Forecasting spare parts demand using condition monitoring information

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Abstract

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The control of an inventory where spare parts demand is infrequent has always been complex to manage because of the randomness of the demand, as well as the existence of a large proportion of zero values in the demand pattern. However, considering the importance of spare parts demand forecasting in production manufacturing and inventory management, several forecasting methods have been developed over the years to allow decision makers in industry to optimize the management of inventory where the demand pattern is infrequent. The Croston method is one of the traditional forecasting method, known because of its ability to take into consideration periods with zero demands. Yet, despite the Croston method's advantage over other traditional methods, there are still shortcomings in the method because it does not consider the condition of the components to be replaced.

This dissertation proposes an alternative forecasting method to the traditional methods, by means of condition monitoring. This method overcomes the Croston method's shortcomings by considering the condition information of the component under operation. A statistical model, the so-called proportional hazards model (PHM), which is a regression model, blending event and condition monitoring data, is used to estimate the risk of failure for the component under analysis, while subjected to condition monitoring. To obtain optimal decision making on spare parts demand, a blending of the hazard or risk with the economics is performed, and an optimal risk point is specified. The optimal risk point guides optimal decision making on spare parts policy for the component under analysis.

To generate the data needed to construct the proportional hazards model, a numerical investigation was performed on a fan axial bade where a crack was inserted and

propagated to estimate the fatigue crack life and corresponding natural frequencies. The simulation was run using MSC.MARC/MENTAT 2016 software. To validate the finite element model, an experiment was run by using a 50kN Spectral Dynamics electrodynamics shaker to apply base excitation to the fan axial blade specimens. The treatment and computation of data generated from experimental and numerical approaches allowed the construction of the proportional hazards model, with the fatigue lifetime as event data and the blade natural frequencies as covariates or condition monitoring information. The baseline Weibull parameters were estimated by maximizing the likelihood function using the Newton Raphson method and the MATLAB package. This allowed the computation of an objective function to determine the shape, scale and location parameters. Instead of defining the covariate behaviour needed to build the cost function by means of the Markov process, a simulation procedure was utilized to define the cost function and determine the optimal risk which minimizes the cost. Furthermore, as the proportional hazards model depends on both, time and covariates, it was also shown how the PHM behaves when time or covariates carry more weight.

The added value of the proportional hazard model as forecasting spare parts method lies in the fact that it allows one to proactively gather failure information which enables a ‘just in time’ supply of spare parts as well as an optimal maintenance plan.

Forecasting spare parts demand, using condition information, performs better than traditional methods because it reduces an overly large spare parts stock pile. By allowing a ‘just in time’ part availability, the spare parts management becomes more related to the condition of the asset. Additionally, the supply chain management and maintenance cost are optimized, and the preventive replacement of components is optimized compared to the time-based method where a component can be replaced while still having a useful life.

Contents

Notation

ADI	Average inter- demand interval
AHM	Additive hazards model
ARL	Applied research laboratory
AFTM	Accelerated failure time models
CM	Condition monitoring
CMS	Condition monitoring system
CV	Coefficient of variation
C_f	Cost of unexpected failure renewal
C_p	Cost of planned preventive renewal
DIC	Digital image correlation
$E(F_t)$	Expected value
EWMA	Exponentially weighted moving average
FEM	Finite element method
FGP	Fault growth parameters
F_{t+1}	Forecast demand per period at a given time
FCL	Fatigue crack length
G_t	Time inter demand at time t
$h\big(t, Z(t)\big)$	Instantaneous conditional probability of failure at time t, given the value of the covariate.
IMS	Intelligent maintenance system
IPDSS	Intelligent prediction decision support system
K	Stress intensity factor
K-S test	Kolmogorov Smirnov test
L	Likelihood
ML	Maximum likelihood
MDTB	Mechanical diagnostic test bed
PHM	Proportional hazards model
POM	Proportional odds model
P_t	Time inter demand interval
PWP	Prentice William Peterson model

$Q(d)$	Probability that failure replacement will occur
R (T_i)	Reliability of the component function of time
$R(T, Z(t))$	Reliability at time T_i considering the time dependent covariate
RNN	Recurrent neural network
RUL	Remaining useful life
SBA	Syntetos Boylan approximation
SES	Single exponential smoothing (SES)
TPM	Transition probability matrix
$W(d)$	Expected time until replacement
$\phi(d)$	Expected average cost per unit time
X_t	Actual demand at a given time
Z_t	Magnitude of the demand
$\boldsymbol{\alpha}$	Smoothing constant
β	Weibull shape parameter
γ	Weibull location parameter
η	Weibull scale parameter
μ	Mean of historical demand

Chapter 1 Introduction

1.1 Problem statement

Nowadays, the management of assets is becoming a point of central interest for the competitiveness of organizations. One of the most important life-cycle phases in asset management is the operation and maintenance of the asset. An efficient maintenance program also assumes proper management of spare parts inventory.

When managing an asset, it is critical to plan and control the spare parts inventory to avoid premature part replacement and overstocking of unnecessary spare parts (Yam, et al., 2001). That is why forecasting the demand of spare parts is important. In fact, forecasting is vital to every business organization and for every spare parts inventory, for it enables estimating the spare parts stock as accurately as possible. A better forecasting technique might allow a more efficient spare parts management policy, as well as cost optimization.

However, several traditional forecasting methods, applied for spare parts management, are inefficient for intermittent demand patterns and cannot accomplish reliable forecasting results. This includes methods such as the time series method, the Croston method and the exponential smoothing method.

Instead of using the classical methods to forecast spare parts demand, recent research proposes an integrated method that combines condition monitoring information with event data associated with the spare parts. The advantage related to this integrated method is the precision estimation of parts failure, and it also avoids downtime of machinery and stock-out. It detects potentially broken parts sufficiently early and allow a just-in-time maintenance and spare parts availability when managing a supply system (Hellingrath & Cordes, 2014).

The aim of this dissertation is to propose an alternative forecasting method based on condition-based maintenance instead of using the traditional methods. The proposed alternative method will be mixing both, event and condition monitoring data by means of a statistical model called the proportional hazards model which is a prognostic model, able to estimate the risk of failing for a component subject to condition monitoring.

The added value brought by this alternative method is that it improves the shortcomings and bridges the gap present in the traditional approach method, for the condition monitoring will track the progressive advance of failure in the component.

Afterward, as soon as the prognostics result from the proportional hazards model is available, the result will serve as input to effectively forecast the spare parts demand. The proactive failure information received through the condition monitoring model allows just-in-time maintenance and spare parts availability to be regulated in such a way that the inventory management avoids premature part replacement and overstocking of unnecessary spare parts.

1.2 Literature review

1.2.1 Spare parts forecasting overview

During the life cycles of equipment, they are used and eventually become obsolete, or fail because of age related failure mechanisms such as fatigue, which necessitates component replacement (Callegaro, 2010). Nowadays, with the growth of technology in industry, the problem of spare parts management is becoming important in maintenance, not only from a technical perspective but also from financial and economic points of view.

Companies where capital goods are made or used, typically have large inventories (Dekker, et al., 2011). In the aerospace and automotive industries, a wide range of service parts are held in stock, and the implication of holding spare parts in the inventory is important for the equipment performance. Wang and Syntetos (2011), reported that in the United States Air Force (USAF), the cost of recoverable spare parts amounted billions of dollars in the past years, which represents 52 percent of the total cost inventory.

The interest in forecasting spare parts demand is therefore growing at an unprecedented rate. Given that insufficient inventory stock lead to the extension of the equipment downtime, and excessive inventory stocks lead to the immobilization of money, it is important to determine an optimal level of spare parts to keep the equipment operating profitably. This makes forecasting spare parts demand a crucial field for researchers.

1.2.2 Spare parts features, demand pattern and classifications

a. Spare parts features

There are characteristics that distinguish spare parts from all other materials in the industries or service system (Callegaro, 2010). The main characteristic resides in the consumption aspect: the demands of spare parts in a company can follow very different patterns. One of the patterns described by the demand of spare parts is intermittency (it means the demand takes place irregularly with variable quantity). Another distinctive characteristic of spare parts concerns the specificity of their use. They must be used only for the use of the function for which they have been acquired. This exposes one to the risk of obsolescence which is faced when decisions are made on replacement of capital equipment. Very often a set of spare parts cannot be re-used on newly acquired equipment (Callegaro, 2010).

b. Spare part demand and classification

Service spare parts are complex in modern companies. According to the type of maintenance which is performed (i.e. preventive or corrective maintenance) it is important to highlight that the demand which arises from preventive maintenance can be scheduled, but remains stochastic in terms of size, whereas demand arising from corrective maintenance is stochastic in terms of failure occurrence but deterministic in size (Wang & Syntetos, 2011). However, both preventive and corrective maintenance imply the intermittent nature of the demand.

Often, spare parts forecasting is complicated because the demand takes place with irregular times, as well as the number of spare parts also vary with every instance. Such type of demand is also intermittent, meaning that the demand occurs infrequently with long periods of time without demand at all.

In the study of spare parts forecasting, intermittent demand patterns are very complex to deal with because of the dual sources of variation, namely, demand arrival and demand size (Wang & Syntetos, 2011). In the following paragraph, attention will be paid to determine parameters which affect the spare parts demand pattern.

To evaluate and classify the two main sources of variation, namely demand arrival and demand size causing the complexity of dealing with the intermittent demand, two parameters are generally used:

- The average inter-demand interval (ADI): As the name indicates, it is the average time interval between two spare part demands.

$$ADI = \frac{\sum_{i=1}^{N} t_i}{N} \tag{1.1}$$

- The coefficient of variation (CV): This parameter expresses the standard deviation of the spare parts over the average demand.

$$CV = \left(\frac{\sqrt{\sum_{i=1}^{N} t_i}}{N / \varepsilon} \right) \tag{1.2}$$

where

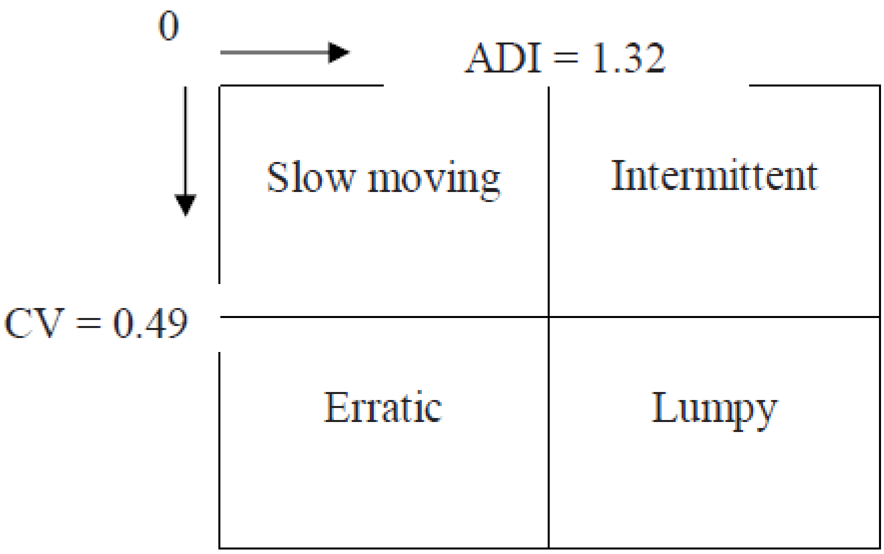
$$\varepsilon = \left(\frac{\sum_{i=1}^{N} \varepsilon_i}{N} \right) \tag{2.3}$$

In the average inter-demand interval (ADI) formula, the denominator N expresses the number of periods with non-zero demand whereas N in the CV formula expresses the total number of periods, ε_i the consumption of spare parts and t_i the interval for two consecutive demands.

Ghobbar and Friend (2003) state that there are cut-off values of CV and ADI that allow the categorization of the spare parts demand pattern. Wang and Syntetos (2011) suggested ADI =1.32 and CV=0.49 for cut-off values. In addition, Synthetos et al. suggested that:

- For ADI less than or equal to 1.32 and CV greater than 0.49 the demand is said to be erratic ($ADI \leq 1.32, CV > 0.49$). Erratic demand is characterized by a high quantity of demand but a constant demand in terms of distribution over time.

- For ADI strictly greater than 1.32 and CV strictly greater than 0.49 the demand is said lumpy ($ADI > 1.32, CV > 0.49$), lumpy demand is one of the more complex demand patterns to control because of many intervals with zero demand as well as great change in the quantity.
- For ADI less than or equal to 1.32 and CV less than or equal to 0.49 $(ADI \leq 1.32, CV \leq 0.49)$ the demand pattern is said to be smooth moving which is characterized by low rotation of the system.
- For ADI, strictly greater than 1.32 and CV less than or equal to 0.49 $(ADI > 1.32, CV \leq 0.49)$) the demand pattern is said to be intermittent. The categorization of the demand pattern will be based on the characteristics of demand data derived from the CV and ADI parameters.



Source: From thesis: “Forecasting method for spare part demand” (Callegaro, 2010).

1.2.3 Traditional forecasting method

Forecasting can be classified into four basic types: Causal relationship, qualitative, time series and simulation (Jacobs & Chaise, 2013).

When considering the future demand of spare parts in the production industries, decision makers use classical statistical methods to forecast future spare parts demand. Some of the well-known forecasting methods are exponential smoothing and regression analysis. However, it is crucial to highlight the uncertainty in forecasting spare parts because of the long period with zero demand. The Croston method is one of the common methods to address the intermittent demand pattern problem.

Croston (1972) found shortcomings in the single exponential smoothing methods. He showed that a bias related to putting the most weight on the most recent demand, led to the highest demand estimates just after the occurrence of the demand and lowest before one (Callegaro, 2010). Croston proposed a solution to the problem by using the average interval between demand and the average size of non-zero demand. Johnston and Boylan (1996) worked on a revision of the Croston method by establishing that the ADI must be greater than 1.25 for seeing the benefit of Croston over exponential smoothing. Furthermore Syntetos & Boylan (2005) highlighted an error in the derivation done by Croston and introduced a factor to correct the Croston formula. The modified Croston method by Syntetos and Boylan is known as Syntetos Boylan approximation (SBA).

The focus of the following section is the time series which is a type of forecasting that is based on data relating to past demand. Several methods belong to the time series class, such as: simple moving average, weighted moving average, and exponential smoothing. The following section addresses only the Croston method which is used for intermittent demand of spare parts.

Croston Method

The single exponential smoothing method (SES) did not explicitly consider the important parameter of the period with zero demands, whereas this is most common for spare parts (Dekker, et al., 2011). The Croston method proposes a solution to cope with this problem by using an alternative approach that considers both demand size and inter-arrival time between demands. The Croston method is today well-known in industry and is incorporated in several forecasting software packages (Teunter & Sani, 2009).

Several authors have assessed the Croston method since 1972. In 1973 Rao corrected some expressions in the Croston paper but this did not affect the conclusion. Syntetos and Boylan (2005) found that Croston method is biased. In 2005 they proposed an improved version of Croston’s method, the SBA. A new Croston type method was proposed by Teunter and Sani (2009) but did not affect considerably the conclusion of the original one.

The Croston method consists of two steps: firstly, the calculation of the time inter demand P_t and the magnitude of the demand Z_t.

$$Z_t = \alpha \times X_t + (1 - \alpha) \times Z_{t-1}$$

$$P_t = \alpha \times G_t + (1 - \alpha) \times P_{t-1} \tag{1.6}$$

where X_t is the actual demand at the time t, G_t Time inter demand at time t, α, is a smoothing constant between 0 and 1.

Therefore, the relationship of the forecast demand per period at time t is:

$$F_{t-1} = \frac{Z_t}{P_t} \tag{1.7}$$

The above formulas show two main factors when forecasting spare parts demand by means of the Croston method: the average time inter demand interval and the magnitude of the demand. This means that the Croston method is easily implemented where there is a significant set of failure data. However, when there is not enough historical failure data the implementation could become more difficult. A certain number of shortcomings are identified when dealing with Croston method and other traditional methods:

- They are based on historical failure and usage trends and do not adequately consider the condition of components in use.
- They do not allow to make decision with high precision when it is applied under certain condition such that for data with very high coefficient of variation (CV).

To overcome these weaknesses, methods have been considered that combine the use of historical failure trends and condition monitoring data. Before describing these methods, the next two sections introduce the concept of condition monitoring, as well as one of the models that has been developed to achieve this combination, namely the proportional hazards model (PHM).

Time series methods

The focus of the following section is the time series which is a type of forecasting that is based on data relating to past demand. Time series forecasting predict the future using past data. A certain number of methods belong to time series, such as: simple moving average, weighted moving average, exponential smoothing etc. The following section addresses only methods used for intermittent demand for spare parts.

Single exponential smoothing (SES)

When forecasting the future by the mean of the SES method, the most recent occurrences are more important than the distant past data (Jacob et al., 2014).

Concerning spare parts forecasting, SES is particularly suited for low period forecast and uses a series of weights, where the values of the weights are decreasing in an exponential manner.

1.2.4 Condition monitoring

Condition monitoring can be described as using external parameters such as vibration, acoustics, oil analysis, temperature, pressure, moisture, humidity, weather, or environmental data to measure the condition of a system (Hellingrath & Cordes, 2014)

Considering the present growth of competitiveness in the industrial environment, most organizations plan to increase their performance and productivity. However, for them to reach the set goal and deliver the intended service required by customers, attention must be focused on the condition of assets in the organization. To better assess the condition of the asset or component, condition monitoring is viewed as the most effective tactic. Over the past few decades condition monitoring became popular because of its efficient role in detecting potential failures, and the use of condition monitoring results in the improvement of the availability of the plant production as well as the decrease of the cost of downtime.

Condition monitoring is a cornerstone of condition-based maintenance. When dealing with condition-based maintenance, which is a proactive maintenance strategy, two aspects should be considered: diagnostics and prognostics. Diagnostics uses recorded condition information to identify, detect and isolate a fault condition whereas prognostics consists of predicting the occurrence of the failure and estimate the remaining useful life of the asset or component to make a suitable decision concerning the optimal replacement time of the component.

In this dissertation, a case study is presented that focuses on constructing a prognostic model for fan axial blades, prone to fatigue failure.

