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Tricorns and Multicorns in Noor Orbit With s-Convexity

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ABSTRACT In today's world, complex patterns of the dynamical framework have astounding highlights of fractals and become a huge field of research because of their beauty and unpredictability of their structure. The purpose of this paper is to visualize anti-Julia sets, tricorns, and multicorns by means of the Noor iteration with s-convexity. Various patterns are displayed to investigate the geometry of antifractals for antipolynomial $\bar{z}^{k+1} + c$ of complex polynomial $z^{k+1} + c$, for $k \geq 1$ in Noor orbit with s-convexity.

INDEX TERMS Noor iteration, s-convexity, Julia set, tricorn, escape criterion.

I. INTRODUCTION

In 1918, French mathematician Julia [1] attained a Julia set by exploring the iteration procedure of complex mapping $z \to z^2 + c$, here c is a complex number. The object Mandelbrot set presented by Mandelbrot in 1979 by utilizing c as a complex parameter in complex mapping $z \to z^2 + c$ [2]. In 1983, Crowe *et al.* [3] examined in formal closeness with Mandelbrot set and called it "Mandelbar sets" and exhibited its appearance bifurcations on circular segments rather at points. Milnor instituted the word "Tricorn" for the connectedness locus for antiholomorphic polynomials $\bar{z}^2 + c$, which plays out a transitional job between quadratic and cubic polynomials [4]. The three-cornered nature, the fundamental characteristic of a tricorn, repetition with deviation at distinct scales, follow the similar kind of self-similarity as the Mandelbrot set.

Winters deciphered that boundary of the tricorn comprise of a smooth curve [5]. Lau and Schleicher [6] investigated the symmetries of tricorn and multicorns. Nakane and Schleicher [7] considered different qualities of tricorn and multicorns and extricated that the multicorns are generalized tricorns. They also examined that the Julia set of antipolynomial $A_c(z) = \bar{z}^{k+1} + c$ for $k \geq 1$, either connected or disconnected and if the Julia set of A_c is connected then the arrangement of similar parameters c is known as the multicorn. Tricorn prints, for example, tricorn coffee cups, containers and tricorn T-shirts are being utilized for business reason.

These antifractals have been generalized in a few distinctive ways. One of these speculations is the use of different fixed point iterative procedures from the fixed point hypothesis. In the fixed point hypothesis there exist many estimated techniques for discovering fixed points of a given mapping, that depend on the utilization of various feedback iteration procedures. These procedures can be utilized in the generalization of antifractals. Rani [8], [9] studied and explored the dynamics of antiholomorphic complex polynomials $\bar{z}^{k+1} + c$ for $k \geq 1$, by using Mann iteration which is a one-step iteration process. Chauhan *et al.* [10] introduced relative superior tricorns and relative superior multicorns via Ishikawa iteration which is a two-step iteration process. Antifractals have been studied extensively by Rani and Chugh [11], Kang *et al.* [12] and Partap *et al.* [13] for various fixed point iteration processes. The association of s-convex combination [14] and different iteration procedures examined in a few papers. Mishra *et al.* [15] got fixed point results in formation of tricorns and multicorns through Ishikawa iteration technique with s-convex combination. Nazeer *et al.* [16] handled the Jungck-Mann and Jungck-Ishikawa iteration procedures and Kang *et al.* [17] presented new fixed point results for formation of fractals with s-convexity in

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Jungck-Noor orbit. In [18] Noor iteration and s-convexity used to generate Mandelbrot sets and Julia sets. Recently, Kwun *et al.* [19] presented Mandelbrot sets, Julia sets and tricorns and multicorns via Jungck-CR iteration with s-convexity.

In this article we present and exhibit another class of tricorns, multicorns and ant-Julia sets by means of Noor iteration process with s-convex combination which is a further generalization of Noor iteration. The results of this paper are the extension of results presented in [18].

This paper is organized as: In section II we present some fundamental definitions. Section III contains the escape criterion for tricorn and multicorns by means of Noor iteration process with s-convexity. In section IV we visualize images of anti-Julia sets, tricorns and multicorns by utilizing proposed three-step iterative procedure with s-convex combination. Finally, section V contains some concluding comments.

II. PRELIMINARIES

Definition 1: (Multicorn [20]) Let $A_c(z) = \bar{z}^m + c$ where $c \in \mathbb{C}$. The multicorn A^* for A_c is defined as the collection of all $c \in \mathbb{C}$ for which the orbit of 0 under the action of A_c is bounded, i.e.,

$$A^* = \{c \in \mathbb{C} : A_c^n(0) \text{ does not tend to } \infty\}$$

where $\mathbb{C}$ is a complex space, A_c^n is the nth iterate of the function $A_c(z)$.

It is noticed that at $m = 2$, multicorns reduce to tricorn.

