University student performance in multiple choice questions: an item analysis of mathematics assessments

by

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DECLARATION

I, the undersigned, declare that the dissertation, which I hereby submit for the degree Magister Educationis at the University of Pretoria, is my own work and has not previously been submitted by me for a degree at this or any other tertiary institution.

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ABSTRACT

The University of Pretoria has experienced a significant increase in student numbers in recent years. This increase has necessarily impacted on the Department of Mathematics and Applied Mathematics. The department is understaffed in terms of lecturing staff, which impacts negatively on postgraduate study and research outputs. The disproportion between teaching staff and the lecturing load and research demands has led to an excessive grading and administrative load on staff.

The department decided to use multiple choice questions in assessments that could be graded by means of computer software. The responses of the multiple choice questions are captured on optical reader forms that are processed centrally. Multiple choice questions are combined with constructed response questions (written questions) in semester tests and end-of-term examinations.

The quality of the multiple choice questions has never before been determined. This research project asks the research question: *How do the multiple choice questions in mathematics, as posed to first-year engineering students at the University of Pretoria, comply with the principles of good assessment for determining quality?*

A quantitative secondary analysis is performed on data that was sourced from the first-year engineering calculus module WTW 158 for the years 2015, 2016 and 2017. The study shows that, in most cases, the questions are commendable with well-balanced indices of discrimination and difficulty including well-chosen functional distractors. The item analysis included determining the cognitive level of each multiple choice question. The problematic questions are highlighted and possible recommendations are made to improve or revise such questions for future usage.

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CHAPTER 1 INTRODUCTION

1.1 Introduction and background

The purpose of this study is to conduct an investigation into the quality of provided response questions (PRQs), of which a subset is multiple choice questions (MCQs), in first-year engineering mathematics as practised at the Department of Mathematics and Applied Mathematics at the University of Pretoria. The research aims to contribute to the existing body of knowledge on assessment.

The South African Qualifications Authority (SAQA) views assessment as “a structured process for gathering evidence and making judgments about an individual’s performance in relation to registered national standards and qualifications” (SAQA, 2001). For an assessment to be credible, some practices and procedures need to be in place. According to SAQA (2001), the credibility of an assessment must be governed by four principles. Firstly, the assessment must be fair. Unfairness would implicate bias, inequality of opportunities and comparison of students’ work with respect to race, gender and values. Secondly, an assessment needs to be valid. Validity refers to the measuring of knowledge, skills, content and information as the assessment was intended to do. Thirdly, an assessment needs to be reliable. Reliability refers to consistency in testing. Lastly, an assessment needs to be practicable, which implies taking into account available facilities, equipment and resources (SAQA, 2001; Canonigo, 2012).

According to Gareis and Grant (2015), assessment is the process of using different tools and techniques to collect information regarding students’ learning. It provides us with evidence on how successful the teaching and learning was. Assessments can be formal, informal, formative and summative. Informal assessments can also be seen as formative since a student’s body language, facial expressions or comments can give an unplanned, immediate feedback regarding the teaching and learning. Formative assessment (also known as *assessment for learning*) takes place during the process of learning and teaching (Bennett, 2011). Summative assessments are the written assessments students do at the end of a cycle or at the end of a semester. Summative assessment is used for making a judgement about a student’s achievement at the end of a learning programme.

According to Brown, Bull and Pendlebury (1997), assessment contributes to institutional management. Summative as well as formative assessment can be used for quality assurance of an educational system. Assessment is used to provide judgement on the educational system in order to evaluate the effectiveness of the learning environment as well as to monitor the quality of an education institution over a period of time. It also provides feedback to staff on the effectiveness of their teaching and assesses the extent to which the learning outcomes of a programme have been achieved (Brown, Bull & Pendlebury, 1997; Canonigo, 2012; Gareis & Grant, 2015).

Different assessment instruments can be utilised by an assessor to assess a student. Amongst others, assessing can be done by observations, questioning, practicals, orals and written assessments. Each brings their own value to the validity and reliability of an assessment (Gareis & Grant, 2015).

The quality of teaching and learning of mathematics can further be enhanced by other forms of assessment. Students can interact in the lecture room when using clickers as a form of formative assessment. Clickers are better known as CRS (classroom response systems). Multiple choice questions can be answered in the lecture room by using clickers; each student submits an answer to the question that the lecturer poses to them. The results, which can be viewed immediately afterwards, provide a direct indication to the lecturer of the extent to which learning took place (Bruff, 2017).

The written forms of assessment questions can be divided into two basic forms: PRQs and CRQs. PRQs form an important part of these assessments and make up about 30 to 40% of the total mark. Constructed response questions (CRQs) are paper-based questions requiring students to write full steps on paper. A third form of assessment was introduced recently at the Department of Mathematics and Applied Mathematics: students have to answer short questions and no written steps are required.

At the Department of Mathematics and Applied Mathematics, where this study was conducted, different forms of systematic monitoring and evaluation are administered, as in many other universities. Students are assessed in many different forms, including semester tests (term tests) and final examinations. For this research the term *assessments* will be replaced by semester tests and final examinations since all the written assessments were either a semester test or an end-of-term examination.

1.2 Statement of the problem and rationale

Barrington (2004) draws attention to the worldwide increase in the number of students entering the higher education system, and in particular the concomitant increase in the diversity of the student population. Teaching and learning in the Western higher educational institutions give privilege to certain ways of thinking and do not always allow for socio-cultural differences. The diversity also varies between countries and their respective institutions. We find diversities in race, class, ethnicity, gender and academic preparations. Luckett and Sutherland (2000) alert to the worldwide globalisation and massification of education, and higher education becoming more flexible, open and responsive to students' needs.

The tendency of increased student numbers worldwide is also cognisable in South Africa. The University of Pretoria has experienced a significant increase in student numbers in recent years, which has necessarily impacted on the Department of Mathematics and Applied Mathematics. The Department is understaffed in terms of lecturing staff, which could potentially impact negatively on research outputs. In 2012 the student to staff ratio was 24.68 in in 2016 it increased to 40.88. The academic staff has not increased proportionally to the lecturing load and research demands and this disproportion has led to an excessive grading and administrative load on staff.

In South Africa, the wider accessibility of higher education follows in reaction to the historical inequalities of the apartheid era. Only a certain number of black students could enrol at traditional "white" universities (Makoni, 2000). With the new democracy that came into being in 1994, economic and political changes took place and resulted in rethinking the educational philosophies that underlay the higher education system.

Engelbrecht and Harding (2002, 2006) observed an increase in the number of first-year students enrolled in mathematics in South African universities over the preceding decade. Since mathematics is regarded as a prerequisite to many different courses such as engineering and actuarial science, an increase in students taking mathematics modules is inevitable.

The increase in student numbers leads to certain challenges faced by academic staff. First of all it leads to an increase in class size and their teaching load, which in turn leads to an increase in their marking load. The increase in class size could be overcome by using digital technology in the lecture hall, but the increase in marking load is problematic, especially if the university is understaffed. The essential act of continuous assessment with large undergraduate mathematics classes became increasingly difficult because of the huge marking load. Alternative assessment forms that would reduce the marking load had to be explored.

Table 1.1 provides an indication of how student numbers have increased between 2011 and 2016 for the undergraduate engineering calculus module (WTW 158), which was under investigation in this study.

Table 1.1. Student numbers at the Department of Mathematics and Applied Mathematics for WTW 158

YEAR	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**
WTW 158	1287	1255	1398	1576	1652	1575

A decision was taken at the department a number of years ago to make use of MCQs in tests and examinations, which could be graded by means of computer software. The responses of the MCQs are captured on optical reader forms that are processed centrally. With MCQs the marking load is decreased significantly and the grading process can be done in less time, allowing for timeous feedback to students (Engelbrecht & Harding, 2006).

However, the use of MCQs is criticised by a small minority of the department's staff since these questions are not always on the same cognitive level as the paper-based constructed response questions (CRQs). Gareis and Grant (2015) state that addressing appropriate cognitive levels is just as important as addressing appropriate content. A compromise will jeopardise the validity of the test. Studies have shown that MCQs are indeed well suited for assessing lower- and higher-level thinking; however, is it hard to set such questions (Gareis & Grant, 2015).

Although the practice of using multiple choice questions for tests has become common at the Department of Mathematics and Applied Mathematics at the University of Pretoria, this practice has been subjected to criticism. Such criticism stems mainly from the fact that the MCQs are home grown, i.e. designed by staff members who, in most cases, have had little formal training in setting such items. Yet the staff members have, by setting MCQs, gained experience to a certain level of expertise through the years. Although different staff members set MCQs with different levels in mind when designing the items, for the sake of this study the focus will be on one module with the same level in mind.

It became imperative to conduct an investigation into the quality of MCQs posed to first-year engineering students. If we ask what makes a good quality or bad quality question, we need to consider different parameters to see how each contributes to a question being good or poor. Three different parameters were identified to investigate the quality of the MCQs under investigation. The quality of MCQs is unpacked in chapter 2 of the dissertation.

The research question of how student performance in MCQs compare to that in CRQs (paper-based questions) was addressed in an honours project in 2015 (Brits, 2015). The study showed that, in most cases, performance in the MCQs exceeded performance in the CRQs. As a result of this phenomenon, students' marks are positively influenced by the multiple choice questions in semester tests and examinations.

Consequently the question to consider now is why do students perform better in MCQs than in CRQs? The quality and cognitive level of MCQs can influence students' performance. According to Hingorjo and Jaleel (2012), if MCQs are properly constructed, it can assess higher-order cognitive levels as depicted in Bloom's taxonomy (Bloom, 1956). The questions can go beyond just the recalling of facts. Questions can move between interpretations, syntheses and applications.

It becomes clear that individual items from the MCQ section need to be investigated. A proper item analysis of the test items has to be performed to provide information regarding the quality of a particular MCQ. The item analysis in the current study will include three parameters, namely the difficulty and discrimination indices and the distractor efficiency. The outcome of the study was to provide feedback to the designers of the MCQs regarding which items are exemplary and what the failings are of other test items by providing some assessment guidelines. Besides the three parameters mentioned, the cognitive level of each MCQ was determined. The literature review discusses the parameters and different assessment components.

1.3 Significance of the study

The conventional method of open-ended CRQs, which was the predominant method of assessment, resulted in large marking loads. The increase in student numbers necessitated the introduction of alternative approaches in grading. The use of MCQs was considered. The quality of MCQs was investigated to assist educators and assessors in improving on their assessment practices and to enhance student learning in mathematics.

It became imperative to conduct a study on the quality of MCQs because the test designers need feedback to provide them with useful information when setting MCQs. It is important that MCQs test knowledge in skills on different cognitive levels. The results from this study will provide valuable information when MCQs have to be set for semester tests and examinations.

1.4 Context of the study

In this study, the quality of MCQs was determined by classifying the questions in terms of their cognitive demand and, secondly, by utilising three measuring criteria.

This research study took place at the University of Pretoria, specifically at the Department of Mathematics and Applied Mathematics. The MCQs under investigation were taken from the assessments for WTW 158, the calculus module for first-year engineering students.

The University of Pretoria

The University of Pretoria (UP) is one of several universities in South Africa and is situated in Pretoria, the capital, in the province of Gauteng. Although research projects are an essential part of the university's conduct teaching and learning is equally important. There are seven campuses, namely Hatfield, Mamelodi, Groenkloof, Prinshof (Faculty of Health Science), Onderstepoort (Faculty of Veterinary Science), Gordon Institute of Business Science (GIBS) and L C de Villiers Sports Campus. All campuses are situated in and around Pretoria, except GIBS, which is in Melville, Johannesburg. The university is situated in close proximity to the National Research Foundation (NRF) and the Council for Science and Industrial Research (CSIR) (University of Pretoria, 2015).

Research is of utmost importance in the university's long-term strategic plan, UP 2025. The aim of the strategic plan is to make the University of Pretoria a leading research-intensive university in Africa. The university also aims to be internationally visible and strives to have a socio-economic impact on post-apartheid, developing South Africa. The strength of the University of Pretoria is underscored by a wide range of research projects and postgraduate education (University of Pretoria, 2015).

The Department of Mathematics and Applied Mathematics

The Mathematics Department is one of nineteen departments in the Faculty of Natural and Agricultural Sciences (NAS). Not only is it one of the largest departments on campus, but also one of the largest mathematics departments in South Africa comprising more than 20 000 students enrolled in mathematics modules. It has around 60 members of staff, which include several temporary and contractual appointees. The department prides itself in excelling in both research and community-based activities such as mathematics competitions and Olympiads.

Several courses are presented at the Department of Mathematics and Applied Mathematics. For this study the MCQs, as employed in the engineering calculus module WTW 158, will be investigated and described. The engineering calculus module is taken by students enrolled in a degree in engineering.

Engineering calculus module WTW 158

Students taking the engineering calculus module need a National Senior Certificate (NSC) with an admission point score (APS) of $\geq$ 42 with mathematics on Level 7 (between 80% and 100%).

The prescribed textbook is the latest edition of *Calculus: Early Transcendentals* by James Stewart. This textbook is used worldwide and is considered informative and easy to use with multiple representations of mathematical objects presented as real-life applications of many mathematical concepts. The textbook is useful to make the content applicable to the engineering field.

Students attend four lecture periods per week of 50 minutes each, as well as one tutorial period of 180 minutes during which recommended problems must be solved and students get time to consult with lecturers, assistant lecturers and tutors. Because the lecture classes are big – more than 300 students at a time – students divide into smaller groups of 50 for tutorial periods, since these periods provide more attention to individual students. Apart from the two semester tests during the course of the semester, the students also take tests on ClickUp (the university's online communication system) and on WebAssign (students' engagement on completing assignments and smaller tests) to enhance continuous assessment. Students are advised to spend at least 14 hours per week on this calculus module. At the end of the semester a final three-hour examination is written for which examinees need to obtain at least 50% to pass the module.

In all semester tests and examinations there is a Section A, consisting of MCQs, and a Section B, consisting of written questions. As of 2017 a third section is being introduced with short, one-step solutions that should be written on paper. Section A usually makes up about 25 to 35% of the marking total and Section B usually makes up about 60 to 70% of the paper. The new Section C with short questions is proportionally the smallest of the paper but the grading is effortless and helps the total grading process.

The engineering calculus module covers the following content:

- Functions (14 periods)
- Limits and continuity (9 periods)
- Differentiation (7 periods)
- Applications of differentiation (9 periods)
- Integration (6 periods)
- Vector algebra (5 periods)

1.5 Research questions

The main research question for this study was formulated as follows:

1.5.1 Primary research question

- How do the multiple choice questions (MCQs) in mathematics, as posed to first-year engineering students at the University of Pretoria, comply with the principles of good assessment for determining quality?

In order to answer the primary research question the following secondary questions were formulated:

1.5.2 Secondary research questions

- Which levels of cognitive demand are addressed in the multiple choice questions of the sample under investigation?
- How do the multiple choice questions measure with respect to the discrimination index, difficulty index and distractor efficiency?

1.6 Research methodology

Quantitative research was conducted and secondary analysis was executed on existing data. All data was sourced from the Department of Mathematics and Applied Mathematics. The marks that engineering students taking the WTW 158 calculus module obtained in Section A of the tests were studied and analysed. Section A is the multiple choice questions section of a test. The MCQs in Semester test 1, Semester test 2 and the exam of the three consecutive years 2015, 2016 and 2017 were investigated by performing an item analysis.

The quality of the MCQs under investigation was determined by examining their cognitive level and by determining how the questions perform in terms of three parameters: the difficulty index, the discrimination index and the distractor efficiency. The information gathered from this research project served as feedback to the designers of the MCQs at the department in terms of which questions were exemplary and which had deficiencies. Item analysis was performed on each of the MCQs and basic statistical analysis was performed to determine how well the MCQs discriminate, what the difficulty levels are, how many distractors are non-efficient (non-functional) or functional, and what cognitive levels are met with each item.

1.7 Outline of the study

Chapter 1 gives an overview of what can be expected in subsequent chapters. Firstly the introduction and background describe the scene from where the research is founded. The research focuses on the changing tendencies in teaching higher education, especially in South Africa. The use of MCQs is introduced to help the grading process but these questions must be tested for their quality. The problem statement and rationale explains why the research is imperative and why the data will be significant and valuable for the Department of Mathematics and Applied Mathematics. The primary research question is given along with the secondary research questions.

In Chapter 2 the literature review gives information on different assessment models for higher education (Biggs, 1999) as well as the development of different mathematics taxonomies (Bloom, Englehart, Furst, Hill & Krathwohl, 1956; Smith, Wood, Coupland, Stephenson, Crawford & Ball, 1996) to test the cognitive levels in different MCQs. Different mathematics assessment components (Niss, 1993) are reviewed. MCQs are discussed with regard to their advantages and qualities. The chapter ends with the conceptual framework, which describes the assessment framework that will be used for the purposes of the current study. It is developed to form a way of measuring the qualities of a *good mathematics question*. The conceptual framework forms the basis against which the primary and secondary research questions are measured.

Chapter 3 presents the quantitative investigation, which forms part of the research design and methodology. The epistemological assumptions are discussed from which the research design was made. The methodology of a secondary analysis is discussed and explained as the preferred method for this research study. Additionally, data collection and documentation are discussed. Attention is given to research ethics, the reliability of the results and the face validity of the tests.

Chapter 4 presents the research findings and discussion of results. A description of the data follows, which specifies the tests written, the number of MCQs per test and number of students per test. A component analysis within the different assessment components is presented. Scatter and radar plots summarise the quality parameters of each item. The quality of the MCQs is discussed by presenting the good quality and poor quality items.

In Chapter 5 conclusions are drawn regarding the mathematics assessment components and how it contributes to the cognitive levels of different MCQs. The research questions are addressed and the limitations of the research are discussed. The chapter ends by pointing out implications for further study.

CHAPTER 2 LITERATURE REVIEW AND CONCEPTUAL FRAMEWORK

2.1 Introduction

This study proposes to investigate the quality of multiple choice questions (MCQs) as it is utilised in first-year engineering mathematics at the Department of Mathematics and Applied Mathematics of the University of Pretoria. In Section 2.2, the literature review pays attention to the worldwide paradigm shift of the tertiary assessment model over the past decades. Section 2.3 looks into assessing and learning and their different forms for improving educational strategies. Section 2.4 focuses on generally accepted principles of good assessment. Section 2.5 discusses the levels of cognitive demand as introduced initially by Bloom et al. (1956) and later developed into a mathematics-related taxonomy. In Section 2.6, the mathematics assessment components, as a tool to determine the cognitive level of an MCQ, are introduced. Examples are given of questions that can be found in each mathematics assessment component. Section 2.7 discusses the quality of an MCQ and how it can be measured. The literature review in Sections 2.8 and 2.9 elaborates on MCQs as a subset of PRQs. Section 2.10 discusses the advantages when using MCQs in tests. In Section 2.11, the assessment framework is discussed as a base from which to specify the parameters that were utilised to determine the quality of the MCQs under investigation. Finally, Section 2.12 discusses the optimum number of options that can be given in an MCQ without reducing the quality of the question.