1.2.5 Introduction to the proportional hazards model

(Cox, 1972) presented a model to estimate mortality risk, called the proportional hazards model (PHM). The PHM incorporates the effects of covariates or explanatory variables on the distribution of the lifetimes. Covariates are any measured parameters that are thought to be related to the lifetimes of components. For each given time, the covariate provides an increase or decrease in the hazard. proportional to the baseline hazard rate.

The model proposed by Cox (1972), was first applied for biomedical data. Some years later the model was considered as a revolution in reliability engineering. In this context PHM is defined as a statistical procedure for the estimation of the risk for a component to fail when its condition is monitored (Jardine & Tsang, 2013).

The PHM is now one of the most popular statistical models used for survival analysis. Its popularity arises from the fact that the proportional hazards model is part of a broader class of survival analysis which provides information on the duration of time between the identifiable start and the occurrence of an event (Leclere, 2005). A key feature when using a proportional hazards model is that it can utilize time series variation in the covariates. The information can be provided based on the change in explanatory variables over time, that influence the probability of the event occurring.

The PHM is often presented in terms of the hazard model formula:

$$h(t, Z(t)) = h_0(t)e^{\sum_{i=1}^{p} \gamma_i z_i(t)} \tag{2.1}$$

where $Z_i(t)$ is the explanatory or predictor variable expressing the hazard at time t for an item or a component with a given specification of a set of predictor variables denoted by covariate. The $h_0(t)$ part is the baseline hazard; it includes time but not covariates, the second part $e^{\sum^{p} \gamma_i z_i}$ which is the exponential part includes covariates but not time, therefore the Cox model equation says that the hazard at a given time is the product of two important quantities whose the baseline hazard function and the exponential expresses the linear sum of $\gamma_i Z_i$.

The PHM formulation assumes that:

- The renewal times (event times) are iid (independent and identically distributed.
- All the significant covariates must be part of the model.

The PHM provides the possibility of incorporating condition monitoring results into the calculation of failure risk, where the condition parameters will be considered as covariates. As discussed in the next section, it is considered as one of the possible techniques that may be used to integrate the use of condition monitoring data into spare parts forecasting.

1.2.6 Integrating condition monitoring and spare parts forecasting

To overcome the shortcomings of the traditional spares forecasting methods, (Hellingrath & Cordes, 2014) explored the conceptualization of an approach for integrating condition monitoring information and spare part forecasting methods. In this work it was first shown that progress has been made in maintenance by forecasting the occurrence of failure for a component or a technical system, estimating the remaining useful life (RUL) using models such as the Proportional hazards models, neutral networks, etc. In addition, it was shown that the main problem today lies on forecasting spare part demand. In fact, several classical forecasting methods exist and are used, such as the time series, explanatory variable and hybrid methods; but these methods present a certain number of limitations which reduce the quality and accuracy of forecasting, to solve the problem related to the accuracy of the spare parts demand. Hellingrath and Cordes (2014) in their work decided to integrate condition monitoring information captured from the intelligent maintenance system (IMS) with the "traditional" forecasting methods.
To be able to implement the integrated model, many factors should be considered (Hellingrath & Cordes, 2014):

- The category of spare parts (for each category, different forecasting methods are used)
- The type of output data from the IMS (it affects the modality)
- Identification of the parameters that must be adapted

Regarding the above, it is important to notice that for each forecasting method, numerous requirements and parameters can be identified, independent of the type of the IMS output data. This implies that it is difficult to establish a guideline or general approach for the integration needed (Hellingrath & Cordes, 2014).

Nevertheless, the spare parts demand forecasting can be addressed in different ways (Hellingrath & Cordes, 2014):

- The first, which is the focus of this dissertation, consists of building a proportional hazards model from the condition-based information, then determine from there the ordering decision for spare parts when the related component is monitored by a condition monitoring system (CMS).
- The second way, which was the aim of Hellingraph and Cordes (2014) consists of integrating CM data with the classical forecasting model. This approach is called CBMF and follows a sequence of steps proposed by Bacchetti & Saccani (2011).

Pre-processing is performed to categorize of the spare parts as slow moving, intermittent, erratic or lumpy. In addition, the main idea in this step consists of integrating CM information and forecasting methods to generate a hybrid two step estimation (Hellingrath & Cordes, 2014). The first step refers to the determination of the forecasting parameters. The CM information is analysed regarding the distribution parameter of potential breakdowns, for the second step, a Bayesian approach is used to provide a probability function of the spare parts demand.

Wang and Aris (2011) worked on linking forecasting to equipment maintenance. Their approach consisted of answering two main questions:

- Why is the demand for spare parts intermittent?
- How can we use models developed in maintenance research to forecast such demand?

Furthermore, it was shown in their work that it is difficult to forecast intermittent demand patterns because of the dual source of variation (demand arrival and demand size). In addition, their work attempts to answer the second question by comparing demand forecast methods and maintenance-based method (time delay forecasting methods).

Forecasting spare parts demand is becoming a huge area of research in maintenance, the main purpose in this work is to improve quality of the spare parts forecasting by making it as accurate as possible.

Considering the weaknesses related to the usual traditional methods, Romeijnders et al. (2012) proposed a method called two step forecast method. The advantages related to this method is first the fact that it considers the type of component repaired, moreover contrary to other methods, the two-step method can use information on planned maintenance and repair operations to reduce forecast error by up to 20 % Romeijnders et al. (2012). The first step of the method it is all about forecasting, for each type of component the number of repairs per time unit and the number of spare part needed per repair. Secondly these forecasts are combined to forecast demand of the spare part Romeijnders et al. (2012).

Real data from Fokker Services (which is a company that maintains and repairs aircraft components) captured for a period of 10 years was used to compare the two-step method with several traditional methods.

Even though the two-step method offers better results than the Croston forecasting method and the Syntetos Boylan approximation, which are among the best, the two- step method still does not consider the actual condition of the component (condition information) but it uses the historical data set.

Bacchetti and Saccani (2011) explored spare parts classification and demand forecasting for stock control. Finally, they concluded that a gap still exists between research and practice concerning the field addressed in this work. In their investigation, they recognize that several aspects concur in making demand and inventory management for spare parts a complex matter. Some of these aspects are the high number of parts managed, and the presence of intermittent or lumpy demand patterns.

It is important to highlight that little progress has been made to date in terms of integrating condition monitoring information into spare parts management. Bacchetti and Saccani (2011) report that there still exists a gap between research and practice in spare parts management. Integrating condition monitoring information captured from a computerised maintenance management system with the traditional forecasting method, promises possible improvement of the traditional forecasting method. The following table displays the classification of forecasting methods for sporadic demand referring to Hellingrath and Cordes (2014).

Table 1.1: Classification of forecasting methods for sporadic demand (Hellingrath & Cordes, 2014)

Forecasting method	Classification				Consideration of the sporadic characteristic of spare parts demand	Usage of condition related information
	T	E	H	O		
SMA, SES	×				No	No
EWMA	×				No	No
Holt and Holt-Winters	×				Yes	No
Croston and its modifications	×				Yes	No
Bootstrapping	×				Yes	No
Filtering /clustering	×				Yes	No
Advance demand information				×	No	No
Failure rate analysis		×			Yes	Utilizing historical data of the installed base of technical systems
Operating condition analysis		×			Yes	Considering influence of the environment (e.g. temperature)
Regression			×		Yes	No
Neural networks			×		No	No
Bayesian approaches			×		Yes	Condition information is used to adjust to the demand value
Proportional hazards model			×		Yes	Condition information is used to adjust the demand value
Installed base forecasting	×	×	×	×	Yes	Utilizing data about the condition of the installed base of technical systems

Forecasting methods in table 1.1 are classified in time series (T), explanatory (E), hybrid (H), and other methods (O), (Bacchetti & Saccani, 2011). The proportional hazards model is the focus in this research, for reasons outlined below.

1.2.7 Selection of the proportional hazards model for this work

Several traditional forecasting methods applied to spare parts management are inaccurate and cannot accomplish appropriate forecasting results. Methods such as Croston, exponential smoothing, moving average and single exponential smoothing are traditional time series method and still the most commonly used in business practice. However, the issue with these methods is that they overestimate the mean level of intermittent demand if applied immediately after a demand occurrence. The aim of the present study is to develop an integrated method that combines condition monitoring information and spare parts forecasting methods by means of PHM, as per the highlighted forecasting method shown in Table 1.1 above. The advantages of such an integrated model would be the precise estimation of part failure because it considers the condition of the component, thereby avoiding downtime of machinery and stock out, by sufficiently early detection of potential failures and allowing a just in time maintenance and spare parts availability.

1.3 Scope of the work

The demand for spare parts in industry can follow different patterns. Forecasting intermittent demand patterns with a long period of zero demand remains particularly challenging. One of the traditional forecasting methods which manages to address the matter properly is the Croston method, but the shortcoming of the Croston method is that it does not consider the condition of the component. To deal with this weakness, the present dissertation proposes an alternative method to overcome the problem.

The approach developed in this study consists of integrating condition monitoring data with event data by means of a proportional hazard model (PHM), to estimate the risk of failure occurring for a component subject to condition monitoring. The statistical model called PHM serves to forecast the spare parts demand and define spare parts management policy.

Knowing that building a PHM requires event and condition data, both experimental and numerical investigations were run to generate the data needed to build a PHM.

Optimal decision making is performed by means of the cost function built and based on the PHM. It is important to highlight at this point that this dissertation does not address aspects of the spare parts management, such as lead time, stock holding etc. It only serves to give to the inventory management the best information possible, required to make optimal decisions.

The following approach is adopted in this dissertation:

- A numerical investigation was conducted which consisted of a modal analysis performed with MSC.MARC2015.0 nonlinear finite element software, to determine the coupling between natural frequency and mode shape for a 30 and 40-degree axial fan blade. A 2mm crack was initiated in the blade, then propagated to failure. Information such as natural frequencies and mode shapes were recorded as the crack propagated into the axial fan blade. For the purposes of this dissertation only the natural frequency was considered as a covariate to build the PHM.
- An experimental investigation run in the laboratory consisted of estimating the lifetime and Paris law material constants. The setup was designed in such a way that an initiated crack in the axial fan blade was propagated and measurements were performed using digital image correlation (DIC). The stress intensity factor was calculated analytically, and the measured crack length was used to determine the Paris law constants. Furthermore, a statistical analysis was performed on the determined material constants and lifetimes. This study was done as a separate master's degree study by (Brits, 2016). The experiment served not only for validation of the finite element model (FEM) but also to determine the Paris material constants and lifetimes which served as event data to build the PHM.
- Both the natural frequencies generated by the FEM and the lifetimes from the experimental investigation served as covariates and event data respectively in the PHM.
- Instead of establishing the covariate behaviour and specifying the probability of shifting from one state to another by means of the transition probability matrix (TPM), a simulation procedure was performed to determine the cost function.
- Optimal decision making is performed by means of the cost function built and based on the PHM. An optimal risk point d was set up and served as input to define a spare parts demand policy. It is important to highlight at this point that

this dissertation does not address aspects of spare parts management which deal with the lead time, stock holding etc. It only provides the inventory manager information needed to make right demand of the component in a right time.

When the process described above is properly performed, it results in reduction of the overestimation of spare parts demand, compared to the traditional forecasting methods and a just in time spare parts management and maintenance policy is established. Moreover, an early indication of failure provides more time for proper maintenance planning and scheduling.

1.4 Document overview

Traditional forecasting methods as well as limitations related to these methods are discussed in chapter 1. The advantages that these methods offer is also discussed in the chapter. The proportional hazard model (PHM) is subsequently introduced in chapter 2 as an appropriate statistical model to allow the integration of condition information to the spare parts forecasting method. Chapter 2 also describes the proposed forecasting method based on the PHM and its economics approach.

In chapter 3, an overview of a case study is presented focused on the generation of data by means of numerical and experimental investigation. Condition monitoring data are generated by means of the MSC.MARC/MENTAT 2016.0 software package. As the PHM requires two types of data to be built, the event data in this work was supplied by the experiment which is the number of loading cycles, whereas condition monitoring data, which comprise natural frequencies, are generated by running 30-degree axial fan blades with a 2mm initial crack inserted in the FEM.

Both event and condition monitoring data being available, in chapter 4, the implementation of the proposed method on the case study is described. Chapter 4 also deals with the important matter of the construction of the PHM, and the goodness of fit testing, using the K.S test.

After constructing the PHM in chapter 4, an estimate of the risk of failure for the case study components (fan blades) is known. Chapter 5 then discusses how to use information from the PHM to obtain economic benefits which will lead one to define a suitable policy for the demand or the replacement of the blades.

The work is concluded in chapter 6 by showing how to use the PHM outcome for the need of component replacement (spare parts demand). Recommendations for future work are also made in this chapter.

Chapter 2 An integrated spare parts forecasting method using condition monitoring

2.1 Introduction

Over the past few decades, preventive maintenance decisions have been optimized by means of statistical analysis of failure data, while condition-based maintenance has been optimized by utilizing sophisticated methods such as vibration and oil analysis. The present research consists of building a mixed model which combines event and condition monitoring data into a mathematical model to predict the risk of failure occurrence for an asset, and then use the outcome from the prediction model to forecast spare parts demand.

Reliability analysis is known as the analysis of event data only, which consists of fitting event data to a time between probability distribution, and the fitted distribution can be utilized for further analysis (Vlok, 1999). However, it is beneficial to combine event data and condition monitoring data by building a mathematical model that allows maintenance decision support (diagnostics or prognostics). In this dissertation a time dependent proportional hazard model (PHM), which is a popular regression model is described and utilized as a tool to forecast spare parts demand.

Renewal theory consist of estimating the reliability of a component using the recorded time to failure and computing the renewal time that minimize the mean life cycle cost of the future components (Vlok, 1999). When dealing with renewal theory the reliability concepts such as failure density, cumulative failure density, reliability function and the instantaneous failure rate are important to model the history of data in possession.

To model the reliability function of a renewable system, several approaches are used:

- A probabilistic modelling approach;
- A non-probabilistic modelling approach;
- A regression modelling approach.

The following paragraph addresses the regression modelling and particularly the proportional hazard model.

2.1.1 Regression modelling approach

Regression modelling entails merging probabilistic and non-probabilistic modelling approaches. The following properties define the regression modelling approach:

- Like non-probabilistic models the regression models directly recognize the existence of the survivor function or hazard rate but do not utilize the existence of an underlying failure distribution as primary assumption.
- The regression models are not only the primary use parameter modelled but also the concomitant information surrounding failure or covariates.

Several regressions models were identified in the literature for renewal theory:

- Accelerated failure time models (AFTM) during 1966;
- Proportional hazard model (PHM) during 1972;
- Prentice William Peterson model (PWP model) during 1981;
- Proportional Odds model (POM) during 1983;
- Additive hazard model (AHM) during 1990.

Literature shows that all the five named regression models have the same structure. The baseline function first which is a time-based part estimated either as parametric or non-parametric techniques, secondly an explanatory part, this part has a direct influence on the baseline function to estimate the overall reliability of the system.

(Vlok, 1999) presented a decision matrix showing that the proportional hazard model is the most suitable out of all the named regressions models. The criteria of evaluation were: (1) Theoretical foundation; (2) Previous practical success in reliability modelling; (3) Potential to lead to the dissertation objective; (4) Achievability of numerical implementation; (5) Future potential in reliability modelling.

2.2 Proportional hazards model (PHM)

2.2.1 Development of the proportional hazards model

a. Cox proportional hazards model

The PHM is a regression model for survival time that allows for covariates, but he did not impose a parametric form for the distribution of survival times (Crumer, 2008). Cox (1972) assumed that the survival distribution satisfies the condition given by the formula (2.1).

b. Extension of the Cox proportional hazards model for time dependent variables

With the extended Cox proportional hazards model, covariate Z is considered as time dependent variable. Time dependent variables are defined as variables whose values may differ over time t, whereas time independent variables are variables which remain constant over time.

When modelling the hazard function $h(t)$, the baseline hazard function $h_0(t)$ can be represented in parametric or non-parametric form. A commonly used parametric baseline hazard function is the Weibull hazard function. To model the PHM is like the process of regression analysis. A set of significant covariates is needed and only the significant covariates are inserted in the models.

For a given PHM, the choice of the type of covariate to be used depend on the theoretical assumption about the relationship between the covariate value and the hazard function (Leclere, 2005). When the hazard function is mostly dependent on the value of the covariates at time zero or some fixed time point, then time independent covariates are the right choice. But when the covariates change over time and the hazard function depends more on the current values of the covariates, then the time dependent covariates are the right choice.

Considering errors yielded by the situation where covariates change over time, many studies ignore the time dependence and deal with time dependent covariates as time independent, by fixing its value at a given point in time or setting the value of the covariate to an average value for the period that is studied. Likely problems when using time dependent covariates as time independent or time invariant covariates are:

- As several covariates are likely to change before the advent of the event, the variation is eliminated, and important information is lost.
- Several phenomena are generated by dynamic, longitudinal processes, because the value of a covariate along the time path affects the probable event happening.
- The model does not include the value of the covariate observed at the time of event occurrence, although it may be this actual value that generate the event.

A few notes are relevant:

- With the availability of software today, there are some which directly deal with time dependent variables and the need for considering time dependent variables as time independent is reduced.
- For the purposes of this research, event and covariate data are generated by laboratory experiments because of the difficult access to industry data. This is dealt with in Chapter 3.

For this dissertation, the covariates are considered time dependent and the PHM will be addressed as follows:

- First determine the Weibull parameters (β, η, γ) constituting the baseline function. This computation is done by applying the maximum likelihood estimation method.
- Secondly the changes in the measurements of the covariate characteristics in the explanatory part will not be modelled according to the semi–Markov process, but through a simulation procedure.
- The third step deals with the economics - it is all about specifying the optimal inspection time that minimizes the cost.

In the parametric PHM one of the most important operations to be done is to estimate the $\gamma's$ to access the effect of explanatory variable, the corresponding estimate parameters are determined by means of the maximization of the likelihood function Kleinbaum (1999).

