THE STABILITY OF ASSIGNMENT RESULTS

Bloy, L.A.K.

VKE Consulting Engineers.

ABSTRACT

The most common technique used in the traffic assignment phase of transportation modelling is the user equilibrium method. This method takes the effect of congestion on travel times into account by implementing Wardrop's first principle. This principle states that if two or more routes between an origin and destination pair are used then the travel times on these routes will be equal, i.e. in equilibrium.

The equilibrium assignment technique is an iterative one, and one of the problems for transportation modellers is to decide on the number of iterations that should be performed. The majority of programs that implement the equilibrium assignment technique include default values for stopping the iterative process. These stopping conditions are usually the number of iterations or one or more types of stopping criteria.

This paper presents the results of a study where a number of different values for stopping the iterative process were tested. It was found that the degree of convergence required for stable results is dependent on the reason for doing the assignment. More iterations are required if the results are to be used in an economic evaluation than if volumes are needed for road design purposes.

The paper provides recommended values that should be used as stopping criteria when doing equilibrium assignments.

1. INTRODUCTION

Trip assignment is the stage of the transportation modelling process where the trip matrix or matrices that have been forecast are allocated to the transportation network (roads, rail lines, etc). This paper deals with the assignment of private vehicle trips onto the road network.

The paper starts with a brief history of trip assignment including descriptions of some early assignment techniques. The user equilibrium assignment technique is also discussed. This is probably the most commonly used and realistic method, and takes the effect of traffic congestion into account in assigning vehicles onto the road network. This is an iterative process and the question is how far the process should be continued in order to obtain stable or consistent results (the stopping criterion).

There are basically two reasons for carrying out traffic assignments. The first is to obtain data in order to perform an economic evaluation of one or more proposed road projects. The other is to provide future traffic volumes for use in engineering design, either to determine where new roads are required, or to provide volumes that can be used to decide on factors such as the number of lanes and pavement design required for new roads. The paper examines both cases and finds that different stopping criteria should be used for the two cases.

Proceedings of the 22nd Southern African Transport Conference (SATC2003) 14 – 16th July 2003
ISBN Number: 0-958-46096-5 Pretoria, South Africa
Proceedings produced by: Document Transformation Technologies Conference Organised by: Conference Planners

2. A BRIEF HISTORY OF TRIP ASSIGNMENT

2.1 The effect of traffic volumes on travel time

The links of a transportation network have associated with them a travel impedance, which can include many factors such as travel time, safety, cost of travel, etc. The major component of this impedance is travel time, which is often used as the only measure of impedance. The travel time on a link depends on the volume travelling on the link and is an increasing function of the volume or flow on the link.

In a modelled transportation network therefore, each link should have associated with it a performance function rather than a constant travel time. The performance function (also called a volume delay function) relates the travel time on the link to the flow on the link. A typical performance function is shown in Figure 1.

2.2 The concept of equilibrium in transportation networks

The idea of equilibrium in the analysis of transportation networks arises from the dependence of the link travel times on the link flows. Assuming that a number of motorists wish to travel between a given origin and a given destination that are connected by a number of possible paths, how will the motorists be distributed among the possible paths? If all the motorists were to use the same path (initially the shortest path in terms of travel time), this path would become more congested. This would result in the travel time on this path increasing. A point might be reached where this is no longer the shortest path. Some of the motorists would then divert to an alternative path that, however, might also be congested. The system will be in a state of equilibrium when no motorists will want to change from their present path to an alternative path.

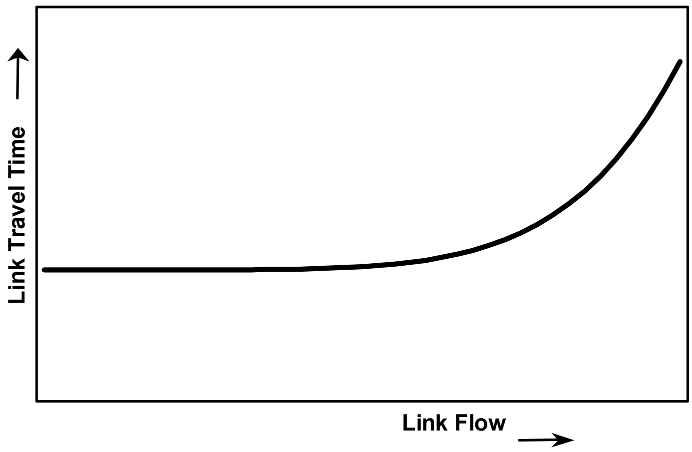

Figure 1. A Typical Link Performance Function.