2.2 A paradigm shift of the university assessment model

2.2.1 The paradigm shift in higher education

Within the last 20 years, the academic paradigm in higher education has shifted from a teaching focus to an emphasis on demonstrated learning (Garceau, Pointer & Tarnoff, 2015). Garceau et al. (2015) add that it was initially assumed that if a student was exposed to adequate infrastructure and a traditional curriculum in an able faculty, learning would be natural. Additionally, the teaching paradigm was to be accredited and supported by different accreditation bodies at all levels, such as national and regional. The paradigm shift occurred when the learning paradigm changed from instructor-centric to learner-centric, where a new emphasis was placed on assessment activities to demonstrate the learning that took place (Garceau et al., 2015). This paradigm shift of the preceding decade, where the focus on teaching was slowly making a shift towards learning, was also mentioned by Rust (2002) and Leung (1988).

Universities in the United Kingdom took the lead in the alignment between teaching, learning and assessment. Briggs, Alonzo, Schwab and Wilson (2010:11) state that "the fundamental principle of constructive alignment is that a good teaching system aligns teaching method and assessment to the learning activities stated in the objectives so that all aspects of this system are in accord in supporting appropriate student learning". Rust (2002) reports on the production of new learning and teaching strategies regarding the development of general, transferable skills as well as the creating of course units in terms of student learning outcomes.

Altmann and Ebersberger (2013) summarise the changing university paradigm as the transition from certain and simple (pure knowledge and research-based paradigm) to complex and uncertain (highly leveraged, entrepreneurial and innovation-driven) through massification and globalisation. Universities move from national and regional interests to international, interdisciplinary, networked and extensively partnered interests.

2.2.2 History of the South African schooling system

South Africa had its own share of challenges in the schooling system during the past few decades. The era since the 1980s was marked by economic, social and political changes. There was a massive resistance inland and abroad against the apartheid ideology of the white minority government. During the apartheid era, before 1994, South Africa was divided into four racial groups: white, black, Indian and coloured. The ideology was that the four groups had to live and develop independently and have separate educational systems and schools. For white learners there was the House of Assembly, Indian learners were under the control of the House of Delegates, and coloured learners were in schools represented by the House of Representatives. Black learners had no system of their own and were under the control of the Department of Education and Training (DET) (Fiske & Ladd, 2004).

Of the four groups (white, black, Indian and coloured), the black (African) students were the most educationally disadvantaged; the pupil–teacher ratios were bad, the teachers were the least qualified and the matriculation pass rates were the worst compared to the other three groups. During the late 1980s, about 24% of African learners of the DET received education in farm schools in rural areas with little or no infrastructure (Hofmeyr & Buckland, 1992).

After the end of the apartheid era and the first democratic elections in South Africa in 1994, a new government and dispensation brought expectations of a new era where all citizens can have equal opportunities. The new, democratically elected government sought to establish a single, national department of education. The aim of the National Department of Education (DoE) was to support, develop and maintain an effective education and training system. The main goal was for everyone to be able to exercise their right to basic education, which includes adult basic education and further education (DoE, 2001).

Some huge challenges the government faced initially included the problem of equal access to schools, unequal education opportunities, inadequate finances, materials and resources, and a shortage of qualified teachers (Botha, 2002). Although the government attempted to improve on all the shortcomings, especially at the previously disadvantaged schools, the situation did not improve much as initially expected. According to the Education For All Global Monitoring Report (2015), students did not develop the necessary skills of problem solving and critical reasoning.

A governmental body, South African Qualifications Authority (SAQA), was conceptualised in 1995 to be responsible for the development and management of the National Qualifications Framework (NQF). The board comprised members appointed by the Ministers of Education and Labour. The task of the members was to oversee the development of the NQF and to formulate and publish policies and criteria for education and training standards and qualifications. In addition, SAQA had to accredit bodies responsible for the monitoring and delivery of standards in terms of such standards and qualifications (Human Sciences Research Council, 2012).

The National Qualifications Framework (NQF) was intended to serve fundamentally to restructure the education and training system. The NQF helped to organise learners in order to facilitate a developmental progression for learners, regardless of the engaged field of learning. Learning standards could be upgraded, qualifications would be evaluated and new curricula would be encouraged. The levels of learning form the basis for progression and serve as a mechanism for achieving a system of education and training (Human Sciences Research Council, 2012). Table 2.1 summarises the NQF levels and qualifications.

Table 2.1. The National Qualifications Framework

NQF level	**Band**	**Types of Qualifications or Certificates**
1	General Education and Training (GET)	• Grade 3 (ABET Level 1) • Grade 5 (ABET Level 2) • Grade 7 (ABET Level 4) • Grade 9 (ABET Level 4)
2 3 4	Further Education and Training (FET)	• Grade 10, 11 and 12 • School, college, trade certificates
5 6 7 8	Higher Education and Training (HE)	• Diplomas and occupational certificates • First degrees and Higher Diplomas • Higher degrees and professional diplomas • Doctorates and further research degrees

Note. ABET = Adult Basic Education and Training

In March 1997, the launching of Curriculum 2005 was announced by the then Minister of Education, Sibusiso Bengu. Curriculum 2005 was introduced to address the crisis of imbalances and changing demands of society. Outcomes-based education (OBE) was introduced with a main shift from a teacher-centred to a learner-centred approach (Botha, 2002). The supporters of OBE suggested that learners would be equipped with competencies and knowledge needed to successfully leave school. OBE is characterised as a learner-centred approach where the emphasis is not on what the teacher wants to achieve but rather on what the learner should know, understand, demonstrate and become (Botha, 2002).

According to the Curriculum 2005 review committee with Prof Linda Chisholm as chair, steps were to be taken in respect to the implementation of the new curriculum. The implementation started in 2001 for grades 4 to 8. By 2005 the curriculum would have been fully implemented and known as Outcomes-based education (Chisholm et al, 2000)

One of the proposals of the new OBE curriculum was that all learners had to take mathematics. The previous higher grade and standard grade mathematics was changed to pure mathematics and mathematical literacy. The latter was supposed to equip learners with basic mathematical skills for life in general (Chisholm et al, 2000).

The South African Qualifications Authority (SAQA), who was responsible for the development and management of the National Qualifications Framework (NFQ), elaborated on the paradigm changes in the assessment models as proposed by OBE. According to SAQA (2001), learning was no longer something that a learner would do, but rather something in which a learner would be actively involved – for example, by identifying and solving problems through critical thinking, effectively working as a team using critical and creative thinking, and being able to organise and manage themselves while busy with learning activities. Students had to be able to collect, organise, analyse and evaluate gathered information and be able to communicate effectively. Students also had to be aesthetically and culturally sensitive across the diverse range of socio-economical contexts they might have found themselves in (SAQA, 2001).

The role of the assessor also changed from controlling the learning process, to being a supportive guide so that a learner can gain access to further learning. It remained important that NQF standards and qualifications were met. Assessors needed to plan and prepare assessments, prepare students for assessments and conduct and record assessments. Moreover, assessors needed to gather, evaluate and judge evidence and provide sufficient feedback (SAQA, 2001).

A few years after 1997, it became clear that OBE was not successful and that universities suffered the consequences of learners not being ready for tertiary studies (Jansen, 1998; Engelbrecht, Harding & Phiri, 2009). Jansen (1998) advocated some major reasons why OBE could have a negative impact on South African schools, the first being the language of innovation associated with OBE. The language was confusing and complex. One needed to come to terms with more than 50 different concepts such as *not yet competent* and *the candidate does not meet the learning outcomes*. To understand "outcomes" one needed to understand learning programmes, assessment criteria, levels, phases, curriculum frameworks, and their relationship to SAQA, the NQF, and the Education and Training Qualification Agencies (ETQA) to name but a few.

According to Jansen (1998), a second criticism of OBE was that there was no evidence yet that a change in curriculum will lead to changes in national economies of a country. South Africa, as a developing country, cannot be compared to developed, first world countries such as North America and Western European countries. Resources, infrastructure and state funding are not sufficient.

A third principal criticism of OBE according to Jansen (1998) was flawed assumptions regarding the level of educated teachers in schools of rural areas. To make a success of OBE you needed qualified and experienced teachers. The majority of teachers neither had access to information regarding OBE, nor understood OBE as a curriculum policy.

A fourth objection to OBE was the multiplication of administrative burdens placed on teachers. Additionally, a school needed to have enough resources and sufficient resources to be able to implement the curriculum. For OBE to be successful a radical revision of the system of assessment had to be implemented (Jansen, 1998).

The many challenges experienced in the school educational system of South Africa before and after 1994 flowed over to the higher educational system. In an article presented by HESA (Higher Education in South Africa), MacGregor (2014) reports on higher education in the 20th year of democracy. The achievements and challenges are scrutinised as they are experienced by vice-chancellors of several universities in South Africa. The vice-chancellors identified six key issues they considered critical to the future health of higher education and the economic and social development of the democracy. The six key issues are:

1. Student access, success and opportunity
2. Research and postgraduate education
3. Epistemological transformation
4. The next generations of academics
5. The landscape of the higher education institutions
6. The funding challenge

Table 2.2 gives a summary of some of the findings of the HESA report regarding the gross participation rate in higher education (HE) when taking into consideration that 17% of the population entered the higher education sector in 1990.

Table 2.2. Gross participation rate in higher education

YEAR	African	Coloured	Indian	White
1990	9%	13%	40%	70%
2013	70%	6%	6%	18%

Although black (African), Indian and coloured people make up 89% of the population, they make up only about 52% of a total of 473 000 students, according to the 1993 HESA report. Several mechanisms have since been put into place to address the imbalances of participation rates: all forms of racial and gender discrimination have been abandoned and extended curriculum programmes for poor students, affirmative action and state-funded National Student Financial Aid Schemes have been implemented (MacGregor, 2014).

Improvements were marginal, though. Table 2.3 gives the percentage representation of the population versus percentage enrolments in higher education.

Table 2.3. Percentage representation versus percentage higher education enrolment

2013	**African**	**White**
Percentage of the population	83%	6%
HE enrolments	16%	54%

After the start of democracy the "drop-out" rates made it clear that equity of opportunity as well as substantial improvements remained to be achieved for African students. Table 2.4 gives the percentage success rate, as opposed to the "drop-out" rate for undergraduate levels in 2010 (MacGregor, 2014).

Table 2.4. Percentage success rate

YEAR	**African**	**White**
2010	71%	82%

Internationally, the graduation rate for a three-year degree programme is 25% (MacGregor, 2014).

Table 2.5 gives the percentage graduation rate in 2010.

Table 2.5. Percentage graduation rate

YEAR	**African**	**White**
2010	16%	22%

The ever-growing participation rates in HE is expected to increase from 950 000 heads in 2012 to a staggering 1.6 million in 2030. These numbers will have enormous implications for South Africa's resources, development, social inclusion and equity (MacGregor, 2014).

Students that enrolled at the Department of Mathematics and Applied Mathematics at the University of Pretoria over the last decade experienced radical changes because of the OBE curriculum. The changes negatively influenced their mathematical preparedness (Engelbrecht et al., 2009).

Engelbrecht et al. (2009) raised concerns that students enrolled in mathematics at the University of Pretoria do not meet the expectations met by their predecessors before OBE was introduced. The 2009 intake of mathematics students was the first group who had followed the OBE curriculum from grade 1 to grade 12. The previous years of students had a partial exposure to the OBE curriculum.

Keeton (2009, as cited in Engelbrecht, Harding & Phiri, 2010) points out that the 2009 intake of mathematics students performed better in their matriculation mathematics examination of 2009 than learners did the year before. More than 47% of learners who wrote the National Senior Certificate (NSC) Mathematics exam papers passed the subject in 2009, compared to 43% in 2008. In addition, about 63 000 learners obtained marks above 50% in 2009 compared to about 25 000 that obtained marks above 50% in 2008 (Engelbrecht, Harding & Phiri, 2010).

The Department of Education appointed a panel of experts to evaluate the 2008 NSC Mathematics papers. The Ministerial Panel (2009) reported that the learners who scored above 50% in the 2008 NSC Mathematics paper would have passed at 40% in the Higher Grade Mathematics papers before 2008. By further investigation the Ministerial Panel (2009) found that the papers did not provide enough questions at the “knowledge” level of taxonomy.

Engelbrecht et al. (2009) advocated further research to analyse the problem in order to make the transition from school to university smoother. Some of the solutions they proposed were school and university curriculum changes, placing more students in extended programmes at universities, cooperation between secondary and higher education authorities, creating more support programmes at universities and increasing the admission rates for university mathematics. An increased admission rate will help limit student numbers.

Table 1.1 gives a summary of the increase in student numbers for the WTW 158 engineering calculus module for the years 2011 to 2016. The situation has worsened for universities ever since and created bigger challenges for students and lecturers. Changes in the assessment strategies had to be implemented to support lecturers with their heavy loads of grading and administrative tasks (Engelbrecht & Harding, 2003; Barrington, 2004).

2.3 Assessment and learning

Assessment is the process of obtaining information that can be used to make educational decisions and give feedback to students regarding their progress, weaknesses and strengths (Braun & Kanjee, 2006; Gareis & Grant, 2015).

Testing, according to Braun and Kanjee (2006), refers to the process of administering a test to measure some concepts under specific, repeatable conditions, to determine how much students have learned.

The United Nations Educational, Scientific and Cultural Organization (UNESCO) (2000) calls for the improvement of educational systems globally and proposes steps on how to achieve this. Measurable learning outcomes have to be achieved especially in literacy, numeracy and general life skills. Developing countries need to monitor their progress towards the goals and strategies set by UNESCO's global movement Education for All (EFA) at all regional, national and international levels. The sustainable development of developing countries, the alleviation of poverty and the restoring of human dignity is only possible when the educational system of a country is successful (UNESCO, 2000).

According to SAQA (2001), there is a close relationship between setting learning outcomes (curriculum planning) and the teaching and assessment methods in a learning programme. The three must always be aligned. The following diagram illustrates the relationship between curriculum planning, assessment, teaching and the learning outcomes. Teaching, assessment and curriculum planning are linked together. The connection between them provides the achievement of learning outcomes.

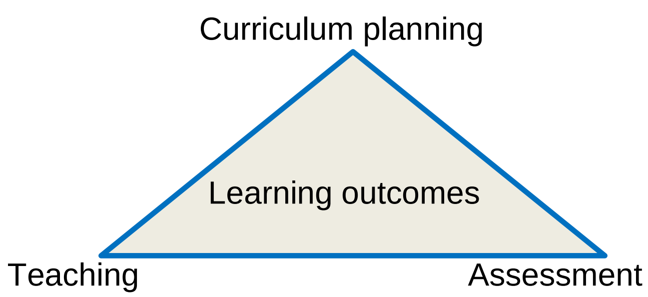


Figure 2.1. The relationship between curriculum planning, assessment, teaching and learning outcomes

Braun and Kanjee (2006) expand on improving education through testing and evaluation. South Africa can be seen as a developing country and, together with other developing countries, shows certain patterns such as substantial inequalities in opportunities for learning and achievements. It can be seen in social class, race, gender and language, amongst others. There are also many impediments to progress such as lack of resources and facilities, especially in rural areas. For these disparities to be addressed assessment policies and practices are important to successfully improve educational strategies. Obtaining assessment data on teaching and learning is necessary to be able to improve, monitor and evaluate educational systems. One recommendation for developing countries such as South Africa is to participate internationally as collaborator in assessments to give access to expertise, to develop, to build capacity and to analyse tests without the political consequence of being exploited because of possible poor performance (Braun & Kanjee, 2006).

According to Cohen and Bloom (2005), every child in the world must be provided with a good basic and secondary education. The expansion of high-quality education is essential to the political, social and economic well-being of developing nations in developing countries. Education for All (EFA) is an initiative that propagates a framework for conceptualising the roles that assessment plays in education.

Braun and Kanjee (2006) delineate the four essential attributes of an educational system. The first attribute of an educational system is access. The concept refers to entry *into* and eventually *through* a formal school system. The second attribute is quality and refers to what happens *in* the school and *in* the classroom, i.e. where and how teaching and learning take place. The third attribute of an educational system is efficiency. Efficiency refers to the optimal use of educational facilities and resources as well the quality of education that takes place. Equity is the fourth attribute of an educational system. All learners should have the same opportunity to learn. Learners are not supposed to be subjected to unfair discrimination or excluded from the educational system because of reasons such as their language, gender, disability, race or socio-economic background.

Gareis and Grant (2015) explain that testing can be formal (such as written tests and examinations) or informal (such as questioning, facial expressions and body language). Testing can happen at different stages of the learning process. Firstly, pre-testing occurs prior to teaching. This happens when the teacher or lecturer asks questions to determine the students' current pre-knowledge regarding the content. Pre-assessments can be done by pre-tests, discussions and interviews. Secondly, formative assessment happens while teaching takes place. Formative assessment can be done by observations, responses, practical tasks and quizzes. Sadler (1998) remarks that formative assessment has the intention to generate feedback on students' performance to accelerate and improve learning. Formative assessment and feedback should be used to empower students to become self-regulated learners (Pintrich & Zusho, 2002). Summative assessment happens after learning has taken place, at the end of a period of teaching. Summative assessments are executed by means of written tests, demonstrations and standardised tests. All different forms of assessment are interconnected and contribute to intended student learning. Students' learning can be improved by giving proper and sufficient feedback. The effective use of assessments can directly be related to students' engagement and learning (McMillan, 2013).

The link between teaching, learning and assessment is illustrated by the following diagram:

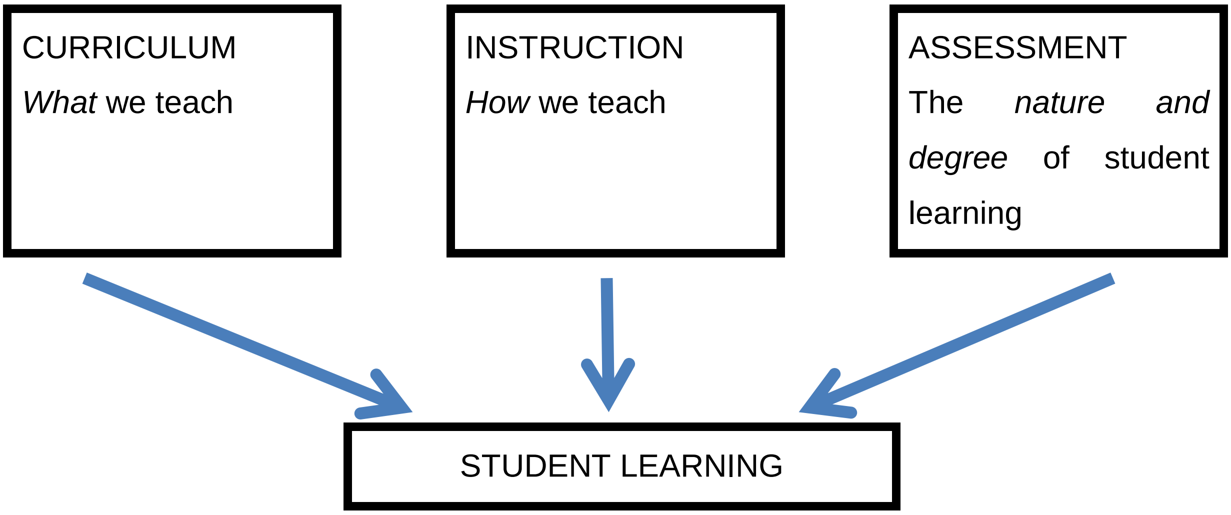


Figure 2.2. The link between teaching, learning and assessment
Adapted from *Teacher made assessments* by C.R. Gareis and L.W. Grant, 2015, New York: Routledge.

