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A
Study on
Integrated Transportation
And Facility Location Problem

By

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Abstract

The focus of this thesis is the development and solution of problems that simultaneously involve the planning of the location of facilities and transportation decisions from such facilities to consumers. This has been termed integrated distribution planning problems with practical application in logistics and manufacturing. In this integration, different planning horizons of short, medium and long terms are involved with the possibility of reaching sub-optimal decisions being likely when the planning horizons are considered separately.

Two categories of problems were considered under the integrated distribution models. The first is referred to as the Step-Fixed Charge Location and Transportation Problem (SFCLTP). The second is termed the Fixed Charge Solid Location and Transportation Problem (FCSLTP). In these models, the facility location problem is considered to be a strategic or long term decision. The short to medium-term decisions considered are the Step-Fixed Charge Transportation Problem (SFCTP) and the Fixed Charge Solid Transportation Problem (FCSTP). Both SFCTP and FCSTP are different extensions to the classical transportation problem, requiring a trade-off between fixed and variable costs along the transportation routes to minimize total transportation costs.

Linearization and subsequent local improvement search techniques were developed to solve the SFCLTP. The first search technique involved the development of a hands-on solution including a numerical example. In this solution technique, linearization was employed as the primal solution, following which structured perturbation logic was developed to improve on the initial solution. The second search technique proposed also utilized the linearization principle as a base solution in addition to some heuristics to construct transportation problems. The resulting transportation problems were solved to arrive at a competitive solution as regards effectiveness (solution value) compared to those obtainable from standard solvers such as CPLEX.

The FCSLTP is formulated and solved using the CPLEX commercial optimization suite. A Lagrange Relaxation Heuristic (LRH) and a Hybrid Genetic Algorithm (GA) solution of the FCSLTP are presented as alternative solutions. Comparative studies between the FCSTP and the FCSLTP formulation are also presented. The LRH is demonstrated with a numerical example and also extended to hopefully generate improved upper bounds. The CPLEX solution generated better lower bounds and upper bound when compared with the extended LRH. However, it was observed that as problem size increased, the solution time of CPLEX increased exponentially. The FCSTP was recommended as a possible starting solution for solving the FCSLTP. This is due to a lower solution time and its feasible solution generation illustrated through experimentation.

The Hybrid Genetic Algorithm (HGA) developed integrates cost relaxation, greedy heuristic and a modified stepping stone method into the GA framework to further explore the solution search space. Comparative studies were also conducted to test the performance of the HGA solution with the classical Lagrange heuristics developed and CPLEX. Results obtained suggests that the performance of HGA is competitive with that obtainable from a commercial solver such as CPLEX.

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List of Abbreviations and Acronyms

CPLEX	IBM ILOG Optimization Studio
CPLP	Capacitated Plant Location Problem
CFLP	Capacitated Facility Location Problem
FCTP	Fixed Charge Transportation Problem
FCNFP	Fixed Charge Network Flow Problem
FCSTP	Fixed Charge Solid Transportation Problem
FCSLTP	Fixed Charge Solid Location and Transportation Problem
FLP	Facility Location Problem
GA	Genetic Algorithm
HGA	Hybrid Genetic Algorithm
IBM	International Business Machines
LP	Linear Programming
LRH	Lagrange Relaxation Heuristic
MIP	Mixed-Integer Programing
MILP	Mixed-Integer Linear Programming
NP-	Non-deterministic Polynomial Time
RPH	Row Perturbation Heuristic
SFCTP	Step Fixed Charge Transportation Problem
SPLP	Simple Plant Location Problem
SFCLTP	Step Fixed Charge Location and Transportation Problem
STP	Solid Transportation Problem

Chapter 1

Introduction

1.1 Background and Motivation for Integrated Distribution Problems

Data-driven decisions have become necessary for organizations that seek a competitive advantage over other similar organizations today (Tian et al., 2019). These decisions are often based on small to large sizes of data captured from the day to day operations of the business (Muñuzuri et al., 2019). Also, Halldórsson (2019) noted that decision making is very important in business planning problems such as production planning, sales and operations planning as well as in logistics planning. Business decisions are commonly categorized as operational, tactical and strategic. Operational decisions include day to day business decisions of fulfilling customer orders, supplier and vendor management, while tactical decisions could deal with making policies on establishing distribution channels (e.g. route planning), inventory management and supplier determination. Strategic decisions, on the other hand, involve activities such as facility location, employee recruitment, etc. The decisions are of various planning horizons of short-term, medium-term and long-term. An integrated problem results when these different planning horizons are jointly considered. Therefore, a global solution which reduces the chances of global sub-optimality consequent to the individual local optimisations arising from individual consideration of the different planning horizons would be required to tackle the integrated problems.

1.2 Selected Distribution Problems

The distribution problems considered in this study are concerned with the movement of items between sources and destinations. These problems are essentially transportation problems in which decisions have to be made on the quantities of items to be moved from one location to a particular destination. The distribution or transportation problems considered in this thesis are the transportation problem with a staircase cost structure (referred to as the step-fixed charge transportation problem) and the multi-index transportation problem (also referred to as the fixed charge solid transportation problem).

1.2.1 Step-fixed charge transportation problem

The Step-Fixed Charge Transportation Problem (SFCTP) can be classified under network design problems with nonconvex piecewise linear cost structure. Besides, the problem is defined as having a cost structure which considers more than one fixed charge in the route planning decisions and thereby showing economies of scale in the cost structure. In the SFCTP, the cost structure essentially behaves like a staircase or step function and it extends from the FCTP. This problem-type is relevant in our modern world and is easily observed in the logistic industry such as postage companies, courier companies, delivery companies and shipping companies. In these companies, incremental discounts, full truck consignment or less-than-full truck consignment are usually used to determine their business pricings. The relevance of this model in imposing taxes or determining subscription fees of utilizing services has also been established. The main problem in step-fixed charge and transportation models is to determine the quantity of items to ship from a fixed set of locations to a fixed set of destinations under more than one route fixed cost requirement to minimize the total of fixed and variable transportation costs. The cost objective function basically distinguishes the FCTP from the SFCTP.

1.2.2 Fixed charge solid transportation problem

This problem is an extension of the Solid Transportation Problem (STP) described as a multi-index simple transportation problem. Since this problem is a variant of the simple transportation problem, it can also be categorized under network design problems. Applications of STP have been noted both in production and distribution systems. Its significance has been noted in raw material and blending transportation model. Also, its importance has been established in creating a shipment plan that assists operational managers to simultaneously select from different transportation sources and also determine the number of products to move from production or warehouse locations to decrease the total transportation costs. In the Fixed Charge Solid Transportation Problem (FCSTP), a single fixed cost is considered in the route planning decisions. Additionally, the main problem is that of determining the quantity of items to move from a set of fixed locations to a set of fixed destinations, while selecting from a set of potential transportation sources to minimize the total of fixed and variable transportation costs.

1.3 Overview of Model Formulations

Integrated facility location and transportation models can also be categorized under network design models. Network design models consist of several integrated models which are often used in the operations and management of a company's supply chain. Examples of some basic network design models already established in the literature are the general transportation problems, fixed charge problems and facility location problems. A key aspect of network design models is the structure of the objective function. These structures can either have a linear, piecewise linear (concave), or staircase cost structure. Integrated facility location and transportation problems as considered in this thesis have continuous and discrete variables in the cost structure with the standard transportation problem-type constraints of moving items from source to destination. Also, the piecewise linear transportation problem and the fixed charge solid transportation problem when considered in the context of facility location have been categorized as an integrated facility location and transportation problem in this thesis. The decision in facility location models is usually to choose from a set of potential capacitated sources or locations, an optimal set of sources from which products or services will originate. Both problems mentioned earlier, therefore share a commonality of integrating facility location decisions into a growing field of transportation problem variants. The basic difference between the two problems is that unlike the piecewise linear transportation problem, the fixed charge solid transportation problem considers that resources such as transportation mediums (or conveyances) can be limited and should be considered in the source to destination distribution decisions. Figure 1-1 and 1-2 below present a schematic representation of the two problems being considered in this thesis without loss of generalization.

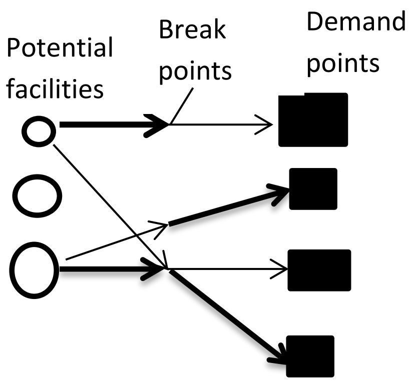

Figure 1-1 Illustration of the problems solved in Chapter 3

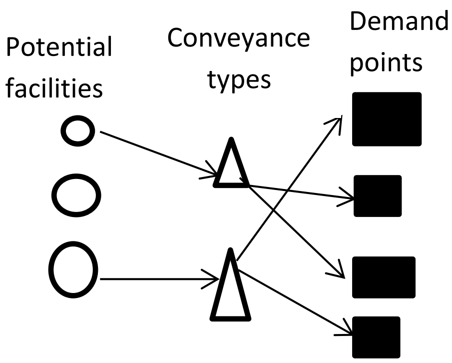

Figure 1-2 Illustration of the problems solved in Chapters 4 and 5

1.4 Overview of Solution Methods

Network design models have generally been solved using exact solution approaches and heuristic solution methods. This thesis considers both exact and heuristic solution methods in the solution of the integrated facility location and transportation problem. This problem can also be categorized as a mixed-integer problem due to the facility location fixed costs and the presence of one or more fixed costs in the transportation term of the objective function.

1.4.1 Exact solution approach

Exact solution methods such as branch and bound, branch and cut are known to guarantee optimality in providing a solution to the mixed-integer problems. However, exact solution methods can become inefficient when solving certain optimization problems such as the NP-hard (Non-deterministic Polynomial-time) problems. Optimization problems are classified as

NP-hard if the computational time increases exponentially as problem sizes increase. Network design problems with fixed charges are optimization problems usually regarded as NP-hard.

Exact solution methods are currently being incorporated into the solver engines of standard optimization software such as the IBM ILOG CPLEX, LINGO, and AMPL etc. As observed in the operations research literature, the IBM CPLEX has been the most common and widely used optimization software when considering exact solutions. Bixby (2002) stated that over-the-years research into algorithm and machine speed (memory) had helped to improve the solving power of the IBM ILOG CPLEX. According to Lima (2010), the CPLEX uses the branch and cut method to solve Mixed-Integer Problems (MIP). The Branch and Cut algorithm is based on the branch and bound algorithm, problem pre-processing and probing techniques, and the addition of cutting planes. The pre-processing and probing steps ensure problem reduction, reformulation and the fixing of variables to observe any improvements in the solution bounds. The processing and probing techniques for reducing the number of feasible solutions have been discussed by Savelsbergh (1994) and Wolsey et al. (1998).

1.4.2 Heuristic solution approach

Heuristics are solution methods that do not guarantee optimality but provide a good and acceptable solution to the optimization problem considered. Few resources such as computation time and computer memory are often utilized by heuristics when providing solutions. Besides, the inefficiencies of exact solutions for solving large problem sizes in a reasonable time have increased the research into the development of heuristics. Genova and Guliashki (2011) generally classified heuristics into the constructive and local improvement types. They referred to constructive heuristics as greedy heuristics which uses the problem data to generate a sequential solution and often reports no solution until the heuristics terminate. On the other hand, they described local improvement heuristics as providing a solution without necessarily reaching the termination of the problem. They further stated that local improvement heuristics search out a feasible solution, to begin with while searching the problem neighbourhood of any improvement in the current solution. Heuristics generally have a good possibility of terminating at a local optima solution.

Another class of heuristics have gained popularity over the years. These are called metaheuristics. Metaheuristics unlike heuristics usually are not problem-specific and can be applied to several optimization problems. Fernandes et al. (2014) established that metaheuristics are general frameworks constructed to ensure heuristics are perturbed out of any local optima. Metaheuristic implementation could be population-based such as the nature-inspired Genetic Algorithm (GA), Ant Colony Optimization (ACO) and Scatter Search (SS) and single solution based metaheuristics such as Simulated Annealing (SA) and Tabu Search (TS).

1.5 Aim and Scope

- Firstly, this research seeks to develop models that integrate facility location into transportation problem variants such as the step-fixed charge transportation problem and the fixed charge solid transportation problem.
- Secondly, the research seeks to develop a range of solution heuristics that can generate good upper and lower bounds to solve the identified integrated distribution problems. These solution heuristics utilize both the classical relaxation approaches and the modern solution search framework. Furthermore, comparative studies are also conducted to assess the performance of the heuristics developed when compared with possible exact solutions obtainable with standard solvers.

The basic assumptions for the formulation of each model developed are presented in Chapters 3, 4 and 5. However, a summary of assumptions is restated below.

A single planning period is considered which is often annual. A two-echelon problem of source to destination which identifies the distribution cases from point to point is modelled. The point to point scenarios considered can either be plants (sources) to distribution centres (destinations), plants to customers, or distribution centres to customers. In addition, a single item is being distributed between the echelons to capture the flow of resources between the points. The parameters considered such as the costs and capacities are known ahead of the optimization and randomly generated from a uniformly distributed data having a lower and upper limit. The lower and upper limits utilized are selected from the literature and shown to give a good sense of the parameters encountered in reality. For the step-fixed charge distribution case, a two step-fixed charge is presented without loss of generality, as this solution can easily extend to multi step-fixed charge cases.

1.6 Structure of the Thesis

This section discusses the different chapters making up this thesis.

Chapter 1 presents background with some motivations on the integrated distribution problem of facility location and transportation. The problem structure and type of solutions considered under integrated facility location and transportation problem are discussed in this section. The chapter ends with the general aims and limitations of all the models considered under the integrated facility location and transportation problem.

Chapter 2 reviews the fundamental models for the facility location problems and the transportation problems studied in this thesis. A comprehensive review of related works on existing models that integrate facility location and such transportation problems are also presented. Furthermore, the exact solutions considered and heuristic solutions developed to solve these problems are discussed. A summary of the research gap identified in the literature is presented. Finally, the motivations for the models and solution techniques developed in this thesis are also established in this section.

In chapter 3, two model formulations for the facility location and step-fixed charge transportation problem are presented. The basic differences in both formulations are on the modelling of the objective functions and the improved solution methods discussed. The first improved solution presents a hands-on method at arriving at a good solution. The second method shows more computation efforts and consisted of comparative studies with solutions obtained from standard solvers. No superiority exists between the different formulations which both have their initial solutions based on the linear programming relaxation.

In chapter 4, another integrated facility location and distribution problem described as Fixed Charge Solid Location and Transportation Problem (FCSLTP) is formulated and solved. This chapter is also segmented into two subchapters (4a and 4b). Subchapter 4a presents a small-sized numerical example to show the workings of a Lagrange relaxation heuristic to solve the FCSLTP. In subchapter 4b the FCSLTP optimization problem is presented and solved using the CPLEX commercial optimization suite. An extension of the Lagrange relaxation heuristic developed in subchapter 4a is proposed as an alternative solution. Computational studies are done to determine the performance of the CPLEX and Lagrange relaxation heuristic. The FCSLTP and the FCSTP formulations are further compared

Chapter 5 presents a hybrid metaheuristic solution to solve the FCSLTP. The hybrid solution integrates a greedy heuristic based on cost relaxation and a modified stepping stone method into the genetic algorithm framework. Genetic algorithm is used to determine the best combination of facilities to open, while the allocations into open facilities are done using a greedy heuristic. Furthermore, allocations made are consolidated using a modified stepping stone. Some comparative studies are shown to test the performance of the solution method.

The research study is concluded in Chapter 6 by summarizing and restating the major contributions of the models and solutions developed as presented in earlier chapters. In addition, future perspectives on possible extensions of these models and solutions proposed are also presented.

Chapter 2

Literature Review on Related Models

2.1 Related Facility Location Models

The optimization problems presented by Kuehn and Hamburger (1963), Ray (1966) and Sá (1969) are among the early models developed in the discrete facility location literature. Kuehn and Hamburger (1963) presented a facility location model where the decision was to sequentially determine the number and location of large scale warehouses. His model also involved perturbation through selected locations for possible improvements. Ray (1966) described facility location as a plant location problem where a fixed cost or fixed charge problem is associated with each plant or sources and the amount shipped from any selected or "open" location varies nonlinearly with the quantity shipped. He presented a pure integer formulation of the problem and solved it using the branch and bound solution method. Sá (1969) presented a facility location model and termed it as a capacitated plant location problem. Given marginal production or location costs and linear transportation costs, he decided to select from potential projects that solve demand requirements. He modelled the problem as a MIP and solved it using both exact and approximate solutions.

The application and use of the mixed-integer models to represent facility location models instead of the pure integer models were introduced by (Elson, 1972). He concluded that continued use of such MIP models would remain valid in future location models to be developed. Nauss (1978) solved the classical Facility Location Problem (FLP). The FLP was represented with fixed and variable cost objectives, demand and supply constraints. His model included the supply and demand capacities, fixed costs for opening facilities and unit transportation costs to supply customers. A Lagrangian relaxation method was proposed to solve the problem.

Neebe (1978) distinguished between the p-median location problem and the classical or Capacitated Warehouse Location Problem (CWLP). He noted that CWLP could be obtained from p-median location problem by removing the p-median constraint and replacing with the fixed cost of each supply points. His model essentially was on solving a p-median transportation problem and a Lagrangian relaxation method was developed to provide good lower bounds to be used in the branch and bound problem. Rosing et al. (1979) also distinguished between the plant location problem and the P-median problem. The major difference presented was that the facilities costs are included in plant location problem objective and the numbers of facilities to be opened are included in the constraints of p-median problems.

Guignard and Spielberg (1979) presented a Mixed Plant Location Problem (MPLP) which was an extension of the classical FLP discussed by Neebe (1978). The extension was on the inclusion of inequality constraints that impose an upper bound on the number of items shipped from plant i to customer j. In addition, they included constraints imposing restrictions on the amount of open or closed plants to be selected from the possible set of plants. Their model was solved using a greedy heuristic.

In the models described above, facility location problems were captioned as either a plant location or warehouse location problem. Since the establishment of models that represent plant location problems, variants of such models have emerged. Kelly and Khumawala (1982) discussed the capacitated warehouse location problem in which economies of scale (nonlinear cost) are associated with the fixed cost at the warehouse site. Cornuéjols et al. (1983) described the uncapacitated FLP in which no supply capacities are associated with the fixed facilities. An integer model formulation was presented. Krarup and Pruzan (1983), in their review paper, identified four problems that have been formulated and discussed under location problems. They mentioned the Simple Plant Location Problem (SPLP), p-median problem, p-center problem and the quadratic problem. The p-median problem and p-center problem were described by them as being easily transformed into the SPLP. It was also noted that the SPLP attracted more attention based on its generic contribution to many complex decision problems. The generic contribution of SPLP as they noted meant that other models such as nonlinear, dynamic and multiproduct models could be formulated based on the SPLP. The SPLP was also reported to be NP-hard and several heuristics were discussed to solve the problem. Klincewicz and Luss (1986) presented an FLP where not more than one facility is assigned to each customer and termed it as a single source constraint problem. They formulated the FLP as an integer programming problem.

Sridharan (1995) presented an exhaustive review paper on the Capacitated Facility Location Problem (CFLP). They described the basic problem being solved as determining a subset of potential facilities such that the total of fixed and transportation costs are minimized, while also ensuring that supply and demand capacities are not violated. They stated that the SPLP presented by Krarup and Pruzan (1983) was an uncapacitated FLP. They further expressed that the SPLP could be extended to the CPLP by including capacities of locations in the problem.

Since the comprehensive review submitted by Sridharan (1995), the CPLP has been utilized as a basis for the development of new facility location models. A significant amount of papers was found on the various extensions made to the capacitated facility location problem. Reference to the works of Revelle and Laporte (1996), Tragantalerngsak et al. (2000), Wu et al. (2006) , Correia et al. (2010), Ulukan and Demircioğlu (2015), Gadegaard et al. (2017) and Srinivasan and Khan (2018) out of many others have shown the integration of new objective functions, constraint type and scenarios to the development of the CPLP. In addition, the development of efficient solution heuristics, have also been complementary to the development of new models. Ulukan and Demircioğlu (2015) presented a comprehensive review on discrete facility location problems and variants, with exact, heuristic and metaheuristic solution methods. The CPLP and its variants were also reported to have had a large application in distribution planning systems and supply chain network design models.

Integrated distribution problems, in which facility location decisions form an integral part, have been noted in the literature and occur both in production and distribution systems. Wu et al. (2017) presented the facility location problem with plant size and selection model. They simultaneously considered an optimization model of plant locations and sizes, with product flows through depot locations and the consumers. Puga and Tancrez (2017) developed a location inventory model and considered the joint optimization of location, allocation and inventory decisions for a large supply chain network. Hiassat et al. (2017) presented a location inventory and routing model where the decision was to jointly determine the location and quantity of warehouses, inventory level, and routes for perishable products. Veenstra et al. (2018) optimized facility location and routing problems simultaneously in health care logistics where the decision was to determine which drug lockers to open, the routing of patients to both open and unopen drug lockers. Also, Amiri et al. (2018) considered integrating time windows in their off/onshore supply vessel while determining the location of warehouse and marine transportation simultaneously. The recent work of these authors among several others have shown the growing emphasis of integrating facility location decisions into new and existing production and distribution systems. Recently, Fauzan and Hisjam (2019) considered the capacity and location of health care facilities to meet demand. The problem was formulated as an integer programming problem defined as a model of capacitated hierarchical maximal covering.

2.2 Related Transportation Models

The classical transportation problem was originally described by Hitchcock (1941). This problem is being solved using the transportation tableaux and considers only the unit variable cost (no fixed charges) in deciding the optimum shipping patterns arising in the distribution network between sources and destinations. However, the reality of fixed charge problems in business decisions as indicated by Hirsch and Dantzig (1968) have resulted in the emergence of variants of the transportation problem which removes the assumption of no fixed charge in the transportation routes. This subsection reviews the base fixed charge transportation problem, the emergence of the SFCTP and the FCSTP.

2.2.1 Fixed and step-fixed charge transportation models

A special structure of the fixed charge problem was studied by Balinski (1961). This problem consisted of fixed and variable costs of transportation. Gray (1971) also considered the same problem and established it as a variant of the standard transportation problem which considers fixed costs and variable costs along transportation routes. The objective was that of minimizing the total of fixed and variable costs. They developed an exact solution that decomposed the problem into a master integer and series of transportation problems. It was observed that an optimal solution to the FCTP exists at extreme points of the constraint sets. Walker (1976) also confirmed the FCTP to be special consideration of fixed cost or fixed charge problem in transportation problems. They expressed the concave structure of the objective function of models involving fixed charges and also acknowledge the possibility of the linearized solution suggested by Balinski (1961).

Barr et al. (1981) described the FCTP as a network design problem with fixed charges on each arc (routes). They studied FCTP with fixed charges on some of the transportation routes (or arcs). This was termed uncapacitated FCTP. The similarity of the FCTP to a special case of the plant location problem with a concave cost was established by them. Sandrock (1988) considered a related FCTP in which fixed charges are associated with sources or locations and not on the usual routes. They provided hands-on heuristics to solve the problem.

Since the establishment of the FCTP, the development of both exact methods and solution heuristics has been a major contribution to this problem area. Palekar et al. (1990) developed a conditional penalty method to reduce the branch and bound enumeration techniques for solving the FCTP. They noted the FCTP to be NP-hard and also observed that problems with very large or very small fixed to supply capacity ratio were much harder to solve than the intermediate problem. Lamar and Wallace (1997) further improved on the condition penalty as reported by Palekar et al. (1990). A hybrid solution of tabu search metaheuristic and a local search technique based on the simplex method was implemented by Sun et al. (1998) to solve the FCTP.

Kim and Pardalos (1999) considered an FCTP modelled as a network problem of nodes and arcs. This consisted of one index notation (j) that represented the transportation routes or flows unlike the typical FCTP with (i, j) notation of source to destination. This problem was termed Fixed Charge Network Flow Problem (FCNFP) and was also stated to be NP-hard. Their solution was based on a recursive gradient linearization approach. Their solution was based on a proof of an existing LP problem whose optimal solution is equivalent to the optimal solution of the FCNFP.

A fixed charge network flow problem with more than one fixed charge on the arcs of the network was described by Kim and Pardalos (2000) as a Piecewise Linear Network Flow Problem (PLNFP). This problem was found to be NP-hard. They also described the occurrence of the PLNFP in logistics, transportation and the academic use of approximating a smooth non-convex function was highlighted. They modified their previous FCNFP gradient linearization method to solve the PLNFP which they established as being an FCNFP comprising of multiple arcs. Croxton et al. (2003) studied the incremental, multiple-choice and convex combination formulation of PLNFP. The three formulations were termed nonconvex piecewise linear minimization problems. They showed the equivalence of the three formulations concerning each model having an LP relaxation approximating the lower convex envelope. Kowalski and Lev (2008) discussed a special PLNFP with a source to destination notation (i, j) in which the objective function behaves like a step function and called it the Step Fixed Charge and Transportation Problem (SFCTP).

Since the SFCTP was considered to be an extension of the NP-hard FCTP, it has been described as being an NP-super hard problem. The SFCTP was solved using an extension of the linearization technique suggested by (Balinski, 1961). A metaheuristic to solve the SFCTP called the Mutation Artificial Immune algorithm (MAI) was developed by El-Sherbiny (2012). They observed the good possibility of MAI searching for the feasible solution domain better than the existing linearized cost approach. Altassan et al. (2013) improved on the initial linearized solution of Kowalski and Lev (2008) for the SFCTP. They suggested three equations for the relaxed cost matrix to capture the scenarios of above and below breakpoint load distribution. Christensen and Labbé (2015) considered another formulation of SFCTP which was referred to as a piecewise linear transportation problem.

They proposed an exact solution which was based on Dantzig-Wolfe reformulation and the addition of upper bound inequalities. They compared the efficiency of their solution to using a commercial solver such as the CPLEX solver. Yousefi et al. (2018b) researched on an FCTP with discounts or price breaks both on the route fixed cost and the variable cost. They proposed both heuristic and metaheuristic solution techniques to solve the problem. Balaji et al. (2019) recently studied an FCTP in which truckload constraint is considered. In their model, the possibility of shipment lot exceeding conveyance capacity thereby increasing the amount of fixed charge incurred is factored in. The problem was solved using a GA and Simulated Annealing.

2.2.2 Fixed charge solid transportation problem

Haley (1962) considered an extended transportation problem which they referred to as a multi-index transportation problem. This was stated to be different from the usual transportation problem of having two (2) major indices for the numbers of sources and destinations. They suggested a practical illustration for the problem such as the determination of the minimum cost of operation of a manufacturer having n -factories, producing m-different soaps to be distributed to p-different customers. They solved the problem using an adapted Modified Distribution (MODI) method for solving the classical transportation problem.

An extension to the solid transportation problem in which a fixed charge or cost is associated was discussed by Basu et al. (1994). This was termed solid fixed-charge transportation problem. Since their description of this problem, several variants and extensions of the Fixed Charge Solid Transportation Problem (FCSTP) have been considered in the literature. Yang and Liu (2007) studied a non-deterministic FCSTP with the objective of minimizing both transportation cost and time. They solved the problem using goal programming. Ojha et al. (2010) considered the FCSTP in which the variable costs, fixed cost and vehicle costs are discounted. This was solved using a genetic algorithm. A fixed charge solid transportation problem in which the model parameters such as supply, demand and conveyance capacities are considered uncertain variables was modelled by Zhang et al. (2016). This was solved using a hybrid metaheuristic.

Halder et al. (2017) presented a special extension to the FCSTP with a maximization objective. They considered a scenario involving damage and substitute items when data such as variable transportation cost, route fixed cost and conveyance capacities are fuzzy and crisp. An extension to the deterministic fixed charge solid transportation problem having more than one fixed costs associated with the conveyance capacities was studied by Sanei et al. (2017). They made conclusions that since the FCTP and SFCTP have been considered to be NP-hard problems, the SFCSTP and implicitly the FCSTP which are extensions of both SFCTP and FCTP are NP-super hard problems. They employed Lagrange relaxation heuristic to solve the SFCTP.

2.3 Related Integrated Facility Location and Transportation Models

Melkote and Daskin (2001) introduced a combined facility location and fixed charge transportation problem which was termed Capacitated Facility Location and Network Design Problem (CFLNDP). They cited the NP-hard nature of the problem with practical application in telecommunication, power transmission and distribution. The objective of the formulation was to minimize total network transportation cost which comprised unit travel cost, location cost of facilities and the route fixed charge or link construction cost. They solved the problem by proposing valid inequalities such as flow conservation, number of facilities and upper bound limits on routes to strengthen the LP relaxation of the problem.

Another integrated facility location with fixed charge transportation problem was studied by Correia et al. (2010). This problem was referred to as the capacitated location problem with modular distribution costs. In their formulation there existed different modules or fixed costs with corresponding capacities on each source to the destination link. They presented a traditional and discretized formulation of the problem. The discretized formulation considered a specific quantity of products or items being shipped through the source to destination route. The basic problem that was solved in both formulations was that of minimizing total distribution costs which consisted of facilities location costs, route fixed costs per module and variable cost between source and destination. They concluded that the discretized formulation could produce valid inequalities and better LP relaxation than that of the traditional formulation.

In the network design problems with piecewise linear cost function studies presented by Christensen (2013), they considered an integrated formulation of facility location with more than one fixed charge present on the transportation routes. This integrated problem was named Capacitated Facility Location Problem with Piecewise Linear Transportation Costs (CFLP with PLTC). They established that the piecewise linear transportation cost function in the facility location context had been given little consideration in literature unlike piecewise linear cost structure in production. It was noted that the piecewise linear structure was similar to that of the step function or staircase cost structure. Two models of the CFLP with PLTC were formulated. The first was similar to a multi-item facility location and step-fixed charge transportation problem. The second model also referred to as the discretised CFLP with PLTC was an extension of Correia et al. (2010) capacitated location problem with modular distribution costs. They added valid inequalities into CFLP with PLTC formulation and solved the problem using Lagrange relaxation heuristics. Recently, Das et al. (2019) studied an integrated facility location and solid transportation problems without fixed charges. They described the problem as solid transportation-p-facility location problem (ST-*p*-FLP). They developed to solution methods to solve this problem, namely a locate-allocate heuristic and an approximate heuristic

2.4 Solution Methods

This subsection reviews the solution methods used in this thesis. The solution methods considered are based on Linear Programming (LP) relaxation, Lagrangian Relaxation Heuristic (LRH) and Genetic Algorithm (GA). These methods have been adapted and implemented to solve the integrated facility location and transportation problems.

2.4.1 Linear programming relaxation method

Linear programming relaxation can be used as a starting or initial solution when dealing with network design and 0-1 mixed-integer problems. This initial solution oftentimes generates a lower bound to which an improved solution will be desired to resolve the relaxed 0-1 integer variables. In addition, various authors have established the existence of certain LP problems that closely approximates the equivalent mixed-integer problems in the literature.

Balinski (1961) developed a linearization technique to solve the 0-1 mixed-integer formulation of the FCTP. His method was described by Walker (1976) and Barr et al. (1981) as a rough approximation solution which would need additional improvement. Barr et al. (1981) further described the underlying assumption used by Balinski (1961) in applying the linearization as the requirement of a demand point being satisfied by a single-origin due to fewer supply points than demand points. Linear relaxation solutions have been used by Palekar et al. (1990) and Lamar and Wallace (1997) to obtain lower bounds on which they built their improved conditional penalty techniques in a branch and bound solution for the FCTP. Kim and Pardalos (1999) applied LP relaxation in their dynamic slope scaling procedure (DSSP) to solve the FCNFP. They showed proof of the existence of an LP problem that has the same optimal solution as the FCNFP within the same feasible domain. It was further suggested that their method could be applied to problems with concave piecewise linear objective functions. A modification to the DSSP in solving piecewise linear network flow problems was presented by Kim and Pardalos (2000). Melkote and Daskin (2001) studied the use of additional valid inequalities to strengthen the LP relaxation of their integrated facility location and network design problems.

Adlakha and Kowalski (2003) presented a single step perturbation method to improve on the initial solution of Balinski (1961) which was an LP relaxation to solve FCTP. They also suggested a similar solution for a concave function problem having transportation-type constraints. Croxton et al. (2007) improved on the LP relaxation of their basic model to solve the piecewise linear capacitated network flow problem. They also justified the use of an LP approach due to the LP relaxation approximating the piecewise linear cost function by its lower convex envelope. The lower convex envelope was defined as the line joining the origin to the maximum demand point for a graph of cost function against demand flow. It was also stressed that the lower convex approximation was poor and the need to improve on the solution was necessary. Their improvement was based on providing valid upper bound inequality for the load distribution. Kowalski and Lev (2008) also considered the lower convex principle and extended the linearization approach suggested in Adlakha and Kowalski (2003) for transportation problems in which the objective function behaved like a step or staircase. They suggested some upward or downward shift of load patterns to further improve on the initial linearized solution.