2.2.3 The fully parametric PHM and maximum likelihood

Before addressing the maximum likelihood method, it is important to first understand the notion of fully parametric. The PHM is totally parametrized by assuming a continuous distribution for the baseline (Vlok, 1999). For the purpose of this work the Weibull distribution is considered. This is given by the expression:

$$h[t, Z(t)] = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}\exp\left\{\sum_{i=1}^{m}\gamma_i Z_i(t)\right\} \qquad (2.2)$$

a. Statistical Model

(Vlok, 1999) highlighted that fewer numerical issues arise when dealing with Weibull PHM to determine the baseline parameters. However, the following steps present Vlok's approach to determine the three Weibull parameters:

Consider the general Weibull distribution formula for time dependence

$$f(t) = \frac{\beta}{\eta}(\frac{t}{\eta})^{\beta-1}\exp[-(t/\eta)^{\beta}] \tag{2.3}$$

The hazard rate function corresponding to the probability density function (pdf) given by (2.3) is:

$$h(t) = \frac{\beta}{\eta}(\frac{t}{\eta})^{\beta-1} \tag{2.3}$$

with beta (β) and eta (η) being the shape and scale parameters of the distribution respectively. By using the Weibull distribution as the baseline hazard rate of the PHM according to (2.1), the formula becomes:

$$h\left(t,\overline{Z(t)}\right) = \frac{\beta}{\eta}(\frac{t}{\eta})^{\beta-1}\exp(\bar{\gamma} \times \overline{Z(t)}) \tag{2.4}$$

Considering the reliability theory, it is stated that the reliability of a component under the influence of ageing only, before renewal at time T_i is given by:

$$R(T_i) = \exp(-\int_0^{T_i} h(t)dt)=\exp(-(\frac{T_i}{\eta})^{\beta}) \tag{2.5}$$

If $U_i = (\frac{T_i}{\eta})^\beta$, U_i has a unit negative exponential distribution. As for (2.5), at time T_i the reliability of the component under the influence of time independent covariates according to the PHM is estimated by:

$$R(t,\bar{Z})\text{=}exp\left[-\int_0^{T_i}\frac{\beta}{\eta}(\frac{t}{\eta})^{\beta-1}\text{dt}\exp(\bar{\gamma}\times\overline{Z)}\right] \quad (2.6)$$

By solving (2.6) it gives:

$$R(t,\bar{Z})\text{=}exp\left[-(\frac{T_i}{\eta})^\beta\exp(\bar{\gamma}\times\bar{Z}\right] \quad (2.7)$$

Equation (2.6) is about the time independent covariate. For the time dependent $U_i = (\frac{T}{\eta})^\beta\exp(\overline{\gamma},\overline{Z}_\iota)$, again with unit exponential distribution. When dealing with this case with time dependent covariates, the reliability at time T_i for the component, considering the time dependent covariate will be:

$$R(t,\overline{Z(t)}\text{=}exp\left[-\int_0^{T_i}\frac{\beta}{\eta}(\frac{t}{\eta})^{\beta-1}\exp(\bar{\gamma}\times\overline{Z(t)}dt\right] \quad (2.8)$$

Equation (2.8) gives:

$$R(t,\overline{Z(t)}\text{=}exp\left[-\int_0^{T_i}\exp(\bar{\gamma}\times\bar{Z}_i\,(t)d(\frac{t}{\eta})^\beta\right] \quad (2.8)$$

Considering $U_i\text{=}\int_0^{T_i}\exp(\bar{\gamma}\times\bar{Z}_i\,(t)d(\frac{t}{\eta})^\beta$, with unit negative exponential distribution. In practice (2.8) and (2.9) are approximated by:

$$R(t,\overline{Z(t)}\text{=}exp\left\{\sum_{k=1}^{i}\exp(\overline{\gamma}\times\overline{Z_i^*}(t_k))\times\left[(\frac{t_{k+1}}{\eta})^\beta-(\frac{t_k}{\eta})^\beta\right]\right\} \quad (2.9)$$

with $0\text{=}t_0 < t_i < \cdots < T_i$ inspection points where covariate measurement was performed and $Z_i^* = 0.5\times\left(\overline{Z_\iota\,(t_k}\right)+\overline{Z_\iota\,(t_{k+1}))}$.

a. Maximum likelihood (Parameter estimation)

As indicated in the literature, the maximum likelihood of the Cox model parameters is found by maximizing a likelihood function. The likelihood function is a mathematical expression which describes the joint probability of obtaining the data observed on the subjects in the study as a function of the unknown parameters (the $\gamma' s$) in the model being considered (Kleinbaum, 2000). Some literature such as, (Vlok, 1999), addressed the optimization of the likelihood equation to determine the Weibull parameters.

The Weibull parameters are estimated by maximizing the likelihood equation given by:

$$L(\beta,\eta,\bar{\gamma})=\prod_i h(T_i,\overline{Z_\iota\,(T_\iota)}) \times \prod_j R(T_j,\overline{Z_J(t)}) \tag{2.10}$$

with the i index referring to failure times and where $j = 1,2 \ldots \ldots \ldots .. n$ indicate failure and suspension times. It is important to highlight that for the aim of this dissertation it deals with complete data.

The Weibull parameters β, η, γ which maximize (2.19), can also maximize $\log(L(\beta,\eta,\gamma)$ or $l(\beta,\eta,\gamma)$. It is numerically appropriate to maximize $l(\beta,\eta,\gamma)$ which is given by:

$$l(\beta,\eta,\bar{\gamma}) = rln\left(\beta/\eta\right) + \sum_i ln\left[(T_i/\eta)^{\beta-1}\right] + \sum_i \bar{\gamma} \times \overline{Z_\iota\,(T_i)} - \sum_j \int_0^{T_j} \exp(\bar{\gamma} \times \overline{Z}_j(t)\; d(t/\eta)^\beta \tag{2.11}$$

where r is the number of failure renewals.

In this dissertation, equation (2.11) or (2.12) are solved numerically using a Newton-Raphson optimization procedure.

$$l(\beta,\eta,\bar{\gamma}) = r(-\beta ln\eta) + rln\beta + (\beta-1) \times \sum_{i=1}^{r} lnt_i + \sum_{b=1}^{m} \gamma_b B_b - \left[\exp(a) \times (\sum_{i=1}^{n} \gamma_g Z_{jg}^i) \times (t_{i(j+1)}^\beta - t_{ij}^\beta\ )\right] \tag{2.12}$$

To maximize equation (2.12) and estimate the three Weibull parameters, a number of techniques have been tested successfully. Among these are:

- A Nelder-Mead method
- A BFGS Quasi-Newton method
- Snyman's dynamic trajectory method
- A modified Newton-Raphson method

The performance of the above-mentioned methods was assessed regarding their economy, which means according to the number of iterations needed to converge, the number of objective function evaluations and the number of partial derivative evaluations, as well as robustness. The outcome from the evaluation of the above-mentioned methods was such that the Newton Raphson method was found more suitable and economic for optimization of the maximum likelihood function. This dissertation uses the Newton Raphson method to optimize the equation (2.12).

b.1 Newton Raphson method for a 3 parameters Weibull

Vlok (1999) proposed a template to simplify the computation of the Newton Raphson optimization technique for vibration monitoring data. Referring to the suggested template, n expresses the number of histories, which is seven for this dissertation, and i indicates the history number such that: $i = 1,2 \dots\dots n$.

The time to failure or suspension in each history, as expressed by T_i, and C_i, are used as indications making the difference between failure and suspension. For C_i=1, T_i is a failure and for $C_i = 0$, T_i is a suspension. For the aim of this dissertation, data are complete, means without suspensions.

The number of inspections k_i must be set to be able to model the scenario associated to the time dependent covariate which is the natural frequency. For the aim of this dissertation a 50000 cycle is set as interval between inspection to build the proposed templates.

Below in table 2.1 at the sample of the template associated to our data is given.

Table 2.1: Template of inspection time and covariate corresponding

Inspection Time	Covariate
t_{i0}	Z_{01}^{i}
t_{i1}	Z_{11}^{i}
	.
	.
	.
t_{iki}	Z_{ki1}^{i}

The above template is adjusted according to our data which deals with a unique covariate as it is the case in this dissertation. The Weibull parameters are estimated by optimizing the objective function (2.12), considering the complexity of the objective function, a MATLAB algorithm called `fmincon` is used to optimize and compute the objective function in the dissertation.

b.2 Maximum likelihood for a simple Weibull (2 parameters)

This section is all about determining the shape and scale parameters related to the axial fan blade data. Firstly, it is important to notice that the Weibull parameter estimates can be defined using different methods such as the graphical method, by means of probability plotting paper, or the analytical method, using either least squares or maximum likelihood (Tan, 2009). The probability plotting method requires less mathematics and is suitable for a small sample size. Furthermore, Tan et al. (2009) present many advantages making the maximum likelihood method more attractive. Among its properties could be mentioned:

- It is asymptotically consistent, efficient and unbiased.
- There is the possibility to handle survival and interval data better than rank regression.

Considering that the lifetime T of the axial fan blades follows a Weibull distribution with β and η parameters, the probability density function could be given by:

$$f(t) = \frac{\beta}{\eta}(\frac{t}{\eta})^{\beta-1}e^{-(\frac{t}{\eta})^{\beta}} \tag{2.13}$$

with t, the failure time, beta the shape parameter strictly greater than zero and eta the scale parameter. Considering $N = 7$ failures as shown in the data, the log likelihood function is given by:

$$\Lambda = Nln(\beta) - N\beta\ln(\eta) + (\beta - 1)\sum_{i=1}^{N}\ln(t_i) - \sum_{i=1}^{N}(\frac{t_i}{\eta})^{\beta} \tag{2.14}$$

Referring to the Newton Raphson method, the above (2.14) log likelihood function maximization, gives:

$$\frac{1}{\beta} = \frac{\sum_{i=1}^{N} t_i^{\beta} lnt_i}{\sum_{i=1}^{N} t_i^{\beta}} - \frac{1}{N}\sum_{i=1}^{N} lnt_i \tag{2.15}$$

As the log likelihood function maximization is dealt with numerically, a MATLAB optimization code is used to solve (2.15).

The estimated parameters obtained from the likelihood function maximization are utilized to build the PHM. The PHM obtained is tested to know how well it fits the data, therefore the goodness of fit is applied to assess the constructed model.

2.2.4 Economical approach with the PHM

The PHM provides us with the approximate risk of failing for the component based on the age and covariates (the natural frequency for the case study in this dissertation). The information which is made available by the PHM should be utilized to obtain economic benefits

a. How to use PHM outcome for economic benefit?

Vlok (1999) states: "Economical benefits from a statistical failure analysis can be guaranteed with a high confidence level if the minimum long-term life cycle cost LCC of a component is determined and pursued''.

Long term life cycle cost (LCC) concept

The LCC in renewal analysis arise from two important quantities in practice:

- The cost of unexpected renewal (failure cost C_f)
- The cost of preventive replacement (C_p)

Equilibrium must be obtained between the risk of having to spend C_f and the advantages in the cost difference between C_f and C_p without wasting useful life of a component. The optimum economic preventive renewal time will be at this balance point.

b. LCC for Weibull PHM

For optimal decision making with the PHM in reliability, Makis and Jardine (2013) made a model available. The model specifies the optimal renewal policy in terms of an optimal hazard leading to the minimum LCC. To be able to determine the hazard rate which leads to the minimum LCC it is needed to predict the behaviour of covariates.

Makis and Jardine's model assumes the covariate behaviour to be stochastic and approximating it by a non - homogeneous Markov chain in a finite space. Referring to that model, the expected average cost per unit time is a function of the threshold risk level given by:

$$\emptyset(d) = \frac{C_p + KQ(d)}{W(d)} \tag{2.16}$$

where, $Q(d) = P(T_d \geq T)$ represents the probability that failure replacement will occur and $W(d)$ the expected time until replacement and $K = C_f - C_p$.

Jardine et al. (1997) state that the calculation of the functions defined by the probability that failure replacement will occur Q(d) and the expected time until replacement W(d), can sometimes take a long time, due to the covariates quantity and structure, sometimes a simulation procedure could be used to determine the cost function, in this project such a simulation procedure is used to determine the cost function and the optimal risk point which minimizes the risk.

2.2.5 Goodness of fit for the PHM

The assumptions characterizing PHM are well defined for the time independent covariates: (1) Renewal times are iid (identically distributed); (2) The influential covariates are inserted in the model building; (3) the ratio of two hazard rates for given covariates should be constant over time.

Several approaches can be used to evaluate the goodness of fit for the PHM, more often residual analysis using graphical methods as well as statistical tests are used to assess at which point the PHM fits the data.

The advantage of the analytical method is that it provides statistical tests with a corresponding p-value to assess the PHM assumptions for covariates. It also gives the ability to make a correct and clear decision (Kleinbaum, 2000).

a. Graphical methods

To test the assumptions of the PHM, several graphical methods can be used. These include:

- Cumulative hazard plots
- Average hazard plots
- Residual plots

Out of the three mentioned categories of graphical methods mentioned, residual plots are the more common. To construct these residual plots, the Cox- generalized residuals for PHM are used.

Several methods are performed to calculate the residual in Cox regression model, among them are (1) Schoenfeld; (2) score residuals; (3) Martingale and (4) deviance. Each of these has a specific utilization, such as goodness of fit, which serves to identify possible outliers and the influential observations (Jin, 2014).

In survival analysis the diagnostics procedure for the model checking is focused on residuals. In this dissertation graphical techniques will not be used to assess the goodness of fit for the PHM even though in many publications residual plots are often used under different ways such as: (1) the residual against order of appearance; (2) ordered residuals against expectation etc.

a. Analytical methods

The use of graphical tests is often mixed with the analytical or statistical test as it is the case with the EXAKT software which uses the graphical residual analysis and the K-S test. However, because of the diversity of interpretation from analysts, the analytical approach seems more advantageous for decision making. Several statistical tests can be used, below are discussed some:

b.1 Wald test

The Wald test allows one to assess the quality of the parameters obtained from the maximum likelihood. Therefore, for the PHM, this method can test the values of β, η and γ that are obtained. The Wald test statistic for a given coefficient is given by:

$$W_i = \frac{n(\theta_i)^2}{var(\theta_i)} \tag{2.17}$$

$var(\theta_i)$ being the variance of the regression coefficient for a sample size expressed by n. The calculation of the p-value is made from the χ^2 distribution.

b.2 K-S test (Kolmogorov Smirnov)

The K-S test is a statistical hypothesis test. It is a non-parametric method used to generally compare the actual data to a normal distribution; the cumulative probability function of the data is compared with the cumulative probability function of a theoretical normal distribution.

However, in the context of the PHM this test is applied on the residual of the PHM. As it is known that the residual of the proportional hazard model must have an exponential distribution, the K-S test is then used to compare the cumulative distribution function of the PHM, residuals and the cumulative distribution function of an exponential distribution.

The null hypothesis: The cumulative distribution function of the PHM residuals is equal to the cumulative distribution function of an exponential distribution fitted on the residuals.

The null hypothesis testing is made by checking whether the critical value $D_{\alpha,}$ which is found in the K-S table according to the level of significance, set is less or greater than D which is the calculated (D-statistic).

The $D-$statistics is defined as the largest absolute difference between the PHM residuals cumulative distribution function and the cumulative exponential distribution.

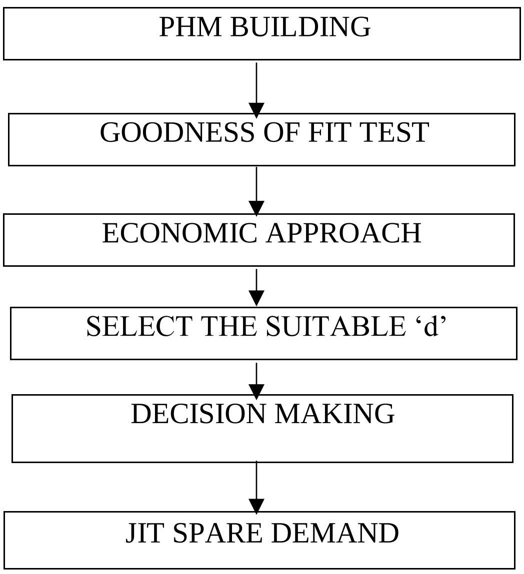
The $p-$value is the probability of obtaining a sample more extreme than the ones observed.

Acceptance criteria: If $D < D_{\alpha}$ for a given significance level, the null hypothesis should be accepted;

Rejection criteria: If $D > D_{\alpha}$ the null hypothesis should be rejected.

2.3 Flowchart illustration of the integrated method

The following diagram expresses the use of PHM to forecast spare parts demands:



Step 1: It consists of building the PHM with the outcome from the maximum likelihood function, in this dissertation a MATLAB algorithm allowed the computation of the Newton Raphson objective function.

Step 2: The goodness of fit testing is performed to assess how well the PHM fits the data, the Kolmogorov Smirnov is the statistical test used in this dissertation.

Step 3: The blending of the PHM and economic consideration is performed at this level. The outcome from this step is the optimal risk point that minimizes the cost during the simulation procedure d.

Step 4: The selected d point allows gives the critical number of loading cycle corresponding to each component.

Step5: The information obtained from the previous step is used to make decisions about the right time to make the component replacement.

Step 6: The replacement is performed according to the critical point pre-defined, which means there is no need of stocking too much spares because the right time for replacement is known, means JIT (just-in-time) spare parts demand.

The integrated forecasting method being proposed in this chapter, before the implementation of the given method in a case study in chapter 4, the following chapter introduces the case study and describes the generation of data needed to implement the new method on the case study.

Chapter 3 Case study description

3.1 Introduction

This chapter addresses the numerical and experimental investigation carried out to make available event and condition monitoring (CM) data needed to build the PHM. The case study focuses on a turbomachinery 30-degree fan axial blade.

The reason for considering the turbomachine blade failure case in this study, was simply to capitalise on the numerical models and experimental results that were already available from a prior study conducted by Brits (2016). In his work Brits worked on estimating the fatigue crack life (FCL) of turbomachine blades by means of a fatigue tests in the laboratory. As part of this study Brits conducted extensive numerical investigations and a very comprehensive experimental study. Because of the dearth of results of this nature in the open literature, these results were used for the current investigation. The author of this dissertation also assisted Brits in executing the experiments described here, to make sure that he has a full understanding of the intricacies of the data.

However, unlike the work by Brits where the main goal was to estimate the fatigue crack life of turbomachinery blades, here the same blades were considered with a focus on updating the finite element model to get the natural frequencies corresponding to the FCL. Then both the FCL and natural frequencies obtained were used as inputs to build a PHM prognostic model. The choice of natural frequency as covariate is since it is easy to measure, compared to the actual crack size which is difficult to directly measure in practice. The numerical investigation which was conducted by the current author, using the models generated by Brits, allowed calculation of the natural frequencies related to the crack propagation.