In 1952 Wardrop (8) expressed the above when he proposed the following principle of route choice that would result in a state of equilibrium: “The costs on all routes used between any given pair of end points are equal and not greater than the cost experienced by a single vehicle on an unused route between them”.

2.3 Some early assignment techniques

The simplest form of traffic assignment is the all-or-nothing assignment. In this procedure, every origin-destination flow between origin node r and destination node s, is assigned to the links that are on the minimum-travel-time path connecting r and s. All other paths connecting r and s are not assigned any flow.

During this process, the link travel times are assumed to be fixed (not dependent on the flow on them). The travel times on the links after the flows have been assigned to them will, in most cases, be different from the times on which the assignment was based. Since this assignment method does not take into account the dependence between flows and travel time, it, in effect, ignores the equilibrium problem.

Initially, the computer power was not available to solve the equilibrium assignment problem and approximate methods were used to try and obtain equilibrium solutions.

These included the following two methods:

- Capacity restraint assignment. This heuristic technique is also sometimes called the iterative assignment technique. This method involves a number of all-or-nothing assignments in which the travel times resulting from the previous assignment are used in the current iteration. The algorithm is terminated after a given number of iterations, N. The equilibrium flow pattern is then taken to be the average flow for each link over the last four iterations.
- Incremental assignment. This is another heuristic technique where a portion of the origin-destination matrix is assigned to the network at each iteration using the all-or-nothing method. After each assignment, the travel times on all links are recalculated taking the assigned flows into consideration, before the next portion of the matrix is assigned.

Sheffi (5) showed that neither of the two techniques described above, necessarily result in an equilibrium solution.

2.4 The equilibrium assignment problem

Beckmann et al (2) showed that the equilibrium assignment problem could be transformed into an equivalent optimisation problem. This could be solved using the Franke-Wolfe algorithm to combine the results of successive all-or-nothing assignments in an iterative manner. Each all-or-nothing assignment uses the link travel times obtained using the link volumes resulting from the previous iteration of the process.

It has been proved that that this process converges to a unique solution (see Sheffi, 5). Since it is an iterative procedure, the question is how many iterations need to performed? This question is addressed in the remainder of this paper.

3. STOPPING CRITERIA FOR THE EQUILIBRIUM ASSIGNMENT PROBLEM

Evans (3) proved that the addition of the-all-or nothing auxiliary assignment always leads to an improved solution. He also showed that provided enough iterations are performed, the Frank-Wolfe procedure converges to the equilibrium solution. However, the successive improvements become smaller with each iteration and it may take a very large number of iterations to reach convergence in a real-world network. The question then is, how many iterations are necessary in order to reach a satisfactory solution?

Thomas (7) lists the following three basic types of stopping rules:

- Those that look at the differences between estimates of quantities, usually link flows or costs, derived in successive iterations and, on the basis of the differences, decide whether or not continuation of the process is likely to bring about significant changes.
- Those which measure the agreement between the latest assumed link costs and assigned flows and the assumed cost/flow relationships.
- Those that consider the potential improvement that may result from continuing with more iterations.

Sheffi (5) has the following to say concerning convergence and the number of iterations required:

In solving the user equilibrium problem over a large network, each iteration involves a significant computational cost, due primarily to the effort required to calculate the shortest paths. It is important then, that, a good answer is achieved after a relatively small number of iterations.

In practice, this is not a major problem for two reasons. First, the convergence pattern of the convex combinations algorithm is such that the first few iterations are the most "cost effective." In other words, the flow pattern after only a few iterations is not very far from equilibrium. Second, the convergence criteria used in practice are not very stringent and thus convergence can be achieved after only a small number of iterations. This is because the accuracy of the input data does not warrant the effort needed to obtain an extremely accurate equilibrium flow pattern.

Sheffi provides a figure similar to Figure 2 that shows the convergence rate for three levels of congestion on what he calls a "medium-sized" network. In Figure 2, low, high and very high refer to the level of congestion in the network.

