \tag{4.6.11}$$

An expression for the weight of the processed inventory at any time is determined by substituting Equation (4.6.11) into Equation (4.6.10) and rearranging the terms. The result is

$$I(t) = \left\{\frac{\delta(1-\psi)}{2L}\Big[t^2+2L(T-t)-T^2\Big]\right\}^{\frac{1}{1-\psi}} \tag{4.6.12}$$

When a new replenishment cycle starts, the retailer receives an order of processed inventory weighing Q_1. This implies that the boundary condition $I(0)=Q_1$ is binding. The weight of the retailer's lot size is determined by substituting the boundary condition into Equation (4.6.12). Hence,

$$Q_1 = I(0) = \left[\frac{\delta(1-\psi)\big(2LT-T^2\big)}{2L}\right]^{\frac{1}{1-\psi}}. \tag{4.6.13}$$

Since $Q_1 = xyw_1$, the number of mature items in the retailer's lot is therefore

$$y = \frac{1}{xw_1}\left[\frac{\delta(1-\psi)\big(2LT-T^2\big)}{2L}\right]^{\frac{1}{1-\psi}}. \tag{4.6.14}$$

The retailer's cyclic holding cost, HC_r, is computed by multiplying the area under the retailer's processed inventory system profile by the holding cost per weight unit per unit time. Hence,

$$HC_r = h_r \int_0^T I(t)\,dt = h_r \int_0^T \left\{ \frac{\delta(1-\psi)}{2L} \left[t^2 + 2L(T-t) - T^2 \right] \right\}^{\frac{1}{1-\psi}} dt. \tag{4.6.15}$$

After integration, the cyclic holding cost becomes

$$HC_r = \frac{h_r \delta(1-\psi)}{2L} \left\{ T + \left[\left(\frac{1-\psi}{2-\psi} \right) \left(2^{\frac{1}{1-\psi}} L^{\frac{1}{1-\psi}} T^{\frac{2-\psi}{1-\psi}} \right) \right] + T^{\frac{3-\psi}{1-\psi}} \right\}. \tag{4.6.16}$$

The retailer's cyclic profit, TP_r, is computed by subtracting the total cyclic inventory management cost (which includes the holding, the ordering and the procurement costs) from the cyclic revenue and the result is

$$\begin{aligned} TP_r = p_r \left[\frac{\delta(1-\psi)\left(2LT - T^2\right)}{2L} \right]^{\frac{1}{1-\psi}} - K_r - p_p \left[\frac{\delta(1-\psi)\left(2LT - T^2\right)}{2L} \right]^{\frac{1}{1-\psi}} \\ - \frac{h_r \delta(1-\psi)}{2L} \left\{ T + \left[\left(\frac{1-\psi}{2-\psi} \right) \left(2^{\frac{1}{1-\psi}} L^{\frac{1}{1-\psi}} T^{\frac{2-\psi}{1-\psi}} \right) \right] + T^{\frac{3-\psi}{1-\psi}} \right\}. \end{aligned} \tag{4.6.17}$$

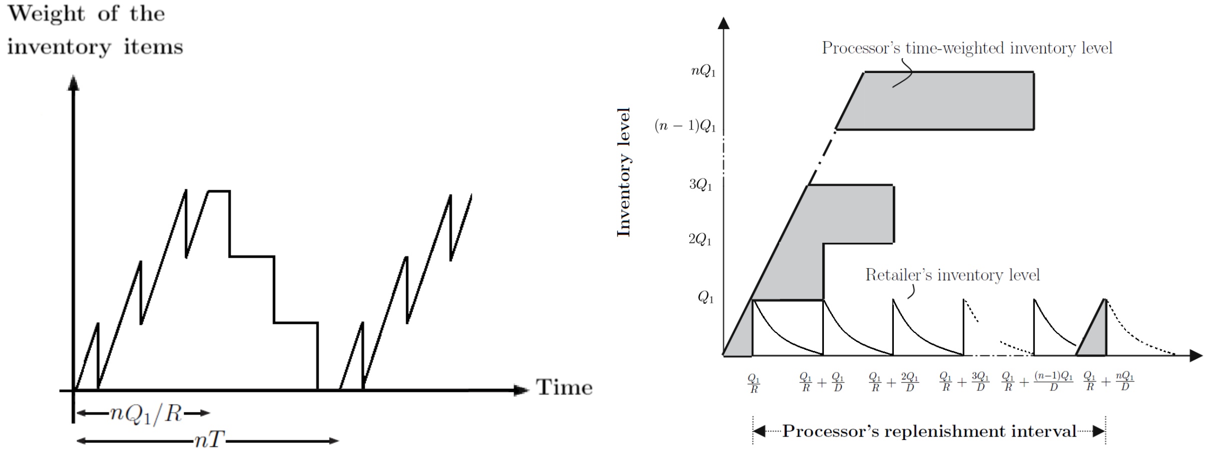
The first term in Equation (4.6.17) is the retailer's revenue per cycle which is computed by multiplying selling price per weight unit of processed inventory charged to consumers (p_r) by the weight of the processed inventory sold per cycle (Q_1). The second term is the fixed cost of placing an order (K_r) which the retailer incurs at the beginning of each cycle. The third term is the retailer's procurement cost per cycle which is computed by multiplying the procurement cost per weight unit of processed inventory procured from the processor (p_p) by the weight of the processed inventory procured per cycle from the processor (Q_1). The last term is the retailer's holding cost per cycle as determined in Equation (4.6.16).

4.6.3.2 Processing echelon profit

At the start of a new replenishment cycle, the processor receives an order of live mature inventory from the farmer. The weight of the mature live inventory in the order is $nQ_1 = nxyw_1$. The live inventory is transformed into processed inventory at a processing rate of R. Throughout the processing cycle, the processor delivers n shipments of processed inventory, each weighing $Q_1 = xyw_1$, to the retailer. Figure 4.6.3b is a depiction of the processor's processed inventory system profile. The figure is a redrawn version of Figure 4.6.3a which represents the processor's inventory system profile. It is easier to derive the area under the graph of the redrawn figure than the original figure because the original's irregular shape. This area is used to determine the processor's holding costs. This method of redrawing the figure is adopted from Yang et al. Yang et al. (2007) who studied a integrated vendor-buyer inventory system operating on a SSMD policy, similar to the delivery policy between the processor and the retailer in this section.

The processor's holding costs per cycle, HC_p, is the product of the area under the processor's inventory system profile and the holding cost per unit time. The area is derived as

$$
\begin{aligned}
\text{Area}_p &= \text{Processor's time-weighted inventory} \\
&= \frac{nQ_1^2}{2R} + Q_1^2\left(\frac{1}{D} - \frac{1}{R}\right) + 2Q_1^2\left(\frac{1}{D} - \frac{1}{R}\right) + \cdots + (n-1)Q_1^2\left(\frac{1}{D} - \frac{1}{R}\right) \qquad (4.6.18) \\
&= \frac{nQ_1^2}{2R} + \frac{n(n-1)Q_1^2}{2}\left(\frac{1}{D} - \frac{1}{R}\right).
\end{aligned}
$$



(a) Original. (b) Redrawn (modified from Yang et al. (2007)).

Figure 4.6.3: Changes to the weight of the processor's processed inventory level.

The expression in Equation (4.6.18) depends on the demand rate D. However, the demand rate is varies with time because it is a function of the inventory level and the freshness index. The variability of the demand function increases the problem's complexity. To counter this, a static approximation of the demand rate is used. Given that the processor ships orders of processed inventory weighing Q_1 to the retailer every T time units in order to meet a demand rate D, the retailer places $\approx D/Q_1$ orders per unit time. This implies that $T \approx Q_1/D$ and thus, $D \approx Q_1/T$. The processor's holding cost per cycle is therefore

$$HC_p = h_p\left[\frac{nQ_1^2}{2R} + \frac{n(n-1)Q_1^2}{2}\left(\frac{T}{Q_1} - \frac{1}{R}\right)\right]. \qquad (4.6.19)$$

The processor's total profit per cycle, TP_p, is defined as the total revenue per cycle less the total inventory management cost per cycle. The total cost of managing inventory at the processing echelon is comprised of the setup, procurement and holding costs. It follows that

$$
\begin{aligned}
TP_p = p_p n\left[\frac{\delta(1-\psi)\big(2LT - T^2\big)}{2L}\right]^{\frac{1}{1-\psi}} &- K_p - p_f n\left[\frac{\delta(1-\psi)\big(2LT - T^2\big)}{2L}\right]^{\frac{1}{1-\psi}} \\
&- \frac{h_p n}{2R}\left[\frac{\delta(1-\psi)\big(2LT - T^2\big)}{2L}\right]^{\frac{2}{1-\psi}} \\
- \frac{h_p n(n-1)}{2}\left[\frac{\delta(1-\psi)\big(2LT - T^2\big)}{2L}\right]^{\frac{2}{1-\psi}} &\left\{T\left[\frac{\delta(1-\psi)\big(2LT - T^2\big)}{2L}\right]^{\frac{1}{\psi-1}} - \frac{1}{R}\right\}.
\end{aligned}
\qquad (4.6.20)
$$

The first term in Equation (4.6.20) is the processor's revenue per cycle which is computed by multiplying selling price per weight unit of processed inventory charged to the retailer (p_p) by the weight of the processed inventory sold to the retailer per cycle (nQ_1). The second term is the fixed cost of setting up for processing (K_p) which the processor incurs at the beginning of each cycle. The third term is the processor's procurement cost per cycle which is computed by multiplying procurement cost per weight unit of live inventory procured from the farmer (p_f) by the weight of the processed inventory procured per cycle from the farmer (nQ_1). The last two terms represent the processor's holding cost per cycle as determined in Equation (4.6.19).

