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Automated Investment Assessment: An investment decision making model for the ordinary person

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Executive summary

The ordinary person invests to achieve their long-term financial goals. Choosing where to invest and what to invest in can be challenging for the ordinary person and paying a financial broker can be expensive and reduces the returns received. Thus, it was decided that the ordinary person requires a decision support tool to aid them in deciding where to invest. It was decided to investigate investing in the shares of 40 Johannesburg Stock Exchange (JSE) companies.

Factors that influence which shares to invest in were investigated and it was discovered that there are four main metrics that should be considered simultaneously. These metrics are the market liquidity of a company, the market risk of the company, the financial risk tolerance of the ordinary person and the expected financial growth of the company. As these metrics must be considered together, it was discovered that this is a multi-objective optimisation (MOO) problem and a customised MOO model should be developed and built. Three simple MOO methods were selected, the lexicographic, weighted sum and weighted product methods, to ensure that the ordinary person using the model will be able to understand the model logic. The metric data used by the model was collected for three years, and as such it was decided that a weighed-moving average metric value should be calculated for each metric for each company to account for stochasticity.

A mathematical model for selecting investment portfolios consisting of at most ten companies from the JSE 40 was developed. Furthermore, pseudocode was written that was used to develop a model that selects investment portfolios for the ordinary person. This model selects investment portfolios using each of the three suggested MOO methods as well as based on only individual metric values over a range of risk tolerance score (RTS) values, using six different weighting combinations of factors.

As different methods are used to select investment portfolios, with six different sets of weightings over a range of 44 different RTS values, the model produced a total of 13 464 model portfolios. The return on investment was calculated for each portfolio and it was found that the majority of model portfolio had similar returns. This is as a result of the fact that the model only selected 16 of the JSE 40 companies and these were selected multiple times as part of multiple portfolios.

These model returns were then compared to the returns on investment received by various unit trusts over the same time period. It was found that the model portfolios significantly outperformed the unit trusts and thus produces worthwhile results. The model was rerun three time with different assumptions and a sensitivity analysis was performed. It was found that the model is not very sensitive to its model assumptions, although this may again be result of the limited sample of companies.

The payout received by the unit trusts and model portfolios for various investment periods were calculated and it was discovered that the model portfolios were significantly outperformed. For this reason, it recommended that the ordinary person should invest in unit trust with reasonable brokerage fees.

Declaration of originality

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Declaration

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Contents

List of Figures

List of Tables

List of Symbols

AMS	Anglo American Platinum
AGL	Anglo American PLC
ANH	Anheuser-Busch InBev SA/NV
APN	Aspen Pharmacare
ATR	asset turnover ratio
ATA	average total assets
BGA	Barclays Africa Group Limited
BIL	BHP Billiton PLC
BV	book value
BM	book-to-market value
BTI	British American Tobacco PLC
CPI	Capitec Bank
$CFFO$	cash flow from operations
CFR	Compagnie Financire Richemont
CN	conditional value at risk number
COS	cost of sales
CA	current assets
CL	current liabilities
CR	current ratio
DSY	Discovery Limited
EFG	expected financial growth
FS	F_ Score
FRT	financial risk tolerance
FSR	FirstRand Limited
TFG	Foschini Group Ltd
GLN	Glencore PLC
GMR	gross margin ratio
GM	gross margin
GRT	Growthpoint Properties Limited
HMN	Hammerson PLC
P_{High}	highest amount of money for which a share was sold during a specific day
IPL	Imperial Holding Ltd
IA	intangible assets
ITU	Intu Properties PLC
INL	Investec
KIO	Kumba Iron Ore
LIX	liquidity index
P_{Low}	lowest price for which a share was sold during a day

MC	market capitalisation of the company
ML	market liquidity
MR	market risk
MV	market value
MEI	Mediclinic International PLC
MNP	Mondi PLC
MRP	Mr Price Group Limited
MTN	MTN Group
NPN	Naspers Limited
NED	Nedbank Group
NRP	NEPI Rockcastle PLC
NI	net income before extraordinary items
C	number of returns calculated
SO	number of shares outstanding
TD	number of trading days in a year
OMU	Old Mutual PLC
PS	preference shares
PSG	PSG Group Limited
RMI	Rand Merchant Insurance Holdings Ltd
i	rank of the objective function
RDF	Redefine Properties
REM	Remgro Limited
R	return on a share
RM	risk measure
RTS	risk tolerance score
RMH	RMB Holdings Limited
SLM	Sanlam
SOL	Sasol Limited
SP	share price
SHP	Shoprite
SBK	Standard Bank Group Ltd
BVT	The Bidvest Group Limited
P_{Close}	the price at which the last shares of the day were sold
TBS	Tiger Brands Limited
TA	total assets
TL	total long-term debt
N	total number of objective functions
TS	total sales
U	Utility Function
$F_i(x)$	value of objective function i
VOD	Vodacom Group Limited
V	volume of shares traded
w_i	weight of objective function i
WHL	Woolworths Holdings Limited

Acronyms

AHP analytic hierarchy process

ARIMA autogressive and moving average

BM book-to-market

CFO operating-cash-flow to total assets ratio

CVaR conditional value-at-risk

DST decision support tool

ELECTRE elimination and choice expressing reality

EW equal weights

FAISA Financial Advisory and Intermediary Services Act

JSE Johannesburg Stock Exchange

MOO multi-objective optimisation

RE rank exponent

ROA return on assets

ROC rank order centroid

RR rank reciprocal

RS rank sum

RTS risk tolerance score

SRTS self-assessed risk tolerance score

TOPSIS technique for order of preference by similarity to ideal solution

TER total expense ratio

VaR value-at-risk

Chapter 1

Introduction

1.1 Background

Investing is the act of committing money or capital to an endeavour with the expectation of gaining an additional income or profit. The practice of investing has existed for millennia with numerous methods, principles and practices for effective investing developing and evolving over time. The ordinary person invests to achieve long-term financial goals such as building a reserve fund for retirement, repaying a mortgage early, paying university fees for their children and many others. Although there are many different forms of investments such as shares, bonds, investment funds, savings, options, insurance and others, each carries its own risk and return. Choosing where to invest and what to invest in is a constant balancing act between the potential gain the investor can receive and the risk of losing all your hard-earned money.

Bartering has been an agent of trade for as long as time itself, but the earliest recorded occurrence of investing appeared in Ancient Mesopotamia around 1700 BC when the Code of Hammurabi was written and implemented. Although this is the first recorded formal use of investing, investment as understood in today's modern society was only established in 1602 when the first stock exchange, the Amsterdam Stock Exchange, opened. The establishment of the formal stock exchange allowed potential investors and businesses to connect and offered liquidity, published value, broadcast availability, and lowered transaction costs. This allowed for easier, cheaper and more standardised investing.

Investment in the twenty-first century has become a complex science. Countless measures and metrics are calculated, analysed and compared by advanced financial software and algorithms to determine whether an investment is worthwhile or not. Despite these changes, the underlying principle has remained the same; to make your hard-earned money work for you. With the development of the internet, financial information is readily available, but as the ordinary person is not trained to determine and understand these measures and metrics they may get lost in the noise of information and lose the essence of what is required to make a worthwhile investment.

Financial brokers are trained financial professionals that use market data to perform scientific comparative analyses between different investment options and are then paid to invest on their clients' behalf. However, as financial brokers act as sales people working for commissions it may be that they have their own personal interests rather than those of their clients at heart. This could be harmful to their clients. Furthermore, a portion of their clients' returns are appropriated by them for services rendered, decreasing the investor's return.

A sure-fire way to ensure that a person is receiving the full rewards of their investment is for them to invest for themselves. The introduction of the internet has made investing for oneself easier than ever. However, there is a sea of information and investing techniques available, many of which may not be reliable, and if used may result in misinformed and potentially detrimental investments made by the investor.

This leads to the research question for this study:

> *How can an ordinary person use publicly available data to develop a customised investment strategy without a finance degree?*

1.2 Research Design

The problem being addressed is that the ordinary person needs a decision support tool (DST) to assist them in making wise and informed investments so as to achieve their long-term financial goals. As there are thousands of possible investment options available, it was decided to focus on investing in the shares of companies listed on the Johannesburg Stock Exchange (JSE). When investing, there are four metrics that must be considered together to select a profitable investment. These four metrics are the market liquidity of the company, the expected financial growth of the company, the market risk of the company and the financial risk tolerance of the ordinary person. These metric values are determined using the JSE market data. Furthermore, each of these metrics have their own objective and as they must be considered together, this is a multi-objective optimisation (MOO) problem. In investing, the historical performance of an investment is considered to be good indication of the future performance of that investment. As such, it is also necessary to consider the historical values of the four metrics to select worthwhile investments. This project proposes to develop and build a custom MOO model (henceforth the model) that will enable the ordinary person to develop a customised investment strategy.

1.3 Research methodology

The design research methodology that Manson (2006) suggested for Operations Research projects is followed in this project. The research methodology specifies how the idea for the project is translated into a well-developed and evaluated solution.

1.3.1 Awareness of the problem

This project addresses an opportunity rather than a problem in identifying that the ordinary person needs a MOO model to help them make profitable investments as discussed in Section 1.1.

1.3.2 Suggestion

Based on the literature study it was found that when solving similar problems, other researchers have used financial metrics and mathematical modelling to develop a solution. Based on these finding it is suggested that a customised MOO model be developed and built to address the opportunity being investigated.

1.3.3 Development

The artefact that will be created by this project is a financial MOO model as described in Section 1.2. This is a prescriptive model as its output will be a recommendation of a possible course of action for the ordinary person. The model should calculate the four metrics, as discussed in Section 1.2, for a sample of JSE companies. The data that will be used to perform these calculations is the market data of the sample companies which is obtained from the financial statements of these companies and Google Finance. As the historical metric values must be considered to select profitable investments, the market data for these companies is collected for a period of five years. This introduces stochasticity into the model and as such it is necessary to incorporate stochastic modelling into model. Furthermore, as each metric has its own objective, the model must make use of MOO to select the best investment options for the ordinary person. Lastly, as this model is intended for use by the ordinary person, the methods and logic followed must be easy to follow and understand.

1.3.4 Evaluation

In this step, the model is verified and validated to ensure that it behaves as expected. As data is available for the last five years, the model will be built using the first three years of data. This is the training set of the model. As described in Section 1.3.2, the model generates several investment portfolios and it is assumed that the ordinary person invests an equal amount of capital in each company in the portfolio. The return on investment is then calculated for each of these portfolios using the data available for the remaining two years. This is the testing set of the data. These return on investment results are compared with one another to determine whether a signal model configuration produces the highest return on investment. Furthermore, to validate these return on investment results, the model returns are compared to the returns on investment received by various unit trusts. This comparison is done to determine whether the model produces worthwhile results. Should the model returns, after accounting for all cost, be greater than the unit trusts' returns after costs, it will be beneficial for the ordinary person to use the model to develop a customised investment portfolio. However, if the opposite is shown to be true, it would be better for the ordinary person to pay a financial broker and invest in unit trusts.

1.4 Document structure

In the remainder of this report, there are four chapters. Chapter 2 consists of a comprehensive literature study on the four investment metrics, different MOO methods and stochastic modelling. A possible solution to the research problem posed in Section 1.1 is also presented in this chapter. In Chapter 3, the conceptual model that is used to solve the research problem is presented. It is shown how all the metrics are calculated and how MOO and stochasticity is accounted for in the model. The mathematical model for the problem is presented and pseudocode that can be used to create the model is given. Lastly, it is discussed how the developed model will be evaluated to determine if the model produces valid and meaningful results. Chapter 4 presents and discusses the results produced by the model. The implications of these results for the ordinary person investing are also discussed in this Chapter. In Chapter discusses 5 whether using the model to select investments is beneficial for the ordinary person, the conclusions of this research project are given and suggestions for further research are presented. An extensive Appendix that the ordinary person can be use as a guide to understand and develop the model is given at the end of this report.

Chapter 2

Literature review

In literature, when financial metrics are investigated they are usually done so in isolation. Academics spend decades investigating the effects of a single metric on the expected returns that an investor can receive. However, returns on investment are subject to the influence of many factors, each having a unique and essential impact that cannot be captured by just a single metric. As such, it is essential to investigate more than one metric in conjunction to determine whether an investment is worthwhile or not. As multiple metrics with often conflicting objectives must be considered in conjunction, it is evident that to solve a problem involving multiple metrics, a multi-objective optimisation (MOO) method must be used to find the most suitable solution.

In this chapter the various metrics pertaining to the research problem described in Chapter 1 as well as different MOO methods for solving this problem will be presented.

2.1 Metrics

From the research it was found that when investing there are four metrics, namely market liquidity, expected financial growth of a company, market risk and financial risk tolerance, that have the greatest combined effect on the returns on investment. These metrics should thus be considered in combination to determine whether an investment is a worthwhile investment for the ordinary person or not. These four metrics will be discussed and explained in the following Sections with the aid of an illustrative example.

> Company ABC is a Johannesburg Stock Exchange (JSE) listed company that designs and manufactures computers. ABC's shares are currently trading on the JSE for R 100 per share. A further analysis of the financial statements of ABC revealed that ABC has a book-to-market (BM) value of 0.05 and an F_{-} Score of 0 (explained below). The evaluation of the share price data of ABC revealed that ABC has a market liquidity (LIX) of 7.5 and a market risk value (MR) of 70%, which is considered an aggressive (high) market risk value. Investor MVN is interested in investing in company ABC and wishes to avoid experiencing any financial losses when investing.

2.1.1 Market liquidity

Market liquidity is a concept that generally refers to how easy it is to buy and sell shares without seeing a change in the price (Brunnermeier and Pedersen, 2008; Pástor and Stambaugh, 2003). When investing, an investor is interested in investments that will result in returns. In terms of market liquidity, this implies that an investor is interested in financial assets that have a resale value similar to, or more desirably higher than, the purchase value (Danyliv et al., 2014). In the example, if MVN purchases shares in ABC for R100 a share and upon sale a few *days* later, manages to sell the shares for R100 per share, those shares would be considered perfectly liquid and would thus have a market liquidity of zero. If MVN manages to sell the shares for any amount above R100, the shares are considered liquid and MVN will make a profit on the sale. The higher the selling price of the shares are, the higher the market liquidity of the shares will be. However, if MVN sells the shares for less than R100, the shares are considered illiquid. If MVN is unable to sell the shares at all (there is no longer a demand for ABC's shares), the shares will be considered perfectly illiquid and MVN will experience a substantial loss.

The above describes the day-to-day effects of market liquidity on returns, however, the ordinary person is interested in the long-term returns of an investment. As such it is important to consider the effect of market liquidity on these returns and how market liquidity can be used to select investments that will yield long-term returns. Through the use of statistical analysis, Brunnermeier and Pedersen (2008); Pástor and Stambaugh (2003) found that less liquid stocks have higher average returns. Thus, the ordinary person should invest in shares that have a low liquidity at present.

Lybek and Sarr (2002) show that liquid shares should possess five characteristics: (i) *tightness*, which refers to low transaction cost; (ii) *immediacy*, which represents a high speed of order execution; (iii) *depth*, which refers to the existence of limit orders; (iv) *breadth*, meaning small market impact of large orders; and (v) *resiliency*, which means a flow of new orders to correct market imbalances. Danyliv et al. (2014) proposed a liquidity measure called the Liquidity Index (LIX) which is calculated using Equation (3.2). It was shown that this liquidity measure has interconnected components related to market breadth, market depth, market resilience and immediacy making it a carefully composed and balanced measure of market liquidity. This liquidity index is measured on a linear scale from five for illiquid shares to around ten for the most liquid shares. If the LIX value of a company is equal to 7.5, the shares of this company would be considered perfectly liquid. This is because a LIX value of 7.5 is equivalent to a market liquidity of zero. Furthermore, calculating the LIX value of a company is simple as all the information that is needed to calculate it is readily available and easy to find online. These are properties that make the LIX an ideal metric to be used by the ordinary person (Danyliv et al., 2014).

For the example this means that if only the market liquidity is considered, MVN should invest in ABC as it has a LIX of 7.5 which implies that ABC's shares will have a high value in the future.

Although market liquidity indicates whether or not a company's shares will have high returns in the future, it does not indicate what returns an investor can expect based on the future value of the company. As receiving returns in the future is the main objective of an investor is necessary to consider what the value of a company will be in the future. One way of doing this is to evaluate the expected financial growth of the company.

2.1.2 Expected financial growth

Expected financial growth is the percentage by which the value of a financial asset, such as shares, is expected to increase in the future. It is a fundamental principle for investing and the ordinary person is interested in this metric as investing in shares that have a high expected financial growth will ensure a greater return on investment. One factor in determining the expected financial growth of a company is the BM value of the company.

It indicates the value of a company and is determined by comparing the value of the company in its accounting books (the book value, BV) to the current trading market price (the market value, MV) (Piotroski, 2000).

Fama and French (1995) found that a low BM value indicates that a company has high average returns on capital and are thus growth shares. A BM value below 1 indicates that investors are willing to pay more for a company than its net assets are worth. This could be an indication of healthy future profit projections and the investors are willing to pay a premium for that possibility. Thus, the ordinary person is interested in investing in shares that have a BM below 1 as this indicates that future profits will be healthy and the company is experiencing financial growth.

However, the work by Piotroski (2000) as well as Lee and Swaminathan (2000) show that using only the BM value as an indication of expected growth on its own is insufficient to determine whether the value of a company is growing. Piotroski (2000) showed that measurements of a company's financial position are generally useful in predicting the future performance of the company. He identified three areas that are indications of the financial condition of a company, namely, profitability, financial leverage/liquidity and operating efficiency. A profitability measure provides information about the company's ability to generate funds internally. Financial leverage/liquidity measures changes in capital structure and whether a company has the ability to meet future debt obligations. Operating efficiency is the ability of a company to deliver products or services in the most cost-effective manner possible without compromising the quality of the products and services. A company that is able to generate funds, meet its debt obligations and provide operating efficiency is a healthy company. Furthermore, increasing profitability, decreasing financial leverage/liquidity and increasing operating efficiency are indications that a company is experiencing financial growth and is expected to continue to grow (Piotroski, 2000).

Piotroski (2000) used this reasoning to identify nine financial statement signals that measure the three areas of a company's financial condition. Each financial statement signal is assigned an indicator variable and the result of each signal is classified as either "good" or "bad". The indicator variable is allocated a value of zero for a "good" result and a value of one for a "bad". An aggregate signal measure, F_ Score , which is the sum of nine binary digits, was defined and indicates the overall strength of a company's financial position. The lower the aggregate score, the stronger the financial position of the company, indicating that the value of that company is expected to grow in the future.

Company ABC has a low BM value of 0.05 and the minimum F_ Score of 0. These results indicate that ABC is experiencing financial growth and as such will experience higher returns in the future. As an investor is interested in receiving higher returns this would be a good company to invest in if expected financial growth was the only consideration.

The expected financial growth of a company indicates whether a company will experience growth leading to an increase in the value of its shares. Although this metric indicates how much of a return an investor can expect, it does not take into account the inherent risk, called market risk, that comes with every investment. This market risk can result in substantial losses for an investor if not considered and should thus be incorporated into an evaluation of investment options. This market risk is discussed in the next Section.

2.1.3 Market risk

Dowd (2007) defines market risk as the possibility that an investor will experience a financial loss, or gain, due to unforeseen changes in market prices. When investing, an individual is interested in the market risk so as to diminish or accept the risk they will be taking with an investment. When determining market risk, financial professionals use the value-at-risk (VaR) or conditional value-at-risk (CVaR) method.

VaR is a statistical technique used to measure and quantify the level of financial risk within a company over a specific time frame. It is measured at three intervals, 95%, 99% and 99.9% which represent the certainty of the risk being calculated. For example, if VaR(95) was calculated to be 60% it means that there exists a 5% chance that an investor will lose 60% or more of the money they invested. VaR is a measure of losses due to normal market movements and is widely used in industry (Dowd, 2007). However, it is unstable and difficult to work with numerically when losses are not normally distributed as it does not take the tail ends of a distribution into account when determining risk. VaR produces a large range of potential losses, rather than one risk value, and it is thus difficult to account for the market risk when using this measure. This most often leads to calculated market risk being underestimated and thus, should losses be experienced, they will be substantial in value. As the majority of financial losses experienced are not normally distributed it is necessary to use a risk measure that does account for the tail ends of a distribution (Dowd, 2007; Rockafellar and Uryasev, 2002).

CVaR is a measure that quantifies the losses that may be experienced for any distribution and is thus a better measure of market risk. CVaR is measured at three intervals, at 95%, 99% and 99.9%, and measures the average expected loss of an investment as opposed to the VaR which produces a large range of potential losses which can be difficult to account for. CVaR(95) indicates that in the worst 5% of returns, what the average loss will be. Similarly, CVaR(99) indicates that in the worst 1% of returns, what the average loss will be and CVaR(99.9) indicates that in the absolute worst-case scenario what the average loss will be. Moreover, it can be expressed as a minimisation formula which can easily be incorporated in optimisation problems that aim to minimise risk or shape it within bounds (Rockafellar and Uryasev, 2002).

Market risk is divided into five categories namely, conservative risk, moderately conservative risk, moderate risk, moderately aggressive risk and aggressive risk. These risk categories indicate the level of risk involved with an investment and encompass market risk values of 0 – 37%, 38 – 45%, 45 – 57%, 58 – 66% and 67 – 100% respectively.

The market risk value is determined from the return received by a company's shares. As a company may experience a loss, the market risk value may be a negative value. However, CVaR is a measure that indicates the percentage of risk associated with an investment. Thus, when considering the market risk, an investor is interested in the absolute CVaR value (Rockafellar and Uryasev, 2002).

In the example, the market risk of ABC was calculated to be 70% using the CVaR(95) method. This indicates that there exists a 5% likelihood that an investment in ABC will result in an investor losing an average of 70% of the money that they have invested in ABC.

Market risk indicates the risk associated with the company but does not incorporate the level of risk that the ordinary person is comfortable with taking. It is necessary to match the market risk of investments with a metric that encompasses the level of risk the ordinary person is comfortable with taking when finding the optimal investments for that person. The metric used in industry for this purpose is financial risk tolerance.

2.1.4 Financial risk tolerance

Hallahan et al. (2004) define financial risk tolerance as the maximum amount of uncertainty that a person is willing to accept when making a financial decision. In other words, it describes how willing an investor is to experience a financial loss should an investment be unsuccessful.

There are two methods for determining financial risk tolerance, namely self-assessed risk tolerance and a financial risk tolerance assessment. A self-assessed risk tolerance involves asking an individual to estimate their financial risk resulting a self-assessed risk tolerance score (SRTS) for the individual. A financial risk tolerance assessment is a psychometric attitude test in which the individual is required to give responses to different financial situations and these responses, in conjunction with other personal factors such as gender, age, marital status, level of income, level of education and current assets owned, are then used to determine a risk tolerance score (RTS) for the individual (Hallahan et al., 2004). Both SRTS and RTS are measured on a scale of twenty (20) to sixty-nine (69), with a higher score indicating a higher propensity towards taking risk. Furthermore, risk tolerance scores are grouped into three categories, namely aggressive risk tolerance, moderate risk tolerance and conservative risk tolerance. A RTS of 20 – 38 indicates a low tolerance for risk, 39 – 42 a below-average tolerance for risk, 43 – 48 an average/moderate tolerance for risk, 49 – 52 an above-average tolerance for risk and a RTS of 53 – 69 indicating a high tolerance for risk.

