A new double sampling control chart for monitoring an abrupt change in the process location

J.C. Malela-Majika[a1], C.M. Motsepa[a] and M.A. Graham[b]

[a] *Department of Statistics, College of Science, Engineering and Technology, University of South Africa, PO Box 392 UNISA 0003, Pretoria, South Africa*

[b] *Department of Science, Mathematics & Technology Education, University of Pretoria, South Africa*

Abstract

This paper develops a new double sampling (DS) monitoring scheme, namely, the side-sensitive DS chart, to monitor the process mean. The operational procedure is presented first followed by the exact form of the probability of the in-control process under the normality assumption. Finally, the performance of the new scheme is investigated by minimizing the out-of-control average run-length and extra quadratic loss function. It was observed that the proposed chart presents a better overall performance than the existing DS chart. An illustrative example is given to facilitate the design and implementation of the new chart.

Keywords: DS control chart; statistical process monitoring; side-sensitive DS scheme; overall performance measures; run-length distribution

1. Introduction

A double sampling (DS) chart is one of the most powerful tools used in statistical process monitoring (SPM) to detect any abnormality in various types of processes (such as business, health and manufacturing processes) as quickly as possible. The DS control chart was first presented by Croasdale (1974) as an attempt to improve the standard Shewhart chart in

[*] *Correspondence to: Jean-Claude Malela-Majika, Department of statistics, College of Science, Engineering and Technology, University of South Africa, PO Box 392 UNISA 0003, Pretoria, South Africa*
Email address: malelm@unisa.ac.za

detecting small and moderate shifts in the process mean. Croasdale's DS chart is a two-stage chart based on two unconnected samples. Daudin et al. (1990) and Daudin (1992) modified Croasdale's DS chart by connecting the first and the second sample at the second stage. Since then, many authors have contributed to the design of the DS chart. Irianto and Shinozaki (1998) presented an optimization model to minimize the out-of-control (OOC) performance of the DS chart. Daudin (1992) and Costa (1994) compared the DS control chart with some well-known control charts. They found that the DS control chart outperformed the Shewhart, exponentially weighted moving average (EWMA), cumulative sum (CUSUM), variable sampling interval (VSI) and variable sample size (VSS) charts in many cases. He et al. (2002) designed double and triple sampling control charts using a genetic algorithm. Carot et al. (2002) combined the DS and VSI (DSVSI) charts and showed that the combined scheme is more sensitive to small and moderate shifts. Later on, the economic design of the DSVSI charts was presented by Lee et al. (2012). Khoo et al. (2011) developed a synthetic DS chart, which was found to be more sensitive than the standard synthetic charts and DS charts for small and moderate shifts. More recently, Costa (2017) investigated the performance of the DS range chart and found out that the DS range chart performs better than the *R* and the DS charts. You (2017) investigated the performance of the synthetic DS chart in terms of the expected run-length and average run-length values for an unknown shift size. Haq and Khoo (2018) constructed a DS chart based on correlated auxiliary variable for monitoring the process mean.

To further increase the sensitivity of control charts, the SPM literature suggests the use of improved schemes such as the synthetic and runs-rules schemes. These schemes could be classified into two main categories, which are the non-side-sensitive (NSS) and side-sensitive (SS) schemes, respectively. The NSS *w*-*of*-(w+v) scheme (with integers and) gives an OOC signal when *w* nonconforming points out of w+v successive points plot outside of the control limits, no matter whether some (or all) of the *w* nonconforming points plot above the upper control limit (*UCL*) and others (or all) plot below the lower control limit (*LCL*), which are separated by, at most, *v* conforming points that plot between the *LCL* and the *UCL*. Alternatively, the SS *w*-*of*-(*w*+*v*) scheme gives a signal when *w* nonconforming points out of *w*+*v* successive points plot on or above (below) the *UCL* (*LCL*), which are separated by, at most, *v* points that plot below (above) the *UCL* (*LCL*), respectively. Klein (2000) and Shongwe and Graham (2016) showed that the SS scheme not only improves the sensitivity of the basic (i.e. 1-*of*-1) control chart in detecting small shifts, but also outperforms the corresponding NSS scheme.

In this paper, we propose the side-sensitive double sampling (SSDS) chart with known process parameters in order to improve the existing DS chart (which is a NSS scheme) in detecting small (and moderate (shifts without affecting its sensitivity in detecting large shifts (.

The remainder of this paper is organized as follows: In Section 2, we present the operation of the proposed chart and the exact expressions of the probability of the in-control (IC) process and average run-length (*ARL*). Section 3 presents the measures of the overall performance. In Section 4, we evaluate the IC and OOC performances of the proposed chart and compare their overall performances with some well-known charts. In Section 5, we give an illustrative example using real-life data to demonstrate the implementation and design of the proposed chart. Finally, some concluding remarks and future recommendations are given in Sections 6 and 7, respectively.

2. Operation and design consideration

2.1. Operation of the SSDS control chart

Assume that the observations of the quality characteristic *X* are independently and identically distributed (iid) from a *N*(,) distribution, where and represent the IC mean and the IC variance, respectively. Let and (with) be the warning and control limits of the first sample at Stage 1, respectively; and (with) be the control limit of the combined samples at Stage 2. Therefore, the SSDS control chart is divided into eight intervals, i.e. A =, ,), B = ,], C = [,], D = [, , E = , , F =], G = , and H = ,.

<Insert Figure 1>

By referring to Figure 1, the operational procedure of the SSDS chart is as follows:

1. Take a sample of size and calculate the sample mean at the sampling time of the first sample.
2. If C, the process is considered as IC.
3. If A, the process is said to be OOC and then the necessary corrective action must be taken to find and remove the assignable causes.
4. If (or), take a second sample of size () and calculate the sample mean at the sampling time of the second sample.
5. At the sampling time, calculate the combined-sample mean .
6. The process is declared IC at stage 2:

   (a) If B and , or

   (b) If D and .

Note that the process is declared OOC at stage 2 if B and or D and .

Remarks:

- If the charting statistic falls in region B at Stage 1, then at Stage 2, we consider the charting regions E = , and F =] only (i.e., the upper scheme with control limit).

- However, if the charting statistic falls in region D at Stage 1, then at Stage 2, we consider charting regions G = , and H = , only (i.e., lower scheme with control limit).

Figure 2 provides a graphical summary of the operation of the proposed double sampling scheme.

<Insert Figure 2>

2.2. Design of the SSDS chart with known process parameters

At Stage 1, the DS-type charts give a signal if the charting statistic plots in region A. Unlike the conventional DS chart, the SSDS chart gives a signal at Stage 2 if both charting statistics and plot on one side of the chart in regions B and E (or D and G), respectively. Therefore, four types of events can be defined:

(i) the Stage 1 IC event when "",
(ii) the Stage 2 IC event when either " and " or " and ",
(iii) the Stage 1 OOC event when "" and
(iv) the Stage 2 OOC event when either " and " or " and ".

Assume that and are known (Case K) and let be the probability that the process is regarded as IC at Stage where = 1, 2. Then,   is the probability of the IC process, where:

(1)

and

(2)

Hence,

(3)

where $\Phi(.)$ and $\phi(.)$ are the cumulative distribution function (cdf) and probability density function (pdf) of the standard normal random variable, respectively. In this paper, δ = represents the magnitude of the standardised mean shift with the OOC mean , , , , and , .

Given that the SSDS chart is a Shewhart-type control chart, its run-length (*RL*) distribution, denoted by , is defined by the geometric distribution. Therefore, the cdf of the *RL* distribution is obtained as

(4)

where .

Then, the percentile of the *RL* distribution, is given by

and (5)

The , standard deviation of the run-length () and the average sample size () at each sampling time are given by

(6)

(7)

and

, (8)

respectively, where

Then, the average number of observations to signal (*ANOS*) is given by

(9)

There are five parameters () that need to be specified in order to design the SSDS chart. The efficiency of the proposed SSDS chart depends on the combinations (). We have two steps in the optimal design of the proposed chart. Firstly, the nominal IC *ARL* () is set to high recommended values such as 370.4 or 500. Secondly, the () combination, which provides an attained IC (), is set much closer to the and the smallest OOC *ARL* () for a given mean shift is considered as an optimal combination. Therefore, the optimization model is presented as follows

(10)

subject to

(11)

(12)

where *n* represents the expected IC *ASS* (denoted as). Moreover, and in order to ensure that the probability of getting a false alarm () is not greater than (Type I error) and the probability of not detecting a shift in the process mean is not greater than (Type II error).

3. Measures of the overall performance

Although the value is the most used metric measurement in statistical process control and monitoring (SPCM), many authors have advocated against the use of the *ARL* as a performance measure (see, for example, Graham et al. (2014) for a recent discussion on this issue). In addition to the arguments made by Graham et al. (2014), a number of authors have shown that if a control chart is designed based on one specific size of a mean shift, it would perform poorly when the actual size of the shift is significantly different from the assumed size (Reynolds and Lou, 2010; Ryu et al., 2010). This makes the *ARL* deficient in assessing the overall performance of a control chart. To solve this problem, a number of researchers have suggested the use of quality loss functions (*QLF*s) instead of the to assess the performance of a monitoring scheme (see, for example, Machado and Costa, 2014; Chiu, 2015). A *QLF* describes the relationship between the shift size and the quality impact. Therefore, when the aim of a study is to measure the overall performance of a control chart over a range of shifts (i.e.), the objective function must be defined in terms of the extra quadratic loss () function given as (see Taguchi, 1986; Kackar, 1989; Wu et al., 2008; Machado and Costa, 2014)

(13)

with

where is the upper boundary of the range of shifts under consideration and (with) represents the weight function associated with . It is generally assumed that all location shifts (mean shifts) occur with equal probability. Therefore, a uniform distribution of δ is implied.