He continues by saying: *In actual applications, only four to six iterations are usually sufficient to find the equilibrium flow pattern over large urban networks. This number reflects common practice in terms of trade-offs among analytical accuracy, data limitations, and budget, given typical congestion levels.*

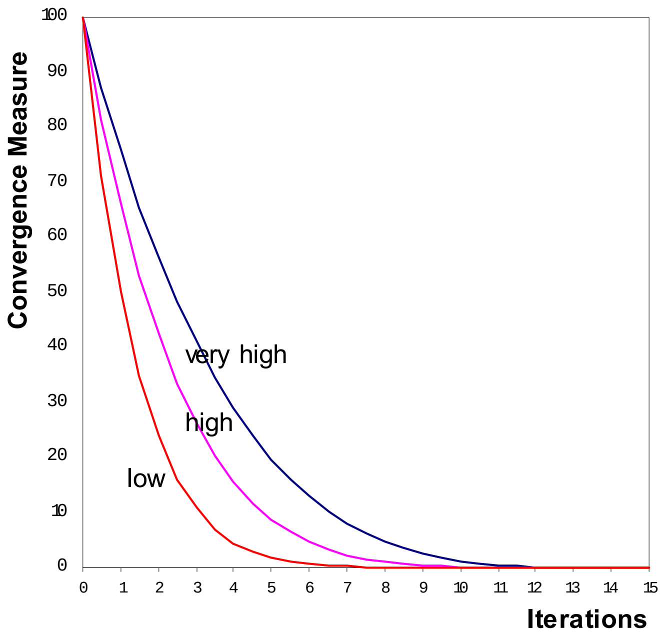

Figure 2. Rate of convergence depending on level of congestion.

Results obtained when plotting the results obtained using the Gautrans model appear to support Sheffi's statements. Figure 3 shows the value of the objective function of the optimisation problem plotted against the number of iterations.

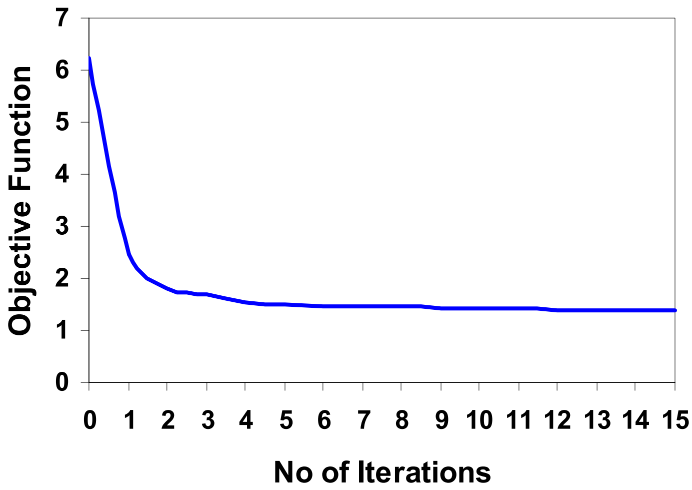

Figure 3. Objective function vs no. of iterations: Gautrans Model.

Probably the most popular transportation modelling program in the world, EMME/2, (more than 1500 licences worldwide) provides the following three stopping criteria as defaults:

- Maximum number of iterations (default = 15).
- The value, as a percentage, for the relative gap (default = 0,50). The relative gap is an estimate of the difference between the current assignment and a perfect equilibrium assignment, in which all paths used for a given O-D pair would have exactly the same time. This estimate is based on the values of the objective function.
- The value, in minutes, for the normalized gap (default = 0,50). The normalized gap, or trip time difference, is the difference between the mean trip time of the current assignment and the mean minimal trip time. The mean trip time is the average trip time used
- in the previous iteration while the mean minimal trip time is the average time computed using the shortest paths of the current iteration.

The relative gap decreases strictly from one iteration to the next, whereas the trip time difference does not necessarily have this property. In a perfect equilibrium assignment, both the relative gap and the normalized gap are zero. (4)

Boyce- et al (1) investigated the relative gap required in order to produce stable link flows when comparing two scenarios to decide whether a new project should be built or not. They concluded that a relative gap of 0,01 was required to obtain the desired level of stability in the link flows.

It should also be remembered that with the increased computing power that is now available, the cost of extra iterations is much lower than it was in 1985 when Sheffi wrote his book.

It is thought that there could be two possible stopping criteria, depending on what the results of the assignments are to be used for.