Bennett and Gitomer (2009) propagate the design of an alternative system for accountability testing wherein deeper learning can prosper. Lecturers need to think differently about the nature of proficiency and how to assess it when designing a comprehensive system of assessment. The comprehensive system needs to capitalise on new technology to make testing more efficient and effective. Students need to master and develop deeper skills when an alternative assessment system is applied. According to Bennett and Gitomer (2009), this system builds on formative assessment, accountability assessment and professional support.

Different test formats can be utilised and in any combination. Three distinctive formats are: (1) multiple choice questions (MCQs), where different options are given and students must choose the correct option, (2) learner constructed response items (CRQs), where students write full steps on paper, and (3) portfolios or extended work samples (Braun & Kanjee, 2006). The choice of format lies in the objective to be tested.

Briggs et al. (2010) describe the development, analysis and interpretation of a novel assessing item format, which is called ordered multiple choice (OMC). The OMC items are linked to a model of cognitive development for student understanding and interpretation of student item responses. The difference between an OMC and a traditional MCQ is the gathering of formative information about the developmental understanding of students. A diagnostic interpretation can be made of each student's response. For each OMC the student must provide a reason for having chosen a specific option. By analysing these responses the lecturer can understand the deeper level of knowledge that the students have instead of only receiving a right or wrong answer to the MCQ.

2.4 Generally accepted principles of good assessment

According to Gareis and Grant (2015), good assessment is in line with a basic tenet consisting of three important legs, namely the curriculum, the instruction and the assessment.

For creating good assessments one firstly needs to unpack the intended learning outcomes. This includes the explicit, implicit and conditional content. Secondly, one needs to create a table of specifications. The content and level of cognitive demand must be identified. Thirdly, it must be clarified why and when the assessment of the student learning will take place. Fourthly, the appropriate type of test items must be determined. The test can consist of PRQs or CRQs or a combination of each. In the fifth step the appropriate number and weight of test items must be determined. For step six, test items must be created and selected that are valid and reliable. Appropriate questions to ask here are: what cognitive levels are targeted and if the items are free from systematic errors (Gareis & Grant, 2015).

Canonigo (2012) mentions some principles of high-quality assessment. First of all is the clarity of the learning targets. The outcomes of the test need to be feasible and stated clearly. The learning targets include skills, reasoning, knowledge and products that can be observed through concrete evidence of students' behaviour. These abilities can be cognitive, affective and/or psychometric. Another principle of high-quality assessment is the appropriateness of the applied assessment methods. These assessment methods can be written tests, essays, checklists, oral questioning or observations. Additionally, validity, reliability and fairness contribute to good assessment principles. If a test is valid, it measures what it is supposed to measure. The reliability of a test refers to the consistency of the test. If the results can be trusted and the same results would be obtained if the test is repeated by a different person, the test can be regarded as reliable. A test can be reliable without being valid, meaning a test can give consistent results, but if it does not measure what it is supposed to measure, it is not valid.

Assessment can be enhanced when avoiding bias in assessment tasks. An assessment must provide an opportunity to learn and there must be no prerequisite knowledge and skills needed to perform the assessment. Moreover, an assessment must lead to positive consequences such as effective feedback and motivation to improve. The positive consequences include other role players such as parents and teachers. Another principle of quality assessment is the practicality and efficiency of the assessment. Tests can be made more practical by making it more objective. Lastly, to produce tests of high quality some ethics must be taken into consideration. Informed consent from the students is needed and the anonymity and confidentiality must be ensured when gathering, recording and reporting data (Canonigo, 2012).

Another principle of good assessment is validity. A test can be regarded as construct valid if it tests what it is supposed to test. This can include skills, content, knowledge or behaviour. The procedures of the test, instruments and materials have to match for a test to be valid (SAQA, 2001). Validity can also refer to the extent to which the research design is scientifically sound and appropriately conducted (Struwig & Stead, 2013). Different forms of validity exist, such as content validity, construct validity, predictive validity, external validity and face validity. Face validity is the appearance of validity without explicit evidence (Gareis & Grant, 2015). Maree (2014) states that face validity cannot be quantified or tested. Experts should scrutinise the test for a high degree of face validity.

Reliability is another principle of good assessment. Reliability refers to consistency. If the test is repeated under the same conditions at another time, the outcomes or results must be the same. Variables that can affect the reliability are bias (in its many different forms), different assessors, different interpretations and inconsistent grading (SAQA, 2001). According to Black (1998), different threats to reliability exist, such as a student's performance on a specific day, the set of questions in the test, the particular examiner, the grading process by a specific examiner, the checking of collated marks, and analysing the results. Threats to reliability can occur at the data collection stage, during data analysis strategies and through the researcher's role in the study (McMillan & Schumacher, 1993).

Another principle of good assessment is practicability. It refers to the consideration of available facilities, equipment, time and available finances (SAQA, 2001). According to SAQA (2001), principles of good assessment is equivalent to credibility. Credibility is the sum of fairness (lack of bias), validity, reliability and practicability.

2.5 Level of cognitive demands

An examiner has the challenging task of creating multiple choice questions that will address higher- and lower-order cognitive levels. According to Gareis and Grant (2015), developing MCQs successfully at higher cognitive levels can be difficult, but it is not impossible. Braun and Kanjee (2006) state that questions of higher cognitive levels can be utilised in MCQs but the highest cognitive levels such as reasoning, integration and argumentation can only be achieved with CRQs.

A leader in the use of a taxonomy for test construction and standardisation was Ralph W. Tyler who, in 1931, claimed to have found eight major types of objectives. These were: information, reasoning, location of applicable data, characteristic skills of subjects, technical performances, reports, consistency in the application of points of view, and character (Tyler, 1931).

Tyler did not link these objectives to a specific behaviour. He also did not arrange the behaviour in order of complexity. However, in 1949 he specified seven types of behaviour objectives. These were:

1. The understanding of important facts and principles
2. Being familiar with dependable sources of information
3. The ability to interpret data
4. The ability to apply principles
5. The ability to study and report results of a study
6. Broad and mature interests
7. Social attitudes.

A follow up on this was done in 1956 by Benjamin Bloom, who organised the educational objectives into a taxonomy, which he dedicated to Tyler. The taxonomy would attempt to assist teachers by fitting all school subjects. In Bloom's *Taxonomy of Educational Objectives*, objectives were separated by *domain* (cognitive, affective and psychomotor), related to *educational behaviours*, and arranged in hierarchical order from simple to complex:

The diagram below shows the educational objectives from simple to complex.

Simple educational objectives

Level 1: Knowledge

Level 2: Comprehension

Level 3: Application

Level 4: Analysis

Level 5: Synthesis

Level 6: Evaluation

Complex educational objectives

Figure 2.3. Bloom's educational objectives

Bloom's learning outcomes are summarised in Table 2.6 in which each learning outcome is linked to terminology used for measuring that specific outcome in a test:

Table 2.6. Learning outcomes and the terms used to measure them

Learning outcome	**Terms for measuring the outcome in a test**
Knowledge	Name, list, define, describe, state, match
Comprehension	Explain, extend, predict, summarise, defend
Application	Compute, demonstrate, predict, solve, prepare
Analysis	Differentiate, distinguish, identify, relate, select
Synthesis	Combine, compile, generate, modify, revise
Evaluation	Criticise, describe, justify, interpret, relate

Note. Adapted from *Taxonomy of Educational Objectives* by B.S. Bloom et al., 1956, New York: Longmans, Green.

The cognitive categories range from simple to complex. The simple behaviours become more and more integrated to form the more complex behaviours. Each successive learning outcome goes to a deeper understanding of the content and proves deeper learning. To avoid surface learning, lecturing and assessing should deploy different cognitive levels by using different teaching and assessment methods. Although Bloom's taxonomy is good for structuring assessment tasks, Freeman and Lewis (1998) suggest that Bloom's taxonomy is not very helpful in identifying which levels of learning are involved.

Anderson and Krathwohl's (2001) revised version of Bloom's taxonomy is a useful tool for creating assessments of different cognitive levels. Bloom's revised taxonomy consists of six cognitive levels: remember, understand, apply, analyse, evaluate and create. *Remembering* focuses on recalling facts and terms from long-term memory. Verbs associated here are: name, list, label and find, amongst others. *Understanding* is the constructing from oral, written and graphic communication. Some verbs associated here are: describe, discuss, explain and identify. To *apply* means using knowledge to demonstrate, to implement and to carry out a procedure. To *analyse* means breaking down a whole into smaller parts to be able to understand the role of each part as well as the relationship to the overall purpose. To *evaluate* implies to make judgements and justify it based on specific criteria. Finally, when *creating* a new form, elements must be put together to form a coherent whole. Here you can be expected to create and to design.

Bloom's taxonomy was modified by Smith et al. (1996) and called the MATH taxonomy. The taxonomy was adapted for mathematics and consists of eight categories that fall into three main groups. According to the MATH taxonomy, Group A requires factual knowledge, comprehension and the use of basic procedures, Group B students must be able to apply knowledge in a new way, and Group C must apply the skills of interpretation, evaluation and justification. The MATH taxonomy is context specific and classifies tasks by the nature of the activity rather than in terms of difficulty as in Bloom's taxonomy (Smith et al., 1996). Table 2.7 presents the three groups of MATH taxonomy:

Table 2.7. The MATH taxonomy

Group A	**Group B**	**Group C**
Factual knowledge	Information transfer	Skills of interpretation
Comprehension	Applications of knowledge in new situations	Implication, conjectures and comparisons
Routine use of procedures		Evaluation

Note. Adapted from "Constructing mathematical examinations to assess a range of knowledge and skills" by Smith et al., 1996, *International Journal of Mathematical Education in Science and Technology*, *27*(1), 65-77.

According to Huntley (2008), evidence from a doctoral study showed that MCQs can be used successfully as an assessment method for mathematics assessment components that require a deeper learning approach. Components involving higher cognitive skills can successfully be applied to MCQs. Although it is more difficult to set good quality questions, especially at higher cognitive levels, the results from Huntley's (2008) study showed that MCQs can add value to the assessments at all levels.

The mathematics assessment components (MACs) that can be successfully utilised in mathematics are: technical, disciplinary, conceptual, logical, modelling, problem solving and consolidation (Huntley, 2008). For this research the different assessment components will be tested along the different MCQs to determine which cognitive skills are being tested currently. According to Huntley (2008), higher levels of cognitive demand can be obtained only with constructed response questions (CRQs) where students write their solutions in full steps on paper. The research will attempt to verify the current status of MCQs as it is utilised in first-year engineering mathematics at the University of Pretoria.

2.6 Conceptual Framework

2.6.1 Mathematics assessment components (MACs)

An assessment taxonomy specifically for mathematics was proposed by Huntley (2008) for a PhD thesis. The taxonomy consists of a set of seven items (assessment components) which are ordered by their cognitive level and the nature of the mathematical tasks associated with each component. The seven levels of the proposed taxonomy are hereafter referred to as the mathematics assessment components. Based on the literature review in Section 2.5 the taxonomy is a combination of Bloom's taxonomy and the MATH taxonomy. The link between the different taxonomies is given in Table 2.9.

A summary of the mathematics assessment components is given in Table 2.8. The table indicates the different cognitive skills that are connected to each assessment component.

Table 2.8. Mathematics assessment component taxonomy and cognitive skills

Mathematics assessment components	Cognitive skills
1. Technical	• Manipulation • Calculation
2. Disciplinary	• Recall (memory, formulae) • Knowledge (facts, theorems)
3. Conceptual	• Algebraic • Verbal • Numerical • Visual (graphical)
4. Logical	• Ordering • Proofs
5. Modelling	Translating words into mathematical symbols
6. Problem solving	Identifying and applying a mathematical method to arrive at a solution

7. Consolidation	• Analysis • Synthesis • Evaluation

Questions involving calculations and manipulations can be regarded as *technical*. Technical questions are on the lowest cognitive level. Questions under the *disciplinary* component will rely on recalling of knowledge regarding facts and memory of formulae and theorems. Comprehension skills are needed for the *conceptual* component, which involves verbal, numerical and graphical skills. The use of graphs is a useful concept. For mathematical proofs the *logical* component is utilised. The logical ordering of steps to prove a proof needs higher levels of thinking. For the translation of words into mathematical symbols, *modelling* is applied. Here word sums or story sums related to real-life situations are utilised. To apply and identify mathematical methods to reach a solution, *problem solving* is involved. Higher-order thinking is needed here and a student needs to have a global picture of all the work. The processes of analysing, synthesis and evaluation form the *consolidation* assessment component. Consolidation is the highest cognitive level of the mathematics assessment component.

By using Bloom's taxonomy as a starting point, together with the adapted MATH taxonomy, the seven mathematics assessment components can be classified according to their level of cognitive demand. Table 2.9 indicates how the three different taxonomies link with each other.

Table 2.9. The link between the taxonomies of Bloom, MATH and the seven mathematics assessment components

Mathematics assessment components	**Bloom's taxonomy**	**MATH taxonomy**
1. Technical • Manipulations • Calculations 2. Disciplinary • Recall memory and formulae • Knowledge (facts & theorems)	1. Knowledge	Group A • Lower-order cognitive level • Involves recalling of knowledge, facts, definitions and observations
3. Conceptual • Algebraic, verbal, numerical & graphical 4. Logical • Ordering & proofs	2. Comprehension 3. Application	Group B • Middle-order cognitive level • Involves the application of existing knowledge to a new context
5. Modelling • Translating words into mathematical symbols 6. Problem solving • Identifying and applying a mathematical method to arrive at a solution 7. Consolidation • Analysis, synthesis & evaluation	4. Analysis 5. Synthesis 6. Evaluation	Group C • Higher-order cognitive level • Goes beyond knowledge. Makes use of skills and principles to predict and create

The MATH taxonomy classifies tasks by nature of activity rather than in terms of difficulty.

2.6.2 Examples of questions in the mathematics assessment components

The classification of test items with respect to the seven mathematics assessment components has the goal of determining which cognitive levels are utilised by each MCQ. The rationale behind this was to check the range of questions to include the seven mathematics assessment components from lower-order to higher-order cognitive skills. It is important to mention that CRQs are not included in this research study.

<u>Mathematics Assessment Component 1: Technical</u>

Determine the value of $(\ln e)^3 - \ln(e^3) + \ln\left(\frac{1}{e}\right)$			
(A) -3	(B) -2	(C) -1	(D) 1
(E) 2	(F) 3	(G) None of these	

WTW 158, March 2014, Question 14

In this question students need to apply the exponential laws as well as the properties of the natural logarithm to calculate the value of the expression.

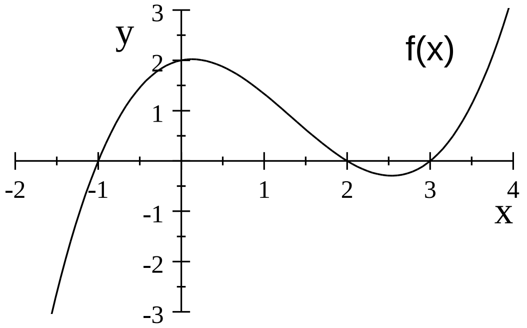
<u>Mathematics Assessment Component 2: Disciplinary</u>

The domain of $f(x) = \sqrt{\frac{1}{1+x}}$ is		
(A) $[-1, \infty)$	(B) $(-\infty, -1) \cup (-1, \infty)$	(C) $(-1, \infty)$
(D) $(-\infty, \infty)$	(E) $(-\infty, -1) \cup (\frac{1}{2}, \infty)$	(F) None of these

WTW 158, May 2014, Question 1

In this question students need to recall the properties of the domain of a root function. Another fact to take into consideration is that the composite function consists of a rational function inside a root function. Lastly all these need to be intersected to determine the final solution.

Mathematics Assessment Component 3: Conceptual



If $g(x) = -\frac{1}{2}f(x) + 1$, the y-intercept of g is

(A) $y = -2$ (B) $y = -1$ (C) $y = 1$

(D) $y = 0$ (E) None of these

WTW 158, June 2014, Question 5

In this question students need to perform different transformations of the given function $f(x)$. First of all there is a vertical shrink by factor 2, then a reflection about the x-axis and lastly the function must shift one unit vertically upwards. It is a graphical, conceptual assessment component since no calculations are needed.

Mathematics Assessment Component 4: Logical

Let g be a function defined for all real numbers. In which of the following cases does g not have an inverse function?

(A) $g(x) = ax + b, a \neq 0$ (B) g is increasing

(C) g is decreasing (D) g is one-to-one

(E) g is an even function (F) None of these

WTW 158, March 2014, Question 6

For this question students need to apply several theorems by checking each option to see if it is applicable to the initial properties of a function with no inverse. The students have to understand the properties of a function with an inverse.

Mathematics Assessment Component 5: Modelling

No test items of the multiple choice section could be classified in terms of Assessment Component 5 since modelling falls outside the scope of the engineering calculus module.

Mathematics Assessment Component 6: Problem solving

No test items in this section could be classified in terms of Assessment Component 6, since identifying and applying a mathematical method to arrive at a solution could be found only in Section B of the engineering calculus tests where students answer in full steps on paper.

Mathematics Assessment Component 7: Consolidation

No test items in this section could be classified in terms of Assessment Component 7, since analysis, synthesis and evaluation could be found only in Section B of the engineering calculus tests.

2.7 Determining the quality of an MCQ

The quality of an MCQ can be determined by performing an item analysis by using different assessment components to see which components are utilised. For this study the indices of discrimination, difficulty and distractor efficiency will be employed, as well as the mathematics assessment components (MACs) as discussed in the literature review. According to Hingorjo and Jaleel (2012), an MCQ can be regarded as a good question if the difficulty index is average, the discrimination index is high and if the MCQ has a few functional distractors (Hingorjo & Jaleel, 2012).

Huntley (2008) developed a model, named the Quality Index (QI), to ascertain the quality of MCQs. The QI model Huntley used is based on the three assessment components of expert opinion, confidence index and the discrimination index. Each of these criteria represents three arms of a triangular-shaped radar plot. All three criteria are also considered equally important in their contribution to the QI for each item. With the QI model the quality of a test item can be represented both quantitatively and visually. In the model the shape and the area are indicated by triangular-shaped radar plots.