It is important to note that blade lifetime was not obtained from the finite element model (FEM). Only the stress intensity factors were used as input to the Paris Law model and a modal analysis was run by means of MSC.MARC/MENTAT 2016.

The experimental investigation by Brits was carried out in the C-AIM Labs at the University of Pretoria and entailed the use of a 50 kN spectral dynamics electrodynamics shaker to apply base excitation to the axial fan blade specimens. The fatigue lifetime recorded from the experimental approach served as event data required to build the PHM.

After having obtained the outcomes from both numerical and experimental investigations, the PHM could be constructed from both types of data made available through numerical and experimental investigations.

Tables and curves associated with both the numerical and experimental approaches are provided in this dissertation. The CM and event data generated are computed as a 'likelihood'. The outcome from the likelihood function are Weibull estimate parameters needed to build the PHM.

3.2 Numerical investigation

(Brits, 2016) followed a FEM approach to identify the natural frequencies corresponding to the crack growth. Furthermore, from the FEM, he calculated the stress intensity factor (SIF) that correlate to specific surface crack lengths. The calculated SIF and material constant obtained after the experiment served as input to a Paris law growth model to specify the crack growth rate. The number of loading cycles were correlated to the crack propagation. Figure 3.1 shows the mains steps characterizing the numerical investigation:

- FEM set up
- Crack insertion
- SIF calculation
- Growth rate
- Life prediction

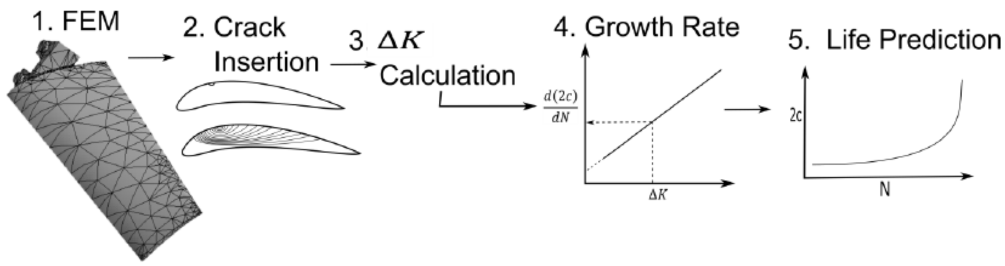


Figure3.1: Numerical investigation approach (Brits, 2016)

This dissertation utilized the FEM designed by Brits (2016), the FEM was performed to estimate the fatigue crack life of an axial fan blade. However, for the aim of this dissertation the mentioned FEM was extended to obtain the natural frequency corresponding to the propagation of the crack, then the obtained natural frequencies served as covariate to the PHM.

3.2.1 FEM set up

Two types of the axial fan blade specimen are considered in this dissertation: the 30 and 40-degree but only the 30-degree is used for building the PHM at the end of the work. Considering the computational cost, a static structural analysis was run with a set periodic tip displacement of ±10mm. The base of the blade was clamped in all directions at the attachment point, whereas a single point displacement is utilized at the extreme tip of the blade. The location of that point is selected similarly as the laser displacement transducer was in the experiments.

Table 3.1 shows the material properties, and figure 3.2 the model of a 40-degree blade, including the boundary condition.

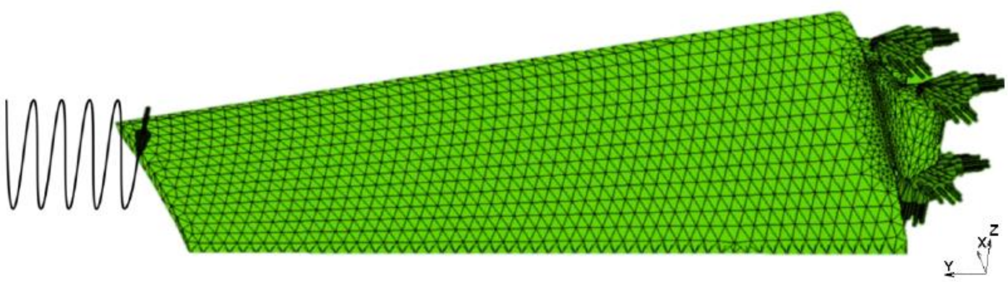


Figure 3.2**:** Finite Element Model of a 40-degree blade with boundary conditions (Brits, 2016)

The table below gives the material properties for the finite element model

Table3.1: Material properties chosen for FE model

Structural Property	Values
Elasticity Modulus E	69 GPa
Tensile Strength	220 MPa
Yield Stress	165 MPa
Density	2830 kg/m^3
Poisson Ratio	0.33

The following flowchart shows systematic process to build the finite element model for an axial fan blade using MSC.MARC/MENTAT2015.0 with an initiated and propagated crack.

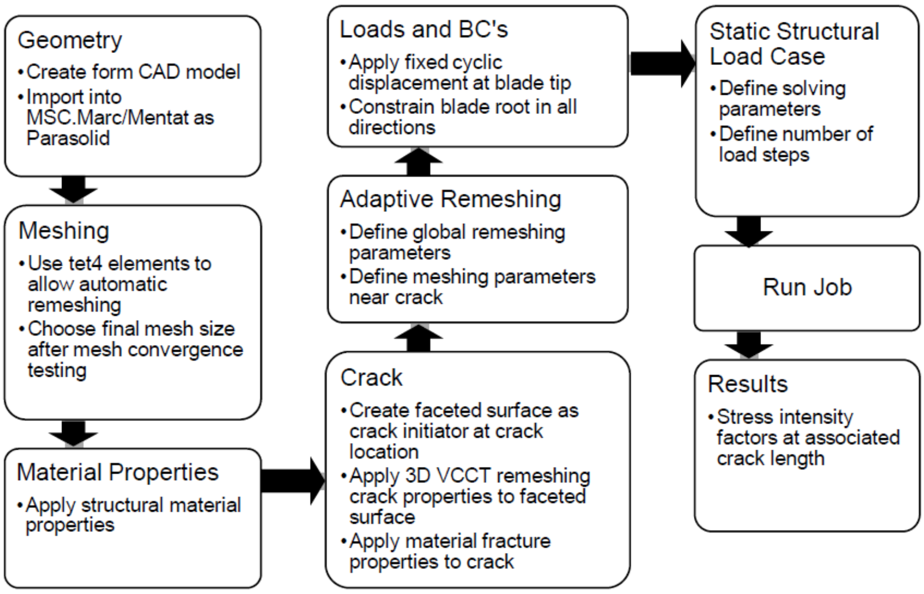


Figure 3.3: Flowchart of model set up in MSC.MARC/ MENTAT 2016 (Brits, 2016).

The validation of the boundary conditions and material properties were done by means of mesh convergence and modal analysis. To establish the maximum size of 4-nodded tetrahedral elements for having an accurate result within a reasonable computational time, a mesh convergence study was performed.

The result of the finite element and experimental modal analysis was that the first three modes and their natural frequencies are similar. The FEM natural frequency result shows a maximum error of 7.92% on the second mode compared to the experimental modal analysis result, which is the mean between the test specimens.

Table 3.2 results indicate that the modelling parameters selected approach those of the real blades, thus, it is used further in the study.

Table 3.2: Natural Frequency results for 40-degree blades

	Experiment (Hz)	FEM (Hz)	Error (%)
Mode 1	105,3	104,92	3,19
Mode 2	428,5	462,44	7,92
Mode 3	667,5	674,82	1,1

For final validation, the experimental and numerical strains at 10mm tip displacement are compared. Figure 3.4 to Figure 3.7 below respectively show the strain field before the initiation of the crack for the experimental and numerical approaches.

When there is no load applied, the noise floor of the readings is measured, since it is an offset in the strain readings. Considering figure 3.4 the determination of the strain noise gives +0.0793 percent and –0.0042 percent. The maximum strain of 0.274 percent was measured on the base of the blade as shown at figure 3.5.

The major strain field was obtained from the numerical approach, where the base of the blade gives a maximum strain of 0.2078 percent. Regarding the noise floor, the experimental and numerical strain fields for the test specimen differ by 6.73 percent for the maximum major strain. The results in our possession shows that the finite element model set up is right and can accurately represent the axial fan blades.

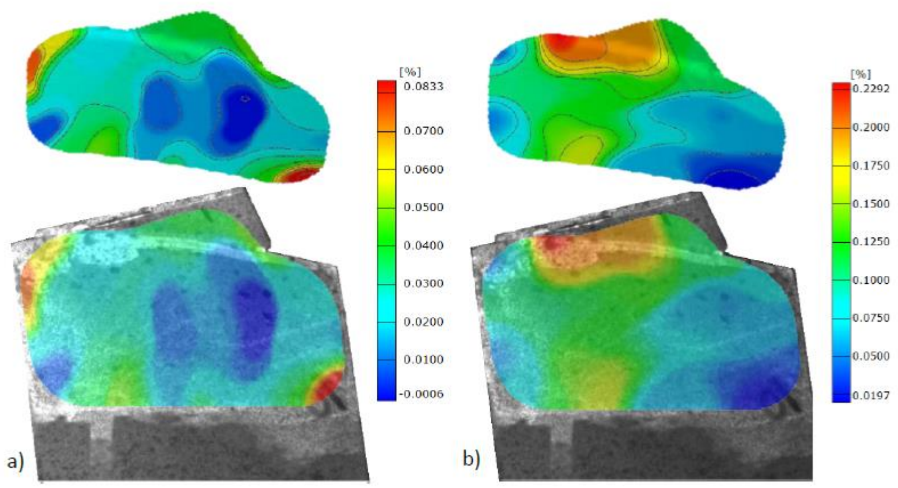


Figure 3.4: Experimental maximum principal strain fields of a 40-degree blade at (a) Zero load and at (b) 10mm tip displacement.

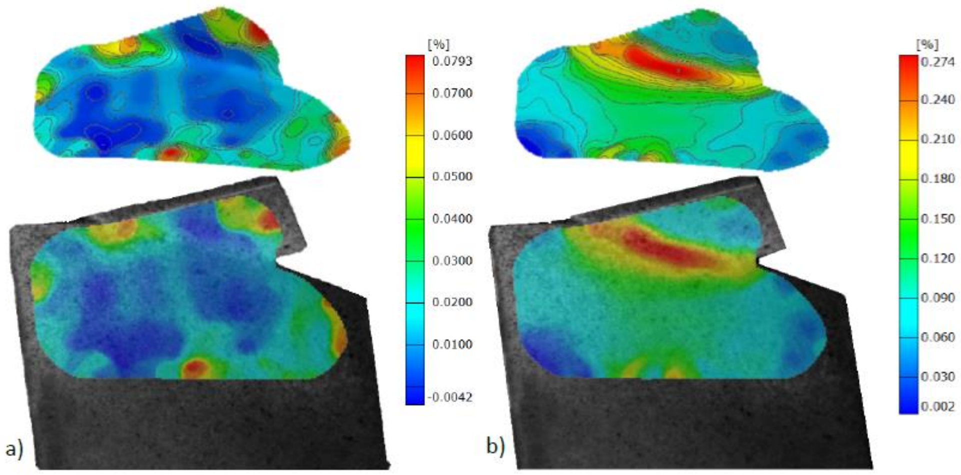


Figure 3.5: Experimental maximum principal strain fields of a 30-degree blade at (a) Zero load and at (b) -10mm tip displacement.

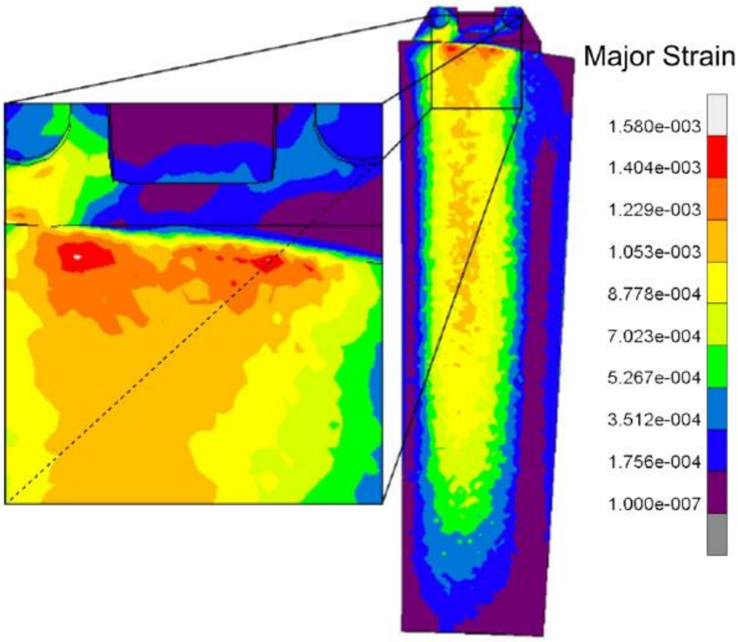


Figure3.6: Numerically computed major strain field of a 40-degree blade at -10mm tip displacement (Brits, 2016).

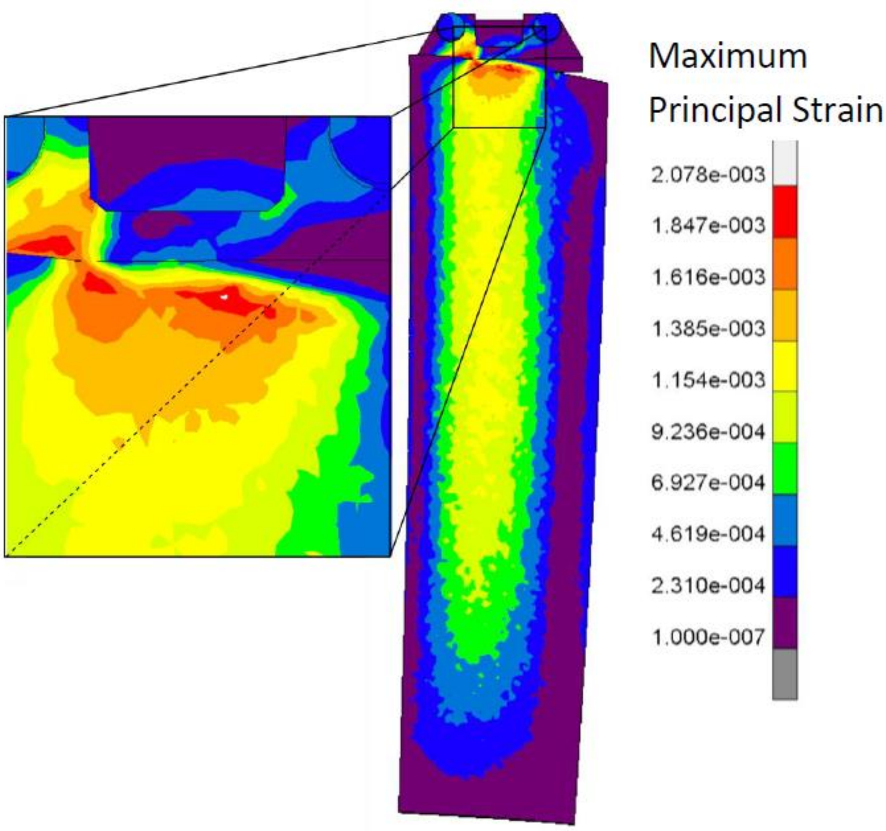


Figure 3.7: Numerically computed major strain field of a 30-degree blade at -10mm tip displacement (Brits, 2016).

3.2.2 Crack insertion

The MSC.MARC/ MENTAT software gives the user the ability to freely add cracks into the model, the size and shape of the crack to be added are arbitrary. Regarding the model under analysis in this dissertation, a crack with a surface length of 2 mm was seeded. The crack propagation was used to obtain the natural frequencies and was not used to obtain lifetime information.

After each load cycle at an increment of 1 mm, the crack propagates from the initial seed. The crack growth is done by the means of a scaling function which scales the crack by taking the relationship of the stress intensity factor (MSC software, 2016, pp.158-162).

$$d = d_0 (\frac{\Delta K_I}{\Delta K_{MAX}})^m \qquad (3.1)$$

Where d is the scaled crack growth size, and d_0 is the user defined crack growth size per increment.

The software automatically does the remeshing, while the crack is propagating. The remeshing works along with the global meshing and focuses on the mesh at and around the crack front.

The cracked finite element axial fan blade showing an extended crack with its meshes shown below.

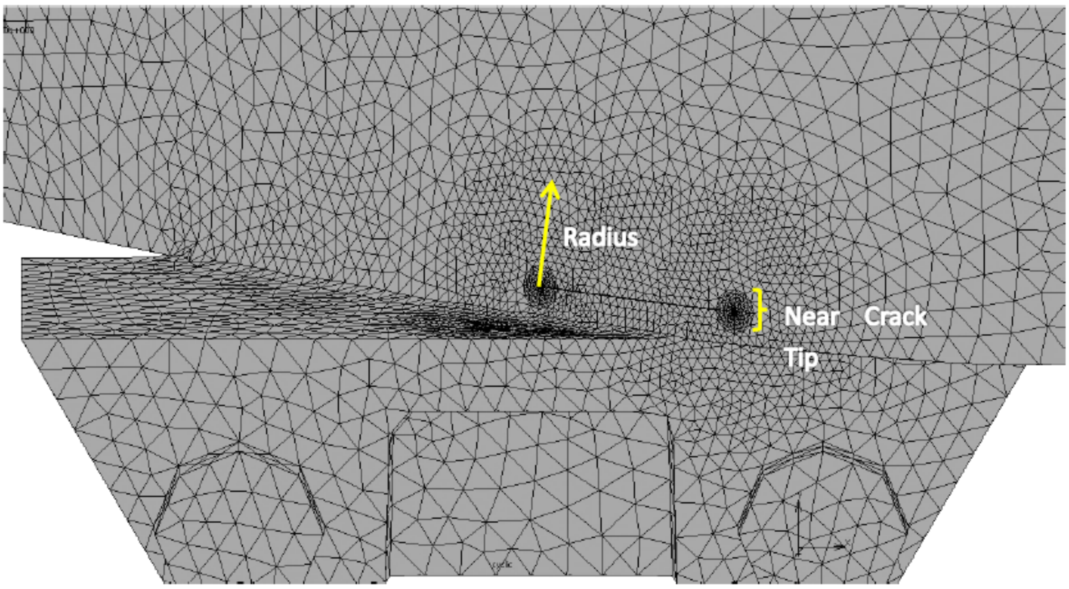


Figure 3.8: Cracked FE model axial fan blade showing extended crack with mesh. (Brits, 2016) To reduce errors when remeshing, the direction of the crack growth was chosen only as mode 1. Same results were obtained when the maximum hoop stress theory was used to determine the crack growth.