The research of Hallahan et al. (2004) shows that the RTS is preferred over the SRTS. They report that when comparing the RTS and SRTS, there is an average difference of 5.33 points. This indicates that the RTS as calculated with the financial risk tolerance assessment is approximately 5 points higher than that estimated by the individual. Which suggests that individuals usually underestimate their risk tolerance score and thus the RTS should be used as the baseline when determining whether to invest in a certain investment. In their research, Grable and Lytton (1999) found, in a sample of 1 075 participants of varying age, gender, education and earnings, that the average RTS was 37, with a standard deviation of 6.40, and a range of 20 – 63. The minimum possible RTS is 20 and the maximum possible RTS is 69. As no distribution is specified it is assumed that these results followed a normal distribution.

In the example, investor MVN took a risk tolerance assessment and was found to have a moderate risk tolerance, thus MVN's RTS is between 43 and 48. As the high market risk of ABC falls outside the bounds of MVN's financial risk tolerance, this would not be a good investment for MVN. This is because MVN would not be comfortable with accepting a 70% chance that she would lose all the money she invested.

In terms of investor MVN's financial risk tolerance this is not a good investment, however, earlier it was shown that in terms of market liquidity and expected financial growth this is a good investment option. This illustrates how choosing the ideal investment option for the ordinary person involves making trade-offs between multiple, sometimes conflicting, objectives. In reality it is impossible to always meet all the objectives in determining which companies to invest. For this reason, it is necessary to use mathematical techniques and methods that find the best possible solution while taking all the objectives into account. This is the aim of MOO, to find an optimal solution from various possible solutions with competing objectives. As such it can be concluded that choosing which shares to invest in based on the market liquidity, expected financial growth, market risk and financial risk tolerance is a MOO problem.

2.2 Multi-objective optimisation

In real-world problems finding the best solution may not be a simple task. This is because a problem may consist of many different, competing and hierarchical objectives that must be met to find the optimal solution (Gerasimov and Repko, 1978). The process of systematically and simultaneously optimising a group of objective functions is called MOO. In a MOO problem there is usually no single global solution and a solution is found by making sets of compromises. For this reason, it is often necessary to determine a set of points that fit a predetermined definition for an optimum. For MOO the concept that defines an optimal point is that of *Pareto Optimality* (Marler and Arora, 2004). Farahani et al. (2010) define a solution point as pareto optimal when it is not possible to move from that point and improve at least one objective function without worsening any other objective function. One is thus interested in a MOO method that is classified as having a sufficient condition for pareto optimality. If a method has a sufficient condition for pareto optimality it means that the using this will always result in a pareto optimal solution (Marler and Arora, 2004).

Various methods exist to solve MOO problems, some of which are more complex than others and may not easily be understood by the ordinary person. Simpler methods that are easily understood include the lexicographic method, the weighted sum method and the weighted product method. More complex method such as elimination and choice expressing reality (ELECTRE), the technique for order of preference by similarity to ideal solution (TOPSIS) and analytic hierarchy process (AHP) may produce mathematically superior results but unfortunately require advanced programming skills, sophisticated software and a deep understanding of advanced mathematics. As such these methods are not considered appropriate for a model that is to be used by the ordinary person, but the interested reader is referred to Chen (2000), Figueira and Roy (2002) and Winston (2003). Thus, the methods that will be used are the lexicographic method, the weighted sum method and the weighted product method. All three methods will be implemented so as to increase the robustness to the analysis that will be performed.

2.2.1 Lexicographic method

The lexicographic method is one of the simplest methods for solving MOO problems. In this method the objective functions are arranged in order of importance, with the most important objective function being first. The objective functions are then solved one at a time starting with the most important objective. This method provides a sufficient condition for pareto optimality indicating that the solution obtained is always a good solution (Marler and Arora, 2004). A big flaw that the lexicographic method has is that it does not consider by how much one objective may be more important than other. It may not fully reflect the optimal solution space and less optimal solutions may be found. These relative importances can have a definite impact on the results being produced and as such it is necessary to use a method in which they are considered by the model. One such method is the weighted sum method.

2.2.2 Weighted sum method

The weighted sum method is the most used approach for solving MOO problems (Marler and Arora, 2004, 2010). It involves determining a weight, w_i, for each of the objective functions in order to convey the importance of each objective relative to the other objectives. A score for each alternative can then be calculated by multiplying the individual weights, w_i, determined for an objective function with the value of that objective function, $F_i(x)$, and then summing these products over all the objective functions. The optimal solution is then found by optimising the composite function defined by Equation (2.1).

$$U = \sum_{i=1}^{N} w_i F_i(x) \tag{2.1}$$

where $\sum w_i = 1$, $w_i \geqslant 0$, N is the total number of objective functions in the problem and i is the index of each individual objective function (Marler and Arora, 2010; Yoon and Hwang, 1995).

The question arises as to how these weights can be set so as to ensure that as little information as possible is lost and "correctness" is maintained (Danielson and Ekenberg, 2016). There are two main methods for setting weights for the objective functions, namely, rating methods and ranking methods. Rating requires the decision-maker to assign independent values of relative importance to each objective function based on his/her experience. Ranking methods involve assigning a rank, on a scale of 1 to the N, to each objective function rather than a specific value. Weights are then derived from this ranking (Marler and Arora, 2010). Danielson and Ekenberg (2016) found that it is better to use the ranking method as it is easier for the average decision-maker to provide a rank than it is to provide a precise number. Thus, MOO methods that use ranking methods produce more accurate results than rating methods.

In the ranking method, the value of one is assigned to the most important objective function, the value of two is assigned to the second most important objective function, and this continues until the rank of last objective function is equal to N. An example of this ranking method for the metrics discussed in Section 2.1 can be found in Appendix E.1. Next, it must be determined how the weights can be derived from the ranking of the objective functions. Danielson and Ekenberg (2016) present six methods for determining the weights of objective functions from the ranking of objective functions and show that they are very efficient methods. These methods are: the equal weights (EW) method, the rank sum (RS) method, the rank reciprocal (RR) method, the rank order centroid (ROC) method, the SR method and the SIMOS method. The SR method is an additive combination of the RS and RR methods, hence the SR method and the SIMOS method is named after its developer, J. Simos (Danielson and Ekenberg, 2016; Figueira and Roy, 2002).

The EW method is essentially not a ranking method as it assumes that all objective functions are equally important, but it is used as the bases for the other five weighting methods. In the ROC method the weight value is a value that estimates the distance between adjacent ranks on a normalized scale. The RS method is based on the idea that the weights obtained should be a direct representation of the ranking given by the decision-maker. In the RR method the weights used have a similar origin as the RS weights but are based on the reciprocals of the rank order for each item ranked. The SR method was developed to reduce extreme values being produced as a result of assumptions about how the objective functions were ranked by the decision-maker. The SIMOS method was developed to try to provide decision makers with a simple method of determining weights that does not require any prior knowledge about decision analytical techniques (Danielson and Ekenberg, 2016). To improve the robustness of an analysis when using the weighted sum method, it is suggested that (2.1) be minimised six times, each time using the weights from one of the aforementioned methods of obtaining weights.

The weighted sum method is an appropriate method for the ordinary person to use to solve MOO problems as it is easy to understand and computationally simple. Furthermore, if all the weights are positive (as will be the case if the above-mentioned methods of obtaining weights are used), minimising (2.1) will result in a Pareto optimal solution which is the aim of any MOO method (Marler and Arora, 2004). However, this method does not allow for objectives with differing measurement units to be compared as these objectives cannot be added. For this reason, a common numerical scaling system such as normalisation is needed to allow for the addition among the objective values (Yoon and Hwang, 1995).

The weighted sum method is not able to compare objectives that are measured with different units and normalising all the objective values in an analysis can result in accuracy being lost in the MOO selection. As such, it would be useful to use a method in which objectives with differing units can be compared without any additional data manipulation. One such method is the weighted product method.

2.2.3 Weighted product method

The weighted product method is a method for solving MOO problems that possesses sound logic and a simple computational process (Yoon and Hwang, 1995). Like the weighted sum method, it is a scoring method in which weights are used to determine a score for each alternative which can then be compared to find the optimal solution. These weights indicate the relative importance of the objective functions. In the weighted product method, the weight of each objective function becomes the exponent of the value of that objective function. The values of all the objective functions, $F_i(x)$, to the power of their weights, w_i, are then multiplied with each other to determine the score of an alternative. The optimal solution is then found by optimising the composite function defined by (2.2) (Yoon and Hwang, 1995).

$$U = \prod_{i=1}^{N} [F_i(x)]^{w_i} \tag{2.2}$$

The methods used to determine the weights of the objective functions are the same as those used in the weighted sum method. This method addresses one of the key drawbacks of the weighted sum method in that it is capable of comparing objectives of differing units of measurement without having to transform objective functions or perform normalisation (Marler and Arora, 2004). This method is computationally easier to implement than the weighted sum method but is by no means superior as both methods will produce pareto optimal solutions. Gerasimov and Repko (1978) successfully used this method to solve a MOO of a truss and found that this method produces a *valid compromise*. As this method is easy to understand and implement and will always produce a pareto optimal solution it is appropriate to solve the financial MOO problem posed in Chapter 1.

The methods discussed in this section can be used to find an optimal solution when there are multiple objective functions that need to be considered. By using all three of the above-mentioned methods, the robustness of the analysis can be increased. However, these methods do not account for the uncertainty that may be present in a problem. The uncertainty of year's metric values to use as inputs influences the results that will be obtained and as such must be incorporated into the model. This can be achieved using stochastic modelling.

2.3 Stochastic modelling

Real-world problems are filled with uncertainty and thus to obtain robust results it is essential that this uncertainty be taken into account. Stochastic modelling is used when there is uncertainty present in the situation being modelled. Various stochastic modelling methods exist that all have these three things in common: these methods reflect all aspects of the problem being studied, a probability is assigned to each event in the model and these probabilities can be used to determine a solution to the problem being modelled (Breuer, 2014).

Sinha and Prasad (1979) show that there are five main methods for modelling stochastic problems, these are: (i) linear regression, (ii) exponential regression (iii) polynomial curve fitting (iv) autoregressive and moving-average models of different orders assuming stationarity and (v) non-stationary time series models of the autogressive and moving average (ARIMA) type. From these five methods, twelve different stochastic models emanate.

Deciding which type of stochastic model to use depends on the type of data that is being entered into the model and for how long this data is available. A stationary time series is one in which the statistical properties such as mean, variance, autocorrelation, etc. remain constant over time. However, many times series found in industry, business and economics are non-stationary, and as such have no natural constant mean level over time (Box et al., 2015). The weighted-moving average method is a non-stationary method and Box et al. (2015) show that when working in economic models, using a weighted-moving average method to account for stochasticity is appropriate. Furthermore, of all the methods mentioned above, the weighted-moving average is the least sensitive to the amount of data that is used and as such is a wise choice when data is limited.

2.4 Problem context

There are four main metrics that must be considered cooperatively in order to make a meaningful investment that will yield returns in the long-term. The ordinary person should invest in shares that have low market liquidity values as these shares have higher average returns. Shares that have a market risk equal to the financial risk tolerance of the ordinary person investing should be selected. This ensures that should a loss be experienced, it is a loss that the ordinary person can afford. Lastly, as the ordinary person invests to receive returns that will fulfil their long-term goals, it is imperative the ordinary person invest in companies that are experiencing growth. Companies that are experiencing growth receive higher returns in the future and thus investing in these companies will be beneficial to the ordinary person. Currently no established method for evaluating these four metrics together exists and as such a customised solution is required.

The data needed for this evaluation is stochastic and finding the optimal ranking of investments for the ordinary person requires stochastic modelling. As the data is a time series and consists of a maximal training set of only three years, a weighted-moving average method will be used to determine the metric values to be used by the model. Furthermore, as each metric has its own objective, to find the optimal investment, it is necessary to find an investment that has the combined best result of all four metrics. Thus, a MOO model should be used to help the ordinary person make meaningful investments.

Although many MOO modelling methods exists they may not all yield pareto optimal solutions. Three simple methods were selected, namely the lexicographic method, the weighted sum method and the weighted product method. These methods were selected as each one is easy to understand and produces reliable results, making them ideal to be used in a model that will be used by the ordinary person.

In this project a customised MOO model will be built using the aforementioned three MOO methods. This model will consider the four main metrics, calculated using the weighted-moving average method, and their respective objectives simultaneously to produce a result that will help the ordinary person select the best investment options. Furthermore, as it is possible for the ordinary person to have a RTS between 20 and 63, the model will determine the best investment options at each of these RTS values. The model will determine the return on investment for the suggested investment options and these results will be compared to the returns produced by various unit trust in order to determine the validity of the model.

In the following chapter the model that was built is presented. It is explained how each of the four metrics is calculated and how the selected modelling approaches can be used to solve the research problem. Furthermore, how the stochastic elements of the data were handled is explained.

Chapter 3

Conceptual Model

In this chapter the conceptual model that was built in order to solve the research problem posed in Chapters 1 and 2 is presented. A diagram of this model can be seen in 3.1.

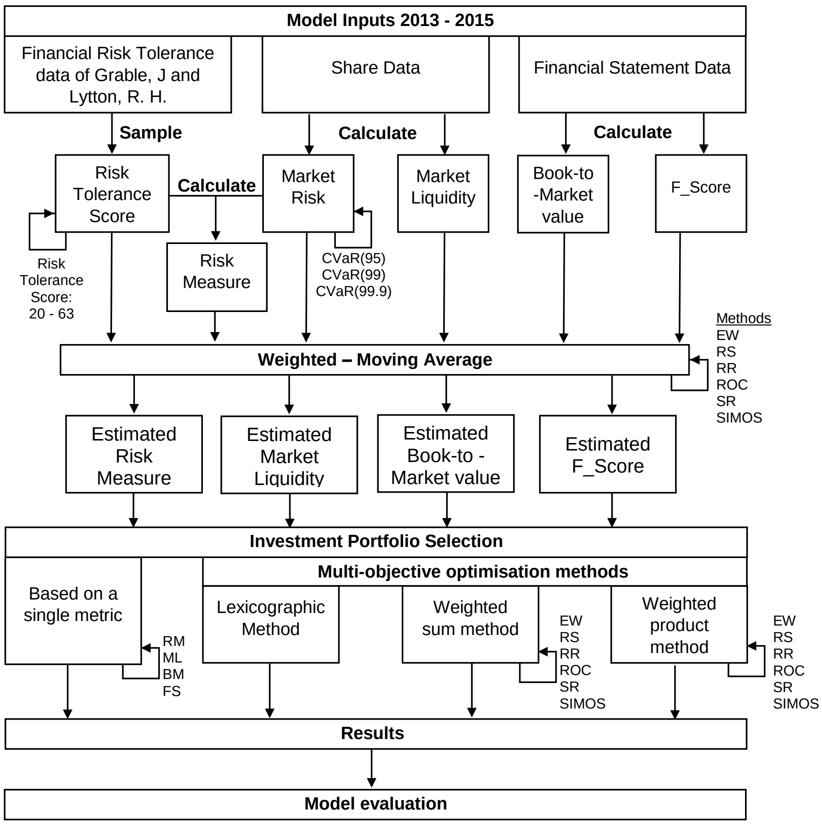

Figure 3.1: Diagram of the model that was built

This chapter also discusses how the companies that will be considered by the model were selected and how data was obtained for these companies. Furthermore, the methods for determining the individual metrics as discussed in Sections 2.1.1, 2.1.3, 2.1.4 and 2.1.2 from this data is presented. The mathematical model for solving the research problem as well as the modelling approaches for solving this multi-objective optimisation (MOO) problem are discussed. Many of the modelling approaches used as well as the weighted-moving average method require the use of weights. For this reason, in this chapter it is shown how these weights are obtained. Lastly, what the model produces and how these results are evaluated is discussed. [1]

3.1 Data gathering and processing

In this section the methods that were followed to gather all the data needed to calculate the metrics described in Sections 2.1.1, 2.1.3, 2.1.4 and 2.1.2 are explained. Two main sets of data had to be obtained, the financial statement data and share data for the companies. The financial statement data of the companies used by the model were obtained from the annual reports of these companies. The share price data was found on Google Finance and was extracted into a usable spreadsheet using Google Sheets. Furthermore, it is explained how the companies that are considered in the model were chosen.

3.1.1 Selection of companies

When investing, most investment firms suggest investing in the companies of a known market index. A market index is a weighted average of different shares from a section of the stock market and measures the value of groups of shares. The Johannesburg Stock Exchange (JSE) has two indices which it measures, the JSE Top 40 and the JSE Top 100. The JSE Top 40 is the most well-known index in South Africa and includes the 40 largest companies, by market capitalisation, listed on the JSE. This index is monitored by most people as an overall benchmark for the local exchange.

As the JSE Top 40 is used as the benchmark for investments in South Africa and consists of a wide variety of companies from different sectors, it was decided that these companies will be used by the model to find portfolios for the ordinary person. However, 2 of the JSE Top 40 companies have not been incorporated since the beginning of 2013 and thus they do not have sufficient historical data to be considered by the model. For this reason, a list of 40 companies was created from the JSE Top 100, as at January 2018, that includes all the eligible Top 40 companies and as well as companies 41 and 42. This group of companies is referred to as the JSE 40 and can be seen in Appendix A.

3.1.2 Financial statement data

In order to calculate the book-to-market (BM) value and the F_ Score of a company, information must be obtained from the financial statements of that company. As these are publicly listed companies, by law their financial statements must be made available to the public. Furthermore, these financial statements must be audited by an independent auditor making the information in these financial statements highly reliable.

The total sales, cost of sales, gross and net income before extraordinary items were obtained from the income statement section of the financial statements. The intangible assets, current assets, total assets, total long-term debt, current liabilities and total liabilities of a company were found on the balance sheet. In the statement of cash flows the cash flow from operating activities was found. The value of a company's preference shares is located in the statement of capital note. Lastly, the total number of shares that have been issued by the company and whether shares were issued during the year were found in the long-term debt note to the financial statements. A detailed procedure of how to find and extract these values can be found in Appendix B.1.

[1]This chapter and its appendices contain a level of detail sufficient to enable the ordinary person to repeat the calculations. A conscientious trade-off was made between brevity and achieving the objectives of the study.

3.1.3 Share data

In order to calculate the market liquidity, ML, and market risk, MR, values of a company, the share price and share volume data for that company is required. This information can easily be found on Google Finance. Unfortunately, Google Finance does not have a download option so that this information can be extracted for use. To solve this problem, Google linked Google Finance to Google Sheets. In Google Sheets, information can be gathered from Google Finance by using the GOOGLEFINANCE function. Depending on the parameters which are entered into the GOOGLEFINANCE function, share data can be captured in a spreadsheet.

The close price P_{Close}, high price P_{High}, low price P_{Low} and volume for each company's shares are required to calculate the ML and MR values. The volume indicates how many shares were bought and sold during a day. Furthermore, it is important to know what currency the price values are being reported in to allow for comparisons between different companies.

Following the detailed steps outlined in Appendix B.2, data was imported into a spreadsheet. An example of such a spreadsheet as shown in Appendix B.2 in Figure B.8. This spreadsheet was used by the model to calculate the necessary metric values.

3.2 Model Inputs

This model has three main inputs, namely, the financial statement data and share data for the JSE 40 companies, and the findings of Grable and Lytton (1999) concerning financial risk tolerance. These inputs are used to determine the market liquidity, market risk, financial risk tolerance, risk measure, BM value and F_ Score for each company being considered by the model. How these metrics are calculated is explained in the following sections.

3.3 Metric Calculations

In this section the equations and methods for calculating each of the required metrics, as discussed in Sections 2.1.1, 2.1.3, 2.1.4 and 2.1.2, are presented.

3.3.1 Market Liquidity

The market liquidity for the companies was found using the liquidity index, LIX, developed by Danyliv et al. (2014).

The LIX for a particular day, T, is calculated using Equation (3.1).

$$LIX_T = \log_{10} \frac{V_T P_{Close,T}}{P_{High,T} - P_{Low,T}} \tag{3.1}$$

Where V_T is the trading volume for day T, $P_{Close,T}$ is the closing price of the shares on day T, and $P_{High,T}$ and $P_{Low,T}$ are the highest and lowest selling prices of the shares on that particular day T respectively.

The LIX for a specific year is the average of all the individual LIX_T values for every day, T, in that year. This process is defined in Equation (3.2).

$$LIX = \frac{1}{TD} \sum_{T=1}^{TD} LIX_T \tag{3.2}$$

Where TD is the total number of days on which shares were traded. An example of how the LIX is calculated can be seen in Appendix C.1.

3.3.2 Book-to-Market Value

The BM value of a company can be determined using Equations (3.3), (3.4) and (3.5). An example of how the BM value is calculated can be seen in Appendix C.2.

$$BV = TA - (TL + PS + IA) \tag{3.3}$$

$$MV = MC = SO \times SP \tag{3.4}$$

$$BM = \frac{BV}{MV} \tag{3.5}$$

Where BV is the book value, TA is the total assets, TL is the total liabilities, PS is the value of the preference shares and IA is the value of the intangible assets. Furthermore, MV is the market value, MC is the market capitalisation, SO is the number of shares outstanding and SP is the share price.

3.3.3 F_ Score

The F_ Score was calculated using the method suggested by Piotroski (2000). This method is laid out below and an example of how the F_ Score is calculated can be found in Appendix C.3. The F_ Score is calculated as follows:

$$\begin{aligned} \text{F\_ Score} = {} & \text{F\_ ROA} + \text{F\_ CFO} + \text{F\_ }\Delta\text{ROA} + \text{F\_ ACCRUAL} + \text{F\_ }\Delta\text{LEVER} + \\ & \text{F\_ }\Delta\text{LIQUID} + \text{EQ\_ OFFER} + \text{F\_ }\Delta\text{MARGIN} + \text{F\_ }\Delta\text{TURN} \end{aligned} \tag{3.6}$$

Where each term in Equation (3.6) is a binary variable.

F_ ROA relates to the profitability of a company and provides information about a company's ability to generate money. It is calculated by:

$$\text{F\_ ROA} = \begin{cases} 0 \text{ if ROA } > 0 \\ 1 \text{ if ROA } \leq 0 \end{cases} \tag{3.7}$$

$$\text{ROA} = \frac{NI}{\text{Beginning of the year } TA} \tag{3.8}$$

Where ROA is the return on assets, NI is the net income before extraordinary items and TA is the total assets of the company.

F_ CFO relates to the profitability of a company and provides information about a company's ability to generate money. It is calculated by:

$$\text{F\_ CFO} = \begin{cases} 0 \text{ if CFO } > 0 \\ 1 \text{ if CFO} \leq 0 \end{cases} \tag{3.9}$$

$$\text{CFO} = \frac{CFFO}{\text{Beginning of the year } TA} \tag{3.10}$$

Where CFO is the operating-cash-flow to total assets ratio and CFFO is the cash flow from operations of the company.

F_ ΔROA relates to the profitability of a company and provides information about a company's ability to generate money. It is calculated by:

$$\text{F\_ }\Delta\text{ROA} = \begin{cases} 0 \text{ if } \Delta\text{ROA} > 0 \\ 1 \text{ if } \Delta\text{ROA} \leq 0 \end{cases} \tag{3.11}$$

$$\Delta\text{ROA} = \text{Current year's ROA} - \text{Prior year's ROA} \tag{3.12}$$

F_ ACCRUAL is the relationship between the earnings and cash flow of a company relates to the profitability of the company. It is calculated by:

$$\text{F\_ ACCRUAL} = \begin{cases} 0 \text{ if CFO} > \text{ROA} \\ 1 \text{ if CFO} \leq \text{ROA} \end{cases} \tag{3.13}$$

F_ ΔLEVER relates to the financial performance of a company and indicates whether a company can sufficient internal funds to meet its debt obligations. It is calculated by:

$$\text{F\_ }\Delta\text{LEVER} = \begin{cases} 0 \ \Delta\text{LEVER} < 0 \\ 1 \ \Delta\text{LEVER} \geq 0 \end{cases} \tag{3.14}$$

$$ATA = \frac{TA \text{ (current year)} + TA \text{ (previous year)}}{2} \tag{3.15}$$

$$\Delta\text{LEVER} = \frac{TL \text{ (current year)}}{ATA \text{ (current year)}} - \frac{TL \text{ (previous year)}}{ATA \text{ (previous year)}} \tag{3.16}$$

Where ATA is the average total assets and TL is the total long-term debt.