They are the following:

- The link volumes are to be used in the geometric and/or pavement design of a road.
- A decision has to be made concerning the economic benefit of a new road or a number of proposed new roads have to be ranked according to their economic benefits. These are usually calculated based on construction costs and the reduction in total travel times and distances resulting from the inclusion of the new roads in the network.

4. INVESTIGATION INTO THE EFFECT OF DIFFERENT STOPPING CRITERIA

In investigating the effect of using different stopping criteria on the results of user equilibrium assignments, use was made of the Gautrans Model (also known as the Vectura Model) and its predecessor the 1985 PWV Model. This investigation was carried out using the EMME/2 computer program. The macro writing capabilities of this program made it easy to write a program that could test a number of alternatives overnight and over weekends without having to set up each alternative manually.

In order to test the two possible cases mentioned above, two investigations were carried out.

These were the following:

- A proposed network for the year 2000 had been developed as part of a previous study. This network included those roads that should have been constructed already in order to relieve congestion on the network. These proposed roads were evaluated by removing them from the network, one at a time, and then doing a trip assignment. A benefit-cost ratio for each road was then obtained by calculating the cost of the difference in total travel times and distances that resulted from removing the roads from the network and the estimated costs of constructing the roads. The proposed roads were then ranked according to their benefit-cost ratios. This ranking was carried out using different stopping criteria to see when these rankings stabilized.
- The trip matrix for the year 2000 was assigned on the proposed 2000 network using different stopping criteria. The assigned volumes were compared to screen line counts that were done in 2000. In addition an analysis was carried out in order to determine how the assigned link volumes obtained with different numbers of iterations and different values of relative gap differed from those for the ultimate equilibrium assignment. Two different models were used in this exercise, the 1985 PWV Model and the existing Gautrans Model. Although both models cover the same area, they are different in that they have different volume delay functions and as a result different rates of convergence.

4.1 Effect of number of iterations on the economic evaluation of projects

Boyce et al (1) in their analysis used relative gaps differing by a factor of 10 (i.e. 10, 1, 0,1, etc) and then showed the differences in volumes that were obtained graphically in arriving at their conclusion that a relative gap of 0,01 was required for stable results. The purpose of this part of the study was to use benefit cost ratios of different projects to have a more quantitative evaluation and to evaluate what happens for intermediate values of relative gap.

A total of 32 different projects were proposed to alleviate the congestion on the provincial road network in Gauteng. The economic value of these projects was evaluated by comparing the total vehicles hours and vehicle kilometres on the network with and without the projects in the network. Benefit costs ratios for each project were calculated using average values for time and vehicle operating costs and the estimated costs of the projects. The projects were then ranked according to the resulting benefit cost ratios. This was done for different numbers of iterations and relative gap so as to determine at what point the ranking became stable and/or when they were the same as obtained with the ultimate equilibrium solution. The "ultimate" equilibrium solution was obtained by using a very large number for the number of iterations and very small values to the relative and normalized gaps as stopping criteria. Eventually the computation stops when the magnitude of the improvement in the objective function of the optimization problem is smaller than the precision with which the value of the objective function is stored in the computer. This occurs before the very small values of the stopping criteria are reached. According to the developer of EMME/2, this can be assumed to be the "ultimate" equilibrium solution (Spiess, 6).

The results obtained using between 15 and 30 iterations are shown in Table 1. Table 2 shows the rankings obtained with different values of the relative gap. For reasons of conciseness, the different projects have been assigned numbers from 1 to 32, and only the top 15 in the rankings are included in the tables. For example, in Table 1, project number 12 was the top ranked project after 15, 16 and 17 iterations, while project 7 was ranked second after 15 iterations but only ranked fourth after 16 or 17 iterations.

Table 1. Ranking of projects for different numbers of iterations.

		Number of Iterations															
		15	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
	1	12	12	12	25	12	25	25	25	28	28	5	30	30	30	30	30
	2	7	25	25	12	25	12	12	28	5	5	30	25	25	25	25	25
	3	25	10	10	7	7	10	28	12	12	25	25	5	32	32	32	32
	4	10	7	7	28	10	7	7	10	10	30	17	32	5	5	5	5
R	**5**	28	28	28	10	17	17	10	17	11	12	10	28	7	7	11	17
a	**6**	17	17	17	17	28	28	17	7	17	10	32	7	28	11	7	11
n	**7**	23	11	23	23	23	23	23	11	22	17	7	11	11	1	17	7
k	**8**	21	23	11	11	11	21	11	5	25	22	12	22	8	24	10	10
i	**9**	11	21	21	21	21	1	21	21	21	32	28	12	12	28	19	28
n	**10**	1	1	1	1	1	11	1	22	1	7	22	21	21	22	28	19
g	**11**	31	31	31	31	14	22	22	1	23	23	23	1	1	21	16	16
	12	3	14	14	22	16	2	2	23	2	21	21	24	22	23	23	23
	13	22	20	16	16	22	14	16	2	7	2	11	23	24	17	1	24
	14	2	16	2	2	2	3	14	3	3	3	2	2	4	12	22	22
	15	14	2	3	3	3	14	3	24	32	18	3	3	23	2	2	2

