Are BRICS Exchange Rates Chaotic?

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Abstract

In this paper, we focus on the stochastic (chaotic) attributes of the US dollar-based exchange rates for Brazil, Russia, India, China and South Africa (BRICS) using a long-run monthly dataset covering 1812M01-2017M12, 1814M01-2017M12, 1822M07-2017M12, 1948M08-2017M12, and 1844M01-2017M12, respectively. For our purpose, we consider the Lyapunov exponents, robust to nonlinear and non-stationary systems, in both full—samples and in rolling windows. For comparative purposes, we also evaluate a long-run dataset of a developed currency market, namely British pound over the period of 1791M01-2017M12. Our empirical findings detect chaotic behavior only episodically for all countries before the dissolution of the Bretton Woods system, with the exception of the Russian ruble. Overall, our findings suggest that the establishment of the free floating exchange rate system have made the path of exchange rates more predictable, and hence, the need for policymakers to intervene in the currency markets for the most important emerging market bloc, is not necessarily warranted.

JEL codes: C46, E52.

Keywords: Exchange rate, chaos, Lyapunov exponent.

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1. Introduction

Exchange rate movements are known to potentially influence many other financial and economic variables like interest rates, trade, output, and equity prices (Ruzima and Boachie, 2017). Naturally, predicting exchange rate movements is of paramount importance to various economic agents, and hence, the literature associate with exchange rate predictability is huge, to say the least (see, Balcilar et al., (2016) and Christou et al., (forthcoming) for detailed reviews). In this regard, determining whether exchange rate behavior is characterized by chaos is of paramount importance. Theoretically, a chaotic system is a random-looking nonlinear deterministic process with irregular periodicity and sensitivity to initial conditions. In other words, similar shocks in different situations create alternative future paths. Thus, the behavior of the series is difficult to predict and probably given the complexity of all chaotic systems, any detected causal linkage from exchange rate movements to economic variables, should be attributed to chance rather than concrete predictive relationships.

Against this backdrop, the objective of this paper is to use the Lyapunov exponent to analyze whether the US dollar-based exchange rates for Brazil, Russia, India, China and South Africa (i.e., the BRICS) depict chaotic behavior or not using longest possible available monthly covering the periods of 1812M01-2017M12, 1814M01-2017M12, 1822M07-2017M12, 1948M08-2017M12, and 1844M01-2017M12, respectively. For the sake of comparison, we also look at the behavior of the British pound over the 1791M01 to 2017M12. Note that, the decision to look at the BRICS is motivated by its emergence as a powerful economic force. In 2010, about 25 percent of global output emanated from the BRICS (Government of India, 2012). Also, the contribution to global output from the bloc is expected to surpass that of current world economic powers like G7 countries by 2050 (Wilson and Purushothaman, 2003). Trade by these economies with the rest of the world has been growing at a fast rate, with the high economic performance of these economies attributed to the high level of foreign direct investment, especially in the private sector (Government of India, 2012). Naturally, unpredictable exchange rate movements are likely to affect the growth potential of these economies, and with them that of the world economy, given the growing dominance of this bloc. Therefore, the significance of an investigation into the chaotic behavior of exchange rates of the BRICS cannot be overstated, which in turn, we aim to achieve in this paper, by looking at the longest possible spans of data available on the exchange rates of these economies, to try and capture the entire historical evolution of the exchange rate dynamics.