F_ ΔLIQUID relates to the financial performance of a company. It measures the changes in a company's capital structure and its ability to pay its debts. It is calculated by:

$$\text{F\_ }\Delta\text{LEVER } = \begin{cases} 0 \ \Delta\text{LIQUID} > 0 \\ 1 \ \Delta\text{LIQUID} \leq 0 \end{cases} \tag{3.17}$$

$$CR = \frac{CA}{CL} \tag{3.18}$$

$$\Delta\text{LIQUID} = CR \text{ (current year)} - CR \text{ (previous year)} \tag{3.19}$$

Where CR is the current ratio, CA are the current assets and CL are the current liabilities of the company.

EQ_ OFFER related to the financial performance of a company and indicates whether or not shares were issued by the company. It is calculated by:

$$\text{EQ\_ OFFER} = \begin{cases} 0 \text{ if no common shares were issued during the year} \\ 1 \text{ otherwise} \end{cases} \tag{3.20}$$

F_ ΔMARGIN relates to the efficiency of a company's operations and is calculated by:

$$\text{F\_ }\Delta\text{MARGIN } = \begin{cases} 0 \ \Delta\text{MARGIN} > 0 \\ 1 \ \Delta\text{MARGIN} \leq 0 \end{cases} \tag{3.21}$$

$$GMR = \frac{GM}{TS} \tag{3.22}$$

$$\Delta\text{MARGIN} = GMR \text{ (current year)} - GMR \text{ (previous year)} \tag{3.23}$$

Where GMR is the gross margin ratio, GM is the gross margin and TS is the total sales of the company.

F_ ΔTURN relates to the efficiency of a company's operations and is calculated by:

$$\text{F\_ }\Delta\text{TURN } = \begin{cases} 0 \ \Delta\text{TURN} > 0 \\ 1 \ \Delta\text{TURN} \leq 0 \end{cases} \tag{3.24}$$

$$ATR = \frac{TS}{\text{Beginning of the year } TA} \tag{3.25}$$

$$\Delta\text{TURN} = ATR \text{ (current year)} - ATR \text{ (previous year)} \tag{3.26}$$

Where ATR is the asset turnover ratio of the company.

3.3.4 Market Risk

This subsection explains how to calculate the conditional value-at-risk (CVaR) value for a company for a specific year using the closing price share data for that company.

To calculate the CVaR(95) value for a company's shares, the following formulas are used:

$$R_T = \frac{P_{Close,T} - P_{Close,T-1}}{P_{Close,T-1}} \tag{3.27}$$

$$CN(95) = (1 - 95\%) \times C \tag{3.28}$$

$$\text{CVaR}(95) = \frac{1}{CN} \sum_{T=1}^{CN} R_T \tag{3.29}$$

Where R_T is the return on the shares and $P_{Close,T}$ and $P_{Close,T-1}$ are the closing prices of the shares on day T and day $T-1$, respectively. Furthermore, CN is the CVaR number and C is the total number of returns calculated, in other words, the count.

To calculate CVaR(95) calculate the return, R achieved by the shares of a company by using Equation (3.27) for every day in the trading year. Count the total number of returns calculated and set this value equal is C. The calculated return values must then be arranged in ascending order and each return value is assigned a number, CN, according to its position in this list. The CN number that corresponds to CVaR(95) should then be found using (3.28). The CVaR value is then determined by averaging all the return values from 1 to the CN number. This CVaR value in the market risk value.

To calculate CVaR(99) and CVaR(99.9), the above process is used and the 95% in Equations (3.28) and 3.29 are replaced with 99% and 99.9% respectively. A detailed example of this process can be found in Appendix C.4.

3.3.5 Financial risk tolerance

As explained in Section 2.1.4, Grable and Lytton (1999) found that the average risk tolerance score (RTS) was 37, with a standard deviation of 6.40. The RTS had a range of 20 – 63 and it is assumed that these results were normally distributed. The model will be configured to find investment portfolios for the ordinary person over this range of RTS values.

3.3.6 Risk Measure

The risk measure, RM, is a measure created to match the financial risk tolerance of the user with the calculated market risk, MR, value. It is the difference between the user's RTS and the market risk of a company as shown in Equation (3.30), where RM is the risk measure and MR is the calculated market risk value. Thus, the lower the RM value of a company, the closer the market risk of that company is to the RTS of the user and thus the company's risk is matched to the user's financial risk tolerance.

$$RM = RTS - MR \tag{3.30}$$

There is one main problem with this risk measure and that is that the units of the RTS and the market risk are not the same and can thus not be compared. The RTS is a score out of 69, with a higher score indicating a higher financial risk tolerance. The risk measure is calculated as a percentage with a higher percentage indicating a higher market risk. To consolidate these two metrics so that they can be compared the RTS is converted into a percentage using Equation (3.31).

$$RTS(\%) = \left(\frac{RTS - 20}{49}\right) \times 100 \tag{3.31}$$

As the risk tolerance assessment is out of 69 but the lowest possible score is 20, 20 must be subtracted from the user's RTS and the total $(69 - 20 = 49)$. Equation (3.30) can now be rewritten as:

$$RM = \left(\frac{RTS - 20}{49}\right) - \mid MR \mid \tag{3.32}$$

3.4 Management of the stochastic elements

The data being entered into the model was collected for the years 2013 to 2015, and thus metric values can be determined for each of these years. As such, this makes the data being entered into the model stochastic. As the data is only available for three years it is not practical to sample values from a distribution as this distribution may not be accurate. Thus, some of the other stochastic models as discussed in Section 2.3 will also not produce valid results. Therefore, it was decided to address the stochastic nature of this data using a weighted-moving average.

Danielson and Ekenberg (2016) showed that the methods used to determine weights for the weighted sum MOO method can be used in a weighted-moving average calculation. As such, the weighted-moving average metrics will be calculated using weights obtained using the equal weights (EW), rank sum (RS), rank reciprocal (RR), rank order centroid (ROC), SR and SIMOS methods. These weights can be seen in Table D.2.

For each modelling approach the model was run six times, each time using weights obtained using a different method. By doing this, different combinations of how important the value of a metric for a specific year is can be considered and investigated. An example of how the metric values are obtained using the weighted-moving average can be seen in Appendix D.

3.5 Weights calculations

In this section the equations for determining weights using the six different methods as discussed in Section 2.2.2 are presented. These weights will be used in the weighted-moving average calculations as well as in the weighted sum and weighed product methods. For all these methods, N is the total number of objective functions being considered and i is the rank of the objective function for which the weight is being calculated. Examples of how each of these methods are applied can be seen in Appendix E in Sections E.2, E.3, E.4, E.5, E.6 and E.7.

$$\text{Equal weights method: } w_i^{EW} = \frac{1}{N} \tag{3.33}$$

$$\text{Rank sum method: } w_i^{RS} = \frac{(N+1+i)}{\sum_{j=1}^{N}(N+1-j)} \tag{3.34}$$

$$\text{Rank reciprocal method: } w_i^{RR} = \frac{\frac{1}{i}}{\sum_{j=1}^{N}\left(\frac{1}{j}\right)} \tag{3.35}$$

$$\text{Rank order centroid method: } w_i^{ROC} = \left(\frac{1}{N}\right)\sum_{j=i}^{N}\left(\frac{1}{j}\right) \tag{3.36}$$

$$\text{SR method: } w_i^{SR} = \frac{\frac{1}{i} + \frac{N+1-i}{N}}{\sum_{j=1}^{N}\left(\frac{1}{j} + \frac{N+1-j}{N}\right)} \tag{3.37}$$

$$\text{SIMOS method: } w_i^{SIMOS} = \frac{(N+1-i)^{N+1}}{\sum_{j=1}^{N}(N+1-j)^{N+1}} \tag{3.38}$$

3.6 Mathematical Model

Below is the mathematical model that will be used to solve the research problem as described in Chapter 1 and Section 2.4. An example of how this model is applied can be found in Appendix F.

<u>Decision variables:</u>

Let:

C be the set of JSE 40 companies such that $C = \{1, ..., 40\}$

M be the set of ranking methods such that $M = \{1, ..., 306\}$

T be the set of risk tolerance values such that $T = \{20, ..., 63\}$

S_{mt} be the set of companies selected by the model (model portfolio) for method m at risk tolerance score t such that $S_{mt} \subset C$ and $S_{mt} = \{1, ..., n\}$ where n is the number of selected companies in S_{mt} and $m \in M$, $t \in T$

K_{cmt} be the set of ranks, k_{cmt} , of the JSE 40 companies where $c \in C$, $m \in M$, $t \in T$

$$x_{cmt} \triangleq \begin{cases} 1 \text{ if company } c \in C \text{ is selected by the model using method } m \in M \text{ at an} \\ \quad \text{RTS } t \in T \\ 0 \text{ otherwise} \end{cases}$$

$RM_{cmt} \triangleq$ risk measure of company $c \in C$ calculated using method $m \in M$ and RTS $t \in T$

$BM_{cmt} \triangleq$ book-to-market value of company $c \in C$ calculated using method $m \in M$ and RTS $t \in T$

$ML_{cmt} \triangleq$ market liquidity of company $c \in C$ calculated using method $m \in M$ and RTS $t \in T$

Objective Function:

$$\text{Min Z} = \sum_{c \in \mathrm{C}} \sum_{m \in \mathrm{M}} \sum_{t \in \mathrm{T}} x_{cmt}\mathrm{k}_{cmt} \tag{3.39}$$

Constraints:

subject to:

$$RM_{cm} \geqslant 0 \quad \forall\, c \in C \quad \forall\, m \in M \quad \forall\, t \in T \tag{3.40}$$

$$BM_{cmt} < 1 \quad \forall\, c \in C \quad \forall\, m \in M \quad \forall\, t \in T \tag{3.41}$$

$$ML_{cmt} \geqslant 7.5 \quad \forall\, c \in C \quad \forall\, m \in M \quad \forall\, t \in T \tag{3.42}$$

$$n = ||S_{mt}|| \quad \forall\, m \in M \quad \forall\, t \in T \tag{3.43}$$

$$n \leqslant 10 \tag{3.44}$$

$$x_{cmt} \in \{0; 1\} \quad \forall\, m \in M \quad \forall\, t \in T \tag{3.45}$$

$k_{cmt} =$ different depending on the portfolio selection method used

$$k_{cmt} \in \{1, ..., 40\} \tag{3.46}$$

$$\begin{aligned} &k_{1mt} \neq k_{2mt} \neq k_{3mt} \neq k_{4mt} \neq k_{5mt} \neq k_{6mt} \neq k_{7mt} \neq k_{8mt} \neq k_{9mt} \neq k_{10mt} \\ &\neq k_{11mt} \neq k_{12mt} \neq k_{13mt} \neq k_{14mt} \neq k_{15mt} \neq k_{16mt} \neq k_{17mt} \neq k_{18mt} \neq \\ &k_{19mt} \neq k_{20mt} \neq k_{21mt} \neq k_{22mt} \neq k_{23mt} \neq k_{24mt} \neq k_{25mt} \neq k_{26mt} \neq k_{27mt} \\ &\neq k_{28mt} \neq k_{29mt} \neq k_{30mt} \neq k_{31mt} \neq k_{32mt} \neq k_{33mt} \neq k_{34mt} \neq k_{35mt} \neq \\ &k_{36mt} \neq k_{37mt} \neq k_{38mt} \neq k_{39mt} \neq k_{40mt} \quad \forall\, m \in M \quad \forall\, t \in T \end{aligned} \tag{3.47}$$

$$K_{cmt(1)} = \min(K_{cmt}) \tag{3.48}$$

$$K_{cmt(2)} = \min(K_{cmt} \backslash K_{cmt(1)}) \tag{3.49}$$

$$K_{cmt(3)} = \min((K_{cmt} \backslash K_{cmt(1)}) \backslash K_{cmt(2)}) \tag{3.50}$$

$$...$$

$$K_{cmt(k)} = \min((...((K_{cmt} \backslash K_{cmt(1)}) \backslash K_{cmt(2)})...) \backslash K_{cmt(k-1)}) \;\forall\, k = \{1, ..., 10\} \tag{3.51}$$

$$S_{mt} = \cup_{k=1}^{10} K_{cmt(k)} \quad \forall\, c \in C \quad \forall\, m \in M \quad \forall\, t \in T \tag{3.52}$$

The model selects investment portfolios that consist of no more than ten companies, S_{mt}, for the ordinary person to invest in. This is accomplished by assigning a rank, k_{cmt}, to each of the JSE 40 companies. This rank indicates how desirable a company is for investment and the smaller the value of a company's rank is, the more attractive the company is for investment. For example, if company ABC has a rank of 1 and company DEF has a rank of 3, it is better to invest in company ABC. This rank can be determined using various methods and this model uses a total of 306 different ranking methods, m. These methods are explained in detail in Section 3.7. A specific rank can only be assigned to one company while using a specific method and RTS value, and thus Equation (3.47) is used to ensure that there are no duplicate ranks. Furthermore, as the risk measure is dependent on the RTS of the ordinary person and it is possible to have a RTS, t, of 20 – 63, it is necessary to factor the RTS into the model. As 306 ranking methods are used, the model produces 306 different portfolios at each RTS. This results in a total of 13 464 model portfolios being produced.

The model should select the most desirable companies for all 13 464 model portfolios and the lower the value of a company's rank is, the more desirable it is. For this reason, the model aims to minimise the sum of the ranks of all the companies selected to be part of a model portfolios for all 13 464 model portfolios, as shown in Equation (3.39).

Model portfolios are selected using the procedure described by Equations (3.48), (3.49), (3.50), (3.51) and (3.52). Once the ranks for each company are determined using a specific method and RTS value, the ranks are added to a set, K_{cmt}. To create the model portfolio, it is necessary create a subset of K_{cmt} that contains the ten smallest rank values. This is accomplished by individually finding the ten smallest rank values and then placing these individual values into a new set. This is accomplished as follows:

Equation (3.48) removes the minimum element of set K_{cmt} from the set. Equation (3.49) removes the second minimum element of set K_{cmt} from the set by finding the minimum of the set different of original set and the set created using Equation (3.48). Equation (3.50) removes the third minimum element of set K_{cmt} by finding the minimum value of the set difference between, the set difference of the original set and the minimum value of that set, and the set created using Equation (3.49). This process of removing the minimum value from the set continues and the equation for removing the k^{th} minimum value from the set is explained by Equation (3.51). Finally, a set with the 10 smallest ranking values is created by adding the results of Equation (3.51), using $k = 0$ to 10, together. This is described by Equation (3.52).

Lastly, as explained in Section 2.1, if a company has a risk measure that is greater or equal to 0, a BM that is smaller than 1 and/or a market liquidity that is greater or equal to 7.5, it is a good company to invest in. Thus, the model eliminates all the companies that do not conform to these constraints so as to prevent undesirable companies from being selected by the model.

3.7 Modelling approaches

In this section, pseudocode for a model that can be used to solve the research problem posed in Chapter 1 and Section 2.4 is given. The model selects model portfolios based on the individual metric values as well as by using three different MOO methods is given. These methods are the lexicographic method, the weighted sum method and the weighted product method. An example of these methods are applied can be seen in Appendix F. The base algorithm is followed for all these methods.

Algorithm 1: Base algorithm

Input : Financial statement data, share data and RTS data of Grable and Lytton (1999)

Output: Database of 13 464 model portfolios

repeat
  set RTS equal to 20
  repeat
    Calculate the MR, RM, ML, BM and FS values for each of the 40 companies for the years 2013 to 2015.
    repeat
      Calculate the weighted-moving average $\overline{MR}$, $\overline{RM}$, $\overline{ML}$, $\overline{BM}$ and $\overline{FS}$ values using the EW method for each of the 40 companies.
      if *A company has a* $RM < 0$ **then**
        Eliminate that company from consideration in the model.
      if *A company has a* $ML < 7.5$ **then**
        Eliminate that company from consideration in the model.
      if *A company has a* $BM \geq 1$ **then**
        Eliminate that company from consideration in the model.
      if *Selecting companies based on only one metric value* **then**
        execute algorithm 2;
      else if *Lexicographic method* **then**
        execute algorithm 3 ;
      else if *Weighted sum method* **then**
        execute algorithm 4;
      else
        execute algorithm 5;
      return *The first ten companies from the list produced in the previous step*
    until *The weighted-moving average metrics have been calculated using the EW, RS, RR, ROC, SR and SIMOS methods*;
  until *The* MR *has been calculated using CVaR(95), CVaR(99) and CVaR(99.9)*;
until *Investment Portfolios have been selected using an RTS of 20 to 63*;

3.7.1 Based on individual metric values

When selecting model portfolios based on only a single metric Algorithm 2 is used by the model.

Algorithm 2: Based on only one metric value pseudo code

foreach *Metric value* (RM, ML, BM *and* FS) **do**
  Select the ten companies with the smallest metric values

3.7.2 Lexicographic method

In the lexicographic method the objective functions are solve one at time starting with the most important objective function. When using the lexicographic method, the model executes the base algorithm and Algorithm 3.

Algorithm 3: Lexicographic method pseudo code

Minimise the RM values.

Minimise the ML values.

Minimise the BM values.

Minimise the FS values.

3.7.3 Weighted sum method

In the weighted sum method, a score is calculated for each company by multiplying weight values (that indicate the importance of each objective function) with the objective function values. The products are then summed over all the objective functions. When using the weighted sum method, the model executes the base algorithm and Algorithm 4.

Algorithm 4: Weighted sum method pseudo code

Save the calculated RM values in an array, RM_ array.

Sort the remaining RM values in ascending order.

Let rank_RM be an empty array.

Set counter equal to 0.

foreach RM *value in the RM_ array* **do**
- Increment counter
- Let rank_RM = counter

Save the calculated ML values in an array, ML_ array.

Sort the remaining ML values in ascending order.

Let rank_ML be an empty array.

Set counter equal to 0.

foreach ML *value in the ML_ array* **do**
- Increment counter
- Let rank_ML = counter

Save the calculated BM values in an array, BM_ array.

Sort the remaining BM values in ascending order.

Let rank_BM be an empty array.

Set counter equal to 0.

foreach BM *value in the BM_ array* **do**
- Increment counter
- Let rank_BM = counter

Save the calculated FS values in an array, FS_ array.

Sort the FS values in ascending order.

Let rank_FS be an empty array.

Set counter equal to 0.

foreach FS *value in the FS_ array* **do**
- Increment counter
- Let rank_FS = counter

repeat
- Calculate the score for each company using the weights obtained using the EW method, the values in the four rank_ arrays and Equation (2.1).
- Minimise the score values.

until *The score for each company has been calculated using weights obtained by using the EW, RS, RR, ROC, SR and SIMOS methods.*;

3.7.4 Weighted product method

In the weighted product method, as with the weighted sum method, a score is calculated for each company. This score is calculated by multiplying the values of all the objective functions to the power of their weights. When using the weighted product method, the model executes the base algorithm and Algorithm 5.

Algorithm 5: Weighted product method pseudo code

repeat
- Calculate the score for each company using the weights obtained using the EW method and Equation (2.2).
- Minimise the score values.

until *The score for each company has been calculated using weights obtained by using the EW, RS, RR, ROC, SR and SIMOS methods.*;

3.8 Model output

Every time the model is run, it produces a list of at most ten companies which it has selected to be the best investment options. This list will be an investment portfolio. Although the recommended number of companies in a portfolio is thirty (Statman, 1987), it was decided to use a list of ten companies as the ordinary person does not want to struggle with monitoring and managing such a large portfolio. The inconvenience of having to manage a large portfolio may outweigh the user's desire to save on brokerage fees. This may lead to the ordinary person using the services of a financial broker rather than the model.

A single RTS value will be used in every model run. The model selected model portfolios based on the individual metric values as well as with three different MOO methods. These are the four modelling approaches used in this project. Due to the fact that four different modelling approaches were used with market risk values that are calculated using the three different CVaR methods (CVaR(95), CVaR(99) and CVaR(99.9)) and changing weights, a total of 306 lists of ten companies were produced for a *single RTS value*. This process was be repeated for every RTS between 20 and 63. As there are 44 different RTS, a total of 13 464 portfolios were produced by the model.

How the model determines which portfolio produced by the model is the best and whether the model produces worthwhile results is discussed in Section 3.9.

3.9 Model evaluation

In this step, the model is verified and validated to ensure that it is behaving as expected. The financial statement data and share data for the JSE 40 companies were collected from 2013 to 2015. This data is the training set of the model. For each portfolio it is assumed that the ordinary person invests an equal amount of capital in each of the ten companies.

Once the model has produced 13 464 portfolios, the return on investment for each portfolio was calculated with Equations (3.53) and (3.54) using data collected for 2016 and 2017. This data is the testing set of the model and 2015/12/31 to 2017/12/13 is the testing period.

$$R_c = \frac{P_{Close,2017/12/31} - P_{Close,2015/12/31}}{P_{Close,2015/12/31}} \tag{3.53}$$

$$R_{\text{portfolio}} = \frac{1}{n}\sum_{i=1}^{n} R_c \tag{3.54}$$

Where R_c is the return on a share for a company selected by the model and $R_{portfolio}$ is the return on investment for the model portfolio. The returns on investment for each portfolio are compared with each other to determine which combination of modelling approach, CVaR method and weights produced the highest returns for the ordinary person. An example of how this is done can be seen in Appendix F.

Furthermore, it is necessary to determine whether the model produces worthwhile results. To accomplish this, the model's return on investment results are compared to the returns realised if a person invested in a unit trust. As explained in Section 2.1.4, financial risk tolerance falls into five different risk categories and the model portfolios are generated for different RTS values which also fall into these five risk categories. Thus, it was decided that the return on investment of a model portfolio should be compared to return on investment of units trust that falls in the same risk category as the portfolio. Five large unit trust companies were selected and from each of these companies a unit trust for each of the five risk categories was selected resulting in a total of 25 unit trusts that are used as the benchmark of performance for this project. These unit trusts with their return on investment over the testing period can be seen in Table 3.1. It should be noted that the return on investment for each unit trust listed in Table 3.1 is the return on investment less all investment fees. This was done so as to appreciate the real return on investment that the ordinary person would receiving when investing in a unit trust.

If the returns on investment generated by the model are greater, it is be beneficial for the ordinary person to use the model to develop a customised investment portfolio. However, if the opposite is true, it is better for the ordinary person to pay a financial broker and invest in unit trusts.

The next chapter display the results produced by model developed in this chapter. These results and as well as their implications for the ordinary person are discussed.

Table 3.1: Table of Unit trusts and their return on investments

Risk Category	Unit trust name	Return on investment over the testing period (%)
Conservative	Allan Gray Optimal Fund	15.20
	Coronation Money Market Fund	16.21
	Momentum Money Market Fund	15.56
	Prudential High Yield Bond Fund	12.80
	Stanlib Conservative Fund of Funds	18.62
Moderately conservative	Allan Gray Stable Fund	16.00
	Coronation Balanced Defensive Fund	12.00
	Momentum Diversified Income Fund	18.51
	Prudential Enhanced Income Fund	9.50
	Stanlib Moderately Conservative Fund of Funds	18.47
Moderate	Allan Gray Balanced Fund	17.20
	Coronation Balanced Plus Fund	13.26
	Momentum Enhanced Diversified Growth Fund of Funds	14.27
	Prudential Balanced Fund	9.22
	Stanlib Moderate Fund of Funds	17.71
Moderately aggressive	Allan Gray-Orbis Global Fund of Funds	8.60
	Coronation Market Plus Fund	15.92
	Momentum Enhanced Growth Plus Fund of Funds	14.58
	Prudential Enhanced SA Property Tracker Fund	15.59
	Stanlib Moderately Aggressive Fund of Funds	17.39
Aggressive	Allan Gray Equity Fund	22.60
	Coronation Top 20 Fund	39.71
	Momentum Real Growth Equity Fund	19.89
	Prudential Equity Fund	12.02
	Stanlib Aggressive Fund of Funds	15.09

Chapter 4

Results

In this chapter the results produced by the model developed in Chapter 3 are presented. These results, what they convey and what their implications are for this project are then discussed. Furthermore, the model portfolios' returns on investment are compared to returns received by the unit trusts tabulated in Section 3.9 to determine whether the model produces worthwhile results or not.