4.6.3.3 Farming echelon profit

Whenever a new replenishment cycle commences, the farmer procures ny live items. Each of the items has a weight of w_0 at the time of procurement. Likewise, the weight of all the items in the order is $nQ_0 = nyw_0$ at the time of procurement. Through feeding, the farmer enables the items to grow for a period of T_f time units, at which point the weight of each item would have increased to w_1. Since growing items are living organism and are thus not immune to death, it is assumed that a fraction x of the initially procured items survive throughout the growth period. Therefore, the weight of all the surviving mature items is $nQ_1 = xnyw_1$. Figure 4.6.4 is a depiction of the farmer's live inventory system profile. The farmer's profit per cycle is defined as the profit per cycle less the sum of the procurement, feeding, mortality and setup.

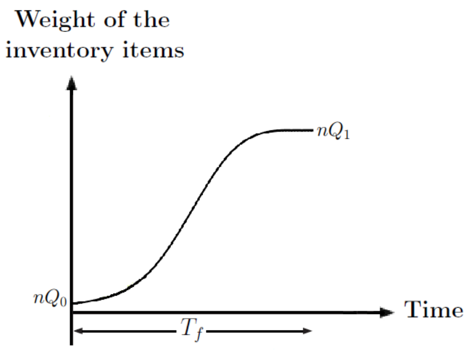


Figure 4.6.4: Changes to the weight of the farmer's live inventory level

The feeding and mortality costs are determined using the area under the inventory system profile graph. The area under the farmer's inventory system profile, as depicted in Figure 4.6.4, is determined as

$$\begin{aligned}\text{Area}_f &= \text{Farmer's time-weighted inventory}\\ &= \int_0^{T_f} nyw(t)\,dt \\ &= ny\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right\}.\end{aligned} \tag{4.6.21}$$

The feeding cost is assumed to be incurred for successfully rearing the live items to maturity. This implies that it is only incurred for the fraction of items that survive (i.e. x), On the other hand, the mortality cost is assumed to be a penalty cost for not successfully rearing the items to maturity, meaning that it is only incurred for the fraction of items that do not survive (i.e. $1-x$). The farmer's feeding cost per cycle, FC_f, is therefore

$$FC_f = c_f xny\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right\}. \tag{4.6.22}$$

Likewise, the farmer's mortality cost per cycle, MC_f, is

$$MC_f = m_f(1-x)ny\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right\}. \tag{4.6.23}$$

The farmer's total cyclic profit, TP_f, is defined as the total cyclic revenue less the sum of the cyclic setup, procurement and feeding, mortality costs. The farmer's total cyclic profit is thus

$$\begin{aligned} TP_f = p_f n\left[\frac{\delta(1-\psi)\left(2LT-T^2\right)}{2L}\right]^{\frac{1}{1-\psi}} - K_f - \frac{p_v n w_0}{x w_1}\left[\frac{\delta(1-\psi)\left(2LT-T^2\right)}{2L}\right]^{\frac{1}{1-\psi}} \\ - ny\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right\}\left[c_f x + m_f(1-x)\right]. \end{aligned} \tag{4.6.24}$$

The first term in Equation (4.6.24) is the farmer's revenue per cycle which is computed by multiplying selling price per weight unit of live inventory charged to the processor (p_f) by the weight of the processed inventory sold to the retailer per cycle (nQ_1). The second term is the fixed cost of setting up for a new growing cycle (K_f). The third term is the farmer's procurement cost per cycle which is computed by multiplying procurement cost per weight unit of live newborn inventory procured from the supplier of the newborn items (p_v) by the weight of the live newborn inventory procured per cycle from the supplier of the newborn items ($nQ_0 = nyw_0$). The last two term represents the sum of the farmer's feeding and mortality costs per cycle as determined by adding Equations (4.6.22) and (4.6.23).

4.6.3.4 Supply chain profit

4.6.3.4.1 Problem formulation

Since the farmer and the processor operate on a SSSD policy, the two parties share the same replenishment interval of nT. On the other hand, the retailer (who receives multiple shipments per single processing cycle) replenishes processed inventory every T time units. Therefore, Equations (4.6.17), (4.6.20) and (4.6.24) are divided by T, nT and nT, respectively, in order to determine the retailer's, processor's and farmer's profits per unit time, respectively. These are then summed to get the total supply chain profit per unit time function (i.e. TPU_{sc}).

Moreover, the fraction of live items that survives throughout the farmer's growth period, x, is considered a random variable with a known probability density function $f(x)$. The expected value of TPU_{sc} along with three constraints (pertaining to the growth period duration, the number of shipments and the available shelf space) are then used to formulate the proposed inventory control system as

$$\begin{aligned}
\text{Maximise:} \quad E[TPU_{sc}] &= \frac{p_r}{T}\left[\frac{\delta(1-\psi)\big(2LT-T^2\big)}{2L}\right]^{\frac{1}{1-\psi}} \\
&- \frac{p_v w_0}{TE[x]w_1}\left[\frac{\delta(1-\psi)\big(2LT-T^2\big)}{2L}\right]^{\frac{1}{1-\psi}} - \frac{K_r}{T} - \frac{K_p}{nT} - \frac{K_f}{nT} \\
-\frac{h_r\delta(1-\psi)}{2LT}&\left\{T+\left[\left(\frac{1-\psi}{2-\psi}\right)\left(2^{\frac{1}{1-\psi}}L^{\frac{1}{1-\psi}}T^{\frac{2-\psi}{1-psi}}\right)\right]+T^{\frac{3-\psi}{1-\psi}}\right\}-\frac{h_p}{2RT}\left[\frac{\delta(1-\psi)\big(2LT-T^2\big)}{2L}\right]^{\frac{2}{1-\psi}} \\
&- \frac{h_p(n-1)}{2T}\left[\frac{\delta(1-\psi)\big(2LT-T^2\big)}{2L}\right]^{\frac{2}{1-\psi}}\left\{T\left[\frac{\delta(1-\psi)\big(2LT-T^2\big)}{2L}\right]^{\frac{1}{\psi-1}} - \frac{1}{R}\right\} \\
-\frac{c_f E[x]+m_f E[1-x]}{TE[x]w_1}&\left[\frac{\delta(1-\psi)\big(2LT-T^2\big)}{2L}\right]^{\frac{1}{1-\psi}}\left\{\alpha T_f+\frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right)-\ln\left(1+\beta\right)\right]\right\} \\
\text{subject to:} \quad T_f \leq nT, &\quad n\in\mathbb{Z}, \quad S \geq \left[\frac{\delta(1-\psi)\big(2LT-T^2\big)}{2L}\right]^{\frac{1}{1-\psi}}. \qquad (4.6.25)
\end{aligned}$$

The first (or growth period duration) constraint is from Equation (4.6.3) and it ensures that the live items are mature enough, in terms of reaching the target weight, for the processing cycle when the growth period ends. The second (or number of shipments) constraint ensures that the problem is practical because it is not possible for the processor to deliver a non-integer number of shipments of processed inventory to the retailer. The third (or available shelf space) constraint, which is essentially $S \geq Q_1$, ensures that the retailer's order quantity Q_1 can be accommodated on the shelves.

4.6.3.4.2 Solution procedure

An iterative solution procedure is used to determine the values of T and n that maximise $E[TPU_{sc}]$. The procedure is as follows:

Step 1 Set n to 1.

Step 2 Find the value of T that maximises $E[TPU_{sc}]$ as given in Equation (4.6.25).

Step 3 Increase n by 1 and find the value of T that maximises $E[TPU_{sc}]$ as given in Equation (4.6.25). Carry on to Step 4.

Step 4 If the latest value of $E[TPU_{sc}]$ increases, go back to Step 3. If the value of $E[TPU_{sc}]$ decreases, the previously calculated value of $E[TPU_{sc}]$ (along with the corresponding T and n values) is the best solution and if this case, carry on to Step 5.

Step 5 Check the solution's feasibility with regard to the growth period duration and available shelf space constraints (i.e. $T_f \leq nT$ and $S \geq Q_1$, respectively). T_f and Q_1 are calculated from Equations (4.6.2) and (4.6.13), respectively. If the solution is feasible, those values of T and n are optimal and if this is the case, carry on to Step 7. If the solution is not feasible, carry on to Step 6.