The literature on chaos in financial markets in general, is huge (see, Tiwari and Gupta (forthcoming) for a review), and this also includes the currency markets (see for example, Serletis and Gogas (1997), Cristescu et al., (2009), BenSaïda and Litimi (2013), Lahmiri (2017)). While, earlier studies have concentrated primarily on developed markets, given the emergence of the BRICS bloc, Kumar and Kamaiah (2016), and Bhattacharya et al., (2018) have recently analyzed chaotic dynamics in the exchange rates of these countries. While, the former study finds underlying chaotic structure for all the five markets, the latter was able to show that the same holds true for Brazil, Russia, India, and China, but not South Africa. Given this, our objective is to provide a definitive answer to the existence or non-existence of chaotic dynamics in the BRICS exchange rates, using the longest spans of data available, and hence, taking out the possibility of the results being sample-specific. We build on this argument further, by carrying out a time-varying (rolling window) analysis of chaos. While, a full-sample analysis is informative, the final results on chaos could be driven by a large sub-sample or sub-samples for which chaos exists or does not exist. In addition, the time-varying approach, would also indicate to the policy-maker of the current status of the currency market in terms of chaos, and whether or not, there is a need for policy intervention to ensure that the future growth paths of the BRICS countries dependent on exchange rate movements are predictable or not. To the best of our knowledge, this is the first paper to analyze chaotic dynamics in the BRICS and the UK dollar-based exchange rates using data, that in some cases spans more than two centuries. The remainder of the paper is organized as follows: Section 2 introduces the econometric methodology, while Section 3 presents the data and results, with Section 4 concluding the paper.

2. Methodology: Lyapunov Exponent

The basic idea behind the detection of chaos lies with the dependence of chaotic systems to initial conditions. The Lyapunov exponent λ measures this difference $\Delta x(X_0, t)$ between the two paths of the same phenomenon generated in time according to different initial conditions. In order to identify a system as chaotic, the corresponding Lyapunov exponent should be strictly positive. In this paper, we follow the procedure described in BenSaïda and Litimi (2013) in order to estimate the maximum Lyapunov exponent. In mathematical notation:

$$x_t = f(x_{t-L} + x_{t-2L} + \cdots + x_{t-mL}) + \varepsilon_t \tag{1}$$

where *L* is the time delay, *f* is an unknown chaotic map, *m* is the *embedding dimension* of the system and ε_t represents the added noise. BenSaïda and Litimi (2013) adopt the Jacobian based approach to compute λ since the direct approach is inefficient in the presence of noise. Briefly, the exponent is given by:

$$\hat{\lambda} = \frac{1}{2M} ln v_i \tag{2}$$

where *M* is an arbitrary selected number of observations often approximating the $^2/_3$ of the total span and v_i is the largest eigenvalue of the matrix $(T_M U_o)(T_M U_o)'$, with

$$U_0 = (1\ 0\ 0\ \ldots 0)' \tag{3}$$

$$T_M = \prod_{t=1}^{M-1} J_{M-t} \tag{4}$$

$$J_t = \begin{bmatrix} \frac{\partial f}{\partial x_{t-L}} & \frac{\partial f}{\partial x_{2t-L}} & \cdots & \frac{\partial f}{\partial x_{t-mL+L}} & \frac{\partial f}{\partial x_{t-mL}} \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \ddots & \vdots & \vdots \\ \vdots & \vdots & \cdots & 0 & 0 \\ 0 & 0 & & 1 & 0 \end{bmatrix} \tag{5}$$

In the case of scalar time series the chaotic map *f* generating the series is usually unknown; as a result the Jacobean matrix in (5) cannot be estimated. Thus, we need to approximate the chaotic map with a data adapting function that can produce an exact approximation of the series. BenSaïda and Litimi (2013) choose to estimate the chaotic map based on a neural network with one hidden layer of neurons and one output layer. In mathematical notation the chaotic map *f* is approximated by the equation:

$$x_t \approx a_0 + \sum_{j=1}^{q} a_j tanh\left(\beta_{0,j} + \sum_{i=1}^{m} \beta_{i,j} x_{t-iL}\right) + \varepsilon_t \quad (6)$$

with q declares the hidden layers of the neural network with a tangent activation function. The order of *(L,m,q)* defines the complexity of the system and is selected according to the triplet that provides the maximum value of the exponent λ.