In this chapter the terms low, low – medium, medium, medium – high and high will be used to refer to the five categories of risk, for both the risk tolerance score (RTS) categories and the market risk categories. The Google Finance ticker names will be used for the names of the Johannesburg Stock Exchange (JSE) 40 companies. For the full company names, see Table A.1. Furthermore, the term return on investment will be used to describe the return received by a portfolio selected by the model. Lastly, the results in these sections are based on the returns on a share of the JSE 40 companies over the testing period (2015/12/31 – 2017/12/31).

4.1 Model results

4.1.1 Selected companies

Due to the different configurations of the model, 306 portfolios were generated for each RTS(20 – 63), resulting in a total of 13 464 portfolios being produced. As shown in Figure 4.1, not all of the JSE 40 companies were selected by the model. Due to the constraints placed on the metrics ($RM \geqslant 0$, $BM < 1$ and $ML \geqslant 7.5$) and the different model configurations it was found that at most 16 of the 40 (40%) companies were eligible for selection by the model. These 16 companies and their Google Finance tickers are given in Table 4.1. This means that due to the research that the model is based on, 60% of the companies in the JSE 40 will not produce high returns on investment *in the long run* and are thus not eligible for selection.

It should be noted that at an RTS of 20 and 21, all the portfolios produced had no companies in them and at an RTS of 22 and 23 many of the portfolios produced had less than 10 companies in them. This is as at these RTS values the ordinary person has such a low risk tolerance that there are not enough companies remaining in the sample that have a market risk lower or equal to these risk tolerances. Thus, many of the remaining companies have a negative risk measure and are thus excluded from consideration by the model. As a result, there are not enough companies that are eligible for selection and consequently empty or partial portfolios were created.

In Figure 4.1 it is shown that RMH was selected for the most portfolios, appearing

Table 4.1: Table of 16 companies selected by the model

Company Name	**Google Finance ticker**
RMB Holdings Limited	RMH
Rand Merchant Insurance Holdings Ltd	RMI
Woolworths Holdings Limited	WHL
Shoprite	SHP
Foschini Group Ltd	TFG
FirstRand Limited	FSR
Discovery Limited	DSY
Aspen Pharmacare	APN
Investec	INL
Vodacom Group Limited	VOD
British American Tobacco PLC	BTI
Standard Bank Group Ltd	SBK
Redefine Properties	RDF
Mr Price Group Limited	MRP
Intu Properties plc	ITU
The Bidvest Group Limited	BVT

in 12 257 of the 13 464 portfolios produced. RMI, WHL, SHP, TFG, and FSR each appeared in more than 11 600 of the portfolios produced and DSY, APN, INL and VOD all appear in more than 50% of the portfolios produced. The limited group of companies that the model had to choose from after applying the model constraints explains why 10 out of 16 of the companies selected by the model appear in more than 50% of the portfolios produced.

When looking at Figure 4.2 it is seen that seven companies (BVT, KIO,AGL, GLN, IPL, NRP and CPI) had returns greater than 100%. KIO had the highest return on a share with a return of 817.8%. That means that if a person had brought shares in KIO on 2015/12/31 and sold these shares on 2017/12/31, they would have received more than eight times the money that they had invested. However, in this Figure and in Figure 4.1 it is shown that of these seven companies only BVT was ever selected by the model. Figure 4.2 also shows that NPN, SOL, VOD, MNP, APN, WHL, INL, HMN and ITU had negative returns over the testing period. In spite of these companies showing negative returns, as shown in this Figure and in Figure 4.1, 5 of these 9 (VOD, APN, INL, WHL and ITU) companies were selected by the model. It is interesting to note that WHL was the third most selected company and was selected as the most desirable investment 1 468 times (10.789% of the portfolios produced) in spite of the fact that it has a negative return on a share. Furthermore, INL and VOD were selected as part of more than 50% of the model portfolios. This raises the question as to why these companies are being selected even though they have negative returns on a share and why companies with very high returns are not being selected.

The testing set for this model only consists of two years, but the model is designed and built to select companies that will have high returns on a share in the long run. Thus, although many of the companies that the model selects may have negative returns at present, they are expected to have high returns in the future. Furthermore, based on the research, companies that are selected by the model that currently have a positive return on a share, will have even higher returns in the future. It should also be noted

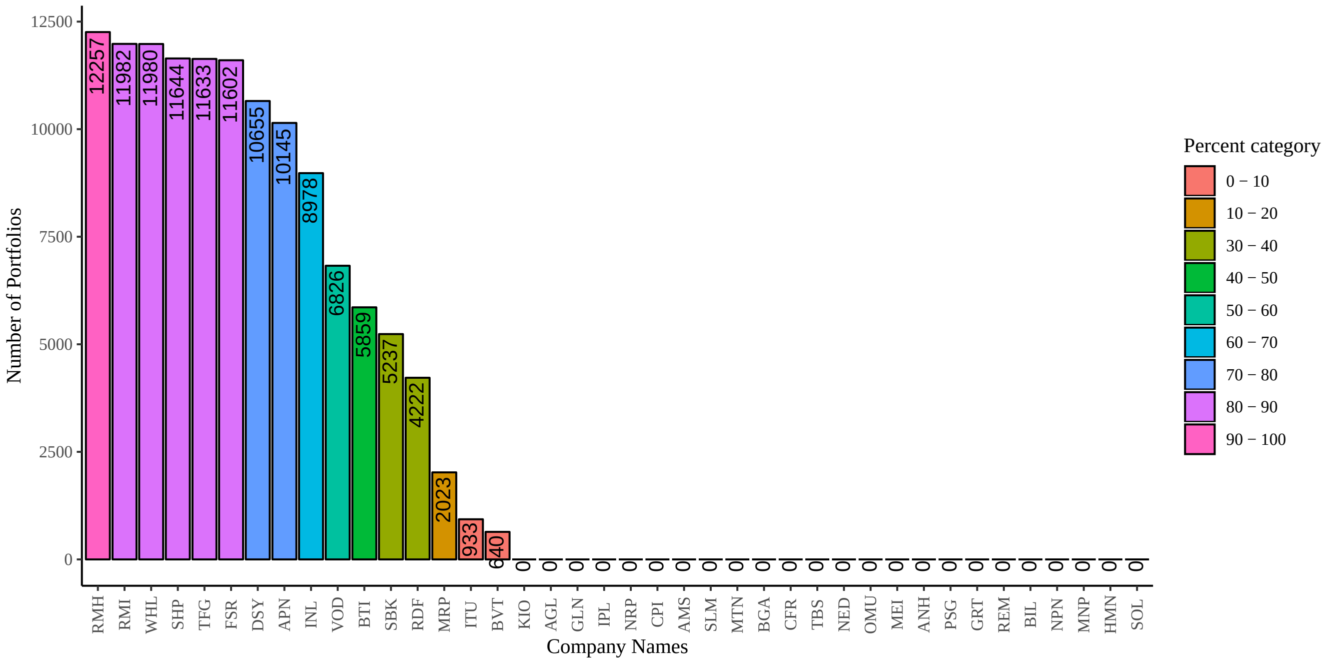

Figure 4.1: The number of portfolios out of 13 464 that each of the JSE 40 companies appear in

that although a company may have very high returns on a share at present, as is the case with KIO, AGL, GLN, IPL, NRP and CPI, they will not necessarily have high returns in the future. This is as a company's growth is not infinite and will eventually stagnate. Companies with high returns at present will continue to grow but will reach their stagnation point relatively soon, whereas companies with lower returns at present will continue growing even after higher return companies have reach their stagnation point. Thus, for long term investments it is more beneficial to invest in companies that have lower returns on a share at present. Evidence of this is presented in Table 4.2.

As shown in the Table, over the last two years WHL had a negative return on a share but over the last ten years this company showed a very high return on a share. The table shows that the opposite is true for KIO. In the short-term, KIO had very high returns but in the long-term, this company's returns were much smaller than those of WHL. This shows that WHL experienced greater financial growth in the long-term than KIO did and is thus a more desirable company to invest. Furthermore, as we use the past as an indication of the future, it is expected that WHL will continue to experience financial growth whereas KIO will have less growth or will stagnate. As explained in Section 2.1.2, companies that are expected to experience growth are desirable investments and this is why the model selected WHL in so many of the portfolios as opposed to KIO.

Lastly, it must be stated that as many of the selected companies have negative returns, the returns on investment for the portfolios produced by the model may be small at present but are likely to have high returns in the future. Thus, it is necessary to examine the returns on investment produced by the model for each of the model portfolios.

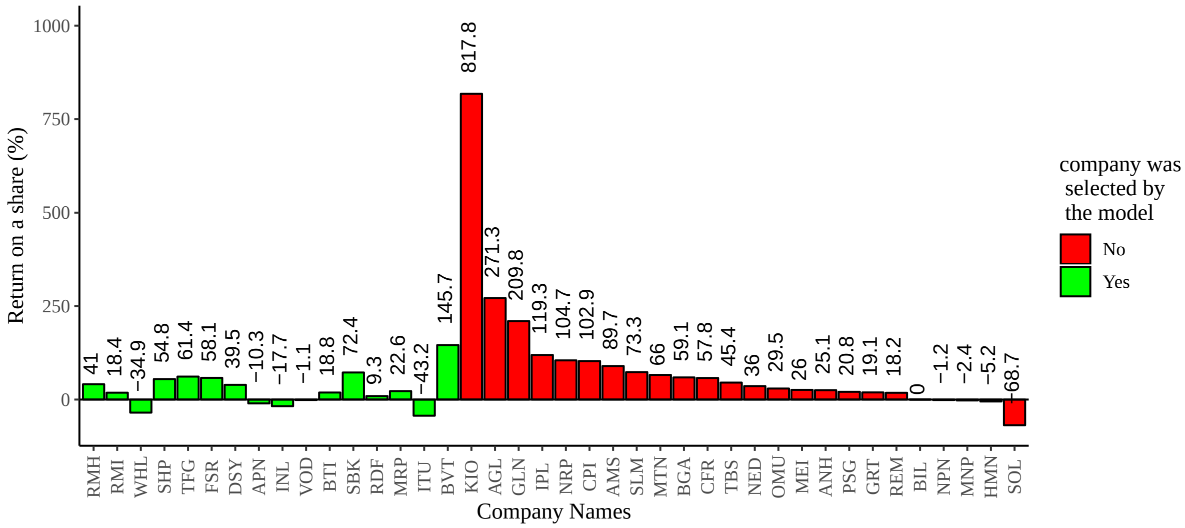

Figure 4.2: The return on a share received by each JSE 40 company over the testing period

Table 4.2: Table of short and long-term return on a share of WHL and KIO

Company	Share Price (ZAC)			Return on a share	
	2007/12/31	2015/12/31	2017/12/31	2015 – 2017	2007 – 2017
WHL	1 534.67	10 015.00	6 531.00	-0.348	3.256
KIO	29 600.00	4 120.00	37 913.00	8.178	0.281

4.1.2 Portfolio returns

The returns produced at all five risk categories were found to follow a Beta distribution with parameters as given in Table 4.3. It was also found that the returns produced at each RTS value are also beta distributed. The Anderson-Darling test was performed on the model returns and for all RTS values and the five risk categories the null hypothesis was rejected with p-values substantially smaller than 0.05. For this reason, these distribution results are significant.

Table 4.3: Table of the Beta distribution parameters for each risk category

	Low	Low – medium	Medium	Medium – high	High
α	21.413	22.212	21.736	21.446	21.275
β	66.417	68.078	66.417	66.008	65.746

As discussed in Section 4.1.1 at an RTS of 20 and 21, all the portfolios produced had no companies in them and thus had a resulting return on investment of 0. Thus, these portfolios have been excluded when analysing the return on investment results.

The portfolios produced at RTS values of 22 and 23 are classified as borderline cases as at these RTS values many of the portfolios produced consist of less than 10 companies. At an RTS of 22, it was found that when the model produced portfolios based on market risk values calculated using the conditional value-at-risk (CVaR)(95) method, the portfolios consisted of 10 companies. However, when the market risk values were calculated using

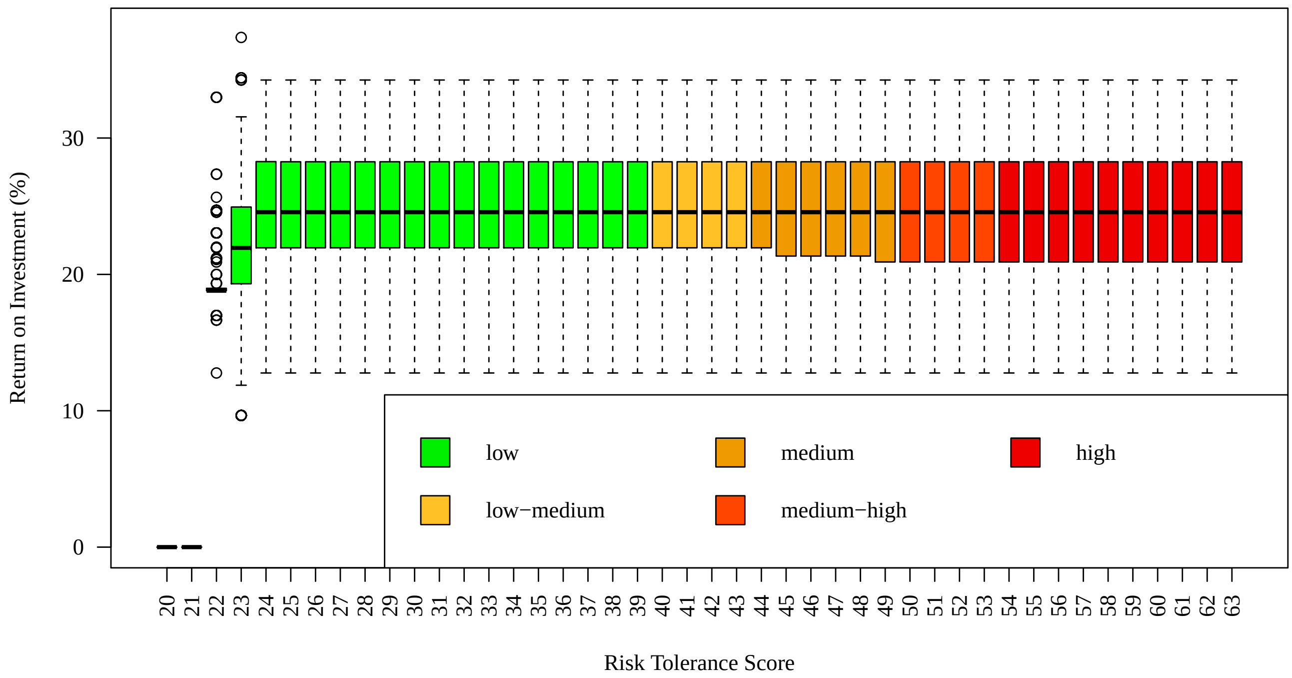

Figure 4.3: The return on investment produced at each RTS over the testing period

the CVaR(99) or CVaR(99.9) methods, the model found that only one company, *BTI*, was eligible and thus produced portfolios with only one company in them. This resulted in 66.667% of portfolios having exactly the same return on investment. This indicates that at an RTS of 22, the model is very sensitive to the certainty with which the market risk metric is determined.

At an RTS of 23, the portfolios with the highest and lowest return on investment were generated as shown in Figure 4.3. These portfolios are listed in Tables 4.4 and 4.5 The highest return on investment of 37.4% was achieved using the weighted product multi-objective optimisation (MOO) method with SIMOS weights, CVaR = 99.9 and equal weights (EW) weighted-moving average metric values. At this configuration, the model selected seven companies that had positive returns and only three companies that had negative returns. The lowest return on investment of 9.7% was achieved when selecting companies using the weighted sum MOO method equal weights, CVaR = 99 and rank reciprocal (RR) weighted-moving average metric values. At this configuration the 5 companies with the lowest returns were selected resulting in a very low return for the portfolio. Both of these portfolios were created only once over all the RTS values and are thus unique portfolios.

As shown in Figure 4.3, from an RTS of 24 – 63, the minimum, median, second quartile and maximum values are exactly the same. The return on investment mean is different at each of these RTS values and gradually decreases as the RTS increases. The only significant changes occur at RTS values of 45 and 49 when the value of first quartile decreases.

The decrease in the first quartile value at a RTS of 45 is explained by the fact that from this RTS, when using the weighted product method with SR weights and CVaR(99.9), the model selected *INL* instead of *TFG*. As *INL* has a negative return on a share and *TFG*

Table 4.4: Highest return portfolio

SHP	54.8
INL	17.7
TFG	61.4
RMI	18.4
VOD	-1.1
RDF	9.3
APN	-10.3
RMH	41.0
SBK	72.4
BVT	145.7
Portfolio return:	37.4

Table 4.5: Lowest return portfolio

WHL	-35.0
INL	-17.7
APN	-10.3
DSY	39.5
RMI	18.4
SHP	54.8
VOD	-1.1
SBK	72.4
BTI	18.8
ITU	-43.2
Portfolio return:	9.6

has a positive return, model portfolios in which *TFG* is replaced by *INL* will experience a decrease in the overall return on investment. The further decrease in the first quartile value at an RTS of 49 is explained by the fact that when using the weighted product method with rank order centroid (ROC) or rank sum (RS) weights and CVaR(99.9), the model selects *SHP* or *INL* instead of *TFG*. As both *SHP* and *INL* have lower returns on a share than *TFG*, portfolios where *TFG* is replaced by *SHP* or *INL*, will have lower return on investments. From a RTS of 45 the number of portfolios with smaller returns increases resulting in decreased first quartile value.

As discussed above and shown in Figure 4.3 the minimum and maximum return on investment portfolios were exactly the same for all RTS values between 24 and 63. These portfolios can be seen in Tables 4.6 and 4.7. This highest return portfolio was generated when the model selected companies based on only the risk measure, using the CVaR(95) method and the SIMOS weighted-moving average metric values. This result makes sense as the risk measure is prioritised in both the model and in calculating the weights used in the MOO methods. This shows that the model is sensitive to the assumptions that it is based on. However, this result contradicts research which urges that all metric objectives should be considered simultaneously when making investment decisions.

The portfolio in Table 4.6 has a lower return than the portfolio in Table 4.4 because this portfolio selects *WHL*. *WHL* has a greater negative return (-0.350) than the sum of the return on share of *INL*, *APN* and *VOD* (-0.291) which were selected as part of the portfolio in Table 4.4. Furthermore, the portfolio in Table 4.4 contains *BVT*, which has the highest return on a share of any company selected by the model.

The portfolio in Table 4.7 was generated when the model selected companies based on only the market liquidity (*LIX* value), using CVaR(95) and the EW weight-moving average metric values. This is interesting as the model is based on the assumption that market liquidity is the second most important metric and not the least important metric. One would expect the smallest return on investment to materialise when selecting companies based on only the book-to-market (BM) value, which is the least important metric. This shows that the 10 companies in low portfolio 2 have the smallest *LIX* values of all the selected companies.

This portfolio has a lower return than the portfolio in Table 4.5 because *RMH* and *TFG* are selected instead of *SBK* and *VOD*. *RMH* and *TFG* have a combined lower return on a share than *SBK* and *VOD*, resulting in an overall decrease in the return on investment.

Table 4.6: Highest return portfolio at RTS values of 24 – 63

MRP	22.6
TFG	61.4
RMH	41.0
FSR	58.1
RDF	9.3
SBK	72.4
WHL	-35.0
SHP	54.8
DSY	39.5
RMI	18.4
Portfolio return:	34.3

Table 4.7: Lowest return portfolio at RTS values of 24 – 63

INL	-17.7
TFG	61.4
APN	-10.3
BTI	18.8
DSY	39.5
ITU	-43.2
RMI	18.4
RMH	41.0
SHP	54.8
WHL	-35.0
Portfolio return:	12.8

The similarity in the boxplots in 4.3 is explained by the fact that from an RTS of 24, all 16 selected companies were eligible for selection and these are the only companies that the model had to choose from. Thus, the model selected the majority of these companies as part of multiple portfolios regardless of the RTS value. As each company has a fixed return on a share value, and many portfolios are the similar, many of the portfolios produced have similar returns on investment.

The sample size was increased from 40 to 52 companies to attempt to advert the similarly in portfolio returns. This only resulted in 3 more viable companies, none of which were ever selected by the model. Thus, the model continued to select only the 16 selected companies from the original sample and these return results remained the same. Thus, increasing the same size had no impact on the model portfolio's returns on investment.

This similarity shows that regardless of the RTS value or MOO method used, the model continuously selected the same companies. From this it is evident that the returns on investment produced by the model are indifferent to the risk tolerance of the ordinary person as well as the MOO method used.

From this it can be concluded that the if the ordinary person uses the model, they can choose any model portfolio to invest in as the model . Thus, the ordinary person should invest in the high portfolio given in Table 4.4 as this is the best performing model portfolio and is expected to continue to have high returns on investment in the future.

It has been shown the model results are insensitive to the RTS of the ordinary person as well as the MOO method used. However, it is still necessary to determine whether or not the model produced meaningful results. In order to do this, the returns produced by the model should be compared to the return on investment received by various unit trusts.

4.2 Model returns versus unit trust returns

In this section, the returns on investment for the portfolios produced by the model are compared to the return on investment achieved by the unit trusts tabulated in Table 3.1. For each risk category the model returns were compared to the returns of 5 different unit trusts over the testing period. The results of the number of model portfolios that had returns on investment higher and lower than those of the unit trusts can be seen in Figure 4.4.