Definition 2: (Julia set [21]) Let $f : \mathbb{C} \longrightarrow \mathbb{C}$ symbolize a polynomial of degree ≥ 2. Let F_f be the set of points in C whose orbits do not converge to the point at infinity. That is, $F_f = \{x \in \mathbb{C} : \{|f^n(x)|, n \text{ varies from } 0 \text{ to } \infty\}$ is bounded$\}$. F_f is called as filled Julia set of the polynomial f. The boundary points of F_f is called as the points of Julia set of the polynomial f or simply the Julia set.

Definition 3: (Mandelbrot set [20]) The Mandelbrot set M consists of all parameters c for which the filled Julia set of Q_c is connected, that is

$$M = \{c \in \mathbb{C} : K(Q_c) \text{ is connected }\}.$$

The Mandelbrot set M for the quadratic $Q_c(z) = z^2 + c$ is defined as the collection of all $c \in \mathbb{C}$ for which the orbit of the point 0 is bounded, that is

$$M = \{c \in \mathbb{C} : \{Q_c^n(0)\}; n = 0, 1, 2, \ldots \text{is bounded}\}.$$

We determine the initial point 0 that is the only critical point of Q_c.

Definition 4: Let $T : \mathbb{C} \to \mathbb{C}$ is a mapping. Then Picard iteration process is defined by the following sequence $\{x_n\}$:

$$\begin{cases} x_0 \in \mathbb{C}, \\ x_{n+1} = Tx_n, \; n \geq 0, \end{cases} \tag{1}$$

Definition 5: Let $T : \mathbb{C} \to \mathbb{C}$ be a mapping. The Mann iteration process [22] is defined by the following sequence $\{x_n\}$:

$$\begin{cases} x_0 \in \mathbb{C}, \\ x_{n+1} = (1 - \eta_n^1)x_n + \eta_n^1 Tx_n, \quad n \geq 0, \end{cases} \tag{2}$$

where $\eta_n^1 \in (0, 1]$.

Definition 6: Let $T : \mathbb{C} \to \mathbb{C}$ is a mapping. Then Ishikawa iteration process [23] is defined by the following sequence $\{x_n\}$:

$$\begin{cases} x_0 \in \mathbb{C}, \\ x_{n+1} = (1 - \eta_n^1)x_n + \eta_n^1 Ty_n, \\ y_n = (1 - \eta_n^2)x_n + \eta_n^2 Tx_n, \; n \geq 0, \end{cases} \tag{3}$$

where $\eta_n^1 \in (0, 1]$ and $\eta_n^2 \in [0, 1]$.

Definition 7: (Noor iteration [24]) Let $T : \mathbb{C} \to \mathbb{C}$ is a mapping. Then a sequence $\{z_n\}$ of iterates for initial point $z_0 \in \mathbb{C}$ such that

$$\begin{cases} z_{n+1} = (1 - \eta_n^1)z_n + \eta_n^1 Tu_n; \\ u_n = (1 - \eta_n^2)z_n + \eta_n^2 Tv_n; \\ v_n = (1 - \eta_n^3)z_n + \eta_n^3 Tz_n; \; n \geq 0, \end{cases}$$

where $\eta_n^1 \in (0, 1]$ and $\eta_n^2, \eta_n^3 \in [0, 1]$. The above sequence is called Noor orbit, that is a function of five tuples $(T, z_0, \eta_n^1, \eta_n^2, \eta_n^3)$.

It is noticed that Noor iteration diminishes to the:

- Ishikawa iteration for $\eta_n^3 = 0$,
- Mann iteration for $\eta_n^3 = \eta_n^2 = 0$.

In the literature convex combination has been generalized in different manners. The s-convex combination is one of them.

Definition 8: (s-convex combination [14]) Let $z_1, z_2, \ldots, z_n \in C$ and $s \in (0, 1]$. The s-convex combination is defined in the following way:

$$\lambda_1^s z_1 + \lambda_2^s z_2 + \ldots + \lambda_n^s z_n, \tag{4}$$

where $\lambda_k \geq 0$ for $1 \leq k \leq n$ and $\sum_{k=1}^{n} \lambda_k = 1$.

It is seen that for $s = 1$ the s-convex combination changed to the standard convex combination. We take $z_o = z \in \mathbb{C}$, $\eta_n^1 = \eta_1, \eta_n^2 = \eta_2$ and $\eta_n^3 = \eta_3$ then the Noor iteration scheme with s-convex combination can be write in the following way, where $Q_c(\bar{z}_n)$ be a quadratic, cubic or $(k + 1)$th degree function.

$$\begin{cases} z_{n+1} = (1 - \eta_1)^s z_n + \eta_1^s Q_c(\bar{u}_n), \\ u_n = (1 - \eta_2)^s z_n + \eta_2^s Q_c(\bar{v}_n), \\ v_n = (1 - \eta_3)^s z_n + \eta_3^s Q_c(\bar{z}_n), \; n \geq 0, \end{cases} \tag{5}$$

where $\eta_1, s \in (0, 1]$ and $\eta_2, \eta_3 \in [0, 1]$. The formula (5) used to obtain anti-Julia sets, tricorns and multicorns. First of all, we establish escape criterion to generate antifractals in Noor orbit with s-convexity.