The expression of the *EQL* given in Equation (13) can also be written as follows

(14)

Note that after using Equations (10) to (12), the optimal parameters are selected using Equation (14), which mean that they yield a minimum *EQL* value.

In order to measure the relative effectiveness of two different charts, Wu et al. (2008) suggest the use of the performance comparison index (*PCI*), which is given as

(15)

where is the of the benchmark control chart. In this paper, the SSDS chart was used as the benchmark control chart. In addition to the and , many authors suggest the use of the average ratio of the (denoted) to measure the overall performance of a benchmark chart against other competitors; see Wu et al. (2008). The is given by

(16)

Note that, if the and/or is larger than one, the competing chart will produce larger values over the range of shifts under consideration. Therefore, the benchmark chart outperforms the competing chart for that specific range; otherwise, the competing chart is more sensitive than the benchmark chart.

Although it is not the scope of this paper, it is worth mentioning that the effectiveness of traditional performance measures should be revisited. Even as far back as 1986, Woodall (1986) had started mentioning flaws in the designs of control charts and, even today, there is still room for improvement. We do not wish to degrade the importance of traditional control charting performance metrics; however, the key common characteristic of these traditional methods is to design the control chart for a pre-specified magnitude of shift. Many researchers have now argued that if a control chart is designed for some pre-specified magnitude of shift, it will perform poorly when the actual shift differs significantly from this pre-specified value (see, for example, Ryu et al., 2010; Machado and Costa, 2014). The recommendation is that overall performance metrics, such as *QLF*s, be used. The exploration into the fact that making use of traditional measures can be misrepresentative is currently under investigation and will be reported on in a future paper.

4. Performance study of the proposed chart

4.1 Optimal design of the SSDS control chart

In this section, we investigate the optimal design of the SSDS control chart in Case K by setting the to some high, acceptable nominal values and minimizing the and / or values with (,) = (0, 2.5) and a step shift of size 0.1. In this study, we set the to 370.4 and 500, respectively, with an (i.e. *n*) {2, 5, 7, 11}. We used Equations (4) to (7) in Mathcad 14 to compute the IC and OOC characteristics of the run-length distribution, respectively. Moreover, the *ASS*, *ANOS* and values were computed using Equations (8), (9) and (14), respectively. Note that there are three main steps in the search of the optimal design parameters:

(i) for some specific sample sizes (i.e., and) and shift (0), find all possible combinations of the design parameters that yield an attained value of 370.4 for a prespecified value of *n* (i.e.,),

(ii) For each combination, calculate the *EQL* value,

(iii) Select the combination that yields the minimum *EQL* value to be the optimal design parameters.

Table 1 presents the optimal design parameters of the proposed chart when 2, 5, 7, 11}. For instance, when (2, 8) with = 5, we compute the optimal design parameters () in order to achieve a specified of 370.4. In this example, we find that () = (2, 8, 0.8856, 3.3526, 3.0085) so that the attained = 370.4 with a minimum value of 33.99. Under the same conditions, and for another choice of design parameters, say () = (2, 8, 0.8815, 3.1026, 3.2731), the proposed chart yields an value of 370.4 with an value of 39.31. In this situation, the design parameters that give a small (i.e. minimum) value are considered to be the winner. From Table 1 it is observed that the sensitivity of the proposed chart is proportional to the first (Stage 1) and second (Stage 2) sample sizes. This means that the larger the sample, the more sensitive the chart is.

<Insert Table 1>

4.2 Performance of the SSDS control chart

Once the optimal parameters are obtained, the OOC performance of the proposed chart can then be investigated. Tables 2 and 3 present the performance of the proposed control chart for different optimal design parameters with = 370.4 when {2, 5} and {7, 11}, respectively. In terms of the *ARL* values, it was observed that the performance of the proposed chart depended on the value, and the first and second sample sizes (i.e., and). When the couple () is kept constant,

(i) as the (i.e.,) value increases, the performance of the proposed control chart improves (see Figure 3),

(ii) in terms of the cost of inspection, the SSDS chart is cost effective for small values (see Figure 4),

(iii) for , when increases, the sensitivity of the proposed chart decreases regardless of the size of the process mean shift, and

(iv) for , when increases, the sensitivity of the proposed chart increases for small and moderate shifts. For large shifts in the process mean, the sensitivity remains the same.

(v) for and , when increases, the sensitivity of the proposed chart increases regardless of the size of the process mean shift (see Figures 3 (a)-(d)). Note that when , the OOC () values remain closer to the value, which makes the proposed chart cost effective (see Figures 4 (a)-(d)).

Therefore, to obtain an optimal and cost effective design of the proposed chart, we suggest keeping the value and as small as possible at an acceptable cut-off point (e.g. = 3 or 4) and increase in order to get an efficient and economic SSDS chart. In terms of the percentile of the run-length (*PRL*) values, for a of 370.4 with an value of 2 and () = (2, 5, 2.9001, 3.0073, 2.9025), the results in Table 2 reveal that when the process is IC (i.e. = 0), there is a 5% chance that the proposed SSDS scheme signals for the first time on the 19th subgroup. However, there is 95% chance that the proposed SSDS chart signals for the first time on the 1110th subgroup. When there is an abrupt mean shift of size 0.2 (i.e. = 0.2), there is 5% chance that the proposed control chart will signal on the 13th subgroup and 95% chance that it will signal on the 768th subgroup. For large shifts, there is 50% chance that the proposed control chart signals on the second subgroup and 75% chance that it will signal on the 5th subgroup. When we increase the value, say 5 for () = (2, 8), () = (0.8856, 3.3526, 3.0085) so that the chart yields a of 370.4. In this case, when there is an abrupt mean shift of size 0.2, there is 5% chance that the proposed control scheme will signal on the 7th subgroup and 95% chance that it will signal on the 387th subgroup. For large sample sizes, there is a very high chance that the proposed scheme signals on the first subgroup. This shows that the sensitivity of the proposed scheme increases as the sample size increases. Consequently, the cost increases as well. It can also be observed that for small values, for instance when the 2, the IC characteristics of run-length are the same for different optimal combinations regardless of the sample sizes. However, for moderate and large values, the IC characteristics of the run-length are not equal for different optimal combinations regardless of the sample sizes. For more details, see Tables 2 and 3.

The results in Tables 2 and 3 also show that the distribution of the is symmetric about (where is the mean shift that produces the maximum value), skewed or relatively constant depending on the triplet (, ,) sizes. Therefore, the of the proposed chart may be considered as an increasing and decreasing function of δ in the ranges [0,] and [,], respectively. Figures 4 (a)-(d) reveal that, for small and moderate shifts, when is kept constant, the design of the proposed chart is cost effective for small . For large shifts, the design of the proposed chart is cost effective for large . When , the proposed control chart is cost effective regardless of the size of the mean shift.

<Insert Tables 2 and 3>

<Insert Figures 3 and 4>

Figures 5 (a)-(d) confirm that for small values of *n,* the proposed chart is cost effective. For instance, in terms of the *ANOS* values, for a mean shift of 0.2, when (and $n = 2$, the proposed control chart signals for the first time on either the 524th or the 525th observation. When (and $n = 5$, the proposed control chart signals for the first time on the either the 634th or the 635th observation. This confirms that the proposed control chart is cost effective for small values of *n*.

<Insert Figure 5>

4.3 Performance comparison

In this section, the proposed control chart is compared to some well-known control charts. In total, five control charts were used, namely, the traditional , -EWMA ($\lambda = 0.1$), VSS , DS and SSDS charts in terms of the , and values. For a fair comparison, these performance measures are computed when *n* {4, 5, 7}, {2, 4}, = 12 and = 2.5 with = 370.4 for each chart. Each competing chart was optimized by minimizing the values resulting in minimum *EQL* values. In terms of the *EQL* values, it can be seen in Table 4 that the proposed SSDS control chart performs better than all competing charts followed by the DS chart (small values of the for the best chart). This is also indicated by the and values, which are equal to one for the best control chart and greater than one for other charts.