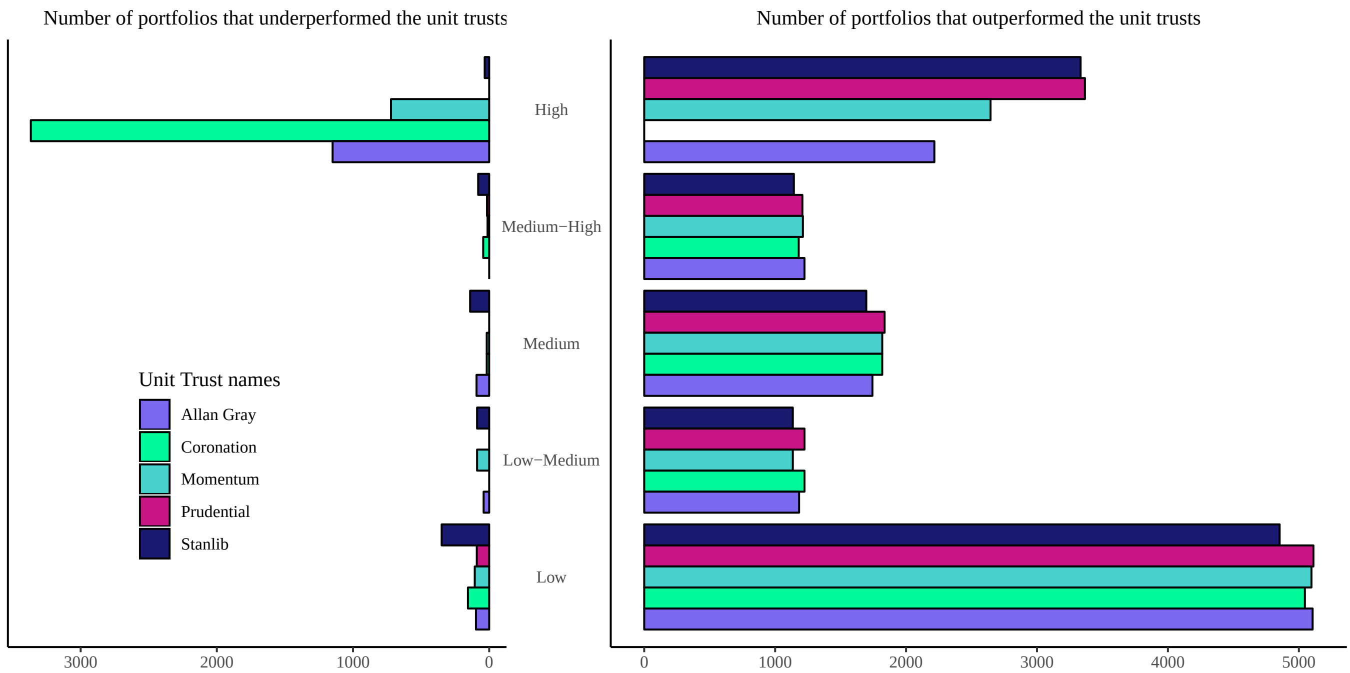

Figure 4.4: The number of model portfolios that outperformed and underperformed the unit trusts

As shown in Figure 4.4, for the low, low – medium, medium and medium – high categories, the model produced far more portfolios that had returns that exceeded those of the units trusts than portfolios that had lower returns than the unit trusts. In these four risk categories, for all 5 unit trusts used as the benchmark, more than 92% of the portfolios produced by the model had returns higher than those of the unit trusts. Furthermore, the medium – high category had the best performance by the model portfolios as in this category 4 of the 5 unit trusts' returns on investment were outperformed by more than 96% of the model portfolios.

Figure 4.4 shows that in the high risk category, for 4 of the 5 unit trusts, the majority of the model portfolios had returns on investment higher than those of the unit trusts. It is interesting that in this risk category the percentage of model portfolios that outperformed the unit trusts in not nearly as high as in the other four risk categories. The Allan Gray unit trust was outperformed by 65.83% of the model portfolios and the Momentum unit trust was outperformed by 78.58% of the model portfolios. Lastly, the Stanlib unit trust performed really poorly and was outperformed by 99.02% of the portfolios produced by the model.

Over all the risk categories, all the model portfolios produced outperformed the Coronation Balanced Defensive Fund, Prudential Enhanced Income Fund, Prudential Balanced Fund, Allan Gray-Orbis Global Fund of Funds and the Prudential Equity Fund. It is interesting to note these are the 5 unit trusts that had the lowest return on investment over the testing period.

Figure 4.4 shows that in the high risk category the Coronation Top 20 Fund, with the highest return on investment of all the unit trusts of 39.71%, outperformed all the model portfolios. It should be noted that this was an unusually high return on investment for this unit trust and it had not received such high returns over any other two year period in its history. Had another two year period been used for the testing set, this unit trust could also have returns less than those of the model portfolios.

These results show that the model produced portfolios had greater returns on investment than 24 of the 25 unit trusts used as benchmarks. Yet, it is unknown by what percentage the model portfolios outperformed these unit trust. As such for each model portfolio, the returns on investment of all 5 unit trusts that fall within the same risk category were subtracted from the portfolio's return. These results are shown in Figure 4.5.

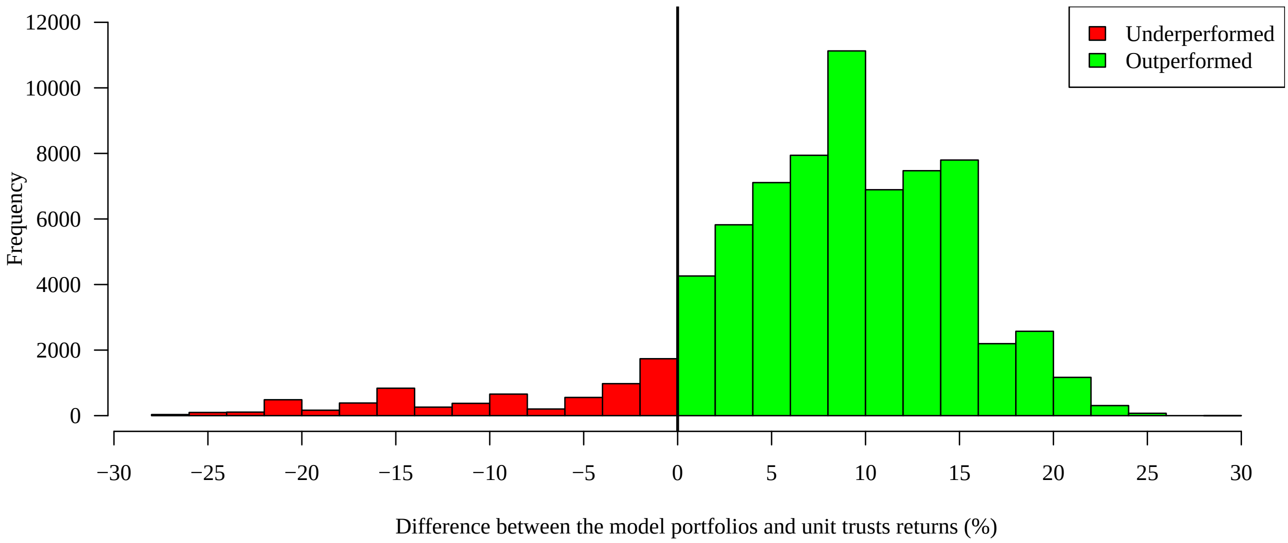

Figure 4.5: The percentage by which the model portfolios outperformed/underperformed the unit trusts over the testing period

As shown in this figure the percentages by which the model portfolios' returns outperformed and underperformed the unit trusts' returns are the same, between 0 and 30%. In spite of this similarity, a significant number of model portfolios outperformed their respective unit trusts as more than 90% of the differences between returns were positive. Furthermore, it can be seen that 71% of the difference between return values were greater than 5%. This indicates that the majority of the model portfolios outperformed the unit trusts by 5 or more percent. As 5% is a significant difference it is evident that if the ordinary person invests in a model portfolio instead of a unit trust they will receive a significantly higher return on investment. These results combined with the results in Figure 4.4 show that it is predominantly the model portfolios in the low, low – medium, medium and medium – high risk categories that significantly outperform their respective unit trusts.

The results in Figures 4.4 and 4.5 show that the model is producing worthwhile results as the model portfolios outperform the 96% of the unit trusts and more than 71% of the model portfolios had returns on investment that were 5 or more percent greater than the respective unit trusts. Therefore, it is shown by investing in a model portfolio, the ordinary person will receive superior returns on investment than those produced by a variety of unit trusts. Furthermore, investing in model portfolios has the added advantage of not having to pay financial brokers, which decreases the return received by the investor. For these reasons, it is shown that the model produces worthwhile results for the ordinary person. Thus, as suggested in Section 4.1.2, the ordinary person should investment in the model portfolio given in 4.4.

It has been shown that the model produces worthwhile results but in Section 4.1.2 it is shown that the model is sensitive to the assumptions on which it is based. As such, it is necessary to investigate how sensitive the model is to these assumptions and whether if it still considered worthwhile to use the model given this sensitivity. This sensitivity analysis is performed in Section 4.3.

4.3 Sensitivity Analysis

In order to investigate how sensitive the model is to the assumptions on which it is based, the model was rerun three times, each time using different model assumptions. The model was changed and named as follows:

- Model revision 1 (rev1): It was assumed that market liquidity is the most important metric, followed by the risk measure, then the F_ Score, with the BM value being the least important metric.
- Model revision 2 (rev2): It was assumed that the F_ Score is the most important metric, followed by the BM value, then the risk, with market liquidity being the least important metric.
- Model revision 3 (rev3): It was assumed that market liquidity is the most important metric, followed by the F_ Score, then the BM value , with risk measure being the least important metric.

The justifications for these assumptions can be seen in Appendix G. It was found that in all three model revisions, the model only selected 16 companies and these were the same 16 companies that were selected by the original model. Yet, the number of times that each company was selected differed between model versions as is shown in Figure 4.6. Of the 16 companies selected, only MRP and RMH were selected exactly the same number of times by each model version. These results show that the model is sensitive to the model assumptions as the results produced change with changes in these assumptions.

It has been shown that the model is sensitive to the assumptions on which it is based, but it is still uncertain how sensitive it is to these assumptions. Thus, the return on investment results produced by the revised models were compared to the original return results as shown in Figure 4.7.

This Figure shows the revised models produced very similar results to each other and that the returns on investment produced by the revised models are lower than those produced by the original model. This is because the results for the revise models have lower quartile and median values than the original model. Nonetheless, from the low – medium risk category the returns produced by each model version have the same range and very similar interquartile ranges. This indicates that regardless of the model configuration, the model produces portfolios with similar returns on investment for all risk categories excluding the low risk category. Furthermore, from low – medium risk category all the model version returns have the same maximum value. Thus, although the model is sensitive to the assumptions on which it is based, this sensitivity is not significant. Once again, this seeming insignificance could be a result of the limited sample of companies the model had to select from.

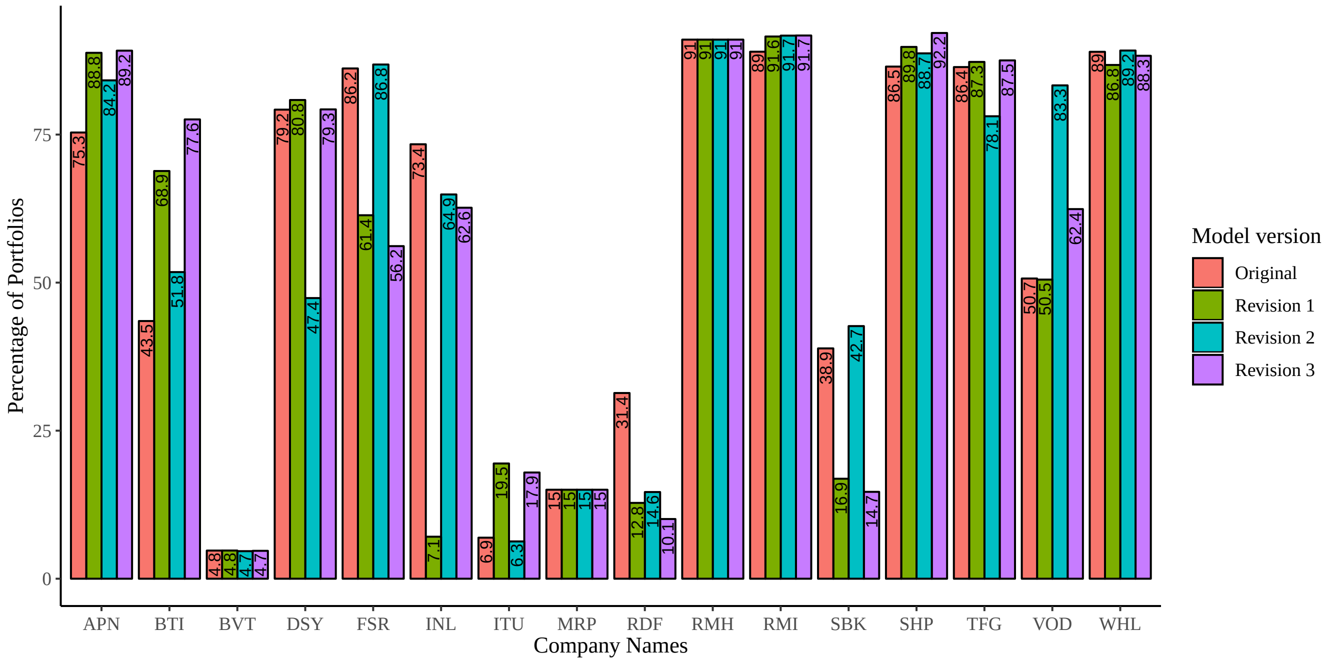

Figure 4.6: Return on investment received by the model portfolios in the different model versions

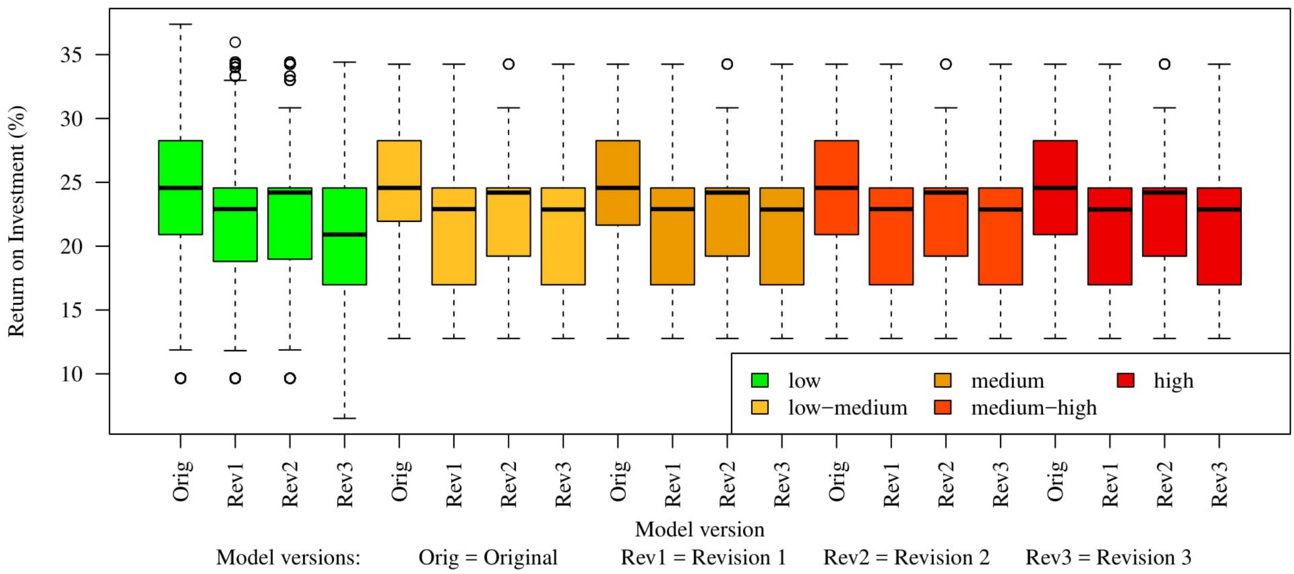

Figure 4.7: Return investment received by the model portfolios for each of the risk categories in the different model versions over the testing period

For all the model version, expect for model revision 1, the highest return on investment was achieved when selecting the model portfolio based on only the risk measure. This supports the original assumption that the risk measure is the most important metric. This shows that the companies with the lowest risk measures tend to have the highest return on investment. Furthermore, as the highest return produced by any model version of 37.4% was produced by the original model, it is still valid to use model configuration for the ordinary person. Thus, the ordinary person should invest in the highest return portfolio as given in 4.4.

It has been shown that the model is relatively insensitive to the model assumptions but still produces worthwhile results. However, using this model requires the ordinary person to invest a copious amount of time and effort into collecting the necessary data for the model. In the next Chapter it will be discussed whether the time and effort required to use this model is worth the returns received by the model portfolios or whether it would better for the ordinary person to invest in unit trusts.

Chapter 5

Discussion and Conclusion

In this chapter the implications of the model and whether it is worthwhile for the ordinary person to use this model are discussed. The conclusions of the project are presented and suggestions of further research for this project are given.

5.1 Discussion of results

In Chapter 4 the results produced by the model are presented and it is shown that the model does produce worthwhile results. However, it must be considered whether it is constructive to use this model to select investment portfolios to achieve one's long-term financial goals. A long-term investment is any investment that is for a period of five or more years.

If a person wishes to use the model they need to spend time and effort to gather the data needed as the inputs for the model. During the completion of this project it was found that it takes a minimum of one hour to gather all the data needed for one Johannesburg Stock Exchange (JSE) 40 company. Furthermore, for every subsequent year that the model is used it takes a minimum of 20 minutes (0.3 hours) to gather all the data needed for one JSE 40 company. As this is an extremely timely task, it must be considered whether this process it truly worth the return on investment received by the model portfolios. The investor must consider whether the time spent on gathering data is worth more in monetary value than paying a financial professional to invest on one's behalf is.

When investing in a unit trust, an investor is required to invest a minimum amount which was found to be R 10 000 for the majority of unit trusts. The annual cost of investing in a unit trust consists of two parts, namely the annual management fee and the total expense ratio (TER). The annual management fee is a set percentage that the investors of the unit trust pay to the trust managers for their services. This fee is paid regardless of whether the trust generates a return or a loss and this cost is usually just deducted from the trust's assets. The TER is the total costs associated with managing and operating the unit trust and these costs are taken from the trust only if the trust receives a return on investment (Gil-Bazo and Ruiz-Verdú, 2009). Thus, if a trust generates a return of 8%, the management fee is 1.75% and the TER is 1.25%, the investor will receive a 5% return. Combined, these two fees usually equate to about 3% of the return on investment of the unit trust.

To determine whether it is better to invest in a model portfolio or in a unit trust it is necessary to consider what return the investor will receive at the end of an investment period if all the cost associated with the investment, monetary and otherwise, are taken into account. This is known as the payout of an investment. Furthermore, as the returns on investment for both the model portfolios and the unit trusts are for a period of two years, it is necessary to find annual return on investment to determine these returns on investment. This return percentage is calculated using the following Equation:

$$R = (1 + R_{\text{annual}})^m - 1 \tag{5.1}$$

where R_a is the effective return on investment rate (the return on investment over the testing period), R_{annual} is the annual return on investment and m is the number of years in the testing period (Newnan et al., 2009). It should be noted that this Equation assumes that the return on investment rate remains constant for the investment period, which is not accurate in reality but will hold for this report.

The payout was calculated for the best performing unit trust and model portfolio in every risk category. *Please note: the second-best performing unit trust was used for the high risk category on account of the unusually high return received by the best performing unit trust.* These unit trust and model portfolios with their return on investment over the testing period and graphing names (due to space limitations on the graph) are given in Table 5.1. The payout results can be seen in Figure 5.1 and the detailed calculations and results are given in Appendix H.

Table 5.1: Table of the unit trusts and model portfolios for which the returns after costs were calculated

Risk category	**Name**	**Graphing name**	**Return on investment (%)**
	Unit trusts		
Low	Stanlib Conservative Fund of Funds	Unit trust 1	18.62
Low – medium	Momentum Diversified Income Fund	Unit trust 2	18.51
Medium	Stanlib Moderate Fund of Funds	Unit trust 3	17.71
Medium – high	Stanlib Moderately Aggressive Fund of Funds	Unit trust 4	17.39
High	Allan Gray Equity Fund	Unit trust 5	22.60
	Model Portfolios		
Low	High portfolio in Table 4.4	High portfolio 1	34.30
All excluding low	High portfolio in Table 4.6	High portfolio 2	37.40

From Figure 5.1 it evident that for various long-term investment periods, all the unit trusts received greater payouts than the model portfolios. Furthermore, when the “cost” of the ordinary person’s time is taken into account, the model portfolios receive very low payouts and over longer periods of time experience a loss. From this it is evident that although the model portfolios have significantly higher returns on investment, as shown in Section 4.2, the “cost” of the ordinary person’s time considerably more than the cost of investing in a unit trust. Thus, the model portfolios do not produce higher payouts than the unit trusts.

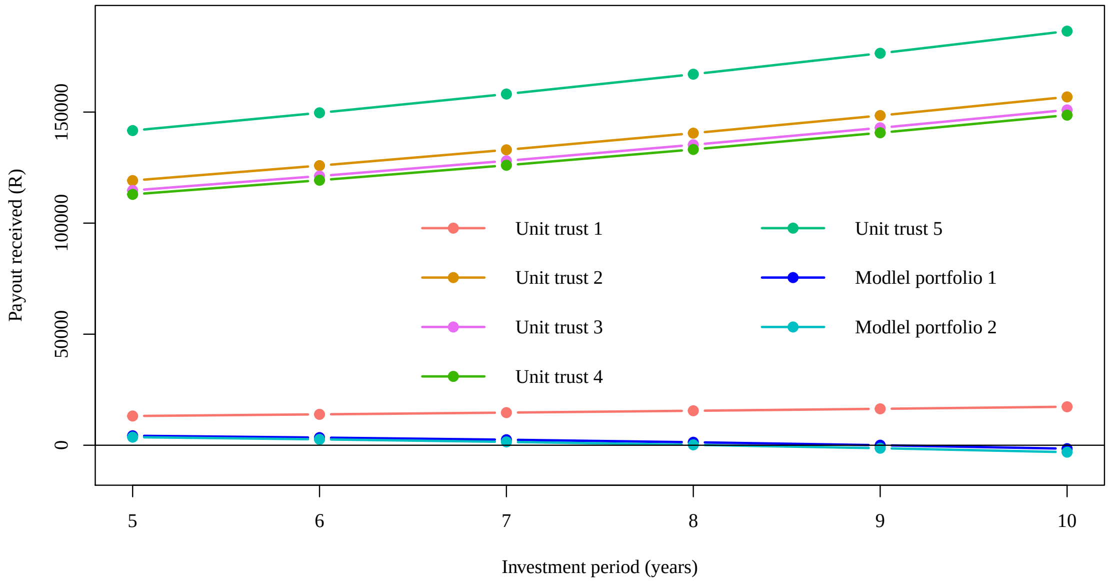

Figure 5.1: Return on investment received by the investor after costs for various unit trusts and model portfolios for different investing period

Another factor that must be considered is the fact that unit trusts are operated and managed by fund managers. These fund managers are trained financial professionals who constantly monitor the investments they make. They track share behaviour, analyse investment trends, anticipate market behaviour and make adjustments to the unit trust's investments as they deem fit or necessary. If an ordinary person invests in a unit trust, they simply invest their money and after a period collect their returns. They do not need to worry about their investment during the investment period. If the ordinary person invests for themselves, they are responsible for managing their portfolio. This may be a challenging and potentially disastrous task for the ordinary person as they are not trained in how to monitor financial behaviour and respond accordingly.

It should also be stated that the model developed in this project is for long-term investments, however, this model is yet untested over the long-term. Thus, although grounded in research, no numerical data exists to supports the premise of this model. Furthermore, all investment guidance in South Africa must be regulated by the Financial Advisory and Intermediary Services Act (FAISA). For this reason, the model will have to be licensed and adhere to a professional code of conduct with specific enforcement measures. As this model is untested in the long-term, it may prove difficult to obtain FAISA accreditation for it. Thus, the model would be impractical for use by the ordinary person to achieve their long-term financial goals.

It has been shown that when all costs are accounted in the determination of an investment's payout, various unit trusts significantly outperform the model portfolios. Furthermore, the model is untested in the long-term, and using it would require that the model receive FAISA accreditation. Lastly, using the model would require that the user manage their own investment portfolio, which may be to their detriment. For these reasons it is not worthwhile for the ordinary person to invest in an investment portfolio produced by the model. It will be more beneficial for the ordinary person to pay a financial professional to invest on their behalf and manage their investments.

5.2 Conclusion

The ordinary person invests to meet their long-term financial goals. From this report it is seen that when investing there are four main metrics, and their objectives must be taken into consideration together to determine what the best investment option will be. These four metrics are the market liquidity, market risk, financial risk tolerance of the ordinary person and the expected growth of a company. As no current method exists for considering these options together it was decided that a customised multi-objective optimisation (MOO) model should be developed for this purpose. Further research was conducted and it was concluded that in order to allow for easy understanding and use by the ordinary person, simple MOO methods should be used. Three methods were selected, namely the lexicographic method, the weighted sum method and the weighted product method, as they are easy to use and understand. Furthermore, as these methods all provide a sufficient condition for pareto optimality, they will always produce an ideal solution. It was concluded that the model created should be evaluated against unit trusts so as to see whether using this model will result in increased returns for the ordinary person.