Assuming the existence of chaos as the null hypothesis, we can reject it in favor of the non-existence of chaos based on a one-sided statistical test.[1] In this way, a system is identified as chaotic when both assumptions are met: a) we find a positive Lyapunov exponent close to unity and b) we are unable to reject the null hypothesis on the existence of chaos.

3. Empirical results

We compile a dataset of nominal exchange rates for BRICS and the U.K. expressed as local currency to U.S. dollar compiled from the Global Financial Database[2] and take logarithms. We begin our analysis examining both the entire time span and rolling windows of 40%, 50% and 60% of the total length with a sliding window of one. With this smooth transition in time we uncover time patterns that may exist during distinct periods, but are typically hidden during the examination of the entire sample.

As part of an initial analysis, as reported in the Appendix of the paper, we found that theHurst exponents of all the six exchange rates for the full-sample tends to be greater than 0.5, i.e., suggesting a random walk behaviour, with that of Brazil, Russia and China being even close to 1, i.e. the three exchange rates being exceptionally persistent, with the possibility of them not returning to equilibrium after a shock. Given this, we proceed in examining the chaotic behaviour of all exchange rates using the Lyapunov exponent. A

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[1] For more information on the derivation of the test, the interested reader is referred to BenSaïda and Litimi (2013).

2 The descriptive statistics are reported in Table A1 of Appendix. As can be seen from the Jarque-Bera test of normality, the null is overwhelmingly rejected in all cases.

Table 1: Lyapunov exponents											
	Entire sample		40% sample			50% sample			60% sample		
Country	λ	Obs	Percentage of positive λ in the window	Rejections of chaos where λ is positive	Window Obs	Percentage of positive λ in the window	Rejections of chaos where λ is positive	Window Obs	Percentage of positive λ in the window	Rejections of chaos where λ is positive	Window Obs
Brazil	-0.07*	2479	1.95	0	992	2.75	0	1240	3.52	0	1487
Russia	0.03	2447	79.65	0	979	100	0	1224			1468
India	-0.32*	2345	0.35	0	938	0.17	0	1173	0	--	1407
China	0.13	832	27.49	0	333	0	--	416	0	--	499
South Africa	-0.15*	2087	12.93	0	835	3.64	0	1044	0	--	1252
UK	-0.38*	2723	23.67	0	1089	24.89	0	1362	21.19	0	1634

Note: * denotes rejection of the null hypothesis about the existence of chaos at the 5% level of statistical significance. The estimation of the eigenvalue for the largest window in Russia was not possible.

positive Lyapunov exponent indicates the existence of chaotic dynamics in the data generating process (Table 1). Given that we are interested in both the sign and the statistical significance of our results, we follow a different approach from the Hurst exponent results. More specifically, we report the percentage of instances that the exponent is positive in all examined windows, instead of its minimum, average and maximum value. In those instances that the exponent is positive, we also test the statistical significance of the exponent in order to infer whether we can reject the null hypothesis about the existence of chaos.

As we observe from Table 1, we detect chaos only episodically and for a limited number of cases. More specifically, only Russia and China exhibit chaotic behaviour on the entire sample, while the detection rate in the rolling windows is high only for Russia[3]. In all other cases the detection of chaos varies from 3% - 30% of windows. In order to observe this, we plot the estimated Lyapunov exponent for all countries and windows in Figure 1.

The Lyapunov exponent for Brazil is positive for the smallest window in the post-Bretton Woods period but only for limited number of months, during the 1990s where Brazil exhibit high inflationary pressures and in the past 1994 period where the Brazilian real is pegged to the U.S. dollar. In contrast, the Russian ruble exhibit a chaotic behaviour with consistency in the entire post WWII period. Both the Chinese yuan and the Indian rupee have a negative exponent in the entire sample and for all windows, so the hypothesis of chaos is strongly rejected. An interesting pattern unveils for the South African rand and the British pound. In the former case, the exponent is below zero until the end of WWII and the beginning of the Bretton Woods period. In the Bretton Woods period, the rand exhibits low chaotic behaviour in a number of instances up to 1971, with a large period after the dissolution of the fixed exchange rates system (we mark the beginning of the dissolution of the Bretton Woods system at August 1971). In the floating exchange rate era, we find a negative exponent for the rand almost consistently for every window. The

[3] In the case of Russia the estimation of the largest window was not possible, since we could not estimate the eigenvalue of the matrix.