III. ESCAPE CRITERION

There exists two distinct kinds of points in functional dynamics. First type of points exists in a stable set of infinity which escape the interval after a limited number of iterations the set of these points is called the escape set and second kind of points never escape the interval after any number of iterations the set of such points is known a prisoner set. These sets perform important role in choosing the escape criterion of polynomials under different fixed point iterative procedures. These escape criteria are important to create the antifractals which are at the core of different applications in PC illustrations. We establish a generalized escape criterion for antipolynomials $Q_c(\bar{z}) = \bar{z}^{k+1} + c$ where $k \geq 1$, in modified Noor orbit.

Theorem 1: If $|z| \geq |c| > (\frac{2}{s\eta_1})^{1/k}$, $|z| \geq |c| > (\frac{2}{s\eta_2})^{1/k}$ and $|z| \geq |c| > (\frac{2}{s\eta_3})^{1/k}$ where c be a complex number. Let $z_\circ = z$, $u_\circ = u$ and $v_\circ = v$ then sequence $\{z_n\}$ define as

$$\begin{cases} z_{n+1} = (1-\eta_1)^s zn_n + \eta_1^s Qn_c(\bar{u}_n), \\ u_n = (1-\eta_2)^s znn_n + \eta_2^s Qn_c(\bar{v}_n), \\ v_n = (1-\eta_3)^s zn_n + \eta_3^s Qnn_c(\bar{z}_n), \ n \geq 0, \end{cases} \tag{6}$$

where $0 < \eta_1, s \leq 1$ and $0 \leq \eta_2, \eta_3 \leq 1$. Then $|z_n| \to \infty$ as $n \to \infty$.

Proof: Suppose $Q_c(\bar{z}) = \bar{z}^{k+1} + c$, $|z| \geq |c| > (\frac{2}{s\eta_1})^{1/k}$, $|z| \geq |c| > (\frac{2}{s\eta_2})^{1/k}$ and $|z| \geq |c| > (\frac{2}{s\eta_3})^{1/k}$ exists then

$$|v| = \left|(1-\eta_3)^s znn + \eta_3^s(\bar{z}^{k+1} + c)\right|.$$

Since $0 < s \leq 1$ and $0 \leq \eta_3 \leq 1$, therefore $\eta_3^s \geq s\eta_3$

$$\begin{aligned} |v| &\geq \left|(1-\eta_3)^s znn + s\eta_3(\bar{z}^{k+1} + c)\right| \\ &\geq \left|(1-\eta_3)^s zn + s\eta_3\bar{z}^{k+1}\right| - |s\eta_3 c| \\ &\geq \left|s\eta_3\bar{z}^{k+1} + (1-\eta_3)^s zn\right| - |s\eta_3 zn| \because |z| \geq |c|) \\ &\geq \left|s\eta_3\bar{z}^{k+1}\right| - \left|(1-\eta_3)^s znn\right| - |s\eta_3 zn|. \end{aligned}$$

By using binomial expansion preferable linear terms of η_3, we obtain

$$\begin{aligned} |v| &\geq \left|s\eta_3\bar{z}^{k+1}\right| - |(1-s\eta_3)z| - |s\eta_3 znn| \\ &\geq \left|s\eta_3\bar{z}^{k+1}\right| - |z| + |s\eta_3 zn| - |s\eta_3 z| \\ &\geq \left|s\eta_3\bar{z}^{k}\right| |\bar{z}| - |z| \\ &\geq \left|s\eta_3\bar{z}^{k}\right| |z| - |z|, (\because |\bar{z}| = |z|) \\ &\geq |z| (s\eta_3 |\bar{z}|^k - 1). \end{aligned} \tag{7}$$

And

$$\begin{aligned} |u| &= \left|(1-\eta_2)^s z + \eta_2^s Q_c(\bar{v})\right| \\ &= \left|(1-\eta_2)^s z + \eta_2{}^s(\bar{v}^{k+1} + c)\right|. \end{aligned} \tag{8}$$

Since $0 < s \leq 1$ and $0 \leq \eta_2 \leq 1$, therefore $\eta_2^s \geq s\eta_2$

$$\begin{aligned} |u| &\geq \left|(1-\eta_2)^s z + s\eta_2(\bar{v}^{k+1} + c)\right| \\ &\geq \left|(1-\eta_2)^s z + s\eta_2((|\bar{z}| (s\eta_3 \left|\bar{\bar{z}}\right|^k - 1))^{k+1} + c)\right| \\ &\geq \left|(1-\eta_2)^s z + s\eta_2((|z| (s\eta_3 |z|^k - 1))^{k+1} + c)\right|. \end{aligned} \tag{9}$$