According to Huntley (2008), when comparing the radar plots of good quality items to poor quality items, their shapes are different. For a good quality item the ideal shape is a triangle with all three arms equal in magnitude. For a poor quality item the three arms of the triangle will be different. The skewness of the shape also tells us which of the three criteria contribute to the poor quality of the test item.

2.8 Provided response questions (PRQs)

The term “provided response questions” – also referred to as PRQs – was introduced by Engelbrecht and Harding (2003). For PRQs, students must choose the one correct option from a selection of given responses. PRQs is the collective name for different forms of questions. Gareis and Grant (2015) elaborates on the different forms of PRQs: One form is MCQs, which is the form where four or more options are given from which the students must choose the one correct option. Another form of PRQs is “match column A with column B”. True/false questions is a two-option type of PRQ where the answers are provided and the student needs to choose the correct option. For “complete the statement” type of questions the student must complete a statement by choosing the correct word or words from a given set of possible answers. Another type of PRQ, called closed response questions, requires one word or short phrase for a given description. Closed response questions can be used in subjects such as life sciences or geography. Another question type of closed response questions has blank spaces positioned within text that have to be filled in to complete the sentence. This is used, for example, in comprehension tests (Gareis & Grant, 2015).

2.9 Multiple choice questions (MCQs)

To answer MCQs students must choose the one correct option from a selection of given responses. These questions are always found in Section A of a question paper and take up between 30% and 40% of the total marks of a semester test or examination.

MCQs were first invented in 1915. During 1916 a psychologist from Stanford University, Lewis Terman, developed the Stanford-Binet IQ test: a test in multiple choice format for assessing the mental ability (intelligence quotient) of individuals. This model of testing has been used for IQ testing ever since. During World War I military recruits were tested for their capabilities, if needed for special tasks, by means of multiple choice tests. Through the development of the IQ test a new era ensued because the test could be used as a means to make decisions about the education and careers of all people (Black, 1998).

In the late 1960s multiple-choice testing came under attack with Hoffmann’s (1962) publication *The Tyranny of Testing*. Critics were of the opinion that students do not learn by picking the correct answers. Effective learning, according to the critics, does not take place when using MCQs (Gifford & O'Connor, 1992). Critics claimed that multiple choice questions do not improve education at all. The danger was that teachers were under pressure to drill students on *how* to pick right answers (Black, 1998).

Since the 1960s multiple choice questions developed into different types of questions. Fixed response and closed response questions, for example, are two types of multiple choice questions. The true/false type of question is typical of fixed response questions, offering only two options. These questions are successfully used in a variety of subjects. MCQs offer specific alternatives from which a respondent must choose one or more correct answers. They are also referred to as closed response questions (Struwig & Stead, 2013).

Struwig and Stead (2013) describe the principles in constructing MCQs. First one needs to avoid overlapping options or alternatives to prevent confusing respondents when setting multiple choice questions. Additionally, the list of alternatives must not be too long. One has to ensure that the respondents understand how many responses they are allowed to select. Also consider to leave scope for the addition of the respondent's own answer by adding an "other" or "none of these" option to the list, especially if the student is uncertain whether all possible alternatives have been listed.

Figure 2.4 is an example of a typical multiple choice question from a first-year calculus paper:

If $y^3x + y^2x^2 = 6x$, then $\frac{dy}{dx} =$	Stem		
A. $\frac{6}{3y^2+4xy}$	Distractor		
B. 0	Distractor		
C. $\frac{6-2x^2y}{3xy^2+2x^2y}$	Distractor	Options	Item
D. $\frac{6-y^3-2xy^2}{3xy^2+2x^2y}$	Key		
E. $None\ of\ the\ above$	Distractor		

Figure 2.4. A sample item from a WTW 158 question paper, March 2014

The multiple choice question is called an item. The item is divided into a stem (the question) and the different possible options, of which there can be two or more. The options are divided into distractors (the wrong choices) and the key (the correct answer to the stem).

2.10 The advantages of using MCQs

The use of MCQs has several advantages for assessment purposes:

- It reduces the threat of human marking and adding errors.
- Markers differ in leniency when marking extended response items especially when marking takes place over a couple of days.
- The excessive grading and administrative load on staff members can be reduced in modules such as engineering calculus where there are more than a 1 000 scripts. At least some part of the test can be marked by a computer.
- Another benefit for the students is that their achievements are not dependent on their writing skills. Students can still get the correct answer irrespective of poor writing skills.
- A large number of questions can be attempted in a given time, which can achieve greater coverage of work.
- Statistical analysis of the scores is relatively straightforward and this useful statistical information can easily be derived (Braun & Kanjee, 2006; Black, 1998; Tamir, 1990).

2.11 The number of options in MCQs

Vyas and Supe (2008) performed a study to determine if the number of multiple choice options could be reduced to three without affecting the quality of the tests. A systematic database search was done using Science Direct, Pubmed, Ovid and ERIC search engines for the period of 1960 to 2007. Theoretical, analytical and empirical studies were done based on the approach of the study. Their conclusion was that MCQs with three options provide similar qualities of tests than when compared with four- or five-option MCQs.

For all the items in the tests under investigation, the number of distractors varies between four and five distractors plus one key. It is argued that the option “none of these” should always be included as one of the options. The rationale for this practice is to provide for instances where students could have made a calculation mistake that results in an answer that is not among the given options. To prevent the student from randomly guessing any option, the option "none of these" may look like the correct option. In some MCQs the option "none of these" is actually the correct option. Another reason why there are sometimes more distractors than the conventional four is because in mathematics a problem can have more than four possible mistakes usually made by students. To accommodate the different possibilities, more options must be available. The following example proves this point:

Example: Determine the domain of $f(x) = \frac{\sqrt{5-x}}{x-1}$

The following options can be given as possible distractors that are usually chosen by students:

(A) $x \in (-\infty, 5)$ (B) $x \in (-\infty, 5]$ (C) $x \in (5, \infty)$ (D) $x \in [5, \infty)$

(E) $x \in (-\infty, 1) \cup (1,5)$ (F) $x \in (-\infty, 1) \cup (1,5]$ (G) None of these

For the analysis of the distractors, it was decided to use the number of *functional distractors* rather than using the number of *non-functional distractors* for each item since we want to determine the quality of each MCQ. The more functional distractors it has, the better the quality of the question. When students did not choose a distractor, the solution was evident.

The literature review started with a discussion of the global paradigm shift of the university assessment model and how the assessment model changed over the last two decades in South Africa. The upcoming and downfall of outcomes-based education (OBE) dominated the South African educational system after the 1994 first democratic elections, which directly and indirectly influenced the higher education system. Assessment and learning had to be re-evaluated, which led to accepted principles of good assessment. Student numbers increased drastically, which led to revising the assessment model and introducing the use of MCQs to reduce the marking load. Generally accepted principles of good assessment was then discussed, where the curriculum, instruction and assessment play equally important roles. This was followed by a discussion of the level of cognitive demand that can be obtained when carefully planning and using MCQs. The literature review was followed by a discussion of PRQs as a collective name for different types of provided response questions, of which MCQs form part. Lastly, the advantages and applications of MCQs were discussed as an alternative form of testing.

CHAPTER 3 RESEARCH DESIGN AND METHODOLOGY

3.1 Introduction

Chapter 3 provides details of the research design and methodology that was followed in this study. Section 3.2 starts with a brief discussion of the epistemological assumptions of this research study, followed by Section 3.3 which describes the quantitative research design that was implemented and the secondary analysis that was executed. Section 3.4 follows with the procedures executed for the data collection and documentation. In Section 3.5 the test reliability and face validity are discussed, followed by a discussion of research ethics in Section 3.6. In Section 3.7 data analysis is done in order to address the primary and secondary research questions. The parameters are defined in Section 3.8, which is followed by the radar plots in Section 3.9. The chapter ends with Section 3.10, a classification of the questions according to the topics covered in the engineering calculus module.

3.2 Epistemological assumptions

According to Maree (2014), epistemology explains how one knows reality. It assumes a relationship between the knower and the known. By the use of scientific methods, positivists believe knowledge can be revealed or discovered. Scientific methods can provide possible explanations for causes of occurrences in the world. Often these happen independently of peoples' intentions. By measurements, experiments and observations the reality can be revealed or discovered. An advantage is that scientific methods will provide answers that are precise, verifiable, systematic and generalisable (Maree, 2014).

Cohen, Manion and Morrison (2011) explain knowledge, amongst others, from a positivist stance. It implies that knowledge is observable as objective facts. In addition, it can be transmitted in a tangible form so that observers can use scientific, statistical and quantitative methods to make assumptions and predictions from it.

According to Neuman (2000), positivism is the research paradigm that combines a deductive approach with precise measurement of quantitative data so that researchers can discover and confirm causal laws that will permit predictions about human behaviour. In addition, Maree (2014) explains that positivism is built on objective, observable facts as it manifests in science and scientific laws.

Although positivism has its limitations and weaknesses, a positivist epistemology is followed, as it centres on the use of data where the measurements are in the form of results of marks obtained from multiple choice questions in first-year engineering calculus papers of the years 2015, 2016 and 2017. All data will be analysed statistically to provide precise and verifiable answers. A post-positivism paradigm which assumes that reality can be measured, but that such findings only provide a partial view of the inquiry's reality, might be a more appropriate philosophical stance.

3.3 Research design

For purposes of this study a quantitative research design will be conducted. According to McMillan and Schumacher (2001), quantitative research involves scientific measurement and the use of statistics. It involves deductive reasoning applied to numerical data, and an explicit description of data collection and analysis procedures. It also provides statements of statistical relevance and probability.

For this research a descriptive research approach will be used by executing a secondary analysis on students' performance marks for MCQs obtained from semester tests and examinations. Secondary analysis is defined by Heaton (2008) as the re-use of pre-existing data that was derived from previous research studies. The data can be presented in any form. A secondary analysis is different from documentary analysis and meta-analysis because a secondary analysis approach involves going back over published findings from previous studies and reworking existing data. Secondary data analysis is the re-analysis of data for purposes outside of its original intent to uncover more meaning from the data. Further, Heaton (2008) states that the researcher may re-use own self-collected data to investigate new or additional matters or to verify previous findings.

A few advantages of executing a secondary analysis include the fact that data archives are increasingly available for further use. The performance results of semester tests and examinations are sourced at the Department of Mathematics and Applied Mathematics and are available for the research. Moreover, several policies exist internationally promoting data retention and sharing for on-going research purposes. A further advantage is the digitisation of data to improve the opportunities for preserving data and giving easier and wider access to datasets online (Corti & Bishop, 2005).

3.4 Data collection and documentation

The data for this study is gathered from performance marks for Semester test 1, Semester test 2 and the examination for the years 2015, 2016 and 2017 for the engineering calculus module WTW 158 at the Department of Mathematics and Applied Mathematics. Most of the tests had two sections – Section A, consisting of MCQs and Section B, consisting of constructed response questions (CRQs) for which students have to provide written answers. In 2017 the semester tests were divided into three sections – Section A, consisting of MCQs, Section B, consisting of single-answer questions and Section C, consisting of questions requiring longer expositions. The MCQ section contributed between 25% and 35% to the total mark of a semester test or end-of-semester examination. For this research study only the MCQs from Section A are considered. All the results are available at the Department of Mathematics and Applied Mathematics at the University of Pretoria.

3.5 Face validity and test reliability

Validity of a test is the degree to which a test measures what it intended to measure (Maree, 2014). Messick (1989:13) notes that "validity is an integrated evaluative judgement of the degree to which empirical evidence and theoretical rationales support the adequacy, the appropriateness of inferences and actions based on the test scores." Validity can also refer to the extent to which the research design (secondary analysis in this case) is scientifically sound and appropriately conducted (Struwig & Stead, 2013).

For the current study, face validity is of importance. The assumption is that the MCQs are without mathematical mistakes, grammatically well written, unambiguous and that the questions test what they set out to do. Since all the MCQs are answered on an optical reader form and graded by a computer program, human errors are eliminated and correctness of the results can be assumed.

An additional face validity exercise was done to determine the mathematics assessment components of each MCQ and for this the expertise of four highly experienced lecturers was used. All of them have at least 15 years of teaching experience in Mathematics. The complete results can be found in Appendix A. The MAC level of each question was determined by examining the question on the basis of Table 2.8 in Chapter 2. For most MCQs the outcome between the lecturers was the same. Where there was a difference in opinion, the option of the majority was chosen.

For purposes of test reliability the computer program used by the University of Pretoria to determine the discrimination uses the upper 27% of the high-scoring students and the lower 27% of the lower-scoring students. The test reliability that was utilised for the MCQs is the Kuder-Richardson Formula 20 (Zaiontz, n.d.-b). The Kuder-Richardson Formula 20 measures internal consistency (Mukherjee & Lahiri, 2015). The Kuder-Richardson formula is given in Figure 3.1:

$$R_{KR20} = \frac{K}{K-1}\left[1 - \frac{\sum_{i=1}^{K} p_i q_i}{\sigma^2}\right]$$

where

$k =$ the number of questions

$p_i =$ the number of examinees in the sample who answered question i correctly

$q_i =$ the number of examinees in the sample who didn't answer question i correctly

$\sigma^2 =$ the variance of the total scores of all the examinees taking the test

Figure 3.1. The Kuder-Richardson Formula 20

Figure 3.2 represents the variance, σ^2, for the KR_{20} formula:

$$\sigma^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n}$$

where

$n =$ the sample size

$X_i =$ the score of individual examinees

$\bar{X} =$ the mean total score

Figure 3.2. The variance for the KR_{20} formula

Mukherjee and Lahiri (2015) explain that the KR_{20} can range between 0 and 1. The closer the value gets to 1, the greater the internal consistency (reliability). High levels above 0.9 is not desirable as it indicates redundant items. According to Maree (2014), if the test items correlate strongly with each other their internal consistency is high. On the other hand if the items correlate poorly, the alpha coefficient will be low. Zaiontz (n.d.-a) provides a generally accepted guideline for the interpretation of the Cronbach's alpha:

- High reliability 0.90
- Moderate reliability 0.80
- Low reliability 0.70
- Acceptable 0.60

According to Maree (2014), reliability and validity for the research instruments are important aspects in a quantitative study such as this. Reliability is the degree of consistency in the data and the repeatability of the study. This implies that different researchers could gather the same results when repeating the study at another time under the same conditions (Maree, 2014). According to McMillan and Schumacher (1993), the reliability in quantitative research refers to a consistency of the test instrument and administration of the study, giving reliable and consistent results.

All test and exam papers were moderated externally before students wrote the tests. Errors in marking were not possible since Section A of the tests was graded by a computer program. This eliminated any human error. Reliability in this study was also enhanced by using a large amount of data collected over a period of three years.

3.6 Research ethics

Ethical clearance for conducting this study was obtained from the Faculty of Education Ethics Committee. The letter of approval is attached in Appendix D. For this research a secondary analysis was done on marks that first-year engineering students obtained in multiple choice questions in semester tests and examinations. To ensure ethical conduct and anonymity, no student names or student numbers were used when using the data. The letter of permission from the Deputy Dean of Research and Postgraduate Studies (Faculty of Engineering, Built Environment and Information Technology) is attached in Appendix E.

3.7 An assessment framework

The use of an assessment framework is to ensure that every question in a test is connected to a taxonomy or an outcome and is suitable for the test (Maree, 2014). Based on an item analysis of each MCQ, improvements, revisions and eliminations can be done.

Basic item statistics can include the following:

- Measures of central tendency such as mean, mode and median
- Measures of dispersion such as standard deviation and variance
- Item difficulty
- Item discrimination
- Distractor efficiency

The measures of central tendency and dispersion are not going to be utilised in this research since these measures are not contributing to determining the quality of individual MCQs. With the assessment framework the research will try to address the two secondary research questions and, subsequently, the primary research question.

Item analysis is an important phase in the development of quality MCQs. According to Mitra, Nagaraj, Ponnudurai and Judson (2009), item analysis is the process of collecting, summarising and using information from students’ responses to MCQs to assess the quality of the questions. The use of an assessment framework helps to give structure to assessment and determine which parameters will be utilised in the process.

For this research the measures of item difficulty, item discrimination and distractor efficiency will be utilised. According to Hingorjo and Jaleel (2012), a good quality MCQ will have an average difficulty, a high discrimination and more that 50% functional distractors.

The discrimination index (DI) measures the extent to which different test items adequately distinguishes between less able and more able students. The difficulty index (p-value) calculates the percentage of correct responses out of the total number of responses to a test item (Maree, 2014). According to Hingorjo and Jaleel (2012), the distractor efficiency indicates whether the test item was well constructed or not. If a distractor is chosen by less than 5% of the examinees, it is considered to be a non-functional distractor (NFD). It is usually the low-performing students, who have not mastered the content, who choose NFDs. The high-scoring students will more likely choose the correct option (key) and therefore ignore the NFDs. When analysing the distractors it becomes clear which of these must be replaced, removed or revised.

A non-functioning distractor makes a question easier to answer since the assumption can be made that students are unlikely to choose such a distractor. Difficult questions tend to have fewer NFDs since the weaker students will guess randomly, thereby using all the distractors (Hingorjo & Jaleel, 2012).

The assumption regarding the three parameters that will be discussed is that:

- All good questions should discriminate well. Item discrimination implies that poor-performing students should not be expected to score well, and high-performing students should be expected to do well.
- The level of difficulty of a question does not make a question good or poor. As challenging questions can be good or poor, easy questions can also be good or poor because the solution becomes evident, which leads to both stronger and weaker students knowing the answer, which will subsequently reduce the discrimination index.

- The efficiency of the distractors can only be seen after a test is written since the statistical analysis will indicate the non-functioning distractors (NFDs).

According to Tarrant, Ware and Mohammed (2009), flawed MCQs affect the performance of high-achieving students more than borderline students. Care should be taken when constructing MCQs; therefore, an item analysis is of utmost importance for subsequent assessments.

3.8 Data analysis for addressing the research questions

In order for the quality of MCQs to be determined it is necessary to do an item analysis of each test item. Mitra et al. (2009) state that item analysis is the process of collecting, summarising and using information from students' responses to assess the quality of the test items. According to Hingorjo and Jaleel (2012), item analysis is a relatively simple procedure to provide information regarding the validity and reliability of a test item. Item analysis also tells us how easy or difficult the questions were, how well the questions discriminated and how the distractors behaved (Mehta & Mokhasi, 2014).

The main research question in this study examines how multiple choice questions in mathematics, as posed to first-year engineering students at the University of Pretoria, comply with the principles of good assessment for determining their quality. In order to answer this question, two sub-questions were posed:

Sub-question 1: Which levels of cognitive demand are addressed in the multiple choice questions of the sample under investigation?

Sub-question 2: How do the multiple choice questions measure with respect to the discrimination index, difficulty index and distractor efficiency?