3.2.3 Summary of the results from Brits (2016) dissertation

With the propagation of the crack in the blade, the stress intensity factors are computed at distinct points in the crack front for each crack size by the means of 3D VCCT.

The meeting point for the free surface and the crack front with the highest stress intensity factor is used as the stress intensity for a given size.

Figure 3.9 below shows a comparison between the analytical and numerical stress intensity factor at the corresponding surface crack lengths. Because of the difference between the experimental and numerical crack front shapes, small differences are seen between the two methods. The FEM crack front deviates from a semi–elliptical shape as the surface crack length increases. For $2c > 33$, the Raju-Newman and FEM stress intensity factors start to move away from one another. During the computation of the stress intensity factor using the Raju-Newman method, an assumption was made that the semi–elliptical crack is in the centre of the plate, while the crack of the FEM is not in the centre of the blade, there is a difference of stress path. The different stress paths from the base to the root and crack front shapes could justify the difference between the stress intensity factors calculated.

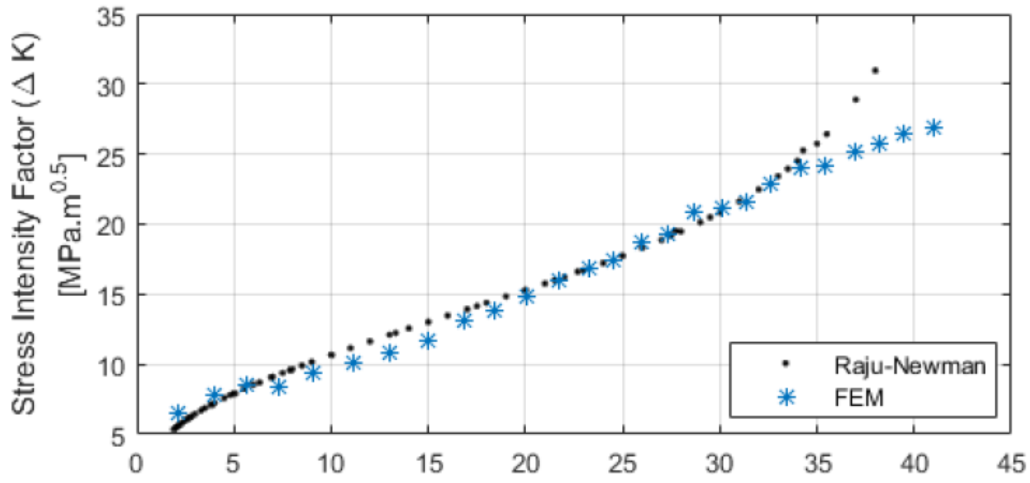


Figure 3.9: Stress Intensity factor at associated crack length of the numerical simulation and analytical calculated results for 40-degree blades (Brits, 2016).

With the stress intensity factor along the propagated surface crack being known. it is then possible to determine the crack growth rate using a growth model and material constants. By means of the Paris law, the growth rate associated with the stress intensity factor is determined. Since the crack lengths between steps are known, the number of loading cycles needed to grow the crack size can be calculated at each step. The life of the blade is estimated by the addition of the cumulative amount of load cycles needed to increase the crack size in between steps.

The surface crack length at the number of load cycles predicted using the mean values of m and B from the 40- degree test specimens as shown in figure 3.11.

Finally, the predicted FCL (fatigue crack length) correlates well with the mean of the experimental FCL result. Thus, the determined Paris law material constants are valid and the approach to predict FCL shows real promise.

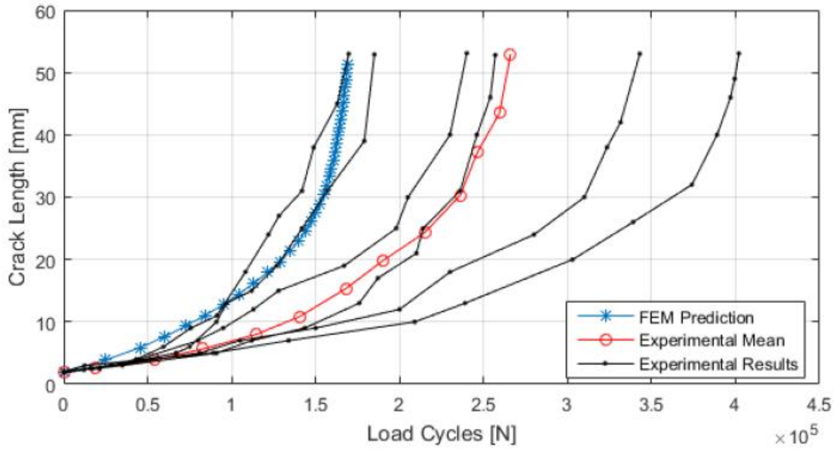


Figure 3.10: Predicted fatigue crack length vs number of load cycles compared to experimental results for a 30-degree blade (Brits, 2016).

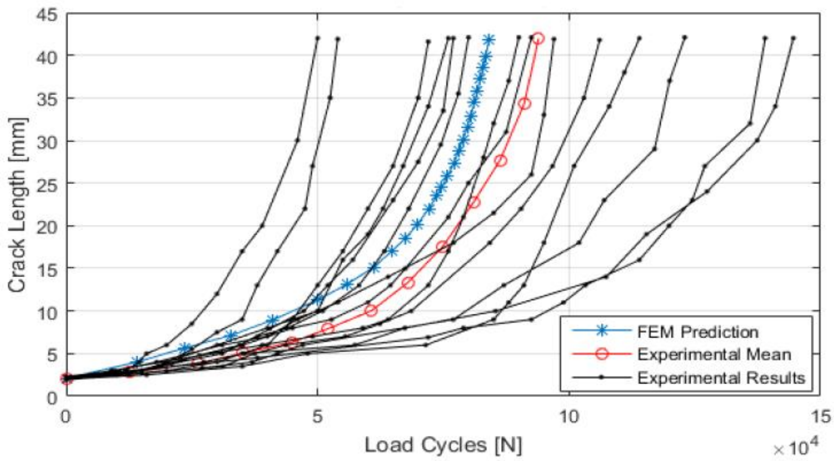


Figure 3.11: Predicted fatigue crack length vs number of load cycles compared to experimental Results for a 40-degree blade (Brits, 2016).

3.2.4 Method validation

For the reason of validation, the 30-degree blade geometry was used with the same model parameters as the 40-degree blade, with the only difference in the validation model the geometry. Boundary conditions, material properties as well as the mesh configuration being the same for the 30 and 40-degree model. This would allow us to determine the model sensitivity for changes in geometry.

Furthermore, this would also provide insight in why the 30-degree test specimen had a longer FCL than the 40-degree test specimens.

a. Natural frequencies

A modal analysis was performed to ensure that the material properties and modelling constraints have been chosen correctly so that the model represents the real blade.

Table 3.3 shows the numerical and the average experimental natural frequencies. A maximum error of 5.8 percent exists between the two, and it can be concluded that the parameters chosen are close enough.

Table 3.3: Natural frequency result for the 30-degree blades

	Experiment (Hz)	FEM (Hz)	Error (percent)
Mode 1	107.2	107.86	4.79
Mode 2	506.3	483.7	4.46
Mode 3	768.8	724.2	5.8

a. Strain

Considering the 30-degree blade, the major strain fields from the FEM and the experiments as shown in Fig 3.4 and 3.5 respectively, the noise floor during the test is + 0.0833 percent and -0.0006 percent. The maximum strain at -10 mm tip displacement is 0.2292 percent at the base of the blade. The maximum computed strain is 0.158 percent, also present at the base of the blade, which means that the numerical strain values differ by 8.5 percent from that of the experiment.

The errors values being small, it can be concluded that the physics-based model of the 30-degree blade does indeed approximate a real 30-degree axial fan blade.

The major strain in the 30-degree test specimens is lower than the 40-degree test specimens, it implies a lower stress and a longer fatigue crack life. In the figure 3.8 is shown the bending stress results for the 30-degree blade, as well as the maximum stress is 123.1MPa, 31.4MPa less than the stress experienced by the 40-degree blade.

c. Life prediction

Similarly, for the 40-degree blade, the stress intensity factor calculates as function of surface crack length and the FCL estimation was done for the 30-degree blade. Figure 3.10 and 3.11 shows the predicted crack growth using FEM and the mean material constant values is obtained in the experimental investigation.

3.3 Experimental investigation

Lifetime and Paris law material constants are obtained by means of the experimental investigation. In this section an experimental setup is designed and utilized for the initiation and propagation of the crack. Figure 3.12 shows an overview of steps undertook to obtain lifetime:

- Experimental set up
- Crack growth measurement
- Stress intensity calculation
- Material constant determination
- Statistical analysis

3.3.1 Experimental set up

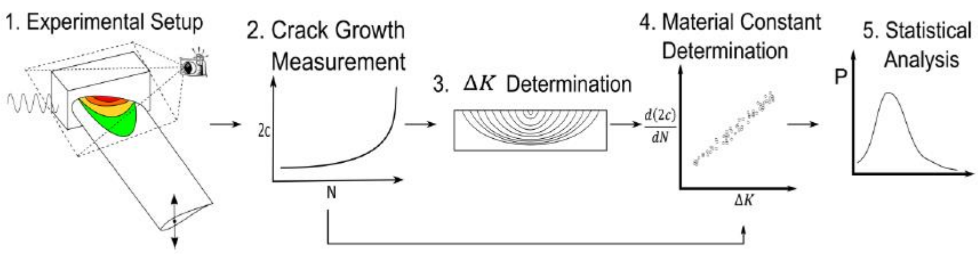


Figure 3.12: Experimental investigation overview (Brits, 2016)

During the experiment, to apply the base excitation to the test specimens, a 50 kN Spectral dynamics electrodynamics shaker was used. The advantage of the chosen shaker is that it has a larger displacement at high frequencies compare to other available equipment in the laboratory. The acquisition of data during the experiment was performed by means of a 4M DIC system from Gesellschaft fur Optische Messtechnik (GOM) by taking images of the test specimens before and during loading. Figure 3.13 shows the experimental setup.

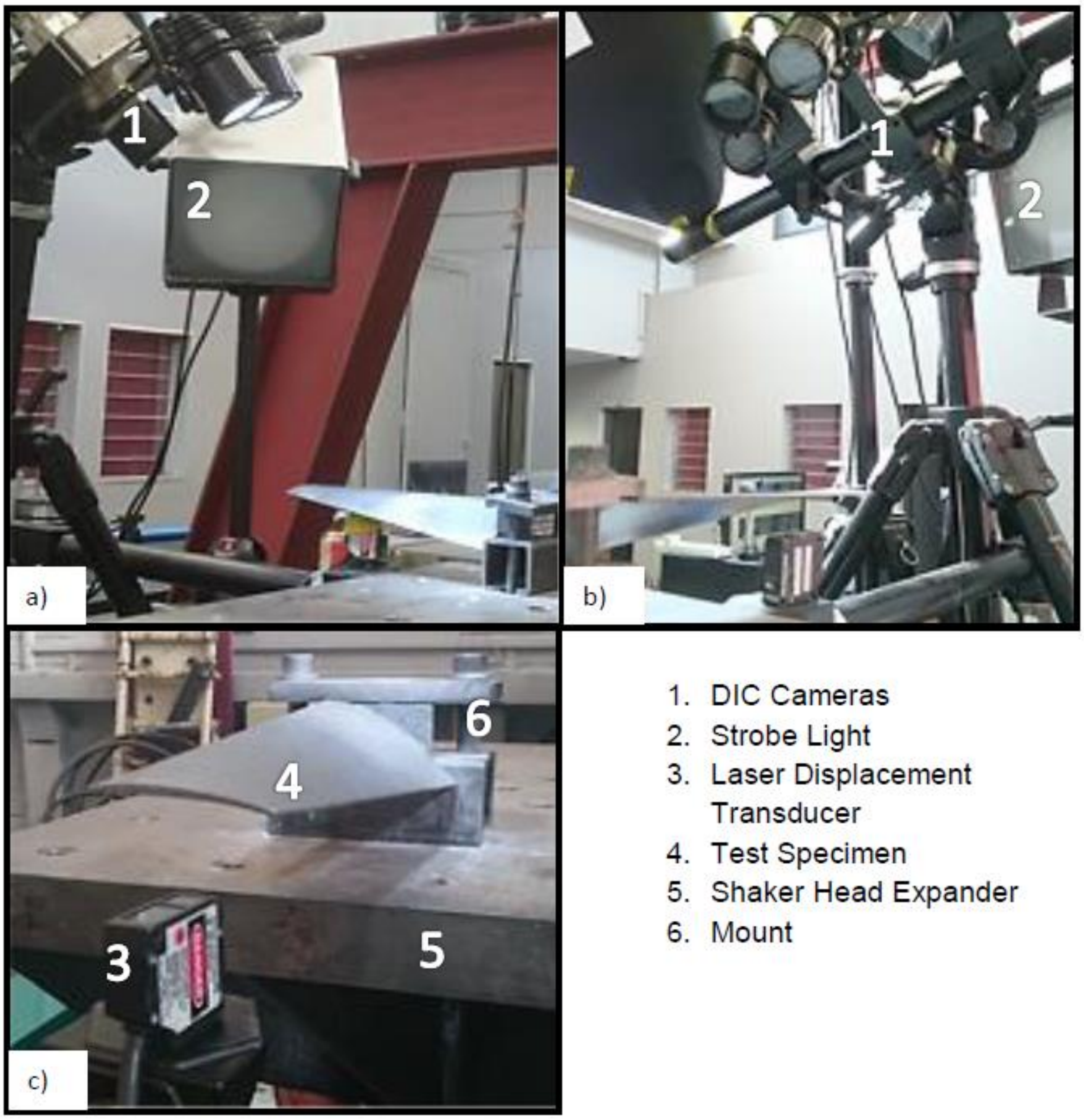


Figure 3.13: Experimental setup showing (a) Right side view, (b) Left side view, (c) The mounted test specimen. (Brits, 2016)

3.3.2 Tables of results generated from finite element model and experiment

In this section the data generated from the experiment performed by Brits (2016) and the natural frequency results were generated by the reviewed finite element model. It is important to highlight that the finite element analysis was extended to allow for the calculation of the natural frequencies for the first mode from the initiated crack to the failure for 30-degree and 40-degree axial fan blade, because this was not included in Brits (2016) study.

Table of experimental and numerical data

Figure 3.12 presents an overview of the steps taken to determine the constants used in the Paris Law to obtain the lifetime. Figure 3.8 shows that the design of the experimental set up was such that a crack initiates and propagates in an axial fan blade. The use of DIC (digital image correlation) allowed the measurement of the crack during a post processing procedure. The measured crack growth and the analytically determined stress intensity factors were then used to determine the material constants for the Paris Law. Several experiments with the same loading parameters were conducted, after which a statistical analysis on lifetime and Paris Law material constant was possible. Tables below represent the results from section 3.2.3 and the extended FEM that was performed in this dissertation to generate natural frequency related to the crack propagation.

Table 3.4: Outcome results from the FEM and experiment for blade 1

Crack length(mm)	Number of cycles	Natural frequencies	Remaining life
2	0	107.9	169700
2.5	21400	107.86	148300
4	43200	107.59	125800
6	59400	107.53	110300
9	75700	107.071	94000
11	91200	106.7	78500
18	108200	105.05	61500
24	121900	102.9	47800
27	128200	100.6	41500
31	141900	98.4	27800
38	148900	91.9	20800
45	162890	81.7	6810
53	169700	73.1	0

Table 3.5: Outcome results from the FEM and experiment for blade 2

Crack length(mm)	Number of cycles	Natural frequencies	Remaining life
2	0	107.9	185000
2.6	16000	107.85	169000
3.5	39000	107.6	146000
5	67000	107.56	118000
6	75000	107.53	110000
10	91000	106.9	94000
13	97000	106.1	88000
15	112000	105.6	73000
19	127000	104.5	58000
25	142000	102.3	43000
31	157000	98.4	28000
39	179000	91.4	6000
52.9	185000	73.1	0

Table 3.6: Outcome results from the FEM and experiment for blade 3

Crack length(mm)	Number of cycles	Natural frequencies	Remaining life
2	0	107.9	240000
2.3	12000	107.86	228000
3	35000	107.79	205000
4	45000	107.59	195000
7	80000	107.4	160000
9	95000	107.071	145000
12	113000	106.53	127000
15	128000	105.6	112000
19	167000	104.5	73000
25	198000	102.3	42000
30	205000	98.8	35000
40	230000	89.8	10000
53.1	240000	73.1	0

Table 3.7: Outcome results from the FEM and experiment for blade 4

Crack length(mm)	Number of cycles	Natural frequencies	Remaining life
2	0	107.9	257000
3	35000	107.19	222000
5	91000	107.5	166000
7	112000	107.4	145000
9	143500	107.071	113500
13	176000	106.1	81000
17	187000	105.1	70000
21	210000	103.8	47000
25	214000	102.3	43000
31	236000	98.4	21000
40	246000	89.8	11000
52.8	257000	73	0

Table:3.8: Outcome results from the FEM and experiment for blade 5

Crack length(mm)	Number of cycles	Natural frequencies	Remaining life
2	0	107.9	343000
2.5	17000	107.86	326000
3	28500	107.79	314500
5	81000	107.5	262000
7	105000	107.4	238000
9	150000	107.071	193000
12	200000	106.53	143000
18	230000	105.05	113000
24	280000	102.9	63000
30	310000	98.8	33000
38	323520	91.9	19480
42	331500	86.6	11500
53	343000	73	0

Table 3.9: Outcome results from the FEM and experiment for blade 6

Crack length(mm)	Number of cycles	Natural frequencies	Remaining life
2	0	107.9	402000
3	12500	107.79	389500
5	89000	107.5	313000
7	134000	107.4	268000
10	209000	106.9	193000
13	239000	106.1	163000
20	303000	104.1	99000
26	339000	101.6	63000
32	374000	97.9	28000
40	389000	89.8	13000
46	397000	79.5	5000
49	399500	75.1	2500
53.05	402000	73	0

Table 3.10: Outcome results from the FEM and experiment for blade 7

Crack length(mm)	Number of cycles	Natural frequencies	Remaining life
2	0	107.9	665000
3	50000	107.79	615000
5	130000	107.56	535000
7	180000	107.4	485000
9	254000	107.071	411000
13	330000	106.1	335000
17	430000	105.1	235000
21	500000	103.9	165000
25	550000	102.3	115000
33	600000	97.4	65000
38	630000	91.9	35000
44	654000	82.7	11000
52.95	665000	73	0

Tables 3.11 and 3.12 document the natural frequencies as functions of crack lengths that were not included in the analyses performed by Brits (2016). He did not focus on presenting the natural frequencies trend as the crack was propagating. However, this project needs two types of data, namely event data (lifetime of the blades) and condition monitoring data (natural frequencies).