Since $|z| > (\frac{2}{s\eta_3})^{1/k}$ implies

$$s\eta_3 |z|^k - 1 > 1,$$

also

$$(s\eta_3 |z|^k - 1)^{k+1} > 1,$$

and

$$|z|^{k+1} (s\eta_3 |z|^k - 1)^{k+1} > |z|^{k+1}.$$

By using this in (9) we have

$$\begin{aligned} |u| &\geq \left|(1-\eta_2)^s z + s\eta_2(|z|^{k+1} + c)\right| \\ &\geq \left|s\eta_2 z^{k+1} + (1-\eta_2)^s zn\right| - |s\eta_2 cn| \\ &\geq \left|s\eta_2 znn^{k+1} + (1-\eta_2)^s zn\right| - |s\eta_2 zn|, (\because |z| \geq |c|) \\ &\geq \left|s\eta_2 znn^{k+1}\right| - \left|(1-\eta_2)^s zn\right| - |s\eta_2 zn|. \end{aligned}$$

By using binomial expansion preferable linear terms of η_2, we obtain

$$\begin{aligned} |u| &\geq \left|s\eta_2 z^{k+1}\right| - |(1-s\eta_2)z| - |s\eta_2 z| \\ &\geq \left|s\eta_2 znn^{k+1}\right| - |z| + |s\eta_2 znn| - |s\eta_2 znn| \\ &\geq |z| (s\eta_2 |z|^k - 1). \end{aligned} \tag{10}$$

Also for

$$\begin{aligned} z_1 &= (1-\eta_1)^s zn + \eta_1^s Qn_c(\bar{u}) \\ |z_1| &= \left|(1-\eta_1)^s znn + \eta_1^s(\bar{u}^{k+1} + c)\right|. \end{aligned}$$

Since $0 < s, \eta_1 \leq 1$, therefore $\eta_1^s \geq s\eta_1$

$$\begin{aligned} |z_1| &\geq \left|(1-\eta_1)^s zn + s\eta_1(\bar{u}^{k+1} + c)\right| \\ &\geq \left|(1-\eta_1)^s zn + s\eta_1((|\bar{z}| (\eta_2 |\bar{z}|^k - 1))^{k+1} + c)\right| \\ &\geq \left|(1-\eta_1)^s znn + s\eta_1((|z| (\eta_2 |z|^k - 1))^{k+1} + c)\right|. \end{aligned} \tag{11}$$

Since $|z| > (\frac{2}{s\eta_2})^{1/k}$ implies

$$(s\eta_2 |z|^k - 1)^{k+1} > 1,$$

and

$$|z|^{k+1} (s\eta_2 |z|^k - 1)^{k+1} > |z|^{k+1}.$$

By using this in (11) we have

$$\begin{aligned} |z_1| &\geq \left|(1-\eta_1)^s znn + s\eta_1(|z|^{k+1} + c)\right| \\ &\geq \left|s\eta_1 zn^{k+1} + (1-\eta_1)^s z\right| - |s\eta_1 c| \\ &\geq \left|s\eta_1 zn^{k+1} + (1-\eta_1)^s znn\right| - |s\eta_1 znn|, (\because |z| \geq |c|) \\ &\geq \left|s\eta_1 zn^{k+1}\right| - \left|(1-\eta_1)^s zn\right| - |s\eta_1 zn|. \end{aligned}$$

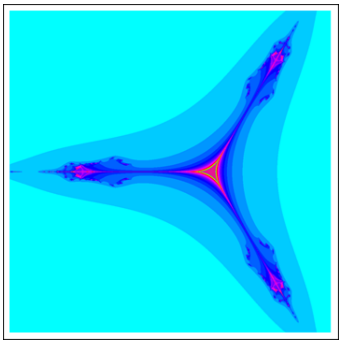

FIGURE 1. Tricorn generated in modified Noor orbit.

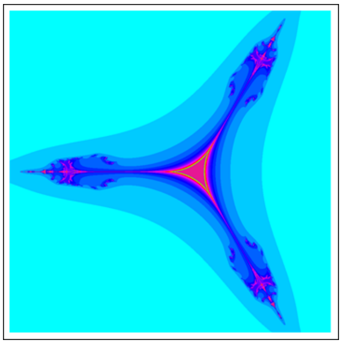

FIGURE 2. Tricorn generated in modified Noor orbit.

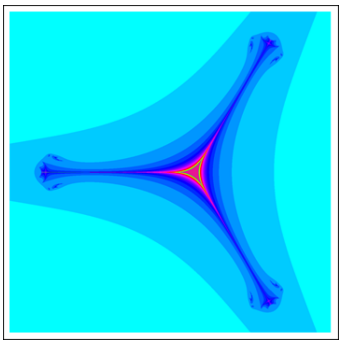

FIGURE 3. Tricorn generated in modified Noor orbit.