Step 6 If the solution is not feasible because of the growth period constraint, carry on to Step 6a. If it is infeasible due to the shelf space constraint, carry on to Step 6b. If both constraints are violated, a solution does not exist and in this instance, carry on to Step 7.

Step 6a Set nT to T_f (by adjusting the value of T) and carry on to Step 7.

Step 6b Set Q_1 to S (by adjusting the value of T) and carry on to Step 7.

Step 7 End.

4.6.4 Profit enhancement mechanism: Non-zero ending inventory

With reference to inventory control models with a demand rate that depends on the inventory level, the higher the level of on-hand inventory, the higher the consumer demand. This implies that displaying more stock on shelves has the potential to increase sales and profits. In order to take advantage of this, it might be beneficial to order larger quantities at the beginning of a replenishment cycle so that demand is increased (as a result of higher on-hand inventory levels). On the other hand, the lower the level of on-hand inventory, the lower the demand (and by extension profits). This means that it is financially detrimental for organisations to display low volumes of stock. This observation motivated Urban (1992) to extend the basic EOQ model for items with an inventory level-dependent demand to a case where the ending inventory is non-zero. While the adoption of this non-zero ending inventory policy increases the size of the order (and thus higher holding costs), it also improved profits because the benefits of keeping higher levels of stock are reaped at the beginning of a cycle (due to the larger order size and thus, increased demand) and the detriments of keeping lower levels of stock are minimised by clearing the stock before it reaches zero at the end of the cycle.

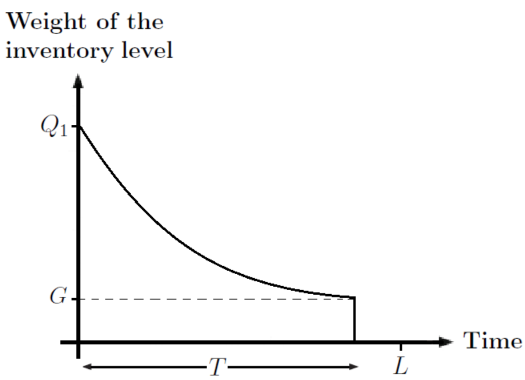


Figure 4.6.5: Changes to the weight of the retailer's processed inventory level with a non-zero ending inventory policy

It might be financially beneficial for the retailer to adopt a non-zero ending inventory policy. In this instance, the retailer would receive an order of processed inventory weighing Q_1 from the processor every T time units. This order would then be used to meet consumer demand which depends on both the inventory level and the freshness index of the inventory. However, the retailer does not wait for the processed inventory level to drop to zero before the receiving a new order from the processor. Instead, the retailer receives a new order when the processed inventory level drops to a certain point G. The retailer's new inventory system profile under this profit improvement mechanism is depicted in Figure 4.6.5. In order to make room available for the new stock, the retailer clears the entire ending stock (i.e. all G weight units) as a single batch through a clearance sale at a salvage price of p_s per weight unit.

Following on from Equations (4.6.8), (4.6.9) and (4.6.10), and using the new boundary condition $I(T) = G$ since the ending inventory level is now G weight units instead of zero, the weight of the retailer's processed inventory at any time t is determined as

$$I(t) = \left\{ \frac{\delta(1-\psi)}{2L} \left[t^2 + 2L(T-t) - T^2 \right] + G^{\frac{1}{1-\psi}} \right\}^{\frac{1}{1-\psi}}. \tag{4.6.26}$$

Likewise, the weight of the retailer's processed inventory is

$$Q_1 = I(0) = \left[\frac{\delta(1-\psi)\left(2LT - T^2\right)}{2L} + G^{\frac{1}{1-\psi}} \right]^{\frac{1}{1-\psi}} \tag{4.6.27}$$

The area under the inventory system profile, as given in Figure 4.6.5, is made up of two sections, namely a triangular-ish portion (as a result of regular demand and deterioration) and a rectangular portion (i.e the non-zero ending inventory that is salvaged). Therefore, the retailer's cyclic holding cost, HC_r, is

$$HC_r = h_r \int_0^T I(t)\, dt + h_r GT = \frac{h_r \delta(1-\psi)}{2L} \left\{ T + \left[\left(\frac{1-\psi}{2-\psi} \right) \left(2^{\frac{1}{1-\psi}} L^{\frac{1}{1-\psi}} T^{\frac{2-\psi}{1-\psi}} \right) \right] + T^{\frac{3-\psi}{1-\psi}} \right\} + h_r GT. \tag{4.6.28}$$

The retailer's total profit per cycle is determined as the sum of the revenue and salvage value of the processed inventory per cycle less the total inventory management cost (i.e. the sum of the setup, procurement and holding costs) per cycle. The retailer's total profit per cycle, TP_r, is thus

$$\begin{aligned} TP_r = p_r &\left\{ \left[\frac{\delta(1-\psi)\left(2LT - T^2\right)}{2L} + G^{\frac{1}{1-\psi}} \right]^{\frac{1}{1-\psi}} - G \right\} + p_s G - K_r \\ &- p_p \left[\frac{\delta(1-\psi)\left(2LT - T^2\right)}{2L} + G^{\frac{1}{1-\psi}} \right]^{\frac{1}{1-\psi}} \\ &- \frac{h_r \delta(1-\psi)}{2L} \left\{ T + \left[\left(\frac{1-\psi}{2-\psi} \right) \left(2^{\frac{1}{1-\psi}} L^{\frac{1}{1-\psi}} T^{\frac{2-\psi}{1-\psi}} \right) \right] + T^{\frac{3-\psi}{1-\psi}} \right\} + h_r GT. \end{aligned} \tag{4.6.29}$$

The first term in Equation (4.6.29) is the retailer's revenue per cycle from the sales of processed inventory and it is determined by multiplying the selling price per weight unit of processed inventory (p_r) by the weight of the processed inventory sold per cycle at the regular selling price ($Q_1 - G$). The second term is the retailer's salvage value per cycle and it is computed by multiplying the salvage value of the processed inventory (p_s) by the weight of the ending inventory level (G) which is cleared at the end of the cycle. The third term is the fixed cost of placing an order (K_r) incurred at the start of each replenishment cycle. The fourth term is the retailer's procurement cost per cycle which is computed by multiplying the procurement cost per weight unit of processed inventory procured from the processor (p_p) by the weight of the processed inventory procured per cycle from the processor (Q_1). The last term is the retailer's holding cost per cycle as determined in Equation (4.6.28).

It should be noted that Equation (4.6.17) is a special case of Equation (4.6.29) when $G = 0$ (i.e. when the traditional zero-ending inventory policy is adopted).

The retailer's total profit per unit time, TPU_r, is therefore

$$TPU_r = \frac{p_r}{T}\left\{\left[\frac{\delta(1-\psi)\big(2LT-T^2\big)}{2L} + G^{\frac{1}{1-\psi}}\right]^{\frac{1}{1-\psi}} - G\right\} + \frac{p_s G}{T} - \frac{K_r}{T}$$
$$- \frac{p_p}{T}\left[\frac{\delta(1-\psi)\big(2LT-T^2\big)}{2L} + G^{\frac{1}{1-\psi}}\right]^{\frac{1}{1-\psi}}$$
$$- \frac{h_r\delta(1-\psi)}{2LT}\left\{T + \left[\left(\frac{1-\psi}{2-\psi}\right)\left(2^{\frac{1}{1-\psi}} L^{\frac{1}{1-\psi}} T^{\frac{2-\psi}{1-\psi}}\right)\right] + T^{\frac{3-\psi}{1-\psi}}\right\} + h_r G. \quad (4.6.30)$$

4.6.5 Numerical results

Two numerical examples are used to solve and analyse the proposed inventory control mechanisms. Both examples consider a three-echelon chicken production supply chain. In the first example, it is assumed that the traditional zero ending inventory policy (at the retail echelon) is adopted while in the second example, the non-zero ending inventory policy (at the retail echelon) is adopted. Both examples make use of the following input parameters: L=4 days; w_0= 0.06kg; w_1= 2kg; R=250 kg/day; K_f=7 500 ZAR; c_f=1 ZAR/kg/day; m_f=2 ZAR/kg/day; K_p=5 000 ZAR; h_p=0.5 ZAR/kg/day; K_r=1 000 ZAR; h_r=1 ZAR/kg/day; α=6.87 kg; β=120; λ=0.12 /day; δ=80 kg/day; ψ=0.2; S=300 kg; p_v=10 ZAR/kg; p_f=20 ZAR/kg; p_p=30 ZAR/kg; p_r=50 ZAR/kg. The survival rate of the newborn items, x, during the course of the farmer's growth period is assumed to be a random variable that is uniformly distributed over [0.8, 1] with a probability density function given by

$$f(x) = \begin{cases} 5, & 0.8 \leq x \leq 1 \\ 1, & \text{otherwise.} \end{cases}$$

This implies that

$$E[x] = \int_{0.8}^{1} 5x \quad dx = 5\left[\frac{(1^2 - 0.8^2)}{2}\right] = 0.9$$

Both examples are solved using the Solver function in Microsoft Excel.