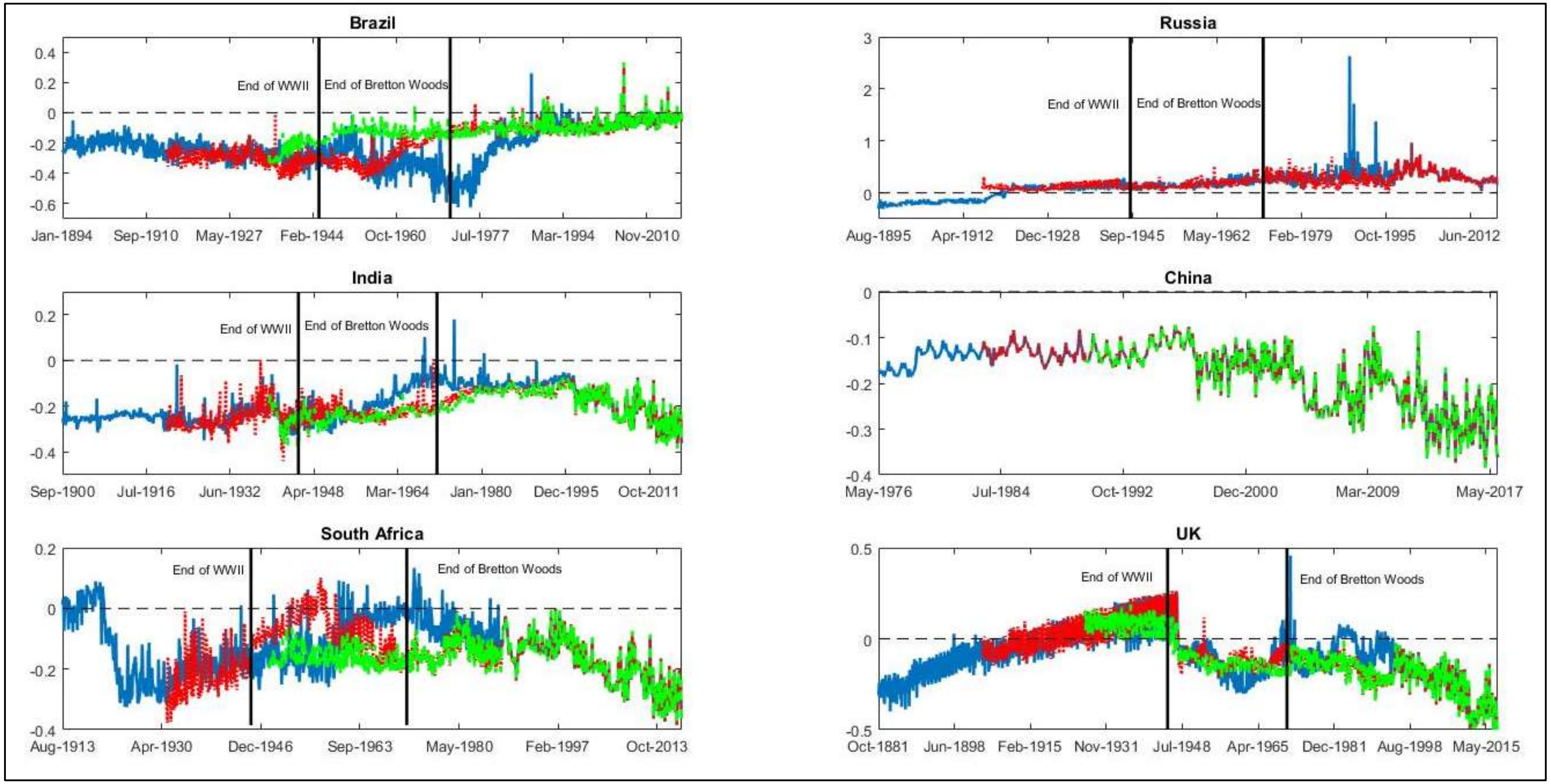

Figure 1: Lyapunov exponents for all exchange rates. The continuous (blue) line depicts the small (40%) rolling window results, the dotted (red) line depicts the mid (50%) window length and the dashed (green) line depicts the large (60%) window length. The estimation of eigenvalues for the largest window in Russia was not possible.

case of the U.K. is way more interesting. In the pre-WWII, we find positive exponents from the beginning of the 19th century up to WWII, where there is an abrupt change in the positive trend and the coefficient turns negative for the entire Bretton Woods period. The coefficient turns positive around 0.5 at the dissolution of the fixed exchange rates system, reverts to a deterministic system up to the turbulent period of the 1980s for the British economy and then reverts back to a non-stochastic system.

Overall, our examination of chaotic behaviour of the exchange rates of the BRICS reveals weak evidence of chaos only in the period before Bretton Woods, while in the modern floating exchange rates era only the Russian ruble keeps its chaotic behaviour, something specifically observed during the period of transition from the Soviet Union to the Russian Federation.

4. Concluding Remarks

In this paper, we focus on the statistical characteristics of exchange rates series for the BRICS and the U.K. in terms of their chaotic behavior using Lyapunov exponents in both full-sample and rolling windows. In doing so, we compile a long-run dataset covering over two centuries of monthly data in three cases (Brazil, Russia, and the UK), and nearly two centuries for India and South Africa, and seventy years for China. The Lyapunov exponent is capable of detecting chaotic dynamics in non-linear and non-stationary