The results shown in Table 1 show that the ranking of the different projects varies widely depending on the number of iterations carried out. An example of this is project number 7, whose position in the rankings fluctuates considerably. Another example is project 30, it is ranked first when 26 to 30 iterations are performed, but only appears in the top fifteen after 24 iterations. In fact, when the number of iterations are from 15 to 21, project 30 is the worst ranked of all the projects, and ends up being ranked 28th when a relative gap 0f 0.01 is used as the stopping criterion.

Table 2. Ranking of projects for different values of relative gap.

		Relative Gap														
		1,0	**0,5**	**0,4**	**0,3**	**0,2**	**0,10**	**0,08**	**0,07**	**0,06**	**0,05**	**0,04**	**0,03**	**0,02**	**0,01**	**Ult**
	1	11	25	17	7	7	25	25	25	25	25	25	25	25	25	25
	2	25	17	10	25	17	7	7	7	7	7	7	7	7	7	7
	3	7	10	7	12	25	17	17	17	17	17	17	17	17	17	17
	4	17	14	11	21	21	21	21	21	21	21	21	21	21	21	21
R	**5**	6	28	25	28	12	28	28	23	28	28	28	28	28	28	28
a	**6**	21	7	21	23	28	23	23	28	23	23	23	23	23	23	23
n	**7**	24	16	28	17	23	12	31	31	31	31	31	31	31	31	31
k	**8**	14	1	32	31	10	31	12	12	12	12	12	12	12	12	12
i	**9**	4	13	23	3	31	10	10	10	10	10	10	10	10	10	10
n	**10**	12	24	12	11	3	1	1	3	1	3	1	1	3	1	1
g	**11**	16	23	31	24	18	3	3	1	3	1	3	3	1	3	3
	12	28	21	1	18	1	16	18	11	11	11	11	16	11	11	11
	13	1	4	3	26	32	24	11	16	16	16	16	11	16	16	16
	14	22	31	14	16	24	11	24	18	24	24	24	24	24	24	24
	15	31	18	13	32	11	18	16	24	18	6	6	18	18	18	18

Looking at Table 2, it can be seen that the rankings obtained using a relative gap of 0.01 are the same as the “ultimate” or final equilibrium assignments. The final equilibrium results were obtained by using a large number of iterations and very small values for the relative and normalized gaps as stopping criteria. For relative gaps of 0.02 and 0.03, the rankings of some projects (1 and 3, 11and 16) alternate in the rankings. It could be argued that the benefit-cost ratios for these projects are almost identical and that therefore these differences are insignificant. With a relative gap of 0,04 the differences in the rankings become more pronounced (project 6 appears in top 15 and there more differences further down the rankings.

The results shown in Tables 1 and 2 and discussed above therefore appear to confirm the conclusions of Boyce et al (1). However, there may be cases where time limitations justify the use of slightly less stringent stopping criteria (a relative gap of 0,02 or 0,03). The question of time requirements will be addressed later in this paper.

4.2 The effect of stopping criteria on the correlation between modelled volumes and traffic counts

As shown above, in order to obtain stable assignment results to be used in the economic evaluation of projects it is necessary to use very small values of the relative gap for the stopping criterion. However, many traffic assignments are used to provide traffic volumes that are used to assist engineers in the design of new facilities. The question therefore is what stopping criterion would be suitable to provide reliable information for this purpose.

One way of doing this would be to compare assigned volumes with traffic counts. In 2000 a number of screen line counts were carried out for the calibration of a new model for Gauteng. A total of 79 stations were counted providing 158 directional counts that could be compared with assignments results. Once again assignments were carried out using a wide range of number of iterations and relative gaps as stopping criteria. These assignments were done using the existing network and the trip matrix for 2000 from the Gautrans model. The results of this investigation are summarized in Table 3.