Table 3.11: Natural frequencies and corresponding crack length for 30- degree fan axial blade

Increment	Crack Length (mm)	Natural Frequencies (Hz)
1	0.0020	107.900
2	0.0037	107.649
3	0.0057	107.595
4	0.0075	107.449
5	0.0093	107.071
6	0.0109	106.938
7	0.0126	106.532
8	0.0141	106.101
9	0.016	105.601
10	0.0177	105.050
11	0.0195	104.525
12	0.0212	103.917
13	0.0229	103.181
14	0.0245	102.389
15	0.0261	101.592
16	0.0278	100.644
17	0.029	99.723
18	0.0304	98.874
19	0.032	97.910
20	0.0347	95.576
21	0.0357	94.228
22	0.0372	92.949
23	0.0386	91.438
24	0.0401	89.892
25	0.0406	87.934
26	0.0423	86.027
27	0.0436	83.935
28	0,0454	81.685
29	0.0469	79.117
30	0.0473	76.446
31	0.0530	73.000

Table3.12: Natural frequencies and corresponding crack length for 40-degree fan axial blade

Increment	Crack Length (mm)	Natural Frequency (Hz)
1	4.194	104.923
2	7.963	103.36
3	11.413	101.774
4	15.197	100.142
5	18.757	97.157
6	21.979	94.560
7	25.075	91.973
8	27.876	89.476
9	30.603	87.06
10	33.437	84.329
11	35.839	80.601
12	38.879	76.832
13	42	73.180

3.8 Conclusion

Having the event data from the experiment and the condition monitoring data available from the FEM, all inputs required to build the PHM are now available. The following chapter therefore deals with the implementation of the PHM for the case study presented in chapter 3.

The choice of natural frequency as covariate is justified by the fact that it is relatively easy to measure (i.e can be measured at different points on the structure without affecting the results). Natural frequency is a global parameter of a structure, as opposed to a local parameter such as mode shape. It is further uniquely related to the stiffness of the structure if one may assume that mass is essentially constant - which is for practical purposes the case except in erosive or very dirty environments. Natural frequency can therefore be indicative of change of stiffness, which may again be assumed to indicate damage.

Chapter 4 Case study implementation of the proposed method

4.1 Introduction

This chapter covers the implementation of the integrated forecasting method by following steps described in section 2.3.2 expressing the use of the PHM to forecast the spare parts demand. The chapter commences by estimating the parameters required to build the proportional hazard model (PHM), after conducting statistical tests to evaluate how well the PHM fits the data. Parameter estimates are determined by maximizing the likelihood function by means of the Newton Raphson method. Risks are then blended with the economics to optimize the decision making with the proportional hazards model, by setting a threshold point which is referred to as the 'd' point. This point may finally be used by decision makers in inventory to forecast spare parts demand and do a just in time spare parts management.

The failure of the 30-degree blade as was described in chapter 3 is used as the case study to demonstrate the proposed method to forecast spare parts demand.

4.2 Maximum likelihood estimate

As described in section 2.2.3 b, maximum likelihood estimation is a well-known method to allow estimation of the regression coefficients needed to build a PHM. Having event and condition data available from the numerical and experimental investigation, the Weibull parameters of equation (2.2) may be estimated by maximization of equation (2.10).

4.2.1 Maximum likelihood for a simple Weibull (2 parameters)

In section 2.2.3, the maximum likelihood of the log function (2.14) gives the following equation:

$$\frac{1}{\beta} = \frac{\sum_{i=1}^{N} t_i^{\beta} \, lnt_i}{\sum_{i=1}^{N} t_i^{\beta}} - \frac{1}{N}\sum_{i=1}^{N} lnt_i$$

To determine the shape parameter β in the above equation requires the log likelihood function maximization. As the equation is dealt numerically, a MATLAB code was written related to the above formula to determine the shape parameter, the output from the MATLAB code gave a shape parameter $\beta = 2.17$.

The differentiation of the equation (4.5) with respect to η gives:

$$\eta = (\tfrac{1}{N}\sum_{i=1}^{N} t_i^{\beta})^{1/\beta} = \tfrac{1}{7}\sum_{i=1}^{7} t_i^{2.17} = 366700.8 \text{ cycles}$$

The following steps are required to determine the simple (2 parameters) Weibull model:

Step 1: With $\beta = 2.17$ and $\eta = 366700.8$ cycles the hazard function for the fan axial blades are given by:

$$h(t) = \frac{2.17}{366700.8}(\frac{t}{366700.8})^{1.17}$$

Step 2: Economical approach for 2 Weibull parameters (Application on the axial fan blade data).

In this section, a time-based approach is presented that can be used to optimize the axial fan blade replacement decision making and the economic implications.

Referring to Jardine et al. (2013), the optimal preventive replacement age of an item subject to breakdown is given by:

$$C(t_p) = \frac{C_p \times R(t_p) + C_f \times (1 - R(t_p))}{t_p \times R(t_p) + M(t_p)(1 - R(t_p))} \quad (4.1)$$

Considering 3/1 cost ratio which describes such that the failure cost C_f in South African Rands (ZAR) is three times the preventive cost C_p with $C_p = 20000\, ZAR$ and $C_f = 60000\, ZAR$, below is given a sample of results table for blade 7.

Table 4.1: Table of result for blade 7 (time-based approach)

	Time (cycles)	Reliability R(t)	Cumulative distribution F(t)	Cost per unit Time C(tp)
1	0	1	0	Inf
2	50000	0,9868	0,0132	0,4151
3	130000	0,9	0,1	0,2026
4	180000	0,8078	0,1922	0,1815
5	254000	0,6372	0,3628	0,1902
6	330000	0,4514	0,5486	0,2185
7	430000	0,2435	0,7565	0,3574
8	500000	0,1409	0,8591	0,5819
9	550000	0,0898	0,9102	0,8319
10	600000	0,0544	0,9456	1,3957
11	630000	0,3993	0,9607	1,9113
12	654000	0,0299	0,9701	2,6871
13	665000	0,0263	0,9737	2,7521

The cost curve corresponding to the above blade is given by:

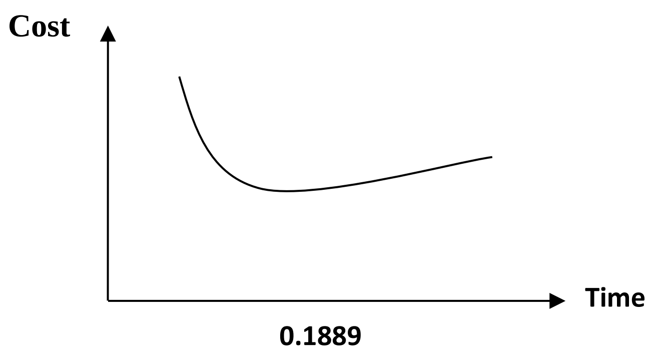


Figure 4.1: Curve of cost versus Time for blade 7

For 2 Weibull parameters or the time-based approach the optimal time which minimizes the cost is 0.1889 ZAR per unit.

The table below 4.2 is the summary of all blade results, showing the optimal replacement time, which minimizes the cost.

Table 4.2: Table of results for all blades (time-based approach)

	Optimal Time	Reliability	Minimal Cost per Unit time
Blade1	169700	0.8	0.18
Blade 2	185000	0.79	0.18
Blade 3	167000	0.83	0.1886
Blade 4	187000	0.79	0.188
Blade 5	200000	0.76	0.188
Blade 6	239000	0.67	0.1889
Blade 7	180000	0.81	0.1815
Average	189571	0.78	0.185

Considering the time-based result presented in table 4.4, it is important to highlight that the simple Weibull calculation, which is a time-based approach, shows that the optimal replacement time for the blades which minimizes the cost varies between 169700 to 239000 cycles, with an average of 189571 cycles. The surprise in the above table is that most of the blades are still reliable at the indicated replacement time. The following section will consider the use of a proportional hazard model.

4.2.2 Maximum likelihood Estimate for 3 Weibull parameters using Newton method

Section 2.2.3 illustrate a template proposed by (Vlok, 1999) adjusting the inspection time to the corresponding covariate. These templates are arranged in such way that they can be easily computed in the objective function given by formula (2.12). Below is given the summary of data for all histories in the proposed template.

History 1: Table of inspection time with corresponding natural frequency

(with natural frequency as covariate)

t_i	Z_k^1	$\sum t_i$	$\sum Z_k^1$	$t_{i(jm)} - t_{ij}$
0	107.9			50000
50000	107.56			50000
100000	105.875			50000
150000	86.8	150000	300.235	50000

History 1: Table of inspection time with corresponding crack size

(with crack size as covariate)

t_i	Z_k^1	$\sum t_i$	$\sum Z_k^1$	$t_{i(jm)} - t_{ij}$
0	2			50000
50000	5			50000
100000	14.5			50000
150000	41.5	150000	61	50000

History 2: Table of inspection time with corresponding natural frequency

(with natural frequency as covariate)

t_i	Z_k^2	$\sum t_i$	$\sum Z_k^2$	$t_{i(jm)} - t_{ij}$
0	107.9			50000
50.000	107.58			50000
100.000	105			50000
150.000	100.35	150.000	312.93	50000

History 3: Table of inspection time with corresponding natural frequency

(with natural frequency as covariate)

t_i	Z_k^3	$\sum t_i$	$\sum Z_k^3$	$t_{i(jm)} - t_{ij}$
0	107.9			50000
50.000	107.5			50000
100.000	106.8			50000
150.000	105.05			50000
200000	100	200000	419..35	50000

History 4: Table of inspection time with corresponding natural frequency

(with natural frequency as covariate)

t_i	Z_k^3	$\sum t_i$	$\sum Z_k^3$	$t_{i(jm)} - t_{ij}$
0	107.9			50000
50.000	107.6			50000
100.000	107.45			50000
150.000	106.5			50000
200000	104.45	200000	426	50000

History 5: Table of inspection time with corresponding natural frequency

(with natural frequency as covariate)

t_i	Z_k^3	$\sum t_i$	$\sum Z_k^3$	$t_{i(jm)} - t_{ij}$
0	107.9			50000
50.000	107.70			50000
100.000	107.4			50000
150.000	107.071			50000
200000	106.53			50000
250000	103.975			50000
300000	100	300000	632.976	50000

History 6: Table of inspection time with corresponding natural frequency

(with natural frequency as covariate)

t_i	Z_k^3	$\sum t_i$	$\sum Z_k^3$	$t_{i(jm)} - t_{ij}$
0	107.9			50000
50.000	107.3			50000
100.000	107.2			50000
150.000	107			50000
200000	106.5			50000
250000	105.9			50000
300000	104			50000
350000	101			50000
400000	73	400000	811.9	50000

History 7: Table of inspection time with corresponding natural frequency

(with natural frequency as covariate)

t_i	Z_k^3	$\sum t_i$	$\sum Z_k^3$	$t_{i(jm)} - t_{ij}$
0	107.9			50000
50.000	107.59			50000
100.000	107.56			50000

150.000	107.52			50000
200000	107.1			50000
250000	107.071			50000
300000	106.1			50000
350000	106			50000
400000	106			50000
450000	104.5			50000
500000	103.9			50000
550000	102.3			50000
600000	97.4			50000
650000	73	650000	1110.5	50000

The computation of all histories in the objective function (2.12) is performed by means of MATLAB package using the `fmincon` algorithm.

4.2.3 Computation of the data using `fmincon` algorithm under MATLAB

To determine the Weibull parameters β, η, γ needed to construct the PHM, the likelihood equation (2.11) is solved numerically using Newton Raphson method which gives the equation (2.12) which is the objective function that has been solved with the algorithm `fmincon`.

`fmincon` algorithm is a nonlinear programming solver which allows finding the minimum of constrained nonlinear multivariable function.

For the aim of this dissertation, the objective function given by the equation (2.12) was minimized using the syntax: $x = fmincon(fun, x_0, A, b, Aeq, beq, lb, ub)$. The results from the simulation gives: (1) $\beta = 1.0012$; (2) $\eta = 7.10e + 05$; (3) $\gamma = 0.0293$.

The PHM construction obtained from the maximum likelihood output is:

$$h(t, z(t)) = \frac{1.0012}{7.1004e + 05} (\frac{t}{7.1004e + 05})^{(1.0012-1)} \exp[0.0293 \times z(t)]$$

In this dissertation, only the K-S test is performed on the residual of the data for a 30-degree blade in the software R to evaluate how well the PHM fit the data, the output results obtained from R was:

$$D = 0.49659 \, , p - \text{value} = 0.06873$$

The above result shows that at 5 percent level of significance the null hypothesis is accepted for D is less than D_{α} which is 0.565, and the $p - \text{value}$ being greater than 0.05, the null hypothesis is accepted which means that the PHM fit well the data.

4.2.4 Optimal decision making with the PHM

It was specified in section 2.2.4 that after receiving the outcome from PHM which presents the risk that the component will fail based on the integration of age and covariate, this outcome from the PHM could only serves when using it for an economical benefit, this introduces the notion of blending the PHM with economics addressed by Jardine and Makis (2013). However, in this dissertation does not address the TPM approach but a simulation approach as shown in section 4.2.5.

4.2.5 Application of the optimal decision making using simulation procedure

a. Tables of the resulting proportional hazard values for the seven experimental blades

Table 4.3: Risk versus loading cycles for blade 1

	N(cycles)	PHM (Risk)
1	0	0
2	21400	1.51E-06
3	43200	1.58E-06
4	56400	1.68E-06
5	75700	1.83E-06
6	91200	1.94E-06
7	108200	2.38E-06
8	121900	2,84E-06
9	128200	3.10E-06
10	141900	3.49E-06

11	148900	4.29E-06
12	162890	5.26E-06
13	169700	6.65E-06

Table 4.4: Risk versus loading cycle for blade 2

	N(cycles)	PHM(Risk)
1	0	0
2	16000	1.52E-06
3	39000	1.56E-06
4	67000	1.63E-06
5	75000	1.68E-06
6	91000	1.89E-06
7	97000	2.06E-06
8	112000	2.18E-06
9	127000	2.46E-06
10	142000	2.93E-06
11	157000	3.49E-06
12	179000	4.41E-06
13	185000	6.63E-06

Table 4.5: Risk versus loading cycle for blade 3

	N(cycles)	PHM (Risk)
1	0	0
2	12000	1.50E-06
3	35000	1.53E-06
4	45000	1.58E-06
5	80000	1.73E-06
6	95000	1.83E-06
7	113000	2.00E-06
8	128000	2.18E-06
9	167000	2.46E-06
10	198000	2.93E-06
11	205000	3.39E-06
12	230000	4.55E-06
13	240000	6.67E-06

Table 4.6: Risk versus loading cycle for blade 4

	N(cycles)	PHM (Risk)
1	0	0
2	35000	1.53E-06
3	91000	1.63E-06
4	112000	1.73E-06
5	143500	1.83E-06
6	176000	2.06E-06
7	187000	2.32E-06
8	210000	2.61E-06
9	214000	2.93E-06
10	236000	3.49E-06
11	246000	4.55E-06
12	257000	6.62E-06

Table 4.7: Risk versus loading cycle for blade 5

	N(cycles)	PHM(Risk)
1	0	0
2	17000	1.51E-06
3	28500	1.53E-06
4	81000	1.63E-06
5	105000	1.73E-06
6	150000	1.83E-06
7	200000	2.00E-06
8	230000	2.39E-06
9	280000	2.85E-06
10	310000	3.39E-06
11	323520	4.29E-06
12	331500	4.82E-06
13	3433000	6.66E-06

Table 4.8: Risk versus loading cycle for blade 6

	N(cycles)	PHM(Risk)
1	0	0
2	12500	1.53E-06
3	89000	1.63E-06
4	134000	1.73E-06
5	209000	1.89E-06
6	239000	2.06E-06
7	303000	2.53E-06
8	339000	3.02E-06
9	374000	3.60E-06
10	389000	4.55E-06
11	397000	5.42E-06
12	399500	5.92E-06
13	402000	6.67E-06

Table 4.9: Risk versus loading cycle for blade 7

	N(cycles)	PHM (Risk)
1	0	0
2	50000	1.54E-06
3	130000	1.63E-06
4	180000	1.73E-06
5	254000	1.83E-06
6	330000	2.06E-06
7	430000	2.32E-06
8	500000	2.61E-06
9	550000	2.93E-06
10	600000	3.31E-06
11	630000	4.29E-06
12	654000	5.12E-06
13	665000	6.65E-06

By fitting time and the corresponding covariate to the constructed PHM we get tables 4.3 to 4.9, these tables represent the risk of failing for all the blades and the corresponding time expressed in term of number of loading cycle.

b. Plotting of the resulting proportional hazard values for the seven experimental blades

The risk $h(t, z(t))$ is an instantaneous conditional probability of failure for the blade at time t, given the value $Z(t)$. Figure 4.2 indicates the plotting of risk versus the number of loading cycles for the obtained PHM represented in section 4.2.3. Tables 4.3 to 4.9 show the fitting of the PHM to the corresponding data which are time and covariate for all blades:

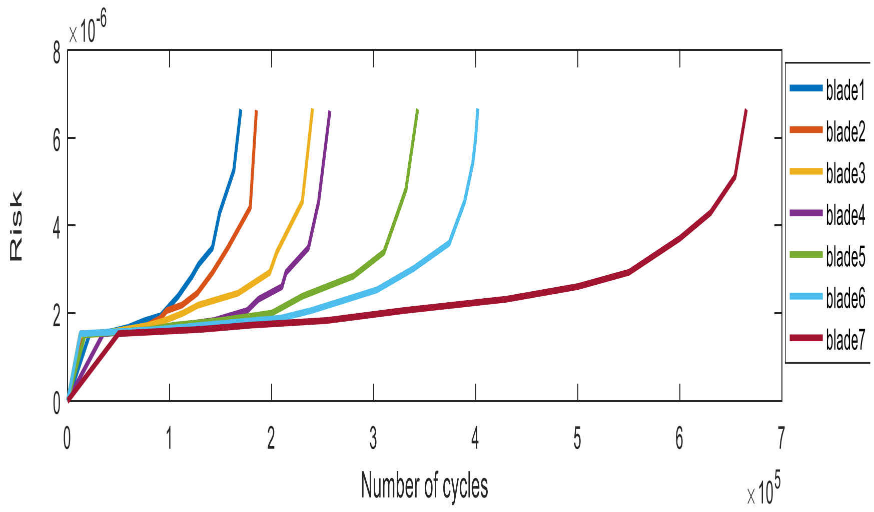


Figure 4.2 Risk versus loading cycles for all the blades.