<Insert Table 4>

5. Illustrative example

To illustrate the implementation and application of the proposed control chart, we consider the well-known data set from Montgomery (2013) on the inside diameters of piston rings manufactured by a forging process. This data set contains 25 retrospective or Phase I samples, each of size five, that were collected when the process was thought to be IC. A goodness of fit test for normality reveals that the data are normally distributed. To illustrate the implementation of the proposed chart, we assume that the process parameters and are known and given by 74.001 and 0.008, respectively. This data set also contains 75 Phase II observations (i.e. 15 sub-groups of size 5). Therefore, we consider the SSDS chart with () = (5, 5) and an of 5. The optimal combination () when = 5 is found to be equal to (5, 5, 3.0, 3.0, 3.0) so that the = 370.4 with = 49.54. For a fair comparison with the DS chart, we also considered the DS chart with () = (5, 5) and an of 5. The optimal combination for this chart is given by (5, 5, 2.51, 3.221, 2.752) so that the the = 370.4 with = 52.46. The standardised statistics for the first sample and combined samples are and , respectively. These values are computed using and . A plot of the charting statistics and for both charts is shown in Figure 6. It can be seen that from the first to the 13th sampling time (i.e. = 1 to 13) of the first sample, the charting statistics of the SSDS chart plotted in region C (. Therefore, at this stage, the process was IC. At the 14th sampling time, the charting statistic () plotted below –*L* (i.e. below – 3). Thus, the SSDS chart gives a signal for the first time on the 14th sampling time. It was observed that for the DS chart, at the first and second sampling times of the first sample, the charting statistics () plotted in region B and their corresponding charting statistics and plotted in the IC region F. Therefore, the process is IC at this stage. On the twelfth sampling time of the first sample, the charting statistic (i.e.,) plots in region D and its corresponding charting statistic plots in region H, which means that the process is IC at this stage. From the fourteenth sampling time of the first sample onwards, all of the charting statistics plotted in the IC regions. Hence, the DS chart did not signal. Therefore, the SSDS chart outperformed the DS chart. These findings confirm the results found in Subsection 4.3. Note that since we needed three times a second sample at the sampling time 1, 2 and 12, the DS chart has 22 sampling time instead of 25.

<Insert Figure 6>

6. Conclusion

In this paper, we proposed a new parametric control chart based on the DS monitoring scheme. At the second stage, the proposed control chart gives a signal if the charting statistic of the first sample and the combined samples plot on one side of the control chart (side-sensitive). This new chart is named as the SSDS chart. The performance of the proposed chart with known process parameters was investigated in terms of the characteristics of the run-length distribution, the , the *ANOS* and the function. It was observed that the SSDS chart outperforms all competing charts considered in this paper, regardless of the size of the mean (location) shift. Moreover, in terms of the *ASS*, the proposed SSDS chart is cost effective when compared to the DS and VSS charts.

However, companies that do not face problems involving large sample sizes are advised to use large sample sizes regardless of the shift as this guarantees a better performance. If the sample size is a major concern for a company, we recommend the use of small sample sizes for small and moderate shifts and large sample sizes for large shifts. We also suggest that companies use 3, 4 or 5 and increase considerably in order to reach the desired efficiency.

7. Future recommendations

The effectiveness of traditional performance measures should be revisited and this is currently under investigation. The results in this article are based on the assumption of normally distributed observations and as part of future research work; we intend to extend this similar investigation for other monitoring schemes as well as for nonparametric schemes. Although synthetic charts have received a lot of criticism in the literature (especially by Knoth (2016)), we plan on doing some future research which will involve synthetic control charts. Specifically, for future research purposes, we will investigate the SSDS chart with estimated process parameters, and the side-sensitive synthetic DS chart with known and unknown process parameters. In addition, we plan on researching the effects of parameter estimation for both SS DS and side-sensitive synthetic DS charts. Knoth (2016) is advising against the use of synthetic charts, however, Knoth (2016) only considered one type of synthetic chart (i.e. the non-side-sensitive synthetic chart) and, it has been shown in Shongwe and Graham (2017), there are actually four types of Shewhart synthetic charts and that the other three types of synthetic charts outperform the chart considered by Knoth (2016). Another future recommendation would be to investigate Knoth (2016)'s conclusion, i.e. add the three other types of synthetic charts to Knoth (2016)'s research and then formulate the generalized synthetic chart of Scariano and Calzada (2009) in zero-state and steady-state by integrating the CUSUM or EWMA chart with a conforming run-length (CRL) sub-chart and check whether these charts perform worse than the basic EWMA and CUSUM charts; which, we believe, will not be the case (see, for instance, Haq et al., 2016; Haq, 2017). Thus, it is of our opinion that synthetic charts should not be discarded, as recommended by Knoth (2016) and this is our motivation to continue working on synthetic papers even after Knoth's (2016) warning not to do so.

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Table 1. Optimal design parameters and values when   {370.4, 500} and *n* == 2, 5, 7 and 11

Attained			370.4		500	
			(, *L*,)	*EQL*	(, *L*,)	*EQL*
2	2	2	(2.9101, 3.0568, 2.4050)	120.11	(2.9631, 3.1688, 2.5120)	138.68
		5	(2.9001, 3.0073,2.9025)	119.97	(2.9337, 3.1021, 2.9611)	134.89
		8	(2.9751, 3.0009, 2.9805)	129.36	(3.0428, 3.0917, 3.0832)	150.93
		11	(2.9908, 3.0002, 2.9925)	131.56	(3.0732, 3.0906, 3.0926)	155.96
5	2	8	(0.8856, 3.3526, 3.0085)	33.99	(0.8868, 3.6788, 3.0367)	35.52
		11	(1.0941, 3.2339, 3.0101)	32.45	(1.0943, 3.3422, 3.0889)	34.46
		14	(1.2377, 3.1693, 3.0126)	32.01	(1.2373, 3.2522, 3.1180)	34.11
	4	4	(1.1491, 3.4180, 3.0412)	35.64	(1.1423, 3.2402, 3.2926)	42.90
		8	(1.5291, 3.2440, 3.0302)	31.11	(1.5293, 3.2602, 3.2027)	34.48
		11	(1.6821, 3.1435, 3.0617)	30.68	(1.6823, 3.2162, 3.1985)	33.07
		14	(1.7906, 3.0989, 3.0773)	30.61	(1.7943, 3.2302, 3.0846)	31.55
	5	5	(2.9934, 3.0008, 2.9998)	49.54	(2.9655, 3.1102, 3.0016)	52.20
		8	(2.9947, 3.0004, 2.9979)	49.56	(2.9935, 3.0992, 3.0301)	52.87
		11	(2.9961, 3.0002, 2.9993)	49.59	(2.9878, 3.0998, 2.9081)	52.15
7	3	8	(0.6740, 3.5671, 3.0013)	29.84	(0.6738, 3.5308, 3.1203)	32.51
		11	(0.9076, 3.5336, 2.9559)	27.60	(0.9078, 3.6105, 3.0541)	29.22
	5	5	(0.8406, 3.4148, 3.0521)	30.99	(0.8406, 3.4405, 3.1610)	33.88
		8	(1.1496, 3.6358, 2.9798)	27.41	(1.1498, 3.8905, 3.0618)	28.96
		11	(1.3342, 3.5663, 2.9306)	26.01	(1.3338, 3.9704, 3.0019)	27.05
	7	8	(2.9952, 3.0005, 2.9993)	37.72	(3.0008, 3.1025, 3.0014)	39.96
		11	(2.9962, 3.0003, 2.9998)	37.73	(3.0002, 3.0994, 3.0001)	39.82
11	3	8	(0.0001, 3.8993, 3.0020)	29.03	(0.0002, 4.2604, 3.0643)	31.07
		11	(0.3486, 3.8211, 2.9827)	26.48	(0.3482, 4.3104, 3.0643)	27.86
	5	8	(0.3185, 3.8526, 3.0049)	26.90	(0.3182, 4.1103, 3.0904)	28.51
		11	(0.6045, 3.8868, 2.9861)	25.08	(0.6039, 4.1006, 3.0750)	26.26

Table 2. Exact *ARL, SDRL*, *ASS*, *ANOS, EQL*, percentile values and optimal design parameters of the proposed chart when the = 370.4, = 2 and 5 with = 2.5