By using binomial expansion preferable linear terms of η_1, we obtain

$$\begin{aligned} |z_1| &\geq \left|s\eta_1 znn^{k+1}\right| - |(1-s\eta_1)z| - |s\eta_1 zn| \\ &\geq \left|s\eta_1 znn^{k+1}\right| - |z| + |s\eta_1 znn| - |s\eta_1 zn| \\ &\geq |z|\,(s\eta_1\,|z|^k - 1). \end{aligned} \tag{12}$$

Since $|z| > (\frac{2}{s\eta_1})^{1/k}$ implies $s\eta_1\,|z|^k - 1 > 1$, there exist a number $\mu > 0$, in such a way $s\eta_1|z|^k - 1 > 1 + \mu > 1$. Therefore

$$\begin{aligned} |z_1| &> (1+\mu)\,|z|\,, \\ |z_2| &> (1+\mu)^2\,|z|\,, \\ &\vdots \\ |z_n| &> (1+\mu)^n\,|z|\,. \end{aligned}$$

Hence $|z_n| \longrightarrow \infty$ as $n \to \infty$ and proved. ■

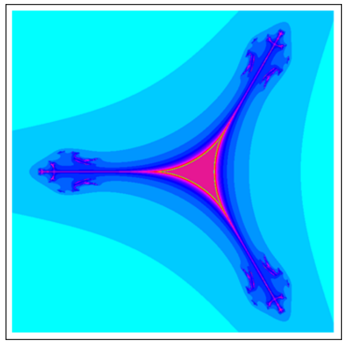

FIGURE 4. Tricorn generated in modified Noor orbit.

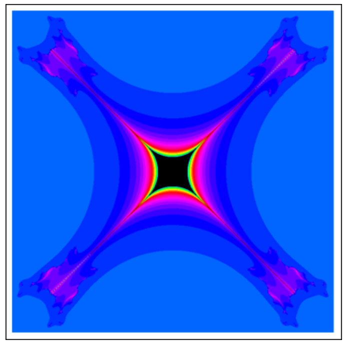

FIGURE 5. Multicorn generated in modified Noor orbit.

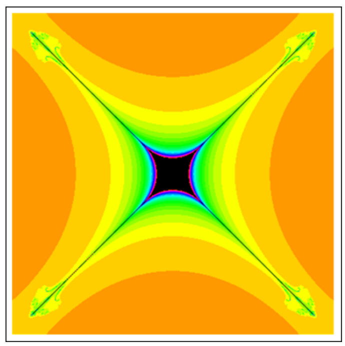

FIGURE 6. Multicorn generated in modified Noor orbit.

Corollary 1: If $|c| > (\frac{2}{s\eta_1})^{1/k}, |c| > (\frac{2}{s\eta_2})^{1/k}$ and $|c| > (\frac{2}{s\eta_3})^{1/k}$ exists, then the orbit $NSO(Q_c, 0, s\eta_1, s\eta_2, s\eta_3)$ escape to infinity.

Following corollary is refinement of escape criterion.

Corollary 2: If $|z| > \max\{|c|\,, (\frac{2}{s\eta_1})^{1/k}, (\frac{2}{s\eta_2})^{1/k}, (\frac{2}{s\eta_3})^{1/k}\}$, then $|z_n| > (1+\mu)^n\,|z|$ and $|z_n| \longrightarrow \infty$ as $n \to \infty$.

IV. GRAPHICAL EXAMPLES

In this section tricorns and multicorns are presented for functions of the form $z \to \bar{z}^{k+1} + c$ for $k \geq 1$ via modified Noor iteration scheme. Also, anti-Julia sets are introduced for quadratic and cubic antipolynomials. To produce the images we applied the escape time algorithm with the general escape criterion implemented in the software Mathematica 9.0. Pseudocode of the tricorns and multicorns generation algorithm

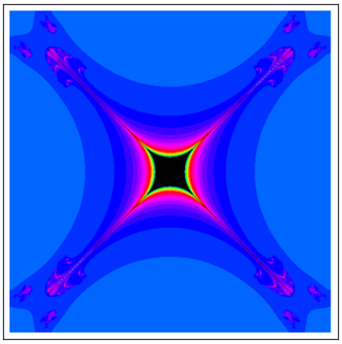

FIGURE 7. Multicorn generated in modified Noor orbit.

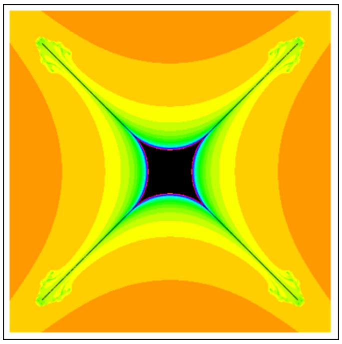

FIGURE 8. Multicorn generated in modified Noor orbit.