Figure 2-1 below illustrates the lower convex envelope in a graph of cost function $g(X)$ against a flow quantity X for a concave (nonconvex) function. Also, Figure 2-2 below shows improvements on using the lower convex principle to solve a step-fixed charge transportation problem with non- convex with a breakpoint at A and Maximum flow at M.

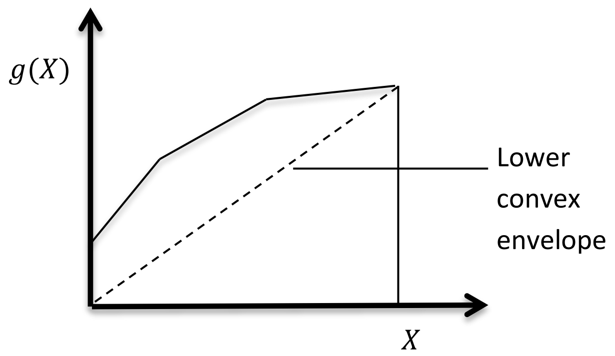

Figure 2-1 Linearization or Lower convex envelope for a concave function Croxton et al. (2007)

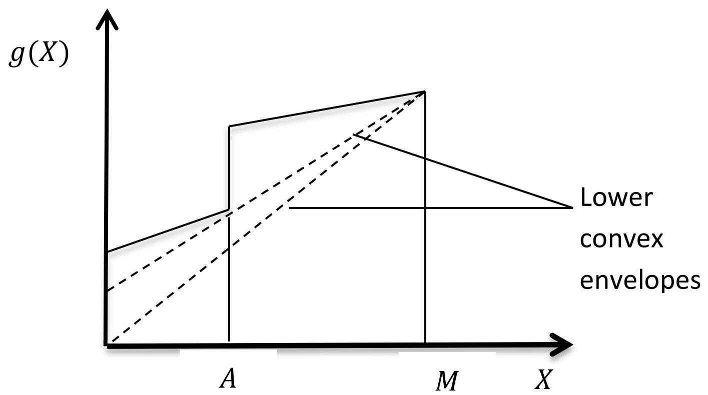

Figure 2-2 Linearization or Lower convex envelopes for a step function problem (Kowalski and Lev (2008) and Altassan et al. (2013))

Altassan et al. (2013) improved on the linearized solution approach for solving SFCTP. They presented initial solutions to address the particular cases of loads being equal to or above breakpoints of the SFCTP requirement which they showed Kowalski and Lev (2008) did not consider. In the dynamic programming solution presented for solving the single-sink, fixed-charge transportation problem presented by Christensen (2013), their initial solutions were based on the use of heuristics and variable pegging on the LP relaxation of the problem. They suggested improvement heuristics such as the actual cost recalculation technique, least cost addition technique and largest cost reduction technique. These were used to calculate upper bounds to the original problem using the LP relaxation as a basis. Yousefi et al. (2018b) considered solving FCTP using linearization by developing four different cost linearization techniques based on the cost and capacity parameters of the problem. They, however, did not present any improvement procedure to the approximate solutions obtained.

2.4.2 Lagrange relaxation heuristic method

Fisher (1981) discussed Lagrange relaxation as better lower bound generating an alternative method to the linear programming relaxation during Branch and bound solutions and that it could also provide a good feasible solution when solving combinatorial optimization problems. He noted that the selection of the constraints and determination of the Lagrange multiplier values to be used during the relaxation influenced the simplicity or difficulty of the relaxation problem. He also indicated that the value of the Lagrange multiplier could be obtained using the multiplier adjustment method, sub-gradient method and simplex column generation method. It was also mentioned that the ease of programming the sub-gradient method and its vast use, made it widely accepted in the determination of either optimal or near-optimal Lagrange multiplier from the Lagrangian dual problem. They concluded by discussing that a structured perturbation of the Lagrange relaxation solution in some instances could provide a feasible solution to the original problem. This was termed a Lagrangian Relaxation Heuristic (LRH).

Both Christofides and Beasley (1983) and Cornuéjols et al. (1983) applied the sub-gradient optimization in the Lagrange relaxation solution of the capacitated warehouse location and uncapacitated FLP respectively. A good lower bound generating capacity was reported in both cases. Klincewicz and Luss (1986) considered the LRH in solving an FLP where a single facility ships to each customer. To obtain the Lagrange relaxation of the original problem, they dualized the supply feasibility constraint and obtained an uncapacitated problem as a result. They used a dual ascent algorithm, an add-heuristic and a final improvement heuristic to arrive at the best feasible solution to the original problem.

Guignard and Kim (1987) and Cornuéjols et al. (1991) both studied a different method of applying Lagrange relaxation which involved creating more than one sub-problems from the original problem and still retain the original constraints. Cornuéjols et al. (1991) referred to this Lagrange decomposition method as variable splitting. Both authors showed proof of how the variable splitting methods could provide bounds that can be as good as the conventional Lagrange relaxation bounds. Cornuéjols et al. (1991) further provided more insight into the complexity of using different constraints for the Lagrange relaxation of CPLP. A linkage constraint such as the type that links the continuous variable to the binary variable in a CPLP was shown to produce a Strongly NP-hard problem when used in the Lagrange relaxation process.

Since the works of the authors presented above, The LRH has been implemented either as a whole solution approach or a base solution on which improvement heuristics are built to solve new problems. Sridharan (1995) in their review of solutions for the CPLP discussed the Lagrange heuristic and variable splitting methods. They suggested greedy heuristics such as the "add" or "drop" procedure to provide feasible upper bounds during the lower bound generating Lagrange relaxation method. Holmberg and Ling (1996) extended the Lagrange relaxation and sub-gradient optimization method to solve FLP with a staircase or step production costs. Holmberg and Ling (1996) discussed a Lagrange relaxation heuristic which incorporated a repeated matching algorithm into a branch and bound algorithm to solve an FLP variant. Tragantalerngsak et al. (2000) used a branch and bound based on Lagrangian relaxation and sub-gradient optimization to solve a two-echelon FLP. Ghiani et al. (2002) used the LRH to solve a plant location problem having facilities with both characteristics of fixed costs and capacities in the same region or site. They emphasized that the upper and lower bound generating capacity of the Lagrange heuristics as being a suitable reason for its continued usage. This observation was also noted by Fisher (2004) and they also gave a comprehensive review of the workings of the sub-gradient optimization method using the generalized assignment problem. The sub-gradient method was defined as a modification of the gradient method in which gradients are substituted with sub-gradients. Below is a brief review on the workings of the sub-gradient method as presented by Fisher (2004).

Given an optimization primal problem with an objective function (Z), i.e. $\min Z$, with constraints $(Ax - b)$ selected to participate in the Lagrange relaxation process. A is a matrix and A$\in R^+$, b is a vector and B$\in R^+$, R is a real number.

The Lagrange relaxation of any of the constraints is given as $Z_D(u)$. Where (u) is the Lagrange multiplier. The next step is the determination of the optimal or near-optimal Lagrange multiplier from the dual problem $Z_D = \max Z_D(u)$. The k- feasible solutions to (u) i.e. (u^k) are determined through the iterative process $u^{k+1} = u^k + t^k\left(Ax^k - b\right)$. Where $Ax^k - b$ is regarded as the sub-gradient at u^k, x^k is an optimal solution of the Lagrange relaxation of Z i.e. $Z_D(u^k)$, t^k is step size and such that $t^k \geq 0$.

The near-optimal or optimal u^k is obtained when $Z_D(u^k) \rightarrow Z_D$. This is possible as $t^k \rightarrow 0$ and $\sum_{i=0}^{k}(t_i \rightarrow \infty)$. The scalar step t^k is well defined in the literature.

The LRH was also used as a base solution for implementing metaheuristics by Chen and Ting (2008) and Li et al. (2009) to solve a facility location problem variant. Christensen (2013) proposed a Lagrange heuristic in addition to other solution techniques such as the addition of valid inequalities and pegging of variables to solve two models of the CFLP with PLTC. The relaxed problem from their Lagrange relaxation, which was obtained by relaxing the demand constraint, was decomposed into a master integer problem and mixed-integer sub-problem. The master problem was reformulated as a knapsack problem with inequalities added to resolve any infeasibilities and solved using CPLEX, variable pegging and through the use of primal heuristics. Due to the similarity in structure with some existing problems in the literature, the dynamic program was employed to solve the sub-problem. Nezhad et al. (2013) also used the Lagrange relaxation heuristic and they considered relaxing a constraint linking two binary variables. They admitted that this special constraint was hard to solve but it, however, ensured that the original problem was decomposed into easily solvable sub-problems. They noted the possibility of generating infeasible solutions when solving the sub-problems obtained and the need to introduce inequalities that can restore feasibilities.

Ulukan and Demircioğlu (2015), Wu et al. (2017) and Sanei et al. (2017) presented recent studies and applications of the LRH. Ulukan and Demircioğlu (2015) in their review of discrete FLP, showed the number of LRH methods that have been successfully applied to variants of problems in this class. Wu et al. (2017) applied hybrid Lagrange heuristic to solve an FLP variant. The application of LRH in a solid transportation problem variant with fixed charges and no consideration for facility location problem was considered in the studies of Sanei et al. (2017). They decomposed the LRH of the original problem into easier sub-problems that had to be solved independently. They also noted the possibility of generating infeasibilities when merging solutions of the sub-problems. Through mathematical deductions and also using sub-gradient optimization to update the Lagrange multipliers, an efficient solution was proposed for the FCSTP.

2.4.3 Genetic algorithm implementation for transportation problem variants and facility location problems

One of the popular evolutionary search techniques employed to solve NP-hard combinatorial problems is the Genetic Algorithm. Vignaux and Michalewicz (1991) described the GA as a probabilistic algorithm which begins with randomly generated feasible solutions called populations and dynamically perturbs towards better solutions through mimicking the genetic evolution process of nature. The basic operations such as the genetic representation of solutions, evaluation function identification, crossover operations, and mutation operations were discussed as necessary in a GA solution replication process. They introduced a matrix representation of the population of solutions (chromosomes) to solve the basic transportation problem. This was further suggested as an alternative to the classical binary representation of chromosomes, which become inadequate to represent nonlinear problems such as fixed charge problems during the GA implementation.

Ho and Ji (2005) developed a GA to solve an extended linear transportation problem known as the general transportation or machine loading problem. A matrix representation was also used to represent the chromosomes (feasible solutions) of the problem. The chromosomes were generated using heuristics in the initialization phase. The roulette wheel approach of selecting chromosomes to participate in the crossover operations was used. Furthermore, their crossover operations and mutation operations were achieved using user-defined crossover and mutation rates.

Gen et al. (2006) used the GA for an extended transportation problem having two stages and modelled as a p-median FLP. They discussed other methods of representing the chromosomes of network optimization problems. Some of the methods include the edge-based encoding, vertex-based encoding and edge-vertex based encoding. They proposed a priority-based encoding to prevent the infeasibility results of an earlier prüfer number representation discussed by Gottlieb et al. (2001). A one cut point crossover operation known as the weight mapping crossover, with an insert and swap mutation method was implemented in their GA.

A comparison of the effectiveness and efficiency of the GA with some other metaheuristics such as the tabu search and simulated annealing was studied by Arostegui Jr et al. (2006). This comparison was done using variants of FLP such as the CFLP. A matrix representation was used and the crossover operation was based on a single cross over point. The mutation operation was randomly achieved using a mutate probability. Conclusively, the GA was observed to perform better for specific problem types and objective functions.

Another extended transportation problem having two stages, with fixed charges was solved using GA by Jawahar and Balaji (2009). They also utilized the matrix representation and generated their population of chromosomes using some allocation heuristics such as least equivalent variable cost. The roulette wheel method was used to select chromosomes to participate in the crossover operation. A summary of their roulette wheel selection is described below in Table 2-1. A two-point crossing over operation was proposed while the mutation operation was achieved using a probability rate to ensure the genes lost can be reintroduced into the chromosomes.

Table 2-1 Sample Roulette wheel selection (Jawahar and Balaji (2009))

Given Chromosomes (solution) c , in population size ($pop\ size$) with objective value ($fitness\ value\ (c)$)
Probability of selecting each chromosome $p(c)$: $p(c) = fitness\ value\ (c) \Big/ \sum_{c=1}^{Pop\ size} fitness\ value(c)$
Cumulative probability of selection of a chromosome $cp(c)$: $cp(c) = \sum_{c=1}^{c} p(c)$
Generation of random numbers R $(0 < R \leq 1)$ such that chromosome c is selected under the condition: $cp(c-1) < R \leq cp(c)$

The GA has also been hybridized with other local search heuristics to strengthen the diversity of the search procedure. Lai et al. (2010) implemented a hybrid GA to solve capacitated plant location problems. The GA was developed to solve the master integer problem generated when using benders decomposition algorithm. The binary representation was utilized since the decision variables of the master problem were binary integers (0 and 1). They also utilized the single-point crossover method and discussed the basic mutation operation of using mutation rates and a complex type referred to as Gen Jam.

A comprehensive review of the use of metaheuristic to solve linear integer problems was presented by Genova and Guliashki (2011). They noted the power of metaheuristics such as the GA as being able to prevent solutions from being trapped in local optima, due to their multi-dimensional search strategies when compared to other classical heuristics. They also observed the solution generating capacity of the GA to be based on the genetic operators and also pointed out a heuristic hybridization advantage of the GA.

Antony et al. (2011) studied a hybrid GA for solving a single-stage FCTP. Heuristics were used to generate their population of chromosomes and to improve on the chromosomes obtained. An improvement technique discussed was a stepping stone method of increasing the number of basic cells or genes of the chromosomes. They also presented a review of the various chromosome representation types that have been proposed to solve fixed charge transportation problems. The differences were based on the number of genes involved in the chromosomes. They showed the matrix representation as possessing the highest number of genes representing the transportation problem i.e. $m \times n$. The prüfer number had the least representation i.e. $m + n - 2$. The roulette wheel was also used in selecting chromosomes for crossover. However, no mutation was done. Their solution approach was also extended to solve two-stage FCTP as implemented in Antony Arokia Durai Raj and Rajendran (2012).

Jawahar and Balaji (2012) extended their earlier GA to solve a multi-period fixed charge distribution problem. However, their chromosomes representation was based on a permutation schedule. Fernandes et al. (2014) also proposed a GA to solve a two-stage capacitated facility location problem. Heuristics were also used to generate chromosomes such as in Jawahar and Balaji (2012) with the chromosomes represented as binary integers.

Both the CFLP and variants of the transportation problems have been solved independently using hybrid GA as discussed by Rahmani and MirHassani (2014), Ulukan and Demircioğlu (2015), Calvete et al. (2016) and Guo et al. (2017). An HGA with another evolutionary algorithm known as the firefly was utilized by Rahmani and MirHassani (2014) to solve CFLP. Ulukan and Demircioğlu (2015) on the other hand presented a review of discrete FLP with the growing use of hybrid evolutionary algorithm as compared to hybrid classical solution methods. Calvete et al. (2016) studied a hybrid GA while incorporating a network simplex algorithm to solve a two-stage transportation problem. A hybrid evolutionary algorithm was also implemented by Guo et al. (2017). They considered the use of GA with a fitness approximation and other local search heuristics.

The use of the GA to solve integrated distribution problems that extends the base distribution problem by incorporating other optimization decisions such as location planning, inventory management, price breaks was described by Ojha et al. (2010), Hiassat et al. (2017) Yousefi et al. (2018a) and Balaji et al. (2019). A GA to solve integrated solid transportation problem with route fixed charge, vehicle cost and price discounts was considered by Ojha et al. (2010). They studied this global optimization problem to emphasize the challenges some organizations are confronted with while creating values for their customers. Furthermore, they suggested the use of real number representation for their chromosomes as against the binary representation due to the non-linearity’s involved in such problems. The roulette wheel method was used to select chromosomes for the cross over operation while they ensured an arithmetic crossover that prevents infeasibilities of new chromosomes or offspring from being utilized. The mutation was randomly done within the range of feasible solutions. Hiassat et al. (2017) on the other hand implemented a GA to solve an integrated location-inventory routing problem with perishable products. Their chromosome representation was presented as an array of numbers that consisted of the different optimization decision considered. They also used the roulette wheel method for crossover operations, the single cross over point and the swap technique for mutation. Yousefi et al. (2018b) compared the GA and Simulated Annealing and linearized heuristics in solving FCTP with incremental discounts. The GA was shown to perform better than the other solution techniques. In addition, Yousefi et al. (2018a) also utilized GA to solve an extended version of FCTP with incremental discounts. They used priority-based encoding scheme for their chromosome representation and employed similar genetic operations of crossover and mutation as described by earlier authors for the new population generation. Balaji et al. (2019) extended their previous GA model in Jawahar and Balaji (2009) to solve a FCTP with the introduction of truckload constraints.

2.4.4 Standard solver (IBM ILOG Suite) method

IBM ILOG optimization suite consists of a set of tools for developing personalized optimizations programs that use the IBM customized solvers. This consists of the IBM ILOG Optimization Programming Language (OPL), the IBM ILOG Optimization Decision Manager (ODM) and Optimization engines such as the IBM ILOG CPLEX for mathematical programming and IBM ILOG Constraint Programming (CP) optimizer. According to Studio (2016), IBM ILOG CPLEX has embedded in it among others the C, C# and Java programming language libraries, that can solve Linear Programming(LP) and similar problems such as MIP. Lima (2010) gave a range on the typical mathematical programming problems solvable by IBM ILOG CPLEX. This includes amongst others the LP, MIP, Mixed-Integer Quadratic Programming (MIQP), Quadratic Constrained Programming (QCP). They further noted that the major solvers used are the Simplex optimizers, Barrier optimizers and the mixed-integer Optimizers. These solvers could be invoked concurrently or individually to solve the optimization problems. He (2012) described the IBM ILOG CPLEX as helpful for building user-defined search heuristics. A key function that helps user-defined heuristics to be developed using the IBM ILOG suite was described by Studio (2016) as the Concert Technology and the Callable Library. They defined Concert Technology as a set of libraries that allow programmers to embed CPLEX optimizers in Java, C++, or .NET applications. The Callable library, on the other hand, allows programmers to embed CPLEX optimizers in applications that can call the C programming language functions.

Some of the search heuristic implementations in this research work was written in the Java programming language and based on the IBM ILOG concert technology for Java users. The Concert technology for Java was used to model and solve the Original MIP problems and also used to solve some linear programming formulations encountered in this thesis.

2.5 Summary of Research Gap

Research in the separate fields of facility location and transportation problems have been well conducted. These have been both in model formulation and solution technique development. However, the field of integrated facility location and transportation problems is gradually receiving attention with a leading work from Melkote and Daskin (2001). Other studies such as the works of Correia et al. (2010) and Christensen (2013) which appeared to have discussed integrated facility location and transportation problems have not considered an integrated facility location and transportation problem in which transport conveyances are considered. The recent work of Das et al. (2019) which appeared to have discussed an integrated facility location problem with transport conveyances have not considered the reality of fixed charges in their models.

Solution techniques which apply a state of the art technique such as hybrid metaheuristic as against classical solution methods have not been utilized to solve integrated facility location and transportation problem in the literature. A classical solution method known as the Lagrange relaxation heuristic is also extended to solve this problem. In addition, a structured perturbation logic based on linear relaxations has also not been used to solve the facility location and step-fixed charge transportation model as considered in this thesis. The models and solution techniques in this thesis are proposed to fill these research gaps. A summary of the gaps and how they have been addressed in the different chapters of this research work is presented in Table 2-2 below.

Table 2-2 Research gap analysis

Selected Authors	Problem Characteristics					Solution Method	
	Variable cost	Route fixed cost	Route Step-fixed cost	Facility Location fixed cost	Conveyance constraint	Type	Class
Adlakha and Kowalski (2008)	✓	✓	✓	✗	✗	Linear Programming Relaxation	Heuristics
Elsherbiny and Alhamali (2013)	✓	✓	✗	✗	✗	Artificial Immune Vs LINGO	Meta-Heuristics
Correia et al. (2010)	✓	✓	✓	✓	✗	Model + Valid In equalities + LP relaxation In CPLEX	Heuristics
Christensen (2013)	✓	✓	✓	✓	✗	Lagrange+ Valid inequality addition vs CPLEX	Heuristics
Sani et al. (2017)	✓	✓	✓	✗	✓	Lagrange Heuristic Vs CPLEX	Heuristics
Das et al. (2019)	✓	✗	✗	✓	✓	locate-allocate heuristic	Heuristics
Chapter 3	✓	✓	✓	✓	✗	LP relaxation + Valid inequality +Location heuristic Vs CPLEX	Heuristics
Chapter 4	✓	✓	✗	✓	✓	Lagrange Heuristic Vs CPLEX	Heuristics
Chapter 5	✓	✓	✗	✓	✓	Genetic Algorithm +greedy heuristic + Modified stepping stone	Meta-Heuristics

2.6 Conclusion

A good number of models and solution methodologies which are formulated to tackle recent NP-hard optimization problems are modifications and extensions of some base models and solution techniques as described above. The linear programming relaxation or lower convex envelope solution have been used as a starting solution for network design problems such as nonconvex piecewise linear transportation problems. However, for piecewise linear cost structure formulations such as the integrated capacitated facility location and step-fixed charge transportation problem, the linear programming relaxation requires the development of improved solution techniques which can further drive the initial solution to optimal or near-optimal solutions.

The Lagrange relaxation heuristic which employs the sub-gradient method has been shown as an alternative starting or complete solution to CFLP variants. The LRH can help to generate lower and upper bounds to the problem being considered. Also, LRH allows the inclusion of improvement heuristics that narrows the solution search pace for faster and optimal or near-optimal solutions. Therefore, in order to solve NP-hard problem such as the integrated capacitated facility location and fixed charge solid transportation problem, extending the LRH with the inclusion of primal heuristics becomes a research area requiring exploration.

Lastly, the use of hybrid heuristics which combines metaheuristics and improvement heuristic methods has formed a new method of solving complex optimization problems. Metaheuristic frameworks such as the GA have been shown in the literature to possess effective abilities of preventing optimization problems from getting stuck in a local optima which classical relaxation approaches may not prevent. A substantial amount of research exists in the literature when solving network design problems using classical and metaheuristic solutions. Therefore, research into hybrid metaheuristic solutions to solve facility location network design problems as compared is an area requiring some considerable research input.

Chapter 3

Solving the Capacitated Step-Fixed charge and Facility Location Problem

Chapter 3.1

On the Capacitated Step-fixed charge and Facility location problem: A row perturbation heuristic

A modified version of this subchapter has been published in Applied Mathematics and Information Sciences Journal.

3.1.1 Introduction

The main objective in this chapter is about minimizing the traditional distribution problem cost of a source to a destination where a minimum number of facilities with known capacities have to be chosen from amongst other competing capacitated facilities or locations with fixed location costs in order to ship an item through routes with step-fixed costs. This SFCTLP emanates also as a variant of the SFCTP in like fashion as the SFCTP and SFCSTP. The formulation of, and reviewing known starting initial solutions for solving the SFCTP is earlier presented before discussing the integrated model for the SFCTLP, followed by a solution heuristic and then a numerical example.

3.1.2 SFCTP model formulation

The classical TP and its variants such as FCTP, SFCTP, SFCSTP are described as m suppliers and n demand point distribution problems, where m denotes the number of sources (factories, warehouses or distribution centers) and n refers to the number of customers or demand points. There are supply and demand requirements which often are represented as capacities S_i and demand D_j for each source i and demand point j respectively over a known time period. The m suppliers incur a unit transportation cost c_{ij} per unit distance and a fixed charge h_{ij} whenever a transportation route is opened (utilized for shipping) under constraints of supply capacity meeting a typical demand of transportation algorithm.

There are more than one fixed charges in the route (i, j) when Step-fixed charges are considered. In the SFCSTP, the fixed charges are represented by the vehicle cost of conveying different volumes of the load. While, in SFCTP, the fixed charges may be incurred either through duties, taxes or vehicle costs of different volumes transported. The number of fixed charges depends on the number of breakpoints in the step function desired. In this case, two steps of fixed charges, h_{ij1} and h_{ij2}, are considered without loss of generality. The fixed charge h_{ij1} is incurred when a route is opened and termed as H_{ij1} in the objective function and the second h_{ij2} is incurred when the shipment load (or transported unit) exceeds an amount A_{ij}, and termed as H_{ij2} in the objective function also. A_{ij} is referred to as the breakpoint and may be fixed or varying per route (i, j) depending on the model under consideration. When there is load distribution in any route i.e. $x_{ij} \geq 0$, h_{ij1} is incurred. While h_{ij2} is incurred when $x_{ij} \geq A_{ij}$.

The standard mathematical model for the SFCTP is represented below:

Min Z =

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\ x_{ij} \ + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{2} g_{ijk}h_{ijk} \qquad (3.1)$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = S_i y_i \qquad \forall\ i = 1 \dots m \qquad (3.2)$$

$$\sum_{i=1}^{m} x_{ij} = D_j \qquad \forall\ j = 1 \dots n \qquad (3.3)$$

$$\sum_{i=1}^{m} S_i y_i = \sum_{j=1}^{n} D_j \qquad \forall\ i = 1 \dots m, \qquad \forall\ j = 1 \dots n \qquad (3.4)$$

Where $g_{ij1} = \begin{cases} 1 & x_{ij} > 0 \\ 0 & Otherwise \end{cases}$, $g_{ij2} = \begin{cases} 1 & x_{ij} > A_{ij} \\ 0 & Otherwise \end{cases}$

$y_i = 0\ or\ 1$

$x_{ij} \geq 0$

Altassan et al. (2013) noted that the solution methods of the SFCTP depend on the breakpoint position i.e. if $A_{ij} < \min(S_i, D_j)$ or $A_{ij} \geq \min(S_i, D_j)$. If $A_{ij} \geq \min(S_i, D_j)$, the optimal solution to the SFCTP is an optimal solution to FCTP . The SFCTP solution heuristic of Kowalski and Lev (2008) and the SFCTLP heuristic presented in this subchapter work through building a relaxed cost matrix which are modifications of Balinski (1961) relaxation for the FCTP. In the model presented in this subchapter, the binary integer z_{ij} associated with the fixed charges h_{ij} (standard SFCTP model above) or H_{ij} (in the model presented below) is replaced by x_{ij}/M_{ij} where $M_{ij} = \min(S_i, D_j)$. Thus a relaxed cost matrix is formed. Kowalski and Lev (2008) followed in a similar fashion to obtain a first relaxed cost matrix $C_{ij} = c_{ij} + \frac{h_{ij1} + h_{ij2}}{M_{ij}}$ or $C_{ij} = c_{ij} + \frac{H_{ij1} + H_{ij2}}{M_{ij}}$ and second relaxed cost matrix $C_{ij} = c_{ij} + \frac{h_{ij2}}{M_{ij} - A_{ij}}$ or $C_{ij} = c_{ij} + \frac{H_{ij2}}{M_{ij} - A_{ij}}$.

To improve their initial solution of SFCTP, Kowalski and Lev (2008) demonstrated that the number of basic variables for a near-optimal solution of the SFCTP having two steps (or tiers) can be greater than $(m + n - 1)$ that is traditionally expected for a classical TP. They considered a minimization model of the step-fixed charge problem and presented a numerical example to support their claim. They also noted that for a two-tier or two step-fixed charge problem where the load distribution x_{ij} is such that $x_{ij} \leq A_{ij}$ or $x_{ij} > A_{ij}$, perturbation moves would result in above or below A_{ij} distribution. This is quite logical as it expected that some optimal load values would occur at the breakpoints. They also established that using the transportation problem would create solutions with $(m + n - 1)$ or less to which a particular perturbation would be needed to redistribute the load units to take advantage of the fixed charges along the routes. In Figure 3-1 and 3-2 below, the cost objective pattern with different fixed cost values and the expected linearization as illustrated by Kowalski and Lev (2008) are shown.

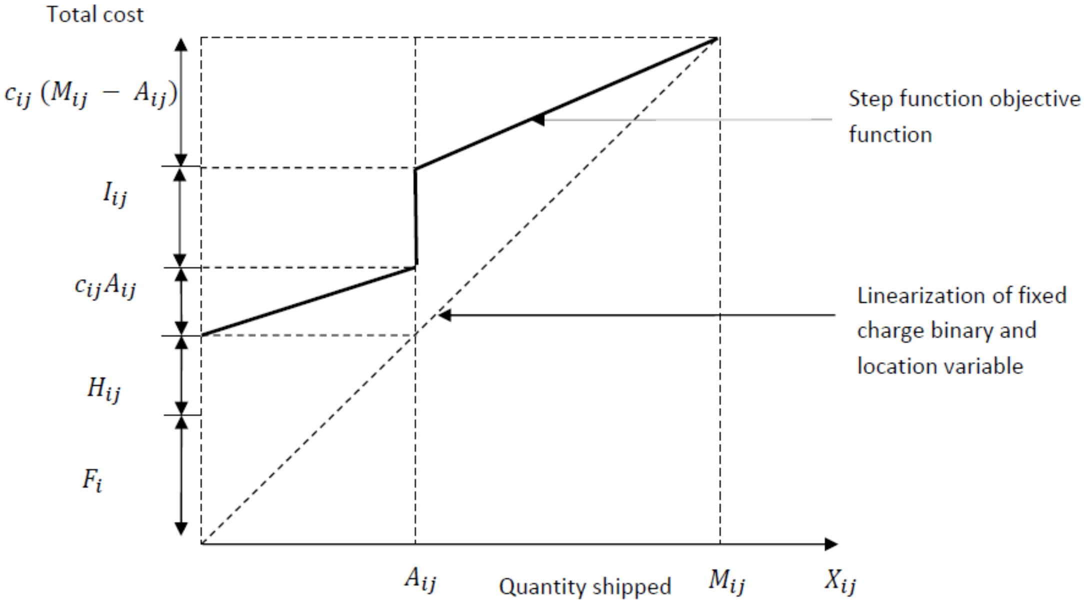

Figure 3-1 Two-step linearization and relaxation Structure (Kowalski and Lev, 2008)

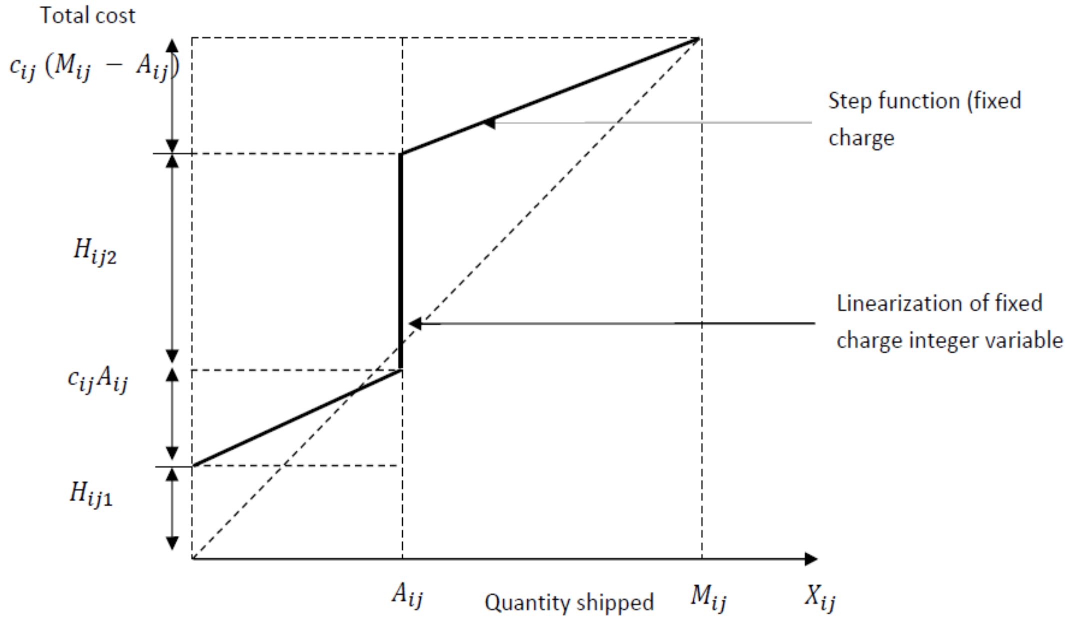

Figure 3-2 Linearization and relaxation structure when $H_{i\,j1} < H_{i\,j2}$

Altassan et al. (2013) however showed the limitation of the second relaxed cost in the works of Kowalski and Lev (2008) when $M_{ij} \leq A_{ij}$ with C_{ij} (relaxed cost) not giving a positive result. They further proposed three formulas for calculating relaxed cost (C_{ij}) which are based on firstly whether $A_{ij} < M_{ij}$ or $A_{ij} \geq M_{ij}$, secondly on A_{ij} being included or not in the formula and thirdly on the number of $M_{ij} - A_{ij}$ shipments done. They used f_{ij1} and f_{ij2} as their route fixed cost in their formulas as represented below.