systems. Our empirical findings show that chaos is observed only episodically, unlike suggested by earlier studies on the BRICS, and that too mostly for the Russian ruble, and in the pre WWII period for the other series. Overall, our findings suggest that the establishment of the free floating exchange rate system have made the path of exchange rates more predictable, and hence, the need for policymakers to intervene in the currency markets for the most important emerging market bloc, barring Russia, and in the developed economy of the UK to reduce volatility (uncertainty), are not warranted. Lack of chaotic behavior in the BRICS exchange rates in general, suggest that future growth prospects of these economies, and the world in general, is likely to be stable. .

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Appendix

Table A1: Descriptive Statistics						
Country	Time Span	Mean	SD	Kurtosis	Skewness	Jarque-Bera test (*p*-value)
Brazil	1812M01-2017M12	1.51	0	25.98	3.42	0
Russia	1814M01-2017M12	1.05	0	257.94	11.60	0
India	1822M07-2017M12	0.15	0	255.25	10.04	0
China	1948M08-2017M12	0.78	0	161.81	11.50	0
South Africa	1844M01-2017M12	0.17	0	42.54	1.99	0
UK	1791M01-2017M12	-0.05	0	234.88	-0.42	0

Note: SD stands for standard deviation.

Hurst ExponentThe Hurst exponent belongs to the broader category of nonparametric analysis methods and was first proposed by Hurst (1951) as a method for analyzing long-range dependence in the hydrology series. The exponent *H* (Hurst exponent) takes values on the range [0, 1]. Values close to zero indicate an anti-persistent series: the series under examination is mean-reverting. Values close to 1 indicate that the series is persistent: the series never returns to equilibrium after an exogenous shock. An $H = 0.5$ indicates a Random Walk (RW).