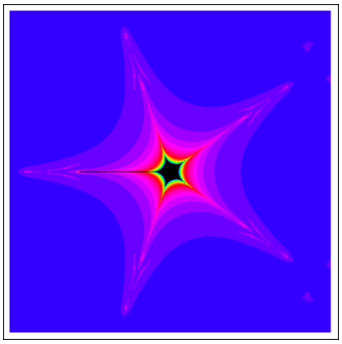

FIGURE 9. Multicorn generated in modified Noor orbit.

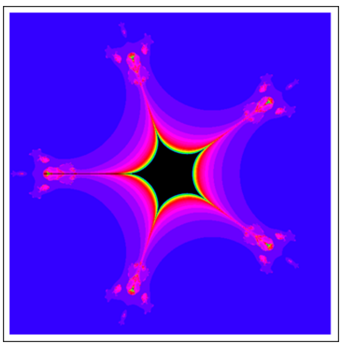

FIGURE 10. Multicorn generated in modified Noor orbit.

is exhibited in Algorithm 1, while Algorithm 2 presents the pseudocode for the anti-Julia set generation algorithm.

Algorithm 1 Generation of Tricorn and Multicorn

Input: $Q_c(\bar{z}) = \bar{z}^{k+1} + c$, where $c \in \mathbb{C}$ and $k \geq 1$, $A \subset \mathbb{C}$ – area, K – iterations, $\eta_1, s \in (0,1]$ and $\eta_2, \eta_3 \in [0,1]$ – parameters for the modified Noor iteration, *colourmap*$[0..H-1]$ – with H colours.

Output: Tricorn or Multicorn for the area A.

for $c \in A$ **do**
- $R = \max\{|c|, (\frac{2}{s\eta_1})^{1/k}, (\frac{2}{s\eta_2})^{1/k}, (\frac{2}{s\eta_3})^{1/k}\}$,
- $n = 0$
- $z_0 = 0$
- **while** $n \leq K$ **do**
  - $v_n = (1-\eta_3)^s zn_n + \eta_3^s Qn_c(\bar{z}_n)$
  - $u_n = (1-\eta_2)^s zn_n + \eta_2^s Qnnn_c(\bar{v}_n)$
  - $z_{n+1} = (1-\eta_1)^s znnn_n + \eta_1^s Qn_c(\bar{u}_n)$
  - **if** $|z_{n+1}| > R$ **then**
    - break
  - $n = n + 1$
- $i = \lfloor (H-1)\frac{n}{K} \rfloor$
- colour c with *colourmap*$[i]$

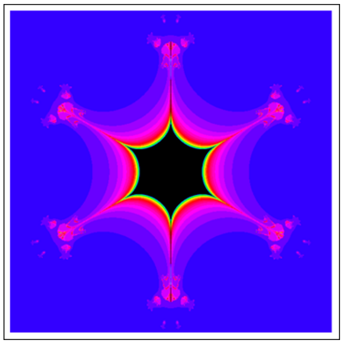

FIGURE 11. Multicorn generated in modified Noor orbit.

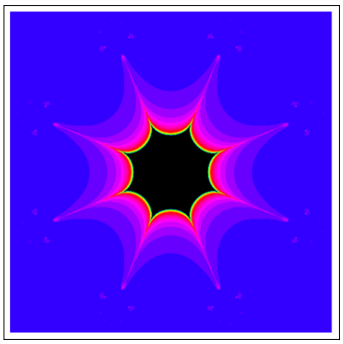

FIGURE 12. Multicorn generated in modified Noor orbit.

A. TRICORNS FOR QUADRATIC FUNCTION $Q_c(\bar{z}) = \bar{z}^2 + c$

In Figs. 1–4, tricorns and multicorns are presented for function $z \to \bar{z}^2 + c$ in modified Noor orbit by taking maximum number of iterations 30 and varying parameters are following:

Algorithm 2 Generation of Anti-Julia Set

Input: $Q_c(\bar{z}) = \bar{z}^{k+1} + c$, where $k \geq 1$, $c \in \mathbb{C}$ and $A \subset \mathbb{C}$ – area, K – iterations, $\eta_1, s \in (0, 1]$ and $\eta_2, \eta_3 \in [0, 1]$ – parameters for the modified Noor iteration, *colourmap*$[0..H-1]$ – with H colours.

Output: Anti-Julia set for the area A.