3.9 Defining the parameters

The following parameters are utilised in the research and will be defined subsequently:

- The discrimination index
- The difficulty index
- The distractor efficiency
- The mathematics assessment components (MACs)

Each of the parameters is now discussed under the assumption that:

- All good, acceptable questions should discriminate well. Good item discrimination implies that poor-performing students should not be expected to score well, and high-performing students should be expected to do well.
- The level of difficulty of a question does not make a question good (acceptable) or poor (not acceptable). As challenging questions can be good or poor, easy questions can also be good or poor especially when investigating in combination with the discrimination index. According to Hingorjo and Jaleel (2012), the normal, accepted behaviour of a question is a decreasing of item discrimination as the difficulty index goes below the 0.3 or above the 0.7 mark. Figure 3.3 illustrates the typical relation between the difficulty and discrimination indices.

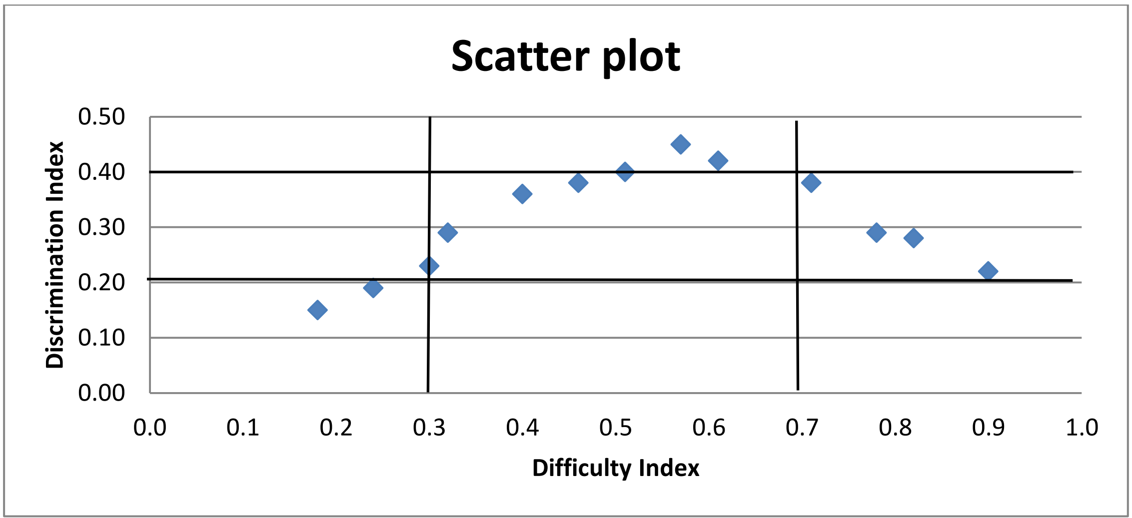


Figure 3.3. The typical relation between the indices of difficulty and discrimination

From the scatter plot it can be observed that as a question's difficulty index increases (the question becomes easier), the discrimination decreases. The opposite is also true; when a question's difficulty index decreases (the question becomes challenging), the discrimination decreases.

- The efficiency of the distractors can only be seen after a test is written, since the statistical analysis will indicate the functional distractors (FDs).
- The mathematics assessment components (MACs) give an indication of the cognitive levels of each question.
- MCQs of good quality should have an average difficulty that goes hand in hand with a good discrimination and some functional distractors (Mehta & Mokhasi, 2014; Mukherjee & Lahiri, 2015). Such questions are acceptable and should be incorporated into future tests.

3.9.1 The discrimination index

One of the basic measures of item quality is the discrimination index (DI). It measures the extent to which different test items adequately distinguishes between less able and more able students. The academically stronger students should answer a particular question correctly, whereas the academically weaker student should answer the question wrongly. If this is the case the question discriminates well. When a question becomes too easy, both the academically stronger and weaker students will answer the question correctly, which makes the gap smaller between the two groups. In this case the question does not discriminate well. The discrimination index (DI) can be computed as follows:

$$\boldsymbol{DI = \frac{C_H - C_L}{N}}$$

Where

C_H= number of students in the high-performance group

C_L= number of students in the low-performance group

N = number of students from both groups

Figure 3.4. The formula for calculating the discrimination index

Steps to calculate the DI:

Step 1: Arrange the marks in descending order, where N is the total number of entries.

Step 2: Separate the marks into an upper group (27%) and a lower group (27%) based on the test scores.

Step 3: Count the number of students in the upper group (C_H) and in the lower group (C_L) who chose the correct option.

Step 4: Record the information.

Step 5: Compute DI by plugging the numbers into the formula.

The discrimination index (DI) ranges between -1.00 and 1.00. If high-performing students select the correct answer for an item more often than low-performing students, the assessment will have a positive DI (between 0.00 and 1.00). If low-performing students responded correctly to a specific item more often than high-performing students, the DI will be negative (between -1.00 and 0.00). If an equal number of students from both groups answer an item correctly, then DI = 0.00

According to Hingorjo and Jaleel (2012), the discrimination and difficulty indices are often related reciprocally as the difficulty index increases. This may, however, not always be true. If a question has a high p-value (easier question), the discrimination is poor since the weaker students will likely answer the question correctly.

Table 3.1 provides the discrimination index and description of each interval that will be applied in the study. The index is divided into four intervals categorising the discrimination as poor, fair, good and excellent. The categorising is used by several researchers like Mitra et al. (2009), Mehta and Mokhasi (2014) and Mukherjee and Lahiri (2015).

Table 3.1. The discrimination index categories and descriptors

Index	**Discrimination description**
$DI < 0.2$	Poor discrimination
$0.2 \leq DI < 0.30$	Fair discrimination
$0.3 \leq DI < 0.40$	Good discrimination
$DI \geq 0.40$	Excellent discrimination

3.9.2 The difficulty index

The difficulty index (p-value) is calculated as a percentage of the total number of correct responses to a test item (Maree, 2014). The higher the number of students who answer a question correctly out of the total number who attempted to answer the question, the higher the difficulty index. Hingorjo and Jaleel (2012) mention that perhaps “item difficulty” should rather be named “item easiness” as it expresses the percentage of students who answered a question correctly. The difficulty index can be calculated with the following formula:

$$p = \frac{R}{N}$$

Where p is the difficulty index

R is the number of correct responses

N is the total number of responses (including both correct and incorrect responses)

Figure 3.5. The formula for the difficulty index

The *p*-values range between 0 and 1. It is recommended that the range of difficulty must fall between 0.3 and 0.7 but some researchers prefer a difficulty index between 0.3 and 0.8 (Mitra et al., 2009). It is generally believed that questions with a difficulty index of $0 \leq p < 0.3$ are difficult since less than 30% of students who attempted the question responded correctly. Questions with a difficulty index of $p \geq 0.7$ are generally regarded as easy since 70% and more of the examinees who attempted the question chose the correct answer (Hingorjo & Jaleel, 2012). Questions with a difficulty index of $0.3 \leq p < 0.7$ are therefore regarded as moderate in difficulty. As mentioned, these values may differ slightly among authors. Figure 3.6 presents the bell-shaped normal distribution, also known as the Gaussian distribution of item difficulty versus item score.

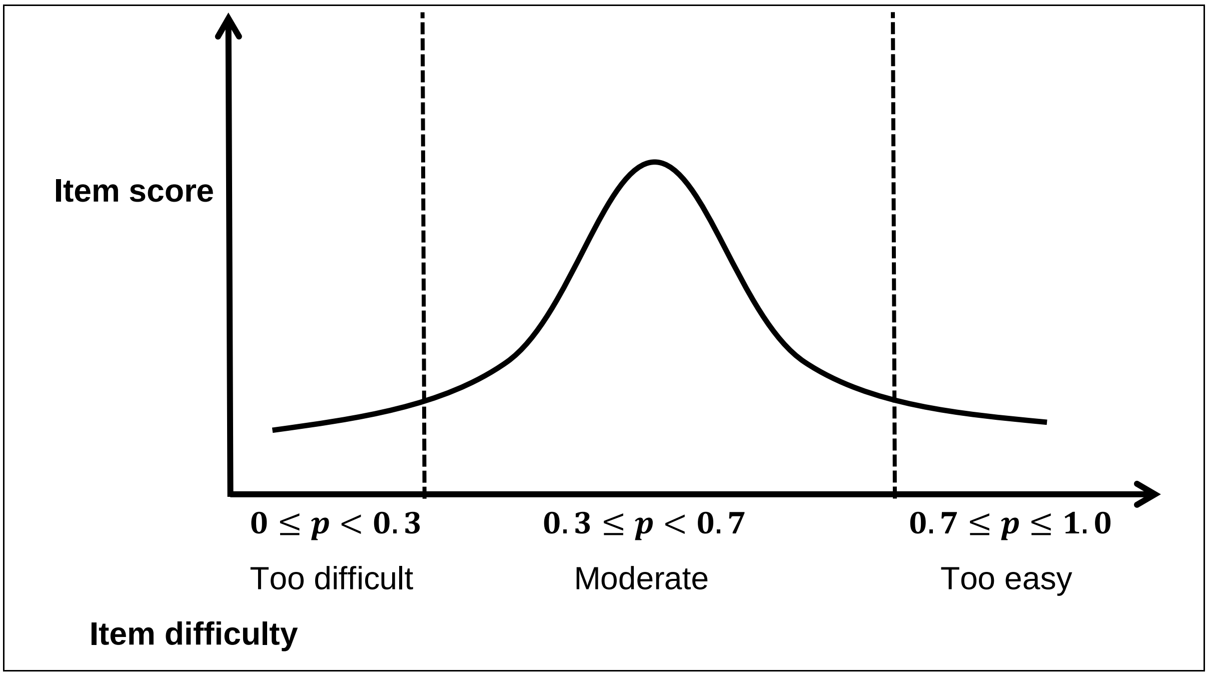


Figure 3.6. Normal distribution of MCQs for item difficulty

For this research, item difficulty will be divided into four categories. Table 3.2 gives the index and difficulty description.

Table 3.2. The difficulty index categories and descriptors

Index	Difficulty description
$0 \leq p < 0.3$	Difficult question
$0.3 \leq p \leq 0.5$	Moderately difficult question
$0.5 < p < 0.7$	Moderately easy question
$p \geq 0.7$	Easy question

3.9.3 The distractor efficiency (DE)

Analysis of the distractors is an important part of item analysis. According to Hingorjo and Jaleel (2012), the distractor efficiency indicates whether the test item was well constructed or not. If a distractor was selected by less than 5% of the examinees, it is considered to be a non-functioning distractor (NFD) and if a distractor was selected by more than 5% of the examinees, it is considered to be a functional distractor (Mukherjee & Lahiri, 2015). It is usually the low-performing students, who have not mastered the content, who choose the NFDs. The high-scoring students will more likely choose the correct option (key) and therefore reject the NFDs. When analysing the distractors it becomes clear which of these must be replaced, removed or revised.

Table 3.3 provides a description of the distractor efficiency. The functional and non-functional distractors of each question under investigation are analysed.

Table 3.3. Distractor efficiency

Non-functional distractor (NFD)	**Functional distractor (FD)**
Chosen by $<$ 5% of examinees	Chosen by $>$ 5% of examinees

A non-functioning distractor makes a question easier to answer since the assumption can be made that students are unlikely to choose such a distractor. Difficult questions tend to have fewer NFDs since the weaker students will guess randomly, thereby using all the given distractors (Hingorjo & Jaleel, 2012).

According to Tarrant, Ware and Mohammed (2009), flawed MCQs affect the performance of high-achieving students more than borderline students. Care should be taken when constructing MCQs; therefore, an item analysis is of utmost importance for subsequent assessments.

The number of distractors varies between four and five depending on the type of mathematical problem and the possible student responses. It is argued that the option “none of these” should always be included as one of the options. The rationale for this practice is to provide for the instances where students could have made a calculation mistake that results in an answer that is not among the given options. To prevent the student from guessing one of the options, the option “none of these” should be included. In most cases there are more than the conventional four options for distractors. The reason for this is the nature of the mathematics calculations involved in the question. A problem can have more possible options because of general mistakes students usually make when solving certain problems. To accommodate the different possibilities, more options must be available. An example to illustrate this phenomenon is:

Question: Determine the domain of function $f(x) = \frac{\sqrt{2-x}}{x^2}$.

The correct solution to this question is $x \in (-\infty, 0) \cup (0,2]$. Because of the nature of this question and the possible mistakes students usually make, the following options (distractors) are possible:

(A) $x \in (-\infty, 0) \cup (0,2]$	(B) $x \in (-\infty, 2)$	(C) $x \in (-\infty 2]$
(D) $x \in [2, \infty)$	(E) $x \in (2, \infty)$	(F) $x \neq 0$

It is clear that a mathematical MCQ can have more than four distractors and it differs from question to question.

For the analysis of the distractors, it was decided to use the number of *functional distractors* rather than using the number of *non-functional distractors* for each item. The functional distractors are presented as a percentage of the total number of distractors. For example, if three out of the five distractors were functional, a value of $\frac{3}{5} \times 100 = 0.6$ (60%) was given. The standardisation of the values ranges between 0 and 1 so that all three parameters (discrimination, difficulty and functional distractors) range between the same values of 0 and 1. The more functional distractors a question has, the better the quality of the question is, as opposed to more non-functional distractors that weakens the quality of the question. When a question is too easy, the assumption is that the majority of students (including the academically weaker students) will ignore the distractors, which will lead to students choosing the correct option (key), which in turn will result in having fewer functional distractors.

3.9.4 Mathematics assessment components (MACs)

Table 3.4 refers to the mathematics assessment component. Each component is given a different shade of grey to distinguish between them when presented as radar plots in Chapter 4. The mathematics assessment components are all linked to a specific cognitive level, as explained in Section 2.6 in the discussion about the assessment framework.

Table 3.4. Mathematics assessment component

Mathematics Assessment Component	**Description**
1	Technical question
2	Disciplinary question
3	Conceptual question
4	Logical question
5	Modelling question
6	Problem solving question
7	Consolidation question

The mathematics assessment components (MACs), as utilised by Huntley (2008), are used to address the first secondary research question, namely which levels of cognitive demand are addressed in multiple choice questions of the sample under investigation? Each MAC level is connected to a cognitive skill and these cognitive skills are connected to both Bloom's taxonomy and the MATH taxonomy, as explained in Section 2.6, Table 2.9.

3.10 The radar plots

All the parameters (namely the difficulty index, discrimination index, and functional distractors) as well as the seven levels of the mathematics assessment components, as aligned to the cognitive levels, are combined in the construction of a radar plot. The rationale behind the radar plots is to visually observe the features of each MCQ under investigation to determine the quality of each. The numerical value of each parameter, together with the size and shape of the triangular radar plot, gives a clear picture of the MCQ with regard to the indices. The values of the three axes range between 0 and 1.

Figure 3.7 and 3.8 are two examples of typical radar plots. Figure 3.7 is an MCQ of which the indices are well balanced and Figure 3.8 is an MCQ of which the indices are imbalanced.

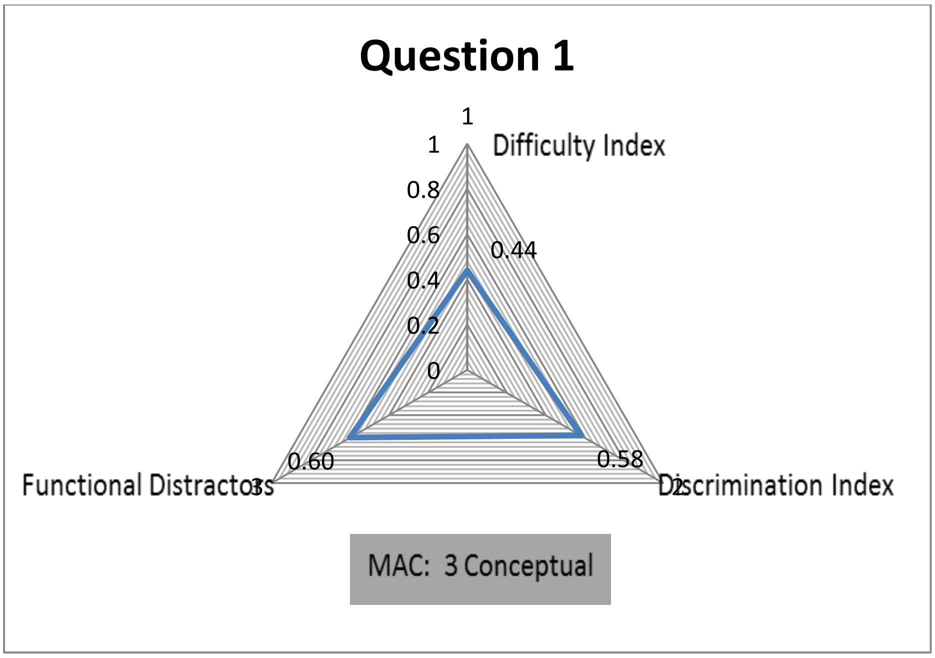


Figure 3.7. A typical MCQ of good quality

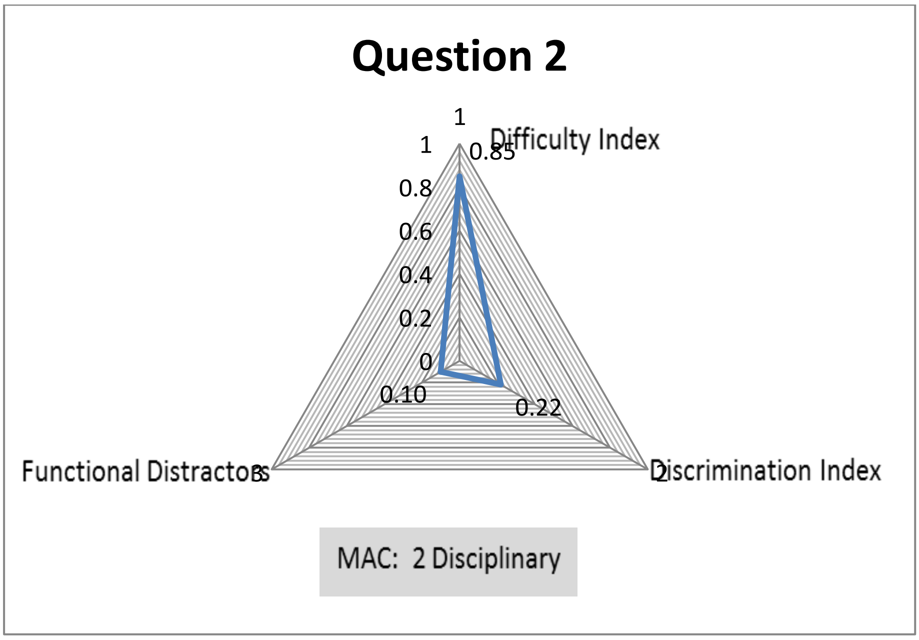


Figure 3.8. A typical MCQ of poor quality

The MCQs of Figure 3.7 and Figure 3.8 are now compared in Table 3.5 to show the basic differences between a generally accepted good quality MCQ and a poor quality MCQ:

Table 3.5. A comparison of a good and a poor quality MCQ

From Figure 3.7: Good quality MCQ	From Figure 3.8: Poor quality MCQ
The area of the triangle is large.	The area of the triangle is smaller.
The three sides of the triangle are proportionally equal in length.	The three sides of triangle are not proportionally equal in length.
Although the question is moderate to difficult, the discrimination between the weaker and stronger students is good and 60% of the distractors were functional.	The question was too easy, resulting in a poor discrimination between the weaker and stronger students, causing only 10% of the distractors to be functional because the weaker students found the distractors obviously wrong.
The question's MAC level of 3 (conceptual) was of a higher cognitive level than for Question 2 (Figure 3.8).	The question's MAC level of 2 (disciplinary) was of a lower cognitive level than for Question 1 (Figure 3.7).
MAC Level 3 has a darker shade of grey than MAC Level 2 of Question 2.	MAC Level 2 has a lighter shade of grey than MAC Level 3 of Question 1.