The first one was given as $$C_{ij} = \begin{cases} c_{ij} + \frac{f_{ij1}}{M_{ij}} & if\ A_{ij} \geq M_{ij} \\ c_{ij} + \frac{f_{ij1} + f_{ij2}}{M_{ij}} & if\ A_{ij} < M_{ij} \end{cases} \quad \forall(i,j) \qquad 3.5(a)$$

The second was given as $$C_{ij} = \begin{cases} c_{ij} + \frac{f_{ij1}}{M_{ij}} & if\ A_{ij} \geq M_{ij} \\ c_{ij} + \frac{f_{ij2}}{M_{ij} - A_{ij}} & if\ A_{ij} < M_{ij} \end{cases} \quad \forall(i,j) \qquad 3.5(b)$$

The third given as $$C_{ij} = \begin{cases} c_{ij} + \frac{f_{ij1}}{M_{ij}} & if\ A_{ij} \geq M_{ij} \\ c_{ij} + \frac{f_{ij2}}{A_{ij}} + \frac{f_{ij1}}{M_{ij} - A_{ij}} & if\ A_{ij} < M_{ij} \end{cases} \quad \forall(i,j) \qquad 3.5(c)$$

From their analyses, they concluded that the first formula gave the best approximation when compared to Balinski (1961) and Kowalski and Lev (2008) and they also made suggestions as to using the other formulas as better starting solutions for the SFCTP.

3.1.3 SFCTLP problem structure and formulation

Two methods are utilized for the linearization and relaxation of the initial solution development of SFCLTP. The first procedure is as described by Balinski (1961), Kowalski and Lev (2008) and Altassan et al. (2013) which employs the transportation model variable cost structure to form a relaxed cost matrix. The motivations for using linearization are described in section 2.4.1. The reason is basically due to the existence of certain LP problems that closely approximates the equivalent mixed-integer problems in the literature. Although, these LP problems still require some perturbations to obtain better results for mixed-integer problems.

As discussed in earlier sections, the position of A_{ij} i.e. $A_{ij} < M_{ij}$ or $A_{ij} \geq M_{ij}$ in developing the relaxed or reduced transportation cost matrix would affect the SFCTP solution found. It is also noted that the breakpoint position i.e. $A_{ij} < M_{ij}$ or $A_{ij} \geq M_{ij}$ for any problem involving a two-tier fixed-charge cost on a route would affect the relaxation and perturbation pattern when seeking for a solution heuristic. Therefore, the model of Balinski (1961), Kowalski and Lev (2008) and more importantly the second formula by Altassan et al. (2013) have been extended by creating the starting SFCTP part of the problem using

$$C_{ij} = \begin{cases} c_{ij} + \frac{h_{ij1}}{M_{ij}} & if\ A_{ij} \geq M_{ij} \\ c_{ij} + \frac{h_{ij1} + h_{ij2}}{M_{ij} - A_{ij}} & if\ A_{ij} < M_{ij} \end{cases} \quad \forall (i,j) \qquad 3.5(d)$$

A summation of route fixed costs $h_{ij1} + h_{ij2}$ or $(H_{ij1} + H_{ij2})$ instead of h_{ij2} (H_{ij2}) alone have been used to account for incurring the fixed cost h_{ij1} (H_{ij1}) whenever a route is opened before h_{ij2} is incurred due to the breakpoint A_{ij}. Also, h_{ij} instead of f_{ij} has been used in the route fixed costs. The second procedure develops an average relaxation method, as indicated by equation (3.14) below. This second method relaxes the location variable y_i by creating an average location variable value ${y_i}^a$ for all the competing locations. Through some perturbation techniques developed on the initial solution, better solutions are obtained.

This problem solved is stated as the Capacitated Facility Location with Step- Fixed Charges along the transportation routes i.e. Step- Fixed Charge Transportation and Location Problem (SFCTLP).

Model assumptions
The following assumptions are made in formulating the SFCLTP model:

1. Deterministic input
2. One stage or two-echelon problem
3. Two step-fixed charge cost
4. Single period and single item distribution problem.

Model Parameters:
i : Index for sources (plants, locations or row)
m : Number of sources (plants, warehouses etc.)
n : Number of destinations (or demand point)
j : Index for demands (destinations or columns)
k : Index for Levels or (number of steps)
c_{ij} : The unit cost of shipment on route (i,j)
S_i : Capacity for each location i
h_{ij1} : the First level fixed cost on route(i,j)
h_{ij2} : the Second level fixed cost on route(i,j)
H_{ij1} : First level step-fixed cost based on load distribution
H_{ij2} : Second level step-fixed cost based on load distribution
x_{ij} : Allocation variable (or load distributions) along the route (i,j)
y_i : Location variable for plant or source (0 or 1)
g_{ij1} : Step-fixed charge variable (determining first or second level of fixed cost)
z_{ij} : Fixed charge variable in the objective function (0 or 1)
A_{ij} : The breakpoint for the fixed costs along the route (i,j)

Mathematical Model (Objective function and Constraints):

(Objective function)

Minimize Z =

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij} + \sum_{i=1}^{m} F_i\, y_i + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{2} H_{ijk} z_{ij} \qquad (3.6a)$$

Where:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} H_{ij1} = \sum_{i=1}^{m}\sum_{j=1}^{n} g_{ij1} h_{ij1}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} H_{ij2} = \sum_{i=1}^{m}\sum_{j=1}^{n} g_{ij2} h_{ij2}$$

∴

$$\sum_{i=1}^{m}\sum_{j=1}^{n} H_{ij1} + \sum_{i=1}^{m}\sum_{j=1}^{n} H_{ij2} = \sum_{i=1}^{m}\sum_{j=1}^{n} g_{ij1} h_{ij1} + \sum_{i=1}^{m}\sum_{j=1}^{n} g_{ij2} h_{ij2}$$

Where:

$$g_{ij1} = \begin{cases} 1 & x_{ij} > 0 \\ 0 & Otherwise \end{cases}, \quad g_{ij2} = \begin{cases} 1 & x_{ij} > A_{ij} \\ 0 & Otherwise \end{cases}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{2} H_{ijk} z_{ij} = \sum_{i=1}^{m}\sum_{j=1}^{n} H_{ij1}\ z_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} H_{ij2}\ z_{ij}$$

Subject to (constraints):

$$\sum_{j=1}^{n} x_{ij} \leq S_i y_i \qquad \forall\ i = 1 \dots m \qquad (3.7)$$

$$\sum_{i=1}^{m} x_{ij} = D_j \qquad \forall\ j = 1 \dots n \qquad (3.8)$$

$$\sum_{i=1}^{m} S_i y_i \geq \sum_{j=1}^{n} D_j \qquad \forall\ i = 1 \dots m, \qquad \forall\ j = 1 \dots n \qquad (3.9)$$

$$x_{ij} \geq 0 \qquad (3.10a)$$

$$y_i = 0\ or\ 1 \qquad z_{ij} = 0\ or\ 1 \qquad (3.10b)$$

Equation (3.6a) is the objective function. The first term is a variable cost, the second term is the facility location cost and the third term is the route step-fixed charge cost. Equation (3.7) is the supply capacity constraint of each location or sources. Equation (3.8) is the demand constraint to be met. Equation (3.9) is the aggregate constraint for supply and demand balance. Equation (3.10a) refers to the non-negativity constraint and (3.10b) refer to the binary integer constraints.

3.1.4 Solution method

The solution method presented iterates through the steps and rules below in seeking an improved solution:

Step 1: An initial solution is developed by linearization and relaxation of the binary variables (y_i *and* z_{ij}) in the model problem.
Step 2: A lower bound for SFCTLP is calculated.
Step 3: Improving the initial solution through a structured perturbation procedure referred to as Row Perturbation Heuristic (RPH)

The RPH works through improving the initial solution method by iterating through the following well-established procedures of moving to a good low-cost solution efficiently. Christensen (2013) discussed how the use of some of the rules below can drive towards a reduced cost solution.

(a) Least cost rule
(b) Utilization rule
(c) Fixed cost elimination rule (Location fixed cost and route selection fixed cost)
(d) Feasibility rule

The heuristic uses the least cost rule to determine which sources to open and where to allocate capacities. Moreover, it allocates load units to reduce the number of fixed costs incurred i.e. facility location cost and route fixed costs by pushing load units to already open sources, closing unneeded locations in the process and also moving away from the higher tier fixed cost. Feasibility rule is been used to ensure capacity and demand constraints are satisfied during the load redistribution.

Initial Solution

This is achieved through the linearization and relaxation of integer (binary) variables i.e. the facility location y_i and fixed charge selection z_{ij} variables. A Relaxed Transportation Problem (RTP) is thus formed as a result.

Using the relaxation of integer variables described earlier: Where: $z_{ij} = x_{ij}/M_{ij}$ and $M_{ij} = \min(S_i, D_j)$

Using equation (3.9), the minimum supply requirement implies that:

$$\sum_{i=1}^{m} S_i y_i \geq \sum_{j=1}^{n} D_j \qquad \forall\ i = 1 \dots m, \qquad \forall\ j = 1 \dots n \qquad (3.11)$$

A new location variable $y_i^{\,a}$, which is the average of $\sum_{i=1}^{m} y_i$ to help relax the location variable y_i is developed. Thus equation (3.11) is restated as

$$\sum_{i=1}^{m} S_i \; y_i^{\,a} = \sum_{j=1}^{n} D_j \qquad (3.12)$$

∴

$$\sum_{i=1}^{m} S_i \; y_i^{\,a} = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} \qquad (3.13)$$

∴

$$y_i^{\,a} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}}{\sum_{i=1}^{m} S_i} \qquad (3.14)$$

Substituting $y_i^{\,a}$ for y_i and $z_{ij} = x_{ij}/M_{ij}$ equation (3.6a) is transformed as:
Minimize Z_{R1}=

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} \; x_{ij} + \sum_{i=1}^{m} F_i \, y_i^{\,a} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{2} H_{ijk} \frac{x_{ij}}{M_{ij}} \qquad (3.6b)$$

Substituting equation (3.14) in (3.6b) gives:

Minimize Z_{R1} =

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} \; x_{ij} + \sum_{i=1}^{m} F_i \left[\frac{\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}}{\sum_{i=1}^{m} S_i}\right] + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{2} H_{ijk} \frac{x_{ij}}{M_{ij}} \qquad (3.6c)$$

Z_{R1} =

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[c_{ij} + \frac{\sum_{i=1}^{m} F_i}{\sum_{i=1}^{m} S_i} + \frac{\sum_{k=1}^{2} H_{ijk}}{M_{ij}}\right] x_{ij} \qquad (3.6d)$$

Therefore

$$Z_{R1} = \sum_{i=1}^{m}\sum_{j=1}^{n}[\,C_{ij}] \;\; x_{ij} \qquad (3.6e)$$

Where;

$$C_{ij} = c_{ij} + \frac{\sum_{i=1}^{m} F_i}{\sum_{i=1}^{m} S_i} + \frac{\sum_{k=1}^{2} H_{ijk}}{M_{ij}} \qquad \forall\,(i,j) \qquad (3.15)$$

However, considering the breakpoint analyses made in section 3.1.3, earlier, equation (3.15) would be limited. Therefore using equation (3.5d), equation (3.6d) can further be stated as

$$Z_{R2} = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[c_{ij} + \frac{\sum_{i=1}^{m} F_i}{\sum_{i=1}^{m} S_i} + \frac{H_{ij1}}{M_{ij}}\right] x_{ij} \qquad if\ A_{ij} \geq M_{ij} \qquad \forall(i,j)$$

or

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[c_{ij} + \frac{\sum_{i=1}^{m} F_i}{\sum_{i=1}^{m} S_i} + \frac{H_{ij1} + H_{ij2}}{M_{ij} - A_{ij}}\right] x_{ij} \qquad if\ A_{ij} < M_{ij} \qquad \forall(i,j) \qquad (3.6f)$$

From equation (3.6f) above, the cost matrix from which the transportation tableau will be constructed is given as:

$$C_{ij} = c_{ij} + \frac{\sum_{i=1}^{m} F_i}{\sum_{i=1}^{m} S_i} + \frac{H_{ij1}}{M_{ij}} \qquad \forall\ (i,j) \qquad if\ A_{ij} \geq M_{ij} \qquad (3.16a)$$

or

$$C_{ij} = c_{ij} + \frac{\sum_{i=1}^{m} F_i}{\sum_{i=1}^{m} S_i} + \frac{H_{ij1} + H_{ij2}}{M_{ij} - A_{ij}} \qquad \forall\ (i,j) \qquad if\ A_{ij} < M_{ij} \qquad (3.16b)$$

The linear Equation (3.6f) above can be solved using an optimal solution technique for transportation model (e.g. method of Modified U-V distribution method). This is present in optimization transportation software such as Tora.

The load distribution obtained from the relaxed cost Z_{R2} in equation (3.6f) is used in calculating Z in equation (3.6a) and would be termed the current best solution(Z^{CB}). After this, necessary perturbations following the rules presented earlier, are employed to arrive at another Z which is compared to the initial Z^{CB}. Comparing the values of Z^{CB} and Z, If $Z^{CB} \leq Z$ keep (Z^{CB}) as the current best, otherwise i.e. $Z^{CB} > Z$, therefore Z is termed as the current best.

Lower bound calculations

Using equation (3.5a) and the average location variable $y_i{}^a$ in equation (3.14), Altassan et al. (2013) best starting solution for SFCTP has been extended. The SFCLTP lower bound is thus calculated below.

$$Z_{LB} = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[c_{ij} + \frac{\sum_{i=1}^{m} F_i}{\sum_{i=1}^{m} S_i} + \frac{H_{ij1}}{M_{ij}}\right] x_{ij} \qquad if\ A_{ij} \geq M_{ij} \qquad \forall(i,j)$$

or

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[c_{ij} + \frac{\sum_{i=1}^{m} F_i}{\sum_{i=1}^{m} S_i} + \frac{H_{ij1} + H_{ij2}}{M_{ij}}\right] x_{ij} \qquad if \quad A_{ij} < M_{ij} \qquad \forall(i,j) \qquad (3.6g)$$

Solution Improvement (using The RPH proposed):

The x_{ij} allocations obtained from the optimal solution of equation (3.6f) is further perturbed using structured combinations of the least cost preference, high utilization of open locations and systematic elimination of fixed cost either by closing an open location or by preventing the use of high fixed charge along the routes. The perturbation technique aims at getting a better solution while using the rules stated in section 3.1.4 as a guide. The Perturbation moves are a top-down load re-distribution along a column of each row to ensure that feasibility in demand is attained as described in the (4 × 4) transportation tableau in Figure 3-3 below.
S_i (location capacity), D_i (Demand capacity). The last column represents relaxed cost summation along a row.

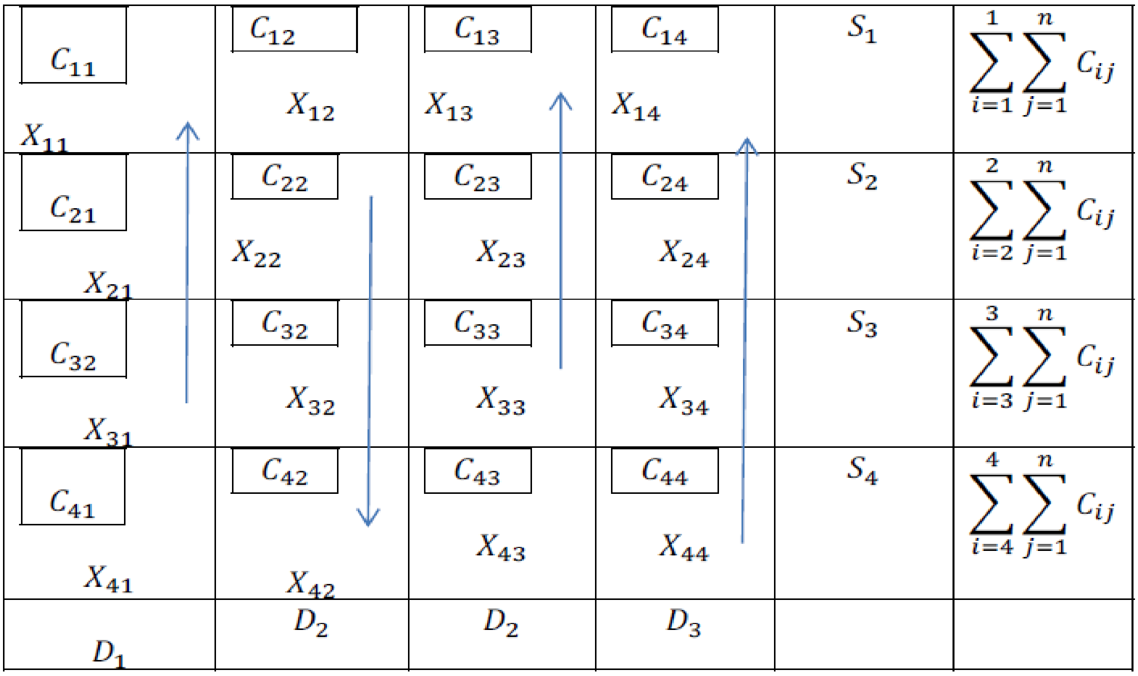

Figure 3-3 Sample perturbation moves

From equation (3.6a) above, it is observed that there are three cost terms in the objective function namely;

(1) Variable cost $(Vc) = c_{ij}x_{ij}$

(2) Location or source fixed cost $(Lc) = (F_i y_i)$

(3) Step fixed cost $(SFc) = (\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{2} H_{ijk}z_{ij})$.

The degree of the values obtained for each of the terms would determine the solution procedure and perturbation technique to be used. The following scenarios out of several possible ones for the structured perturbation logic are therefore noted:

(a) $Vc \gg Lc \ and \ SFc$ (Variable cost having the largest value);
(b) $Lc \gg Vc \ and \ SFc$ (Location cost having the largest value);
(c) $Sfc \gg Vc \ and \ Lc$ (Step-fixed cost having the largest value);
(d) $Vc \cong Lc \cong SFc$ (The three terms being approximately equal)

The Summarized perturbation procedure is given below;

1. Using the linearization in (3.6f) to obtain the starting solution and initial load distribution. The Z obtained is termed the current best (Z^{CB}).

2. **(a)** Calculate the values of the major terms of the objective function i.e. $Vc, \ Lc \ and \ SFc$
(b.1) If $Lc \gg Vc \ and \ SFc$ go to **step (3),**
(b.2) Else If $Sfc \gg Vc \ and \ Lc$ go to **step (4).**
(b.3) Else go to Step 1 and exit Procedure.

3. **(a.1)** For location cost reduction (Lc) if dummy rows are obtained from **step (1)**
(a.2) Yes: ignore row and capacity in calculation. Else go to **Step (3b.1)**
(b.1) Check if $\sum_{i=1}^{m} S_i y_i - \sum_{j=1}^{n} D_j \geq \min(S_{i=1 \cdots m})$, excluding $i = \ dummy \ row.$
(b.2) If true proceed to 3c.1,
(b.3) Else if $\sum_{i=1}^{m} S_i y_i - \sum_{j=1}^{n} D_j = 0$ Stop and exit procedure. Return Z^{CB}
(b.4) Else go to **Step 3h.**
(c.1) Identify whether rows or locations with partially utilized capacities are available
(c.2) arrange in the order of decreasing $\sum_{i=m}^{m} \sum_{j=1}^{n} C_{ij} \ \ for \ i = 1,2 \ldots m$ where $C_{ij} \rightarrow$ relaxed cost matrix (Break ties arbitrarily and select largest.)
(d.1) Identify rows or locations with fully utilized capacities and arrange in the order of decreasing $\sum_{i=m}^{m} \sum_{j=1}^{n} C_{ij} \ \ for \ i = 1,2 \ldots m$ where $C_{ij} \rightarrow$ relaxed cost matrix (Break ties).If yes go to **(3e.1).** If none go to **Step (3g).**
(e.1) Is there an X_{ij} with maximum C_{ij} position according to the row identified in **Step (3d.1)**?
(e.2) Yes: Remove allocations starting with maximum C_{ij} position from open and allocated X_{ij} positions of the fully utilized rows as identified in **Step3(d.1)** or (as per partially utilized row as in **step (3g)**) and add into position (i, j) of the partially utilized rows in decreasing order of $\sum_{i=m}^{m} \sum_{j=1}^{n} C_{ij} \ \ for \ i = 1,2 \ldots m$ to balance the row capacity (break ties as **step3c).**
(e.3) No: maximum position has no load then move to next in rank of C_{ij}, (break ties arbitrarily)
(f) Repeat step **3(e)** until allocations have been completely removed in the fully or partially utilized row identified as per step (3e). Go to **Step (3h)**

(g) Arrange the partially utilized location or row capacity in an order of decreasing $\sum_{i=m}^{m} \sum_{j=1}^{n} C_{ij} \ \ for \ i = 1,2 \ldots m$, (break ties as in **Step3c**) and select Maximum. Repeat **Steps (3e) to (3f).**

(h.1) Use the current load distribution to calculate $Z(new)$. Compare the values of Z^{CB} and $Z(new)$

(h.2) If $Z(new) < Z^{CB}$ Z is termed as the current best and go to **Step (1).**

(h.3) If otherwise i.e. $Z\ (new) > Z^{CB}$, Stop and exit procedure.

4 **(a.1)** For the Step-fixed charge cost reduction, check if any dummy rows?

(a.2) Delete any dummy rows or unutilized locations obtained in **step 1**.

(b.1) Identify if rows or locations with partially utilized capacities are available

(b.2) Arrange in the order of increasing $\sum_{i=m}^{m}\sum_{j=1}^{n} C_{ij}\ \ for\ i = 1,2...m$ where $C_{ij} \rightarrow$ relaxed cost matrix (Break ties arbitrarily).

(c) Identify rows and locations with fully utilized capacity and arrange in the order of increasing $\sum_{i=m}^{m}\sum_{j=1}^{n} C_{ij}\ \ for\ i = 1,2...m$. (Break ties as in **Step4b**).

If none go to **Step (3b).**

(d.1) Check If there are open and allocated x_{ij} positions greater than A_{ij} within the largest row as identified by **step (4c)** at maximum C_{ij} position?

(d.2.) No: If the maximum position has no load move to next in rank of C_{ij}

(d.3) Yes: Check if un-allocated positions x_{ij} of the row as identified by **step (4b.1)** can accommodate the move.

(d.3.1) No: If current capacity cannot accommodate the reallocation, move to the next ranked partially utilized capacity row according to **Step(4b.1)** Proceed till the identified x_{ij} position in **step(4d.1) or (4g)** has been redistributed in a single step of A_{ij}. If no partially utilized row with availability go to **Step (3b).**

(d.3.2) Yes: Redistribute (A_{ij}) identified at **Step(4.d.1)** starting with the x_{ij} with at maximum C_{ij} position

(f.) Repeat **Step (4b) to (4d)** until moves already taken are about to be repeated or till a position $x_{ij} - A_{ij}$ after using **step (4d or 4g)** becomes x_{ij} . Use the current load distribution to calculate Z . go to **Step (3b).**

RPH Flow Chart Description

A flow chart showing the perturbation steps described above and how they iterate to improve the starting solution is presented in Figure 3-4 below. The flow chart symbols utilized have the same meaning as standard flow chart symbols. RPH iterative procedure as shown in the flow chart uses the initial solution to determine quickly whether location Fixed cost elimination or upper-tier route fixed cost elimination would be appropriate to achieve an overall cost reduction. The load redistribution using the order of decreasing $\sum_{i=m}^{m}\sum_{j=1}^{n} C_{ij}$ from the fully allocated routes during Location fixed cost elimination aims to reduce cost from high-cost arcs or routes. Furthermore, reducing cost by value A_{ij} from maximum C_{ij} position at Step 4 prevents incurring upper-tier route fixed cost. Also, load redistribution into locations with increasing order of $\sum_{i=m}^{m}\sum_{j=1}^{n} C_{ij}$ in step 4 ensures lower-cost routes are utilized before higher ones. The flow chart also has the capacity to quickly arrive at a current best solution depending on the problem structure encountered while checking the condition $(\sum_{i=1}^{m} S_i y_i - \sum_{j=1}^{n} D_j = 0)$ and using the exit procedure of Step (2b.3).

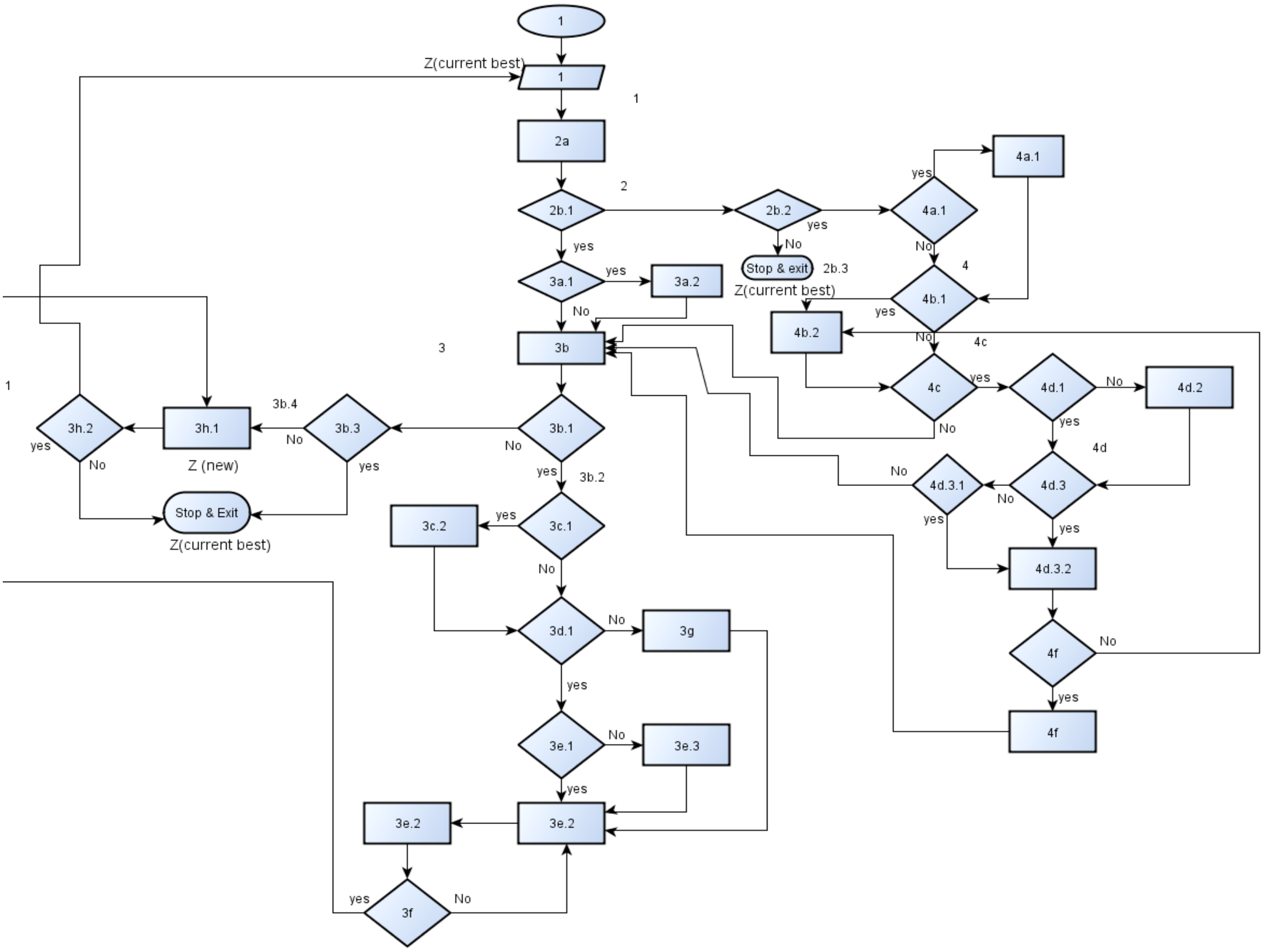

Figure 3-4 Flow chart on row perturbation heuristic improving initial solution

3.1.5 Numerical example

Given the Supply and demand capacities, unit costs and fixed charges as in Table 3-1 and 3-2 below (adapted from Kowalski and Lev (2008)), the workings of RPH are illustrated.

Table 3-1 Supply, demand, location (set up) costs and unit cost parameters

$\boldsymbol{i}$	$\boldsymbol{S_i}$	$\boldsymbol{F_i}$	$\boldsymbol{j}$ **= 1**	**2**	**3**	**4**
			c_{ij}			
1	**25**	**100**	1	3	1	3
2	**25**	**200**	2	2	3	2
3	**25**	**250**	2	1	2	1
4	**25**	**150**	1	3	1	3
$\boldsymbol{D_j}$			**10**	**30**	**20**	**15**

Table 3-2 Two-tier fixed charges on route $\boldsymbol{i,j}$

$\boldsymbol{i}$	$\boldsymbol{h_{ij1}}, \boldsymbol{h_{ij2}}$	$\boldsymbol{h_{ij1}}, \boldsymbol{h_{ij2}}$	$\boldsymbol{h_{ij1}}, \boldsymbol{h_{ij2}}$	$\boldsymbol{h_{ij1}}, \boldsymbol{h_{ij2}}$
1	10 ; 20	10 ; 10	10 ; 30	10 ; 10
2	10 ; 30	10 ; 20	10 ; 20	10 ; 20
3	10 ; 20	10 ; 30	10 ; 10	10 ; 30
4	10 ; 20	10 ; 10	10 ; 30	10 ; 10
	$\boldsymbol{j}$ **=1**	**2**	**3**	**4**

The breakpoint $A_{ij} = 5$ $(constant)$ through route i, j

From equation 6(a) to 10(b) it is noted that;
If $x_{ij} > 0$ and $\leq 5, g_{ij1} = 1, g_{ij2} = 0,$ and $z_{ij} = 1$
Therefore: $H_{ij1}\ z_{ij} + H_{ij2}\ z_{ij} = g_{ij1}h_{ij1}z_{ij} + g_{ij2}h_{ij2}z_{ij} = (1) \times h_{ij1} \times (1) + (0) \times h_{ij2} \times (1) = H_{ij1}\ z_{ij}$

If $x_{ij} > 0$ $and > 5, g_{ij1} = 1$ and $g_{ij2} = 1,$ $z_{ij} = 1$
Therefore: $H_{ij1}\ z_{ij} + H_{ij2}\ z_{ij} = g_{ij1}h_{ij1} + g_{ij2}h_{ij2} = (1) \times h_{ij} \times (1) + (1) \times h_{ij2} \times (1) = H_{ij1}\ z_{ij} + H_{ij2}\ z_{ij}$

If $if\ \ x_{ij}\ = 0,\ z_{ij}\ = 0,$ Therefore:

$H_{ij1}\ z_{ij} + \ H_{ij2}\ z_{ij} = 0$

For (i,j) position (1,1) $M_{11} = 10$, and $A_{11} = 5$ thus $A_{11}\ < M_{11}$

From equation (3.16a and 3.16b) above Equation 3.16b is selected

$$C_{11} = \left[c_{11} + \frac{F_1 + F_2 + F_3 + F_4}{S_1 + S_2 + S_3 + S_4} + \frac{h_{111+}\ h_{112}}{M_{11} - A_{11}}\right]$$

$$C_{11} = \left[1 + \frac{100 + 200 + 250 + 150}{25 + 25 + 25 + 25} + \frac{10 + 20}{10 - 5}\right] = 14$$

For all (i,j) position, $A_{ij}\ < M_{ij}$ Equation 3.16b is selected for $C_{11}, C_{12}... C_{mn}$

C_{ij} relaxed cost matrix for $C_{11}, C_{12}... C_{mn}$ is given in Table 3-3 below;

Table 3-3 C_{ij} relaxed cost matrix

14	11	10.67	12
17	10.5	12	12
15	10	10.33	12
14	11	10.67	12

Initial solution

Tora software which uses the modified u-v distribution method of solving linear transportation models was used to solve the cost matrix above (as a balanced problem) optimally to give the initial solution of the SFCLTP (Z_{R2}) represented in Table 3-4 below.

Table 3-4 Optimal load distribution using the relaxed cost matrix

14 **5**	11 0	10.67 0	12 0	0 **20**	25
17 0	10.5 **5**	12 0	12 **15**	0 **5**	25
15 0	10 **25**	10.33 0	12 0	0 0	25
14 **5**	11 0	10.67 **20**	12 0	0 0	25
10	30	20	15	25	

For the lower bound value for SFCLTP (Z_{LB}), the cost matrix below was obtained from the relaxed unit cost in equation (3.6g) like the relaxed costs of (3.16a and 3.16b). The load distributions after solving optimally with Tora software are presented Table 3-5 below.