Shift ()	**(*ARL, SDRL, ASS, ANOS*)** **(P5, P25, P50, P75, P95)**						
0.00	(370.40, 369.90, 2, 742.82) (19, 106, 256, 513, 1110)	(370.42, 369.82, 2, 741.54) (19, 106, 256, 513, 1110)	(370.36, 369.86, 2, 741.07) (19, 106, 256, 513, 1110)	(370.43, 369.93, 5, 1852) (19, 106, 256, 513, 1110)	(370.38, 369.87, 5, 1852) (19, 0.108, 261, 523, 1118)	(370.40, 369.91, 5, 1887) (19, 108, 261, 522, 1119)	(370.40, 369.90, 5, 1852.0) (19, 106, 256, 513, 1110)
0.20	(257.39, 256.89, 20.1, 516.64) (13, 74, 178, 356, 768)	(261.59, 261.08, 2.01 , 523.83) (13, 74.9, 181, 362, 780)	(262.28, 261.78, 2.02, 524.88) (13, 75, 181, 362, 780)	(130.06, 129.56, 5.15, 669.50) (7, 37, 89, 179, 387)	(122.00, 121.50, 5.20, 634.61) (6, 34, 82, 165, 356)	(117.18, 116.68, 5.23, 625.04) (6, 34, 82, 165, 357)	(177.37, 176.87, 5.01, 886.95) (9, 50, 122, 245, 529)
0.40	(123.33, 122.83, 2.02, 248.25) (6, 35, 85, 170, 367)	(128.97, 128.47, 2.03, 258.53) (7, 37, 89, 178, 385)	(130.08, 129.58, 2.03, 260.44) (7, 37, 89, 179, 385)	(30.63, 30.13, 5.56, 170.37) (2, 9, 21, 42, 90)	(26.23, 25.73, 5.78, 151.49) (1, 7, 17, 35, 76)	(25.51, 25.01, 5.88, 152.66) (1, 7, 17, 35, 76)	(56.33, 55.83, 5.01, 281.74) (3, 16, 38, 77, 167)
0.60	(57.44, 56.94, 2.02, 116.22) (3, 16, 39, 79, 170)	(61.70, 61.20, 2.03, 123.93) (3, 17, 42, 85, 183)	(62.66, 62.15, 2.04, 125.57) (3, 18, 43, 86, 185)	(9.47, 8.95, 6.16, 58.34) (1, 3, 6, 12, 27)	(7.81, 7.30, 6.63, 51.76) (1, 2, 5, 10, 22)	(7.65, 7.13, 6.82, 52.86) (1, 2, 5, 10, 21)	(20.43, 19.92, 5.02, 102.22) (1, 6, 13, 28 , 60)
0.80	(28.46, 27.96, 2.05, 58.09) (1, 8, 19 , 39, 84)	(31.24, 30.73, 2.05, 62.94) (2, 9, 21, 43 , 92)	(31.91, 31.41, 2.04, 64.06) (2, 9, 21, 43, 92)	(3.95, 3.42, 6.85, 27.06) (1, 1, 2, 5, 10)	(3.35, 2.81, 761, 25.50) (1, 1, 2, 4, 8)	(3.22, 2.68, 7.83, 25.44) (1, 1, 2, 4, 8)	(8.79, 8.27, 5.02, 44.01) (1, 2, 6, 11, 25)
1.00	(15.30, 14.79, 2.08, 31.60) (1, 4, 10, 21, 44)	(17.01, 16.50, 2.07, 34.42) (1, 5, 11, 23, 49)	(23.38, 22.87, 2.06, 35.08) (1, 5, 12, 24, 51)	(2.17, 1.60, 7.49, 16.27) (1, 1, 1, 2, 5)	(1.97, 1.38, 8.57, 16.89) (1, 1, 1, 2 , 4)	(1.84, 1.24, 8.60, 15.89 (1, 1, 1, 2,4)	(4.46, 3.93, 5.03, 22.36) (1, 1, 3, 6, 12)
1.20	(8.94, 8.42, 2.10, 18.74) (1, 2, 6, 12, 25)	(9.97, 9.45, 2.10, 20.29) (1, 2, 6, 13, 28)	(10.22, 9.71, 2.08, 20.62) (1, 3, 7, 13, 29)	(1.50, 0.87, 7.98, 11.99) (1, 1, 1, 1, 2)	(1.46, 0.82, 9.34, 13.65) (1, 1, 1 1, 2.)	(1.33, 0.67, 8.87, 11.86) (1, 1, 1, 1, 2)	(2.65, 2.09, 5.03, 13.26) (1, 1, 1, 3, 6)
1.40	(5.64, 5.12, 2.12, 12.04) (1, 1, 4, 7, 15)	(6.26, 5.74, 2.13, 12.83) (1, 2, 4, 8, 17)	(6.41, 5.89, 2.10, 12.97) (1, 2, 4, 8, 18)	(1.23, 0.53, 8.24, 10.10) (1, 1, 1, 1, 2)	(1.25, 0.55, 9.79, 12.20) (1, 1, 1, 1, 1)	(1.13, 0.39, 8.56, 9.71) (1, 1, 1,1, 1)	(1.80, 1.20, 5.05, 9.06) (1, 1, 1, 1, 4)
1.60	(3.83, 3.29, 2.13, 8.302) (1, 1, 2, 5, 10)	(4.20, 3.67, 2.16, 8.67) (1, 1, 3, 5, 11)	(4.29, 3.75, 2.12, 8.71) (1, 1, 2 , 5, 11)	(1.11, 0.34, 8.23, 9.11) (1, 1, 1, 1, 1)	(1.14, 0.40, 9.85, 11.22), (1, 1, 1, 1, 1)	(1.05, 0.24, 7.76, 8.17) (1, 1, 1, 1, 1)	(1.38, 0.73, 5.06, 6.96) (1, 1, 1 , 1, 2)
1.80	(2.77, 2.21, 2.14, 6.082) (1, 1, 2, 3, 7)	(3.00, 2.45, 2.12, 6.22) (1, 1, 2, 3, 7)	(3.05, 2.50, 2.13, 6.21) (1, 1, 2, 4, 8)	(1.05, 0.24, 7.94, 8.36) (1, 1, 1, 1, 1)	(1.08, 0.29, 9.49, 10.25) (1, 1, 1, 1, 1)	(1.02, 0.14, 6.73, 6.87) (1, 1, 1, 1, 1)	(1.18, 0.46, 5.08, 5.90) (1, 1, 1, 1, 2)
2.00	(2.12, 1.54, 2.15, 4.693) (1, 1, 1, 2, 5)	(2.26, 1.69, 2.10 , 4.71) (1, 1, 1, 2, 5)	(2.63, 2.07, 2.11, 4.69) (1, 1, 1, 2, 5)	(1.03, 0.17, 7.39, 7.52) (1, 1, 1, 1, 1)	(1.04, 0.21, 8.78, 9.16) (1, 1, 1, 1, 1)	(1.01, 0.08, 5.74, 5.79) (1, 1, 1, 1, 1)	(1.07, 0.28, 5.09, 5.38) (1, 1, 1, 1, 1)
2.50	(1.36, 0.69, 2.13, 2.957) (1, 1, 1, 1, 2)	(1.40, 0.75, 2.07, 2.91) (1, 1, 1, 1, 2)	(2.30, 1.73, 2.08, 2.88) (1, 1, 1, 1, 2)	(1.00, 0.06, 5.39, 5.41) (1, 1, 1, 1, 1)	(1.01, 0.09, 6.12, 6.16) (1, 1, 1, 1, 1)	(1.00, 0.02, 4.47, 4.32) (1, 1, 1, 1, 1)	(1.00, 0.07, 5.11, 5.02) (1, 1, 1, 1, 1)
EQL	119.97	129.36	131.56	33.99	32.45	31. 11	49.56
(, *L*,)	(2.9001, 3.0073, 2.9025)	(2.9751, 3.0009, 2.9805)	(2.9908, 3.0002, 2.9925)	(0.8856, 3.3526, 3.0085)	(1.0941, 3.2339, 3.0101)	(1.5291, 3.2440, 3.0412)	(2.9934, 3.0008, 2.9998)
(,)	(2, 2, 5)	(2, 2, 8)	(2, 2, 11)	(5, 2, 8)	(5, 2, 11)	(5, 4, 8)	(5, 5, 5)

Table 3. Exact *ARL, SDRL*, *ASS*, *ANOS*, *EQL*, percentile values and the optimal design parameters of the proposed chart when the = 370.4, = 7 and 11 with = 2.5