$R = \max\{|c|, (\frac{2}{s\eta_1})^{1/k}, (\frac{2}{s\eta_2})^{1/k}, (\frac{2}{s\eta_3})^{1/k}\}$
for $z_0 \in A$ **do**
  $n = 0$
  while $n \leq K$ **do**
    $v_n = (1-\eta_3)^s znnn_n + \eta_3^s Qnnn_c(\bar{z}_n)$
    $u_n = (1-\eta_2)^s zn_n + \eta_2^s Qnn_c(\bar{v}_n)$
    $z_{n+1} = (1-\eta_1)^s znn_n + \eta_1^s Qnn_c(\bar{u}_n)$
    if $|z_{n+1}| > R$ **then**
      break
    $n = n + 1$
  $i = \lfloor (H-1)\frac{n}{K} \rfloor$
  colour z_0 with *colourmap*$[i]$

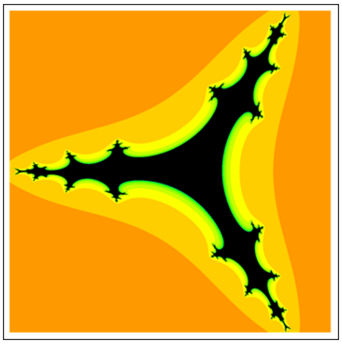

FIGURE 13. Anti-Julia set generated in Noor orbit with s-convexity.

- Fig. 1: $\eta_3 = 0.6, \eta_2 = 0.3, \eta_1 = 0.4, A = [-3.4, 2] \times [-2.7, 2.7]$ and $s = 0.6$,
- Fig. 2: $\eta_3 = 0.7, \eta_2 = 0.4, \eta_1 = 0.5, A = [-2.8, 2.0] \times [-2.4, 2.4]$ and $s = 0.7$,
- Fig. 3: $\eta_3 = 0.2, \eta_2 = 0.3, \eta_1 = 0.3, A = [-3.8, 2.8] \times [-3.3, 3.3]$ and $s = 0.7$,
- Fig. 4: $\eta_3 = 0.6, \eta_2 = 0.3, \eta_1 = 0.8, A = [-2, 1.4] \times [-1.7, 1.7]$ and $s = 0.7$.

B. MULTICORNS FOR THE FUNCTION $Q_c(\bar{z}) = \bar{z}^{k+1} + c$

In Figs. 5–12, multicorns are presented for functions $z \to \bar{z}^{k+1} + c$ for $k \geq 2$ in Noor orbit with s-convexity by taking maximum number of iterations 30 and varying parameters are following:

- Fig. 5: $\eta_3 = 0.3, \eta_2 = 0.4, \eta_1 = 0.2, A = [-1.7, 1.7]^2, s = 0.7$ and $k = 2$,
- Fig. 6: $\eta_3 = 0.6, \eta_2 = 0.7, \eta_1 = 0.3, A = [-1, 1]^2, s = 0.5$ and $k = 2$,

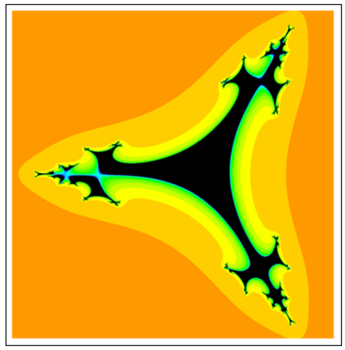

FIGURE 14. Anti-Julia set generated in modified Noor orbit.

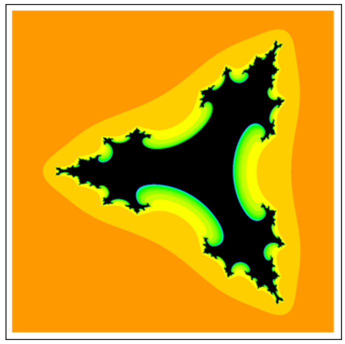

FIGURE 15. Anti-Julia set generated in modified Noor orbit.

FIGURE 16. Anti-Julia set generated in modified Noor orbit.

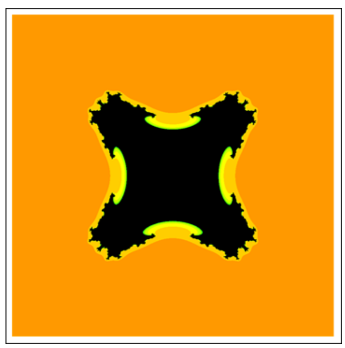

FIGURE 17. Anti-Julia set generated in modified Noor orbit.

- Fig. 7: $\eta_3 = 0.5, \eta_2 = 0.4, \eta_1 = 0.3, A = [-1.3, 1.3]^2, s = 0.6$ and $k = 2$,
- Fig. 8: $\eta_3 = 0.5, \eta_2 = 0.2, \eta_1 = 0.7, A = [-1, 1]^2, s = 0.5$ and $k = 2$,

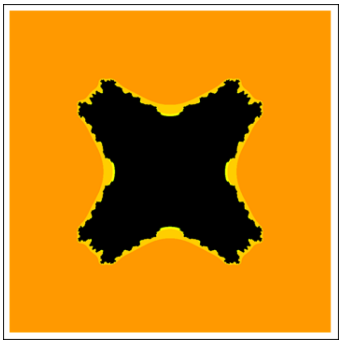

FIGURE 18. Anti-Julia set generated in modified Noor orbit.