Table 3-5 Optimal load distribution for lower bound determination

11 **10**	10.8 0	10 **15**	11.33 0	0 0	25
13 0	10.2 **5**	11.5 0	11 **15**	0 **5**	25
12 0	9.6 **25**	10 0	10.67 0	0 0	25
11 0	10.8 0	10 **5**	11.33 0	0 **20**	25
10	30	20	15	25	

Using equation (3.6f) above;

$$Z_{R2} = (14 \times 5) + (14 \times 5) + (10.5 \times 5) + (10 \times 25) + (10.67 \times 20) + (12 \times 15)$$
$$= 835.9$$

Using equation (3.6e) for the lower bound calculation,

$$Z_{LB} = (11 \times 10) + (10.2 \times 5) + (9.6 \times 25) + (10 \times 5) + (10 \times 15) + (11 \times 15)$$
$$= 766$$

Using the load distribution for both Z_{R2} and Z_{LB} and equation (3.6a) for calculating Z for Z_{R2} and Z_{LB} which is represented as Z (Z_{R2}) and Z (Z_{LB}) respectively,

$$\text{Z}\,(Z_{R2}) = (14 \times 5) + (14 \times 5) + (10.5 \times 5) + (10 \times 25) + (10.67 \times 20) + (12 \times 15) + (100 + 200 + 250 + 150) + (10 + 10 + 10 + 40 + 40 + 30) = 935$$

$$\text{Z}\,(Z_{LB}) = (1 \times 10) + (2 \times 5) + (1 \times 25) + (1 \times 15) + (1 \times 5) + (2 \times 15) + (100 + 200 + 250 + 150) + (10 + 30 + 10 + 40 + 40 + 30) = 955$$

Therefore the current best solution (Z^{CB}) for the SFCLTP Z $(Z_{R2}) = 935$ with a lower bound $Z_{LB} = 766$

Improved solution (Using RPH)

In order to apply RPH solution heuristic, the initial solution Z_{R2} matrix is labelled row and column-wise as in Table 3-6 below:

Table 3-6 Row and Column labelling of initial solution to apply RPH

Column1	Column 2	Column 3	Column 4	Dummy		A	
14 **5**	11.33 0	10.67 0	12 0	0 **20**	25	47.67	Row1
17 0	10.5 **5**	12 0	12 **15**	0 **5**	25	51.5	Row2
15 0	10 **25**	10.33 0	12 0	0 0	25	47.33	Row3
14 **5**	11 0	10.67 **20**	12 0	0 0	25	47.67	Row4
10	30	20	15	25			

From lower bound calculation section and using the initial solution Z (Z_{R2}), it is noted that optimum objective function cost and current best $(\text{Z}^{CB}) = 935$.

Step (1) Current best $(\text{Z}^{CB}) = 935$.

Step (2a) Variable cost (Vc) = $c_{ij}x_{ij}$ = 95

Location or source fixed cost (Lc) = $(F_i y_i)$ = 700

Step fixed cost (SFc) = $(\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{2} H_{ijk}z_{ij})$ = 140

Step (2b.1) Therefore since $Lc \gg Vc$ and SFc → Step 3

Step (3a.1) No dummy rows → Step (3b.1)

Step (3b.1) Check $\sum_{i=1}^{m} S_i y_i - \sum_{j=1}^{n} D_j \geq \min(S_{i=1\cdots m})$ $i.e$ $100 - 75 = 25$ → Step (3c.1)

Step (3c.1) Row 1 and Row 2 are partially utilized→ Step (3c.2).

Step (3c.2) Arranging in decreasing $\sum_{i=m}^{m}\sum_{j=1}^{n} C_{ij}$ $for\ i = 1,2\ldots m$

In decreasing order gives (Row 2, Row1). Row 2 selected as having the largest $\sum_{i=m}^{m}\sum_{j=1}^{n} C_{ij}$

Step (3d.1) Row 3 and Row 4 are fully utilized.

In order of decreasing $\sum_{i=m}^{m}\sum_{j=1}^{n} C_{ij}$ $for\ i = 1,2\ldots m$ (Row4, Row3). Row 4 is selected →Step (3e.1). The Row selection is shown in Table 3-7 below:

Table 3-7 Row selection for perturbation

14 **5**	11.33 0	10.67 0	12 0	47.67	25	Partially utilized
17 0	10.5 **5**	12 0	12 **15**	51.5	25	Partially utilized & selected
15 0	10 **25**	10.33 0	12 0	47.33	25	Fully utilized
14 **5**	11 0	10.67 **20**	12 0	47.67	25	Fully utilized & selected
10	30	20	15			

Step (3e.1) Row 4, has $X_{41} = 5$ at largest $C_{ij} = C_{43}$ and also load at $X_{43} = 20$ →Step (3e.2).

Step (3e.2) Remove allocation at $X_{41} = 5$ and add to the position X_{21}

Remove allocation at $X_{43} = 20$ also, but no capacity to accommodate move at the position X_{21}. Row 1 is selected next in the decreasing order to receive $X_{43} = 20$ →Step (3f).

Step (3f). Allocations have been fully removed go to →Step (3h).

Step (3h.1). While current best $Z^{CB} = 935$. New load distribution is given in Table 3-8 below:

Table 3-8 Load distribution after Applying RPH

14 **5**	11.33 0	10.67 **20**	12 0	25
17 **5**	10.5 **5**	12 0	12 **15**	25
15 0	10 **25**	10.33 0	12 0	25
14 0	11 0	10.67 0	12 0	25
10	30	20	15	

$$Z\,(new) = \; (1\;\times 5) + (2\;\times 5) + (2\;\times 5) + (1\;\times 25) + (1\;\times 20) + (2\;\times 15) + (100 + 200 + 250) + (10 + 10 + 10 + 40 + 40 + 30) = \;\; 790$$

Step (3h.2). $Z(new) < \mathrm{Z}^{CB}$ $\;Z(new)$ is termed as the current best and go to Step (1).
Step (1) Current best ($\mathrm{Z}^{CB}) = 790$.
Step (2a) Variable cost (Vc) = $c_{ij}x_{ij}$ = 100
Location or source fixed cost $(Lc\,)$ = ($F_i y_i$) = 550
Step fixed cost (SFc) = ($\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{2} H_{ijk}z_{ij}$) = 140
Step(2b.1) Therefore since $Lc \;\gg\; Vc \;\; and \;\; SFc$ $\rightarrow$ Step 3
Step(3a.1) Dummy row at Row 4.$\rightarrow$ Step(3a.2)
Step(3a.2) Ignore row in calculation capacity. $\rightarrow$ Step 3b.1
Step(3b.1) Check $\sum_{i=1}^{m} S_i y_i \; - \; \sum_{j=1}^{n} D_j \; \geq \min(S_{i=1\,\cdots m})\; i.e\; 75 - 75 = 0$ $\rightarrow$Step (3b.2)
Step(3b.2) Check $\sum_{i=1}^{m} S_i y_i \; - \; \sum_{j=1}^{n} D_j \; = \; 0 \;\; \mathrm{i.}e\; 75 - 75 = 0$
Stop and exit procedure
Return Z ($\mathrm{Z}^{CB}) = 790$
Therefore Z (RPH) = 790

3.1.6 Discussion of solutions obtained

For the numerical examples in section 3.1.5 above, using the recast/ relaxed cost matrix as stated in equation (3.16a) and (3.16b) Z_{R2} = 835.9. Also, the lower bound calculation from equation (3.6g) gives $Z_{LB} = 766$. In this example, the relaxed value i.e. Z_{R2} gave an upper bound to the objective function $Z(\,Z_{R2}) \;= 935$ obtained by equation (3.6a). From Figures 3-1 and 3-2, it is noted that the ideal relaxed cost matrix is linear in the objective function and should give a lower bound to the SFCLTP objective function. Furthermore, there could be instances where the relaxation type used, could give an upper bound at the breakpoint as seen in Figure 3-2. The lower bound Z_{LB} gave the minimum out of Z_{LB} , Z_2, $Z(Z_{LB})$ and $Z(Z_{R2})$. However, the load distribution of Z_{R2} gave a better starting solution for RPH.

The starting solution for the numerical example $Z(Z_{R2})$ is 935. However, the solution heuristic gave an improved objective value Z (RPH) = 790 with all constraints satisfied. Using RPH solution, location (Row 4) out of the four competing locations with equal supply capacities but different setup costs is closed as unprofitable for shipping through the fixed charges and transportation costs. RPH thus uses a structured combination of Fixed location cost elimination, cheap route variable cost and load consolidation at lower-tier route fixed cost to drive towards an improved solution while also ensuring the feasibility of all constraints are satisfied.

3.1.7 Perspective

An integrated model that combines the fixed location cost and step-fixed charge transportation cost has been proposed in this subchapter. This has been termed Step-Fixed charge Location and Transportation Problem (SFCLTP). In this model, the Fixed charge Transportation Problem of Balinski (1961) has been extended. Moreover, the linearization and relaxation method developed by Kowalski and Lev (2008) and Altassan et al. (2013) have been extended using the normal Transportation Tableau as a starting solution. Through a perturbation technique that uses the variable transportation cost, fixed facility location cost, and step- fixed charge cost along the selected route in deciding the perturbation moves, better solutions than the optimal solution obtained from the relaxed transportation problem was progressively attained. These solutions are considered good enough, and the heuristic has been termed Row perturbation Heuristics (RPH). Future directions on the model presented could be on applying single solution metaheuristics such as simulated annealing, Tabu search or population metaheuristics such as genetic algorithm, particle swarm optimization to evaluate the relative effectiveness and efficiency of RPH to these metaheuristics. Lastly, initial solutions that do not use the relaxation and linearization which have been employed and better improvement solutions for SFCLTP could be investigated on.

Chapter 3.2

On the Capacitated Step-fixed Charge Transportation and Facility Location Problem: A Local search heuristic

A modified version of this Chapter was presented and has been published in the conference proceedings of the International Conference on Industrial Engineering and Operations Management (IEOM) in Pretoria.

3.2.1 Introduction

In this subchapter, the SFCLTP which has been described to be NP-hard is studied and consequently an LP-based relaxation heuristic to solve the problem is developed. Two fixed charges in the route without loss of generality have been considered. Furthermore, more valid inequality and equality approximations to the model of Christensen (2013) is introduced. The LP relaxation heuristic developed adapts the principle of linearized cost by Kim and Pardalos (2000). The solution begins with some violation of the supply capacities but through the minimum total demand flow cost from a source to all destinations for feasible locations, a simple transportation problem is developed that ensures the demand and supply constraints are met. The results of some problem instances solved based on the LP heuristic in comparison to those obtained from using the CPLEX concert technology are presented.

3.2.2 Base model formulation for SFCLTP

The one stage or two echelons SFCTP and SFCLTP are described as m suppliers and n demand point distribution problems, where m connotes the number of sources (factories or distribution centers) and n refers to the number of customers, demand or sink points. There are capacities and demand for each source or locations and demand point respectively over a time period, usually annual. The m suppliers incur a unit transportation cost per unit distance. In addition, there is more than one fixed charge in the route, link or arc depending on the number of breakpoints in the transportation routes. The SFCLTP further consists of fixed location costs attached to several potential locations to which the cheapest locations that satisfy the problem constraints are to be selected. Christensen (2013) presented a mathematical model for the MultiChoice Model (MCM) of the CFLP with PLC which forms a base for the SFCLTP presented in this subchapter: x_{ijl} is a variable defined as the flow on mode l between facility i and customer j. While a binary variable v_{ijl}, selects if the earlier mentioned mode is used with a one and zero otherwise. F_i represents the facility fixed cost at each facility i, with a binary variable y_i determining if a facility is used or not. The variable cost between facility i and customer j based on the mode l is given as (c_{ijl}). The fixed cost between facility i and customer j based on the mode l is given as g_{ijl}.

Min Z =

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{l=1}^{q}(c_{ijl}x_{ijl} + g_{ijl}v_{ijl}) + \sum_{i=1}^{n} F_i\, y_i \qquad (3.17)$$

Subject to (constraints):

$$\sum_{i=1}^{n}\sum_{l=1}^{q} x_{ijl} = D_j \qquad \forall\ j \qquad (3.18)$$

$$\sum_{l=1}^{q} v_{ijl} \leq 1 \qquad \forall\ (i, j\ ) \qquad (3.19)$$

$$\sum_{j=1}^{m}\sum_{l=1}^{q} x_{ijr} \leq \quad S_i y_i \qquad \forall\ i \tag{3.20}$$

$$x_{ijl} \leq L_{ijl}\, v_{ijl} \qquad \forall\,(i,j,l\,) \tag{3.21}$$

$$x_{ijl} \geq L_{ijl}\, v_{ijl} \qquad \forall\,(i,j,l\,) \tag{3.22}$$

$$x_{ijl} \geq \ 0 \tag{3.23}$$

$$y_i \ \in \ \{\,0{,}1\} \qquad \forall\ i \tag{3.24}$$

$$v_{ijl} \ \in \ \{\,0{,}1\} \qquad \forall\,(i,j,l\,) \tag{3.25}$$

Equation (3.17) is the total cost minimization objective function representing location and transportation cost. Equation (3.18) represents the compulsory demand capacity. Equation (3.19) enforces one transportation mode between each pair of source and destination. Equation (3.20) represents supply capacity not being exceeded. Equation (3.21) ensures an upper bound on the load distribution, while Equation (3.22) enforces a lower bound on distribution. Equation (3.23) refers to the non-negativity constraint while equations (3.24 and 3.25) refer to the binary constraints.

3.2.3 Problem structure and formulation for SFCLTP

For the linearization and relaxation of the solution for the SFCLTP, a valid inequality constraint for the breakpoint as used in the work of Sanei et al. (2017) was introduced to the model of Christensen (2013) to further strengthen the LP lower bound relaxation. Secondly, the Balinski (1961) method of binary variable relaxation which Kowalski and Lev (2008) and Altassan et al. (2013) employed in relaxing their binary fixed charge variables was also employed. This method uses some constraints in the problem to relax the binary constraints in the cost objective term. This is a general approach commonly employed for LP relaxations.

Model Assumptions

The following assumptions made in this model are similar to those discussed in section 3.1.3:

Model Parameters

i : Index for the set of Sources (Plants, Warehouses etc.)
j : Index for the set of Destinations (Customers, Warehouses, depots etc.)
m : Number of sources (or plants)
n : Number of destinations (or demand point)
c_{ij} : The unit cost of shipment on route (i,j)
S_i : Supply Capacity for Source i
D_j : Demand Capacity for Destination j
H_{ij} : the First level fixed cost
I_{ij} : The second level fixed cost
A_{ij} : Breakpoint for selecting the route fixed charges

Decision variables

x_{ij} : Allocations (or load distributions) along the route (i,j)
y_i : Location variable for plant or source (0 or 1)
g_{ij} : Binary fixed charge variable before the breakpoint

z_{ij} : Binary fixed charge variable after the breakpoint

Objective function of the Original Problem:

Minimize Z_O =

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\,x_{ij} + \sum_{i=1}^{m} F_i\,y_i + \sum_{i=1}^{m}\sum_{j=1}^{n} H_{ij}\ g_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} I_{ij}\ z_{ij} \qquad (3.26)$$

Subject to (constraints):

$$\sum_{j=1}^{n} x_{ij} \le S_i y_i \qquad \forall\ i = 1 \dots m \qquad (3.27)$$

$$\sum_{i=1}^{m} x_{ij} = D_j \qquad \forall\ j = 1 \dots n \qquad (3.28)$$

$$\sum_{i=1}^{m} S_i y_i \ge \sum_{j=1}^{n} D_j \qquad \forall\ i = 1 \dots m, \quad \forall\ j = 1 \dots n \qquad (3.29)$$

$$x_{ij} \le M_{ij}\,g_{ij} \qquad \forall\ i = 1 \dots m,\ \forall\ j = 1 \dots n \qquad (3.30)$$

$$x_{ij} - A_{ij} \le M_{ij} z_{ij}\ \ \forall\ i = 1 \dots m,\ \forall\ j = 1 \dots n \qquad (3.31)$$

$$x_{ij} \ge 0 \qquad (3.32a)$$

$$y_i = 0\ or\ 1 \qquad z_{ij} = 0\ or\ 1 \qquad g_{ij} = 0\ or\ 1 \qquad (3.32b)$$

$$g_{ij} = \begin{cases} 1 & x_{ij} > 0 \\ 0 & Otherwise \end{cases}, \quad z_{ij} = \begin{cases} 1 & x_{ij} > A_{ij} \\ 0 & Otherwise \end{cases} \qquad (3.32c)$$

$$M_{ij} = \min(S_i, D_j)\ \ \forall\ i = 1 \dots m,\ \forall\ j = 1 \dots n$$

Equation (3.26) is the objective function. The first term is a variable cost, the second term is the facility location cost and the third term is the route step-fixed charge cost. Equation (3.27) is the supply capacity constraint of each location or sources which should not be exceeded. Equation (3.28) is the demand constraint of each customer which must be met. Equation (3.29) is the aggregate constraint for supply and demand balance. Equation (3.30) is a constraint that ensures that an upper bound on the load distribution before the breakpoint is established. Equation (3.31) is a constraint that ensures that a lower bound on the load distribution after the breakpoint is established. Equation (3.32a) refers to the non-negativity constraint while Equation (3.32b) refers to the binary integer constraints. Equation (3.32c) ensures the objective function selects the fixed charges based on the breakpoints.

3.2.4 Solution method

As indicated in earlier sections, the solution method presented in this work is based on linearizing the cost objective function through the relaxation of some of the constraints. The relaxed constraints are used in the objective function to relax the location y_i and the route fixed charge g_{ij} and z_{ij} variables. Figure 3-1 in section 3.1.2 represents the linearization cost structure of the objective function Z_O before and after linearization. The linearized model is solved to optimality as the first transportation problem and a lower bound to the SFCLTP is obtained from the result of the first transportation problem solution. The linearized cost solution obtained, however, may produce some supply capacity infeasibilities at the sources, which have to be resolved before the desired solutions are obtained. The LP heuristic presented, resolves the supply infeasibilities by identifying the combination of potential sources that meet the demand. Using the linearized cost version of equation (3.26) and assuming total flow of the entire demand load from a source (i) and all destinations(j), the total flow for each of the sources ($i = 1 \dots m$) is computed. Croxton et al. (2007) suggested that the LP relaxation of the cost objective function would give an approximation of the original problem with the "lower convex envelope" when the total flow on each route is used in the computation. Finally, combination(s) of the sources (i) that meet all the demand $\sum_{j=1}^{n} D_j \quad \forall\ j = 1 \dots n$ with the least-cost combination using the linearized version of cost equation (3.26) is selected for the final transportation problem. To solve the final transportation problem, the relaxed cost combination of $c_{ij,} H_{ij}$ *and* I_{ij} is used to determine the final load distribution (x_{ij}) to be used in equation (3.26).

Linearized Cost Solution

The principle of linearized cost as used by Kim and Pardalos (2000) is adapted for the linearization to solve the first transportation problem. Constraints (3.27), (3.30) and (3.31) are selected respectively, an equality upper bound is placed on them and they are used in relaxing the fixed location cost and fixed charge variables, y_i , g_{ij} and z_{ij} respectively in the objective function equation (3.17).

The constraints given by (3.27), (3.30) and (3.31) are transformed respectively into

$$\sum_{j=1}^{n} x_{ij} = \quad S_i y_i \qquad \forall\ i = 1 \dots m \tag{3.33}$$

$$x_{ij}/M_{ij} \ = \ g_{ij} \tag{3.34}$$

$$(x_{ij} - A_{ij})/\ M_{ij} = z_{ij} \tag{3.35}$$

Equation (3.33) is substituted into the aggregate constraint of equation (3.29) above to obtain

$$\sum_{i=1}^{m} S_i (\frac{\sum_{j=1}^{n} x_{ij}}{S_i}) \ \geq \ \sum_{j=1}^{n} D_j \qquad \forall\ i = 1 \dots m, \qquad \forall\ j = 1 \dots n \tag{3.36}$$

Substituting equation (3.33) (3.34) and (3.35) into the objective function (equation (3.26)) and using the derived equation (3.36) would give a linear programming problem model of the original problem and given as:

Min Z_{LPO} =

$$\sum_{i=1}^{m} (F_i \ \frac{\sum_{j=1}^{n} x_{ij}}{S_i}) + \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij} \ + \ \sum_{i=1}^{m} \sum_{j=1}^{n} H_{ij} \ \frac{x_{ij}}{M_{ij}} \ + \ \sum_{i=1}^{m} \sum_{j=1}^{n} I_{ij} \ \frac{x_{ij} - A_{ij}}{M_{ij}} \tag{3.37}$$

Subject to

$$\sum_{i=1}^{m} x_{ij} = D_j \qquad \forall\ j = 1 \dots n \tag{3.38}$$

$$\sum_{i=1}^{m} S_i\left(\frac{\sum_{j=1}^{n} x_{ij}}{S_i}\right) \geq \sum_{j=1}^{n} D_j \qquad \forall\ i = 1 \dots m, \qquad \forall\ j = 1 \dots n \tag{3.39}$$

$$x_{ij} \geq 0 \tag{3.40}$$

Equation (3.37) could also be restated as

$$Z_{LP0} = \sum_{i=1}^{m}\left(F_i \frac{\sum_{j=1}^{n} x_{ij}}{S_i}\right) + \sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij} + \frac{H_{ij}}{M_{ij}} + \frac{I_{ij}}{M_{ij}}\right) x_{ij} - \sum_{i=1}^{m}\sum_{j=1}^{n} I_{ij} \frac{A_{ij}}{M_{ij}} \tag{3.41}$$

Equations (3.37) to (3.40) give a linear programming problem which can be solved to optimality by any known optimal linear programming solver.

a. Resolution of Linearization Infeasibility and Final Transportation Problem

It is noted that in the solution of the new relaxed model represented by equation (3.37) to (3.40), there could be supply infeasibilities due to constraints (3.38) and (3.39) not strictly enforcing the supply requirements. These supply infeasibilities are resolved through the heuristic presented below:

Step 1: Select individuals or groups of potential sources ($i \in m$) such that $\sum_{i=1}^{m} S_i\, y_i \geq \sum_{j=1}^{n} D_j$

Step 2: For the individuals or groups selected, Use equation (3.41) only to calculate $Z_{LP0}^{i}\ \forall\ i = 1 \dots m,$ where the load distribution is given as $x_{ij} : x_{ij} = D_j\ \forall\ j = 1 \dots n$. (thus Z_{LP0}^{1} is calculated using $x_{11},\ x_{12},\ x_{13}, \dots\dots x_{1n},$)

Step 3: From the individuals or groups of $i \in m$ selected in step 1, choose the minimum Z_{LP0}^{i} or combinations of Z_{LP0}^{i} ($\sum_{i}^{m} Z_{LP0}^{i}$) if groups are selected. It is noted that Z_{LP0}^{i} values could be positive or negative depending on the problem parameters. This is evident by the third term in equation (3.41) which has the possibility of making the equation negative. If the combinations of Z_{LP0}^{i} are positive the minimum value is selected. However if the combinations of Z_{LP0}^{i} are negative values the minimum absolute value is selected.

Step 4: Solve the transportation problem created by using the sources selected in Step 3 and the relaxed cost combination of ($c_{ij} + \frac{H_{ij}}{M_{ij}} + \frac{I_{ij}}{M_{ij}}$) subject to simple demand and supply constraints to obtain the load distribution x_{ij} for equation (3.26). The transportation problem to be solved is given as

$$Min\ Z_F = \sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij} + \frac{H_{ij}}{M_{ij}} + \frac{I_{ij}}{M_{ij}}\right) x_{ij} \tag{3.42}$$

$$\sum_{j=1}^{n} x_{ij} \leq \quad S_i y_i \qquad \forall\ i = 1 \dots m \qquad (3.43)$$

$$\sum_{i=1}^{m} x_{ij} = \quad D_j \qquad \forall\ j = 1 \dots n \qquad (3.44)$$

$$x_{ij} \geq 0 \qquad (3.45)$$

Step 5: Using the value of Z_O, compare the load distribution x_{ij} obtained by solving the transportation model given by equation (3.35) to (3.38) by any feasible approach such as the North-West Corner (NWC) method and optimally using Simplex or the Modified u-v distribution method known as (Modi). Select the minimum Z_O obtained by the methods.

The Least Cost (LC) approach of solving the transportation problems ensures that maximum allocations are made to the minimum relaxed cost position, while NWC approach ensures that allocations are made to the least cost positions from a northwest corner of the transportation tableau. An optimal transportation cost solution using the Modified u-v distribution method or Modi would perform dual variable (u and v) analysis and ensure that $m + n - 1$ variable positions are occupied.

3.2.5 Numerical solution and computation study

This section consists of the numerical computation done using a random problem size of (4× 4). Furthermore, computation studies were done to further gain insight into the heuristic performance when compared to the standard MILP solver such as CPLEX.

Numerical Example

In order to explain the workings of the LP-based heuristic presented under section 3.2.4, the problem created by Kowalski and Lev (2008) has been adapted. The problem is represented in Tables (3-9) and (3-10) below and the solution is also presented below.

Table 3-9 Supply, demand, location (set up) costs and unit cost parameters

i	S_i	F_i	j = 1	2	3	4
			c_{ij}			
1	25	100	1	3	1	3
2	25	200	2	2	3	2
3	25	250	2	1	2	1
4	25	150	1	3	1	3
D_j			10	30	20	15

Table 3-10 Two-tier fixed charges on the route (i, j)

i	H_{ij}, I_{ij}	H_{ij}, I_{ij}	H_{ij}, I_{ij}	H_{ij}, I_{ij}
1	10 ; 20	10 ; 10	10 ; 30	10 ; 10
2	10 ; 30	10 ; 20	10 ; 20	10 ; 20
3	10 ; 20	10 ; 30	10 ; 10	10 ; 30
4	10 ; 20	10 ; 10	10 ; 30	10 ; 10
	j =1	2	3	4

The breakpoint $A_{ij} = 5$ $(constant)$ through route i, j

From section 3.2.4, using the linearized cost model represented by equation (3.37) to (3.40) the value of $Z_{LPO} = 488.75$ is obtained

25	10	30	20	15
25	0	0	0	0
25	0	0	0	0
25	0	0	0	0
	10	30	20	15

This clearly gives an infeasible solution as noted in section 3.2.4.

Step 1:

Using the first step of the heuristic, the combinations of locations is obtained and used to resolve the infeasible solution obtained above. The combinations of the locations are (1,2,3), (1,2,4), (1,3,4), (2,3,4) and (1,2,3,4) as having $\sum_{i=1}^{m} S_i\, y_i \geq \sum_{j=1}^{n} D_j$

However, it is clear that the combination of (1,2,3,4) would not be cost-saving based on the extra fixed location cost.

Step 2:

The objective value Z_{LP0}^{i} for the four sources is computed (no optimization is done here):

Z_{LP0}^{1} is calculated using $x_{11}, \; x_{12}, \; x_{13}, x_{14}, \; = 488.75$

Z_{LP0}^{2} is calculated using $x_{21}, \; x_{22}, \; x_{23}, x_{24}, \; = \; 794.5$

Z_{LP0}^{3} is calculated using $x_{31}, \; x_{32}, \; x_{33}, x_{34}, \; = \; 867.75$

Z_{LP0}^{4} is calculated using $x_{11}, \; x_{12}, \; x_{13}, x_{14}, \; = \; 638.57$

The location (1,2,3) gives a sum of (488.75+794.5+867.75) = 2151

Similarly locations (1,2,4), (1,3,4), (2,3,4) gives 1921.82, 1995.07 and 2300.82 respectively.

Step3:

Since the combinations of Z_{LP0}^{i} are positive, the minimum value is selected. The location (1,2,4) possesses the minimum sum of linearized cost and thereby chosen.

Step 4:

The relaxed cost combination of $c_{ij} + \frac{H_{ij}}{M_{ij}} + \frac{I_{ij}}{M_{ij}}$ given below is used as the new unit cost to solve the second transportation problem given by equation (3.42) to (3.45) to obtain the final load distribution to be used to calculate Z_O in equation (3.26) above. The relaxed cost combination matrix is presented below.

4	3.8	3	5.166667
6	3.2	4.5	3.416667
5	2.6	3	2.166667
4	3.8	3	5.166667

Step 5:

The Load distribution obtained when Transportation problem (3.42) to (3.45) is solved optimally and using the least-cost approach $Z_F = 259.25\ and\ Z_O = 750$ is presented below.

25	10	15	0	0
25	0	10	0	15
25	0	0	0	0
25	0	5	20	0
	10	30	20	15

Using the Northwest corner approach for solving the transportation problem 3.42 to 3.45 gives

$Z_F = 276.95\ and\ Z_O = 720$. This is shown below.

25	10	0	0	15
25	0	25	0	0
25	0	0	0	0
25	0	5	20	0
	10	30	20	15

The CPLEX Values obtained for Z_O are given below. $Z_O = 710$

25	5	0	20	0
25	0	25	0	0
25	0	0	0	0
25	5	5	0	15
	10	30	20	15

Computation study

To further gain a little insight into the performance of this LP-based heuristic in relation to the solution provided by CPLEX, a small scale computation study was done. This study was however based on the objective values of the problem alone, though articles in this field would also compare the runtime of the different solution methods. The primary interest in this subchapter is to observe the trend of the objective values for the little pilot study. The SFCLTP model was coded using the Java Eclipse environment with the CPLEX 12.8 concert technology as the MILP solver. The LP heuristic was partly coded in Java and the transportation optimizations were solved with Microsoft excel solver and TORA software. A total of 25 randomly generated problems were solved. These problems were classified under two sets of problem instances. The first set of problem instances had two problem sizes (4×4 and 7×7) with 5 problems solved for each size. The LP-based heuristic was solved using the NWC to obtain the final load distribution according to the LP heuristic step 5.

For the second set of problems, three problem sizes (6×4, 8×5 and 9×7) were considered with 5 problem instances generated under each size. The step 5 of the LP-based heuristic was however solved using the NWC, LC and optimally using the modified u-v distribution method (Modi) in TORA optimization software to obtain the final load distribution. This was done to further gain an insight into the performance of the feasible transportation methods and optimal transportation method at arriving at the final load distribution. Table 3-11 below shows the parameter range used for the random problem generation. The orders of magnitude of the parameters utilized for the range have been selected to reflect the reality of the proportions of the parameters compared to one another.

Table 3-11 Data range used for the small scale computation study

Problem size (No. of Instances)	Parameter	Range of values	Breakpoint per problem size
4×4 (5)	F_i	$100 - 550$	$A_{ij} = 5$
	c_{ij}	$1 - 10$	
7 × 7 (5)	S_i	$10 - 100$	$A_{ij} = 5$
6 × 4 (5)	D_j	$5 - 50$	$A_{ij} = 5$
8 × 5 (5)	H_{ij}	$10 - 30$	$A_{ij} = 10$
9 × 7 (5)	I_{ij}	$10 - 30$	$A_{ij} = 25$

The mean value and the 5 different problem instances of the CPLEX and LP heuristic (using NWC solution) generated are shown in Table 3-12 below. Also, the percentage mean difference have been included to show the gap obtained from the CPLEX values. The larger the mean percentage, the poorer the results of the LP heuristic as compared to CPLEX values. Similarly to Table 3-12, Table 3-13 below presents the results obtained using the northwest corner, least cost and modified u-v distribution method using only the mean values of the different 5 problem instances generated.

Table 3-12: First set of results obtained

Problem Size	Instance No	CPLEX (Z_O)	LP heuristic (Z_O) (NWC)	% mean Difference LP heuristic and CPLEX
4×4	1	710	720	2.4%
	2	735	735	
	3	705	710	
	4	685	725	
	5	580	610	
Mean values	$(\overline{Z_O})$	683	700	
7×7	1	1315	1315	1.0%
	2	1425	1480	
	3	1610	1620	
	4	1910	1920	
	5	1755	1765	
Mean values	$(\overline{Z_O})$	1603	1620	

Table 3-13 Second set of results obtained

Problem Size	Mean CPLEX ($\overline{Z_O}$) Value	Mean LP heuristic ($\overline{Z_O}$)(NWC)	Mean LP heuristic ($\overline{Z_O}$) (LC)	Mean LP heuristic ($\overline{Z_O}$) (Modi)	% mean difference LP-NWC and CPLEX	% mean difference LP-LC and CPLEX	% mean difference LP-Modi and CPLEX
6×4	1665	1915	1810	1731	15.01%	8.70%	3.96%
8×5	2471	2964	2514	2494	19.95%	1.74%	0.93%
9×7	2961	3392	3102	3015	14.56%	4.76%	1.82%

3.2.6 Discussion of solutions obtained

From the results of the little computation study conducted, it is easily seen that the CPLEX solution outperformed the LP-based heuristic as per the individual instances and mean values considered both in Tables 3-12 and 3-13. Results obtained for the first set of problems as per Table 13, shows the mean objective value gap difference of 2.4% and 1.0% respectively. A quick analysis using the paired t-test of means for the (4×4) problem size at 0.05 level of significance gave a p-value of 0.09130, showing there is not quite statistical significance in the difference of the means obtained, while the second and larger problem size (7×7) gave a p-value of 0.15440 showing no statistical significance in the difference of means. This further shows that the LP heuristic may have the possibility of having very good solutions as the problem size increases. Furthermore, it is observed that the NWC used to obtain the final distribution load for the LP heuristic had a good performance of being close to a 0% difference.