Shift ()	(*ARL, SDRL, ASS, ANOS*) (P5, P25, P50, P75, P95)						
0.00	(370.48, 369.97, 7.00, 2593) (19, 106, 256, 513, 1110)	(370.37, 369.87, 7.00, 197.37) ((19, 106, 256, 513, 1110)	(370.38, 369.88, 7.00, 2593) (19, 106, 256, 513, 1110)	(370.44, 369.94, 7.00, 2593) (19, 106, 256, 513, 1110)	(370.42, 369.92, 11.00, 4075) (19, 106, 256, 513, 1110)	(370.40, 369.90, 11.00, 4074) (19, 106, 256, 513, 1110)	(370.36, 369.86, 11.00, 4074) (19, 106, 256, 513, 1110)
0.20	(108.02, 107.52, 7.20, 777.51) (6, 31, 74, 149, 322)	(115.65, 115.15, 7.22, 99.33) (1, 33, 79, 159, 342)	(91.59, 91.09, 7.37, 674.57) (5, 26, 63, 126, 271)	(79.18, 78.68, 7.46, 591.04) (4, 22, 54, 109, 235)	(84.43, 83.94, 11.17, 942.86) (4, 24, 58, 116, 249)	(88.48, 87.98, 11.18, 989.44) (4, 25, 61, 122, 264)	(73.18, 72.68, 11.42, 835.79) (4, 21, 50, 100, 218)
0.40	(23.11, 22.61, 178.68) (1, 7, 16, 31, 67)	(25.87, 25.37, 7.78, 39.77) (1, 1, 3, 6, 14)	(17.94, 17.43, 8.33, 149.48) (1, 5, 12, 24, 52)	(14.39, 13.88, 8.73, 125,68) (1, 4, 10, 19, 41)	(15.83, 15.32, 11.61, 183,76) (1, 4, 10, 21, 46)	(17.02, 16.51, 11.63, 197.93) (1, 5, 11, 23, 49)	(12.79, 12.28, 12.48, 159.57) (1, 4, 9, 17, 36)
0.60	(6.99, 6.47, 8.44, 58.98) (1, 1, 4, 9, 19)	(7.89, 7.38, 8.44, 20.84) (1, 1, 1, 3, 6)	(5.34, 4.81, 9.57, 51.12) (1, 1, 3, 7, 14)	(4.31, 3.78, 10.43, 44.94) (1, 1, 3, 5, 11)	(4.69, 4.16, 12.18, 57.14) (1, 1, 3, 6, 13)	(5.04, 4.51, 12.11, 60.99) (1, 1, 3, 6, 13)	(3.77, 3.24, 13.69, 51.67) (1, 1, 2, 5, 10)
0.80	(2.97, 2.42, 9.13, 27.16) (1, 1, 2, 3, 7)	(3.30, 2.75, 8.90, 13.83) (1, 1, 1, 1, 3)	(2.36, 1.79, 10.66, 25.13) (1, 1, 1, 3, 5)	(2.02, 1.44, 12.02, 24.29) (1, 1, 1, 2, 4)	(2.12, 1.54, 12.73, 26.95) (1, 1, 1, 2, 5)	(2.23,1.65, 12.42, 27.66) (1, 1, 1, 2, 5)	(1.78, 1.18, 14.59, 25.99) (1, 1, 1, 1, 4)
1.00	(1.71, 1.10, 9.64, 16.50) (1, 1, 1, 2, 3)	(1.84, 1.24, 9.00, 10.79) (1, 1, 1, 1, 2)	(1.45, 0.81, 11.25, 16.32) (1, 1, 1, 1, 3)	(1.34, 0.68, 12.97, 17.43) (1, 1, 1, 1, 2)	(1.35, 0.69, 13.08, 17.73) (1, 1, 1, 1, 2)	(1.38, 0.73, 12.40, 17.17) (1, 1, 1, 1, 2)	(1.22, 0.51, 14.92, 18.14) (1, 1, 1, 1, 2)
1.20	(1.26, 0.57, 9.84, 12.35) (1, 1, 1, 1, 2)	(1.30, 0.62, 8.67, 9.23) (1, 1, 1,1, 1)	(1.14, 0.40, 10.11, 12.71) (1, 1, 1, 1, 1)	(1.12, 0.36, 12.95, 14.47) (1, 1, 1, 1, 1)	(1.11, 0.34, 13.18, 14.57) (1, 1, 1, 1, 1)	(1.11, 0.34, 11.97, 13.24) (1, 1, 1, 1, 1)	(1.05, 0.23, 14.54, 15.27) (1, 1, 1, 1, 1)
1.40	(1.09, 0.31, 9.67, 10.50) (1, 1, 1, 1, 1)	(1.09, 0.32, 8.00, 8.14) (1, 1, 1, 1, 1)	(1.04, 0.20, 10.36, 10.76) (1, 1, 1, 1, 1)	(1.04, 0.20, 11.96, 12.43) (1, 1, 1, 1, 1))	(1.03, 1.930.18, 12.93, 13.31) (1, 1, 1, 1, 1)	(1.02, 0.15, 11.10, 11.35) (1, 1, 1, 1, 1)	(1.01, 0.1, 13.47, 13.60) (1, 1, 1, 1, 1)
1.60	(1.03, 0.17, 9.15, 9.40) (1, 1, 1, 1, 1)	(1.02, 0.16, 7.16, 7.19) (1, 1, 1, 1, 1)	(1.01, 0.1, 9.12, 9.21) (1, 1, 1, 1, 1)	(1.01, 0.11, 10.31, 10.45) (1, 1, 1, 1, 1)	(1.01, 0.09, 12.31, 12.42) (1, 1, 1, 1, 1)	(1.00, 0.06, 9.86, 9.90) (1, 1, 1, 1, 1)	(1.00, 0.04, 11.82, 11.84) (1, 1, 1, 1, 1)
1.80	(1.01, 0.09, 8.32, 8.40) (1, 1, 1, 1, 1)	(1.00, 0.07, 6.35, 6.37) (1, 1, 1, 1, 1)	(1.00, 0.05, 7.77, 7.79) (1, 1, 1, 1, 1)	(1.00, 0.06, 8.52, 8.55) (1, 1, 1, 1, 1)	(1.00, 0.05, 11.32, 11.36) (1, 1, 1, 1, 1)	(1.00, 0.02, 8.45, 8.47) (1, 1, 1, 1, 1)	(1.00, 0.03, 9.89, 9.90) (1, 1, 1, 1, 1)
2.00	(1.00, 0.05, 7.31, 7.32) (1, 1, 1, 1, 1)	(1.00, 0.03, 5.73, 8.73) (1, 1, 1, 1, 1)	(1.00, 0.02, 6.61, 6.61) (1, 1, 1, 1, 1)	(1.00, 0.03, 7.00, 7.00) (1, 1, 1, 1, 1)	(1.00, 0.03, 10.02, 10.03) (1, 1, 1, 1, 1)	(1.00, 0.01, 7.14, 7.14) (1, 1, 1, 1, 1)	(1.00, 0.01, 8.07, 8.07) (1, 1, 1, 1, 1)
2.50	(1.00, 0.01, 4.78, 4.78) (1, 1, 1, 1, 1)	(1.00, 0.00, 5.07, 5.07) (1, 1, 1, 1, 1)	(1.00, 0.00, 5.20, 5.203) (1, 1, 1, 1, 1)	(1.00, 0.00, 5.24, 5.236) (1, 1, 1, 1, 1)	(1.00, 0.01, 6.36, 6.36) (1, 1, 1, 1, 1)	(1.00, 0.00, 5.33, 5.33) (1, 1, 1, 1, 1)	(1.00, 0.00, 5.48, 5.49) (1, 1, 1, 1, 1)
EQL	29.84	30.99	27.41	26.01	26.48	26.90	25.08
(, *L*,)	(0.6740, 3.5671, 3.0013)	(0.8406, 3.4148, 3.0521)	(1.1496, 3.6358, 2.9798)	(1.3342, 3.5663, 2.9306)	(0.3486, 3.8211, 2.9827)	(0.3185, 3.8526, 3.0049)	(0.6045, 3.8868, 2.9861)
(*n*, ,)	(7, 3, 8)	(7, 5, 5)	(7, 5, 8)	(7, 5, 11)	(11, 3, 11)	(11, 5, 8)	(11, 5, 11)

Table 4. Control charts performance comparison under case K when *n* {4, 5, 7}, {2, 4}, = 12 and = 2.5 with = 370.4

Overall Performance measures	Control charts						(,)	n
		-EWMA	-CUSUM	VSS	DS	SSDS		
EQL	61.34	56.47	55.53	51.22	43.89	**33.07**		
ARARL	1.89	1.74	1.71	1.61	1.42	**1.00**	(2, 12)	
PCI	1.85	1.71	1.68	1.55	1.33	**1.00**		
EQL	42.02	39.86	39.92	34.4	31.93	**30.02**		4
ARARL	1.60	1.49	1.51	1.33	1.19	**1.00**	(4, 12)	
PCI	1.40	1.33	1.35	1.15	1.06	**1.00**		
EQL	46.99	43.82	43.93	38.11	35.67	**32.21**		
ARARL	1.62	1.48	1.50	1.27	1.18	**1.00**	(2, 12)	
PCI	1.46	1.36	1.39	1.18	1.11	**1.00**		
EQL	39.71	36.32	35.92	33.18	32.88	**30.63**		5
ARARL	1.46	1.37	1.33	1.26	1.19	**1.00**	(4, 12)	
PCI	1.30	1.19	1.17	1.08	1.07	**1.00**		
EQL	43.35	41.26	40.96	35.93	33.10	**29.86**		
ARARL	1.49	1.42	1.41	1.25	1.17	**1.00**	(2, 12)	
PCI	1.45	1.38	1.36	1.20	1.11	**1.00**		
EQL	36.41	34.22	35.32	32.07	29.87	**26.21**		7
ARARL	1.50	1.41	1.47	1.34	1.27	**1.00**	(4, 12)	
PCI	1.39	1.31	1.35	1.22	1.14	**1.00**		