4.6.5.1 Example 1: Zero ending inventory

The results from the first example are presented in Table 4.6.1. The two decision variables are used to determine the optimal replenishment and shipment policies to be followed at all three supply chain echelons. The farmer should place an order for roughly ($ny =$) 2 776 live newly born (i.e. one day old) chicks. The weight of the all the ordered newborn chicks (nQ_0) would amount to about 167 kg. When the growth period ends (at T_f=32.5 days), ($E[x] =$) 90% of the initially ordered chicks would have survived and reached the targeted maturity weight. This implies that the weight of the surviving mature chickens (nQ_1) would be roughly 4 998 kg. The farmer should then transfer the entire lot to the processing plant. During the course of the processing cycle, the processor should deliver the processed chickens (now in a consumable form) to the retailer in ($n =$) 20 equally sized batches. Each batch that the retailer receives should have processed chickens weighing about ($Q_1 =$) 250 kg, which is less than the maximum shelf capacity of $S =$ 300 kg. In order to ensure that the processed chickens don't expire (after $L = 4$ days) and the demand rate does not diminish as a result of reduced freshness, the processor should deliver new orders of processed chickens to the retailer every ($T =$) 1.62 days. The farmer and the processor should start new growing and processing cycles every (nT=) 32.5 days. By following this replenishment and shipment policy, the entire supply chain should except to make a profit of about 4 213.77 ZAR/day.

Decision variables and objective function	**Quantity**
T^* (days)	1.62
n^* (shipments)	20
$E[TPU^*_{sc}]$ (ZAR/day)	4 213.77

Table 4.6.1: Results from Example 1

In order to investigate the relative importance of some of the model's input parameters, in terms of effects on the model's objective function and decision variables, a sensitivity analysis was performed. The results from the analysis are presented in Table 4.6.2 and the following observations and managerial insights are note-worthy:

- When the expiration date of the processed inventory (L) is reduced, the model's optimal solution responds by reducing the retailer's replenishment interval and increasing the number of shipments delivered by the processor to the retailer. A reduction in the retailer' cycle time implies that the size of the retailer's order size is reduced as well. This leads to reduced holding costs because the processed inventory will spend less time in stock. On the other hand, the fixed costs (of placing an order and setting up the growing and processing facilities) are increased. Since the demand rate is a function of the freshness index and stock level of the processed inventory, smaller order sizes have a negative effect on the demand rate and by extension, the supply revenue. The negative effects of reduced order sizes on the demand combined with the increased fixed costs supersede the reduced holding costs and consequently, as the expiration date increases, the supply chain profit decreases. In practical terms, management can use this observation to increase profit by prolonging the lifetime of the processed inventory. This can be achieved by investing in advanced refrigeration and preservation technologies.

- When the expected value of the survival rate of the items ($E[x]$) is reduced, the model responds by increasing the retailer's replenishment interval and reducing the number of shipments. When a higher fraction of the live inventory items die, the farmer has to neutralise this (i.e. ensure that the specified demand is met) by ordering more live items. The effect on the supply chain profit is negative not only because of the increased holding cost (due to larger order sizes) but also because there is a penalty cost (i.e. the mortality cost) incurred for every item that dies. Based on this observation, management should take measures aimed at improving the survival rates of the items such as vaccinating them and feeding them with healthier feedstock.

- The optimal solution responds to a reduction in the shelf capacity (S) by prompting the retailer to reduce their replenishment interval. The major benefit of doing this is that the size of the retailer' order fits on the available shelf space. Consequently, the processor will have to deliver more shipments (of smaller sizes) to the retailer.

This results in reduced profit, but the reduction is relatively small. When the shelf capacity increases beyond a certain point, the no benefit in terms of a profit increase. This is because the retailer's optimal order quantity in the base example is very close to the shelf capacity. Management should strive to make use of the available shelf space as far as possible instead of investing in additional capacity because there is no profit benefit.

Table 4.6.2: Results from the sensitivity analysis for Example 1

Parameters	%	**Retailer's cycle time (T^*)**		**Number of shipments (n^*)**		**Total supply chain profit ($E[TPU^*_{sc}]$)**	
	change	days	% change	shipments	% change	ZAR/day	% change
Base case		**1.62**		**20**		**4 213.77**	
L	-40	1.16	-28.6	28	+40.0	3 200.02	-24.1
	-20	1.41	-13.0	23	+15.0	3 776.66	-10.4
	+20	1.83	+12.9	18	-10.0	4 564.30	+8.3
	+40	1.78	+9.7	19	-5.0	4 823.61	+14.5
$E[x]$	-40	1.80	+11.1	18	-10.0	1 991.36	-52.7
	-20	1.71	+5.3	19	-5.0	3 378.41	-19.8
	+20	1.62	0	20	0	4 770.68	+13.2
	+40	1.62	0	20	0	5 168.48	+22.7
S	-40	1.17	-28.2	28	+40.0	4 043.33	-4.0
	-20	1.56	-4.2	21	+5.0	4 209.67	-0.1
	+20	1.62	0	20	0	4 213.77	0
	+40	1.62	0	20	0	4 213.77	0
δ	-40	2.03	+25.0	16	-20.0	1 598.69	-62.1
	-20	1.80	+11.1	18	-10.0	2 817.75	-33.1
	+20	1.55	-4.7	22	+10.0	5 782.79	+37.2
	+40	1.27	-21.7	48	+140.0	7 551.36	+79.2
ψ	-40	1.62	0	20	0	2 339.03	-44.5
	-20	1.62	0	20	0	3 127.79	-25.8
	+20	1.56	-4.0	22	+10.0	5 743.49	+36.3
	+40	1.25	-23.0	58	+190.0	7 786.24	+84.8

- When the scale parameter of the demand rate (δ) is increased, the replenishment interval is reduced and the number of shipments is increased. While this leads to increased fixed setup and ordering costs which negatively affect the profit, it also leads to reduced holding costs and more importantly, increased demand because reduced replenishment intervals ensure that the inventory is kept as fresh as possible which leads to increased sales. Since the scale parameter of the demand represents the asymptotic level of demand attainable when the inventory level is considered most favourable to consumers, from a practical standpoint, management can exploit this observation by improving their marketing efforts which has the potential to increase the number of prospective consumers.

- When the shape parameter of the demand rate (ψ) is increased, the optimal solution responds by reducing the retailer's replenishment interval and increasing the number of shipments. This parameter corresponds to the elasticity of the demand rate with respect to the level of inventory on the retailer's shelves which represents the sensitivity of the demand rate to the inventory level. As this parameter is increased, so does the demand rate and consequently, the supply chain profit is increased as well.

4.6.5.2 Example 2: Non-zero ending inventory

The results from the second example are presented in Table 4.6.3. When the retailer adopts a non-zero ending inventory policy, the remaining inventory is salvaged as a single batch at a discounted price. For this analysis, it is assumed that the number of shipments delivered by the processor remains the same and the retailer is concerned with determining the optimal replenishment cycle time (T^*) and ending processed inventory level (G^*).

Decision variables and objective function	**Salvage value as a % of retailer's selling price**		
	0%	**25%**	**50%**
T^* (days)	1.80	1.79	1.62
n^* (shipments)	20	20	20
G^* (kg)	2.57	10.26	24.77
$E[TPU^*_{sc}]$ (ZAR/day)	4 286.77	4 513.02	5 043.66

Table 4.6.3: Results from Example 2

Three different salvage values were considered, namely, $p_s = 0$ ZAR/kg, $p_s = 12.50$ ZAR/kg and $p_s = 25.00$ ZAR/kg, which correspond to 0%, 25% and 50%, respectively, of the retailer's selling price ($p_r = 50.00$ ZAR/kg). For all three salvage values tested, the expected supply chain profit improved when compared to the traditional zero-ending inventory policy (i.e. Example 1). However, the improvement in profit varied with the salvage value. When the salvage value is 0% of p_r, profit improved by 1.7% (from 4 213.77 to 4 286.77 ZAR/day), at 25% of p_r, profit improved by 7.1% (from 4 213.77 to 4 513.02 ZAR/day) and at 50% of p_r, profit improved by 19.7% (from 4 213.77 to 5 043.66 ZAR/day).

In essence, the salvage value of the ending processed inventory level is the major determinant of the profitability of the non-zero ending inventory policy. As the salvage value is increased, the retailer's replenishment interval is reduced and the ending inventory level is increased. As a result, the profit is increased because frequent replenishment implies that the processed inventory is fresher (than it would have been if the replenishment interval was longer) which intensifies the demand rate.