According to the Detrended Fluctuation Analysis (DFA) in estimating the exponent *H* by Peng et al. (1994), the initial series *X* of length *N* is divided into *q* equally sized parts of length $n = {}^{N}/_{q}$. Each of the new segments *m=1,2,3……,q* is integrated by the cumulative sums:

$$Y_{i,m}^{(n)} = \sum_{j=1}^{i} x_{j,m}^{(n)}\,, \qquad i = 1,2,3,\dots,q \tag{A1}$$

We then estimate the OLS line for the points in each segment $Y_{m,i}^{(n)} = a_m^{(n)} i + b_m^{(n)}$ and calculate the standard deviation residuals:

$$F_m^{(n)} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(Y_{m,i}^{(n)} - a_{m,j}^{(n)} - b_m^{(n)}\right)^2} \tag{A2}$$

The average standard deviation is calculated for all segments of length *n*:

$$F(n) = \frac{1}{q}\sum_{m=1}^{q} F_m^{(n)} \tag{A3}$$

The F_n values are calculated for every partition and plotted against the partition segment size *n* in a log–log scale. The slope of the linear fit expresses the Hurst exponent *H*.

In Table A2, we report the Hurst exponents for the entire sample and the rolling window estimates.

As we observe, in the entire sample the India and the U.K. exhibit almost RW behaviour while South Africa is very close to the 0.5 threshold. Brazil, Russia and China exchange rates exhibit a highly persistent behaviour with values of the Hurst exponent close to 1. The rolling windows estimation exhibit a different pattern, with values of the exponent varying from under the 0.5 threshold to values close to 1 for the smaller and the medium length windows. As expected, the empirical findings of the rolling windows are close to those on the entire sample only for the large window, since most of the variability of the exponent is smoothed out in larger windows. Thus, for the cases that the exchange rates exhibit high persistence, a shock to the exchange rate causes a permanent effect on its level. In figure A1, we depict the time evolution of the Hurst exponent for all windows.

The Hurst exponent for all countries is above the 0.5 threshold in the post WWII period. Especially for India, in the pre-WWII era there exist several windows that the exponent is around 0.5 declaring a stochastic evolution. The exchange rates where the exponents move close to 0.5 are those of South Africa and the U.K., showing that these two countries could exhibit a quasi-efficient market. In contrast, Russia and China are highly persistent exchange rates where effects on their levels have permanent effects, making them more sensitive to external shocks. Overall, the persistence of all exchange rates seem to rise in the post-Bretton Woods period, denoting open markets that are more sensitive to external shocks.

Table A2: Hurst exponents																	
	Entire sample		40% sample					50% sample					60% sample				
Country	H	obs	min	mean	max	sd	Obs	min	mean	max	sd	Obs	min	mean	max	sd	Obs
Brazil	0.96	2479	0.53	0.70	1	0.17	990	0.54	0.72	1	0.17	1240	0.53	0.76	1	0.18	1490
Russia	0.93	2447	0.57	0.91	1	0.14	980	0.59	0.97	1	0.07	1220	0.86	0.97	1	0.03	1470
India*	0.56	2345	0.41	0.60	0.77	0.10	940	0.45	0.59	0.73	0.08	1170	0.47	0.58	0.70	0.06	1410
China*	0.97	832	0.43	0.72	1	0.10	330	0.62	0.75	1	0.06	420	0.54	0.73	1	0.10	500
South Africa*	0.60	2087	0.50	0.63	0.75	0.04	830	0.55	0.64	0.72	0.03	1040	0.56	0.63	0.70	0.03	1250
UK*	0.51	2723	0.45	0.59	0.70	0.05	1090	0.48	0.56	0.65	0.03	1360	0.49	0.56	0.60	0.02	1630

Note: * denotes uncertainty indices that show an unstable behaviour with values varying from below 0.5 to above 0.5, i.e., being from anti-persistent to highly persistent series. This fact denotes significant changes in the persistence and uncertainty of these exchange rates; sd: stands for standard deviation.