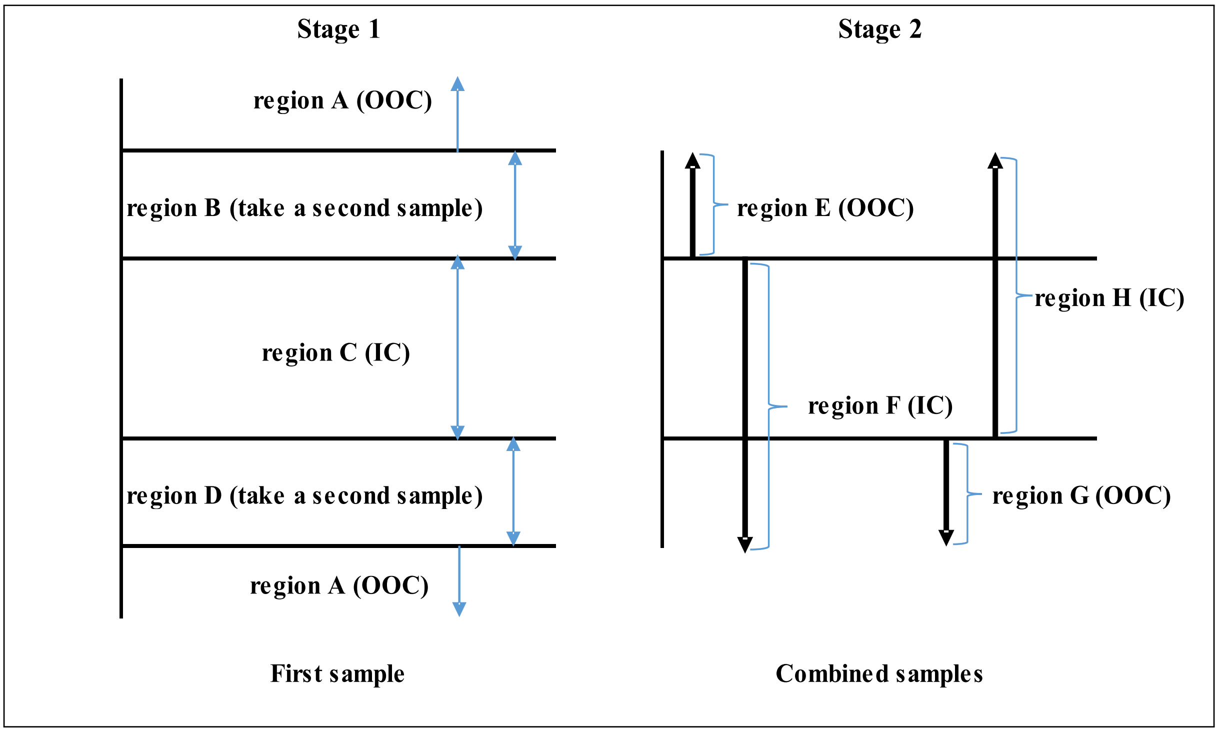


Figure 1. The SSDS control scheme

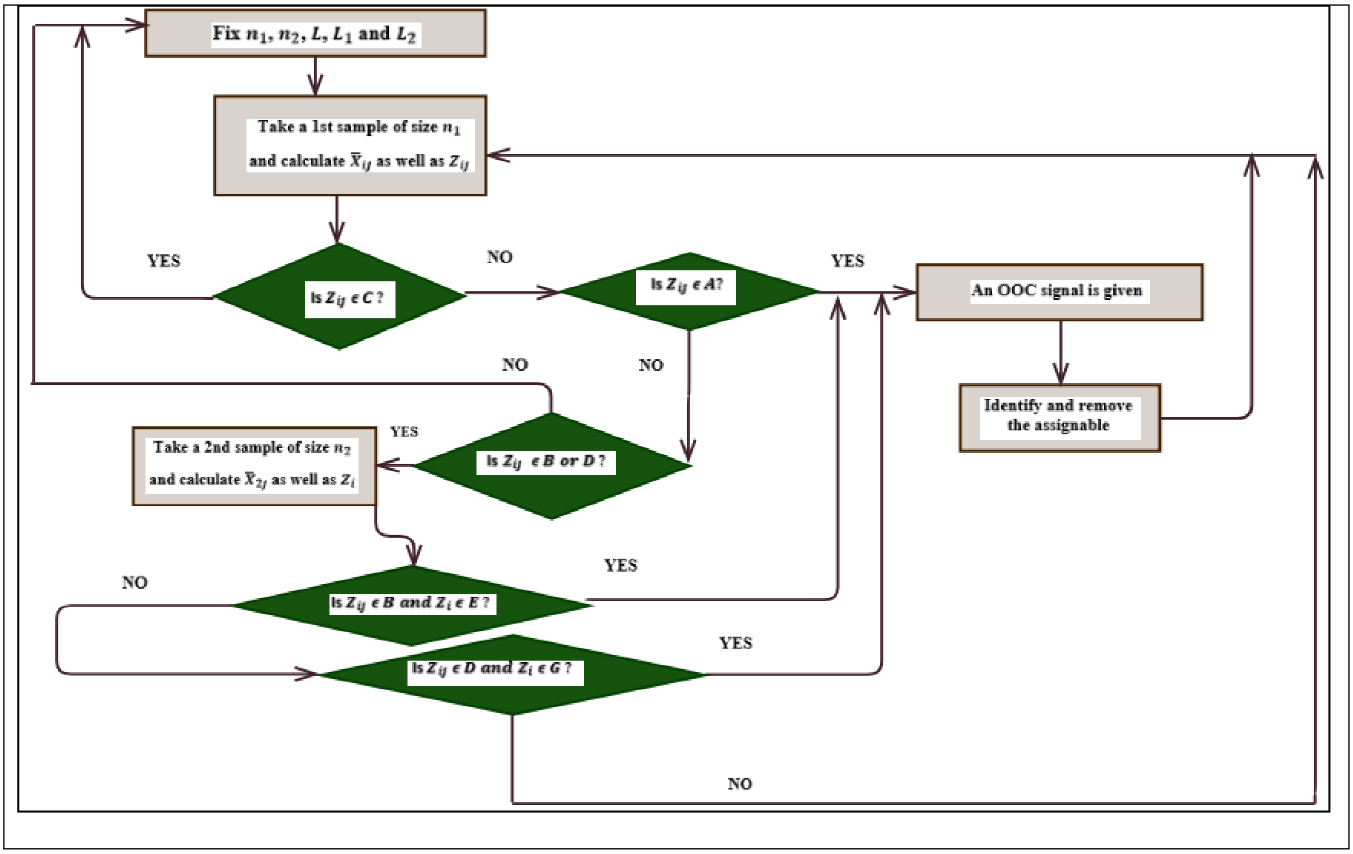
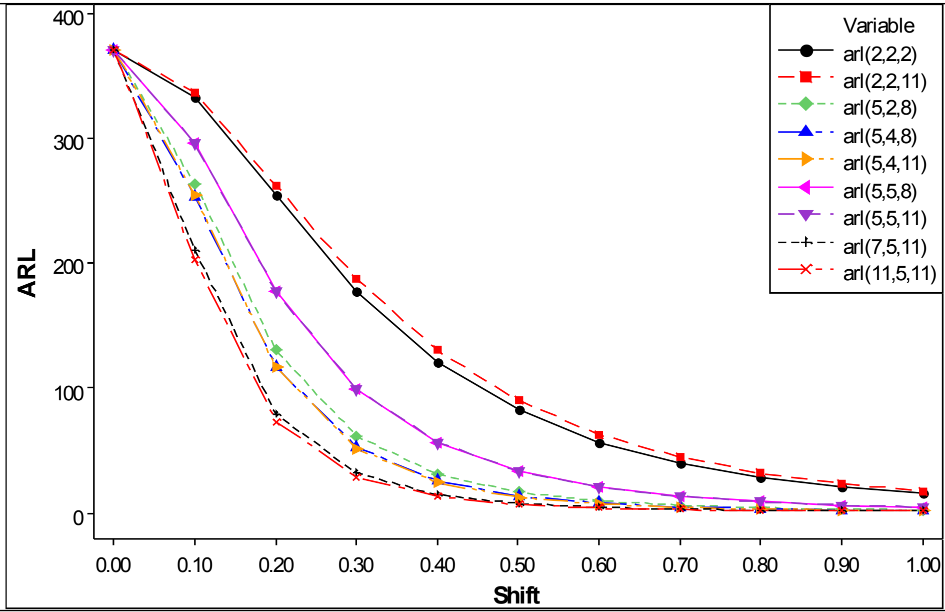
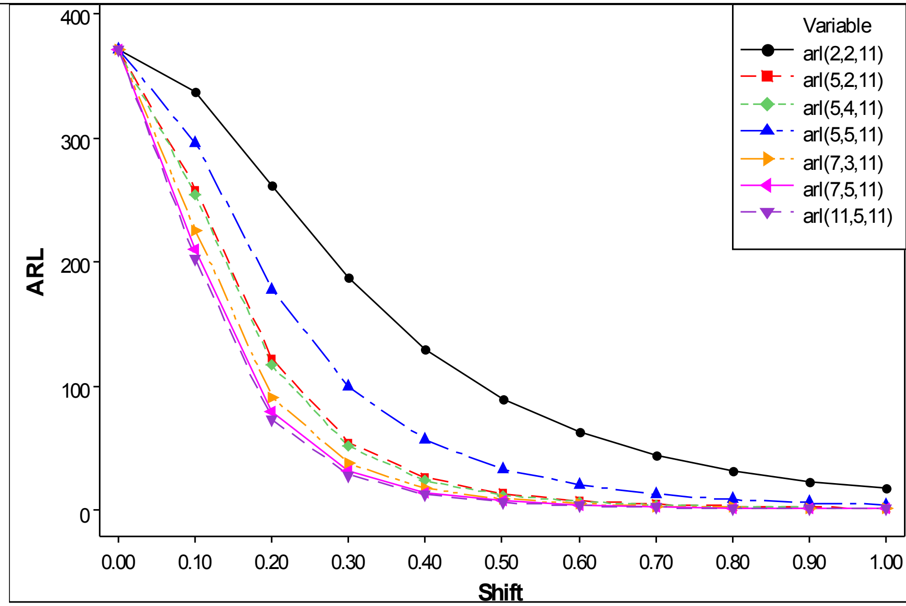