The model described in Chapter 3 was built. To increase the robustness of the analysis, the model was built to select portfolios using the three selected MOO methods as well as based on only the individual metric values over a range of risk tolerance score (RTS) values. It was found that regardless of the model configuration, a maximum of 16 of the JSE 40 were eligible for selection by the model. Furthermore, the majority of these 16 companies appear in more than 50% of the model portfolios. Thus, it was concluded that the model has a very limited sample of companies from which to select portfolios.

When looking at the returns on investment produced by the model portfolios, it was discovered that the highest return on investment, of 37.4%, for any model portfolio was produced at an RTS of 23. The highest return on investment produced for all other RTS values was produced when the model selected portfolios based on only the risk measure. It was thus concluded that the model is producing portfolios that adhere to the assumptions on which it is based. The returns on investment produced by the model portfolios at RTS values of 24 to 63 were equal or similar. As a result of limited sample size, the model selected many of the companies for many of the portfolios resulting in multiple identical or similar portfolio, and thus similar returns.

When comparing the returns on investment produced by the model to the returns on investment received by various unit trusts, it was found that the model portfolios outperformed 24 of the 25 unit trusts. Furthermore, when the unit trust returns were subtracted from the model returns it was found that the majority of the model portfolios had returns on investment that were 5 or more percent higher than their respective unit trusts. Thus, it was concluded that the model produced worthwhile results as the ordinary person can receive significantly higher returns on investment by investing in a model portfolio rather than in a unit trust. Therefore, it was suggested the ordinary person invest in the highest return portfolio produced by the model.

A sensitivity analysis was performed and it was found that the model is sensitive to the assumptions that it is based on. Regardless of this, returns on investment for the various model version were very similar and the highest model returns were generated when selecting companies based on the risk measure. Thus, original assumption that the risk measure is the most important metric was confirmed.

The payout for best performing unit trusts and model portfolios in each risk category were calculated. It was found that the unit trusts have significantly greater payout values and thus it is not worthwhile for the ordinary person to invest in a model portfolio over a unit trust. This is as the ordinary person's time is worth more in monetary value than the cost of investing in a unit trust is.

From this project it is concluded that the portfolios produced by the model produced higher returns on investment than the unit trusts used as benchmarks. Nevertheless, the unit trust provided greater payouts than the model portfolios in the long-term. Thus, it is recommended that the ordinary person invest in a unit trust with reasonable fees to achieve their long-term financial goals. Suggestions of how the research in this report can be expanded are given in the next section.

5.3 Further Research

To expand on the research conducted in this project it is suggested that the sample size of companies that the model has to select from be significantly increased, for example to the JSE Top 100. By increasing the sample size, the model will have more companies to select from and it can be investigated whether returns on investment are truly indifferent towards risk tolerance or if this result was only a consequence of the limited sample size. The model could also be developed for a longer time period to evaluate the long-term impact of the model results. Another suggestion is that the weights in the weighted-moving average calculation be changed to reflect different assumptions. This was already performed three times but as there are twenty-four different possible combinations with which to determine the weighted-moving average weights, this analysis can be expanded even further. By testing all possible weight combinations it can be investigated whether there is another weighting configuration that results in higher returns on investment than the original model. The last suggestion is that the model be designed to only select investments that have the same market risk as the ordinary person's risk tolerance. So, if a person has a high risk tolerance, the model will only select high risk companies for them rather than selecting any company that has a market risk lower to equal to their risk tolerance. By configuring the model in this way, it can be investigated whether investing in higher risk companies will produce higher returns on investment than lower risk companies, as is often theorised.

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Appendix A

JSE 40 companies

Table A.1: Table of the companies in the Johannesburg Stock Exchange (JSE) 40

Company number	Company name	Google Finance ticker
1	Anheuser-Busch InBev SA/NV	ANH
2	British American Tobacco PLC	BTI
3	Naspers Limited	NPN
4	Glencore PLC	GLN
5	Compagnie Financire Richemont	CFR
6	BHP Billiton PLC	BIL
7	Anglo American PLC	AGL
8	FirstRand Limited	FSR
9	Standard Bank Group Ltd	SBK
10	Sasol Limited	SOL
11	Vodacom Group Limited	VOD
12	MTN Group	MTN
13	Old Mutual PLC	OMU
14	Sanlam	SLM
15	Barclays Africa Group Limited	BGA
16	Nedbank Group	NED
17	Shoprite	SHP
18	Mondi PLC	MNP
19	Remgro Limited	REM
20	Aspen Pharmacare	APN
21	Discovery Limited	DSY
22	RMB Holdings Limited	RMH
23	Capitec Bank	CPI
24	Kumba Iron Ore	KIO
25	Growthpoint Properties Limited	GRT
26	Anglo American Platinum	AMS
27	NEPI Rockcastle PLC	NRP
28	The Bidvest Group Limited	BVT
29	Tiger Brands Limited	TBS
30	Mediclinic International PLC	MEI

31	Mr Price Group Limited	MRP
32	Woolworths Holdings Limited	WHL
33	Redefine Properties	RDF
34	Investec	INL
35	Hammerson PLC	HMN
36	Rand Merchant Insurance Holdings Ltd	RMI
37	PSG Group Limited	PSG
38	Intu Properties PLC	ITU
39	Foschini Group Ltd	TFG
40	Imperial Holdings Ltd	IPL

[1]

[1] The companies South32 and Bid Corporation Limited are listed a number 15 and 26 on the JSE Top 40 respectively. However, these companies were only publicly listed in 2015 and as such, historical financial and share information is not available for these companies. For this reason, they will be excluded from consideration in the model and company 41 and 42 will be considered as company 39 and 40 with all other companies being moved up by one rank.

Appendix B

Data gathering methods

B.1 Financial statement data

The process below describes how the financial statement information for each of the JSE 40 companies was obtained.

To find the financial statements of a company:

1. In Google, type the name of the company and financial statements, for example AB InBev financial statements.
2. The first result to appear is usually the company's annual reports web page. Go to this site.
3. The financial statements for a given year are contained in the company's annual report for that year. Find the annual report for the year in question.
4. Locate the financial statements in that report.

Using the method below, the necessary values were found and then read into the spreadsheet shown in Figure B.1.

	Million US dollar														
Year	Revenue (sales)	Cost of Sales	Gross Profit	Profit (EBIT)	Cash flow from operating activities	Intangible Assets	Current Assets	Total assets	Average Total Assets	Non - Current Liabilities	Current liabilities	Total Liabilities	Were shares issued? (if Yes, type 1 otherwise 0)	Shares issued	Preference shares
2011															
2012															
2013															
2014															
2015															

Figure B.1: Table used to read in the values from the financial statements

Reading information from the financial statements.

1. Locate the income statement of the company in financial statements.
2. In the income statement section read off the following values:
   - Total sales, also known as revenue or sales

– Cost of sales, also known as the cost of goods sold
– Gross margin, also known as the gross profit
– Net income before extraordinary items, also known as the profit from operations before non-recurring items, the operating profit or the profit before interest and taxes(EBIT).

For an example the income statement of AB InBev highlighting these values can be seen in Figure B.2.

Consolidated Income Statement

For the year ended 31 December Million US dollar, except earnings per shares in US dollar	Notes	2015	2014[1]
Revenue		**43 604**	**47 063**
Cost of sales		(17 137)	(18 756)
Gross profit		**26 467**	**28 307**
Distribution expenses		(4 259)	(4 558)
Sales and marketing expenses		(6 913)	(7 036)
Administrative expenses		(2 560)	(2 791)
Other operating income/(expenses)	7	1 032	1 386
Profit from operations before non-recurring items		**13 768**	**15 308**
Restructuring	8	(171)	(158)
Business and asset disposal	8	524	157
Acquisition costs business combinations	8	(55)	(77)
Impairment of assets	8	(82)	(119)
Judicial settlement	8	(80)	–
Profit from operations		**13 904**	**15 111**
Finance cost	11	(2 417)	(2 797)
Finance income	11	1 178	969
Non-recurring net finance income/(cost)	8	(214)	509

Figure B.2: Extract from the 2015 AB InBev Income Statement highlighting the revenue, cost of sales, gross profit and profit from operations before non-recurring items

3. Locate the statement of financial position of the company in the financial statements.
4. In the statement of financial position section read off the following values:

– Intangible assets. The total intangible assets of a company are the sum of goodwill and intangible assets as reported on the statement of financial position.
– Current assets
– Total assets
– Total long-term debt, also known as the non-current liabilities
– Current liabilities
– Total liabilities. Should this value not be reported individually on the statement of financial position it can be calculated by summing the current and non-current liabilities.

For an example the statement of financial position of AB InBev highlighting these values can be seen in figures B.3 and B.4.

Consolidated Statement of Financial Position

As at Million US dollar	Notes	31 December 2015	31 December 2014[1]
Assets			
Non-current assets			
Property, plant and equipment	13	18 952	20 263
Goodwill	14	65 061	70 758
Intangible assets	15	29 677	29 923
Investments in associates and joint ventures		212	198
Investment securities	16	48	30
Deferred tax assets	17	1 181	1 058
Employee benefits	23	2	10
Derivatives	27H	295	507
Trade and other receivables	19	913	1 262
		116 341	**124 009**
Current assets			
Investment securities	16	55	301
Inventories	18	2 862	2 974
Income tax receivable		687	359
Derivatives	27H	3 268	1 737
Trade and other receivables	19	4 451	4 712
Cash and cash equivalents	20	6 923	8 357
Assets held for sale		48	101
		18 294	**18 541**
Total assets		**134 635**	**142 550**

Figure B.3: Extract from the 2015 AB InBev Statement of Financial Position highlighting the intangible, current and total assets

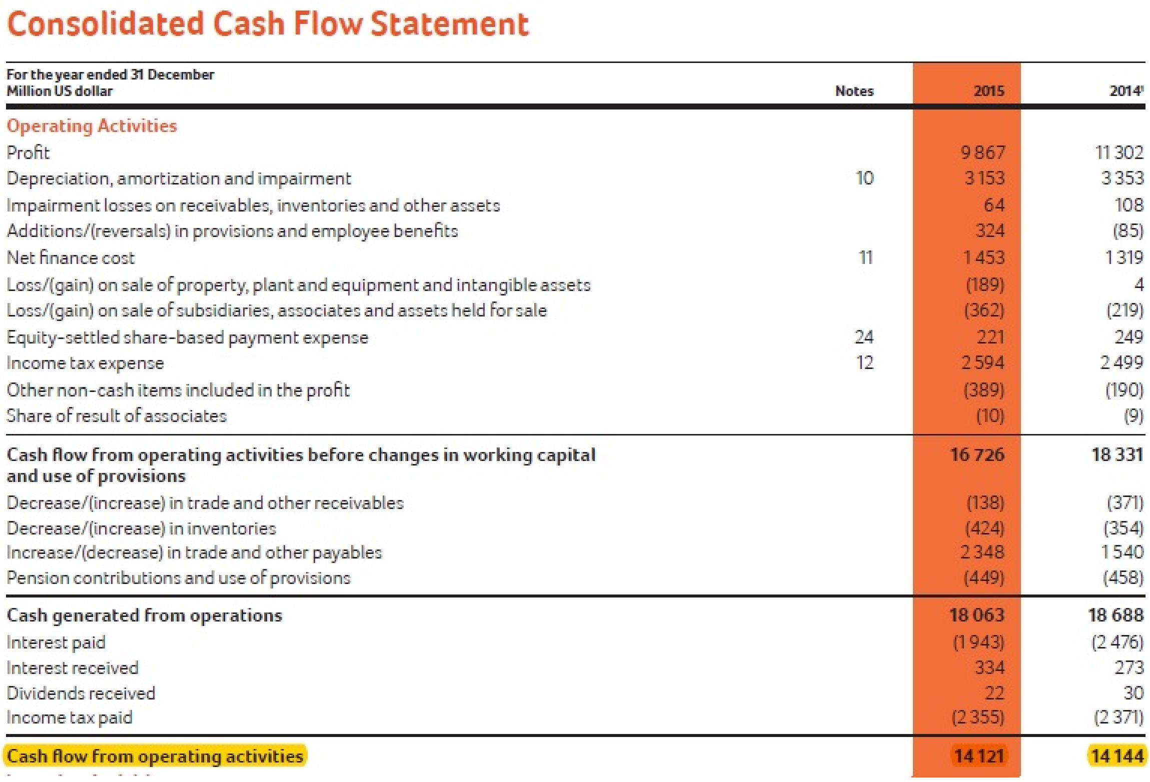
Equity and Liabilities			
Equity			
Issued capital	21	1 736	1 736
Share premium		17 620	17 620
Reserves		(13 168)	(4 558)
Retained earnings		35 949	35 174
Equity attributable to equity holders of AB InBev		**42 137**	**49 972**
Non-controlling interest		3 582	4 285
		45 719	**54 257**
Non-current liabilities			
Interest-bearing loans and borrowings	22	43 541	43 630
Employee benefits	23	2 725	3 050
Deferred tax liabilities	17	11 961	12 701
Derivatives	27H	315	64
Trade and other payables	26	1 241	1 006
Provisions	25	677	634
		60 460	61 085
Current liabilities			
Bank overdrafts	20	13	41
Interest-bearing loans and borrowings	22	5 912	7 451
Income tax payable		669	629
Derivatives	27H	3 980	1 013
Trade and other payables	26	17 662	17 909
Provisions	25	220	165
		28 456	27 208
Total equity and liabilities		**134 635**	**142 550**

Figure B.4: Extract from the 2015 AB InBev Statement of Financial Position highlighting the current and non-current liabilities

5. Locate the cash flow statement of the company in the financial statements.
6. In the cash flow statement section read off the following value:

   – Cash flow from operations, also known as the cash flow from operating activities.

   For an example the cash flow statement of AB InBev highlighting this value see Figure B.5.



Consolidated Cash Flow Statement

For the year ended 31 December Million US dollar	Notes	2015	2014[1]
Operating Activities			
Profit		9 867	11 302
Depreciation, amortization and impairment	10	3 153	3 353
Impairment losses on receivables, inventories and other assets		64	108
Additions/(reversals) in provisions and employee benefits		324	(85)
Net finance cost	11	1 453	1 319
Loss/(gain) on sale of property, plant and equipment and intangible assets		(189)	4
Loss/(gain) on sale of subsidiaries, associates and assets held for sale		(362)	(219)
Equity-settled share-based payment expense	24	221	249
Income tax expense	12	2 594	2 499
Other non-cash items included in the profit		(389)	(190)
Share of result of associates		(10)	(9)
Cash flow from operating activities before changes in working capital and use of provisions		**16 726**	**18 331**
Decrease/(increase) in trade and other receivables		(138)	(371)
Decrease/(increase) in inventories		(424)	(354)
Increase/(decrease) in trade and other payables		2 348	1 540
Pension contributions and use of provisions		(449)	(458)
Cash generated from operations		**18 063**	**18 688**
Interest paid		(1 943)	(2 476)
Interest received		334	273
Dividends received		22	30
Income tax paid		(2 355)	(2 371)
Cash flow from operating activities		**14 121**	**14 144**

Figure B.5: Extract from the 2015 AB InBev Cash Flow Statement highlighting the cash flow from operating activities

7. Locate the notes to the financial statements of the company in the financial statements.
8. In the notes to the financial statements section locate the statement of capital note and read off the following value:

   – The value of the company's preference shares.

   For an example the statement of capital note of AB InBev highlighting this value can be seen in Figure B.6.

9. In the notes to the financial statements section locate the long-term debt note and read off the following value:

   – The total number of shares that have been issued by the company.
   – Whether shares were issued by the company during the year.

Statement of capital

The tables below summarize the changes in issued capital and treasury shares during the year:

	Issued capital	
Issued capital	**Million shares**	**Million US dollar**
At the end of the previous year	1 608	1 736
Changes during the year	–	–
	1 608	**1 736**

Figure B.6: Extract from the 2015 AB InBev statement of capital note highlighting the number of shares issued

For an example the long-term debt note of Sasol (AB InBev does not have preference shares and thus Sasol is used as the example) highlighting this value can be seen in Figure B.7.

for the year ended 30 June	Note	2015 Rm	2014 Rm
18 Long-term debt			
Total long-term debt		**42 066**	25 921
Short-term portion	24	**(2 797)**	(2 502)
		39 269	23 419
Analysis of long-term debt			
At amortised cost			
Secured debt		**8 477**	815
Preference shares		**12 113**	8 106
Finance leases		**1 532**	940
Unsecured debt		**20 331**	16 204
Unamortised loan costs		**(387)**	(144)
		42 066	25 921

Figure B.7: Extract from the 2015 Sasol notes to the financial statements highlighting the preference shares' value

The above process is repeated for each of the JSE 40 companies for the years of 2012 to 2015 and the total assets for 2011 are also obtained.

B.2 Share data

To obtain the share data of the JSE 40 companies, the GOOGLEFINANCE function in Google Sheets was used.
The GOOGLEFINANCE function looks as follows:

GOOGLEFINANCE(ticker, [attribute], [start_ date], [end_ date—num_ days], [interval])

where:

ticker - a unique code assigned to each company that Google Finance uses to identify that company. The ticker for each of the JSE 40 companies can be seen in Table A.1.

attribute - the attribute one is interested in for the shares (for example price)

start_ date - the start date when fetching historical data

end_ date—num_ days - the end date when fetching historical data, or the number of days from start_ date for which to return data

the frequency of returned data; either ”DAILY” or ”WEEKLY”.

For AB InBev the necessary share data was obtained as follows:

Create a new spreadsheet in Google sheets

type =GOOGLEFINANCE(“ANH”, “price”, “12/31/2012”, “1/1/2018”, “DAILY”) into cell A2

type =GOOGLEFINANCE(“ANH”, “volume”, “12/31/2012”, “1/1/2018”, “DAILY”) into cell C2

type =GOOGLEFINANCE(“ANH”, “high”, “12/31/2012”, “1/1/2018”, “DAILY”) into cell E2

type =GOOGLEFINANCE(“ANH”, “low”, “12/31/2012”, “1/1/2018”, “DAILY”) into cell G2

type =GOOGLEFINANCE(“ANH”, “currency”, “12/31/2012”, “1/1/2018”, “DAILY”) into cell I2

The results of using this method can be seen in Figure B.8

	A	B	C	D	E	F	G	H	I
1	Anheuser-Busch InBev SA NV								
2	Date	Close	Date	Volume	Date	High	Date	Low	USD
3	12/31/2012 16:00:00	5.78	12/31/2012 16:00:00	1883962	12/31/2012 16:00:00	5.79	12/31/2012 16:00:00	5.7	
4	1/2/2013 16:00:00	5.9	1/2/2013 16:00:00	1508631	1/2/2013 16:00:00	5.93	1/2/2013 16:00:00	5.82	
5	1/3/2013 16:00:00	5.94	1/3/2013 16:00:00	901974	1/3/2013 16:00:00	5.96	1/3/2013 16:00:00	5.88	
6	1/4/2013 16:00:00	6.02	1/4/2013 16:00:00	1424395	1/4/2013 16:00:00	6.04	1/4/2013 16:00:00	5.9601	
7	1/7/2013 16:00:00	6.03	1/7/2013 16:00:00	780837	1/7/2013 16:00:00	6.05	1/7/2013 16:00:00	6.01	
8	1/8/2013 16:00:00	6.05	1/8/2013 16:00:00	861290	1/8/2013 16:00:00	6.08	1/8/2013 16:00:00	6.03	
9	1/9/2013 16:00:00	6.05	1/9/2013 16:00:00	826691	1/9/2013 16:00:00	6.07	1/9/2013 16:00:00	6.01	
10	1/10/2013 16:00:00	6.08	1/10/2013 16:00:00	593651	1/10/2013 16:00:00	6.08	1/10/2013 16:00:00	6.03	
11	1/11/2013 16:00:00	6.09	1/11/2013 16:00:00	619597	1/11/2013 16:00:00	6.1	1/11/2013 16:00:00	6.06	
12	1/14/2013 16:00:00	6.07	1/14/2013 16:00:00	620254	1/14/2013 16:00:00	6.09	1/14/2013 16:00:00	6.06	
13	1/15/2013 16:00:00	6.09	1/15/2013 16:00:00	861176	1/15/2013 16:00:00	6.11	1/15/2013 16:00:00	6.07	
14	1/16/2013 16:00:00	6.11	1/16/2013 16:00:00	683888	1/16/2013 16:00:00	6.14	1/16/2013 16:00:00	6.09	
15	1/17/2013 16:00:00	6.13	1/17/2013 16:00:00	718697	1/17/2013 16:00:00	6.17	1/17/2013 16:00:00	6.1	
16	1/18/2013 16:00:00	6.16	1/18/2013 16:00:00	779730	1/18/2013 16:00:00	6.16	1/18/2013 16:00:00	6.12	
17	1/22/2013 16:00:00	6.26	1/22/2013 16:00:00	1721576	1/22/2013 16:00:00	6.3	1/22/2013 16:00:00	6.16	
18	1/23/2013 16:00:00	6.27	1/23/2013 16:00:00	833001	1/23/2013 16:00:00	6.29	1/23/2013 16:00:00	6.22	
19	1/24/2013 16:00:00	6.15	1/24/2013 16:00:00	1938865	1/24/2013 16:00:00	6.27	1/24/2013 16:00:00	6.1375	
20	1/25/2013 16:00:00	6.24	1/25/2013 16:00:00	1575594	1/25/2013 16:00:00	6.26	1/25/2013 16:00:00	6.15	

Figure B.8: Share price and volume data for AB InBev in Google Sheets

To obtain these values for the other companies in the JSE 40, the above process is repeated with the Google ticker being changed with every repetition to find the data for a different company.

Appendix C

Metric calculation examples

C.1 Market liquidity example

The information in Table C.1 was obtained from Google Finance as explained in Section 3.1.3 and will be used to demonstrate how the ML value for AB InBev was calculated.

Please note: 2015 had 252 trading days but for the sake of simplicity, only the first ten days will be used to demonstrate how the ML value is calculated.

Table C.1: Prices and volume for AB InBev shares for the beginning of 2015

Day Number	Date	High Price	Low Price	Close Price	Volume
1	2015/01/02	5.31	5.22	5.30	650 871
2	2015/01/05	5.32	5.23	5.24	1 040 754
3	2015/01/06	5.27	5.22	5.24	712 718
4	2015/01/07	5.25	5.22	5.24	463 324
5	2015/01/08	5.25	5.21	5.23	518 928
6	2015/01/09	5.24	5.21	5.23	582 071
7	2015/01/12	5.23	5.18	5.22	656 077
8	2015/01/13	5.26	5.21	5.23	801 696
9	2015/01/14	5.22	5.17	5.20	1 499 407
10	2015/01/15	5.28	5.18	5.22	739 987

For this example:

T = Day Number

$TD = 10$

From Equation (3.1): $LIX_1 = \log_{10}\left(\frac{650871 \times 5.30}{5.31 - 5.22}\right) = 7.584$

Similarly, $LIX_2 = 7.782, LIX_3 = 7.873, LIX_4 = 7.908, LIX_5 = 7.832, LIX_6 = 8.006, LIX_7 = 7.836, LIX_8 = 7.924, LIX_9 = 8.193$ and $LIX_{10} = 7.587$

Therefore

$$\begin{aligned}
\text{From Equation (3.2): } LIX_{2015} &= \frac{1}{10}\sum_{T=1}^{10} LIX_T \\
&= \frac{1}{10}(LIX_1 + LIX_2 + LIX_3 + LIX_4 + LIX_5 + LIX_6 \\
&\quad + LIX_7 + LIX_8 + LIX_9 + LIX_{10}) \\
&= \frac{1}{10}(7.584 + 7.782 + 7.873 + 7.908 + 7.832 + 8.006 \\
&\quad + 7.836 + 7.9234 + 8.193 + 7.587) \\
&= 7.852
\end{aligned}$$

Since the $LIX > 7.5$, AB InBev would be a good company to invest in if market liquidity was the only consideration.