The empirical establishment of the quality of these MCQs and if they are suitable for use in a tertiary environment are of great importance in the current study. Feedback could be given to the designers of the MCQs as to which questions are exemplary and what the deficiencies are of other questions.

3.11 Classification of questions with regard to topics

The multiple choice questions under investigation are grouped into nine topics as it is presented in the WTW 158 engineering calculus syllabus. The rationale behind the classification is that MCQs with similar topics can easily be compared with each other. These topics are:

- **Functions** (number system, intervals, domain, even and odd functions, transformations, compositions, piecewise-defined functions, inverse functions)
- **Trigonometry** (radian measure, trigonometric functions, inverse trigonometric functions, equations)

- **Absolute value and inequalities** (definition, properties, graphs, linear, quadratic and rational functions)
- **Exponential and logarithmic functions** (laws, properties, graphs, equations)
- **Limits and continuity** (limit laws, limits at infinity, horizontal asymptotes, discontinuities, one-sided continuities)
- **Derivatives** (rates of change, derivative as a function, polynomials and exponential functions, differentiation rules, trigonometric functions, implicit differentiation, logarithmic functions, hyperbolic functions)
- **Applications of differentiation** (maxima and minima, mean value theorem, how the derivative affects the shape of the graph, L'Hospital's Rule, curve sketching, optimisation)
- **Integration** (anti-derivatives, the area problem, definite integral, fundamental theorem of calculus, indefinite integrals)
- **Vector algebra** (three-dimensional coordinate system, vectors, dot product, cross product)

CHAPTER 4 RESEARCH FINDINGS AND RESULTS

4.1 Introduction

The research findings and results are presented in Chapter 4. Section 4.2 gives a brief description of the data under investigation. Thereafter the research findings are analysed and discussed guided by the literature review and conceptual framework (Chapter 2). In Section 4.3 the face validity and reliability are discussed and in Section 4.4 all the MCQs are presented on a single scatter plot. The difficulty index versus the discrimination index is presented by the scatter plot. In Section 4.5 the MCQs under investigation are grouped into nine topics for discussion purposes. Nine scatter plots are given, related to each topic. Each scatter plot incorporates the difficulty index versus the discrimination index. Lastly, radar plots show how the MCQs perform with regard to the indices of discrimination and difficulty, the functional distractors and the mathematics assessment components as discussed in Chapter 3. This is followed by a short discussion of the selected MCQs with regard to their quality. Presentable MCQs are selected for discussion to point out problematic as well as acceptable questions.

4.2 Data description

A summary of data relating to Semester test 1, Semester test 2 and the examination for the years 2015, 2016 and 2017 is provided in Table 4.1:

Table 4.1. Summary of the written tests of the years 2015, 2016 and 2017

Year	Test	Number of MCQs	Number of students
2015	Semester test 1	20	1630
	Semester test 2	10	1608
	Exam	10	1534
2016	Semester test 1	13	1575
	Semester test 2	10	1518
	Exam	10	1453
2017	Semester test 1	12	1395
	Semester test 2	12	1378
	Exam	10	1304

4.3 Face validity and reliability

In Chapter 3, section 3.5, face validity was defined and discussed. To ensure face validity the joint expertise of four lecturers was used to determine the cognitive level of each MCQ as discussed in Chapter 3. The outcome was unanimous for the majority of questions. If the lecturers had a difference in opinion, the MAC level chosen by the majority was settled on. Each MCQ was scrutinised to ensure a high degree of face validity.

Test reliability was defined and discussed in Chapter 3, section 3.5. The computer program used by the University of Pretoria determines the internal reliability of each assessment. Table 4.2 summarises the reliability values of all the assessments under investigation.

Table 4.2. Test reliability with Kuder-Richardson Formula 20

Assessment	**Kuder-Richardson Formula 20**
ST1 2015	0.64
ST2 2015	0.55
Exam 2015	0.46
ST1 2016	0.71
ST2 2016	0.47
Exam 2016	0.51
ST1 2017	0.59
ST2 2017	0.62
Exam 2017	0.51

Table 4.2 shows that the internal reliability has values between 0.46 (exam 2015) and 0.71 (ST1 2016). The reason for the low to medium reliability is possibly because only a few items per construct are found in each test. The MCQs in each test assessed different content, different topics and assessed on different MAC levels.

4.4 Scatter plot presenting all the MCQs under investigation

Without going into a deeper discussion regarding each MCQ under investigation, Figure 4.1 presents the difficulty index on the x-axis against the discrimination index on the y-axis so that all 107 MCQs can be seen at a glance.

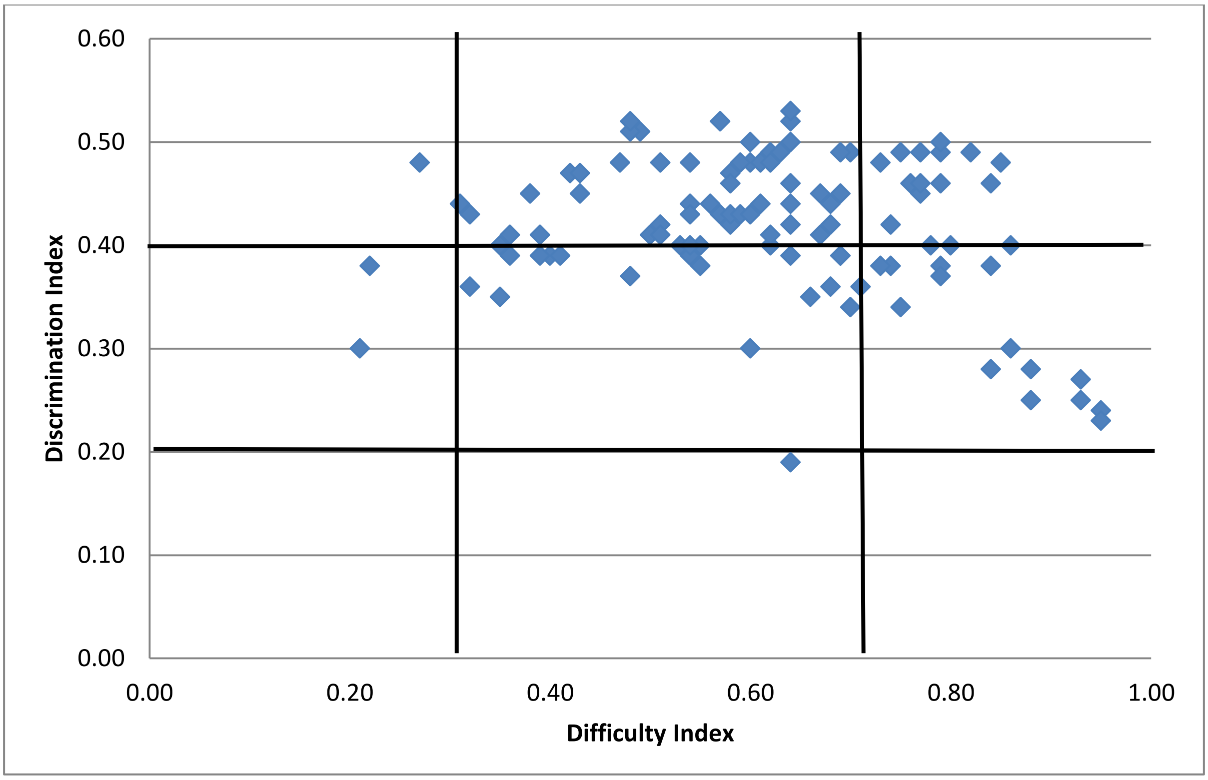


Figure 4.1. Scatter plot of all MCQs under investigation

From the figure it is clear that the vast majority of questions are in the satisfactory pattern that falls within the 0.3 and 0.7 band of the difficulty index. The majority of questions also fall above the 0.40 mark in the discrimination index, which indicates excellent discrimination. Only one question discriminates poorly (DI $<$ 0.20). Eight questions confirm the tendency of a high difficulty index (easy questions) with a low discrimination index. Three questions tend to be difficult with a low difficulty index ($p < 0.3$), but good discrimination (DI $\geq$ 0.30) still persists. A full discussion of the findings will follow shortly.

4.5 Question analysis by topics

All the MCQs under investigation are grouped according to their topics as determined by the engineering calculus module's curriculum. The rationale behind the classification is that MCQs can be compared with other MCQs of the same topic.

The nine topics are:

1. Functions
2. Trigonometry
3. Absolute value and inequalities
4. Exponential and logarithmic functions
5. Limits and continuity
6. Derivatives
7. Applications of differentiation
8. Integration
9. Vector algebra

The following selection of MCQs was singled out because they are the problematic and commendable MCQs. These MCQs are presented by scatter plots and radar plots to visually show how each question presents itself with regard to the discrimination index, the difficulty index and the distractor efficiency. The cognitive level for each question is also given by using the mathematics assessment component levels and the way it is linked to Bloom's taxonomy and the MATH taxonomy discussed in Chapter 2. All other MCQs which are not presented here can be found in Appendix C.

4.5.1 Functions

Table 4.3 presents all the questions that are related to the functions topic.

Table 4.3. Items related to functions

No	ITEM	Question	Difficulty	Discrimination
1	ST1 2015	Q5	0.88	0.25
2	ST1 2015	Q6	0.84	0.38
3	ST1 2015	Q11	0.64	0.19
4	ST1 2015	Q14	0.62	0.40
5	ST1 2016	Q4	0.40	0.39
6	ST1 2016	Q8	0.54	0.44
7	ST1 2017	Q1	0.59	0.43
8	ST1 2017	Q7	0.36	0.41
9	ST1 2017	Q10	0.50	0.41
10	ST1 2017	Q12	0.69	0.39
11	ST2 2017	Q3	0.51	0.41

Figure 4.2 presents a scatter plot of all the questions that are related to the functions topic.

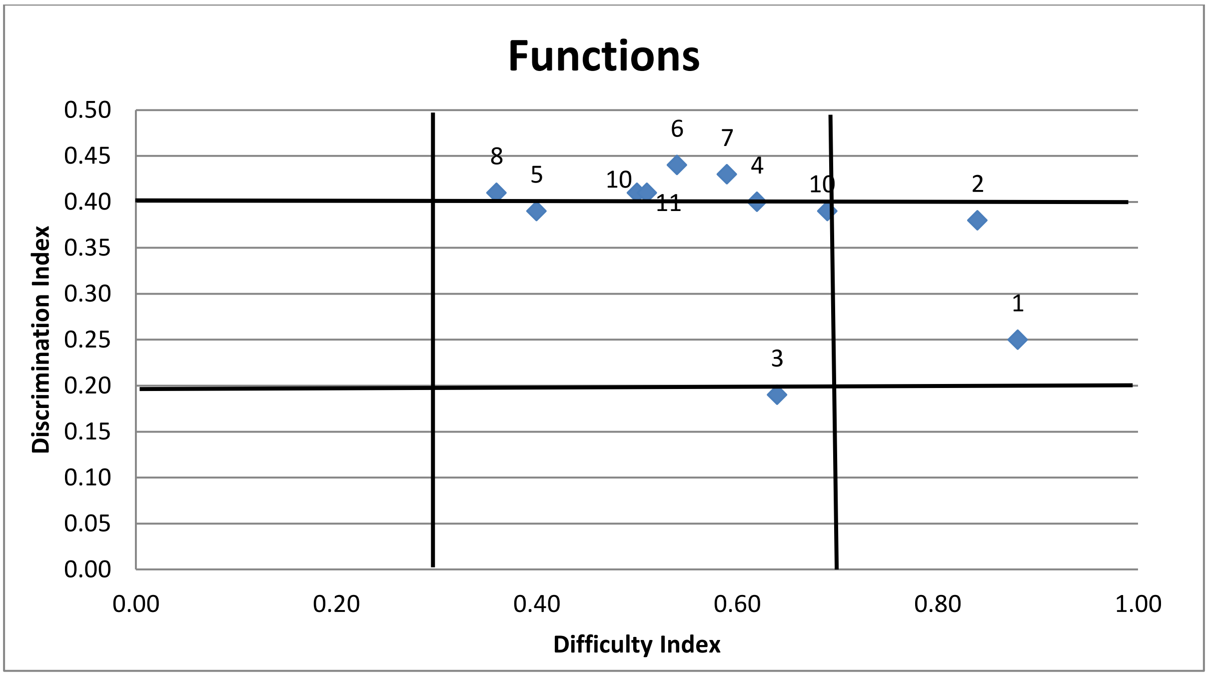


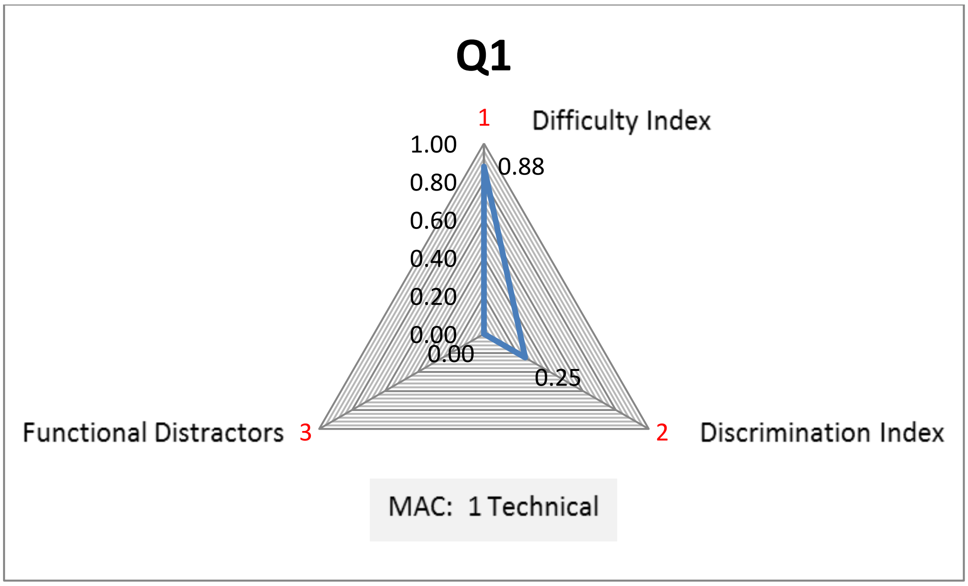
Figure 4.2. Scatter plot of all questions related to functions

Questions 1, 2, 3, 6 and 8 are now discussed. The scatter plot in Figure 4.2 shows these questions to be problematic or commendable.

Q1:

If $f(x) = 2x^2 + 1$ and $g(x) = x - 2$, then $fog(2) =$
(A) -3 (B) 9 (C) 4 (D) 1 (E) 33 (F) None of these

Question 5, Semester test 1, 2015



Q1	Value	Comment
Topic		Functions
Math Assessment Component	1	Technical
Difficulty Index	0.88	Easy question
Discrimination Index	DI = 0.25	Fair discrimination
Distractor efficiency	0.0	No functional distractors

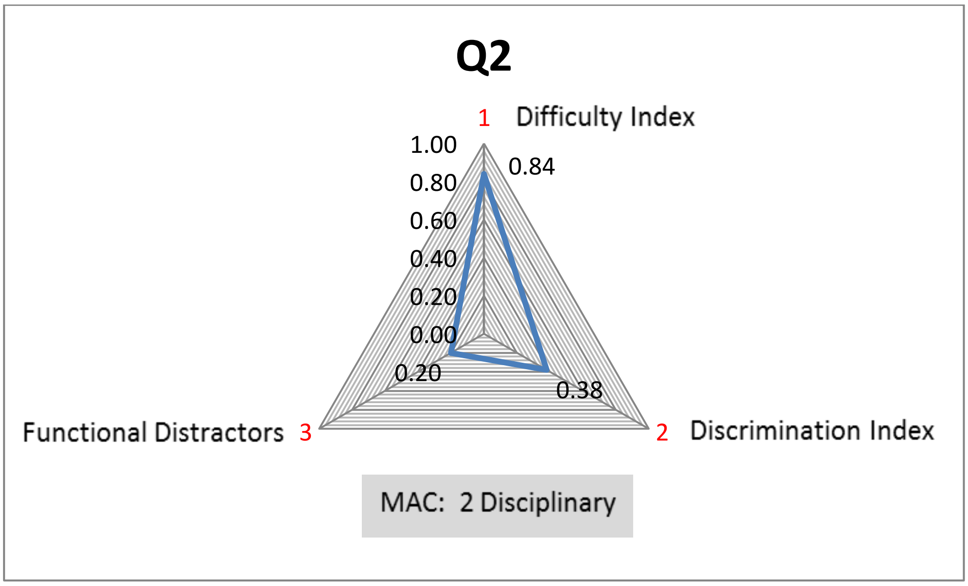
The question can be regarded as easy with a difficulty index of $p = 0.88$. The question has no functional distractors, which shows that the correct answer was evident to most students. The fair discrimination index goes hand in hand with the high difficulty index. The MAC level was 1 (technical), assessing at the lowest cognitive level.

Conclusion: This question is flagged as problematic. The question is direct and easy and does not discriminate well between strong and weak students.

Q2:

If $f(x) = \begin{cases} \lvert x \rvert, & x < 2 \\ x - 3, & x \geq 2 \end{cases}$ then $f(-3) =$
(A) -3 (B) 3 (C) 0 (D) 2 (E) -1 (F) None of these

Question 6, Semester test 1, 2015



Q2	Value	Comment
Topic		Functions
Math Assessment Component	2	Disciplinary
Difficulty Index	0.84	Easy question
Discrimination Index	DI = 0.38	Good discrimination
Distractor efficiency	0.20	20% Functional

The question is easy with a difficulty index of $p = 0.84$. The question shows good discrimination but only 20% of the distractors are functional.

Conclusion: Although this question is easy it discriminates better than the previous question. An obvious functional distractor that was excluded is -6 (obtained by substituting -3 into the second piece of the function). The question could be improved by paying more attention to the functional distractors.