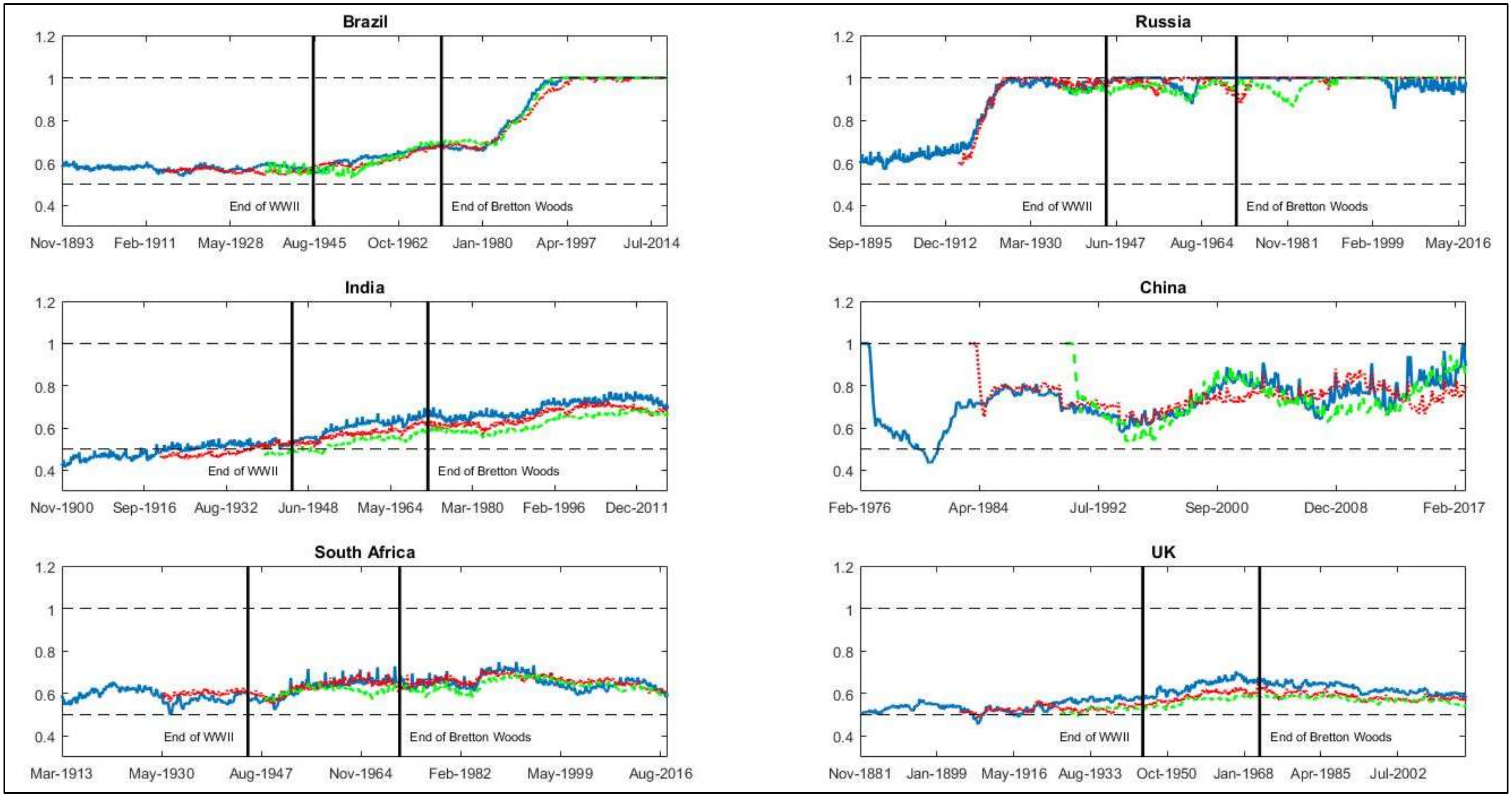

Figure A1: Hurst exponents for all exchange rates. The continuous (blue) line depicts the small (40%) rolling window results, the dotted (red) line depicts the mid (50%) window length and the dashed (green) line depicts the large (60%) window length.