FIGURE 19. Anti-Julia set generated in modified Noor orbit.

- Fig. 9: $\eta_3 = 0.2, \eta_2 = 0.5, \eta_1 = 0.4, A = [-1.2, 1.2]^2$, $s = 0.3$ and $k = 3$,
- Fig. 10: $\eta_3 = 0.8, \eta_2 = 0.5, \eta_1 = 0.4, A = [-1.4, 1.4]^2$, $s = 0.6$ and $k = 3$,
- Fig. 11: $\eta_3 = 0.5, \eta_2 = 0.3, \eta_1 = 0.4, A = [-1.4, 1.4]^2$, $s = 0.7$ and $k = 4$,
- Fig. 12: $\eta_3 = 0.2, \eta_2 = 0.3, \eta_1 = 0.6, A = [-1.3, 1.3]^2$, $s = 0.6$ and $k = 6$.

C. ANTI-JULIA SETS FOR QUADRATIC FUNCTION $Q_c(\bar{z}) = \bar{z}^2 + c$

Anti-Julia sets for quadratic function are presented in modified Noor orbit in Figs. 13 –16. The basic parameters used to create the pictures are: $K = 30$ and $c = 0.02 + 0.04\mathbf{i}$. While, the changing parameters are the following:

- Fig. 13: $\eta_3 = 0.3, \eta_2 = 0.4, \eta_1 = 0.6, A = [-3, 3] \times [-3.5, 2.5]$ and $s = 0.6$,
- Fig. 14: $\eta_3 = 0.3, \eta_2 = 0.8, \eta_1 = 0.4, A = [-3.2, 2.1] \times [-2.7, 2.7]$ and $s = 0.5$,
- Fig. 15: $\eta_3 = 0.6, \eta_2 = 0.4, \eta_1 = 0.4, A = [-3, 2] \times [-2.5, 2.5]$ and $s = 0.6$,
- Fig. 16: $\eta_3 = 0.8, \eta_2 = 0.5, \eta_1 = 0.3, A = [-2.8, 2.2] \times [-2.5, 2.5]$ and $s = 0.7$.

D. ANTI-JULIA SETS FOR CUBIC FUNCTION $Q_c(\bar{z}) = \bar{z}^3 + c$

Anti-Julia sets for cubic function are obtained in modified Noor orbit in Figs. 17–20. The common parameters used to formation of images are: $K = 30$ and $c = 0.001 + 0.005\mathbf{i}$. While, the changing parameters are the following:

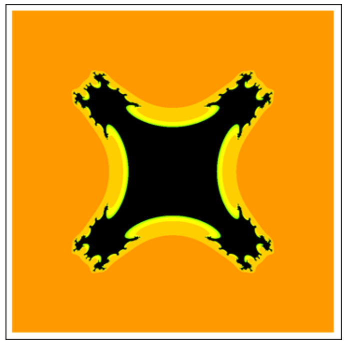

FIGURE 20. Anti-Julia set generated in modified Noor orbit.

- Fig. 17: $\eta_3 = 0.4, \eta_2 = 0.8, \eta_1 = 0.3, A = [-2.3, 2.3]^2$ and $s = 0.5$,
- Fig. 18: $\eta_3 = 0.4, \eta_2 = 0.5, \eta_1 = 0.6, A = [-2.3, 2.3]^2$ and $s = 0.7$,
- Fig. 19: $\eta_3 = 0.3, \eta_2 = 0.4, \eta_1 = 0.6, A = [-2.3, 2.3]^2$ and $s = 0.6$,
- Fig. 20: $\eta_3 = 0.2, \eta_2 = 0.6, \eta_1 = 0.5, A = [-2.3, 2.3]^2$ and $s = 0.5$.

V. CONCLUSIONS

In this paper escape criterion for antifractals has presented with respect to Noor orbit with s-convexity and visualized the pattern of symmetry among them. We attained quite different antifractals from those presnted in Noor orbit by Rani and Chugh [11]. In dynamics of antipolynomials $z \to \bar{z}^{k+1} + c$ for $k \geq 1$, we created a few examples of tricorns and multicorns for a similar estimation of k and various estimations of η_1, η_2, η_3 and s in Noor orbit with s-convexity. We observed that the quantity of branches joined to the main body of the tricorns and multicorns are $k + 2$, where $k + 1$ is the power of $\bar{z}$ for $k \geq 1$. We likewise seen that when $k + 1$ is odd the symmetry of multicorn is around x-axis and y-axis and for $k + 1$ is even the symmetry is preserved just along x-axis. Many connected anti-Julia sets presented for quadratic and cubic functions. Attractive changes can be seen in antifractals generated in Noor orbit with s-convexity for different values of η_1, η_2, η_3 and s. We believe that consequences of this paper will be impress those who are interesting in generating aesthetic graphics automatically.

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