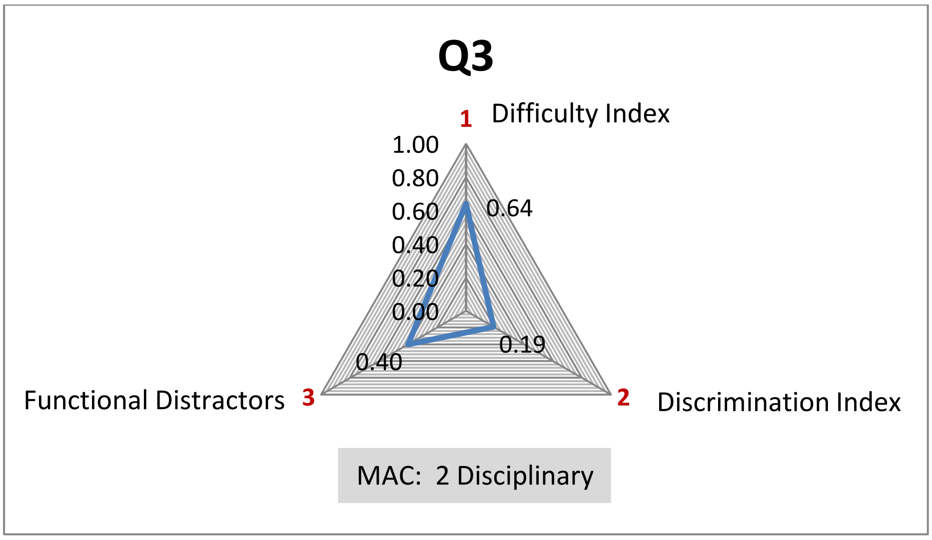
Q3:

The equation of the graph obtained by reflecting the graph of $y = |x - 1|$ by the y-axis, stretching it vertically by a factor 2 and translating it down by 3 is given by

(A) $y = 2|x + 1| - 3$ (B) $y = |x + 1| - 3$ (C) $y = -2|x + 1| + 3$

(D) $y = |x - 1| - 3$ (E) $y = 2|x + 1| + 3$ (F) None of these

Question 11, Semester test 1, 2015



Q3	Value	Comment
Topic		Functions
Math Assessment Component	2	Disciplinary
Difficulty Index	0.64	Moderately easy
Discrimination Index	DI = 0.19	Poor discrimination
Distractor efficiency	0.40	40% Functional

The question has a difficulty index of $p = 0.64$ which makes it moderately easy. The discrimination is poor (DI = 0.19) and 40% of the distractors are functional. Conclusion: This question is flagged as problematic. Although it is not a particularly easy question it does not discriminate well. The question is verbose, using terminology such as reflecting, stretching and translating that may have impeded students' understanding of the formulation. The problem could also lie in the fact that the question includes three different transformations that must be performed on one absolute value function, which may demand too many actions.

Q6:

A function f has a domain $[-1,1]$ and a range $[0,2]$. The inverse g^{-1} of the function $g(x) = f(x-1) + 1$ has

(A) Domain $[0,2]$ and range $[1,3]$

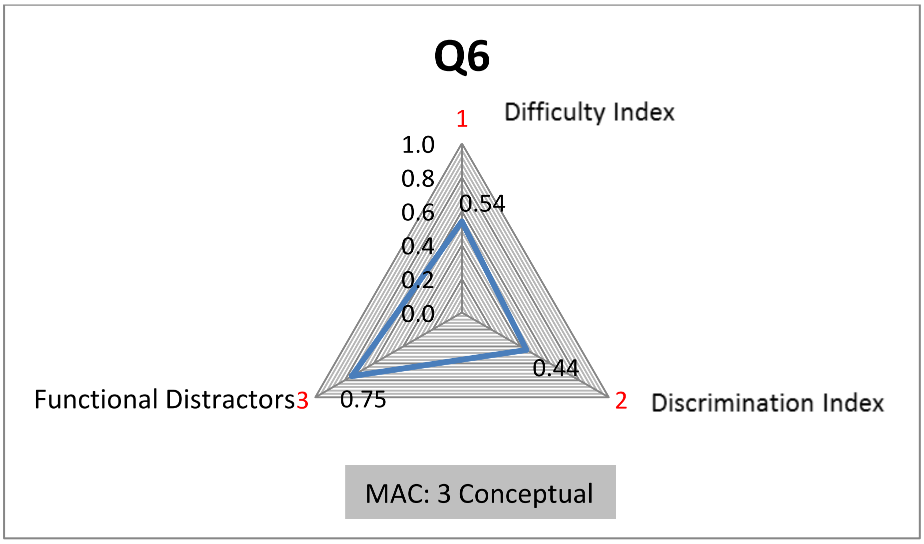
(B) Domain $[1,3]$ and range $[0,2]$

(C) Domain $[1,3]$ and range $[-2,0]$

(D) Domain $[-2,0]$ and range $[0,3]$

(E) None of these

Question 8, Semester test 1, 2016



Q6	Value	Comment
Topic		Functions
Math Assessment Component	3	Conceptual
Difficulty Index	0.54	Moderately easy
Discrimination Index	DI = 0.44	Excellent
Distractor efficiency	0.75	75% Functional

The question tests on a conceptual assessment component and is moderately easy. The question discriminates excellently. The percentage of functional distractors is 75%.

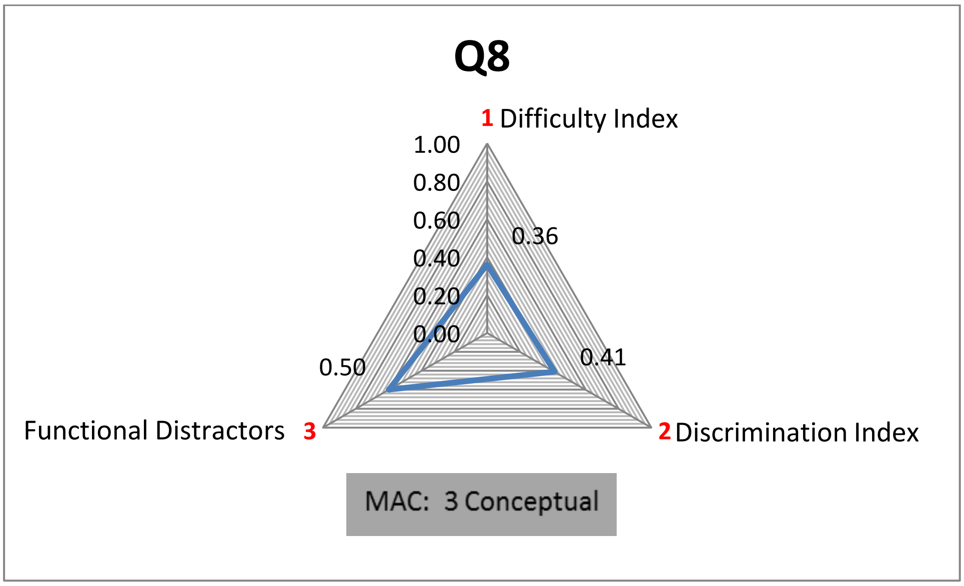
Conclusions: The question is acceptable. It is moderately easy and the indices are well balanced. The functional distractors are well chosen.

Q8:

If $f : A \to B$ and f^{-1} is the inverse of f then $(fof^{-1})(x) = x$ for

(A) $x \in R$ (B) $x \in A$ (C) $x \in B$ (D) $x = 1$ (E) None of these

Question 7, Semester test 1, 2017



Q8	Value	Comment
Topic		Functions
Math Assessment Component	3	Conceptual
Difficulty Index	0.36	Moderately difficult
Discrimination Index	DI = 0.41	Excellent
Distractor efficiency	0.50	50% Functional

The question tests on a conceptual assessment component and is moderately difficult. The question discriminates excellently. The percentage of functional distractors is 50%. For this question the majority of students chose distractor A (the wrong option) instead of option C (the key). It seems that the question was challenging. Possibly the students got confused when determining the domain of the composition of a function and the inverse. It is also possible that they did not understand the question correctly.

Conclusions: The question is acceptable. It is moderately difficult and the indices are well balanced.

4.5.2 Trigonometry

Table 4.4 presents all the questions that are related to the trigonometry topic.

Table 4.4. Items related to trigonometry

No	ITEM	Question	Difficulty	Discrimination
1	ST1 2015	Q7	0.66	0.35
2	ST1 2015	Q8	0.35	0.40
3	ST1 2015	Q9	0.55	0.38
4	ST1 2015	Q16	0.38	0.45
5	Exam 2015	Q1	0.39	0.41
6	ST1 2016	Q6	0.47	0.48
7	ST1 2016	Q7	0.60	0.50
8	ST1 2016	Q12	0.54	0.48
9	ST1 2016	Q13	0.79	0.49
10	ST2 2016	Q1	0.39	0.39
11	ST2 2016	Q2	0.48	0.51
12	Exam 2016	Q1	0.43	0.47
13	ST1 2017	Q3	0.54	0.40
14	ST1 2017	Q4	0.61	0.48
15	ST1 2017	Q6	0.70	0.49
16	ST1 2017	Q9	0.76	0.46

Figure 4.3 presents a scatter plot of all the questions that are related to the trigonometry topic.

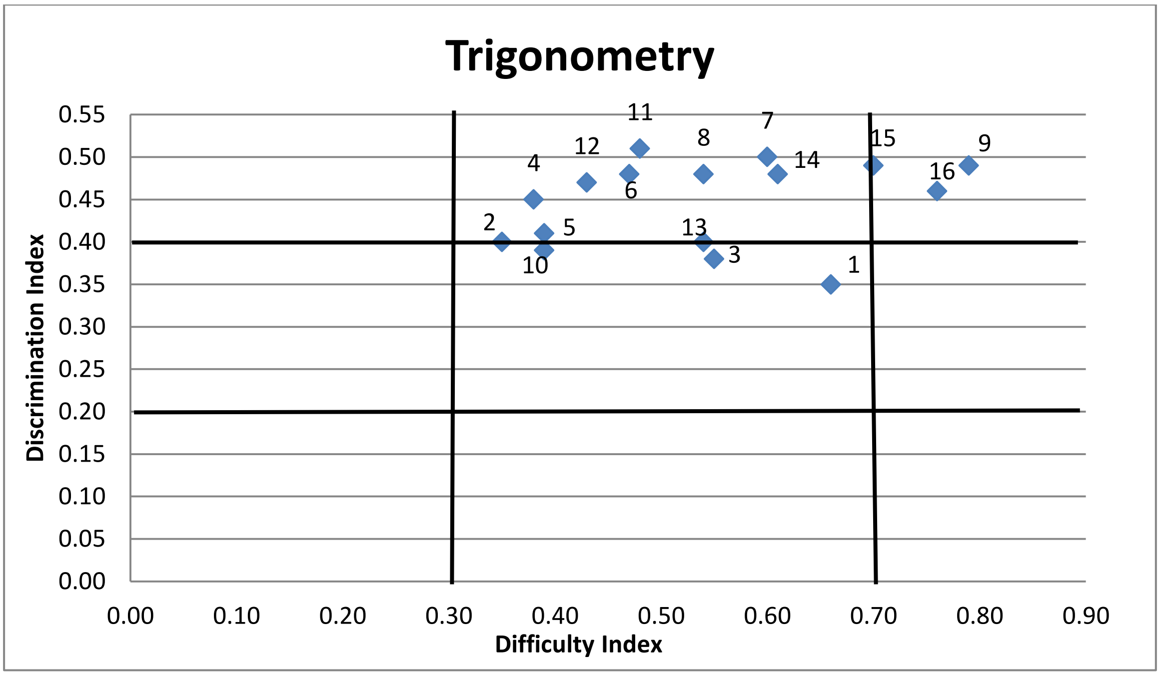


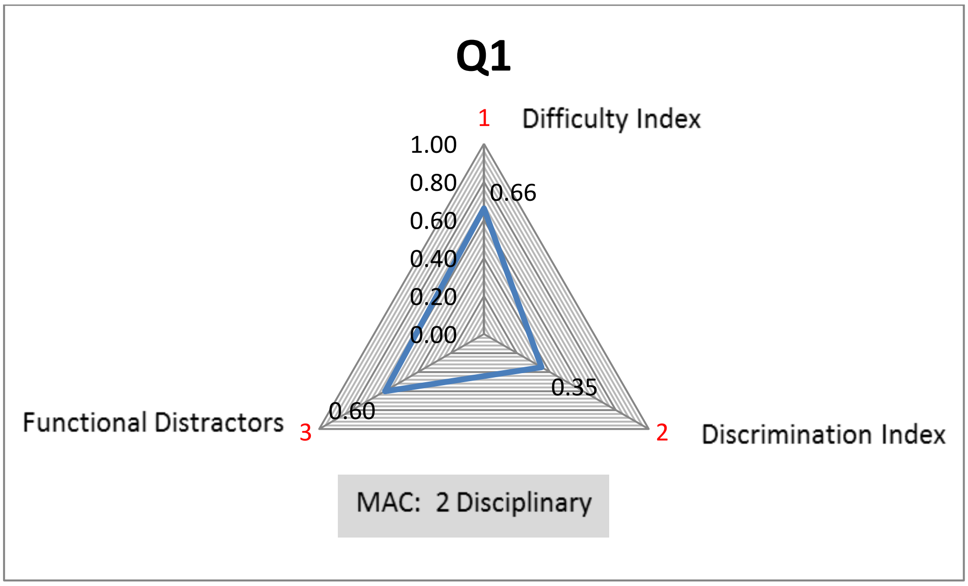
Figure 4.3. Scatter plot of all questions related to trigonometry

Questions 1, 2, 9, 10 and 11 are now discussed. The scatter plot in Figure 4.3 shows these questions to be problematic or commendable.

Q1:

Determine $\tan\left(-\frac{7\pi}{6}\right) =$
(A) $-\sqrt{3}$ (B) -1 (C) $\frac{1}{\sqrt{3}}$ (D) $\sqrt{3}$ (E) $-\frac{1}{\sqrt{3}}$ (F) None of these

Question 7, Semester test 1, 2015



Q1	Value	Comment
Topic		Trigonometry
Math Assessment Component	2	Disciplinary
Difficulty Index	0.66	Moderately easy
Discrimination Index	DI = 0.35	Good
Distractor efficiency	0.60	60% Functional

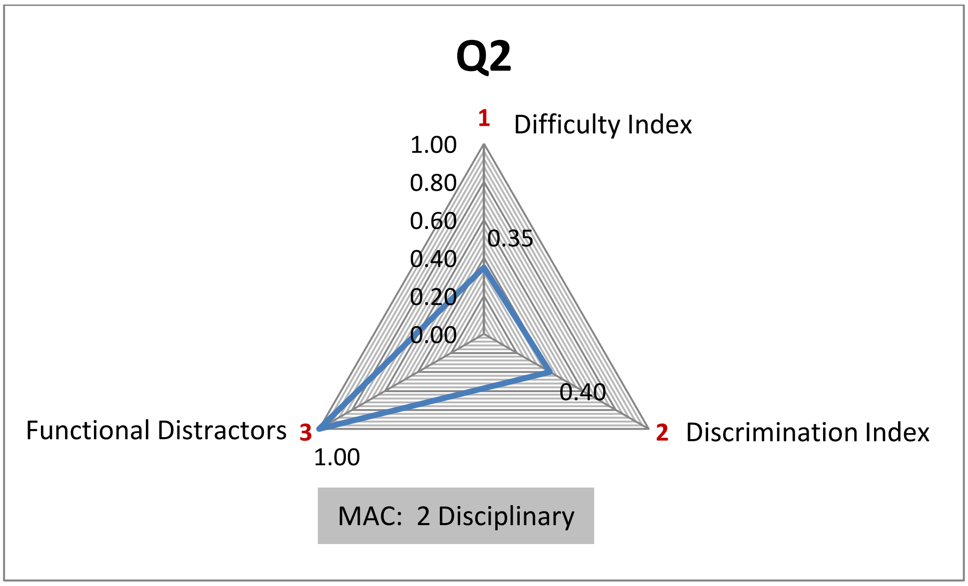
The question has a MAC Level 2 (disciplinary) and the discrimination between stronger and weaker students is good. The percentage of functional distractors is 60% and the question is moderately easy with $p = 0.66$.

Conclusion: The question is acceptable with well-balanced indices.

Q2:

Determine $\operatorname{cosec}\left(\frac{47\pi}{2}\right) =$					
(A) 1	(B) -1	(C) 0	(D) $\frac{\sqrt{3}}{2}$	(E) $-\frac{2}{\sqrt{2}}$	(F) None of these

Question 8, Semester test 1, 2015



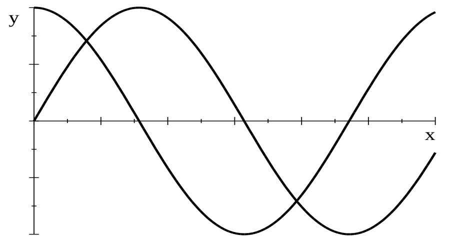
Q2	Value	Comment
Topic		Trigonometry
Math Assessment Component	2	Disciplinary
Difficulty Index	0.35	Moderately difficult
Discrimination Index	DI = 0.40	Excellent
Distractor efficiency	1.00	All distractors functional

By first observation Question 1 and Question 2 look alike, but when comparing the results it seems that the two questions are quite different with regard to the outcome. This question is more challenging with a difficulty index of $p = 0.35$ and discriminates excellently. With all the distractors being functional proves that no distractors were evidently the wrong answers.

Conclusions: The question is acceptable since it discriminates excellently irrespective of its lower difficulty index. The distractors are also well chosen.

Q9:

The graphs of $y = \sin x$ and $y = \cos x$ on $[0, 2\pi]$ are shown on the same set of axes:

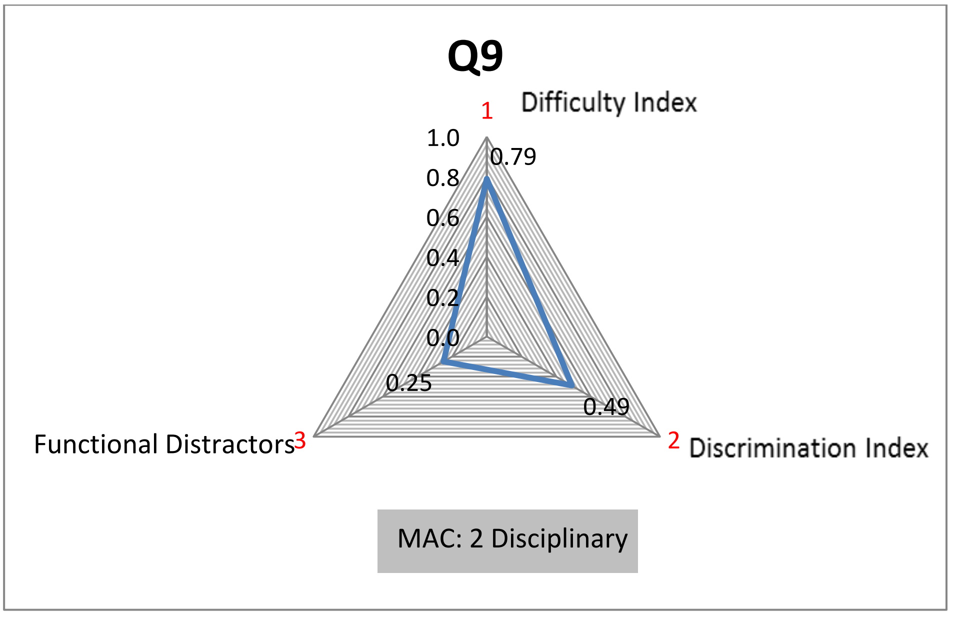


The solution of $\sin x > \cos x$ is

(A) $x \in (\frac{\pi}{4}, \frac{5\pi}{4})$ (B) $x \in (0, \frac{\pi}{4}) \cup (\frac{5\pi}{4}, \frac{7\pi}{4})$ (C) $x \in (\frac{\pi}{4}, \frac{7\pi}{4})$

(D) $x \in (\frac{\pi}{2}, \pi)$ (E) None of these

Question 13, Semester test 1, 2016



Q9	Value	Comment
Topic		Trigonometry
Math Assessment Component	2	Disciplinary
Difficulty Index	0.79	Easy
Discrimination Index	DI = 0.49	Excellent
Distractor efficiency	0.25	25% Functional

The question is moderately easy with a difficulty index of $p = 0.79$. The question discriminates excellently. The use of radians instead of degrees may have contributed to the excellent discrimination. The 25% functional distractors go hand in hand with the high difficulty index, which proves that the answer was evident to the majority of students.