Table 3. Regression results: Assigned volumes vs traffic counts for different stopping criteria.

	No of Iterations		**Relative Gap**				
	15	**30**	**1,0**	**0,5**	**0,2**	**0,1**	**0,01**
R^2	0.750	0.746	0.747	0.746	0.745	0.745	0.744
Intercept	21.237	18.517	20.960	18.475	17.664	17.386	17.959
Slope	0.867	0.867	0.864	0.866	0.866	0.866	0.865

The results shown in Table 3 indicate that the degree of convergence of the equilibrium assignment process has virtually no effect on correlation between modelled and counted volumes. This would seem to agree with Sheffi’s statement that most of the convergence takes place in the first few iterations.

The fact that the degree of convergence has little or nor effect when comparing modelled with counted volumes and yet has a significant effect in the economic evaluation of projects can be explained as follows. Consider a road operating at or near its capacity of 4000 vehicles per hour. A difference of plus or minus 100 vehicles would have a very small effect on the regression coefficients with 158 points being considered. However, the same differences would have a much larger effect on the total time travelled since the road would be operating at the steep section of its link performance function as shown in Figure 1.

The results shown in Table 3 appear to show that, when compared with counts, a relatively small number of iterations are sufficient to provide reliable estimates of traffic volumes.

However, it should be remembered that only a small proportion of the total number of links are included in the comparison. The question that then arose was: how close to the final equilibrium volumes are the link volumes when different stopping criteria are used? It was thought that it would be useful to produce a table that would provide modellers with an idea of the level of confidence they could have in their results compared to the final equilibrium solution.

In order to obtain answers that might be generally applicable, it was decided to test more than a single scenario. Two different models were used, the 1985 PWV Study and the Gautrans models. Although both models modelled the same area, they are different models having different volume delay functions and matrices. As a result, their convergence characteristics are very different. In addition, six different scenarios using each model were investigated. These different scenarios were obtained by using different networks and different matrices to give different congestion levels (e.g. matrices for 1985, 2000 and 2010 assigned to 2000 network). The results of this analysis are shown in Table 4.

An example of how Table 4 should be understood is that with a relative gap of 0,5, 94,9 per cent of the links will have assigned volumes within 10 per cent of those for the final equilibrium solution. It is believed that this table could provide modellers with an indication of the stopping criterion to use so as to obtain a required level of confidence in their results.

Table 4. Cumulative percentage within given percentage of "Equilibrium" link volumes.

	Iterations		**Relative Gap**							
Cum %	**15**	**30**	**1,,0**	**0,5**	**0,4**	**0,3**	**0,2**	**0,1**	**0,05**	**0,01**
0	16,5	21,7	19,2	23,4	25,0	27,4	30,8	37,7	45,8	69,6
1	33,7	47,8	45,0	55,5	59,0	64,0	70,5	80,5	88,3	97,3
2	46,7	63,5	61,5	71,9	74,9	79,1	83,7	90,2	94,5	98,9
5	69,7	83,0	82,1	88,4	90,0	92,0	94,1	96,7	98,3	99,7
10	83,9	91,9	91,8	94,9	95,7	96,7	97,6	98,7	99,4	99,9
25	94,4	97,5	97,5	98,5	98,7	99,1	99,4	99,7	99,8	100
50	97,7	98,9	99,0	99,4	99,6	99,7	99,8	99,9	99,9	100
100	99,0	99,5	99,6	99,8	99,8	99,9	99,9	100	100	100

Table 3 indicates that the stopping criterion used has very little effect on the correlation between counts and assigned volumes. However, Table 4 shows that if one wants the assigned volumes to be near the equilibrium volumes then more stringent stopping criteria should be used. Since modellers generally refer the results as being from an equilibrium assignment they should be fairly close to the actual equilibrium solution. For design purposes, it is not necessary that they be very close, a relative gap of 0,1 or 0,2 would be probably be sufficient since they would mean that approximately 95 per cent of the link volumes are within five per cent of the equilibrium solution.

The results obtained using the number of iterations as the stopping criteria varied much more widely between the different scenarios tested than did those using relative gap. This suggests that relative gap should rather be used as a stopping criterion since it more likely to result in similar degrees of convergence for different models.