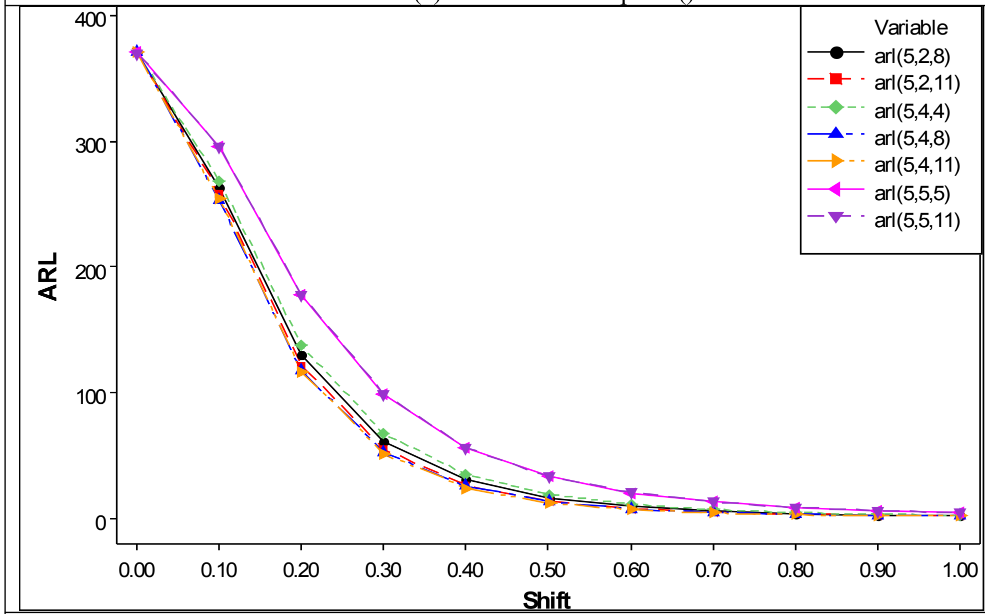
Figure 2. Flow chart for the proposed SSDS control scheme



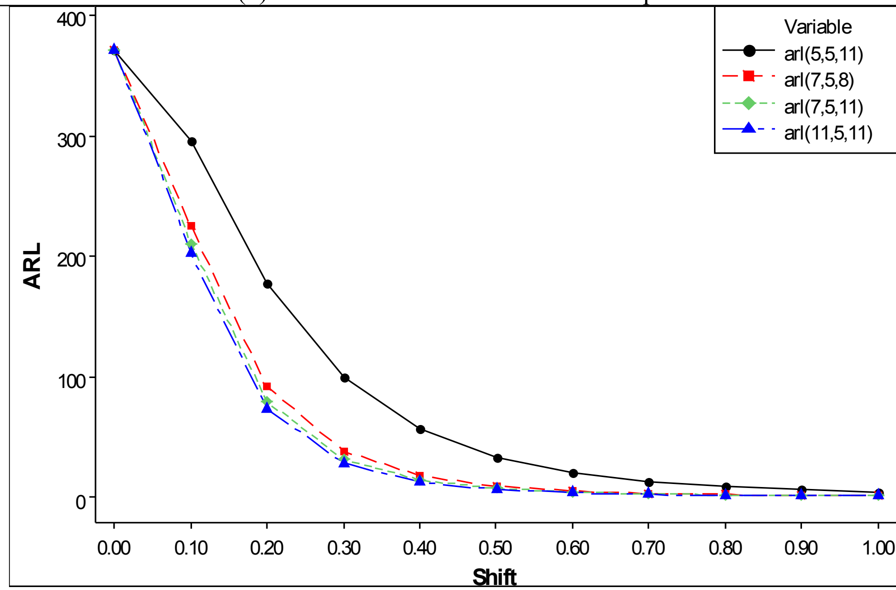
(a) for different triplets ()



(b) when 11 with different and sample sizes

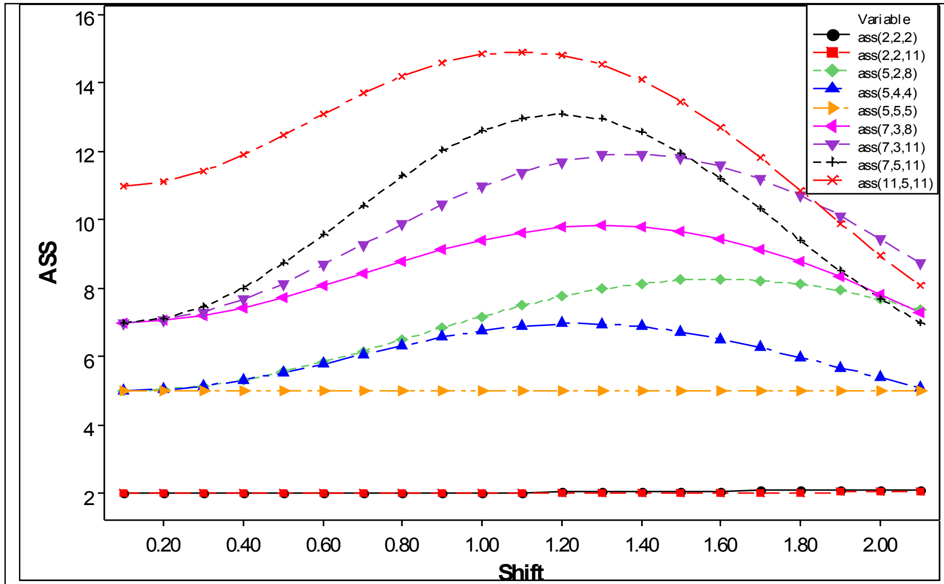


(c) when 5 with different and sample sizes

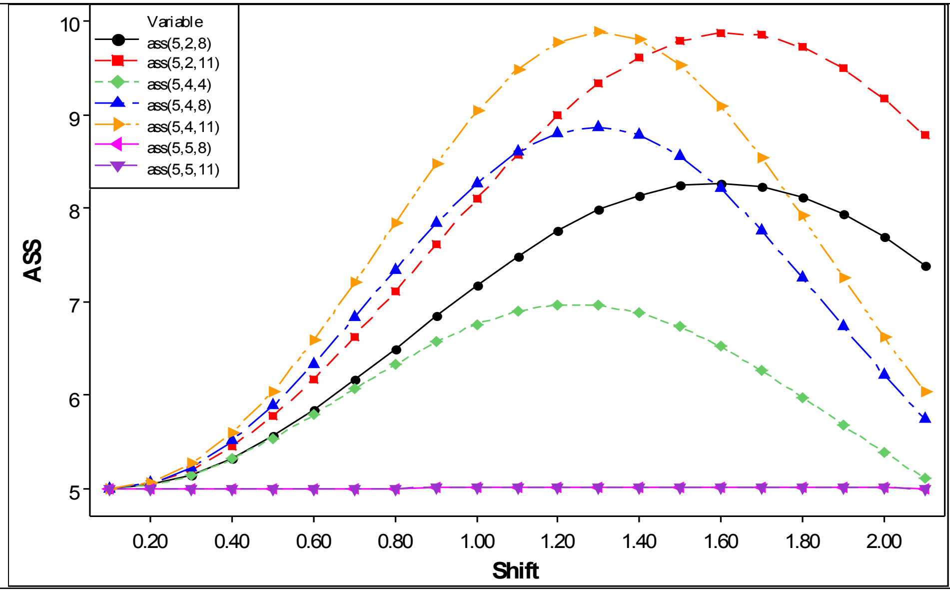


(d) when 5 with different and sample sizes

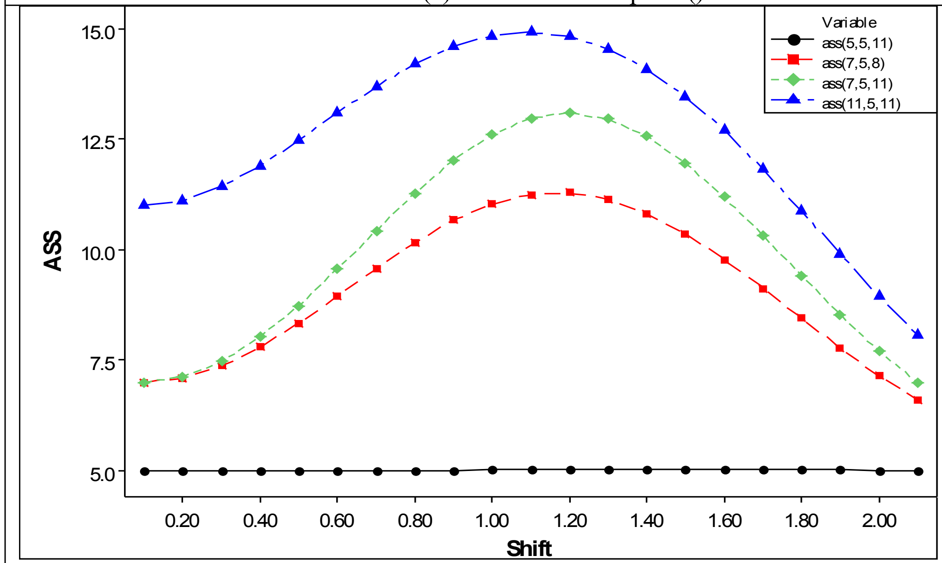
Figure 3. *ARL* values of the SSDS control chart for different sample sizes