4.6.6 Concluding remarks

Managers in charge of procurement and inventory control in perishable food production chains are faced with several unique challenges because of the structure of those supply chains and the perishability of food products. From a supply chain structure perspective, there are often multiple parties involved in the different stages before meeting consumer demand. There is often a farming operation at the upstream end of supply chain where live items are reared while the downstream end is often characterised by the presence of a retail outlet through which end consumer demand for the perishable food products is met. These two ends are often connected by a processing operation where the live items are transformed into a form that is suitable for human consumption. Furthermore, each of these stages have unique challenges of their own as well. For instance, consumer demand at the retail end of the supply chain is often influenced by the expiration date of the products (due to their perishability) and the level of on-hand inventory. At the farming stage, item mortality is an important issue because the inventory items are alive at that stage.

With this in mind, this section developed an integrated model for inventory control in a three-echelon supply chain for growing items whose end demand rate depends on the levels of stock and the expiration date. The model was then extended to a case where the traditional zero-ending inventory policy at the retail end of the supply chain is relaxed in favour of a non-zero ending inventory policy. Under this policy, the retailer sells the processed inventory at the regular selling price throughout the cycle but when the inventory level reaches a certain point, it is salvaged as a single batch at a discounted price. Through numerical experimentation, the adoption of a non-zero ending inventory policy was found to have a positive impact supply chain profit depending on the salvage value.

Granted that this section focused specifically on perishable food supply chains, there a still a few more characteristics of these type of supply chains that might still be exploited in the future as possible extensions to the proposed model. For instance, retailers often carry multiple stock-keeping units (SKU's) of a particular product, this presents an opportunity for further research through the development of an extension that considers an inventory system with multiple-items. Most food retailers are huge corporations with buying and negotiation powers so it might be useful to investigate the effects that different incentive mechanisms such as quantity discounts and revenue-sharing contracts might have on the supply chain. Other characteristics of food supply chains that are worth investigating include pricing decisions, transportation and distribution modes, reverse logistics and quality control.

Chapter 5

Conclusions and Recommendations

5.1 Summary

The main objective of this thesis was to develop inventory models for the management of growing items in multi-echelon supply chains. This was achieved through the development of six lot-sizing models, with each model representing a sub-objective of the thesis, in supply chains with separate farming, processing and retail facilities. The multi-echelon structure of the models, as well as the fact that a vast majority of growing items are the primary source of food products, make these models ideal representations of industrial-scale food production systems. To enhance their practicality, each of the models accounts for specific characteristics of food production systems, such as price-dependent demand, freshness-dependent demand, inventory level-dependent demand, item mortality (of the live growing items), item perishability (of the processed inventory) in the form of expiration dates and quality control. In addition to filling the research gaps identified in the literature, the development of these models can also be of use to operations and supply chain management practitioners in food production and related industries.

5.2 Contributions to knowledge

Effective inventory management in food supply chains is very crucial, not only because it ensures that consumable products are available to meet consumer demand at the right time and right price, but also because a well-managed inventory system has the potential to significantly reduce operational costs. Any form of cost-saving, regardless of its magnitude, is important in food production systems because they are often characterised by relatively low profit margins.

The six models presented in this thesis represent simplified versions of end-to-end food supply chains with separate farming, processing and retail echelons. Two of the models, which consider the probability that some of the processed inventory might be of unacceptable quality, have an additional quality inspection echelon on top of the three basic echelons. Table 5.2.1 provides a summary of a selection of lot-sizing models that are closely related to the six models presented in the thesis. From the table, it is evident that the six models represent the first attempts at developing lot-sizing models for growing items in multi-echelon supply chain settings. The individual contributions made by each of the six models to the literature are as follows:

Section 4.1 of this thesis: A three-echelon supply chain inventory model for growing items.
The novelty of this model is due to the explicit consideration of the following factors simultaneously:

- A three-echelon supply chain involving growing items with farming, processing and retail echelons.
- Growing inventory items are at the upstream end of the supply chain.
- Processed and ready-for-sale inventory is at the downstream end of the supply chain.
- The upstream and downstream ends of the supply chain are joined by the processing echelon where the live inventory items are slaughtered, cut and packaged at a finite rate.

Section 4.2 of this thesis: A three-echelon supply chain inventory model for growing items with price- and freshness-dependent demand.
The novelty of this model is due to the explicit consideration of the following factors simultaneously:

- Separate farming, processing and retail operations with the common goal of jointly maximising profit.
- The demand rate at the retail level is a function of freshness and selling price.
- At the retail level, the inventory has a maximum life time (or expiration date) which is the main determinant of the inventory's freshness index.

Section 4.3 of thesis: A four-echelon supply chain inventory model for growing items with imperfect quality.
The novelty of this model is due to the explicit consideration of the following factors simultaneously:

- A supply chain for growing items with farming, processing, inspection and retail echelons.
- Growing items are reared at the farming echelon provided that a fraction of the items does not survive.
- A fraction of the processed inventory is of poorer quality.
- An inspection echelon is added between the processing and the retail echelons in order to isolate poorer quality processed inventory from that of good quality.

Section 4.4 of this thesis: A three-echelon supply chain inventory model for growing items with expiration dates.
The novelty of this model is due to the explicit consideration of the following factors simultaneously:

- A three-echelon (namely, farming, processing and retail echelons) supply chain for perishable food products.

- Survival rates at the farming echelon are not 100% (i.e. some of the live inventory items do not survive throughout the replenishment cycle).

- Once the processed inventory reaches the retail echelon, it has a maximum shelf life, indicated by its expiration date, beyond which its no longer suitable for human consumption.

Section 4.5 of this thesis: A four-echelon supply chain inventory model for growing items with imperfect quality and errors in quality inspection.
The novelty of this model is due to the explicit consideration of the following factors simultaneously:

- Separate farming, processing, inspection and retail echelons with the aim of maximising the joint supply chain profit.

- At the farming echelon, survival rates for the live items are not 100%.

- At the processing echelon, the processed inventory that is of good quality is not 100% of the initially received order.

- The inspection process at the inspection echelon is not 100% effective and thus makes some classification errors.

Section 4.6 of this thesis: A three-echelon supply chain inventory model for growing items with inventory level- and freshness-dependent demand.
The novelty of this model is due to the explicit consideration of the following factors simultaneously:

- Perishable food products supply chain with farming, processing and retail echelons.

- At the farming echelon, some of the live inventory items do not survive throughout the replenishment cycle.

- At the retail echelon, consumer demand is a function of the processed inventory level and the freshness index which itself is a function of the expiration date.

Table 5.2.1: Characteristics of a selected group of closely related inventory control models in the literature and the contributions made by each of the six models presented in the thesis

References	**Item type**			**Nature of the demand pattern**				**Supply chain echelon(s) in the system**				**Imperfect quality**		**Other characteristics**	
	Conventional	Growing	Perishable	Deterministic constant	Freshness-dependent	Inventory-dependent	Price-dependent	Farming	Processing	Inspection	Retail	With inspection errors	Without inspection errors	Mortality	Expiration dates
Harris (1913)	✓			✓							✓				
Whitin (1955)	✓						✓				✓				
Ghare and Schrader (1963)			✓	✓							✓				
Covert and Phillip (1973)			✓	✓							✓				
Robinson and Lakhani (1975)	✓						✓				✓				
Baker and Urban (1988)	✓					✓					✓				
Urban (1992)	✓					✓					✓				
Salameh and Jaber (2000)	✓			✓							✓		✓		
Cardenas-Barron (2000)	✓			✓							✓		✓		
Khan et al. (2011)	✓			✓							✓	✓			
Sarkar (2012)			✓	✓							✓				✓
Khan et al. (2014)	✓			✓							✓	✓			
Rezaei (2014)		✓		✓							✓				
Wang et al. (2014)			✓	✓							✓				✓
Wu et al. (2016)			✓		✓	✓	✓				✓				✓
Banerjee and Agrawal (2017)			✓				✓				✓				✓
Feng et al. (2017)			✓		✓	✓	✓				✓				✓
Li and Teng (2018)			✓		✓	✓	✓				✓				✓
Gharaei and Almehdawe (2020)		✓		✓							✓			✓	
Khan et al. (2019)			✓				✓				✓				✓
Li and Teng (2019)			✓		✓	✓	✓				✓				✓
Malekitabar et al. (2019)		✓		✓				✓			✓			✓	
Nobil et al. (2019)		✓		✓							✓				
Pando et al. (2019)	✓					✓					✓				
Sebatjane and Adetunji (2019a)		✓		✓							✓				
Sebatjane and Adetunji (2019b)		✓		✓							✓				
Agi and Soni (2020)			✓		✓	✓	✓				✓				✓
Sebatjane and Adetunji (2020c)		✓		✓				✓	✓		✓				
Section 4.1 of this thesis		✓		✓				✓	✓		✓				
Section 4.2 of this thesis		✓	✓		✓		✓	✓	✓		✓				✓
Section 4.3 of this thesis		✓		✓				✓	✓	✓	✓		✓	✓	
Section 4.4 of this thesis		✓	✓	✓				✓	✓		✓			✓	✓
Section 4.5 of this thesis		✓		✓				✓	✓	✓	✓	✓		✓	
Section 4.6 of this thesis		✓	✓		✓	✓		✓	✓		✓			✓	✓

5.3 Recommendations for practitioners

In addition to the contributions to the available literature, the results from this thesis can be of help to operations and supply chain management practitioners in multi-echelon supply chain systems involving growing items. Before discussing some potential benefits afforded to practitioners, a few notable results from the models' numerical experimentation are highlighted. In certain instances, the profit generated across the supply chain may increase by as much as 15% if all members collaborate and integrate their ordering and shipment decisions. Prolonging the shelf life (expiration date) of food products by 40% may increase supply chain profits by as much as 21%. Furthermore, supply chain profits may be increased by as much as 10% and 21%, respectively, if survival rates of live inventory items and acceptable quality levels of the processed inventory are kept close to 100%. Granted that some of these results might not be possible to achieve in reality (for instance, 100% survival rates and 100% acceptable quality levels), operations and supply chain management practitioners should strive to keep them as high as possible. Based on these results, the following recommendations are made for operations and supply chain practitioners:

- With regards to keeping survival rates as high as possible, practitioners should take measures aimed at improving the livelihood of the growing items in farming operations. These measures could include vaccinating the items from common infections and feeding them with healthier feedstock.