However, in the second set of problems as presented in Table 3-13, the NWC performance was consistently worse when compared to LC and Modi. This shows that for certain problem structures the NWC could quickly obtain good results. The LP-based heuristic using the Modi and LC distributes the load using the minimum relaxed cost of the problem parameters, while NWC does not. The LP-Modi in problem size (8×5) obtained the best mean difference of 0.93%. Showing it has the likelihood of generating good results irrespective of problem size and parameter structure. Finally, the numerical problem considered showed the possibility of having near solutions to the CPLEX values when a minimum value among the NWC, LC and Modi is used for the final load distribution of the LP-based heuristic.

3.2.7 Conclusion and future directions

This subchapter considered the Facility Location (FL) and Step-Fixed Charge Transportation Problem (SFCTP). A Linear Programming (LP) based heuristic was developed which was decomposed into two major transportation problems. The relaxed objective function of the first transportation problem in equation (3.41) was used in selecting the combinations of locations to be considered for the final transportation problem. In addition, solving the first transportation problem could help in generating a good lower bound for other computation methods such as Lagrange relaxations. A Numerical solution was presented to show the workings of the heuristic while a small scale pilot computation study was done to gain a little insight into the heuristic performance as regards the objective value obtained. Two sets of problem sizes were considered. The first set of problem sizes was solved using the NWC method as a solution method for the final transportation problem. The statistical results obtained showed a little or no significant differences in the mean values of the CPLEX solution and the LP-based heuristic with increase in problem size. However, the NWC method lagged behind the LC and Modi methods for the second set of problem sizes. The Modi method for the final transportation problem was superior in all the problem instances of the second set of problems. The optimal distribution pattern of the Modi method (minimum cost) seemed to have a strong link to the good results obtained.

Few sized problems have been considered in chapter 3. As problem size increase the linear relaxation and perturbation techniques are expected to converge faster to good and approximate solutions. This is expectedly due to the polynomial growth order of linearization employed within the solution technique and also due to the structured perturbation technique which possesses the capacity to terminate quickly. In addition, the linearization technique reduces the solution search space to provide approximate solutions in a polynomial growth manner.

Conclusively, large scale computation study still have to be conducted to gain proper insight into the performance of the heuristic under different parameter ranges and also due to the reason that exact methods which may be obtainable using the solution methods present in the CPLEX optimization studio could become inefficient as the problem size increases. Also, comparing the runtimes of the different solution methods would also need to be considered for an efficient study. Finally, a comparative study between this LP-based heuristic and solution methods such as the Lagrangian relaxation heuristic, metaheuristics such as genetic algorithm, tabu search and hybrid solutions which uses both LP-based relaxation and metaheuristics would be possible areas for exploring.

Chapter 4

Facility Location and Fixed-charge Solid Transportation Problem

Chapter 4.1

On the Facility Location and Fixed Charge Solid Transportation Problem: A Lagrange relaxation heuristic

A modified version of this subchapter was presented and has been published in the conference proceedings of the International Conference on Industrial Engineering and Operations Management (IEOM) in Pretoria

4.1.1 Introduction

In this subchapter, an integrated distribution problem between the Facility Location Problem (FLP) and the Fixed Charge Solid Transportation Problem (FCSTP) is considered. As noted in chapter 2, most FLPs are formulated as pure integer problems and most TPs as mixed-integer problems. The Facility Location Problem and Fixed Charge Solid Transportation Problem considered in this subchapter is formulated as a mixed-integer problem to show the desired product quantities been distributed by the selected conveyances between the m sources and n destinations. By doing this, more decisions can be taken by the operational personnel involved in decision making. This problem has been termed Fixed Charge Solid Location and Transportation Problem (FCSLTP). The objective for the FCSLTP is to find the optimal locations amongst several competing locations (e.g. Warehouses, depots) to distribute products at a unit cost to meet customers' orders by selecting from competing capacitated conveyances or transport mediums under route fixed charges. Furthermore, the LHR method presented by Sanei et al. (2017) has been adapted to solve the FCSLTP and also compared it with the solution obtainable by a standard general-purpose solver such as the IBM CPLEX. Figure 4-1 below shows a schematic representation of the FCSLTP.

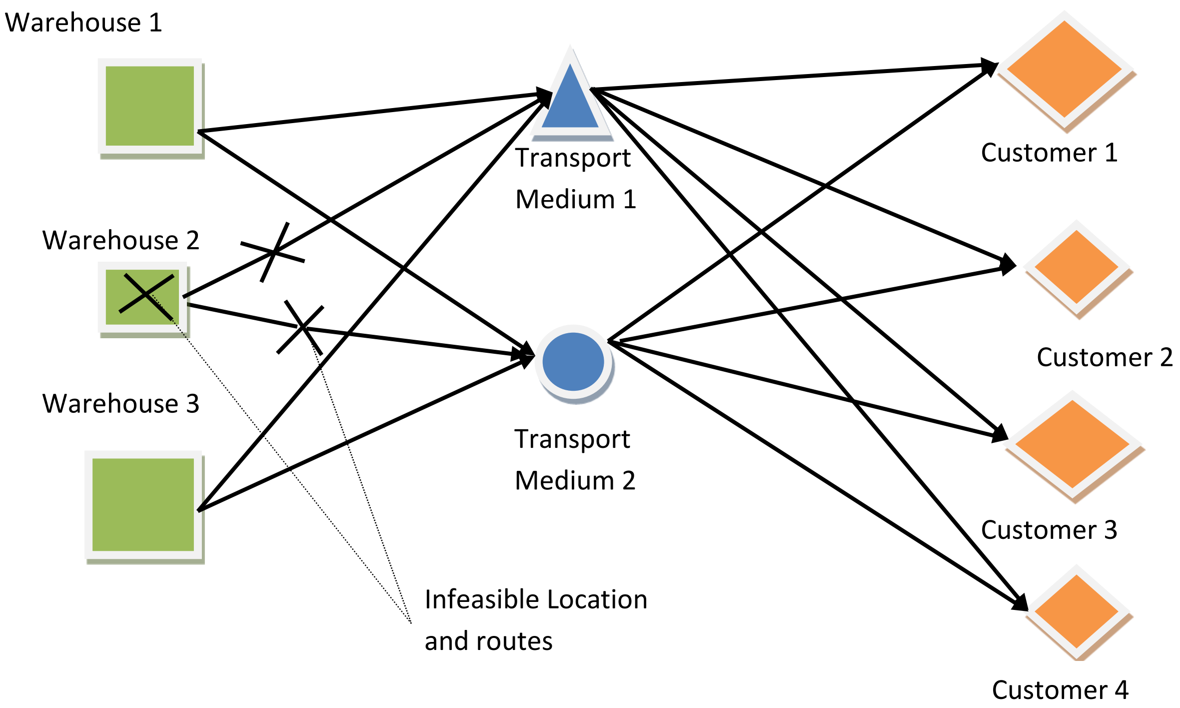

Figure 4-1 Schematic representation of FLP and FCSTP

4.1.2 Mathematical formulation and structure for FCSLTP

As indicated earlier, the FCSLTP is formulated as a mixed-integer problem, with m- sources, n- destinations, a- conveyances. The FCSTP model as presented by Sanei et al. (2017) has been adapted to include the fixed cost of facility location which is seemingly not present in his formulation. The location cost is necessary to build an integrated model of various fixed cost planning horizons.

Assumptions for model development of FCSLTP

The following assumptions were considered in the model presented:

1. Deterministic input.
2. One stage or two-echelon problem.
3. Fixed location cost and fixed charge route cost.
4. One Planning period and single item distribution problem.

a. Parameters For Model Formulation:

Below are the optimization parameters and variables used in the model formulation.

Deterministic parameters

i: Index for sources or location (warehouses, depots etc.)
j: Index for destinations (customers, other warehouses etc.)
r : Index for conveyances (or Transportation mediums)
m : Number of sources
n : Number of destinations
a : Number of conveyances
c_{ijr} : Variable cost of shipment on the route (i, j) using conveyance r.
S_i : Capacity at source i
D_j : Demand at Destination j
T_r : Capacity based on the conveyance r
F_i : Fixed-charge location cost
H_{ijr} : Fixed cost incurred on shipping through route (i, j) based on the conveyance r .

Decision Variables:

x_{ijr} : Quantity of products transported from source (i) to destination (j) using conveyance (r)
y_i : Location variable for selecting sources
y_{ijr} : Fixed-charge variable in selecting which conveyance is utilized on the route (i, j)

Objective Function:

Original Problem (OP)

Min (OP):

$$\sum_{i=1}^{m} F_i\, y_i + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} c_{ijr}x_{ijr} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} H_{ijr}y_{ijr} \quad (4.1)$$

Subject to

$$\sum_{j=1}^{n}\sum_{r=1}^{a} x_{ijr} \le S_i y_i \qquad \forall\ i = 1 \dots m \quad (4.2)$$

$$\sum_{i=1}^{m}\sum_{r=1}^{a} x_{ijr} = D_j \qquad \forall\ j = 1 \dots n \quad (4.3)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijr} \leq T_r \qquad \forall\, r = 1 \dots a \qquad (4.4)$$

$$x_{ijr} \leq M_{ijr}\, y_{ijr} \qquad \forall\ i = 1 \dots m, \quad \forall\ j = 1 \dots n, \ \forall\ r = 1 \dots a \qquad (4.5a)$$

M_{ijr} = min (S_i, D_j, T_r)

$$y_{ijr} = \begin{cases} 1 & x_{ijr} > 0 \\ 0 & Otherwise \end{cases} \quad \forall\ i = 1 \dots m, \quad j = 1 \dots n, \quad \forall\ r = 1 \dots a \qquad (4.5b)$$

$$x_{ijr} \geq 0 \qquad \forall\ i = 1 \dots m, \quad j = 1 \dots n, \forall\ r = 1 \dots a \qquad (4.6)$$

$$y_i = 0 \ or\ 1 \quad \forall\ i = 1 \dots m \qquad (4.7a)$$

$$y_{ijr} = 0 \ or\ 1 \quad \forall\ i = 1 \dots m, \quad j = 1 \dots n, \forall\ r = 1 \dots a \qquad (4.7b)$$

Equation (4.1) is the objective function. The first term is the facility location cost, the second term is the route variable cost per conveyance type and the third term is the route fixed-charge cost per conveyance type. Equation (4.2) is the supply capacity constraint of each location or sources. Equation (4.3) is the demand constraint to be met. Equation (4.4) is the conveyance capacity constraint. Equations (4.5a and 4.5b) are the route fixed-charge requirement constraints. Equation (4.6) refers to the non-negativity constraint for the continuous variables. Equation (4.7a) refers to the binary integer constraints for selecting open locations. Equation (4.7b) refers to the binary integer constraints for selecting open routes.

b. Lower bound formulation of FCSLTP

As noted in earlier sections of this subchapter, the LRH works through the Lagrangian relaxation of some difficult constraints in the OP that makes it difficult to solve. Often, these constraints consist of the integer constraints such as in the OP above. These constraints make the optimization problem NP-hard in nature. Constraints (4.2) and (4.5a) have been selected to apply the Lagrangian multipliers ($\lambda_i\ i.e \sum_i^m \lambda_i$ and $\beta_{ijr}\ i.e \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^a \beta_{ijr}$) respectively. These constraints have been shown by Cornuéjols et al. (1991) to give a strong lower bound though it may be computationally intensive. The Lagrangian multipliers used are such that each $\lambda_i \geq 0$ and each $\beta_{ijr} \geq 0$ in like fashion as Sanei et al. (2017). The Lagrangian Relaxation of the Original Problem (LR of OP) is given below:

LR of OP (λ, β) =

Minimize

$$\sum_{i=1}^{m} F_i\, y_i + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} c_{ijr} x_{ijr} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} H_{ijr} y_{ijr} + \sum_{i=1}^{m} \lambda_i \left(\sum_{j=1}^{n}\sum_{r=1}^{a} x_{ijr} - S_i y_i\right) +$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} \beta_{ijr} (x_{ijr} - M_{ijr}\, y_{ijr})$$

= Minimize

$$\sum_{i=1}^{m}(F_i \; - \lambda_i \, S_i) y_i \; + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} \, (c_{ijr} \; + \beta_{ijr}) \, x_{ijr} \; + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} (H_{ijr} - \beta_{ijr} M_{ijr}) y_{ijr} \; +$$

$$\sum_{i=1}^{m}(\lambda_i * \sum_{j=1}^{n}\sum_{r=1}^{a} x_{ijr} \,) \qquad (4.8)$$

Subject to

Constraints (4.3), (4.4),(4.5b) (4.6) and (4.7)

The Lagrangian relaxation of the Original problem i.e. LR of OP (λ, β) is decomposed into two sub-problems (SP1 and SP 2).

First Sub-problem i.e. SP1 of OP (λ, β) :

Minimize

$$\sum_{i=1}^{m}(F_i \; - \lambda_i \, S_i) y_i \; + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} (H_{ijr} - \beta_{ijr} M_{ijr}) y_{ijr} \qquad (4.9)$$

Subject to

$$y_i \; = 0 \; or \; 1 \;\; , \quad y_{ijr} \; = 0 \; or \; 1 \qquad (4.10)$$

Second Sub-problem i.e. SP2 of OP (λ, β):

Minimize

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} \, (c_{ijr} \; + \beta_{ijr}) \, x_{ijr} \; + \sum_{i=1}^{m}(\lambda_i * \sum_{j=1}^{n}\sum_{r=1}^{a} x_{ijr} \,) \qquad (4.11)$$

Subject to

$$\sum_{i=1}^{m}\sum_{r=1}^{a} x_{ijr} \;\; = \;\; D_j \qquad \forall \; j = 1 \dots n \qquad (4.12)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijr} \;\; \leq \;\; T_r \qquad \forall \, r = 1 \dots a \qquad (4.13)$$

$$x_{ijr} \; \geq 0 \qquad (4.14)$$

It is noted that SP2 of OP is a Linear Programming (LP) problem and could be easily solved to optimality using a general-purpose solver such as CPLEX.

For SP1 of OP which is a pure integer programming problem, two scenarios are noted below which can lead to arriving at the optimal values $y^*_i \; and \; y^*_{ijr}$

Scenario 1: When $(F_i \; - \lambda_i \, S_i) \; < 0$ and $(H_{ijr} - \beta_{ijr} M_{ijr}) < 0$, these imply negative values, therefore

The best minimum will be arrived at when $y^*_i \; = 1 \; and \; y^*_{ijr} = 1$ respectively.

Scenario 2: When $(F_i - \lambda_i S_i) > 0$ and $(H_{ijr} - \beta_{ijr} M_{ijr}) > 0$, this implies positive values, therefore,

The best minimum will be arrived at when $y_i^* = 0$ *and* $y_{ijr}^* = 0$ respectively

To arrive at the LRH Lower bound, the following procedure below is followed;

1. Compute SP2 of OP to generate the optimal x_{ijr}^* and $SP2^*$
2. For SP1 of OP, For all $i = 1 \ldots \mathrm{m}$, if $(F_i - \lambda_i S_i) < 0$ then $y_i^* = 1$
   Else $y_i^* = 0$
3. For SP1 of OP, For all $i = 1 \ldots \mathrm{m}, \mathrm{j} = 1..\mathrm{n}, \mathrm{r} = 1 \ldots \mathrm{a}$, if $(H_{ijr} - \beta_{ijr} M_{ijr}) < 0$ then $y_{ijr}^* = 1$
   Else $y_{ijr}^* = 0$
4. Compute the optimal value for SP1 of OP i.e. $SP1^*$

   Where $SP1^* =$
   Minimize:

$$\sum_{i=1}^{m} (F_i - \lambda_i S_i) y_i^* + \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{r=1}^{a} (H_{ijr} - \beta_{ijr} M_{ijr}) \; y_{ijr}^* \tag{4.15}$$

5. The LB is given as $SP1^* + SP2^*$

c. Upper bound formulation of FCSLTP

In the solution of SP1 of OP to arrive at $SP1^*$ according to equation 4.15, since the x_{ijr}^* values are not directly considered in the selection of the y_i^* and y_{ijr}^* some possible contradictions or infeasibilities would have to be resolved or perturbed through. This will ensure feasibility as noted by Fisher (1981) and also Implemented by Sanei et al. (2017). The resolution of the contradictions is used in generating an upper bound to be used in the LRH. These possible six (6) contradictions are identified and given below;

1. Given that $x_{ijr}^* > 0$, $y_i^* = 0$ and $y_{ijr}^* = 0$
2. Given that $x_{ijr}^* > 0$, $y_i^* = 0$ and $y_{ijr}^* = 1$
3. Given that $x_{ijr}^* > 0$, $y_i^* = 1$ and $y_{ijr}^* = 0$
4. Given that $x_{ijr}^* = 0$, $y_i^* = 1$ and $y_{ijr}^* = 1$
5. Given that $x_{ijr}^* = 0$, $y_i^* = 0$ and $y_{ijr}^* = 1$
6. Given that $x_{ijr}^* = 0$, $y_i^* = 1$ and $y_{ijr}^* = 0$

The following procedure can be used in resolving the contradictions:

For all $i = 1 \ldots \mathrm{m}$, (y_i^*) and For all $i = 1 \ldots \mathrm{m}, \mathrm{j} = 1..\mathrm{n}, \mathrm{r} = 1 \ldots \mathrm{a}$, (y_{ijr}^*)

1 If $x_{ijr}^* > 0$, and $y_i^* = 0$ and $y_{ijr}^* = 0$ then set $y_i^* = 1$ and $y_{ijr}^* = 1$
2 If $x_{ijr}^* > 0$, and $y_i^* = 0$ and $y_{ijr}^* = 1$ then set $y_i^* = 1$
3 If $x_{ijr}^* > 0$, and $y_i^* = 1$ and $y_{ijr}^* = 0$ then set $y_{ijr}^* = 1$

4 If $x^*_{ijr} = 0$, and $y^*_i = 1$ and $y^*_{ijr} = 1$ then set $y^*_i = 0$ and $y^*_{ijr} = 0$
5 If $x^*_{ijr} = 0$, and $y^*_i = 0$ and $y^*_{ijr} = 1$ then and $y^*_{ijr} = 0$
6 If $x^*_{ijr} = 0$, and $y^*_i = 1$ and $y^*_{ijr} = 0$ then set $y^*_i = 0$

Using the values of y^*_i , y^*_{ijr} and x^*_{ijr} in equation (4.1) above and after resolving any infeasibilities as indicated above, the upper bound (UB) is obtained.
UB of OP =

$$\sum_{i=1}^{m} F_i\, y^*_i + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} c_{ijr} x^*_{ijr} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} H_{ijr} y^*_{ijr} \quad (4.16)$$

4.1.3 Lagrange relaxation heuristic procedure

The UB of OP and LR of OP (λ, β) are the first requirements for the computation of this heuristic procedure. Following this is the sub-gradient optimization method which is widely used in determining the necessary Lagrange multipliers for the iterations. The parameters required for the termination procedure and sub-gradient optimization are listed below;

1. A value (ε) that is user-determined (or pre-specified) for algorithm termination. It is usually small-sized positive number such that $(UB_{Best}) - (LB_{Best}) \leq \varepsilon$. The term UB_{Best} refers to the best Upper Bound (UB) and while LB_{Best} is the best Lower Bound (LB).
2. Step size for Lagrange multipliers (λ *and* β) generation is given as θ^t. The symbol t refers to the iteration number.

$$\theta^t = \frac{\delta\,[(UB_{Best}) - (LB_{Best})]}{\sum_i^m\left(\sum_{j=1}^n \sum_{r=1}^a x^t_{ijr} - S_i {y_i}^t\right)^2 + \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^a \left(x^t_{ijr} - M_{ijr}\, y^t_{ijr}\right)^2}$$

3. A value (δ) is chosen from the interval (0, 2) as normally used in the sub-gradient procedure. When $t \geq t_{max}$, $\delta = \delta/2$. t_{max} is the number of maximum iterations allowed per δ used. In this subchapter, a termination condition of $\delta = 0$ for the LRH has been included.
4. $UB_{initial}$, $LB_{initial}$ are the initial values for the Upper Bound and Lower bound respectively.
5. λ^t_i and β^t_{ijr} refer to the Lagrange multipliers at the iteration number t. The initial Lagrange multiplier chosen are $\lambda^t_i = \lambda^*_i$ and $\beta^t_{ijr} = \beta^*_{ijr}$

Iterative steps for LRH:
Step 1 Initialize using the parameters (ε, t, t_{max} , $\lambda^t_i = \lambda^*_i$, $\beta^t_{ijr} = \beta^*_{ijr}$, $UB^t = UB_{initial}$, $LB^t = LB_{initial}$, $\delta = 2$)
$(LB\ of\ OP)^t = (LB\ of\ OP)^* = SP1^* + SP2^*$
$(UB\ of\ OP)^t = (UB\ of\ OP)^*$
Step 2 Solve $(LB\ of\ OP)^t = SP1^t + SP2^t$

$LB_{Best} = max\,[(LB\ of\ OP)^t,\ LB^t, 0\,)]$

Step 3 Find a feasible solution to the Upper bound from $(LB\ of\ OP)^t$ i.e. $(UB\ of\ OP)^t$

$UB_{Best} = min\,[(UB\ of\ OP)^t, \quad UB^t)]$

Step 4 if $(UB_{Best}) - (LB_{Best}) \leq \varepsilon$, Terminate the heuristic. Else

Step 5 Update the Lagrange Multipliers

$$\lambda_i^{t+1} = \lambda_i^t + \theta^t \left(\sum_{j=1}^{n} \sum_{r=1}^{a} x_{ijr}^t - S_i {y_i}^t \right) \qquad \forall\ i \in m$$

$$\lambda_i^{t+1} = Max\,(\lambda_i^{t+1}, 0)$$

$$\beta_{ijr}^{t+1} = \beta_{ijr}^t + \theta^t \left(x_{ijr}^t - M_{ijr}\, y_{ijr}^t \right) \qquad \forall\ i \in m,\ j \in n,\ r \in a$$

$$\beta_{ijr}^{t+1} = Max\,(\beta_{ijr}^{t+1}, 0)$$

Step 6 if no improvement in LB_{Best} at $t \geq t_{max}$, then $\delta = \delta/2$

set $t = 0$ (termination and restart condition)

Step 7 if $\delta = 0$ (The Heuristic is terminated and UB_{Best} is selected)

Step 8 Else $t = t + 1$ (Go to Step 2).

Numerical Computation

In order to show the workings of the solution method of this nature, data is usually randomly generated for various instances of problem sizes following some known probability distribution and coded using some programming languages. A hands-on example of the Lagrange relaxation heuristic to further explain the procedures and observations to note when performing such computations is presented.

The generic model problem size consists of (m) location binary variables, $(m \times n \times a)$ shipment continuous variables and $(m \times n \times a)$ fixed charge binary variables. Furthermore, we have used standard general-purpose optimizer software such as IBM CPLEX version 12.8 which is enabled with a default Mixed-Integer Linear Programming (MILP) solver to compare with the solution obtained with the LRH.

Data generation

The data parameters utilized are of two types. These are the parameters for the FCSLTP and that used in running the LRH. Part of the interest in this chapter is in showing the workings of LRH for good comprehension of the user. Therefore a small-sized sample problem that fits the model parameters has been included. The following values have been used for the Lagrange Heuristic parameters:

$\varepsilon = 0.01$, $UB_{initial} = +\infty$, $LB_{initial} = -\infty$, $t_{max} = 5$, $t = 0$

Numerical example:

Number of sources $m\ (1 \ldots i) = 3$

Number of destinations $n\ (1 \ldots j) = 2$

Number of Conveyances $a\ (1 \ldots r) = 2$

A numerical example is described in Table 4-1 and 4-2 below.

Table 4-1 Location fixed charges, supply and demand capacities, Unit cost per quantity shipped per conveyances

			r= 1		r= 2	
i	F_i	S_i	c_{ij1}		c_{ij2}	
1	150	25	1	3	3	2
2	250	30	2	2	2	1
3	200	40	2	1	1	3
D_j			20	15	20	15
T_r			10		25	

Table 4-2 Route fixed charges per conveyances

	r= 1		r= 2	
i	H_{ij1}		H_{ij2}	
1	6	4	8	6
2	8	6	6	8
3	6	8	6	4
j	1	2	1	2

Numerical solution

To begin the LRH solution, the initial values for the Lagrange multipliers $\lambda_i^t = \lambda_i^*$ and $\beta_{ijr}^t = \beta_i^*$ need to be determined. From equation (4.15) and using the non-negativity constraints for the Lagrange multiplier, it is observed that the values of λ_i^* to arrive at a minimum value of the objective function can either be zero or a positive value of magnitude sufficiently greater than $\frac{F_i}{S_i}$. For the numerical example computation, $\lambda_i = 0$ $(\forall i = 1 \dots 3)$ is used for the first LRH trial run and $\lambda_i^* = (4 * \frac{F_3}{S_3}) = 20$ $(\forall i = 1 \dots 3)$ for the second LRH trial run.

Similarly for the second set of Lagrange multipliers i.e. β_{ijr}^*, and also using equation (4.15), it is observed that β_{ijr}^* could either be fixed at zero (0) or a positive number of magnitude greater than $\frac{H_{ijr}}{M_{ijr}}$. For both the First and Second trial runs, $\beta_{ijr}^* = 1$ $(\forall i = 1 \dots 3,\ j = 1 \dots 2,\ r = 1 \dots 2)$ have been used. This is because all the $\frac{H_{ijr}}{M_{ijr}}$ in the numerical example computations are positive values between 0 and 1. In all the first and second runs for the LRH $t_{max} = 5$ was fixed. The results can be seen in Tables 4-3 and 4-4 below. The symbol u in Table 4-3 and 4-4 refers to the count of iterations done irrespective of restarting the heuristic $(\delta = \delta/2)$ as per the heuristic procedure.

Table 4-3 LRH computation for initial Lagrange values $\boldsymbol{\lambda_i^* = 0 \ and \ \beta_{ijr}^* = 1}$(First run)

u	t	δ	θ	$(LB\ of\ OP)^t$	LB_{Best}	$(UB\ of\ OP)^t$	UB_{Best}	$(UB_{Best}) - (LB_{Best})$
1	0	2		-4	0	655	655	655
2	1		0.95855	-15	0	507	507	507
3	2		1.19294	-76.4707	0	465	465	465
4	3		3.38182	-77.4707	0	465	465	465
5	4		3.38182	-77.4707	0	465	465	465
6	5		3.38182	-77.4707	0	465	465	465
restart								
7	0	1	1.6909	-80.1177	0	465	465	465
restart								
8	0	0.5	0.84545	-80.1177	0	465	465	465
restart								
9	0	0.25	0.422725	-80.1177	0	465	465	465
restart								
10	0	0.125	0.211363	-80.1177	0	465	465	465
restart								
11	41	0.0625	0.105681	-80.1177	0	465	465	465

Table 4-4 LH computation for initial Lagrange values $\boldsymbol{\lambda_i^* = 20 \ and \ \beta_{ijr}^* = 1}$(Second run)

u	t	δ	θ	$(LB\ of\ OP)^t$	LB_{Best}	$(UB\ of\ OP)^t$	UB_{Best}	$(UB_{Best}) - (LB_{Best})$
1	0	2		-604	0	655	655	655
2	1		0.95273	-5	0	263	263	263
3	2		3.50667	-148.909	0	551	263	263
4	3		0.50095	-232	0	527	263	263
5	4		0.50095	-236.523	0	327	263	263
6	5		10.52	-1482.48	0	463	263	263
restart								
7	0	1	0.55368	-80.1177	0	465	263	263
restart								
8	0	0.5	0.27684	-80.1177	0	465	263	263
restart								
9	0	0.25	0.13842	-80.1177	0	465	263	263

It is noted in the LRH results shown in Table 4-3 above that $(LB\ of\ OP)^t$ and LB_{Best} failed to improve after the count = 6, for which $t = 5$ (t_{max}). Hence the solution was restarted. Unfortunately, no better Lower bound solution was obtained on using the sub-gradient rule of $\delta = \frac{\delta}{2}$. Similarly in Table 4-4 above, LB_{Best} values failed to increase above zero(0) with the values of $(UB_{Best}) - (LB_{Best})$ becoming constant as both δ and θ reduced significantly, tending towards zero(0). The LRH search iteration was terminated in both the first and second trial runs using the condition that $\delta = 0$.

4.1.4 Discussion of solutions obtained

The Original Problem was coded into IBM CPLEX version 12.8 and solved with the default Mixed-Integer Linear Programming (MILP) Solver. Table 4-5 below shows the results obtained for the decision variables under the different solution methods. The variables not shown were equal to zero (0) in the solution obtained. It can be well observed in Table 4-5 below that the LRH for the second trial run gave the lowest value of 263. The value (263) was obtained only due to the violation of one of the constraints used for the Lagrange Relaxation. This is a constraint (4.5a) i.e. $x_{ijr} \leq M_{ijr}\, y_{ijr}$. The violation was at $x^*_{312} = 20$ and such that $x^*_{312} > (M_{312} = 15)$. This violation was most likely due to the non-inclusion of possible infeasibility contradiction of the term $x^*_{ijr} > M_{ijr}\, y_{ijr}$ for $x^*_{ijr} > 0$ under Upper bound formulation of FCSTLP. By the infeasibility of constraint (4.5a), a lower bound was obtained. In addition, the different LRH values of λ^*_i and β^*_{ijr} used as the starting solution for the Lagrange multipliers show how these values could help in the final lower bound or optimal solution reached by the heuristic. A simple infeasibility resolution of the lower bound solution obtained by LRH (second trial) to satisfy the constraint (4.5a) while also satisfying demand constraints would result in the value obtained by the CPLEX values.

Table 4-5 Decision variables result (O.P.)

Method	Solver Characteristic	$Min\ (O.P.)$	x_{ijr}	y_i	y_{ijr}
CPLEX	Default MILP solver	**284**	$x_{311} = 5$ $x_{321} = 5$ $x_{312} = 15$ $x_{322} = 10$	$y_1 = 0$ $y_2 = 0$ $y_3 = 1$	$y_{311} = 1$ $y_{321} = 1$ $y_{312} = 1$ $y_{322} = 1$
LRH (First run) Lower bound	$\lambda^*_i = 0\ and\ \beta^*_{ijr} = 1$	**465**	$x_{111} = 10$ $x_{212} = 10$ $x_{222} = 15$	$y_1 = 1$ $y_2 = 1$ $y_3 = 0$	$y_{111} = 1$ $y_{212} = 1$ $y_{222} = 1$
LRH (Second trial) Lower bound	$\lambda^*_i = 20\ and\ \beta^*_{ijr} = 1$	**263**	$x_{321} = 10$ $x_{312} = 20$ $x_{322} = 5$	$y_1 = 0$ $y_2 = 0$ $y_3 = 1$	$y_{321} = 1$ $y_{312} = 1$ $y_{322} = 1$
Resolved LRH (Second trial) **Infeasibility**	$\lambda^*_i = 20\ and\ \beta^*_{ijr} = 1$	**284**	$x_{311} = 5$ $x_{321} = 5$ $x_{312} = 15$ $x_{322} = 10$	$y_1 = 0$ $y_2 = 0$ $y_3 = 1$	$y_{311} = 1$ $y_{321} = 1$ $y_{312} = 1$ $y_{322} = 1$

Values of x_{ijr}, y_{ijr} not shown in the table equals 0 in the solution.

4.1.5 Conclusion and future direction

The use of Lagrangian Relaxation Heuristic (LRH) in solving facility location and fixed charge solid transportation problem has been considered in this subchapter. The lower bound solution obtained for the LRH (second trial) was better compared to the value obtained for LRH (First trial) when the respective values obtained were compared to the CPLEX values. The results of the LRH could still result in a lower bound as was the case of the LRH(second trial) if possible infeasibility contradictions of the upper bound formulations are not well captured. For the LRH (second trial), a careful perturbation to satisfy one of the violated constraints would result in the CPLEX values obtained.