C.2 Book-to-market value example

The information below was extracted from the financial statements of Sasol and obtained using Google Finance as explained in Sections 3.1.2 and 3.1.3. This information will be used to demonstrate how the book-to-market (BM) value for the companies is calculated.

Table C.2: Sasol financial information for 2015

Values in Millions					
TA	TL	PS	IA	SO	SP
R 323 932	R 127 116	R 12 113	R,2 293	653 933 281	R 132.0781

$$\begin{aligned}
\text{From Equation (3.3): } BV &= \text{R}\,323\,932\,000\,000 - (\text{R}\,127\,116\,000\,000 \\
&\quad + \text{R}\,12\,113\,000\,000 + \text{R}\,2\,293\,000\,000) \\
&= \text{R}\,182\,077\,000\,000
\end{aligned}$$

$$\begin{aligned}
\text{From Equation (3.4): } MV &= 653\,933\,281 \text{ shares} \times \text{R}\,132.0781 \text{ per share} \\
&= R\,86\,370\,265\,281
\end{aligned}$$

$$\begin{aligned}
\text{Therefore from Equation (3.5): } BM &= \frac{\text{R}\,182\,077\,000\,000}{\text{R}\,86\,370\,265\,281} \\
&= 2.1081
\end{aligned}$$

Since the $BM > 1$ this is not a company that the ordinary person would want to invest in if only this metric is considered.

C.3 F_ Score example

The F_ Score for 2015 will be calculated below. The information in Table C.3 was read gathered from the financial statements of AB InBev using the methods described in 3.1.2.

Table C.3: Financial Statement information for AB InBev for 2013 to 2015

	Values in million US Dollar									
Year	*TS*	*COS*	*GM*	*NI*	*CFFO*	*IA*	*CA*	*TA*	*TL*	*CL*
2013	43 195	17 594	25 601	14 203	13 864	29 338	18 690	141 666	60 731	25 627
2014	47 063	18 756	28 307	15 308	14 144	29 923	18 541	142 550	61 085	27 208
2015	43 604	17 137	26 467	13 768	14 121	29 677	18 294	134 635	60 460	28 456

$$\text{From Equation (3.8): } \mathrm{ROA}_{2015} = \frac{13\,768}{142\,550} = 0.096$$

Since return on assets (ROA) > 0
F_ ROA $= 0$

$$\text{From Equation (3.10): } \mathrm{CFO}_{2015} = \frac{14\,121}{142\,550} = 0.099$$

Since operating-cash-flow to total assets ratio (CFO) > 0
F_ CFO $= 0$

$$\text{From Equation (3.8): } \mathrm{ROA}_{2014} = \frac{15\,308}{141\,666} = 0.108$$

$$\begin{aligned}\text{From Equation (3.12): } \Delta\mathrm{ROA}_{2015} &= \mathrm{ROA}_{2014} - \mathrm{ROA}_{2015}\\ &= 0.096 - 0.108 \ = -0.011\end{aligned}$$

Since ΔROA < 0
F_ ΔROA $= 1$

$$\text{From Equation (??): } \mathrm{ACCRUAL}_{2015} = \frac{13\,7688 - 14\,121}{142\,550} = -0.002$$

Since CFO $>$ ROA
F_ ACCRUAL $= 0$

$$\text{From Equation (3.15): } ATA_{2014} = \frac{142\,550 + 141\,666}{2} = 142\,108$$

$$ATA_{2015} = \frac{134\,635 + 142\,550}{2} = 138\,593$$

$$\text{From Equation (3.16): } \Delta\text{LEVER}_{2015} = \frac{60\,460}{138\,593} - \frac{61\,085}{142\,108} = 0.006$$

Since ΔLEVER > 0
F_ ΔLEVER $= 1$

$$\text{From Equation (3.18): } CR_{2014} = \frac{18\,541}{27\,208} = 0.681$$

$$CR_{2015} = \frac{18\,294}{28\,456} = 0.643$$

$$\text{From Equation (3.19): } \Delta\text{LIQUID}_{2015} = 0.643 - 0.681 = -0.039$$

Since ΔLIQUID < 0
F_ ΔLEVER $= 1$

No shares were issued in 2015 and therefore EQ_ OFFER $= 0$

$$\text{From Equation (3.22): } GM_{2014} = \frac{28\,307}{47\,063} = 0.60147037$$

$$GM_{2015} = \frac{26\,467}{43\,604} = 0.607$$

$$\begin{aligned} \text{From Equation (3.23): } \Delta\text{MARGIN}_{2015} &= GM_{2015} - GM_{2014} \\ &= 0.607 - 0.601 \\ &= 0.006 \end{aligned}$$

Since ΔMARGIN > 0
F_ ΔMARGIN $= 0$

$$\text{From Equation (3.25): } ATR_{2014} = \frac{47\,063}{141\,666} = 0.332$$

$$ATR_{2015} = \frac{43\,604}{142\,550} = 0.306$$

$$\begin{aligned}\text{From Equation (3.26): } \Delta\text{TURN}_{2015} &= ATR_{2015} - ATR_{2014} \\ &= 0.306 - 0.332 \\ &= -0.026\end{aligned}$$

Since ΔTURN < 0
    F_ ΔTURN $= 1$

$$\begin{aligned}\text{From Equation (3.6): } FS_{2015} = {} & \text{F\_ ROA}_{2015} + \text{F\_ CFO}_{2015} + \text{F\_ }\Delta\text{ROA}_{2015} \\ & + \text{F\_ ACCRUAL}_{2015} + \text{F\_ }\Delta\text{LEVER}_{2015} \\ & + \text{F\_ }\Delta\text{LIQUID}_{2015} + \text{EQ\_ OFFER}_{2015} \\ & + \text{F\_ }\Delta\text{MARGIN}_{2015} + \text{F\_ }\Delta\text{TURN}_{2015} \\ FS_{2015} = {} & 0 + 0 + 1 + 0 + 1 + 1 + 0 + 0 + 1 = 4\end{aligned}$$

C.4 Market risk example

The information in Table C.4 was obtained from Google Finance as explained in Section 3.1.3 and will be used to demonstrate how the MR value for AB InBev was calculated.

Please note: 2015 had 252 trading days to save space only the twenty values will be shown at a time. These values will be used to demonstrate how the MR value is calculated. Unlike the example in C.1 this example cannot be reduced to fewer values.

Table C.4: Closing Prices for AB InBev shares for the end of 2015

Day Number	Date	Close Price
1	2015/01/02	5.30
2	2015/01/05	5.24
3	2015/01/06	5.24
4	2015/01/07	5.24
5	2015/01/08	5.23
6	2015/01/09	5.23
7	2015/01/12	5.22
8	2015/01/13	5.23
9	2015/01/14	5.20
10	2015/01/15	5.22
...	...	...
242	2015/12/16	4.53
243	2015/12/17	4.54
244	2015/12/18	4.50
245	2015/12/21	4.56
246	2015/12/22	4.63
247	2015/12/23	4.62
248	2015/12/24	4.62
249	2015/12/28	4.56
250	2015/12/29	4.43
251	2015/12/30	4.39
252	2015/12/31	4.35

As there are 252 days listed in the table the returns for 251 days can be calculated. Therefore, $C = 251$.

From Equation (3.27): $R_2 = \dfrac{(5.24 - 5.30)}{5.30} = -0.011320755$

Similarly,

$$
\begin{aligned}
&R_2 = \text{-}0.011320755 \\
&R_3 = 0 \\
&R_4 = 0 \\
&R_5 = \text{-}0.001908397 \\
&R_6 = 0 \\
&R_7 = \text{-}0.001912046 \\
&R_8 = 0.001915709 \\
&R_9 = \text{-}0.005736138 \\
&R_{10} = 0.003846154 \\
&\ldots \\
&R_{242} = 0.036613272 \\
&R_{243} = 0.002207506 \\
&R_{244} = \text{-}0.008810573 \\
&R_{245} = 0.013333333 \\
&R_{246} = 0.015350877 \\
&R_{247} = \text{-}0.002159827 \\
&R_{248} = 0 \\
&R_{249} = \text{-}0.012987013 \\
&R_{250} = \text{-}0.028508772 \\
&R_{251} = \text{-}0.009029345 \\
&R_{252} = \text{-}0.009111617
\end{aligned}
$$

Arranging the returns in ascending order results in the following list:

Return	Return value
R_{250}	-0.028508772
R_{249}	-0.012987013
R_{2}	-0.011320755
R_{252}	-0.009111617
.	.
R_{251}	-0.009029345
R_{244}	-0.008810573
R_{9}	-0.005736138
R_{247}	-0.002159827
.	.
R_{7}	-0.001912046
R_{5}	-0.001908397
R_{3}	0
R_{4}	0
R_{6}	0
R_{248}	0
R_{8}	0.001915709
.	.
R_{243}	0.002207506
R_{10}	0.003846154
R_{245}	0.013333333
.	.
R_{246}	0.015350877
R_{242}	0.036613272

Each return should then be numbered. This produces the following list:

Return	Return value	Return number
R_{122}	-0.0366088631984586	1
R_{240}	-0.0333333333333334	2
R_{59}	-0.0325047801147229	3
R_{186}	-0.0311284046692606	4
R_{250}	-0.0285087719298245	5
R_{162}	-0.0270793036750483	6
R_{190}	-0.0242914979757085	7
R_{231}	-0.0207900207900207	8
R_{86}	-0.0173745173745173	9
R_{107}	-0.0173410404624279	10
R_{185}	-0.0172084130019122	11
R_{57}	-0.0167910447761195	12
R_{123}	-0.016	13
R_{173}	-0.0158415841584159	14
R_{12}	-0.0151228733459357	15
R_{232}	-0.0148619957537156	16
R_{213}	-0.0147679324894515	17
R_{212}	-0.0145530145530144	18
R_{209}	-0.0143149284253578	19
R_{207}	-0.0142276422764228	20

Next it is necessary to calculate the number associated with conditional value-at-risk (CVaR) for each percentage, 95%, 99% and 99.9%.

$$\text{Using Equation (3.28): } CN(95) = (1 - 95\%) \times 251 \simeq 13$$
$$CN(99) = (1 - 99\%) \times 251 \simeq 3$$
$$CN(99.9) = (1 - 99.9\%) \times 251 \simeq 1$$

Therefore

$$\begin{aligned}
\text{From Equation (3.29): CVaR(95)} &= \frac{1}{13}\sum_{T=1}^{13} R_T \\
&= \frac{1}{13}(-0.0366 - 0.0333 - 0.0325 - 0.0311 - 0.0285 \\
&\quad - 0.0271 - 0.0243 - 0.0208 - 0.0174 - 0.0173 \\
&\quad - 0.0172 - 0.0168 - 0.016) \\
&= -0.0245
\end{aligned}$$

$$\begin{aligned}
\text{CVaR(99)} &= \frac{1}{3}(-0.0366 - 0.0333 - 0.0325) \\
&= -0.0341 \\
\text{CVaR(99.9)} &= \frac{1}{1}(-0.0366) \\
&= -0.0366
\end{aligned}$$

$$\begin{aligned}
\text{Therefore } MR(95) &= -0.024535384 = -2.45\% \\
MR(99) &= -0.034148992 = -3.41\% \\
MR(99.9) &= -0.0366088631984586 = -3.66\%
\end{aligned}$$

This $MR(95)$ indicates that in the worst five percent of returns the average loss will be 2.45%. The $MR(99)$ indicates that in the worst one percent of returns the average loss will be 3.41%. The $MR(99.9)$ indicates that in the absolute worst case scenario, 0.01% of the returns will result in an average loss of 3.66%

C.5 Risk measure example

This section demonstrates how the risk measure MR is determined.

The risk tolerance score (RTS) = 38 and from Appendix C.4, $MR = -0.02454 \simeq -2.454\%$ using the CVaR(95) method. Since the market risk value is negative it must be added to the RTS.

Therefore

$$\begin{aligned} \text{From Equation (3.32): } RM &= \left(\left(\frac{38-20}{49}\right) \times 100\right) + \mid -2.454 \mid \\ &= 36.735 + 2.454 \\ &= 34.28 \\ &\approx 34 \end{aligned}$$

This means that the market risk associated with AB InBev is much lower than the user's financial risk tolerance and thus this is a company to consider when investing.

Appendix D

Management of the stochastic elements

Here it is demonstrated how the value to be used in the model is determined by using the rank sum method's weights as listed in Table D.2

Table D.1: Table of LIX values

Year	LIX
2013	7.649
2014	7.701
2015	7.672

Using the weights obtained using the rank sum method the LIX value to be used is $= 0.167(7.649) + 0.333(7.701) + 0.500(7.672) = 7.677$

Table D.2: Weighted-moving average weights for the stochastic elements

Year	**EW**	**RS**	**RR**	**ROC**	**SIMOS**	**SR**
2013	0.333	0.167	0.182	0.111	0.010	0.174
2014	0.333	0.333	0.272	0.278	0.163	0.304
2015	0.333	0.500	0.545	0.611	0.827	0.522

Appendix E

Weight calculation examples

E.1 Ranking the objective functions

Table E.1: Table of the Rankings of the objective functions

Objective Function	**Rank**	**Reason for ranking**
Minimise RM	1	The RTS is the only input that the user of the model will input. Thus, it is the only variable that reflects the preference of the user and should thus be the most important input in the model. For this reason, this objective function has the highest rank.
Minimise ML	2	The ordinary person is not interested in buying shares that have no resale as this will result in a loss for them. As such it is important that liquid shares that will result in returns be purchased and as such this objective function has the second highest rank.
Minimise FS	3	FS and BM are both indicators of the expected financial growth of a company. However, Piotroski (2000) shows that if a company has a bad BM value but it has a good F_ Score it can indicate that even though the company is currently struggling, it will perform better in the future. For this reason, the FS is more important than the BM and thus has a higher rank.
Minimise BM	4	This is the last remaining objective function.

E.2 Equal weights example

As there are four objective functions, $N = 4$

From Equation (3.33): $w_1^{EW} = \dfrac{1}{N} = \dfrac{1}{4} = 0.25$

Similarly, $w_2^{EW} = 0.25$, $w_3^{EW} = 0.25$ and $w_4^{EW} = 0.25$

E.3 Rank sum example

As there are four objective functions, $N = 4$

Using Equation (3.34): $w_i^{RS} = \dfrac{(N+1+i)}{\sum_{j=1}^{N}(N+1-j)}$

Therefore

$$\begin{aligned}\sum_{j=1}^{N}(N+1-j) &= (4+1-1)+(4+1-2)+(4+1-3)+(4+1-4)\\ &= 4+3+2+1\\ &= 10\end{aligned}$$

Therefore

$$\begin{aligned}w_1^{RS} &= \frac{(4+1-1)}{10}\\ &= \frac{4}{10}\\ &= 0.4\end{aligned}$$

Similarly, $w_2^{RS} = 0.3$, $w_3^{RS} = 0.2$ and $w_4^{RS} = 0.2$

E.4 Rank reciprocal example

As there are four objective functions, $N = 4$

Using Equation (3.35): $w_i^{RR} = \dfrac{\frac{1}{i}}{\sum_{j=1}^{N}\left(\frac{1}{j}\right)}$

Therefore

$$\begin{aligned}\sum_{j=1}^{N}\left(\frac{1}{j}\right) &= \frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\\ &= 1+0.5+0.3333+0.25\\ &= 2.0833\end{aligned}$$

Therefore

$$\begin{aligned}w_1^{RR} &= \frac{\frac{1}{1}}{2.0833}\\ &= 0.48\end{aligned}$$

Similarly, $w_2^{RR} = 0.24$, $w_3^{RR} = 0.16$ and $w_4^{RR} = 0.12$

E.5 Rank order centroid example

As there are four objective functions, $N = 4$

Using Equation (3.36): $w_i^{ROC} = \left(\frac{1}{N}\right) \sum_{j=i}^{N} \left(\frac{1}{j}\right)$

Therefore

$$\begin{aligned}
w_1^{ROC} &= \left(\frac{1}{4}\right) \sum_{j=i}^{N} \left(\frac{1}{j}\right) \\
&= \left(\frac{1}{4}\right)\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) \\
&= (0.25)(1 + 0.5 + 0.3333 + 0.25) \\
&= 0.5208
\end{aligned}$$

and

$$\begin{aligned}
w_2^{ROC} &= \left(\frac{1}{4}\right) \sum_{j=i}^{N} \left(\frac{1}{j}\right) \\
&= \left(\frac{1}{4}\right)\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) \\
&= (0.25)(0.5 + 0.3333 + 0.25) \\
&= 0.2708
\end{aligned}$$

Similarly, $w_3^{ROC} = 0.1458$ and $w_4^{ROC} = 0.0625$

E.6 SR method example

As there are four objective functions, $N = 4$

Using Equation 3.37: $w_i^{SR} = \dfrac{\frac{1}{i} + \frac{N+1-i}{N}}{\sum_{j=1}^{N}\left(\frac{1}{j} + \frac{N+1-j}{N}\right)}$

Therefore

$$\begin{aligned}
\sum_{j=1}^{N}\left(\frac{1}{j} + \frac{N+1-j}{N}\right) &= \left(\frac{1}{1} + \frac{4+1-1}{4}\right) + \left(\frac{1}{2} + \frac{4+1-2}{4}\right) + \left(\frac{1}{3} + \frac{4+1-3}{4}\right) + \\
&\quad \left(\frac{1}{4} + \frac{4+1-4}{4}\right) \\
&= 2 + 1.25 + 0.8333 + 0.5 \\
&= 4.5833
\end{aligned}$$

Therefore

$$
\begin{aligned}
w_1^{SR} &= \frac{\frac{1}{1} + \frac{4+1-1}{4}}{4.5833} \\
&= \frac{2}{4.5833} \\
&= 0.4364
\end{aligned}
$$

Similarly, $w_2^{SR} = 0.2727$, $w_3^{SR} = 0.1818$ and $w_4^{SR} = 0.1091$

E.7 SIMOS method example

As there are four objective functions, $N = 4$

Using Equation 3.38: $w_i^{SIMOS} = \dfrac{(N+1-i)^{N+1}}{\sum_{j=1}^{N}(N+1-j)^{N+1}}$

Therefore

$$N + 1 = 4 + 1 = 5$$

and

$$
\begin{aligned}
\sum_{j=1}^{N}(N+1-j)^{N+1} &= (4+1-1)^5 + (4+1-2)^5 + (4+1-3)^5 + (4+1-4)^5 \\
&= 1024 + 243 + 32 + 1 \\
&= 1300
\end{aligned}
$$

Therefore

$$
\begin{aligned}
w_1^{SIMOS} &= \frac{(4+1-1)^5}{1300} \\
&= \frac{1024}{1300} \\
&= 0.7877
\end{aligned}
$$

Similarly, $w_2^{SIMOS} = 0.1869$, $w_3^{SIMOS} = 0.0246$ and $w_4^{SIMOS} = 0.0008$

E.8 Results of the different weighting methods

Below is a table that contains the weights that are obtained when using the equal weights (EW), rank sum (RS), rank reciprocal (RR), rank order centroid (ROC), SR and SIMOS methods for four objective functions.

Table E.2: Table of weights obtained from the different weighting methods

		Weights					
Objective Function	**Rank**	**EW**	**RS**	**RR**	**ROC**	**SR**	**SIMOS**
Minimise RM	1	0.25	0.4	0.48	0.5208	0.4364	0.7877
Minimise ML	2	0.25	0.3	0.24	0.2708	0.2727	0.1869
Minimise BM	3	0.25	0.2	0.16	0.1458	0.1818	0.0246
Minimise FS	4	0.25	0.1	0.12	0.0625	0.1091	0.0008

Appendix F

Model example

The metric values given in Table F.1 were calculated using the equal weights weighed moving average method. These metrics will be used as the metric values for this example. This example is for a RTS of 25 and the market risk values were calculated using the CVaR(95) method.

Table F.1: Table of metric values calculated using the equal weights method

Company	RM	BM	ML	FS	Return on investment	Is company eligible?	Reason
ANH	0.075	3.004	7.838	4	0.2506	No	BM >1
BTI	0.078	0.064	7.674	5	0.188	Yes	
NPN	0.081	0.721	6.03	5	-0.0122	No	ML <7.5
GLN	0.043	77.596	7.334	3	2.0975	No	BM >1 and ML <7.5
CFR	0.074	0.194	7.404	3	0.5775	No	ML <7.5
BIL	0.102	6.504	9.058	2	0.0004	No	BM >1
AGL	0.053	1.296	7.977	4	2.713	No	BM >1
FSR	0.067	0.353	8.665	3	0.5813	Yes	
SBK	0.069	0.518	8.198	4	0.7239	Yes	
SOL	0.008	316.257	6.608	5	-0.6871	No	BM >1 and ML <7.5
VOD	0.071	0.025	8.722	4	-0.0112	Yes	
MTN	0.075	0.045	7.019	5	0.6601	No	ML <7.5
OMU	0.066	107.647	8.579	3	0.2951	No	BM >1
SLM	0.056	12.652	8.256	3	0.7331	No	BM >1
BGA	0.049	11.499	7.535	3	0.5914	No	BM >1
NED	0.069	8.721	7.511	5	0.3596	No	BM >1
SHP	0.065	0.178	7.825	4	0.5479	Yes	
MNP	0.086	0.449	6.434	1	-0.0243	No	ML <7.5
REM	0.076	0.216	7.404	4	0.182	No	ML <7.5
APN	0.067	0.057	7.624	4	-0.1034	Yes	
DSY	0.066	0.221	7.694	5	0.395	Yes	
RMH	0.065	0.395	7.774	2	0.4096	Yes	
CPI	0.062	0.263	6.799	4	1.0294	No	ML <7.5
KIO	0.037	0.812	7.175	3	8.1784	No	ML <7.5
GRT	0.069	1.001	8.389	5	0.1907	No	BM >1
AMS	0.052	0.606	6.898	3	0.8967	No	ML <7.5
NRP	0.026	0.175	7.049	5	1.0472	No	ML <7.5
BVT	0.071	0.577	8.13	5	1.457	Yes	
TBS	0.066	0.448	7.352	4	0.4541	No	ML <7.5
MEI	0.038	0.045	7.029	4	0.2598	No	ML <7.5
MRP	0.059	0.074	7.491	3	0.226	No	ML <7.5
WHL	0.061	0.049	8.122	4	-0.3495	Yes	
RDF	0.066	0.79	8.516	4	0.0928	Yes	
INL	0.069	0.834	7.525	3	-0.1773	Yes	
HMN	0.076	1.181	8.059	4	-0.0517	No	BM >1
RMI	0.071	0.283	7.728	3	0.1839	Yes	
PSG	0.066	0.449	6.67	3	0.2081	No	ML <7.5
ITU	0.072	0.977	7.712	5	-0.4318	Yes	
TFG	0.062	0.275	7.591	5	0.6145	Yes	
IPL	0.061	0.275	7.457	6	1.1934	No	ML <7.5

When removing all the ineligible companies, the result is Table F.2.