- Marketing theory has shown that price, among other factors, is an important determinant of demand (Feng et al., 2017). Consequently, pricing decisions are very critical. Effective pricing is a balancing act because, on the one hand, lower prices spike demand but the resulting margins are relatively lower when compared to higher prices which reduce demand. Practitioners should carefully balance these two extremes to get the most benefit.

- Quality levels should be kept as high as possible because the cost of poor quality is high not only because of lost sales but also because of potential liability costs as a result of selling poorer quality processed inventory to consumers. Furthermore, inspection processes should be continually monitored to ensure that they are not incorrectly classifying the inventory. In practical terms, this can be done through regular maintenance of inspection equipment and regular training of inspection personnel.

- Investments in preservation technologies such as more advanced refrigeration should be prioritised because these technologies have the potential to prolong the shelf life (or expiration date) of processed inventory. Longer shelf lives have been shown to increase supply chain profits. While the initial investment will be large in the short term, the long term benefits will outweigh the initial investment.

5.4 Suggestions for future research

There are various avenues that can be explored as potential future research directions for the six inventory models presented in this thesis. These directions include accounting for the presence of multiple parties in each of the supply chain echelons, uncertainty in demand and incentive mechanisms such as profit-sharing agreements.

All six models were formulated under the assumption that there is only one party in each echelon. While this assumption removes some complexity from the modelling process, it is not realistic because supply chains are very complex systems that often involve multiple parties at each echelon. For instance, a processor might receive live items from multiple farmers for processing and the same processor might sell the processed inventory to multiple retailers. This is not accounted for in each of the six models. Therefore, the incorporation of multiple farmers, processors and retailers in a growing items supply chain represents possible future extensions for the six models.

The models presented in the thesis are suited specifically for multi-echelon food production systems involving growing items. Profit margins in these industries are relatively thin and therefore, supply chain members often resort to incentive mechanisms such as profit-sharing agreements. These types of agreements encourage collaboration and transparency, in terms of information sharing, among supply chain members which leads to improved profits for the entire supply chain. If supply chain members share the gains equitably (which can be enforced through profit-sharing agreements), they are incentivised to share more information for the benefit of the supply chain. Game-theoretic methods can be used to incorporate profit-sharing agreements in any of the six models.

The demand rate in four of the six models was assumed to be a deterministic constant. This assumption can be relaxed to accommodate uncertainties in demand. Macroeconomic factors such as unemployment and inflation rates, natural disasters such as tornado's and earthquakes and global pandemics such as the Ebola or coronavirus diseases are some of the few factors that lead to uncertainties in global supply chains. For instance, the ongoing global coronavirus pandemic has highlighted the importance of inventory management in retail outlets, where certain products, particularly canned food products and toilet paper, were out of stock at multiple retail outlets (USAToday, 2020). This shows that demand patterns in food supply chains are very unpredictable and this presents an opportunity for further development. Therefore, the models presented in this thesis can be extended by considering stochastic demand patterns.

5.5 Closing remarks

The six models presented in this thesis add to the increasing body of knowledge on inventory management for growing items. The multi-echelon structure of the models that integrates the separate functions of farming, processing and retail operations not only enhances the models' practicality, which is helpful to operations and supply chain management practitioners, but it also represents, from a research perspective, a new subfield for inventory management for growing items. This presents researchers with a good foundation for developing new extensions by relaxing some of the assumptions made in the development of the six models presented in the thesis. Furthermore, practitioners also stand to benefit from new extensions that are more practical.

References

Agi, M.A.N., Soni, H.N., 2020. Joint pricing and inventory decisions for perishable products with age-, stock-, and price-dependent demand rate. *Journal of the Operational Research Society* 71, 85–99.

Al-Khazraji, H., Cole, C., Guo, W., 2018. Multi-objective particle swarm optimisation approach for production-inventory control systems. *Journal of Modelling in Management* 13, 1037–1056.

Aljazzar, S.M., Jaber, M.Y., Moussawi-Haidar, L., 2016. Coordination of a three-level supply chain (supplier–manufacturer–retailer) with permissible delay in payments. *Applied Mathematical Modelling* 40, 9594–9614.

Andriolo, A., Battini, D., Grubbstrom, R.W., Persona, A., Sgarbossa, F., 2014. A century of evolution from harris' basic lot size model: Survey and research agenda. *International Journal of Production Economics* 155, 16–38.

Baker, R.C., Urban, T.L., 1988. A deterministic inventory system with an inventory level dependent demand rate. *Journal of the Operational Research Society* 39, 823–831.

Banerjee, A., 1986. A joint economic lot size model for purchaser and vendor. *Decision Sciences* 17, 292–311.

Banerjee, S., Agrawal, S., 2017. Inventory model for deteriorating items with freshness and price dependent demand: Optimal discounting and ordering policies. *Applied Mathematical Modelling* 52, 53–64.

Ben-Daya, M., Al-Nassar, A., 2008. An integrated production system in a three layer supply chain. *Production Planning & Control* 19, 97–104.

Ben-Daya, M., Ertogral, M.R., 2008. The economic lot sizing problem: Review and extensions. *European Journal of Operational Research* 185, 726–742.

Cai, X., Chen, J., Xiao, Y., Xu, X., 2010. Optimization and coordination of fresh product supply chains with freshness-keeping effort. *Production & Operations Management* 19, 261–278.

Cai, X., Chen, J., Xiao, Y., Xu, X., Yu, G., 2013. Fresh-prodcut supply chain management with logistics outsourcing. *Omega* 41, 752–765.

Cardenas-Barron, L.E., 2000. Observation on: Economic production quantity model for items with imperfect quality" [int. j. production economics 64 (2000) 59-64]. *International Journal of Production Economics* 67, 1201.

Castellano, D., Gallo, M., Santillo, L., Song, D., 2017. A periodic review policy with quality improvement, setup cost reduction, backorder price discount, and controllable lead time. *Production & Manufacturing Research* 5, 328–350.

Chang, C.T., Cheng, M.C., Soong, P.Y., 2016. Impacts of inspection errors and trade credits on the economic order quantity model for items with imperfect quality. *International Journal of Systems Science: Operations & Logistics* 3, 34–48.

Chen, L.H., Kang, S.H., 2007. Integrated vendor–buyer cooperative inventory models with variant permissible delay in payments. *European Journal of Operational Research* 183, 658–673.

Chen, S.C., Min, J., Teng, J.T., Li, F., 2016. Inventory and shelf-space optimization for fresh produce with expiration date under freshness-and-stock-dependent demand rate. *Journal of the Operational Research Society* 67, 884–896.

Clark, A.J., Scarf, H., 1960. Optimal policies for a multi-echelon inventory problem. *Management Science* 6, 475–490.

Corstjens, M., Doyle, P., 1981. A integrated model for optimizing retail space allocations. *Management Science* 27, 822–833.

Covert, R.B., Phillip, G., 1973. An EOQ model for items with weibull distribution deterioration. *AIIE Transactions* 5, 323–326.

Dani, S., 2015. *Food Supply Chain Management: From Farm to Fork* (1 edn.). Kogan Page.

Dey, O., Giri, B.C., 2019. A new approach to deal with learning in inspection in an integrated vendor-buyer model with imperfect production process. *Computers & Industrial Engineering* 131, 515–523.

Dobson, G., Pinker, E.J., Yildiz, O., 2017. An EOQ model for perishable goods with age-dependent demand rate. *European Journal of Operational Research* 257, 84–88.

Duan, Y., Li, G., Tien, J.M., Huo, J., 2012. Inventory models for perishable items with inventory level dependent demand rate. *Applied Mathematical Modelling* 36, 5015–5028.

Dye, C.Y., Yang, C.T., Wu, C.C., 2018. Joint dynamic pricing and preservation technology investment for an integrated supply chain with reference price effects. *Journal of the Operational Research Society* 69, 811–824.

Feng, L., Chan, Y.L., 2019. Joint pricing and production decisions for new products with learning curve effects under upstream and downstream trade credits. *European Journal of Operational Research* 272, 905–913.