Few sized problems have been considered in this subchapter for illustration purpose. As problem size increase, the solution time is not expected to grow proportionately with the Lagrange relaxation heuristic developed. This is due to the decomposition, mathematical deductions and greedy heuristics employed. These solution techniques reduce the search space and nodes being explored to seek an approximate solution. Terminating conditions of the heuristic also make a fast convergence of solution possible. To further test the performance of the LRH compared to the values obtainable from standard optimization software, extensive computations for various problem sizes and instances still have to be considered. It is also worth to note that a structured perturbation technique could be employed to resolve the possible LRH lower bound infeasibility. Furthermore during the LRH process, metaheuristics such as genetic algorithm, simulated annealing could be used to search for a better upper bound obtained, in case the solution fails to improve before the terminating conditions are reached.

Chapter 4.2

Solving the Fixed charge Solid Location and Transportation Problem

A modified version of this subchapter has been accepted for publication by the Journal of Industrial and Management Optimization.

4.2.1 Introduction

In this subchapter, a distribution problem that simultaneously optimizes facility location and fixed charge solid transportation problem is presented. This has been termed Fixed Charge Solid Location and Transportation Problem (FCSLTP). The objective of the FCSLTP is to minimize total transportation and location costs by determining the optimal allocations from open locations through open routes by a set of conveyances. In order to solve this problem the CPLEX mixed-integer program dynamic solver which utilizes the branch and cut algorithm to search for optimality is utilized. Furthermore, an attempt is made to compare the performance of an alternative solution method against the CPLEX by extending Sanei et al. (2017) LRH feasibility resolution pattern. In addition, FCSLTP and FCSTP are compared in terms of the total cost. This is done to determine if there are any possible cost savings and the magnitude of cost savings obtained when an optimal number of facilities are opened (FCSLTP) compared to opening all facilities (FCSTP). Figure 4-2 below describes the FCSLTP.

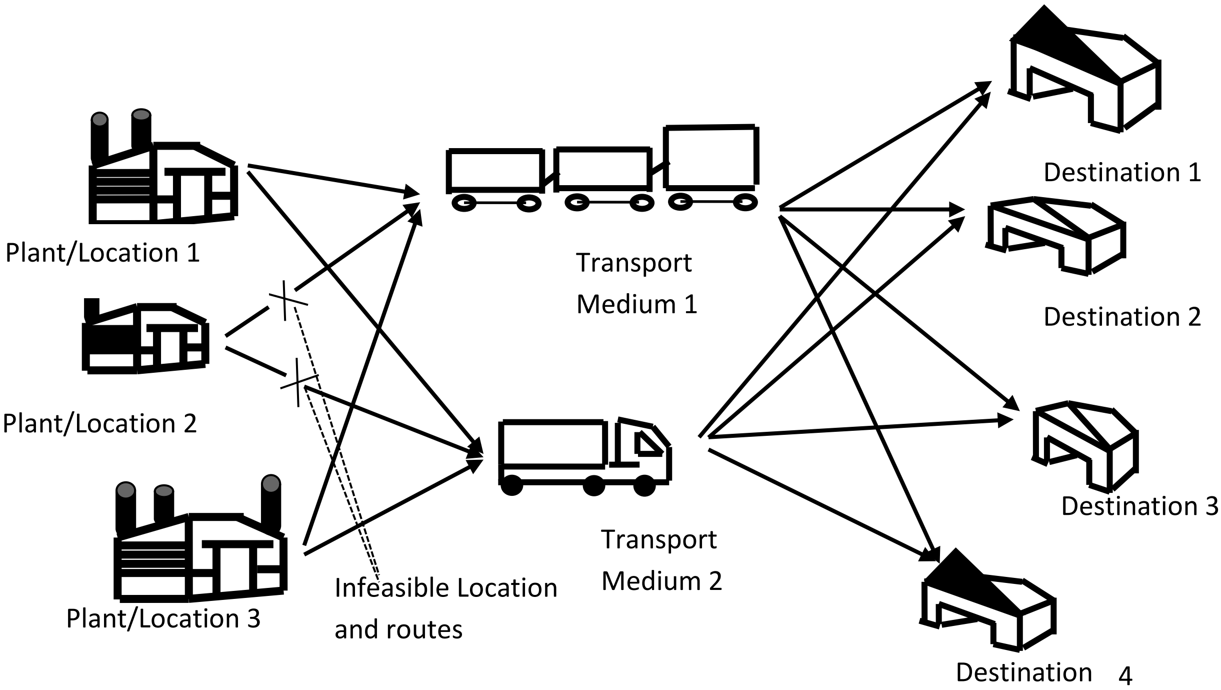

Figure 4-2 Schematic representation of FCSLTP

4.2.2 Mathematical model of FCSLTP

The FCSLTP is formulated as a Mixed-Integer Programming (MIP) problem, with m- sources, n- destinations, and a- conveyances. The model of FCSTP as presented by Sanei et al. (2017) has been extended to include fixed costs of facility location. In their FCSTP a single product is to be shipped through a set of locations to a number of demand points using a set of transport mediums. The capacity of each location to supply products in FCSTP is simply determined by the route fixed and variable costs, and also the problem capacities.

However, in the FCSLTP, fixed location costs, route fixed and variable route costs and problem capacities are simultaneously used in determining whether the locations will be opened or closed for shipment.

Model assumptions for the FCSLTP
The assumptions made in the model formulation are similar to that of section 4.1.2:

Model Development Parameters and Variables for FCSLTP:
The parameters and variables used in the model formulation are given below.

Parameters

i: Index for sources or locations (plants, warehouses, depots etc.)
j: Index for destinations (customers, other warehouses etc.)
r : Index for conveyances (or Transportation mediums)
m : Number of sources
n : Number of destinations
a : Number of conveyances
c_{ijr} : Variable cost of shipment on the route (i, j) using conveyance r.
S_i : Capacity of source i $\quad \forall\ i = 1 \dots m$.
D_j : Demand at Destination j $\quad \forall\ j = 1 \dots n$.
T_r : Conveyance capacity for the conveyance r $\quad \forall\, r = 1 \dots a$
F_i : Fixed cost of opening the facility at location$i$.
H_{ijr} : Fixed-charge cost incurred for shipping through route (i, j) based on the conveyance r.

Decision Variables:

x_{ijr}: Quantity of products transported from source (i) to destination (j) using conveyance (r).
y_i : Location variable for setting source (i) as either opened or closed.
z_{ijr} : Fixed-charge variable in selecting whether conveyance(r), is utilized or not on the route(i,j).

A mathematical model of the FCSTP is described below.

$$min \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} c_{ijr}x_{ijr} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} H_{ijr}z_{ijr} \qquad (4.17)$$

Subject to

$$\sum_{j=1}^{n}\sum_{r=1}^{a} x_{ijr} \leq S_i \qquad \forall\ i = 1 \dots m \qquad (4.18)$$

$$\sum_{i=1}^{m}\sum_{r=1}^{a} x_{ijr} = D_j \qquad \forall\ j = 1 \dots n \qquad (4.19)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijr} \leq T_r \qquad \forall\, r = 1 \dots a \qquad (4.20)$$

$$x_{ijr} \geq 0 \qquad (4.21a)$$

$$z_{ijr} = \begin{cases} 1 & x_{ijr} > 0 \\ 0 & Otherwise \end{cases} \quad (4.21b)$$

Equation (4.17) is the objective function. The first term is the route variable cost per conveyance type and the second term is the route fixed-charge cost per conveyance type. Equation (4.18) is the supply capacity constraint ensuring no supply preference for selected locations. Equation (4.19) is the demand constraint to be met at each destination. Equation (4.20) is the conveyance capacity constraint. Equation (4.21a) refers to the non-negativity constraint for the continuous variables and Equation (4.21b) refers to the binary constraints for the route fixed charge requirement.

Objective Function for FCSLTP:
Original Problem (OP)
Min (OP):

$$\sum_{i=1}^{m} F_i\, y_i + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} c_{ijr}x_{ijr} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} H_{ijr}z_{ijr} \quad (4.22)$$

Subject to

$$\sum_{j=1}^{n}\sum_{r=1}^{a} x_{ijr} \leq S_i y_i \quad \forall\ i = 1 \dots m \quad (4.23)$$

$$\sum_{i=1}^{m}\sum_{r=1}^{a} x_{ijr} = D_j \quad \forall\ j = 1 \dots n \quad (4.24)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijr} \leq T_r \quad \forall\, r = 1 \dots a \quad (4.25)$$

$$x_{ijr} \leq M_{ijr}\, z_{ijr} \quad \forall\ i = 1 \dots m, \quad \forall\ j = 1 \dots n, \ \forall\ r = 1 \dots a \quad (4.26)$$

M_{ijr} = min $(S_i, D_j, T_r\ )$

$$x_{ijr} \geq 0 \quad (4.27a)$$

$$z_{ijr} = \begin{cases} 1 & x_{ijr} > 0 \\ 0 & Otherwise \end{cases} \quad (4.27b)$$

$$y_i = 0\ or\ 1 \quad (4.27c)$$

Equation (4.22) is the objective function. The first term is the facility location cost, the second term is the route variable cost per conveyance type and the third term is the route fixed-charge cost per conveyance type. Equation (4.23) is the supply capacity constraint of each location or source with preference given to selected locations. Equation (4.24) is the demand constraint to be met at each destination. Equation (4.25) is the conveyance capacity constraint. Equation (4.26) refers to an upper bound limit on the continuous variables. It also represents a valid inequality for hopefully improving the solution to the FCSLTP. Equation (4.27a) refers to the non-negativity constraint for the continuous variables. Equations (4.27b and 4.27c) refer to the binary constraint for route fixed charge and facility location requirement.

4.2.3 Solution Approaches

As earlier indicated, the FCSLTP and the FCSTP are optimization problems with the presence of fixed charges. Optimization problems with fixed charges have been noted by Christensen (2013) to be classified under NP-hard network design problems. Usually, these NP-hard problems have a time complexity which makes computational time increase exponentially as problem size increase. In order to solve these NP-hard network design problems, solution techniques such as the branch and bound, branch and cut can be implemented from commercial optimization programmes such as CPLEX, LINGO, AMPL etc. These solution techniques are generally known in the optimization literature to possess a significant capacity in obtaining optimal solutions. An attempt is made to seek optimal solutions to the FCSLTP using the CPLEX optimization tool.

Unfortunately, MIP optimization tools such as CPLEX may be costly to acquire for some category of users and practitioners. In some cases, the commercial solver may not be quickly available to some others requiring urgent solutions to combinatorial problems such as the FCSLTP. This category of users will mostly desire a solution technique which may not guarantee an optimal solution but can help obtain good solutions within appreciable bounds. As a result, a solution technique known as the Lagrange relaxation heuristic has been developed to provide a solution to the FCSLTP. This technique can also help provide the user with an understanding of how feasible solutions to such combinatorial problems are achieved.

(A) Solving the FCSLTP using CPLEX

According to Studio (2016), the IBM ILOG CPLEX is a commercial development platform for modelling and solving combinatorial problems. Some combinatorial problems such as Linear Programming (LP), MIP, and Mixed-Integer Quadratic Problem (MIQP) have been noted to be solvable using CPLEX (Lima, 2010). The quality of solutions provided by CPLEX has been noted by (Lima, 2010) to depend on the problem type and size being solved. The FCSLTP is formulated as a MIP and solved using the MIP dynamic optimizer tool of the CPLEX. The dynamic optimizer tool has the capacity to serially launch a variety of exact solutions to solve a MIP. In order to solve a minimization MIP, the dynamic optimizer uses the continuous relaxation of integrality constraints to obtain a lower bound from which different cuts are applied to improve on the lower bounds obtained. Some cuts used are the mixed-integer rounding cuts, cover cuts and Gomory fractional cut (Studio, 2016). Readers are being referred to Wolsey et al. (1998) on the formulation and application of some of these cuts. The Branch and Cut algorithm is essentially used by CPLEX to obtain its solution to combinatorial problems.

(B) Solving the FCSLTP using Lagrange relaxation heuristic method

An attempt in this section is made to develop an alternative solution method to using CPLEX known as the Lagrange relaxation heuristic. Equation (4.26) is included as a valid inequality to possibly arrive at a good solution to the Lagrange relaxation. In order to use the Lagrange relaxation heuristic, constraints (4.23) and (4.26) have been selected for the application of the Lagrangian multipliers (λ_i $i.e$ $\sum_i^m \lambda_i$ and β_{ijr} $i.e$ $\sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^a \beta_{ijr}$) respectively. These constraints are similar to that used by Cornuéjols et al. (1991) to give a strong lower bound. Moreover, dualizing such constraints as in equation (4.23) and (4.26) have been noted by Nezhad et al. (2013) to leave the OP with a structure that is easy to exploit in finding a solution. In addition, non-negative Lagrangian multipliers ($\lambda_i \geq 0$ and $\beta_{ijr} \geq 0$) have been utilized to help generate a lower bound. The Lagrangian Relaxation of the Original Problem (LR of OP) is given as:

LR of OP (λ, β) =

Minimize

$$\sum_{i=1}^{m} F_i\, y_i + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} c_{ijr}x_{ijr} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} H_{ijr}z_{ijr} + \sum_{i=1}^{m} \lambda_i \left(\sum_{j=1}^{n}\sum_{r=1}^{a} x_{ijr} - S_i y_i\right) +$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} \beta_{ijr}(\, x_{ijr} - M_{ijr}\, z_{ijr}\,)$$

This can be equivalently written as:

$$\sum_{i=1}^{m}(F_i - \lambda_i\, S_i)y_i + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} (c_{ijr} + \beta_{ijr})\, x_{ijr} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a}(H_{ijr} - \beta_{ijr}M_{ijr})z_{ijr} +$$

$$\sum_{i=1}^{m}(\lambda_i * \sum_{j=1}^{n}\sum_{r=1}^{a} x_{ijr}\,) \qquad (4.28)$$

Subject to

Constraints (4.24), (4.25) and (4.27a)

Decomposition Method for LR of OP (λ, β)

The solution to the original problem (OP) starts with the decomposition of the Lagrangian relaxation of the original problem. The decomposition is done based on the separation of the continuous variable x_{ijr} and the binary variables y_i *and* z_{ijr}. This is to utilize the easy problem structures created. This decomposition allows for an easy solution to the original problems through simpler methods of solving the individual sub-problems and aggregating them into one piece. The decomposition into two major sub-problems is given below.

The Lagrangian relaxation of the Original problem i.e. LR of OP (λ, β) is decomposed into two sub-problems (SP1 and SP 2).

First Sub-problem i.e. SP1 of OP (λ, β) :

Minimize

$$\sum_{i=1}^{m}(F_i - \lambda_i S_i)y_i + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a}(H_{ijr} - \beta_{ijr}M_{ijr})z_{ijr} \qquad (4.29)$$

Subject to

$$y_i = 0 \; or \; 1 \;, \quad z_{ijr} = 0 \; or \; 1 \qquad (4.30)$$

Second Sub-problem i.e. SP2 of OP (λ, β):

Minimize

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a}(c_{ijr} + \beta_{ijr})\,x_{ijr} + \sum_{i=1}^{m}(\lambda_i * \sum_{j=1}^{n}\sum_{r=1}^{a}x_{ijr}) \qquad (4.31)$$

Subject to:

$$\sum_{i=1}^{m}\sum_{r=1}^{a}x_{ijr} = D_j \qquad \forall\; j = 1 \ldots n \qquad (4.32)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}x_{ijr} \le T_r \qquad \forall\, r = 1 \ldots a \qquad (4.33)$$

$$x_{ijr} \ge 0 \qquad (4.34)$$

Aggregation Method for SP1 of OP (λ, β) and SP2 of OP (λ, β)

Following the decomposition, the sub-problems SP1 of OP (λ, β) and SP2 of OP (λ, β) created are coupled. The aggregation is based on methods that can generate the lower bounds and upper bounds necessary for utilization in the iterations of the Lagrange relaxation heuristic. The first step is to determine the lower bound to be used in the Lagrange relaxation**.** Thereafter the upper bound is formulated through identified infeasibility resolutions.

Lower bound formulation for LRH

The continuous nature of the variable of LR of OP (λ, β) i.e SP2 of OP gives an LP problem which could be easily solved to optimality using a general-purpose solver such as TORA, Microsoft Excel solver and CPLEX. The integer variable aspect of SP1 of OP gives a pure integer programming problem and is solved using mathematical deductions and scenarios to arrive at possible values $y_i^* \; and \; z_{ijr}^*$ for these integers.

Scenario 1: This considers the fact that a minimization problem is being solved and thus makes efforts to obtain the best location and route fixed charge under the following conditions listed:

(1) If the term $(F_i - \lambda_i S_i) < 0$, which imply a negative term, the best integer variable will be obtained when $y_i^* = 1$.

(2) If the term $(H_{ijr} - \beta_{ijr} M_{ijr}) < 0$, which also imply a negative term, the best integer variables will be obtained when $z_{ijr}^* = 1$.

Scenario 2: Similarly to scenario1, when considering a minimization problem, the listed conditions are also observed.

1. If the term $(F_i - \lambda_i S_i) > 0$ this implies positive values, therefore, the best integer variables will be obtained when $y_i^* = 0$.
2. If the term $(H_{ijr} - \beta_{ijr} M_{ijr}) > 0$, this also implies positive values, therefore, the best integer variables will be obtained when $z_{ijr}^* = 0$.

To arrive at the LRH Lower bound, the procedure below is followed;

1 Compute SP2 of OP to generate the optimal x_{ijr}^* and $SP2^*$ of OP (λ, β):

2 For SP1 of OP, For all $i = 1 \ldots m$, if $(F_i - \lambda_i S_i) < 0$ then $y_i^* = 1$
Else $y_i^* = 0$.

3 For SP1 of OP, For all $i = 1 \ldots m, j = 1 \ldots n, r = 1 \ldots a$, if $(H_{ijr} - \beta_{ijr} M_{ijr}) < 0$ then $z_{ijr}^* = 1$
Else $z_{ijr}^* = 0$

4 Compute the optimal value for SP1 of OP i.e. $SP1^*$

Where $SP1^* =$

$$\sum_{i=1}^{m} (F_i - \lambda_i S_i) y_i^* + \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{r=1}^{a} (H_{ijr} - \beta_{ijr} M_{ijr})\, z_{ijr}^* \tag{4.35}$$

5 The $LB\ of\ OP$ is given as $SP1^* + SP2^*$

Upper bound formulation for LRH

In order to determine the upper bound to be used in the LRH, the method of resolving feasibility contradictions as used also by Sanei et al. (2017) is extended. In the solution of SP1 of OP to arrive at $SP1^*$ according to equation (4.35), since the x_{ijr}^* values are not directly considered in the selection of y_i^* and z_{ijr}^* some possible contradictions or infeasibilities would have to be resolved. This will ensure feasibility as noted by Fisher (1981). The resolution of the contradictions is used in generating an upper bound to be used in the LRH.

These possible seven (8) contradictions are identified and given below;

1 Given that $x^*_{ijr} > 0$, $y^*_i = 0$ and $z^*_{ijr} = 0$
2 Given that $x^*_{ijr} > 0$, $y^*_i = 0$ and $z^*_{ijr} = 1$
3 Given that $x^*_{ijr} > 0$, $y^*_i = 1$ and $z^*_{ijr} = 0$
4 Given that $x^*_{ijr} = 0$, $y^*_i = 1$ and $z^*_{ijr} = 1$
5 Given that $x^*_{ijr} = 0$, $y^*_i = 0$ and $z^*_{ijr} = 1$
6 Given that $x^*_{ijr} = 0$, $y^*_i = 1$ and $z^*_{ijr} = 0$
7. Given that $x^*_{ijr} > M_{ijr}$
8. $\sum_{j=1}^{n}\sum_{r=1}^{a} x_{ijr} > S_i y_i \quad \forall\ i = 1 \ldots m$.

The following procedure can be used in resolving these contradictions respectively

For all $i = 1 \ldots \mathrm{m}, (y^*_i)$ and For all $i = 1 \ldots \mathrm{m}, \mathrm{j} = 1..\mathrm{n}, \mathrm{r} = 1 \ldots \mathrm{a}, (z^*_{ijr})$

1 If $x^*_{ijr} > 0$, and $y^*_i = 0$ and $z^*_{ijr} = 0$ then set $y^*_i = 1$ and $z^*_{ijr} = 1$
2 If $x^*_{ijr} > 0$, and $y^*_i = 0$ and $z^*_{ijr} = 1$ then set $y^*_i = 1$
3 If $x^*_{ijr} > 0$, and $y^*_i = 1$ and $z^*_{ijr} = 0$ then set $z^*_{ijr} = 1$
4 If $x^*_{ijr} = 0$, and $y^*_i = 1$ and $z^*_{ijr} = 1$ then set $y^*_i = 0$ and $z^*_{ijr} = 0$
5 If $x^*_{ijr} = 0$, and $y^*_i = 0$ and $z^*_{ijr} = 1$ then and $z^*_{ijr} = 0$
6 If $x^*_{ijr} = 0$, and $y^*_i = 1$ and $z^*_{ijr} = 0$ then set $y^*_i = 0$
7 To resolve this contradiction a constraint to SP2 of OP (λ, β) i.e $x^*_{ijr} \leq M_{ijr}$ is added.
8 An attempt is made to resolve any possible supply infeasibility by load shifting from open locations with capacity overload into open locations with capacity underload in the order of increasing relaxation cost given as :

$$\left(\frac{F_i}{S_i} + \sum_{j=1}^{n}\sum_{r=1}^{a}\left(\frac{H_{ijr}}{M_{ijr}} + c_{ijr}\right) \right) \quad \forall\ i = 1 \ldots m$$

If there are no open locations with available capacities, new locations are opened to receive load shifting in the order of increasing relaxation cost.
The load shifting is done between locations within the same conveyance capacity constraints (randomly selected) to maintain the feasibility of equation (4.33).
A high relaxation cost shows a possibility of high supply cost from that location and a high possibility of closing the location.

After resolving the eight (8) possible infeasibilities as indicated above, the values of y^*_i, z^*_{ijr} and x^*_{ijr} obtained in equation (6) to arrive at the upper bound (UB) are utilized.

UB of OP =

$$\sum_{i=1}^{m} F_i y^*_i + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} c_{ijr} x^*_{ijr} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} H_{ijr} z^*_{ijr} \tag{4.36}$$

Lagrange Relaxation Heuristic using the Sub-gradient Optimization procedure

The workings of the sub-gradient optimization for developing the Lagrange relaxation heuristic are similar to that presented in section 4.1.3.

4.2.4 Computational Study

In order to assess the effectiveness (objective value of OP) of using CPLEX and the LRH developed, a number of experiments using reference problems obtained in the literature were conducted. The Original problem (OP) and the LRH were coded on ECLIPSE development platform using Java and IBM ILOG Concert Technology Library. IBM CPLEX 12.8 was used as both the MIP solver and LP solver. In addition, a windows 8.1 operating system with 6GB Random Access Memory (RAM) has been used for the computation experiments. The original problem (OP) was solved using the MIP dynamic solver of CPLEX which basically uses the Branch and Cut algorithm to search the solution space for optimality. In addition, the lower and upper bounds and optimality gap obtainable by both CPLEX and the LRH are presented.

Data generation for the Lagrange Relaxation Heuristic and Benchmark data

The following values have been randomly selected for the Lagrange Heuristic initial parameters: $\varepsilon = 0.01$, $UB_{initial} = 1exp.9$, $LB_{initial} = 0$, $t_{max} = 10$, $t = 2$. A search for benchmark data for the facility location and fixed solid transportation problem has not been successful. Therefore, the benchmark data used for the fixed solid transportation problem of Sanei et al. (2017) have been extended by including the facility location fixed cost in the data supplied.

Data generation for the problem sizes

The Benchmark data used by Sanei et al. (2017) basically considers uniformly distributed data randomly generated as integers in a unit square coordinate U[a, b]. The uniform distribution is used to ensure a constant probability of selecting values randomly within the interval. The letter “a” refers to the lower cost limit and “b” is termed the upper-cost limit. They specified the supply capacity, demand capacity, conveyance capacity, unit costs and route fixed charge. For the facility location cost, the method of generating facility location cost instances from the supply capacities considered in the facility location literature as used by Gadegaard et al. (2017), Fischetti et al. (2016) and Guastaroba and Speranza (2014) has been used. In this method, the facility location cost is calculated using $F_i = U(0{,}90) + \sqrt{S_i}\ U(100{,}110)$. A total of 50 problem instances were solved across 10 different problem sizes generated as presented in Table 4-6 below. The data distribution used in generating the parameters is also given subsequently in Table 4-7 below.

Table 4-6 Problem sizes and number of instances used for experimentation

Problem Size No	**Problem Size** $\boldsymbol{m \times n \times a}$	**No of instance**
1	5×5×2	5
2	5×8×2	5
3	7×10×2	5
4	8×8×2	5
5	10×10×3	5
6	10×20×3	5
7	15×30×4	5
8	20×20×5	5
9	25×38×8	5
10	35×42×9	5

Table 4-7 Parameter distribution used for experimentation

Parameter	Distribution
$\boldsymbol{S_i}$	U(200, 400)
$\boldsymbol{D_j}$	U(50, 100)
$\boldsymbol{T_r}$	U(800, 1800)
$\boldsymbol{c_{ijr}}$	U(20, 150)
$\boldsymbol{H_{ijr}}$	U(200, 600)
$\boldsymbol{F_i} = \boldsymbol{U(0, 90) + \sqrt{S_i}\, U(100, 110)}$	
$\boldsymbol{M_{ijr} = \min(S_i, D_j, T_r)}$	

4.2.5 Experimentation and Results

The following tests were conducted in the computational study conducted

a) Using the problem sizes 1 to 6 in Table 4-7, the mean lower bounds, mean upper bounds, and gap% obtainable by both methods were computed. A selection of the better solution method as regards best optimality gap (gap%) was done and used for further comparing the FCSLTP and FCSTP.

gap% is defined as follows:

$$\text{gap\%} = \left(\frac{UB_{Sm} - LB_{Sm}}{LB_{Sm}}\right) \times 100$$

$UB_{Sm} =$ Best upper bound found by CPLEX and LRH

$LB_{sm} =$ Best lower bound found by either CPLEX or LRH

b) Comparative study of FCSLTP and FCSTP to investigate possible cost savings resulting from either formulation. Comparing both the FCSLTP and the FCSTP seems to provide a biased comparison because they both have different objective functions. An FCSTP makes an assumption that all locations are opened for shipment, while this may not hold for the FCSLTP. An FCSTP with total supply matching total demand is termed as balanced with all locations opened. However, when solving an unbalanced FCSTP with total supply capacity greater than total demand requirement, the possibility of having locations without any allocation exits. Unbalanced transportation problems seem to show more real-world applications than balanced transportation problems. This may due to the competition of resources in meeting demand requirements, demand uncertainties that require inventory keeping at various locations or unplanned disruptions that limits supply capacities. Therefore, when in-active locations of an FCSTP are closed, possible cost savings are made and the assumption that all locations are opened does not hold.

Based on these observations, an equivalent FCSLTP is defined from an FCSTP and termed it as FCSTP-EQ. This is done by computing normally the FCSTP without the facility location constraint and the facility location costs. Subsequently, the load distributions obtained are used to compute the equivalent facility location costs and added to the FCSTP objective function. This procedure is shown in Figure 4-3 below.

Compute the load distribution x_{ijr} using FCSTP model formulation

If locations have $\sum_{j=1}^{n}\sum_{r=1}^{a} x_{ijr} > 0 \quad \forall\ i = 1 \ldots m$,.

Compute facility location decision variable $y_i = 1$

Else

$y_i = 0$

If $x_{ijr} \geq 0 \quad \forall\ i = 1 \ldots m,\ \forall\ j = 1 \ldots n,\ \forall\ r = 1 \ldots a$

$z_{ijr} = 1$

Else

$z_{ijr} = 0$

using x_{ijr} , z_{ijr} and y_i Compute objective function FCSLTP (equation 4.22)

Figure 4-3 Procedure for computing the FCSTP-EQ

Under experimentation (a), results from Table 4-8 below show that for the Six (6) problems considered, the CPLEX solution obtained better mean lower bounds per problem size compared to the LRH. In addition, the mean upper bounds obtained by CPLEX solution were superior compared to the LRH. This is also supported by the gap% representing the optimality gap obtained in Table 4-9. The superior lower bounds achieved by CPLEX have been used to compute the optimality gap as shown in Table 4-9. The worst gap% obtained by CPLEX solution for the problem sizes considered was 0.1% while that of the LRH was 15.98%. The superior performance as displayed by CPLEX is likely connected to the several cutting planes solutions of improving the lower bounds obtained from the initial linear relaxations as earlier indicated in section 4.2.3. On the other hand, the LRH is an alternative option developed but have not obtained significant results when compared with the quality of solutions obtainable using CPLEX. The best optimality gap obtained using the LRH for the problem sizes considered was 3.62 %. Therefore, based on the superior performance of CPLEX solution for the problem sizes considered, the CPLEX optimization tool is used for a comparative study between the FCSTP-EQ and FCSLTP.

Table 4-8 Mean values for best Lower bound and upper bound computation per solution method

Problem Size No.	**Problem Size** $\boldsymbol{m \times n \times a}$	**Total Problem Instances**	**mean** $\boldsymbol{LB_{LRH}}$ **(best)**	**mean** $\boldsymbol{UB_{LRH}}$ **(best)**	**mean** $\boldsymbol{LB_{CPLEX}}$ **(best)**	**mean** $\boldsymbol{UB_{CPLEX}}$ **(best)**
1	5×5×2	5	10879.80	18505.79	17859.01	17860.29
2	5×8×2	5	17322.20	28333.22	25534.45	26333.42
3	8×8×2	5	15736.80	29614.83	25534.45	25667.43
4	7×10×2	5	22063.60	35494.88	33925.34	33925.34
5	10×10×3	5	18764.20	34423.95	29758.67	29758.67
6	10×20×3	5	39061.60	62065.47	58664.97	58813.86

Table 4-9 Mean Gap% of each solution method using the best mean lower bound (CPLEX)

Problem Size No.	Problem Size $m \times n \times a$	mean LB_{CPLEX} (best)	Gap% LRH	Gap % CPLEX
1	5×5×2	17859.01	3.62%	0.007%
2	5×8×2	25534.45	7.72%	0.1%
3	8×8×2	25534.45	15.98%	0.05%
4	7×10×2	33925.34	4.63%	0.00%
5	10×10×3	29758.67	15.68%	0.00%
6	10×20×3	58664.97	5.8%	0.03%

Under experimentation (b) the equivalent FCSLTP developed (using the FCSTP) described as FCSTP-EQ (Figure 4-3 above) and the original formulation (equations 4.22 - 4.27c) termed as the FCSLTP were compared. A computation time limit of 9000 seconds to obtain solutions was placed. Table 4-10 below shows the results obtained. It was observed that the original formulation (FCSLTP) performed considerably better than the FCSTP-EQ through the 10 different problem sizes considered. Problems Size (10) in particular showed a 25% reduction in total costs when using the FCSLTP formulation. Figure 4-4 below shows a trend of the increase in cost savings as the problem size increased, with the lowest cost savings at 3% in the smallest sized problem (1) and 25% in the largest problem size (10). This trend is possibly connected to the increase in solution search space of the FCSTP-EQ formulation when compared with the FCSLTP as problem size increased. The feasible solution search space of FCSTP-EQ is wider due to less limitation on the number of locations to open for shipment. In addition, the FCSTP-EQ obtains its load distribution assuming all locations are opened for shipment. Although during actual allocations, there are possibilities of some locations not shipping to any destinations, therefore requiring the need for closure of such in-active location as done in the FCSTP-EQ computation.

It was also observed that when solving the FCSLTP, the FCSTP formulation could possibly be used as an initial feasible solution. The percentage total cost savings obtained (Table 4-10 below) between FCSLTP and FCSTP-EQ could also be interpreted as an optimality gap when using the FCSTP as starting solution to compute the FCSLTP. In addition, the run time of FCSTP is much lower compared to the FCSLTP as shown in Figure 4-5 below. This is expected although the magnitude of difference may not be easily quantified unless experimentally conducted. The reduction in computation time of FCSTP is likely due to the reduced number of computations required to be performed compared to the FCSLTP formulation. Therefore, the reduced time could be an insight into the development of a hybrid improvement solution or heuristic using the FCSTP as a basis to solve the original formulation of FCSLTP. The run time of FCSLTP using CPLEX shows significant exponential time complexity than the FCSTP for larger problem sizes. Figure 4-5 presents a comparison between the mean run time of the FCSTP and the FCSLTP as problem size increase.