Table F.2: Table of metric values for the eligible companies

Company	RM	BM	ML	FS	Return on investment
BTI	0.078	0.064	7.674	5	0.188
FSR	0.067	0.353	8.665	3	0.5813
SBK	0.069	0.518	8.198	4	0.7239
VOD	0.071	0.025	8.722	4	-0.0112
SHP	0.065	0.178	7.825	4	0.5479
APN	0.067	0.057	7.624	4	-0.1034
DSY	0.066	0.221	7.694	5	0.395
RMH	0.065	0.395	7.774	2	0.4096
BVT	0.071	0.577	8.13	5	1.457
WHL	0.061	0.049	8.122	4	-0.3495
RDF	0.066	0.79	8.516	4	0.0928
INL	0.069	0.834	7.525	3	-0.1773
RMI	0.071	0.283	7.728	3	0.1839
ITU	0.072	0.977	7.712	5	-0.4318
TFG	0.062	0.275	7.591	5	0.6145

These companies are then evaluated by the individual modelling approaches to select the best investment portfolio.

F.1 Based on only the risk measure

When selecting a model portfolio based on only the risk measure, the model arranges according to the risk measure in ascending order. The smaller the risk measure value is, the more desirable the company is and thus the model then selects the ten companies with the smallest risk measure values.

Arranging the companies according to the risk measure, the result is:

Company	RM	BM	ML	FS	Return on investment	Company's rank
WHL	0.061	0.049	8.122	4	-0.3495	1
TFG	0.062	0.275	7.591	5	0.6145	2
SHP	0.065	0.178	7.825	4	0.5479	3
RMH	0.065	0.395	7.774	2	0.4096	4
DSY	0.066	0.221	7.694	5	0.395	5
RDF	0.066	0.79	8.516	4	0.0928	6
FSR	0.067	0.353	8.665	3	0.5813	7
APN	0.067	0.057	7.624	4	-0.1034	8
SBK	0.069	0.518	8.198	4	0.7239	9
INL	0.069	0.834	7.525	3	-0.1773	10
VOD	0.071	0.025	8.722	4	-0.0112	11
BVT	0.071	0.577	8.13	5	1.457	12
RMI	0.071	0.283	7.728	3	0.1839	13
ITU	0.072	0.977	7.712	5	-0.4318	14
BTI	0.078	0.064	7.674	5	0.188	15

Thus the portfolio created using this method consists of: WHL, TFG, SHP, RMH, DSY, RDF, FSR, APN, SBK and INL. The return on investment for this portfolio is calculated using Equation 3.54 as follows:

$$\begin{aligned}R_{portfolio} &= \frac{1}{10}(-0,3495 + 0,6145 + 0,5479 + 0,4096 + 0,395 + 0,0928 + 0,5813 - 0,1034 \\ &\quad + 0,7239 - 0,1773) \\ &= 0,273\end{aligned}$$

F.2 Based on only the BM value

When selecting a model portfolio based on only the BM value, the model arranges according to the BM value in ascending order. As the smaller the BM values are, the more desirable a company is considered to be, the model then selects the ten companies with the smallest BM values.

Arranging the companies according to the BM, the result is:

Company	RM	BM	ML	FS	**Return on investment**	**Company's rank**
VOD	0.071	0.025	8.722	4	-0.0112	1
WHL	0.061	0.049	8.122	4	-0.3495	2
APN	0.067	0.057	7.624	4	-0.1034	3
BTI	0.078	0.064	7.674	5	0.188	4
SHP	0.065	0.178	7.825	4	0.5479	5
DSY	0.066	0.221	7.694	5	0.395	6
TFG	0.062	0.275	7.591	5	0.6145	7
RMI	0.071	0.283	7.728	3	0.1839	8
FSR	0.067	0.353	8.665	3	0.5813	9
RMH	0.065	0.395	7.774	2	0.4096	10
SBK	0.069	0.518	8.198	4	0.7239	11
BVT	0.071	0.577	8.13	5	1.457	12
RDF	0.066	0.79	8.516	4	0.0928	13
INL	0.069	0.834	7.525	3	-0.1773	14
ITU	0.072	0.977	7.712	5	-0.4318	15

Thus the model portfolio consists of VOD, WHL, APN, BTI, SHP, DSY, TFG, RMI, FSR and RMH. Using Equation 3.54 it was found that the return on investment for this model portfolio is 0.2456.

F.3 Based on only the market liquidity

When selecting a model portfolio based on only the market liquidity, the model arranges according to the market liquidity in ascending order. As the smaller the market liquidity is, the more desirable a company is considered to be, the model then selects the ten companies with the smallest market liquidity values.

Arranging the companies according to the market liquidity value, the result is:

Company	RM	BM	ML	FS	Return on investment	Company's rank
INL	0.069	0.834	7.525	3	-0.1773	1
TFG	0.062	0.275	7.591	5	0.6145	2
APN	0.067	0.057	7.624	4	-0.1034	3
BTI	0.078	0.064	7.674	5	0.188	4
DSY	0.066	0.221	7.694	5	0.395	5
ITU	0.072	0.977	7.712	5	-0.4318	6
RMI	0.071	0.283	7.728	3	0.1839	7
RMH	0.065	0.395	7.774	2	0.4096	8
SHP	0.065	0.178	7.825	4	0.5479	9
WHL	0.061	0.049	8.122	4	-0.3495	10
BVT	0.071	0.577	8.13	5	1.457	11
SBK	0.069	0.518	8.198	4	0.7239	12
RDF	0.066	0.79	8.516	4	0.0928	13
FSR	0.067	0.353	8.665	3	0.5813	14
VOD	0.071	0.025	8.722	4	-0.0112	15

Thus the model portfolio consists of INL, TFG, APN, BTI, DSY, ITU, RMI, RMH, SHP and WHL. Using Equation 3.54 the return on investment for this model portfolio was found to be 0.1277.

F.4 Based on only the F_Score

When selecting a model portfolio based on only the F_Score, the model arranges according to the F_Score in ascending order. As the smaller the F_Score is, the more desirable a company is considered to be, the model then selects the ten companies with the smallest F_Score values.

Arranging the companies according to the F_Score, the result is:

Company name	RM	BM	ML	FS	Return on investment	Company's rank
RMH	0.065	0.395	7.774	2	0.4096	1
INL	0.069	0.834	7.525	3	-0.1773	2
RMI	0.071	0.283	7.728	3	0.1839	3
FSR	0.067	0.353	8.665	3	0.5813	4
APN	0.067	0.057	7.624	4	-0.1034	5
SHP	0.065	0.178	7.825	4	0.5479	6
WHL	0.061	0.049	8.122	4	-0.3495	7
SBK	0.069	0.518	8.198	4	0.7239	8
RDF	0.066	0.79	8.516	4	0.0928	9
VOD	0.071	0.025	8.722	4	-0.0112	10
TFG	0.062	0.275	7.591	5	0.6145	11
BTI	0.078	0.064	7.674	5	0.188	12
DSY	0.066	0.221	7.694	5	0.395	13
ITU	0.072	0.977	7.712	5	-0.4318	14
BVT	0.071	0.577	8.13	5	1.457	15

Thus the portfolio consists of RMH, INL, RMI, FSR, APN, SHP, WHL, SBK, RDF and VOD. Using Equation (3.54), the return on investment for this model portfolio was found to be 0.1898.

F.5 Lexicographic method

In the lexicographic method, the objective functions are solved one at a time starting with the most important objective function. At a RTS of 25, this results in a ranking of:

Company	RM	BM	ML	FS	Return on investment	Company's rank
VOD	0.071	0.025	8.722	4	-0.0112	1
WHL	0.061	0.049	8.122	4	-0.3495	2
APN	0.067	0.057	7.624	4	-0.1034	3
BTI	0.078	0.064	7.674	5	0.188	4
SHP	0.065	0.178	7.825	4	0.5479	5
DSY	0.066	0.221	7.694	5	0.395	6
TFG	0.062	0.275	7.591	5	0.6145	7
RMI	0.071	0.283	7.728	3	0.1839	8
FSR	0.067	0.353	8.665	3	0.5813	9
RMH	0.065	0.395	7.774	2	0.4096	10
SBK	0.069	0.518	8.198	4	0.7239	11
BVT	0.071	0.577	8.13	5	1.457	12
RDF	0.066	0.79	8.516	4	0.0928	13
INL	0.069	0.834	7.525	3	-0.1773	14
ITU	0.072	0.977	7.712	5	-0.4318	15

Thus the model portfolio consists of VOD, WHL, APN, BTI, SHP, DSY, TFG, RMI, FSR and RMH. Using Equation (3.54) it was found that the return on investment for this model portfolio is 0.2456.

F.6 Weighted sum method

In the weighted sum method as score is calculated for each company using Equation (2.1). However, this score is not calculated using the calculated weighted average metric values but rather the rank of the companies as determined for each metric value as done in Appendices F.1, F.2, F.3 and F.4. The results of these ranking are as follows:

Company name	RM	BM	ML	FS
APN	5	5	5	5
BTI	12	12	12	12
BVT	15	15	15	15
DSY	13	13	13	13
FSR	4	4	4	4
INL	2	2	2	2
ITU	14	14	14	14
RDF	9	9	9	9
RMH	1	1	1	1
RMI	3	3	3	3
SBK	8	8	8	8
SHP	6	6	6	6
TFG	11	11	11	11
VOD	10	10	10	10
WHL	7	7	7	7

In this example, the score for each company is determined using RS weights. Using Equation (2.1), the score for APN is:

$$\begin{aligned}\text{Score}_{\text{APN}} &= (0.4 \times 5) + (0.2 \times 5) + (0.3 \times 5) + (0.1 \times 5) \\ &= 5\end{aligned}$$

Similarly $\text{Score}_{\text{BTI}} = 12$, $\text{Score}_{\text{BVT}} = 15$, $\text{Score}_{\text{DSY}} = 13$, $\text{Score}_{\text{FSR}} = 4$, $\text{Score}_{\text{INL}} = 2$, $\text{Score}_{\text{ITU}} = 14$, $\text{Score}_{\text{RDF}} = 9$, $\text{Score}_{\text{RMH}} = 1$, $\text{Score}_{\text{RMI}} = 3$, $\text{Score}_{\text{SBK}} = 8$, $\text{Score}_{\text{SHP}} = 6$, $\text{Score}_{\text{TFG}} = 11$, $\text{Score}_{\text{VOD}} = 10$ and $\text{Score}_{\text{WHL}} = 7$
The model selects the ten companies with the lowest score and thus the portfolio consists of RMH, INL, RMI, FSR, APN, SHP, WHL, SBK, RDF and VOD. Using Equation (3.54), the return on investment for this portfolio was found to be 0.1898.

F.7 Weighted produce method

In the weighted product method, the model also selects companies based on a score values, but this method uses that metric values as inputs. The score for each is determined using Equation (2.2) and the metric values in F.1. In this example, the score values for each company is determined using the RS weights as follows:

$$\text{Score}_{\text{RMH}} = (0.065^{0.4}) + (7.774^{0.3}) + (0.395^{0.2}) + (2^{0.1}) = 3.671$$

Similarly $\text{Score}_{\text{APN}} = 3.412$, $\text{Score}_{\text{BTI}} = 3.477$, $\text{Score}_{\text{BVT}} = 3.890$, $\text{Score}_{\text{DSY}} = 3.652$, $\text{Score}_{\text{FSR}} = 3.727$, $\text{Score}_{\text{INL}} = 3.904$, $\text{Score}_{\text{ITU}} = 4.021$, $\text{Score}_{\text{RDF}} = 3.952$, $\text{Score}_{\text{RMI}} = 3.653$, $\text{Score}_{\text{SBK}} = 3.834$, $\text{Score}_{\text{SHP}} = 3.589$, $\text{Score}_{\text{TFG}} = 3.682$, $\text{Score}_{\text{VOD}} = 3.369$ and $\text{Score}_{\text{WHL}} = 3.400$

The model selects that ten companies with the smallest score values and thus the portfolio consists of VOD, WHL, APN, BTI, SHP, DSY, RMI, RMH, TFG and FSR. Furthermore, using Equation (3.54) the return on investment for this model portfolio was found to be 0.24561.

Appendix G

Model revision assumptions

G.1 Model revision 1

Table G.1: Table of the Rankings of the objective functions for model revision 1

Objective Function	**Rank**	**Reason for ranking**
Minimise ML	1	The ordinary person is not interested in buying shares that have no resale as this will result in a loss for them. As such it is important that liquid shares that will result in returns be purchased and as such this objective function has the highest rank.
Minimise RM	2	The RTS is the only input that the user of the model will input. Thus, it is the only variable that reflects the preference of the user and should thus be the most important input in the model. For this reason, this objective function has the second highest rank.
Minimise FS	3	FS and BM are both indicators of the expected financial growth of a company. However, Piotroski (2000) shows that if a company has a bad BM value but it has a good F_ Score it can indicate that even though the company is currently struggling, it will perform better in the future. For this reason, the FS is more important than the BM and thus has a higher rank.
Minimise BM	4	This is the last remaining objective function.

G.2 Model revision 2

Table G.2: Table of the Rankings of the objective functions for model revision 2

Objective Function	Rank	Reason for ranking
Minimise FS	1	FS and BM are both indicators of the expected financial growth of a company. The ordinary person is interested in purchasing shares that will yield high returns in the future and thus share that have a high expected financial growth should be purchased. However, Piotroski (2000) shows that if a company has a bad BM value but it has a good F_ Score it can indicate that even though the company is currently struggling, it will perform better in the future. For this reason, the FS is more important than the BM and thus has the highest rank.
Minimise BM	2	This is an indication of expected financial growth which is considered to be the most important metric but described above the F_ Score is superior to the BM value. Therefore, this objective function has the second highest rank.
Minimise RM	3	The RTS is the only input that the user of the model will input. Thus, it is the only variable that reflects the preference of the user and should thus be the most important input in the model. For this reason, this objective function has the third highest rank.
Minimise ML	4	This is the last remaining objective function.k.

G.3 Model revision 3

In this model revision, it is assumed that taking the user input into account in the model, by means of a risk measure is the least important objective function. Furthermore, the other assumptions as given for the original model remain the same but move up by one rank.

Table G.3: Table of the Rankings of the objective functions for model revision 3

Objective Function	**Rank**	**Reason for ranking**
Minimise ML	1	The ordinary person is not interested in buying shares that have no resale as this will result in a loss for them. As such it is important that liquid shares that will result in returns be purchased and as such this objective function has the highest rank.
Minimise FS	2	FS and BM are both indicators of the expected financial growth of a company. However, Piotroski (2000) shows that if a company has a bad BM value but it has a good F_ Score it can indicate that even though the company is currently struggling, it will perform better in the future. For this reason, the FS is more important than the BM and thus has a higher rank.
Minimise BM	3	This is an indication of expected financial growth which is considered to be the most important metric but described above the F_ Score is superior to the BM value. Therefore, this objective function has the third highest rank..
Minimise RM	4	This is the last remaining objective function..

Appendix H

Actual return on investment calculations

H.1 Unit trusts

To demonstrate how the payout for a unit trust is calculated for various investment periods, consider the Stanlib Conservative Fund of Funds:

$$
\begin{aligned}
\text{Initial investment} &= \text{R}\,10\,000 \\
\text{Return on investment} &= 18.62\,\% = 0.1862 \\
\text{Investment fee percentage} &= 3\% = 0.03
\end{aligned}
$$

$$
\begin{aligned}
\text{From Equation (5.1)} : R &= (1 + R_{\text{annual}})^m - 1 \\
\text{Therefore } R_{\text{annual}} &= \sqrt[m]{R+1} - 1 \\
&= \sqrt[2]{0.1862+1} - 1 \\
&= 0.089
\end{aligned}
$$

$$
\begin{aligned}
&\textbf{For a 1 year investment:} \\
\text{Return receive by the unit trust} &= \text{R}\,10\,000 + (\text{R}10\,000 \times 0.089) \\
&= \text{R}10\,891.28 \\
\text{Investment fees} &= \text{R}10\,891.28 \times 0.03 \\
&= \text{R}326.74 \\
\text{Payout} &= \text{R}10\,891.28 - \text{R}326.74 \\
&= \text{R}10\,564.54
\end{aligned}
$$

$$
\begin{aligned}
&\textbf{For a 2 year investment:} \\
\text{Initial amount} &= \text{ending amount of the previous year} \\
&= \text{R}10\,564.54
\end{aligned}
$$

$$\begin{aligned}
\text{Return receive by the unit trust} &= \text{R}10\,564.54 + (\text{R}10\,564.54 \times 0.089) \\
&= \text{R}11\,506.14 \\
\text{Investment fees} &= \text{R}11\,506.14 \times 0.03 \\
&= \text{R}345.18 \\
\text{Payout} &= \text{R}11\,506.14 - \text{R}345.18 \\
&= \text{R}11\,160.96
\end{aligned}$$

For a 3 year investment:

$$\begin{aligned}
\text{Initial amount} &= \text{ending amount of the previous year} \\
&= \text{R}11\,160.96 \\
\text{Return receive by the unit trust} &= \text{R}11\,160.96 + (\text{R}11\,160.96 \times 0.089) \\
&= \text{R}12\,155.71 \\
\text{Investment fees} &= \text{R}12\,155.71 \times 0.03 \\
&= \text{R}364.67 \\
\text{Payout} &= \text{R}12\,155.71 - \text{R}364.67 \\
&= \text{R}11\,791.04
\end{aligned}$$

Similarly, Payout(4 years) = R 12 456.69, Payout(5 years) = R 13 159.93, Payout(6 years) = R 13 902.86, Payout(7 years) = R 14 687.74, Payout(8 years) = R 15 516.92, Payout(9 years) = R 16 392.92 and Payout(10 years) = R 17 318.37.

The payouts received by the Stanlib Conservative Fund of Funds (unit trust 1), Momentum Diversified Income Fund (unit trust 2), Stanlib Moderate Fund of Funds (unit trust 3), Stanlib Moderately Aggressive Fund of Funds (unit trust 4) and the Allan Gray Equity Fund (unit trust 5) for investment periods of 5 to 10 years are given in Table H.1.

Table H.1: Table of the payout (R) received for various unit trusts over a 5 to 10 year investment period

	Investment period (years)					
	5	6	7	8	9	10
Unit trusts						
Unit trust 1	13 159.93	13 902.86	14 687.74	15 516.92	16 392.92	17 318.37
Unit trust 2	119 166.08	125 893.51	133 000.73	140 509.19	148 441.53	156 821.69
Unit trust 3	114 718.83	121 195.19	128 037.17	135 265.42	142 901.72	150 969.13
Unit trust 4	112 935.69	119 311.39	126 047.03	133 162.92	140 680.53	148 622.55
Unit trust 5	141 671.69	149 669.66	158 119.15	167 045.65	176 476.08	186 438.91

H.2 Model portfolios

To demonstrate how the payout for a model portfolio is calculated for various investment periods, consider the highest return portfolio as listed in Table 4.4:

$$\begin{aligned}
\text{Initial investment} &= \mathrm{R}\,10\,000 \\
\text{Return on investment} &= 37.40\% = 0.3740 \\
\text{From F (5.1)} : R &= (1 + R_{\text{annual}})^m - 1 \\
\text{Therefore } R_{\text{annual}} &= \sqrt[m]{R+1} - 1 \\
&= \sqrt[2]{0.3740+1} - 1 \\
&= 0.1722
\end{aligned}$$

$$\begin{aligned}
\text{Time to gather data for 1 company in year 1} &= 1 \text{ hour} \\
\text{Time to gather data for 1 company in subsequent years} &= 0.3 \text{ hour} \\
\text{Number of companies} &= 40 \\
\text{The average professional person's hourly rate} &= \mathrm{R}\,130/\text{hour}
\end{aligned}$$

$$\begin{aligned}
\text{Cost of the ordinary person's time in year 1} &= 1 \text{ hour/company} \times \\
&\quad 40 \text{ companies} \times \mathrm{R}\,130/\text{hour} \\
&= \mathrm{R}\,5\,200.00 \\
\text{Cost of the ordinary person's time in following years} &= 0.3 \text{ hour/company} \times \\
&\quad 40 \text{ companies} \times \mathrm{R}\,130/\text{hour} \\
&= \mathrm{R}\,1\,560.00
\end{aligned}$$

$$\begin{aligned}
&\textbf{For a 1 year investment:} \\
\text{Return receive by the portfolio} &= \mathrm{R}\,10\,000 + (\mathrm{R}10\,000 \times 0.3740) \\
&= \mathrm{R}11\,721.77 \\
\text{Cost of investing} &= \mathrm{R}\,5,200.00 \\
\text{Payout} &= \mathrm{R}11\,721.77 - \mathrm{R}\,5,200.00 \\
&= \mathrm{R}\,6\,521.77
\end{aligned}$$

$$\begin{aligned}
&\textbf{For a 2 year investment:} \\
\text{Initial amount} &= \text{ending amount of the previous year} \\
&= \mathrm{R}\,6\,521.77 \\
\text{Return receive by the portfolio} &= \mathrm{R}\,6\,521.77 + (\mathrm{R}\,6\,521.77 \times 0.3740) \\
&= \mathrm{R}\,7\,644.68 \\
\text{Cost of investing} &= \mathrm{R}\,1\,560.00 \\
\text{Payout} &= \mathrm{R}\,7\,644.68 - \mathrm{R}\,1\,560.00 \\
&= \mathrm{R}\,6\,084.68
\end{aligned}$$

For a 3 year investment:

$$\begin{aligned}
\text{Initial amount} &= \text{ending amount of the previous year} \\
&= \text{R}\,6\,084.68 \\
\text{Return receive by the portfolio} &= \text{R}\,6\,084.68 + (\text{R}\,6\,084.68 \times 0.3740) \\
&= \text{R}\,7\,132.32 \\
\text{Cost of investing} &= \text{R}\,1\,560.00 \\
\text{Payout} &= \text{R}\,7\,132.32 - \text{R}\,1\,560.00 \\
&= \text{R}\,5\,572.32
\end{aligned}$$

Similarly, Payout(4 years) = R 4 971.75, Payout(5 years) = R 4 267.77, Payout(6 years) = R 3 442.59, Payout(7 years) = R 2 475.32, Payout(8 years) = R 1 341.52, Payout(9 years) = R 12.50 and Payout(10 years) = R -1 545.35.

The payouts received by the highest return portfolios in each risk category as listed in Tables 4.6 (High portfolio 1) and 4.4 (High portfolio 2) for investment periods of 5 to 10 years are given in Table H.2.

Table H.2: Table of the payout (R) received for various model portfolios over a 5 to 10 year investment period

	Investment period (years)					
	5	6	7	8	9	10
Model Portfolios						
High portfolio 1	3 632.26	2 649.34	1 510.27	190.22	-1 339.56	-3 112.39
High portfolio 2	4 267.77	3 442.59	2 475.32	1 341.52	12.5	-1 545.35

Appendix I

Industry sponsorship letter

To whom it may concern:

Melissa van Niekerk, student number 14048885, is currently an industrial engineering final year student and is completing her BPJ 420 project under my supervision. The project, titled Automated Investment Assessment: An investment decision making model for the ordinary person, focuses on helping the ordinary person to make wise investments so that they may achieve their long-term financial goals. It focuses on investing in the shares of companies listed on the Johannesburg Stock Exchange (JSE) and investigates how the ordinary person can invest for themselves without the aid of financial consultants. In this project, a model was designed and built that selects investment portfolios for the ordinary person based on multiple and often conflicting objectives. A return on investment is then calculated for each of these model portfolios which are then compared to the returns of unit trusts to determine whether worthwhile results are produced by the model.

The model is based on research publish in various articles and all the information used throughout the completion of this project is publicly available. For this reason, this project does not have a formal industry sponsor. I am fully aware of the situation and find it acceptable.

In order to develop and build this model, Melissa has had to employ a variety of industrial engineering techniques and principles as well as display the needed competences to complete a final year project. It is my opinion that this project is broad enough for a final year project and does encompass the scope of the desired outcomes for the BPJ 420 module. For this reason, I am acting as the industry sponsor for this project.

Kind regards
Dr. Nadia M. Viljoen

28/08/2018