After determining the PHM and plotting it as shown in Figure 4.2 for all the 30- degree axial fan blades, the following step is to establish optimal replacement decisions, including economic considerations, based to the PHM values, where these values would be a function of both age of the components, as well as the condition parameter. This basically entails finding an optimum PHM risk value at which components would be replaced. It is normally expected that such optimisation would be achieved by balancing the risk of expensive failures (when the replacement PHM value is chosen at a too high a level and some components may fail before reaching this value) and wasting remaining useful life of components (when the replacement PHM value is chosen at a low level).

The trends of the PHM shown in Figure 4.2, however, show that, in this case, the optimisation problem seems to be trivial and that the optimal replacement PHM level would be at almost the constant PHM failure value (with some small safety factor) and that this result seems to be independent of age. With such PHM trend, the application of PHM for spare part demand becomes useless because time does not have influence and the forecast could be performed straight with covariate trending. In Figure 4.2 all the blades have the risk of failing at almost the same PHM level. Table 3.4 to Table 3.10 confirm the PHM trend because the outcome from the experiment and FEM is showing that all the blades failed at almost equal crack length 52 to 53mm and the natural frequency at that failure point was around 72 to 73 Hz, independent of age. This is discussed further in the next chapter.

Chapter 5 Interpretation of results

The event and condition monitoring data generated in Chapter 3 for the axial flow fan by means of the experiment and numerical simulations were processed in Chapter 4. The Weibull parameters were obtained by minimizing the objective function given by equation (2.12). The optimal Weibull parameters are: (1) the shape parameter $\beta = 1.0012$, (2) the scale parameter $\eta = 7.1004e + 05$ and the location parameter $\gamma = 0.0293$. The mentioned Weibull parameters results served as input to construct the PHM shown in Chapter 4, section 4.2.3.

$$h\big(t, z(t)\big) = \frac{1.0012}{7.1004e + 05}\left(\frac{t}{7.1004e + 05}\right)^{(1.0012-1)} \exp(0.0293 \times z(t)) \tag{5.1}$$

The model presented in equation (5.1) was tested to see how well it represents the data. For this the K-S test served to verify how well the PHM fits the data. At 5% significance level with $D_\infty = 0.565$, the calculated D-statistic was equal to 0.49659 with a corresponding p −value of 0.06873. The following paragraph proposes results interpretation starting by the K-S test.

5.1 Interpretation of the results

5.1.1 Interpretation of the K-S test results

The test statistic D applied is simply the maximum absolute difference between two cumulative distributions, and the p-value the area under the cumulative distribution. For the PHM in this dissertation the null hypothesis is that the cumulative distribution function of the PHM residuals is equal to the cumulative distribution function of an exponential distribution. The inferences on the goodness of fit for the model (5.1) is made on the D-statistic an p −value. The following is the meaning of the results:

The calculated D-statistic obtained is 0.49659 and at 5% significance level the $D_\infty = 0.565$, the corresponding p-value obtained is 0.06873. The calculated D-statistic being less than 0.565 implies that the null hypothesis should not be rejected, moreover the p-value obtained also goes in the same direction than the D-statistic test. By not rejecting the null hypothesis, it means that the cdf of the PHM residuals is equal to the cumulative distribution function of the exponential distribution; therefore, the PHM constructed fits the data well.

5.1.2 Interpretation of the obtained PHM curve

Figure 4.2 expresses the resulting PHM curves for the seven experimental blades, the implication of the characteristics of the PHM displayed in that figure is that time has no influence, only the covariate has influence in the obtained model. This means that the decisions can be made based only on the covariate. In this case it would be sufficient to advice that spare parts must be ordered when the covariate reaches a critical value. The following curve in figure 5.1 demonstrate that the baseline of the obtained PHM is a constant.

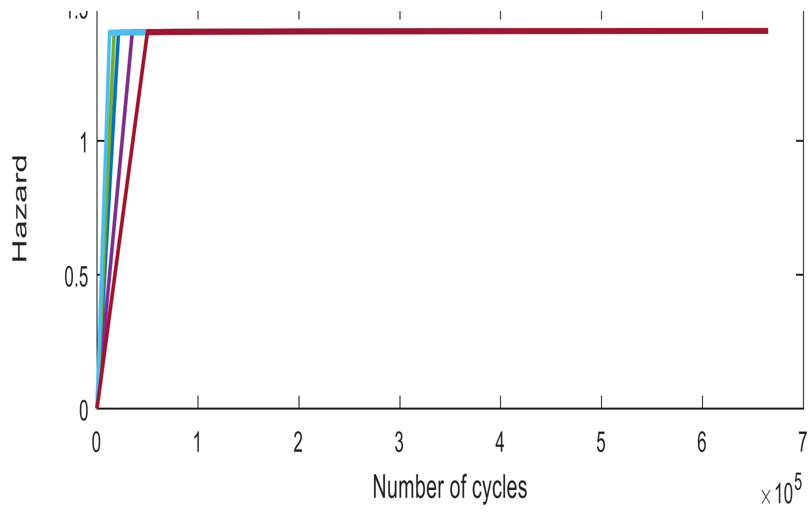


Figure 5.1 Hazard curve, without considering the covariate (Only baseline of the PHM).

Figure 5.1 indicates how the baseline of the PHM behaves, and since the baseline of the Weibull PHM is essentially time based, it can be concluded that time does not have influence on the PHM, only the covariate has an influence. Therefore, the economical optimisation will be trivial, for the decision can be made only based by the observation of the covariate. It is important to highlight that this situation where age does not have an influence, would lead to the most economical replacement strategy, since replacement decisions are then based on a highly predictive covariate measurement, with little risk of failure or wasted life.

This result does demonstrate the universality of the PHM method for a wide range of situations where, on the one extreme, the predictive capability of the covariate is very high (implying that the mathematics reduces the influence of age in the parameter solution, by solving to β= 1), towards the other extreme, where the predictive capability of the covariate is very low (and the parameter solution would yield β >1 and γ very small, to reduce the influence of the covariate on the age-based hazard rate).

In practice, it may be expected that to have a condition measurement which would be so accurate in predicting the failure (which in the present case was done in laboratory conditions), would not be common. To be able to demonstrate the application of the PHM method (where both condition monitoring results and age plays a role) for decision-making with economical consideration, it was decided to introduce noise or randomness progressively in the PHM so that the risk for the seven blades is different. Two methods were considered, namely to introduce noise on the covariate (simulating inaccurate (real-life) measurements and secondly, to randomise the failure points (the crack size at which failure would take place).

5.1.3 Introduction of noise in the covariate of the PHM

To introduce the noise in the covariate, we assumed the data subjected to a Gaussian process with each of the covariate taken singularly as mean, and the standard deviation being the product of each mean by the value which expresses the percentage. For example, given, if the natural frequency equals 72Hz or the crack size equal to 52 mm, the noise levels are defined as follows:

- Mean = 72 and the standard deviation = 72 × 0.1 means 10 % noise;
- Mean = 52 and the standard deviation = 52 × 0.3 means 30 % noise.

The expected outcome of the investigation is to see that as the covariate is randomised, it would lose its influence on the PHM by a relative reduction of the covariate weight parameter and the shape parameter increasing progressively from the initial value.

In a first attempt, the noise level as defined in the previous section was varied between 0.1 and 0.9 percent. Figure 5.2 is a sample for 0.2% of noise, table 5.1 expresses the corresponding shape, weight and scale parameters corresponding to the noise level.

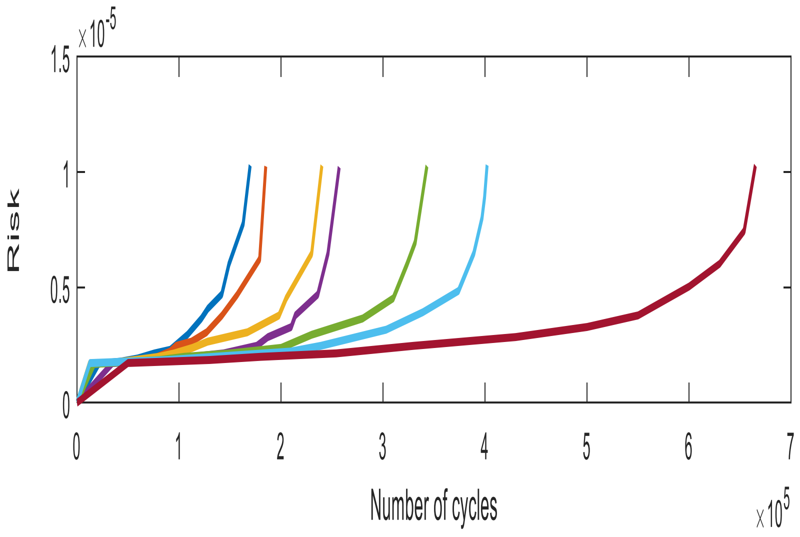


Figure 5.2: Risk versus loading cycles for data expressing 0.2% noise level.

Table 5.1 below displays the result of all the Weibull parameters corresponding to each level of noise, from 0.1 to 0.9 percent.

Table 5.1: Level of noise results

Noise (%)	Beta (β)	Eta (η)	Gamma (γ)
Initial	1.0012	7.10E+05	0.0293
0.1	1.0016	6.23E+05	-0.0582
0.2	1.0014	6.45E+05	0.0357
0.3	1.0201	8.32E+05	0.1523
0.4	1.0188	8.36E+05	0.1535
0.5	1.0014	6.07E+05	-0.0748
0.6	1.0014	6.53E+05	-0.0265
0.7	1.0014	6.88E+05	0.0109
0.8	1.0014	6.45E+05	0.0407
0.9	1.0016	6.32E+05	-0,0501

As may be observed, the weak noise introduction did not make any significant difference to the PHM results. Higher noise levels caused instabilities in the PHM parameter solving algorithms, which could not be solved.

5.1.4 Randomising the failure level

As the final purpose of this work is to use the PHM as a tool to make optimal decision for axial fan blades replacement when managing spare parts, it is at least important to present a scenario which better approaches a practical scenario. Significant differences in the failure time of the blades are required and therefore we randomised the failure levels (critical crack sizes). This was done in a similar way than with the introduction of noise on the covariate (using the typical failure level as a mean and introducing a variance of a percentage of this mean. For each blade, the failure level is then randomly sampled from the arising normal distribution.

The result with 10% randomisation is illustrated by figure 5.4 below.

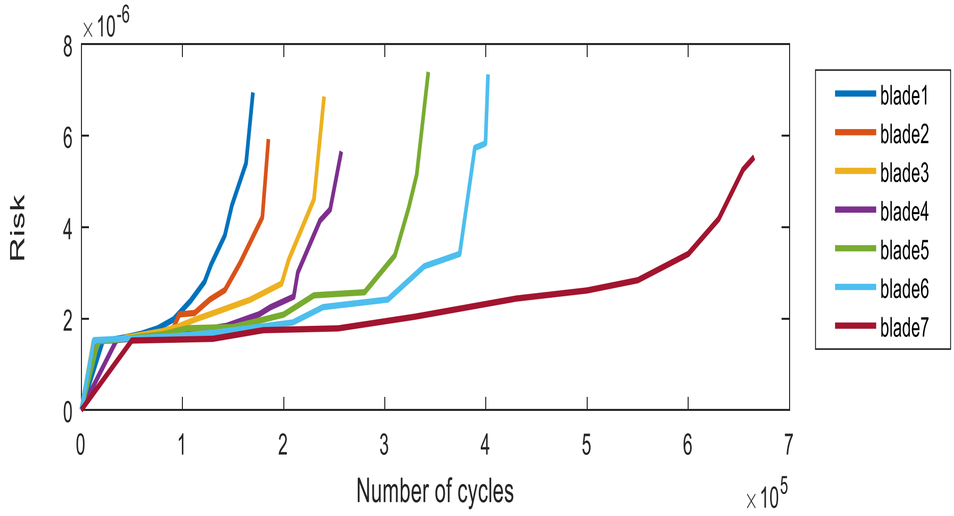


Figure 5.4: PHM at 10% failure level randomisation

Compared to Figure 4.2 and Figure 5.2, Figure 5.4 represents a situation where the PHM is no longer based on the covariate only with time being constant. In contrast it shows a situation where there is an influence of time and the covariate. This situation requires the economic approach to determine the optimal risk point because the blades are failing at different risk levels. If we set the replacement risk level (d) at 4E-06 all blades will be replaced before failure, but there will be some blades with significant remaining life. If we set the risk point at 6.5 E-06, blade 2, blade 4 and blade 7 will fail before reaching the risk point that has been set, which will be expensive. The purpose is then to find an optimal choice for (d).

Optimal decision policy with PHM using simulation procedure (with randomised failure data).

Makis and Jardine, (2013) addressed the optimal decision making with PHM successfully. To build the cost function, they stated that the determination of the risk value which will lead to an optimal cost requires the prediction of the covariate behaviour. Their model was constructed based on the hypothesis that the covariate behaviour was stochastic and approximating a non-homogeneous Markov chain in a finite state space. The covariate behaviour was demonstrated using a Transition Probability Matrix (TPM).

Instead of using the approach that track the covariate behaviour using a Markov Chain, a simulation approach is utilized. Referring to Figure 5.6, the approach consisted of:

- Selecting from the lowest to the highest a given value of risk expressed by 'd'
- Draw a straight line passing through the selected 'd'
- Interpolate in the x-axis the intercession of 'd' to the risk curve to find either the number of cycle (time) for preventive replacement or for failure replacement.
- Apply the following formula to calculate the cost per unit time

$$Cost/unit\ time = \frac{\left(A\ \times C_p\right) + (B\ \times C_f)}{Average\ cost} \tag{5.2}$$

In this equation A is the number of blades falling under the preventive replacement time, B is the number of blades falling under the failure time, and C_p and C_f respectively are the preventive replacement cost and failure cost.

Among the set of risk values selected, choose the optimal, means the one that is minimizing the cost. Below is given an illustration applying the simulation procedure on the 0.2 % noise level data.

The risk of failing at a time t given the covariate is expressed by d.

After doing the computation, the following results were obtained:

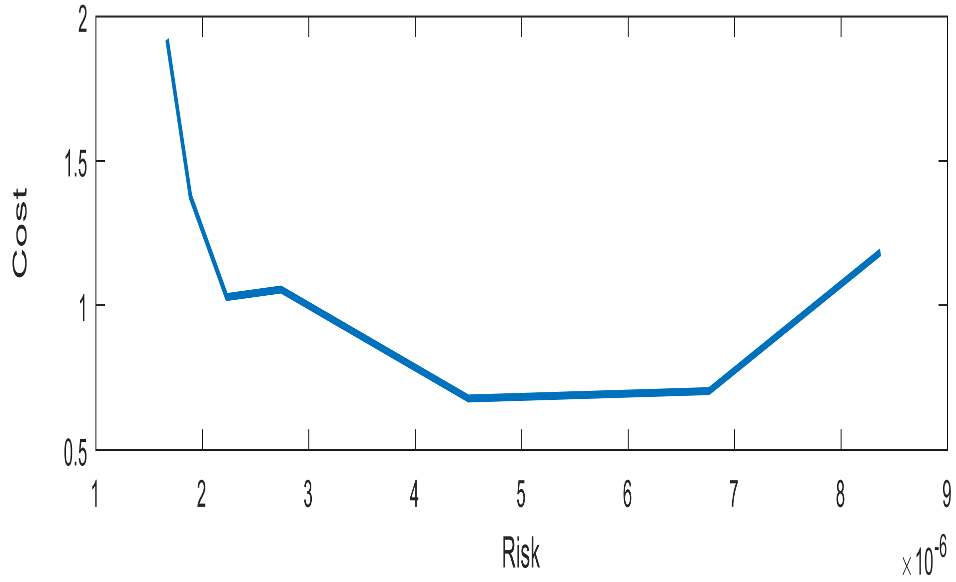


Figure 5.5 Cost per unit time versus risk point for 10% noise level.

Figure 5.5 above shows that the optimum Risk level for replacement can be obtained between 4.5E-06 and 6.76E-06 because in that region the values of the optimal cost are almost the same, however, for less wasted life 6.76E-06 better.

For the sample of data that was treated for 10 % level of randomisation, the optimal risk point which minimizes the cost per unit time was found to be at 6.76e-06. Referring to the set risk point in terms of each blade, the following results are obtained:

- For blade 1 the optimal risk level corresponds to the crack length varying between 50 to 58.332 mm crack length and 165650 loading cycles. This implies less wasted life. The decision maker could adjust the replacement of the blade accordingly, then the management of the spare parts can be done efficiently.
- For blade 2, the optimal risk level corresponds to the crack length varying between 42.6 and 54.168 mm, 184620 loading cycles, therefore less wasted life.
- For blade 3 it failed before reaching the optimal risk level.
- For blade 4 it failed before reaching the optimal risk level.
- For blade 5 the optimal risk level corresponds to the crack length varying between 60 to 64.5 mm, 336870 loading cycles.
- For blade 6 it failed before reaching the optimal risk.

- For blade 7 the optimal risk level corresponds to the crack length between 55 to 60 mm, 660680 loading cycles.