Table 4-10 Comparison between the FCSTP EQ and FCSLTP using CPLEX under 9000secs computation time

Problem No	**Problem Size** $m \times n \times a$	**Total no. of Instances**	$FCSTP$ **EQ mean**	$FCSLTP$ **mean**	**Cost Difference**	**% Cost Difference**
1	5×5×2	5	18480.39	17860.29	620.10	3%
2	5×8×2	5	28333.22	26333.42	1999.80	8%
3	8×8×2	5	29064.07	25667.43	3396.64	13%
4	7×10×2	5	35807.10	33925.34	1881.76	6%
5	10×10×3	5	32147.15	29758.67	2388.48	8%
6	10×20×3	5	62797.43	58813.86	3983.57	7%
7	15×30×4	5	85778.98	77653.71	8125.27	10%
8	20×20×5	5	61498.56	50054.12	11444.44	23%
9	25×38×8	5	106532.31	89098.73	17433.58	20%
10	35×42×9	5	120932.51	96508.73	24,423.78	25%

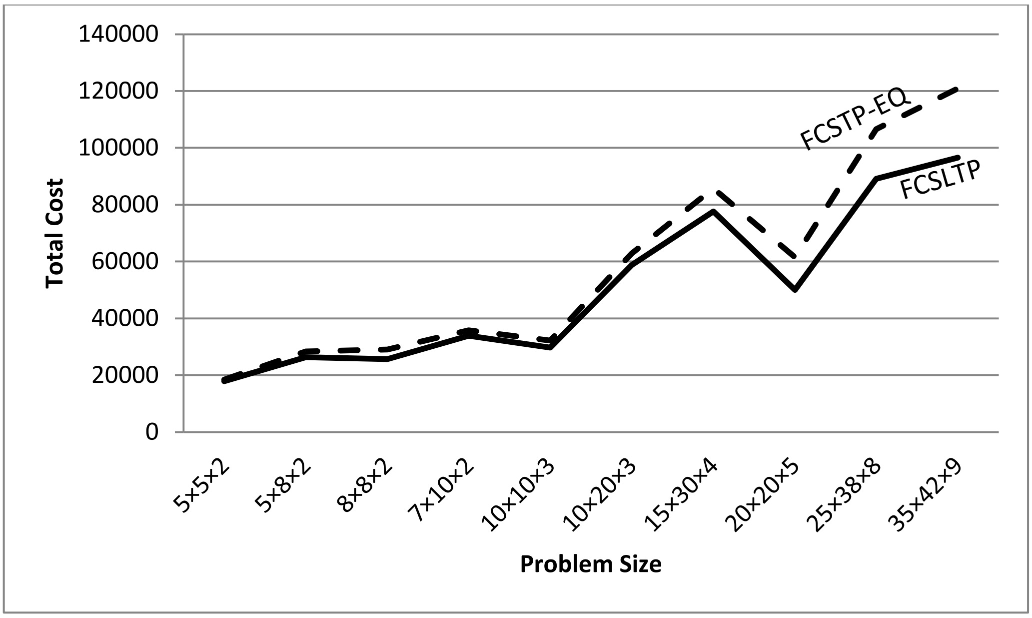

Figure 4-4 FCSLTP and FCSTP-EQ

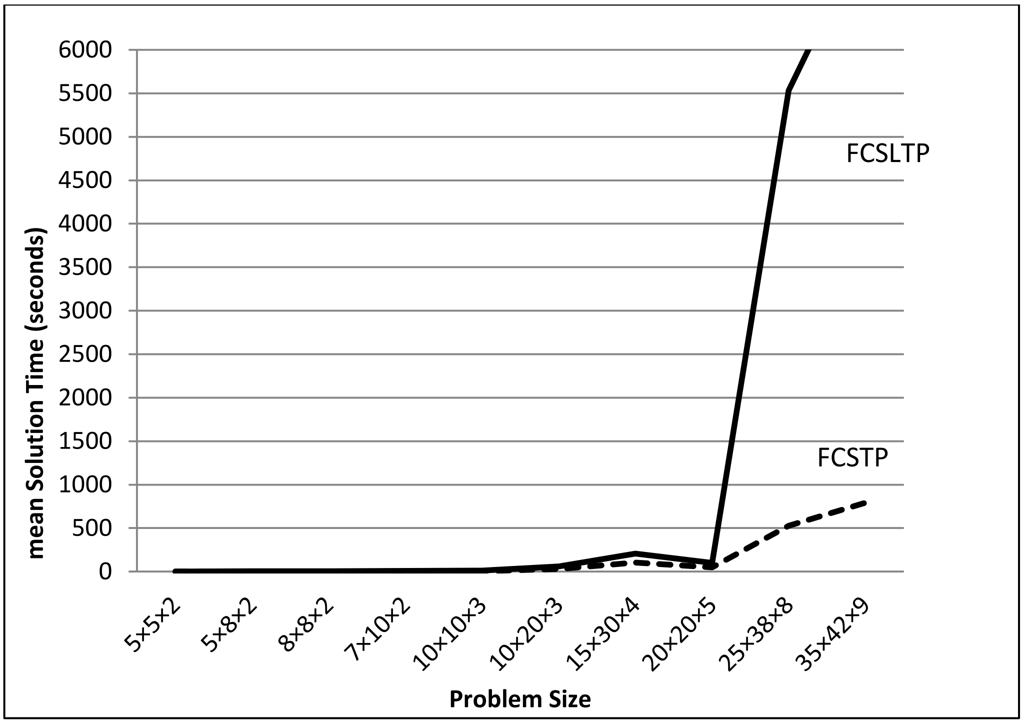

Figure 4-5 Solution time of FCSLTP and FCSTP

4.2.6 Conclusion and Future Direction

An optimization problem that integrates facility location decisions into a distribution problem known as the Fixed Charge Solid Transportation Problem (FCSTP) has been studied. This problem was termed Fixed Charge Solid Location and Transportation Problem (FCSLTP) and was solved using CPLEX optimization tool. An LRH was developed as an alternative solution for users who possibly may not have access to commercial optimization solvers for certain MIP due to costs or other constraints. The LRH was compared to the CPLEX solution. Results obtained presents CPLEX solution as more effective with regard to a lower optimality gap for all the problem sizes considered. However, the LRH could still give some solutions within an optimization gap of 4%.

A formulation of FCSLTP using the load distribution obtained from FCSTP was discussed and termed an equivalent FCSLTP (FCSTP-EQ). The basic motivation for this formulation was stated as the possibility of some locations turning out to be in-active for shipping out when solving an unbalanced FCSTP and the cost savings obtained when such locations are closed. The FCSTP-EQ was compared to the original formulation of FCSLTP denoted as FCSLTP. Using total cost as a measure of comparison, the FCSLTP formulation presented better cost savings compared to the FCSTP-EQ. This possibly was attributed to a narrower feasible solution search space used by the FCSLTP as compared to the FCSTP-EQ. The percentage total cost difference was also interpreted as an optimality gap between FCSLTP and FCSTP-EQ. Based on the cost savings or optimality gap obtained, the load distribution of an FCSTP was considered as a feasible starting solution to solving the FCSLTP for larger problem sizes. This is further supported by the very low solution time obtained when solving an FCSTP as compared to the FCSLTP for larger problem sizes. Suggested improvement for using the FCSTP as a starting solution might be the requirement of a hybrid improvement heuristic to obtain improved upper bounds within the solution time limit of the FCSLTP.

Possible extensions to improve the quality of solution of the LRH can be strengthening the upper bound search using other heuristics or possible metaheuristics such as genetic algorithm and/or simulated annealing. Metaheuristics have the capacity to further increase the solution search space and make the hybrid solution become very competitive with solution methods obtainable in commercial optimization tools such as in CPLEX. As noted in the experiments, the time complexity of CPLEX solution is exponential as the problem size increase. Consequently, this might strengthen the reason for a search for an efficient alternative solution method.

Chapter 5

Hybrid Genetic Algorithm Solution of the facility location and fixed charge solid transportation problem

5.1 Introduction

In this chapter, a variant of the FCSTP, referred to as the Fixed Charge Solid Location and Transportation Problem (FCSLTP) is considered. In addition, a hybrid metaheuristic solution is proposed. This hybrid metaheuristic uses the GA process to select a combination of feasible facility locations while allocation from the feasible locations is achieved using a constructive greedy heuristic. An improvement heuristic termed modified stepping stone algorithm has been used. This was used to further consolidate load distribution for cost reduction and improve the search for a better solution. In order to test the effectiveness (objective function) and efficiency (solution time) of the hybrid GA method, a comparison was with the solutions provided by CPLEX, a commercial solver.

5.2 Model Formulation

The FCSLTP is modelled as a mixed-integer linear programming problem consisting of m feasible sources or locations, n destinations or customers, and a conveyances or transport sources. The FCSLTP basically differs from the FCSTP discussed by Sanei et al. (2017) in that location costs, location capacities, route costs and route capacities are simultaneously used in determining whether locations will be open or closed when servicing customers. Moreover, the model formulation and assumptions considered are similar to those presented in Oyewole and Adetunji (2018) and rehashed in this chapter. The FCSLTP seeks to minimize total transportation and location costs by determining the optimal allocations from selected open locations through open routes via a set of conveyances.

5.2.1 Model parameters

i: Index for sources or facility locations (warehouses, depots etc.)
j: Index for destinations (customers, other warehouses etc.)
r : Index for conveyances (or Transportation mediums)
m : Number of sources
n : Number of destinations
a : Number of conveyances
c_{irj} : Variable cost of shipment from source i through conveyance r to destination j.
S_i : Capacity at source i.
D_j : Demand at Destination j .
T_r : Capacity of conveyance r.
F_i : fixed charge for keeping a facility location open.
H_{irj} : Fixed cost (fixed charge) incurred for shipping from source i through conveyance r to destination j.
Decision Variables:
x_{irj}: Quantity of products transported from source i through conveyance r to destination j.
y_i : Variable indicating which facility location is opened.

z_{irj} : Variable indicating which conveyance means is utilized en route (i, j).

Objective Function (minimum cost function):

Minimize (Z) =

$$\sum_{i=1}^{m} F_i \, y_i + \sum_{i=1}^{m}\sum_{r=1}^{a}\sum_{j=1}^{n} c_{irj} x_{irj} + \sum_{i=1}^{m}\sum_{r=1}^{a}\sum_{j=1}^{n} H_{irj} z_{irj} \qquad (5.1)$$

Subject to

$$\sum_{r=1}^{a}\sum_{j=1}^{n} x_{irj} \leq S_i y_i \qquad \forall \ i = 1 \dots m \qquad (5.2)$$

$$\sum_{i=1}^{m}\sum_{r=1}^{a} x_{irj} = D_j \qquad \forall \ j = 1 \dots n \qquad (5.3)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{irj} \leq T_r \qquad \forall \, r = 1 \dots a \qquad (5.4)$$

$$x_{irj} \geq 0 \qquad \forall \ i = 1 \dots m, \ \forall \ j = 1 \dots n, \forall \, r = 1 \dots a \qquad (5.5)$$

$$z_{irj} = \begin{cases} 1 & x_{irj} > 0 \\ 0 & Otherwise \end{cases} \qquad \forall \ i = 1 \dots m, \ \forall \ j = 1 \dots n, \forall \, r = 1 \dots a \qquad (5.6a)$$

$$y_i = 0 \; or \; 1 \qquad \forall \ i = 1 \dots m \qquad (5.6b)$$

Equation (5.1) is the objective or the cost function, which to be minimized. The first term computes the total facility location cost, the second term computes the total route variable cost and the third term computes the route total fixed charge. Equation (5.2) is the supply capacity constraint of each facility location or sources. It also ensures that the capacities of closed facilities are not utilized. Equation (5.3) is the demand constraint indicating the destination demands should be met. Equation (5.4) is the conveyance capacity constraint. It ensures that the capacities of selected conveyances are not exceeded. Equation (5.5) refers to the non-negativity constraint for the continuous variables and Equations 5.6a and 5.6b are binary constraints indicating whether a facility is opened or not and whether a conveyance is selected or not.

5.3 Genetic Algorithm

The Genetic Algorithm(GA) is a multi-dimensional search strategy defined by Fernandes et al. (2014) as a framework that imitates the evolutionary principle of nature to provide solutions to NP-hard combinatorial problems. The GA has also been viewed as a probabilistic or stochastic search technique due to the probability rates normally associated with the genetic operations involved in producing solutions during the search process. As noted by Jawahar et al. (2012) and Pérez-Salazar et al. (2015), the successful implementation of the GA depends on the user-defined solution representations, initialization, genetic operations and terminating conditions. The solution representation basically is concerned with how to encode and decode the feasible solution of the combinatorial problem taking part in the genetic operations. These feasible solutions are usually referred to as chromosomes. In addition, the representation of each individual variable type making up the chromosomes (i.e. genes) has to be properly captured. This is because optimization variables could either be continuous, binary or integer. The representation types used have been noted by several authors to determine how sensitive the GA will be in converging to the solution desired. The Genetic operations consist of the chromosome selection method, crossover operation and mutation operation used to ensure necessary diversity in the search process. The stages of the GA implementation include initialization, crossover, mutation and termination.

Initialization

The initialization conditions include the determination of the desired fitness function (objective function) for the GA procedure, chromosome representation, initial population size and the terminating condition of the GA, including the number of generations.

Crossover operation

The aim of the crossover is to generate and promote the replication of good solutions (chromosomes) while rejecting the bad ones. Before crossover is performed, chromosomes are selected using some assigned probabilities known as the crossover rate. The roulette wheel technique is a popular selection technique used in the literature to achieve the selection (Jawahar and Balaji (2009), Ojha et al. (2010), Pérez-Salazar et al. (2015)). The crossover operation ensures the reproduction of new offspring or children solution from parent solutions. Different cross over operation types have been discussed in the literature as noted by Jawahar and Balaji (2009). These are either based on a single point or two-point crossovers such as the partially mapped crossover and the ordinal mapped crossover.

Mutation operation

The mutation operation involves perturbation of some of the genes (variables) of a chromosome, based on some assigned probabilities known as the mutation rate. Genes are also randomly selected using a user-defined mutation rate. The mutation operation or gene replacement essentially gives the GA its power of arriving at other new solutions not possible with the crossover and have the potential of being better than existing solutions.

Termination

Terminating conditions usually involve the stopping criteria normally employed in optimization problems such as the number of desired iterations and optimization time desired. For the GA the number of generations employed can also be utilized as a stopping criterion.

5.3.1 Solution representation

Choosing a suitable representation for the candidate solutions of the original problem has been considered by several authors to be based on the optimization problem structure and the ease of performing the genetic operations of the GA. The matrix and vector (binary) representation were discussed by Vignaux and Michalewicz (1991). A Priority-based encoding was proposed by Gen et al. (2006). This was to prevent likely infeasibility during genetic operations observed with the prüfer number technique of representing chromosomes discussed by Gottlieb et al. (2001). Antony et al. (2011), while discussing solutions to a m-number of sources and n -number of destinations FCTP, underscored the differences between the matrix, permutation, prüfer number and direct representation. The differences were based on the number of genes involved in the chromosomes. They showed the matrix representation as possessing the highest number of genes representing the transportation problem which is $m \times n$, while the prüfer number had the least i.e. $m + n - 2$. A hybrid chromosome representation that presents both the continuous and the binary variables of the original mixed-integer problems as an array was discussed by Pérez-Salazar et al. (2015) and Hiassat et al. (2017).

In this chapter, a vector of binary numbers is used to represent the facility locations while a matrix of continuous numbers is used to represent the candidate solution, which is essentially the allocated quantity from the facility locations (sources) and to the points of demand (destinations). A typical matrix representation used for a sample feasible solution to an original problem with 3 candidate facility locations, 4 demand destinations and two conveyances is shown in Figure 5.1 below. The facility location vectors are encoded as the GA chromosomes and manipulated through the various GA operations while the constructive greedy heuristics and improvement modified stepping stone heuristics work on the allocation matrix. The allocation (shown in Figure 5.1) is made based on the result from the GA operations on the facility location vector (as shown in Figure 5.2). The fixed charges are incurred when the continuous variable part of equation (5.1) is non-zero.

$y_1=1$ S_1	x_{111} ,$z_{111}=1$	0	x_{113} ,$z_{113}=1$	0	T_1
	0	0	0	x_{124} ,$z_{124}=1$	T_2
$y_2=0$ S_2	0	0	0	0	T_1
	0	0	0	0	T_2
$y_3=1$ S_3	x_{311} ,$z_{311}=1$	0	0	x_{314} ,$z_{314}=1$	T_1
		x_{322} ,$z_{322}=1$	0	0	T_2
	D_1	D_2	D_3	D_4	

Figure 5-1 Typical candidate solution representation for a 3-source, 4-destination and 2-conveyance problem

$i=1$	$i=2$	$i=3=m$
1	0	1

Figure 5-2 Sample chromosome representation for an FCSLTP with 3 sources

In Figure 5.2 above, a value of zero allocated to the second source($\boldsymbol{i} = \mathbf{2}$) means there can be no load allocation to the matrix member of that row.

5.3.2 Initialization

Fitness function

The fitness function to be used in the GA is the objective function of the original problem. This is same as equation (5.1) above.

Initial population and candidate feasible solution generation

Given the location fixed cost F_i of dimension (m), route fixed cost H_{irj} of dimension $(m \times a \times n)$, variable cost c_{irj} of dimension $(m \times a \times n)$, population size (p) and number of generation (g), the generation of the candidate feasible solutions to the original problem $(c_1 \dots c_p)$ and the initial population of chromosomes are described below and illustrated in Figures 5-3 and 5-4 below respectively.

1. Random generation of the combination of facilities or locations that possess sufficient capacity to meet demand. A selection of y_i $(i = 1 \dots m)$ is made such that the feasibility $\sum_{i=1}^{m} S_i \, y_i \geq \sum_{j=1}^{n} D_j$ is checked and uniqueness of each combination of facilities is ensured. A matrix $(m \times p)$ is created to store each feasible combination of the facilities and checked for uniqueness of each solution. The matrix $(m \times p)$ is also referred to as the population of chromosomes in this chapter and represented in Figure 5-4 below.
2. Creation of a Relaxed Average Variable Cost (RAVC) matrix of dimension $(m \times a \times n)$. This is based on the integration of the route fixed cost, the variable cost of the

problem and minimum of all capacities and it is used to allocate capacities. This is similar to the least equivalence variable cost discussed by Jawahar and Balaji (2009). The RAVC is stated as

$$RAVC\ (irj) = \frac{Route\ fixed\ cost\ \left(H_{irj}\right)}{\min\left(S_i, D_j, T_r\right)} + Variable\ cost\ \left(c_{irj}\right) \qquad (5.7)$$

3. Creation of the matrix of candidate feasible allocation (Illustrated with Figure 5-3 below). A three-dimension matrix of dimension $(ma \times n \times p)$ is created. This is called the candidate feasible solution allocation matrix. The procedure for creating the three-dimensional matrix is stated below.
   (a) From the earlier two-dimension matrix $(m \times p)$ of feasible combinations of facilities (chromosomes), select each feasible chromosome (m) from the (p) rows of population.
   (b) Compute the RAVC as stated above to obtain the matrix$(m \times a \times n)$.
   (c) Apply the constructive greedy heuristic (illustrated with Figure 5-5 below) to make the initial allocation. The greedy heuristic utilizes the m rows of the matrix obtained in step (3a) above and the RAVC computed in step (3b) above to allocate into the first layer of the candidate feasible solution three-dimensional matrix $(ma \times n \times 1)$.
   (d) Use the improvement heuristic (the modified stepping stone algorithm) (illustrated in Figure 5.6 below). This is based on the actual route fixed cost and variable cost matrix, to improve allocations obtained in step(3c) above. This gives the initial candidate feasible solution (candidate solution , $c = 1$) of dimension $(ma \times n \times 1)$.

   Anthony (2011) proposed a stepping stone method which attempts to increase the number of current basic variables (current solution) of the initial solution by exploring the possibilities of non-basic variables or other feasible solutions entering the basis. However, the modified stepping stone method consolidates on basic variable positions only. This is done in order to check for possible cost savings through route fixed cost and variable cost trade-off by either eliminating route fixed costs and/or possible reduction in variable costs subject to capacity re-allocation.
   (e) Repeat step (3a) to (3d) for all the candidate feasible solution ($c_1 \dots c_p$) to obtain the candidate feasible three-dimension matrix $(ma \times n \times p)$.
4. Computation of the candidate feasible solution fitness function using the actual cost parameters and the allocation of the three-dimensional matrix obtained in step (3e) above.

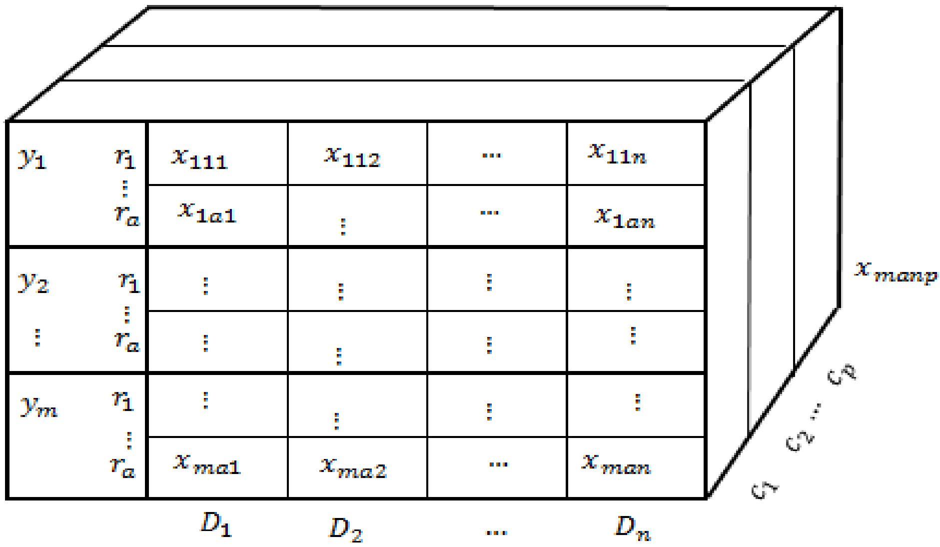

Figure 5-3 Candidate feasible solution allocation matrix ($\boldsymbol{ma} \times \boldsymbol{n} \times \boldsymbol{p}$)

$m \times P$

$i = 1$	$i = 2$	$\cdots$	$i = m$	
1	0	$\cdots$	1	$pop = 1$
1	0	$\cdots$	0	$pop = 2$
		$\vdots$		$\vdots$
		$\vdots$		
0	1	$\cdots$	1	$pop = p$

Figure 5-4 Sample populations of Chromosomes ($\boldsymbol{m} \times \boldsymbol{p}$)

Greedy allocation heuristic

For every entry of demand, $\boldsymbol{j} = \mathbf{1}:\boldsymbol{n}$,
find the minimum RAVC from ($\boldsymbol{i} = \mathbf{1} : \boldsymbol{ma}$)
While demand ($\boldsymbol{j}$) $> \mathbf{0}$
If $\boldsymbol{T_r} > \mathbf{0}$ and $\boldsymbol{S_i} > \mathbf{0}$
$\boldsymbol{x_{irj}}$ = allocation = $\mathbf{min}(\boldsymbol{S_i}, \boldsymbol{D_j}, \boldsymbol{T_r})$
Subtract $\boldsymbol{x_{irj}}$ from $\boldsymbol{D_j}$
Subtract $\boldsymbol{x_{irj}}$ from $\boldsymbol{S_i}$
Subtract $\boldsymbol{x_{irj}}$ from $\boldsymbol{T_r}$
Else
Move to the next minimum RAVC ($\boldsymbol{i} = \mathbf{1} : \boldsymbol{ma}$)

End if
Update $\boldsymbol{j}$: $\boldsymbol{j} = \boldsymbol{j} + \mathbf{1}$
End for

Figure 5-5 Greedy heuristic to populate Initial solution

Improvement Heuristic (Modified stepping stone method)

A set of acronyms are defined below in the modified stepping stone method for comprehension purposes and presented below.

$i \ r \ and \ j$ are already defined under section 5.2.1 above.
$m \ n \ and \ a$ is as stated in the original problem in section 5.2.1 above.
Define source indices $(i \ and \ u) : \ i < u \leq m$
Define destination indices $(j \ and \ p) : \ j < p < n$
Define conveyance indices $(r \ and \ v) : \ r < v < a$
$x_{i,r,j}$ => Variable allocation at positon (irj). (Similarly for x_{irp} , x_{uvp} , x_{uvj})
H_{ijr} => Route fixed cost at position (irj).(Similarly for H_{irp} , H_{uvp} , H_{uvj})
min_alloc => Minimum of allocation.
variable_cost change at position (irj)= $(c_{irj} + c_{uvp}) - (c_{irp} + c_{uvj})$
(based on $x_{irj}, x_{irp}, x_{uvp}, x_{uvj}$ and Illustrated in Figure 5-7)

The improvement heuristic is illustrated in Figure 5-6 below, while an Illustration of the selection of variables for the improvement heuristic (modified stepping stone consolidation) is presented in Figure 5-7 below.

Improvement Heuristic (Modified stepping stone method)

For every source$(i = 1, r = 1)$ to $(i = m, r = a)$
Source = source 1
For every destination $j = 1{:}n - 1$, if $x_{irj} > 0$
Source, Destination = source1, dest 1
(# Source, Destination combination is explained in Figure 7 below #)
If for any destination $p > j$, $x_{irp} > 0$
Source, Destination = source 1, dest 2
If for any source $(u > i \; and \; v >= r)$ OR source $(u >= i \; and \; v > r)$, $x_{uvp} > 0$
Source, Destination = source 2, dest 2
If $x_{uvj} > 0$
Source, Destination = source 2, dest 1
Find $min\_alloc = min\,(x_{irj}, x_{irp}, x_{uvp}, x_{uvj})$
If H_{irj} < variable_cost change (check 1) THEN
(consolidation step1)
$x_{irj} = x_{irj} - min\_alloc$
$x_{uvp} = x_{uvp} - min\_alloc$
$x_{irp} = x_{irp} + min\_alloc$
$x_{uvj} = x_{uvj} + min\_alloc$
(Repeat check1 for fixed costs positions H_{irp}, H_{uvp} and H_{uvj} and apply the pattern of consolidation step1 if true)

ELSE If H_{irj} > variable_cost change (check 2)THEN
(consolidation step2)
$x_{irp} = x_{irp} - min\_alloc$
$x_{uvj} = x_{uvj} - min\_alloc$
$x_{irj} = x_{irj} + min\_alloc$
$x_{u\,vp} = x_{u\,vp} + min\_alloc$
(Repeat check2 for fixed cost positions H_{irp}, H_{uvp} and H_{uvj} And apply the pattern of consolidation step2 if true)

Else (no improvement for cost position irj)

Figure 5-6 Improvement heuristic (modified stepping stone procedure).

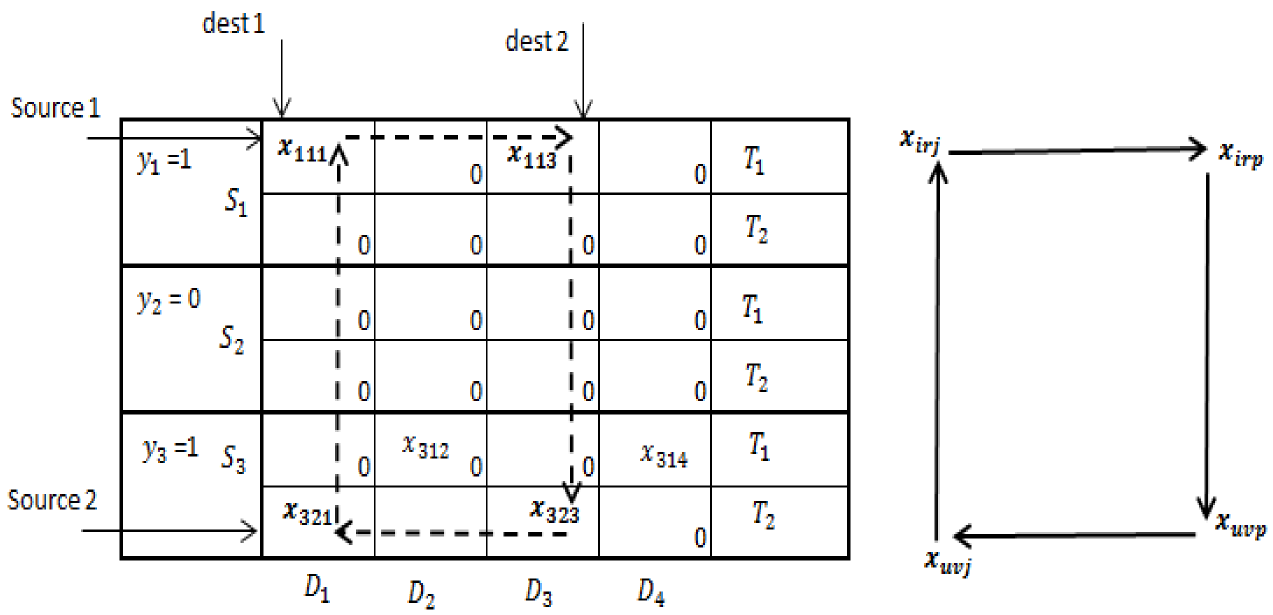

Figure 5-7 Selection of variables for load consolidation

5.3.3 Generation of new population

The generation of new chromosomes is discussed in this section. An emphasis is put on the best fit solution over the weak solutions during the crossover and mutation operations as also indicated by Jawahar and Balaji (2009) and Balaji et al. (2019) in their solution. As described in section 5.3.2 the generation of the initial population utilizes random search in generating a combination of facilities such that $\sum_{i=1}^{m} S_i\, y_i \geq \sum_{j=1}^{n} D_j$. The random search is implemented for the binary facility location term in this chapter and stored in matrix of dimension $(m \times p)$ as shown in Figure 5-4 above.

Inputs for new population

The new population generation function takes as input the following

a) The parent matrix $(m \times p)$ generated in section 5.3.2 above.
b) A vector of sort index of chromosomes $(m \times p)$ in increasing order of cost for each chromosome (dimension$p$).
c) The crossover rate $(cross\ rate)$
d) The mutation rate $(mut\ rate)$
e) The source and demand capacities corresponding to the matrix $(m \times p)$.

Genetic operations procedure

The matrix of the old facilities opened (parent) contains p chromosomes, each chromosome being a set of binary values indicating which supply points were opened or closed. This matrix was crossed over and mutated to create a new population on which the allocation and improvement heuristics were applied. This cycle was repeated until the numbers of generations (g) were completed.

This procedure is described below.

1. Determine the number of chromosome of the old population to keep from the crossover rate ($cross\ rate$).
2. Populate the discarded chromosomes to build up a matrix of a new population of size ($m \times\ p$) using the procedure below.
    a. Copy the retained chromosomes into the relevant positions in the new population matrix, keeping the least cost chromosome in position 1.
    b. Use rank based roulette wheel selection (as shown in Figure 5-8 below) to select the two mating chromosomes among the retained chromosomes. The rank-based roulette wheel selects two chromosomes to be used for crossover from the chromosomes retained from the population. It receives as input a population of chromosomes ranked based on fitness function from the best ranked in the first position to the worst ranked in the last position.
    c. Randomly generate the crossover point for the mating chromosomes as described in Figure 5-9 below.
    d. Perform crossover (Figure 5-9 below) and store the two new offsprings in the next two positions in the new population matrix.
    e. Repeat step (d) until the new population matrix is fully constructed
3. Use the mutation rate to determine the number of genes to mutate by flipping the binary value ($0\ to\ 1\ or\ 1\ to\ 0$). This is described in mutation section below.
4. Randomly generate the two index positions to mutate in the new matrix and flip the gene in the location while preserving the least cost gene in position 1 unchanged
5. For every chromosome in position 2 till the last, check for feasibility illustrated below)
    a. If a chromosome is not feasible, randomly locate a position that is closed and open until the chromosome has a number of opened sites that is feasible for demand allocation
6. Once a new allocation matrix is complete, pass matrix to the greedy algorithm to allocate demand and consolidate the allocation using the modified stepping stone algorithm
7. Repeat all steps until the number of generation is complete.

Rank-based roulette selection

rank = ranking of chromosome in the population
p = population size
cumProb = sum of probability up until the current member of the population, initialised to zero
sumRank = sum of the rank of all members initialised to zero
chrom 1 = First chromosome selected for crossover
chrome 2 = Second chromosome selected for crossover
 	
for all members of the population,
   sumRank = sumRank + rank of chromosome
end for
 	
for all members of population,
   cumProb = cumProb + ((*p* – rank + 1) / sumRank)
end for
 	
Generate the mating chromosomes
      number = random between 0 and 1
      start from the first member of the population
            while number =< cumProb
                  then chrom 1 = current chromosome
                  go to the next member
            end while
      Repeat for chrom 2 what was done for chrom 1
 	
Return chrom 1 and chrom 2

Figure 5-8 Rank based roulette selection

Crossover operation

Given length of chromosome = chromLength ,

crossover point = random integer between 1 and chromLength = crossPoint

For the first chromosome (chrom 1) in the pair, copy gene from the second chromosome starting from crossPoint to the end and put into the same position in the first chromosome and assign to offspring 1.