Feng, L., Chan, Y.L., Cardenas-Barron, L.E., 2017. Pricing and lot-sizing polices for perishable goods when the demand depends on selling price, displayed stocks, and expiration date. *International Journal of Production Economics* 185, 11–20.

Gan, S.S., Pujawan, I.N., Suparno, V.I., Widodo, B., 2018. Pricing decisions for short life-cycle product in a closed-loop supply chain with random yield and random demands. *Operations Research Perspectives* 5, 174–190.

Geetha, K.V., Uthayakumar, R., 2014. Evaluation of supply chain with quality improvement under trade credit and freight rate discount. *Opsearch* 51, 463–478.

Gharaei, A., Almehdawe, E., 2020. Economic growing quantity. *International Journal of Production Economics* 223, 107517.

Ghare, P.M., Schrader, G., 1963. A model for an exponentially decaying inventory. *Journal of Industrial Engineering* 14, 238–243.

Giri, B.C., Bardhan, S., 2012. Supply chain coordination for a deteriorating item with stock and price-dependent demand under revenue sharing contract. *International Transactions in Operational Research* 19, 753–768.

Glock, C.H., 2012. The joint economic lot size problem: A review. *European Journal of Operational Research* 135, 671–686.

Goyal, S.K., 1977. An integrated inventory model for a single supplier-single customer problem. *International Journal of Production Research* 15, 107–111.

Goyal, S.K., 1988. A joint economic-lot-size model for purchaser and vendor: A comment. *Decision Sciences* 19, 236–241.

Goyal, S.K., Cardenas-Barron, L.E., 2002. Note on: Economic production quantity model for items with imperfect quality- a practical approach. *International Journal of Production Economics* 77, 85–87.

Goyal, S.K., Chang, C.T., 2009. Optimal ordering and transfer policy for an inventory with stock dependent demand. *European Journal of Operational Research* 196, 177–185.

Goyal, S.K., Huang, C.K., Chen, K.C., 2003. A simple integrated production policy of an imperfect item for vendor and buyer. *Production Planning & Control* 14, 596–602.

Harris, F.W., 1913. How many parts to make at once. *Factory, The Magazine of Management* 10, 135–136.

Hill, R.M., 1999. The optimal production and shipment policy for the single-vendor single-buyer integrated production-inventory problem. *International Journal of Production Research* 37, 2463–2475.

Hillier, F.S., Lieberman, G.J., 2001. *Introduction to Operations Research* (7 edn.). McGraw-Hill.

Ho, C.H., Ouyang, L.Y., Su, C.H., 2008. Optimal pricing, shipment and payment policy for an integrated supplier–buyer inventory model with two-part trade credit. *European Journal of Operational Research* 187, 496–510.

Hou, K.L., Lin, L.C., 2006. An EOQ model for deteriorating items with price- and stock-dependent selling rates under inflation and time value of money. *International Journal of Systems Science* 37, 1131–1139.

Hsu, J.T., Hsu, L.F., 2013a. An EOQ model with imperfect quality items, inspection errors, shortage backordering, and sales returns. *International Journal of Production Economics* 143, 162–170.

Hsu, J.T., Hsu, L.F., 2013b. An integrated vendor–buyer cooperative inventory model in an imperfect production process with shortage backordering. *International Journal of Advanced Manufacturing Technology* 65, 493–505.

Hsu, P.H., Wee, H.M., Teng, H.M., 2007. Optimal ordering decision for deteriorating items with expiration date and uncertain lead time. *Computer & Industrial Engineering* 52, 448–458.

Huang, C.K., 2002. A simple integrated production policy of an imperfect item for vendor and buyer. *Production Planning & Control* 14, 596–602.

Jaggi, C.K., Gupta, M., Kausar, A., Tiwari, S., 2019. Inventory and credit decisions for deteriorating items with displayed stock dependent demand in two-echelon supply chain using stackelberg and nash equilibrium solution. *Annals of Operations Research* 274, 309–329.

Jaipuria, S., Mahapatra, S.S., 2019. A study on behaviour of bullwhip effect in (r, s) inventory control system considering dwt-mggp demand forecasting model. *Journal of Modelling in Management* 14, 385–407.

Jauhari, W.A., Saga, R.S., 2017. A stochastic periodic review inventory model for vendor–buyer system with setup cost reduction and service–level constraint. *Production & Manufacturing Research* 5, 371–389.

Khalilpourazari, S., Pasandideh, S.H.R., 2019. Modeling and optimization of multi-item multi-constrained EOQ model for growing items. *Knowledge-Based Systems* 164, 150–162.

Khan, M., Jaber, M.Y., Ahmad, A.R., 2014. An integrated supply chain model with errors in quality inspection and learning in production. *Omega* 42, 16–24.

Khan, M., Jaber, M.Y., Bonney, M., 2011. An economic order quantity (EOQ) for items with imperfect quality and inspection errors. *International Journal of Production Economics* 133, 113–118.

Khan, M., Jaber, M.Y., Zanoni, S., Zavanella, L., 2016. Vendor-managed-inventory with consignment stock agreement for a supply chain with defective items. *Applied Mathematical Modelling* 40, 7102–7114.

Khan, M.A.A., Shaikh, A.A., Panda, G.C., Konstantaras, I., Taleizadeh, A.A., 2019. Inventory system with expiration date: Pricing and replenishment decisions. *Computer & Industrial Engineering* 132, 232–247.

Khouja, M., 2003. Optimizing inventory decisions in a multi-stage multi-customer supply chain. *Transportation Research Part E: Logistics and Transportation Research* 39, 193–208.

Konstantaras, I., Goyal, S.K., Papachristos, S., 2007. Economic ordering policy for an item with imperfect quality subject to the in-house inspection. *International Journal of Systems Science* 38, 473–482.

Kreng, C.H., 2011. A multiple-vendor single-buyer integrated inventory model with a variable number of vendors. *Computers & Industrial Engineering* 60, 173–182.

Kreng, V.B., Tan, S.J., 2011. Optimal replenishment decision in an EPQ model with defective items under supply chain trade credit policy. *Expert Systems with Applications* 38, 9888–9899.

Krommyda, I.P., Skouri, K., Konstantaras, I., 2015. Optimal ordering quantities for substitutable products with stock-dependent demand. *Applied Mathematical Modelling* 39, 147–164.

Lambert, D.M., 2008. *Supply Chain Management: Processes, Partnerships, Performance* (3 edn.). Supply Chain Management Institute.

Levin, R.I., McLaughlin, C.P., Lamone, R.P., Kottas, F.J., 1972. *Production/Operations Management: Contemporary Policy for Managing Operating Systems*. McGraw-Hill.

Li, R., Chan, Y.L., Chang, C.T., , Cardenas-Barron, L.E., 2017. Pricing and lot-sizing policies for perishable products with advance-cash-credit payments by a discounted cash-flow analysis. *International Journal of Production Economics* 193, 578–589.

Li, R., Teng, J.T., 2018. Pricing and lot-sizing decisions for perishable goods when demand depends on selling price, reference price, product freshness, and displayed stocks. *European Journal of Operational Research* 270, 1099–1108.

Li, R., Teng, J.T., 2019. Optimal credit term, order quantity and selling price for perishable products when demand depends on selling price, expiration date, and credit period. *Annals of Operations Research* 280, 377–405.

Lu, L., 1995. A one-vendor multi-buyer integrated inventory model. *European Journal of Operational Research* 81, 312–323.

Ma, X., Wang, S., Islam, S.M., Liu, X., 2019. Coordinating a three echelon fresh agricultural products supply chain considering freshness-keeping effort with with asymmetric information. *Applied Mathematical Modelling* 67, 337–356.

Maddah, B., Jaber, M.Y., 2008. Economic order quantity for items with imperfect quality: Revisited. *International Journal of Production Economics* 112, 808–815.

Malekitabar, M., Yaghoubi, S., Gholamian, M.R., 2019. A novel mathematical inventory model for growing-mortal items (case study: Rainbow trout). *Applied Mathematical Modelling* 71, 96–117.

Mandal, B.N., Phaujdar, S., 1989. An inventory model for deteriorating items and stock-dependent consumption rate. *Journal of the Operational Research Society* 40, 483–488.

Nobil, A.H., Sedigh, A.H.A., Cardenas-Barron, L., 2019. A generalized economic order quantity inventory model with shortage: Case study of a poultry farmer. *Arabian Journal for Science and Engineering* 44, 2653–2663.

Oliveira, B.B., Carravilla, M.A., Oliveira, J.F., 2018. Integrating pricing and capacity decisions in car rental: A matheuristic approach. *Operations Research Perspectives* 5, 334–356.

Ouyang, L.Y., Wu, K.S., Ho, C.H., 2004. Integrated vendor–buyer cooperative models with stochastic demand in controllable lead time. *International Journal of Production Economics* 92, 255–266.