4.3 Computational effort

The equilibrium assignment process is an iterative process and as such the improvement resulting from each successive iteration gets progressively smaller. Therefore, the relationship between the relative gap and the number of iterations is not a linear one. Table 5 shows the time taken to achieve different relative gaps.

It should be noted that these are specific to the Gautrans Model as it existed at the time of the investigation, (668 zones, 7326 nodes including centroids and 18133 links), and the computer on which the model was run (1,20 Ghz processor). However, the table could be used to give a modeller an indication of the times required once they had done an assignment using one of the stopping criteria, for example relative gap = 0.5. Boyce et al (1) stated that they thought that a reasonable solution time for a large scale network would be an overnight run, or up to 12 – 14 hours. Using the EMME/2 macro language it is easy to test a number of alternatives overnight or over a weekend.

Table 5. Computational effort required for different stopping criteria (Gautrans Model).

Relative Gap	No. of Iterations	Time (min:sec)
1,90	15	0:56
0,50	40	2:24
0,40	45	2:42
0,30	55	3:16
0,20	72	4:16
0,10	113	6:44
0,05	184	10:58
0,04	218	13:26
0,03	275	16:28
0,02	376	22:12
0,01	692	40:10

5. CONCLUSIONS AND RECOMMENDATIONS

The conclusions reached from this investigation are the following:

- Modellers should be very wary about using default values for stopping criteria when doing assignments.
- An objective measure such as the relative gap is preferable to the number of iterations as a stopping criterion.
- Different stopping criteria can be used depending on the purpose of the assignment (economic evaluation or inputs for design).
- The speed of modern computers means that doing extra iterations is no longer the drawback that it used to be.

The following recommendations are made:

- When doing assignments where the results will be used in economic evaluation, a relative gap of 0,01 is the most appropriate stopping criterion.
- If the results of the assignment are to be used as input in the design of roads, the values contained in Table 4 should be used as indication of the relative gap needed to provide the required confidence that the results are close to the equilibrium solution.
- The number of iterations should not be used as a stopping criterion in equilibrium assignment.

The views expressed in this paper are those of the author, and do not necessarily reflect those of either VKE Consulting Engineers or Gautrans.

6. ACKNOWLEDGEMENTS

The Gauteng Department of Public Transport, Roads and Works (Gautrans) for their permission to use their models in the research and to publish the results. The work described in this paper forms part of the work being done for a Master of Science degree in Operations Research at Unisa.

7. REFERENCES.

[1] Boyce, D., Ralevic-Dekic, B. and Bar-Gera, H, 1988 Convergence of Traffic Assignments: How Much is Enough? The Delaware Valley Case Study. Presented at the 16th Annual International EMME/2 Users' Conference, Albuquerque, New Mexico.

[2] Beckmann, M.J., McGuire, C.B., and Winsten, C.B, 1956. Studies in the Economics of Transportation, Yale University Press, New Haven, Connecticutt.

[3] Evans, S.P, 1976. Derivation and analysis of some models for combining trip distribution and assignment. Transportation Research, vol. 10, p. 37-57.

[4] INRO Consultants,1998 EMME/2 User's Manual Release 9. Montreal, Canada.

[5] Sheffi, Y, 1985. Urban Transportation Networks. Englewood Cliffs, New Jersey: Prentice-Hall, 398p.

[6] Spiess, H, 2003. Personal communication.

[7] Thomas, R, 1991. Traffic Assignment Techniques. Avebury, Aldershot, England, 385p.

[8] Wardrop, J.G, 1952. Some theoretical aspects of road traffic research. Proceedings of the Institute of Civil Engineering, Part II, vol. 1, p. 325-378.

THE STABILITY OF ASSIGNMENT RESULTS

Bloy, L.A.K.

VKE Consulting Engineers.

Biography

Matriculated at Dundee High School (1967).
B Sc. Civil Engineering – University of Natal (1972)
M S (Traffic & Transportation) – University of Florida (1980)
B Sc. and Honours B Sc (Operations Research) – Unisa (1989 and 1992 respectively)
Joined the then National Institute for Road Research of the CSIR in 1972. Seconded to the then TPA Roads Department in Pietersburg (now Polokwane) in 1973 and 1974. Joined VKE in September 1975. Has been there ever since, apart from the year spent at the University of Florida. Initially in the Geometric Design Section and since 1983 in the Transportation Section.
Registered as both a Professional Engineer and a Professional Natural Scientist.