For the second chromosome (chrom 2) in the pair, copy gene from the first chromosome starting from position 1 to position crossPoint and put into the same position in the second chromosome and assign to offspring 2.

Return offsprings 1 and offspring 2

Mutation operation

p = number of a row of the matrix, equal to the population size.
numSource = number of genes in a chromosome, total equal to the number of sources (m).
mutPercent = percentage of matrix genes to mutate.
mutNum = numbers of genes to mutate = (p) × (numSource) × mutPercent

number 1 = Generate a random number between 1 and numSource.
number 2 = Generate a random number between 2 and p.
for 1 till mutNum

flip the gene in position (number 1, number 2) of the matrix

endfor

Check for feasibility

This section checks every chromosome representing a combination of opened sites out of all possible sources to ensure feasibility. It accepts as inputs the vector of demand at each destination, vector of supply capacity at each source and the matrix of opened sites.

sumDem = sum of all destination demands (from demand vector).
sumOpenedCap = sum of capacities of all opened sites, based on a chromosome.
numSource = number of source sites available, equal to the number of columns of the matrix and also equal to the length of a chromosome(m).
population size (p) = equal to the number of rows of the matrix.
for all chromosomes (rows)

while sumproduct(current chromosome vector and source capacity vector) < sumDem
randomly flip one of the zeros to 1
end while
end for

A summary of the working procedure of the HGA is illustrated in Figure 5-9 below

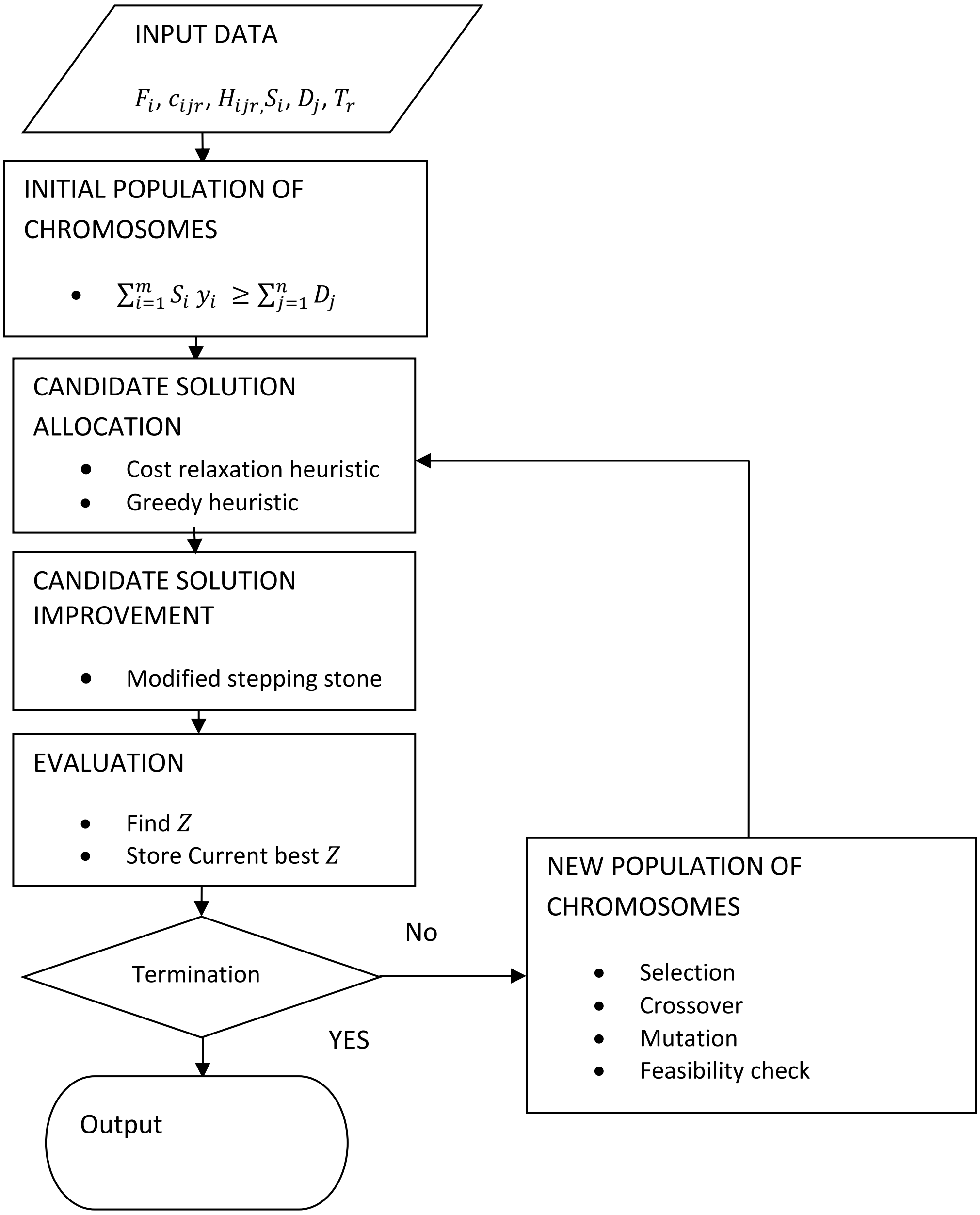

Figure 5-9 Hybrid Genetic Algorithm flow chat

5.4 Computation Study

The computational study was done in two stages. The first stage is preliminary experimentation while the second is the main experimentation. The preliminary experimentation was performed to obtain necessary parameters to effectively implement the HGA. In addition, it was done to identify the most influential parameters of the HGA by observing the relative effectiveness. The parameters such as the population size (p), number of generations (g), crossover rate ($cross\ rate$) and mutation rate ($mut\ rate$) implemented, have been shown in the literature to affect the convergence of a GA solution. Studies in the literature have shown that important parameter settings for a GA to be based on tuning the population size and number of generations such as in Ho and Ji (2005), Fernandes et al. (2014) and Guo et al. (2017). In the main experimentation, the quality of solutions of the HGA and CPLEX will be assessed. This is based on measures of performance described in section 5.5.

5.4.1 Preliminary experimentation

In this section the population size, number of generation, crossover rate and mutation rate are varied. The problem sizes have been stated in the order of ($m \times n \times a$). Where m = number of sources (or locations), n = number of destinations or demand points and a = number of transport sources or conveyances. In conducting parameter tunings for the HGA, two problem sizes (7×10×2 and 15×30×2) which represent a sample of small to medium-sized problems comprising the test data were selected. Population sizes considered for the smaller problem were smaller than the medium-sized problem to account for the uniqueness of the solution required to populate each population size. In both instances, initial crossover and mutation rates were randomly selected and kept constant while the population size and number of generations were varied in increasing order. The population size and the number of generations that showed a quick convergence were retained. Similarly, the best performing population size and number of generations were kept constant while the crossover rate and mutation rate were progressively varied. Tables 5-1 and 5-2 below show the parameter variations and convergence of the test problem size (7×10×2). The values of the minimum cost obtained (equations 5.1) for the first and last iterations are recorded as MinCost (first) and MinCost (last) respectively.

Table 5-1 First test result for problem size (7×10×2)

Problem Specification	$\boldsymbol{p}$	$\boldsymbol{g}$	**MinCost first**	**MinCost Last**
7X10X2	10	8	27689	27058
$\boldsymbol{cross\ rate}$ **= 0.5**	20	20	27006	27006
$\boldsymbol{mut\ rate}$ **= 0.1**	30	50	27006	27006

Table 5-2 Second Test results for problem Size (7×10×2)

Problem Specification	$\boldsymbol{cross\ rate}$	$\boldsymbol{mut\ rate}$	**MinCost First**	**MinCost Last**
7X10X2	0.7	0.3	28059	27006
$\boldsymbol{p}$ **= 30**	0.3	0.3	27342	27006
$\boldsymbol{g}$ **= 50**	0.3	0.1	27024	27006

Table 5-3 and 5-4 below show the parameter variations and convergence of the test problem size (15×30×2).

Table 5-3 First Test results for problem Size (15×30×2)

Problem Specification	***p***	***g***	**MinCost First**	**MinCost Last**
15X30X2	20	8	77240	71408
Cross rate = 0.5	50	50	75100	70927
Muta rate = 0.1	50	100	73084	70927

Table 5-4 Second Test results for problem Size (15×30×2)

Problem Specification	***cross rate***	***mut rate***	**MinCost First**	**MinCost Last**
15X30X2	0.7	0.3	73915	70927
***p* = 50**	0.3	0.3	75150	70927
***g* = 100**	0.3	0.1	75085	70927

Results obtained indicate that the parameter combinations all converged to the same minimum cost except in the case of p =10 and g =8 in Table 5.1. However, some parameter combinations obtained lower minimum cost from the initial generation (iteration). This possibly could indicate a quick convergence when using such parameter combinations for the HGA. For the test problem size (7×10×2), the results in Table 5.1 and 5.2 showed that the population size (30), number of generations (50), crossover rate (0.3) and mutation rate (0.1) converged rather quickly for the minimum cost value compared to other parameters used. Results of the problem size (15×30×2) as displayed in Table 5.3 and Table 5.4 showed that population size (50), number of generations (100), crossover rate (0.5) and mutation rate (0.1) converged more quickly than with other parameters. In summary, the population size and the number of generations seemed to be very effective in determining the final minimum cost value obtained.

5.4.2 Data generation for experimentation

A modification to Sanei et al. (2017) experimental data was used to test the different solution methods. Their benchmark data was extended to capture the cost of facility location which was not considered in their model. For the facility location cost, the method of generating facility location cost instances from the supply capacities considered in the facility location literature as used by Gadegaard et al. (2017), Fischetti et al. (2016) and Guastaroba and Speranza (2014) have been used. In this method, the facility location cost is calculated using $F_i = U(0,90) + \sqrt{S_i}\ \ U(100,110)$. Uniformly distributed data randomly generated as integers in a unit square coordinate U[a, b] were considered for the experiments. The letter "a" refers to the lower cost limit and "b" is termed the upper cost limit.

A total of 45 problems instances across 9 different problem sizes were considered for the main experimentation. Problem size number (1) to (4) and (5) to (9) have been termed as small and medium-sized problems respectively. A summary of the problem sizes considered and the parameters used for data generation are given in the Tables 5-5 and 5-6 respectively.

Table 5-5 Parameter distribution used for computations

Problem Size No.	**Problem Size** $m \times n \times a$	**No of instances**
1	5×5×2	5
2	5×8×2	5
3	7×10×2	5
4	8×8×2	5
5	9×10×2	5
6	10×10×2	5
7	10×20×2	5
8	15×30×2	5
9	30×30×2	5

Table 5-6 Parameter distribution used for experimentation

Parameter Distribution	
S_i	U(200, 400)
D_j	U(50, 100)
T_r	U(800, 1800)
c_{ijr}	U(20, 150)
H_{ijr}	U(200, 600)
$F_i =$ $\mathrm{U}(0, 90) + \sqrt{S_i}\, U(100, 110)$	
$M_{ijr} = \min(S_i, D_j, T_r)$	

5.4.3 Solution methods

The IBM CPLEX has been utilized as a solution method in this chapter. IBM CPLEX utilizes the conventional branch and cut algorithm and also implements a dynamic search algorithm. According to the IBM reference manual (Studio, 2016), the dynamic search algorithm basically uses the Branch and Cut algorithm as the optimization technique. It also indicates that the dynamic search algorithm consists of LP relaxation, branching, cuts and heuristics. At the default settings, CPLEX decides whether to provide a solution using the conventional branch and cut or the dynamic search algorithm based on the model formulation (Studio, 2016). The default settings of IBM CPLEX 12.8 has been used as a solution method to the original problem. This can imply a possible conventional Branch and Cut or dynamic search could be used by CPLEX to find a solution.

The HGA was coded using Matlab 7.4.0. Based on the results from the preliminary experimentation, the HGA was computed with population size (30), number of generations (50), crossover rate (0.3) and mutation rate (0.1) for the small problem sizes. The medium problem sizes, the HGA was computed with population size (50), number of generations (100), crossover rate (0.5) and mutation rate (0.1). The Solution methods were implemented on a Windows 8.1 Laptop with 6GB RAM and a processor speed of 2.5 GHz.

5.5 Experimentation and Discussion of Results

The performance of each solution method was determined under the following test categories.

a) Preliminary experimentation to determine the HGA parameters for the main experimentation. This was computed in section 5.4.1 above.
b) Mean of each problem size. This was calculated based on the effectiveness and efficiency of each solution method. The mean value was expressed with the notations: $CPLEX_{\,mean}$ and $HGA_{\,mean}$.

$$CPLEX_{\,mean} = \frac{\sum_{i=1}^{5} CPLEX\ \ SM_i}{5}$$

$$or$$

$$HGA_{\,mean} = \frac{\sum_{i=1}^{5} HGA\ \ SM_i}{5}$$

$$i = index\ of instance\ number\ of\ each\ problem\ size$$

$$CPLEX\,SM\ \ => \ \ CPLEX\,minCost\ \ or\ \ CPLEX\,time$$
$$HGA\,SM\ \ => \ \ HGA\,minCost\ \ or\ \ HGA\,time$$

The mean values give an indication of the problem size effectiveness or efficiency

c) Instance and mean gap computation. This was also calculated based on the effectiveness and efficiency of each solution method. The Instance and mean gap computation were computed as percentages respectively. These were expressed with the notations % gap $_{\text{i}}$ and % gap $_{\text{mean}}$ respectively.

$$\%\ \text{gap}_{\ \text{i}} = \left(\frac{HGA\ SM_{\ i} - CPLEX\ SM_{\ i}}{CPLEX\ SM_{\ i}}\right) \times 100$$

$$\%\ \text{gap}_{\ \text{mean}} = \left(\frac{HGA\ SM_{\ mean} - CPLEX\ SM_{\ mean}}{CPLEX\ SM_{\ mean}}\right) \times 100$$

$i =$ Instance number index of each problem size

Other notations are previously indicated in (b) above.

The term % gap $_{\text{i}}$ or % gap $_{\text{mean}}$ values obtained can either be zero, positive number or negative number. A zero value indicates equivalent performance from both methods. A positive value indicates that CPLEX obtained better results. A negative value indicates that the HGA obtained better results. Based on the effectiveness measure of performance, CPLEX should expectedly obtain better results than the HGA. This is because of the conventional Branch and Cut implemented by CPLEX. The Branch and Cut has been noted to be an exact algorithm that can generate optimal solutions like Branch and Bound (Wolsey et al., 1998).

CPLEX uses the branch and cut algorithm. Therefore, it can be expected that the solutions provided by CPLEX should be at least as good as that provided by the HGA algorithm. However, the use of some other search heuristics to speed up the convergence of CPLEX in its dynamic search function can lead to some less superior results than the HGA. This is possible because of some nodes that could have been fathomed away, especially in instances where there are many nodes at a given search level with close possible final solution in their exploration, whose exploration may be considered not to hold so much promise, but can significantly increase the time of convergence of the solution.

The results obtained for each of the problem instances based on the defined measures of effectiveness and efficiency are presented in Table 5-7 below. The values obtained for the individual cases and the categorical averages are presented next. Result presentation begins with the instance observations in Table 5-7 below, followed by the problem size averages in Table 5-8 below.

Table 5-7 Problem instance effectiveness, efficiency and % gap $_i$

Problem Size	Instance no	*CPLEX minCost*	*HGA minCost*	*CPLEX time*	*HGA time*	% gap $_i$ (Min Cost)	% gap $_i$ (Time)
5X5X2	1	18417.92	18417.92	1.14	0.13	0.0%	-88%
	2	18801.22	18534.22	1.12	0.13	-1.4%	-88%
	3	15369.09	21830.55	1.12	0.13	42.0%	-89%
	4	18828.75	23005.75	1.13	0.13	22.2%	-89%
	5	17090.47	16662.33	1.13	0.12	-2.5%	-89%
5X8X2	1	31320.91	31320.91	3.06	0.17	0.0%	-94%
	2	26301.38	30143.11	3.00	0.17	14.6%	-94%
	3	24209.66	31825.38	3.02	0.16	31.5%	-95%
	4	25539.24	25539.24	3.05	0.17	0.0%	-94%
	5	25864.92	25798.92	3.01	0.17	-0.3%	-94%
7X10X2	1	34676.97	34806.92	4.71	0.20	0.4%	-96%
	2	34760.30	33612.17	4.57	0.20	-3.3%	-96%
	3	32667.86	34587.04	4.53	0.23	5.9%	-95%
	4	36182.85	35841.88	4.72	0.20	-0.9%	-96%
	5	31334.72	31214.63	4.54	0.20	-0.4%	-96%
8X8X2	1	26952.66	26902.27	2.96	0.16	-0.2%	-95%
	2	26434.64	25816.64	2.97	0.18	-2.3%	-94%
	3	23830.67	23830.67	2.94	0.18	0.0%	-94%
	4	27032.24	27225.24	2.93	0.16	0.7%	-95%
	5	25864.92	25316.20	2.96	0.17	-2.1%	-94%
9X12X2	1	39844.21	37944.21	6.52	1.09	-4.8%	-83%
	2	35472.49	36024.36	6.61	1.06	1.6%	-84%
	3	39697.67	43278.10	6.88	1.11	9.0%	-84%
	4	41005.10	42701.06	6.70	1.04	4.1%	-84%
	5	40125.84	40749.84	6.53	1.03	1.6%	-84%
10X10X2	1	30471.92	30411.76	4.67	0.89	-0.2%	-81%
	2	28252.39	28252.39	4.82	0.99	0.0%	-79%
	3	35462.65	34160.77	4.63	0.88	-3.7%	-81%
	4	27648.69	27648.69	4.67	0.90	0.0%	-81%
	5	29211.92	29496.92	4.72	0.95	1.0%	-80%
10X20x2	1	64045.09	65361.83	18.00	1.77	2.1%	-90%
	2	57654.61	58505.61	17.40	1.63	1.5%	-91%
	3	62724.33	66508.88	18.19	1.56	6.0%	-91%
	4	63389.91	67396.36	18.12	1.79	6.3%	-90%
	5	68527.51	74904.08	17.90	1.81	9.3%	-90%
15X30X2	1	82136.32	93036.10	54.26	2.62	13.3%	-95%
	2	85684.45	87040.73	56.17	2.56	1.6%	-95%
	3	85515.94	89097.15	55.61	2.59	4.2%	-95%
	4	81937.51	81588.46	55.20	2.71	-0.4%	-95%
	5	83554.61	89640.24	55.46	2.66	7.3%	-95%
30X30X2	1	76649.71	75140.45	59.25	2.83	-2.0%	-95%

	2	81897.82	82452.20	54.88	2.94	0.7%	-95%
	3	81475.84	81725.59	64.07	2.92	0.3%	-95%
	4	78941.52	79702.55	57.16	2.74	1.0%	-95%
	5	88615.06	89680.65	57.33	2.95	1.2%	-95%

The limits of a significant difference in effectiveness performance are defined as about 5% based on the popular alpha level applied on many empirical tests. This implies that any difference in values between the results posted by CPLEX and HGA that is in this range is probably not significant enough. If this approach is followed, it can be seen in Table 5-7 above that the instances tend to show less significant differences with an increase in problem sizes, while more significant differences are observed with the smaller problem sizes. In addition, the percentage difference in solution time($\%\,\text{gap}_i$ (Time)) across the problem instances indicates that the HGA solution seems much faster than CPLEX.

Table 5-8 Mean effectiveness, mean efficiency and $\%\ \mathbf{gap}_{\mathbf{mean}}$

Problem Size	$CPLEX_{mean}$ (Min Cost)	HGA_{mean} (Min Cost)	$CPLEX_{mean}$ (Min Cost)	HGA_{mean} (Time)	$\%\ \mathbf{gap}_{\mathbf{mean}}$ (Min Cost)	$\%\ \mathbf{gap}_{\mathbf{mean}}$ (Time)
5X5X2	17701.49	19690.16	1.13	0.13	11.2%	-89%
5X8X2	26647.22	28925.51	3.03	0.17	8.5%	-95%
7X10X2	33924.54	34012.53	4.61	0.21	0.3%	-96%
8X8X2	26023.03	25818.20	2.96	0.17	-0.8%	-94%
9X12X2	39229.06	40139.52	6.65	1.07	2.3%	-84%
10X10X2	30209.51	29994.11	4.70	0.92	-0.7%	-80%
10X20x2	63268.29	66535.35	17.92	1.71	5.2%	-90%
15X30X2	83765.77	88080.54	55.34	2.63	5.2%	-95%
30X30X2	81515.99	81740.29	58.54	2.88	0.3%	-95%

The group results as shown in Table 5-8 above also indicate that there seems not to be any significant difference in effectiveness as the problem size increases while efficiency seems to favour HGA. Figure 5-10 shows a plot of the solution time of the HGA and CPLEX as the problem size increases. The increase in solution time trend indicates the possibility of the HGA obtaining solutions faster than CPLEX when interpreted as probably being equivalent as stated earlier.

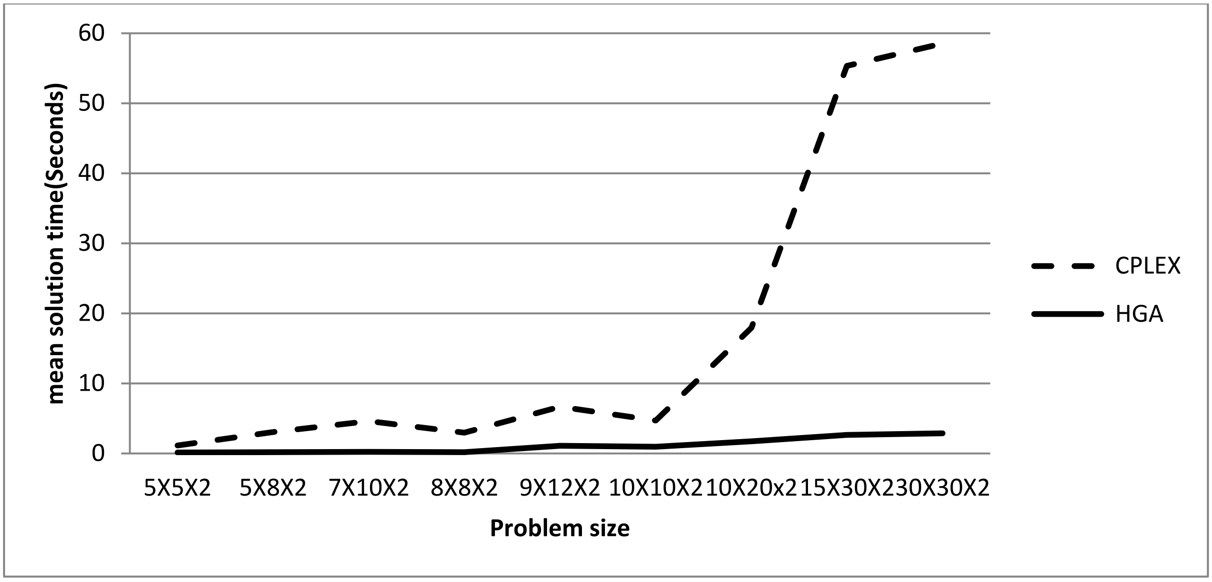

Figure 5-10 Computation time between CPLEX and HGA per problem size

In Figure 5-11 below a representation of the number of times, each solution method obtained comparable results for each problem size is shown. By comparable results, it is implied that the results obtained by HGA are within a neighbourhood of between (0% and -5%) of CPLEX values. In Figure 5-11, a representation of the results of CPLEX that were significantly better than HGA is presented. By being significantly better we imply the computations $\% \, gap_i$ (Min Cost) greater than 5%. These analyses were done for the minimum cost computation (effectiveness).

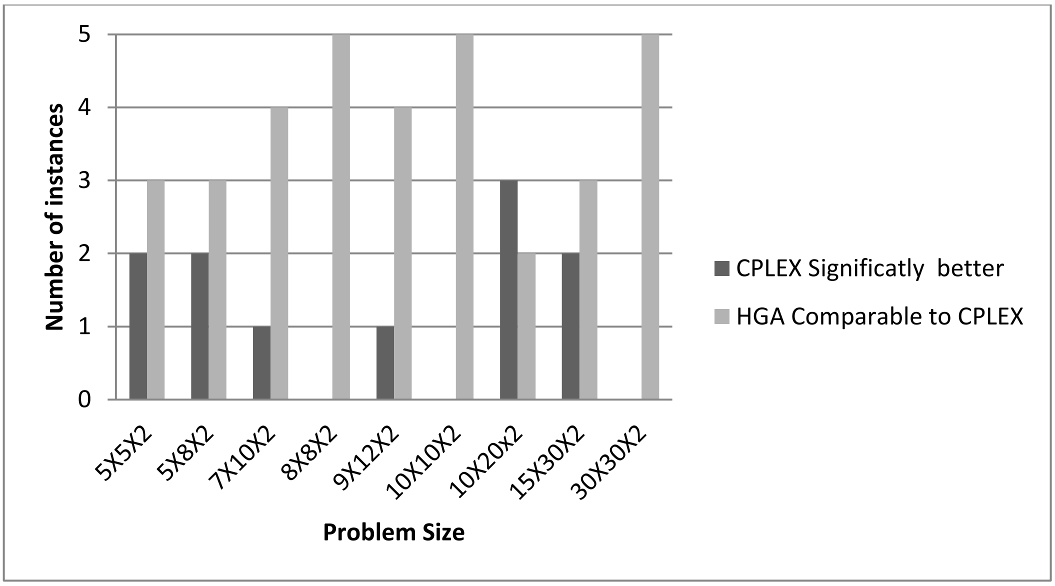

Figure 5-11 Number of Instances CPLEX obtained comparable and significantly better results than HGA

A summary of the results presented showed that CPLEX seems able to obtain better results than the HGA most of the times. Nonetheless, CPLEX could become computationally intensive as problem size increases and may return good values in an exponential time when larger sized problems are considered. Therefore, the HGA which may not guarantee the best solutions most of the times but converges faster with good performances for many problem instances can be of significant value for solving the FCSLTP.

5.6 Conclusion and Future Direction

An optimization problem that integrates facility location and the fixed charge solid transportation problem was considered in this chapter. This problem was termed Fixed Charge Solid Location and Transportation problem (FCSLTP). In order to solve this problem, solutions from CPLEX commercial solver and the hybrid genetic algorithm (HGA) were considered. The HGA utilizes the Genetic Algorithm framework to generate a population of feasible facility locations, while a greedy heuristic which uses cost relaxations implements the load allocation. After the allocations are done, an improvement heuristic is used to further search the solution space for better results. Some measures of performance such as mean, percentage instance and mean gap were used to assess the HGA and the CPLEX solutions. The solution time and mean solution time were also used to assess both solution methods. The HGA demonstrated competitive performance in obtaining solutions within the neighbourhood of CPLEX values based on the effectiveness measure. In addition, the solution time of the HGA seems much faster than CPLEX through all the problem sizes considered, and this even more so as the problem size increases. However, overall effectiveness results indicate that CPLEX can obtain results comparable to and sometimes much significantly better than HGA. This, however, could be computationally intensive for CPLEX as problem size increases.

Possible extensions to the HGA include using a modified stepping stone that can search the non-basic positions of the load allocations. Furthermore, the GA or other metaheuristics could also be used to perturb the load allocations in search of better results. The reduced computation time of the HGA makes it suitable as a hybrid heuristic to other solution methods to obtain better results.

Chapter 6

General Conclusion

6.1 Research Summary in Perspective

The development of models and solutions for problems which integrate facility location into some variants of the basic transportation problem was the primary objective of this thesis. The integrated model can be segmented into the transportation and facility location sections. The transportation sections comprised variants of the basic transportation problem such as the step-fixed charge transportation problem and the fixed charge solid transportation problem. The facility location section considered the capacitated facility location model. Two categories of integrated models were essentially developed in this thesis.

- The first category of the integrated model was aimed at solving the step-fixed charge location and transportation problem. Two solutions were developed to tackle this problem. The first solution proposed was a hands-on and low-cost heuristic that primarily improves on the initial solution obtained using the lower cost envelope (linearization) principle for approximating piecewise linear models. The second solution heuristic was also based on the same linearization principle with an emphasis on deriving a reduced number of transportation problems to obtain the final solution. The second solution, however, presented comparative studies with regard to effectiveness (optimal solution) to solutions obtainable from standard solvers such as CPLEX.

- In the second category of the integrated model developed, the fixed charge solid location and transportation problem was formulated. This problem was solved using the CPLEX optimization suite. Two other alternative solutions usable by personnel possibly not having access to standard solvers such as CPLEX due to reasons such as cost were presented. These solutions are the Lagrange Relaxation Heuristic (LRH) and Hybrid Genetic Algorithm (HGA).

  The Lagrange relaxation heuristic was presented with a numerical example to demonstrate the workings of the heuristic. The different iteration steps and the effect of using different initial Lagrange multipliers in achieving lower and upper bounds were discussed. The numerical example showed that depending on the upper-bound heuristic integrated into the sub-gradient algorithm, the possibility of generating solutions comparable to CPLEX using a Lagrange relaxation heuristic was achievable. Comparative studies were performed using an extended version of the earlier Lagrange heuristic and CPLEX. The CPLEX outperformed the Lagrange relaxation heuristic in the upper bound and lower bound generation through the problem sizes considered. The FCSLTP and an equivalent FCSLTP termed FCSTP-EQ (defined in Chapter 4) was also compared based on total cost or optimality gap. The FCSLTP formulation consistently produced better results than FCSTP-EQ. Solving the FCSLTP using CPLEX showed a time complexity for large problem sizes. The FCSTP was experimentally shown to be a starting solution to efficiently arrive at good solutions to the FCSLTP for large problem sizes.

The Hybrid Genetic Algorithm (HGA) solution technique integrates heuristics into the genetic algorithm framework to search the solution space. Feasible locations were determined using genetic operations. Allocations and consolidations were done using a greedy heuristic and an improvement heuristic. Comparative studies were also done to compare the performance of the hybrid solution to solutions obtainable by CPLEX, a commercial optimization solver. The HGA demonstrated competitive performance in obtaining solutions within the neighbourhood of CPLEX values based on the effectiveness measure. In addition, the solution time of the HGA seemed much faster than CPLEX through all the problem sizes considered, and this even more so as the problem size increased.

6.2 Major Contribution

The main contributions of the models and solutions developed in the thesis are both to advance knowledge in the facility location literature and for direct application by operational personnel involved logistic planning. These contributions are sectioned based on the category of the models developed and the respective solutions proposed.

Based on the first category of integrated model:

- A hands-on and low-cost improvement technique usable by logistic operational planners to assist in the planning of the location of facilities in environments where more than one fixed costs are present along the transportation routes is proposed.

- Secondly, a linearization approach and an improvement technique that reduces the feasible solution space and obtained competitive results in terms of effectiveness (objective value) with solutions obtainable by standard solvers such as CPLEX is proposed.

Based on the second category of integrated model:

- The Fixed Charge Solid Location and Transportation problem is formulated. The classical Lagrange relaxation heuristic framework is newly applied as an alternative solution to the problem. With improved upper bound generating heuristics, the possibility of having the Lagrange providing a competitive solution with CPLEX is demonstrated.

- A Hybrid Genetic Algorithm (HGA) is newly applied to solve the facility location and fixed charge solid transportation problem. Results obtained suggests that the performance of HGA is competitive with that obtainable from a commercial solver such as CPLEX.

6.3 Future directions and Perspectives

The development of integrated combinatorial problems and the application of both classical and modern heuristics to solve such problems have been considered in this thesis. The models developed can be extended to a hierarchical type consisting of two or more stages (echelons). In addition to the optimization decisions considered in this research work, inventory and routing decisions are possible areas that could be integrated into the developed models.

In order to solve the step-fixed charge facility location and transportation problem, new initial solutions which neither uses the linearization nor improvement techniques discussed in this research work could be investigated. Furthermore, computation studies with different ranges of data can be conducted to further gain insight into the performance of the heuristics that competed effectively with CPLEX. In addition, metaheuristics which have the power of lifting solutions out of local optima solutions usually associated with classical heuristics could be applied to further search the solution space.

On the fixed charge solid location and transportation problem, extending the Lagrange relaxation heuristics into a hybrid solution technique could be studied. Metaheuristics could also be integrated to the Lagrange relaxation to search for better upper bounds. Furthermore, the hybrid genetic algorithm developed can also be integrated into another evolutionary algorithm that can further ensure diversification in the solution search space.

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