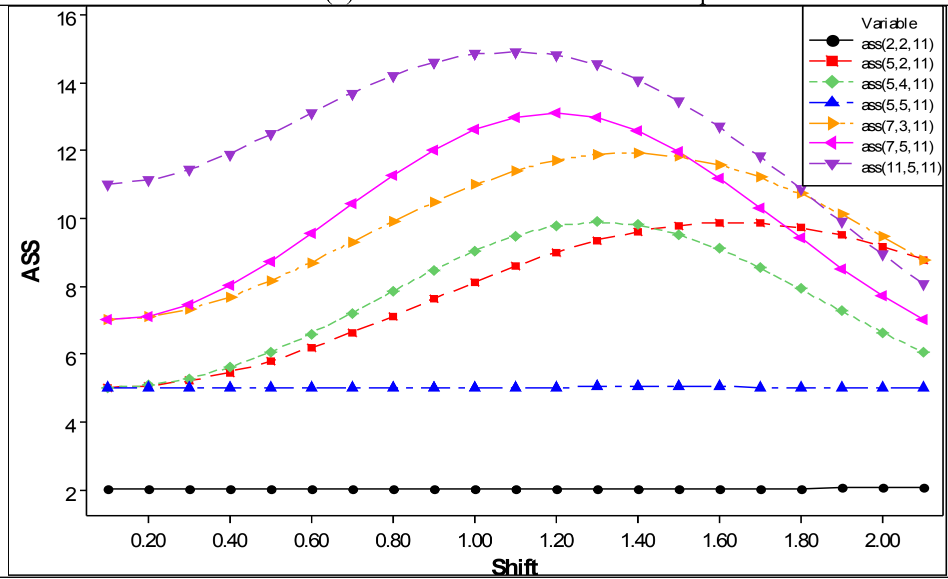
(a) for different triplets ()



(b) when 5 with different and sample sizes

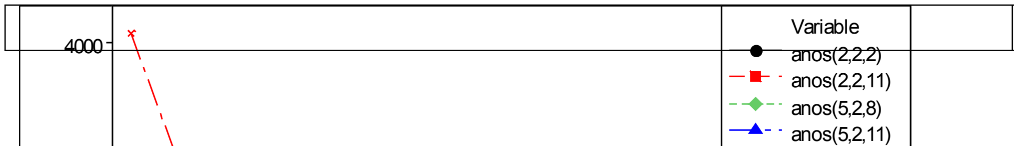


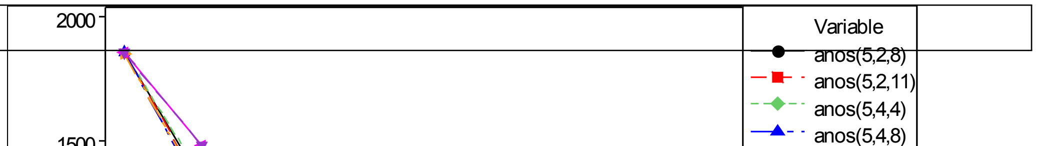
(c) when 5 with different and sample sizes



(d) when 11 with different and sample sizes

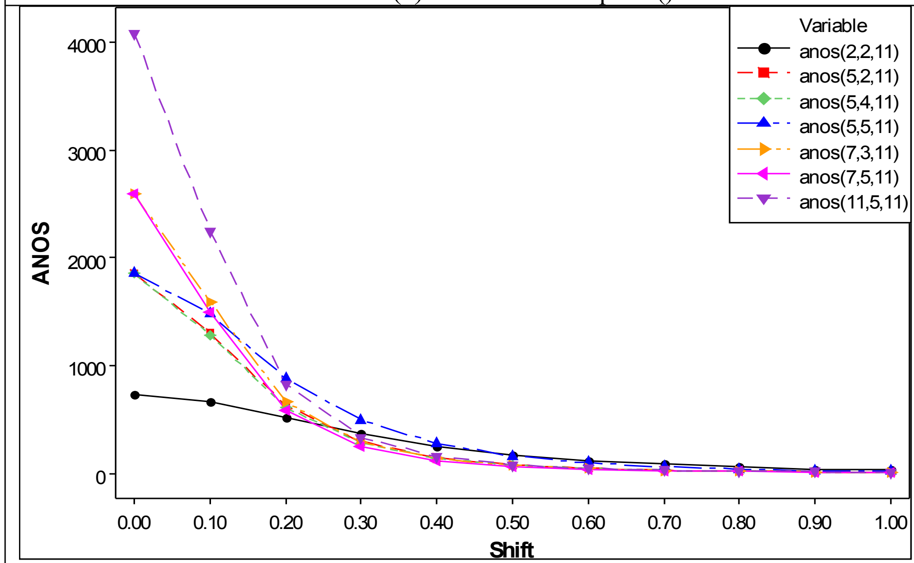
Figure 4. *ASS* values of the SSDS control chart for different sample sizes



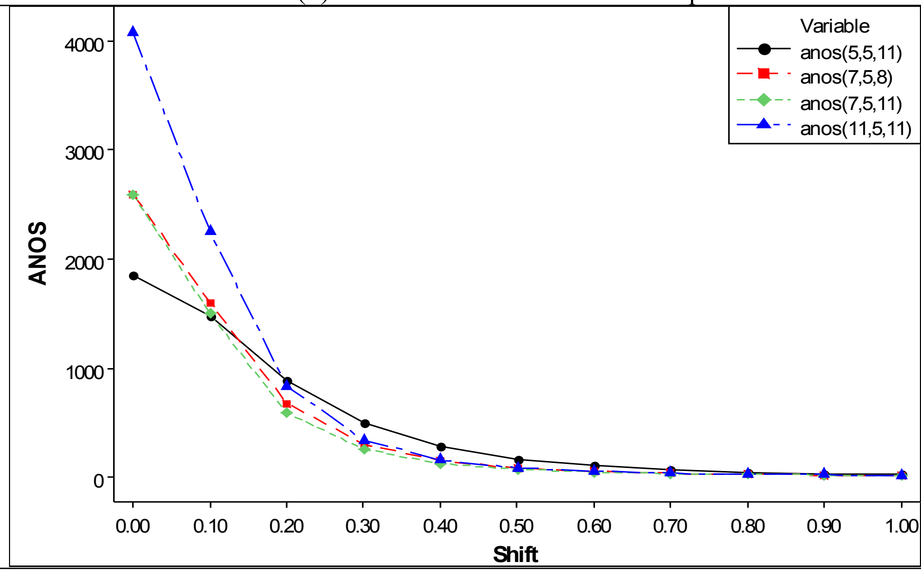


(a) for different triplets ()

(b) when 5 with different and sample sizes



(c) when 11 with different and sample sizes



(d) when 5 with different and sample sizes

Figure 5. *ANOS* values of the SSDS control chart for different sample sizes

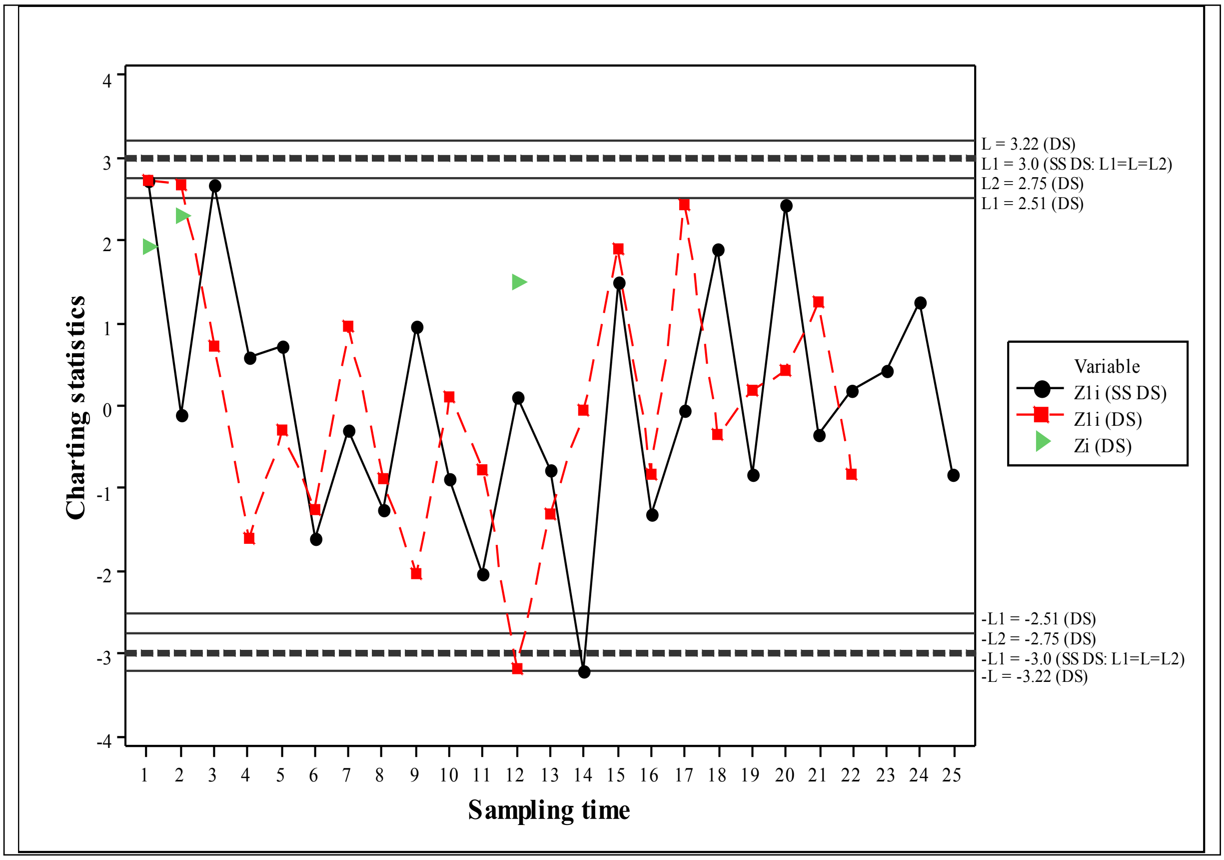


Figure 6. The SSDS and DS control charts for the piston ring data