These results illustrate the replacement policy for each of the blades taken individually the decision maker managing the demand of the blades, can use these results and optimize the spare parts (blades) demand. As soon as the crack size which is linked to the natural frequency, or the number of cycles reach the mentioned value for each of the blades it will be known that replacement should be performed which is related to the demand of the blades. Figure 5.6 presents the PHM curves with a cutting line at 6.76E-06 optimal risk value.

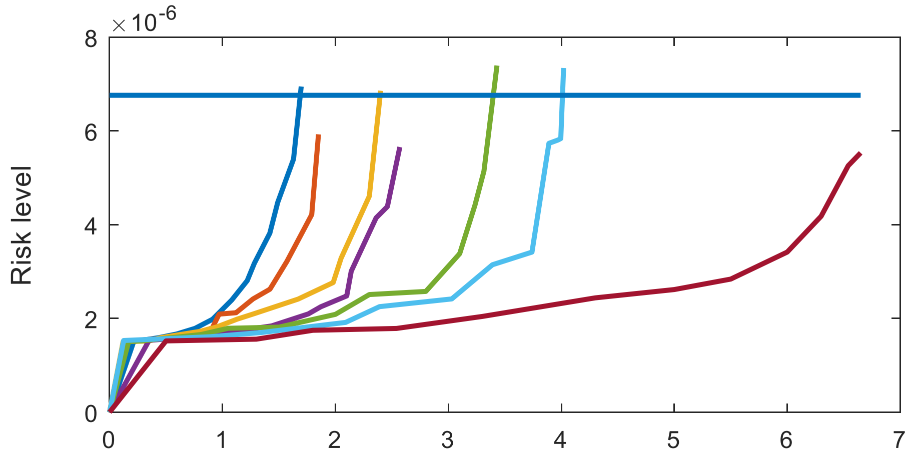


Figure 5.6 PHM curves with optimal cost per unit cut off line.

Chapter 6 Conclusion and recommendations

6.1 Conclusion

This dissertation originated from an inventory management challenge, where the demand for spare parts is infrequent. The randomness of the demand when managing the inventory makes it difficult to forecast spare parts. Several forecasting methods have been developed over the years aiming to address the challenge. One of the more efficient traditional forecasting methods that tries to address the challenge is the Croston method. However, despite its performance, it does not consider the condition of the component to be replaced which is inefficient.

The purpose of this dissertation was to develop an alternative forecasting method to the traditional method. To reach this goal, a PHM approach was suggested that integrates condition-based maintenance with to the spare parts forecasting method, so that the condition of the component is also considered to motivate the demand. A PHM was used with condition monitoring data to calculate the risk of failure for the component under monitoring. The added value of this new method is that it tracks the failure arrival and makes the forecasting more accurate because of the condition of the component which is well known but also it is suitable for critical component where there is not enough historical data to forecast.

To demonstrate the expected solution from the PHM and to be able to determine the optimal risk point used to forecast the spare parts, an investigation was performed to calculate the demand for 30-degree fan axial blades from Fatigue Crack Life (FCL) data. Fatigue tests was performed by Brits (2016) on the 30-degree fan axial blades which resulted in cracks to develop and to grow until the blades failed. FCL data, consisting of crack length over the number of loading cycles, were acquired during the tests. In this work, a finite element model is used to estimate the natural frequencies of the blades over crack length and time from the FCL data. Both FCL data and natural frequency data served as inputs in a PHM to predict the failure arrival which is essential for forecasting spare parts.

The procedure used in this dissertation on the 30-degree fan axial blades and the benefits of using it, are summarized as follows:

- Estimate the parameters needed to construct the PHM by means of maximizing the likelihood function. The maximization was performed with the Newton-Raphson method.
- To test how well the PHM fits the data, the K-S test was used with a 5% level of significance. the obtained D- statistic and p −value obtained with the R package confirmed that the PHM fits the data well.
- The economic approach was investigated because the outcome from the PHM could be useless when applied without the context of economic considerations. A blending of the PHM with the economics allows one to determine the optimal risk level which minimizes the cost. The optimal risk point found was the main tool to define a spare parts management policy.
- The proposed procedure has the benefit that it uses natural frequency data as opposed to the Paris law parameters used in the work by (Brits, 2016) to predict fatigue crack life. Another contribution of this work is that in the previous work by Brits (2016) the optimal point to replace a blade was not investigated.
- The benefits of this proposed alternative forecasting method is that it gives the ability to proactively have information which can allow a 'just- in- time' supply of spare parts. This implies that a component can be replaced without wasting useful life because the component replacement is no longer time- based only, but also condition - based.

6.2 Recommendations

From the observations and experiences obtained during this dissertation, the following recommendations are made for future investigation:

- As the spare parts approach in this dissertation was oriented to a single component replacement, it is required to extend the application to more than one component because most of the machines in the industry have more than one critical component. Parameters such as lead time, stock holding, and cost related needs to be considered as well.
- Compare the PHM outcome with other regression models which also consider the condition of the component. An example is the Prentice William Peterson model (PWP) model which has additional benefits to the PHM because it

considers also the previous replacement of the item under analysis as well. Vlok, (2006) briefly presented the benefits of this model in his work.

- Investigate on the influence of increasing the noise in the covariate and evaluate its impact on the three Weibull parameters and give physical meaning related to that. Because we assume that the noisy data are closer to the real situation than the experimental data which can be submitted to some constraint due to the measurement condition.

References

Al-Najjar, B. (2000). Impact of real-time measurements of operating conditions on effectiveness and accuracy of vibration-based maintenance policy – A case study in paper mill. Journal of Quality in Maintenance Engineering, vol.6, pp.275- 287.

Al-Najjar, B. (2007). The lack of maintenance and not maintenance which cost: A model to describe and quantify the impact of vibration-based maintenance on company's business. International Journal of Product Economics, vol. 107, pp. 260-273.

Adebar, P. (1994). Testing structural concrete beam elements. Materials and Structures, vol. 27, pp. 445-451.

Bacchetti, A., Saccani, N. (2012). Spare parts classification and demand forecasting for stock control: Investigating the gap between research and practice. Journal Omega, pp.722-737.

Barata, J., Soares, C.G., Marseguerra, M.& Zio, E. (2002). Simulation modelling of repairable multi-component deteriorating systems for 'on condition' maintenance optimization. Reliability Engineering & System Safety, vol. 76, pp. 255-264.

Borgonovo, E., Marseguerra, M.& Zio, E. (2000). A Monte Carlo methodological approach to plant availability modeling with maintenance, again and obsolescence. Reliability Engineering & System safety, vol. 67, pp. 61-73.

Brits, J.C.P. (2016). An experimental and stochastic approach to estimate the fatigue crack life of a turbomachinery blade using finite element modelling. Pretoria: University of Pretoria.

Bunday, B.D., Kiri, V.A.& Stoodley, K.D.C. (1992). The Reliability of artificial heart valves- A case study for the proportional hazards model. International Journal of Quality & Reliability Management, vol. 9, pp. 56-68.

Callegaro, A., 2010. Forecasting methods for spare parts demand. Universita Degli Studi di Padova.

Cheng, T., Qiao, R., & Xia, Y. (2004). A Monte Carlo simulation of damage and failure process with crack saturation for unidirectional fiber reinforced ceramic composites. Composites Science and Technology, vol. 64, pp. 2251-2260.

Cox, D.R. (1972). Regressions models and life tables. Journal of the Royal Statistics Society, Imperial college, vol.34, pp.187-220.

Croston, J.D. (1972). Forecasting and stock control for intermittent demands. Operational Research Quaterly.

Crumer, A.M. (2011). Comparison between Weibull and Cox proportional hazards models. Master Thesis, Kansas: Southeast Missouri State University.

Dekker, R., Pince, C., Zuidwijk, R., Jalil, M.N. (2013). On the use of installed base information for spare parts logistics: A review of ideas and industry practice. Int. J. Production Economics. 143, pp. 536-545.

De Almeida, A.T. (2001). Multicriteria decision making on maintenance: spare and contracts planning. European Journal of Operational Research, vol. 129, pp. 235-241.

Dohi, J.W. (2003). Estimating the mixture of proportional hazards model with incomplete failure data. Journal of Quality in Maintenance Engineering, vol. 9, pp. 265-278.

Elfaki, F.A.M., Daud, I.B., Ibrahim, N.A.& Abdullah, M.Y., Usman. M. (2007). Competing risks for reliability analysis using Cox's model. International Journal for Computer – Aided Engineering and Software vol. 24, pp. 373-383.

Elwany, A.H., Gebraeel, N.Z. (2008). Sensor-driven prognostic models for equipment replacement and spare parts inventory. IIE Transactions, pp. 629-639.

Grall, A., Berenguer C., and Dieulle, L. (2002). A condition-based maintenance policy for stochastically deteriorating systems. Reliability Engineering & System Safety, vol. 76, pp. 167-180.

Gardin, C., Fiordalisi, S., Baudoux, C.S., Gueguen, M., & Petit, J. (2016). Numerical prediction of crack front shape fatigue propagation considering plasticity-induced crack closure. International Journal of Fatigue, vol. 88, pp. 68-77.

Galar, D., Gustafson, A., Tormos, B. & Berges, L. (2012). Maintenance Decision Making based on different types of data fusion. Maintenance and Reliability, 14(2), pp.135-144.

Ghobbar, A.A., Friend, C.H. (2003). Evaluation of forecasting methods for intermittent parts demand in the field of aviation a predictive model. Computers & Operation Research, vol.30, pp. 2097-2114.

Golmakani, H.R. (2011). Cost-effective condition-based inspection scheme for condition-based maintenance. IEEE, pp. 327- 330.

Haitao, L., Rausch, M. (2010). Spare part inventory control driven by condition-based maintenance. IEEE, pp. 1-6.

Hertzberg, R.W., 1996. Deformation and fracture mechanics of engineering materials. 4th ed, John Wiley &sons.

Hecker, M. Hentschel, R., Hensel, M. & Lehr, M.U (2011). Crack propagation and delamination analysis within the die by camera- assisted double cantilever beam technique. IEEE, pp. 1-7.

Huynh, T.K., Barros, A., Berengeur, C. & Castro, I.T. (2010). Value of condition monitoring information for maintenance decision-making. IEEE, pp. 1-6.

Hellingraph, B., Cordes, A.K. (2014). Conceptual approach for integrating condition monitoring information and spare parts forecasting methods. Production & Manufacturing Research, pp. 725-737.

Jardine, A.K.S. & Tsang, A.H.C. (2013). Maintenance, replacement, and reliability: theory and applications. 2nd ed, Boca Raton, USA: CRC Press.

Jardine, A.K.S., Lin, D. & Banjevic, D. (2006). A review on machinery diagnostics and prognostics implementing condition- based maintenance. Mechanical Systems and Signal Processing, vol. 20, pp. 1483-1510.

Jardine, A.K.S., Banjevic, M., Wiseman, S.& Joseph, B.T. (2001). Optimizing a mine haul truck wheel motors' condition monitoring program use of proportional hazards modelling. Journal of Quality in Maintenance Engineering, vol.7, pp. 286-302.

Jardine, A.K.S., Banjevic, D., Makis, V. (1997). Optimal replacement policy and the structure of software for condition-based maintenance. Journal of Quality in Maintenance Engineering, vol. 3, pp. 109-119.

Jardine, A.K.S., Zhang, F. & Yan, H. (1996). Enhancing system reliability through maintenance decision making. IEEE, pp. 1004-1007.

Jardine, A.K.S. (2002). Optimizing condition-based maintenance decision. IEEE, pp. 90-97.

Jacobs, F.B., Chaise, R.B. (2013). Operations and supply management. 13 ed, Global Edition.

Jin, T.S., Song, G.X., Qiang, Y.C., Jie, Z.Z., Fa, Z.Z., Cheng, Z.B. (2014). Real time remaining useful life prediction based on nonlinear Wiener based degradation with measurement error. Journal of Central South University, vol. 21, pp. 4509-4517.

Johnston, F.R. & Boylan, J. (1996). Forecasting for items with intermittent demand. Journal of the operational Research Society. Journal of the Operational Research Society, vol.47, pp.113-121.

Jun, D., Jianchen, Q. (1989). On crack initiation life during low cycle fatigue. Fatigue of Engineering Materials, vol. 12, pp. 627- 630.

Kobayashi, K., Kaito, K. (2011). Random proportional Weibull hazard model for large-scale information systems. Journal of Quality in Maintenance Engineering, vol. 29, pp. 611-627.

Klysz, S., Gmurczyk, G., Lisiecki, J. (2010). Investigations of some properties of material samples taken from the aircraft withdrawn from service. Fatigue of Aircraft Structures, Institute of Aviation Scientific Publication, pp.52-58.

Kleinbaum, D.G., (1999). Survival analysis: A self-learning text. Springer-Verlag, New York.

LeClere, M.J. (2005). Time-dependent and time-Invariant covariates within a Proportional hazards model: A financial distress application. Review of Accounting and Finance, Vol.4, pp. 91-109.

Li, Z., Zhou, S., Sievenpiper, C.& Choubey, S. (2010). Change detection in the Cox proportional hazards models from different reliability data. Quality and reliability Engineering international, pp. 677-689.

Marseguerra, M., Zio, E., Podofillini, L. (2002). Condition-based maintenance optimization by means of genetic algorithms and Monte Carlo simulation. Reliability Engineering & System Safety, vol. 77, pp. 151-166.

Meng, M.Y.G. (2011). Updated proportional hazards model for equipment residual life prediction. International Journal of Quality & Reliability Management, Vol. 28, pp. 781-795.

Michael, W., Steinberg, L.L. (2001). Maintenance of mobile mine equipment in the information age. Journal of Quality in Maintenance Engineering, Vol.7, pp. 264-274.

Montgomery, N., Lindquist, T., Garnero, M.A., Chevalier, R.& Jardine, A.K.S. (2006). Reliability function and optimal decisions using condition data for EDF primary pumps. IEEE, pp. 1-6.

Montanari, M.B.A. (2004). Mutti- attribute classification method for spare parts inventory management. Journal of Quality in Maintenance Engineering, Vol. 10, pp. 55-65.

Moon, S., Hicks, C., Simpson, A. (2012). The development of a hierarchical forecasting method for predicting spare parts demand in the South Korean Navy – A case study, Int. J. Production Economics, vol. 140, pp. 794-802.

Makis, V., Jiang, X., & Cheng, K. (2000). Optimal preventive replacement under Minimal repair and random repair cost. Mathematics of Operations Research vol.25, pp. 141-156.

Makis, V., Jardine, A.K.S. (1992). Optimal replacement policy for a general model with Imperfect Repair. The Journal of the Operational Research Society, vol.43, pp. 111-120.

Romeijnders, W., Teunter, R. & Jaarsveld, W.V. (2012), A two-step method for forecasting spare parts demand using information on component repairs. European Journal of Operational Research, vol. 220, pp. 386-393.

Sheng Si, X., Wang, W., Hu, C.H., Zhou, D.H (2011). Remaining useful life estimation – A review on the statistical data driven approaches. European Journal of Operational Research, pp. 1-14.

Salunkhe, T., Jamadar, N.J. & Kivade, S.B. (2014). Prediction of remaining useful life of mechanical components-A review. International Journal of Engineering Science and Innovative Technology, Vol. 3, pp. 125-135.

Shinozuka, M. (1972). Monte Carlo solution of structural dynamics. Computers & structures, vol. 2, pp. 855-874.

Syntetos, A.A., Boylan, J. (2005). The accuracy of intermittent demand estimate. International Journal of Forecasting, vol.21, pp. 303-314

Tanaka, K., Matsuoka, S. (1977). A tentative explanation for two parameters, C and m, in Paris equation of fatigue crack growth. International Journal of Fracture, vol. 13, pp. 563-583.

Teunter, R., Sani, B. (2009). On the bias of Croton forecasting method. European Journal of Operational Research, pp. 177-183.

Vlok, P.J. (1999). Vibration covariate regression analysis of failure time data with the Proportional Hazards Model. Master thesis, University of Pretoria.

Vlok, P.J. (2006). Dynamic residual life estimation of industrial equipment based on failure intensity proportions. PhD thesis, University of Pretoria.

Vlok, P.J., Coetzee, J.L. (2006). Advances in renewal decision- making utilising the proportional hazards model with vibration covariates. University of Pretoria.

Wang, Yi, Wang, H. & He, B. (2010). Calculation of power equipment reliability for condition-based maintenance decision-making. International Conference on Power System Technology, pp.1-7.

Wang, X., Chiang, M.Y.M., & Snyder, C.R (2004). Monte-Carlo simulation for the fracture process and energy release rate pf unidirectional carbon fiber-reinforced polymers at different temperatures. Composites Part A: Applied Science and Manufacturing, Vol. 35, pp. 1277-1284.

Wang, W., Syntetos, A.A. (2011). Spare parts demand: Linking forecasting to equipment maintenance. Transportation Research Part, vol.47, pp. 1194-1209.

Waeyenbergh, G.M.H., Pintelon, L. (2002). A framework for maintenance concept development. International Journal of Production Economics, 77(3), pp.299-313.

Waeyenbergh, G.M.H., Pintelon, L. (2003). Maintenance concept development: A case study. International Journal of Production Economics, 89(3), pp.395-313.

Willman, T.R., Smart, C.N., Schwarz, H.F. (2004). A new approach to forecasting intermittent demand for service parts inventories. International Journal of Forecasting 20, pp. 375-387.

William, W.S. Wei. (2006). Time series analysis: Univariate and multivariate methods. 2nd Edition. Pearson Education.

Xu, X., Tang, J. (2005). Analysis of response of the micro-machined poly-silicon cantilever subjected to vibration environment. IEEE, pp.1-5.

Xue, Y. modeling fatigue small-crack growth with confidence- A multistage approach. International Journal of Fatigue, vol.32, pp.1210-1219.

Yam, R.C.M., Tse, P. W., Li, L. & Tu, P. (2001). Intelligent predictive decision support system for condition- based maintenance. The International Journal of Advanced Manufacturing Technology, vol 17, pp. 383-391.

Zhou, Y., Baseer, M.A., Mahfuz, H. Jeelani, S. (2006). Monte Carlo simulation on tensile failure process of unidirectional carbon fiber reinforced nano-phased epoxy. Materials Science and Engineering, pp. 63-71.