Ouyang, L.Y., Wu, K.S., Ho, C.H., 2007. An integrated vendor–buyer inventory model with quality improvement and lead time reduction. *International Journal of Production Economics* 108, 349–358.

Ouyang, L.Y., Wu, K.S., Ho, C.K., 2006. Analysis of optimal vendor-buyer integrated inventory policy involving defective items. *International Journal of Advanced Manufacturing Technology* 29, 1232–1245.

Pan, J.C.H., Yang, J.S., 2002. A study of an integrated inventory with controllable lead time. *International Journal of Production Research* 40, 1263–1273.

Pando, V., Garcia-Laguna, J., San-Jose, L.A., Sicilia, J., 2012. Optimal lot-size policy for deteriorating items with stock-dependent demand considering profit maximization. *Computers & Industrial Engineering* 117, 81–93.

Pando, V., San-Jose, L.A., Sicilia, J., 2019. Profitability ratio maximization in an inventory model with stock-dependent demand rate and non-linear holding cost. *Applied Mathematical Modelling* 66, 643–661.

Priyan, S., Manivannan, P., 2017. Optimal inventory modeling of supply chain system involving quality inspection errors and fuzzy defective rate. *Opsearch* 54, 21–43.

Raza, S.A., Govindaluri, S.M., 2019. Pricing strategies in a dual-channel green supply chain with cannibalization and risk aversion. *Operations Research Perspectives* 6, 100118.

Rezaei, J., 2014. Economic order quantity for growing items. *International Journal of Production Economics* 155, 109–113.

Richards, F., 1959. A flexible growth function for empirical use. *Journal of Experimental Botany* 10, 290–301.

Robinson, B., Lakhani, C., 1975. Dynamic price models for new-product planning. *Management Science* 21, 1113–1122.

Rout, C., Kumar, R.S., Chakraborty, D., Goswami, A., 2019. An EPQ model for deteriorating items with imperfect production, inspection errors, rework and shortages : a type-2 fuzzy approach. *Opsearch* 56, 657–688.

Saha, S., Modak, M.M., Panda, S., Sana, S.S., 2018. Managing a retailer's dual-channel supply chain under price- and delivery time-sensitive demand. *Journal of Modelling in Management* 13, 351–374.

Sajadieh, M.H., Thorstensson, A., Jokar, M.R.A., 2010. An integrated vendor-buyer model with stock-dependent demand. *Transportation Research Part E: Logistics and Transportation Review* 46, 963–974.

Salameh, M.K., Jaber, M.Y., 2000. Economic production quantity model for items with imperfect quality. *International Journal of Production Economics* 64, 59–64.

Sargut, F.Z., Isik, G., 2017. Dynamic economic lot size model with perishable inventory and capacity constraints. *Applied Mathematical Modelling* 48, 806–820.

Sarkar, B., 2012. An EOQ model with delay in payments and time-varying deterioration rate. *Mathematical and Computer Modelling* 55, 367–377.

Sarkar, B., 2013. A production-inventory model with probabilistic deterioration in two-echelon supply chain management. *Applied Mathematical Modelling* 37, 3138–3151.

Sarmah, S.P., Acharya, S.K., Goyal, S.K., 2006. Buyer vendor coordination models in supply chain management. *European Journal of Operational Research* 175, 1–15.

Sebatjane, M., Adetunji, O., 2019a. Economic order quantity model for growing items with imperfect quality. *Operations Research Perspectives* 6, 100088.

Sebatjane, M., Adetunji, O., 2019b. Economic order quantity model for growing items with incremental quantity discounts. *Journal of Industrial Engineering International* 15, 545–556.

Sebatjane, M., Adetunji, O., 2020a. Optimal inventory replenishment and shipment policies in a four-echelon supply chain for growing items with imperfect quality. *Production & Manufacturing Research* 8, 130–157.

Sebatjane, M., Adetunji, O., 2020b. A three-echelon supply chain for economic growing quantity model with price- and freshness-dependent demand: Pricing, ordering and shipment decisions. *Operations Research Perspectives* 7, 100153.

Sebatjane, M., Adetunji, O., 2020c. Three-echelon supply chain inventory model for growing items. *Journal of Modelling in Management* 15, 567–587.

Silver, E.A., Peterson, R., 1985. *Decision Systems for Inventory Management and Production Planning*. John Wiley & Sons.

Stevenson, W., 2018. *Operations Management* (13 edn.). McGraw-Hill.

Taleizadeh, A.A., Yadegari, M., Sana, S.S., 2019. Production models of multiple products using a single machine under quality screening and reworking policies. *Journal of Modelling in Management* 14, 232–259.

Teng, J.T., Cardenas-Barron, L.E., Chang, H.J., Wu, J., Hu, Y., 2016. Inventory lot-size policies for deteriorating items with expiration dates and advance payments. *Applied Mathematical Modelling* 40, 8605–8616.

Teng, J.T., Chang, C.T., 2005. Economic production quantity models for deteriorating items with price- and stock-dependent demand. *Computers & Operations Research* 32, 297–308.

Tiwari, S., Cardenas-Barron, L.E., Goh, M., Shaikh, A.A., 2018a. Joint pricing and inventory model for deteriorating items with expiration dates and partial backlogging under two-level partial trade credits in supply chain. *International Journal of Production Economics* 200, 16–36.

Tiwari, S., Daryanto, Y., Wee, H.M., 2018b. Sustainable inventory management with deteriorating and imperfect quality items considering carbon emission. *Journal of Cleaner Production* 192, 281–292.

Urban, T.L., 1992. An inventory model with an inventory-level-dependent demand rate and relaxed terminal conditions. *Journal of the Operational Research Society* 43, 721–724.

Urban, T.L., 2005. Inventory models with inventory-level-dependent demand: A comprehensive review and unifying theory. *European Journal of Operational Research* 162, 792–804.

Urban, T.L., Baker, R.C., 1997. Optimal ordering and pricing policies in a single period environment with multivariate demand and price markdowns. *European Journal of Operational Research* 103, 573–583.

USAToday, 2020. Canned goods and non-perishables are selling out—here's where you can still buy them. `https://www.usatoday.com/story/tech/reviewedcom/2020/03/16/heres-where-you-can-still-buy-canned-goods-and-non-perishables-online/5061455002/`. Date accessed: 2020-04-02.

Valentini, G., Zavanella, L., 2003. The consignment stock of inventories: industrial case and performance analysis. *International Journal of Production Economics* 81-82, 215–224.

Vats, P., Soni, G., Rathore, A.P.S., Yadav, S.P., 2019. A demand aggregation approach for inventory control in two echelon supply chain under uncertainty. *Opsearch* 56, 840–868.

Wang, W.C., Teng, J.T., Lou, K.R., 2014. Seller's optimal credit period and cycle time in a supply chain for deteriorating items with maximum lifetime. *European Journal of Operational Research* 232, 315–321.

Wangsa, I.D., Wee, H.M., 2018. An integrated vendor–buyer inventory model with transportation cost and stochastic demand. *International Journal of Systems Science: Operations & Logistics* 5, 295–309.

Whitin, T.M., 1955. Inventory control and price theory. *Management Science* 2, 61–68.

Wu, J., Ouyang, L.Y., Cardenas-Barron, L.E., Goyal, S.K., 2014. Optimal credit period and lot size for deteriorating items with expiration dates under two-level trade credit financing. *European Journal of Operational Research* 237, 898–908.

Wu, J., Tang, C.T., Cheng, M.C., Teng, J.T., Al-Khateeb, F.B., 2016. Inventory management for fresh produce when the time-varying demand depends on product freshness, stock level and expiration date. *International Journal of Systems Science: Operations & Logistics* 3, 138–147.

Wu, J., Tang, C.T., Teng, J.T., Lai, K.K., 2017. Optimal order quantity and selling price over a product life cycle with deterioration rate linked to expiration date. *International Journal of Production Economics* 193, 434–351.

Wu, J., Teng, J.T., Chan, Y.L., 2018. Inventory policies for perishable products with expiration dates and advance-cash-credit payment schemes. *International Journal of Systems Science: Operations & Logistics* 5, 310–326.

Wu, Q., Mu, Y., Feng, Y., 2015. Coordinating contracts for for fresh product outsourcing logistics channel with power structures. *International Journal of Production Economics* 160, 94–105.

Yang, P.C., Wee, H.M., 2000. Economic ordering policy of deteriorated item for vendor and buyer: An integrated approach. *Production Planning & Control* 11, 474–480.

Yang, P.C., Wee, H.M., Yang, H.J., 2007. Global optimal policy for vendor buyer integrated inventory system within just in time environment. *Journal of Global Optimization* 37, 505–511.

Zanoni, S., Mazzoldi, L., Zavanella, L.E., Jaber, M.Y., 2014. A joint economic lot size model with price and environmentally sensitive demand. *Production & Manufacturing Research* 2, 341–354.

Zhou, Y., Chen, C., Li, C., Zhong, Y., 2016. A synergic economic order quantity model with trade credit, shortages, imperfect quality and inspection errors. *Applied Mathematical Modelling* 40, 1012–1028.