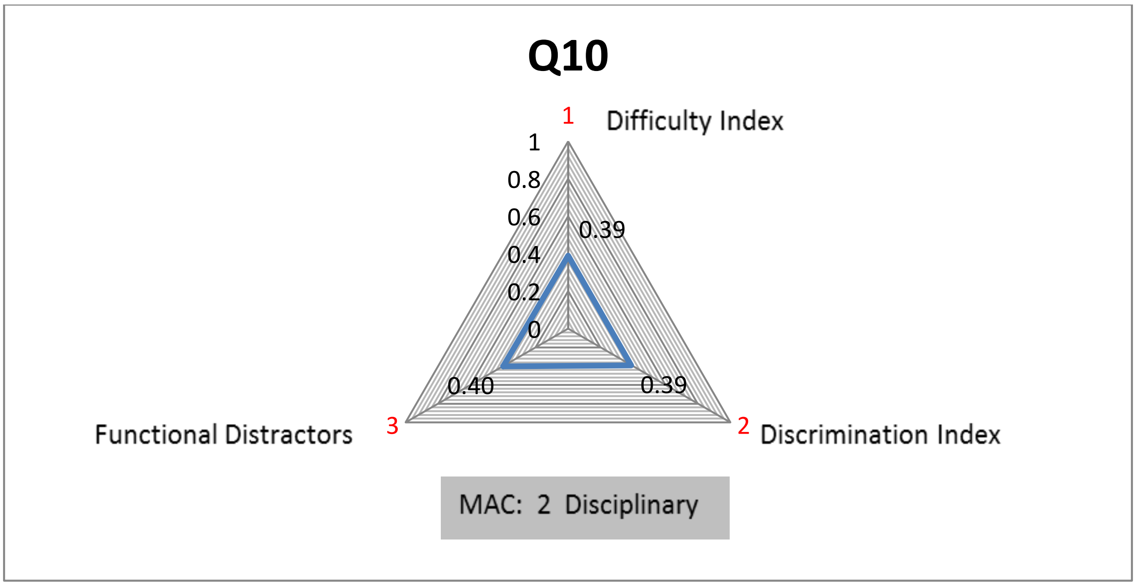
Conclusion: The question is acceptable.

Q10:

$$\cos^{-1}\left(\cos(-\frac{\pi}{3})\right) =$$

(A) $-\frac{2\pi}{3}$ (B) $-\frac{\pi}{3}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$ (E) π (F) None of these

Question 1, Semester test 2, 2016



Q10	Value	Comment
Theme		Trigonometry
Math Assessment Component	2	Disciplinary
Difficulty Index	0.39	Moderately difficult
Discrimination Index	DI = 0.39	Good
Distractor efficiency	0.40	40% Functional

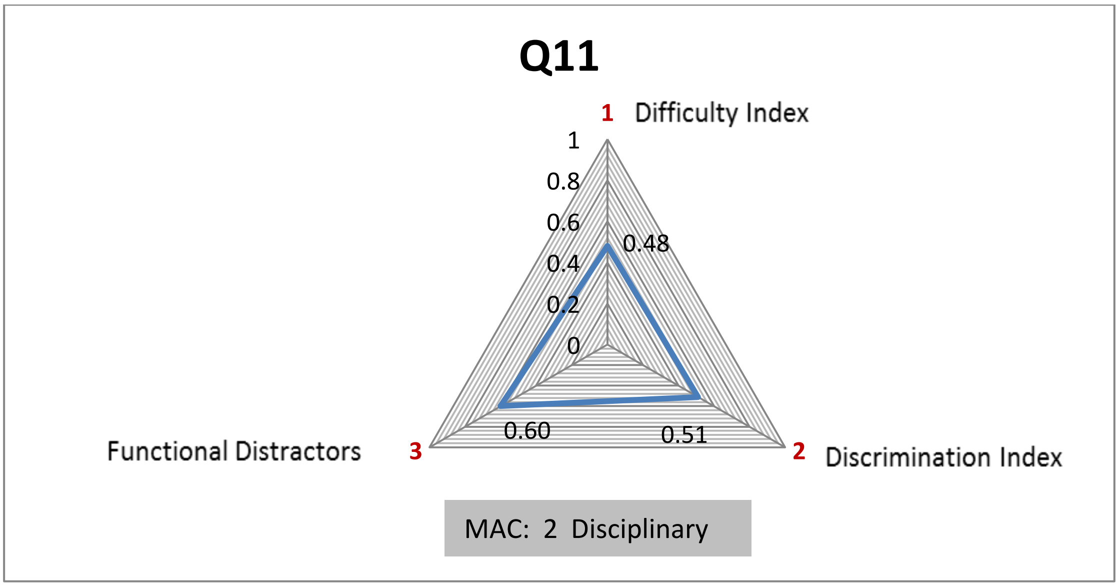
The question can be regarded as moderately difficult with a difficulty index of $p = 0.39$. The discrimination is good and 40% of the distractors are functional. It seems that the compounded expression of a trigonometric function with its inverse function can be challenging.

Conclusion: The question is acceptable but can be enhanced by changing the composition to two different trigonometric functions.

Q11:

The domain of $f(x) = \sin^{-1}(2x+1)$ is		
(A) $(-\frac{\pi}{2}, \frac{\pi}{2})$	(B) $[-\frac{\pi}{2}, \frac{\pi}{2}]$	(C) $(-1,1)$
(D) $[-1,0]$	(E) $(-1,0)$	(F) None of these

Question 2, Semester test 2, 2016



Q11	Value	Comment
Topic		Trigonometry
Math Assessment Component	2	Disciplinary
Difficulty Index	0.48	Moderately difficult
Discrimination Index	DI = 0.51	Excellent
Distractor efficiency	0.60	60% Functional

The question has a MAC Level 2 (disciplinary) and discriminates excellently between stronger and weaker students. The percentage of functional distractors is 60%. With a difficulty index of $p = 0.48$ the question is not particularly easy. It seems that the domain of a transformed, inverse trigonometric function can be challenging.

Conclusion: The question is acceptable with well-balanced indices and well-chosen distractors.

4.5.3 Absolute value and inequalities

Table 4.5 presents all the questions that are related to the absolute value and inequalities topic.

Table 4.5. Items related to absolute value and inequalities

No	ITEM	Question	Difficulty	Discrimination
1	ST1 2015	Q2	0.84	0.28
2	ST1 2015	Q3	0.74	0.38
3	ST1 2015	Q4	0.70	0.38
4	Exam 2015	Q5	0.71	0.36
5	ST1 2016	Q1	0.74	0.42
6	ST1 2016	Q3	0.82	0.49
7	ST1 2016	Q11	0.63	0.49
8	ST1 2017	Q8	0.48	0.52
9	Exam 2017	Q1	0.62	0.48

Figure 4.4 presents a scatter plot of all the questions that are related to the absolute value and inequalities topic.

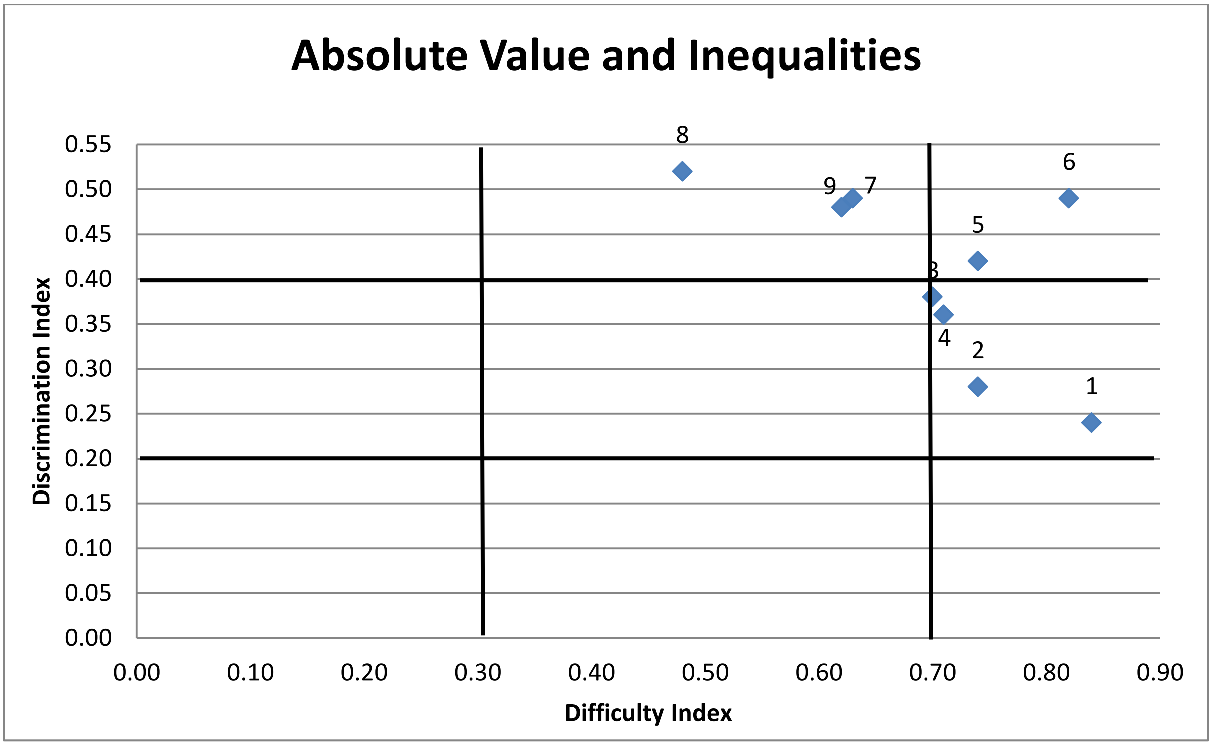


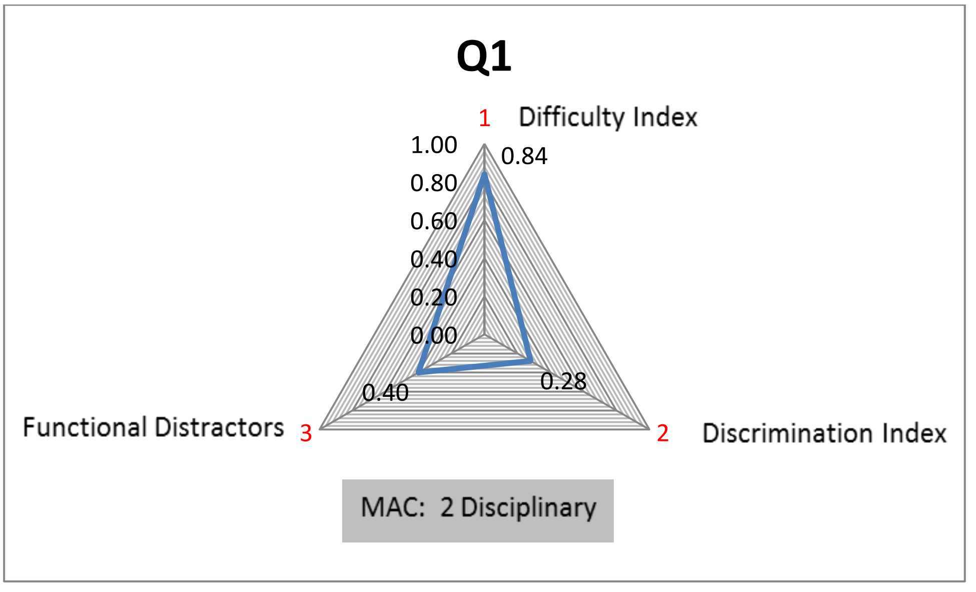
Figure 4.4. Scatter plot of all questions related to absolute value and inequalities

Questions 1, 2, 4, 6 and 8 are now discussed. The scatter plot in Figure 4.4 shows these questions to be problematic or commendable.

Q1:

The solution of $\lvert 3x-1\rvert \geq 5$ is
(A) $x \geq -\frac{4}{3}$ or $x \leq 2$ (B) $x \leq -\frac{4}{3}$ or $x \geq 2$ (C) $x \geq 2$ or $x \leq \frac{4}{3}$
(D) $x \leq -2$ and $x \geq \frac{4}{3}$ (E) $x \geq -\frac{4}{3}$ and $x \leq 2$ (F) None of these

Question 2, Semester test 1, 2015



Q1	**Value**	**Comment**
Topic		Absolute value & inequality
Math Assessment Component	2	Disciplinary
Difficulty Index	0.84	Easy
Discrimination Index	DI = 0.28	Fair
Distractor efficiency	0.40	40% Functional

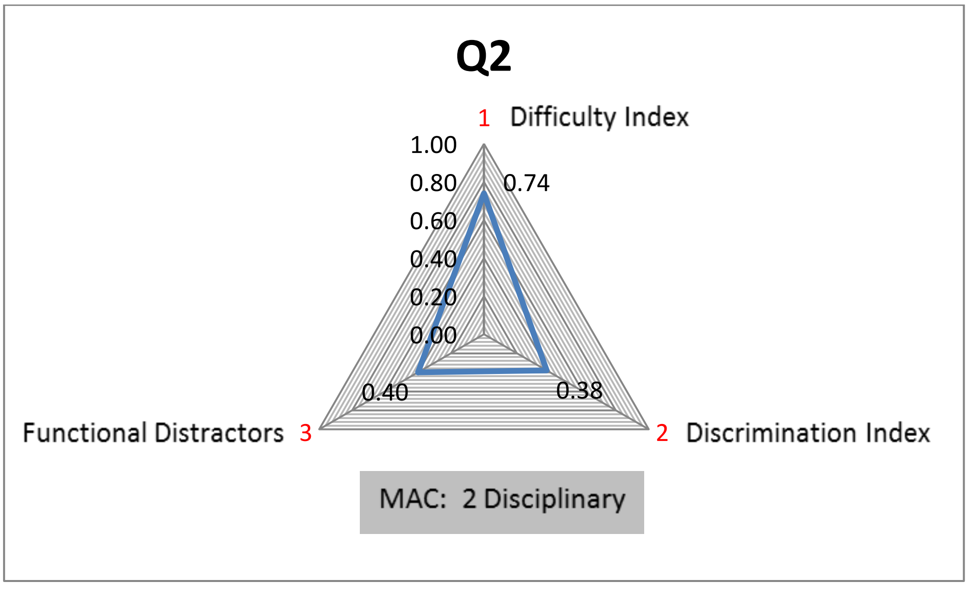
The question can be regarded as easy with a difficulty index of $p =$ 0.84. The question has 40% functional distractors, which show that the correct answer was evident to most students. The fair discrimination index goes hand in hand with the high difficulty index. The MAC level is 2, assessing at the disciplinary cognitive level.

Conclusion: This question is flagged as problematic. The question is easy and does not discriminate well between strong and weak students.

Q2:

If $-x^2 + 4x - 3 > 0$, then		
(A) $x \in (0,3)$	(B) $x \in (1,3]$	(C) $x = 2$
(D) $x \in (1,3)$	(E) $x \in (-\infty, 1) \cup (3, \infty)$	(F) None of these

Question 3, Semester test 1, 2015



Q2	Value	Comment
Topic		Inequality
Math Assessment Component	2	Disciplinary
Difficulty Index	0.74	Easy
Discrimination Index	DI = 0.38	Good
Distractor efficiency	0.40	40% Functional

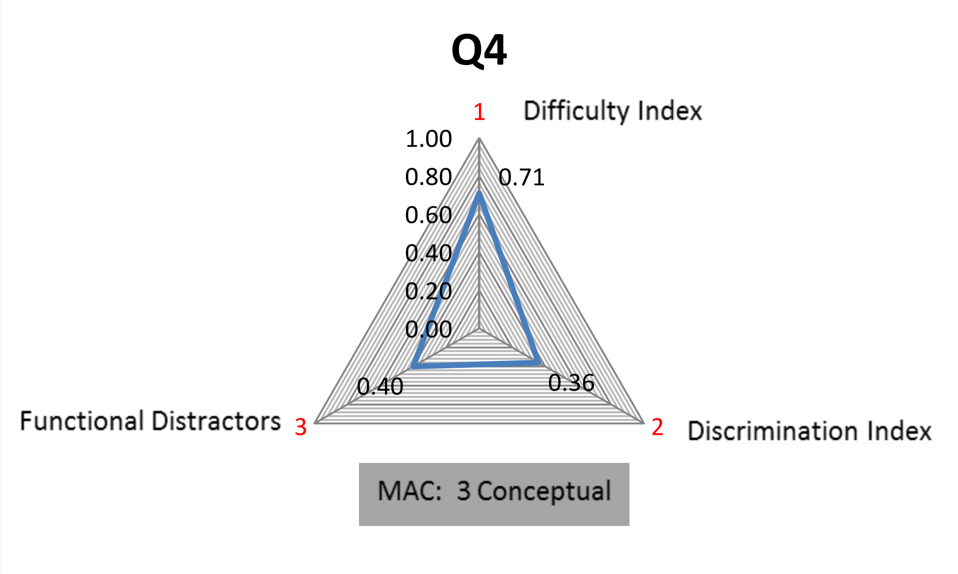
The question has a difficulty index of $p = 0.74$. The correct answer is evident, which led to the 40% functional distractors. The discrimination between stronger and weaker students is also good.

Conclusion: The question can be made more challenging by using a more complicated inequality function.

Q4:

The solution of $\frac{x+3}{\lvert x-2 \rvert} \le 0$ is
(A) $-3 \le x < 2$ (B) $x \le -3$ or $x > 2$ (C) $-3 < x < 2$
(D) $x \le -3$ and $x > 2$ (E) $-3 \le x < 2$ (F) None of these

Question 5, exam, 2015



Q4	Value	Comment
Topic		Absolute value & inequality
Math Assessment Component	3	Conceptual
Difficulty Index	0.71	Easy
Discrimination Index	DI = 0.36	Good
Distractor efficiency	0.40	40% Functional

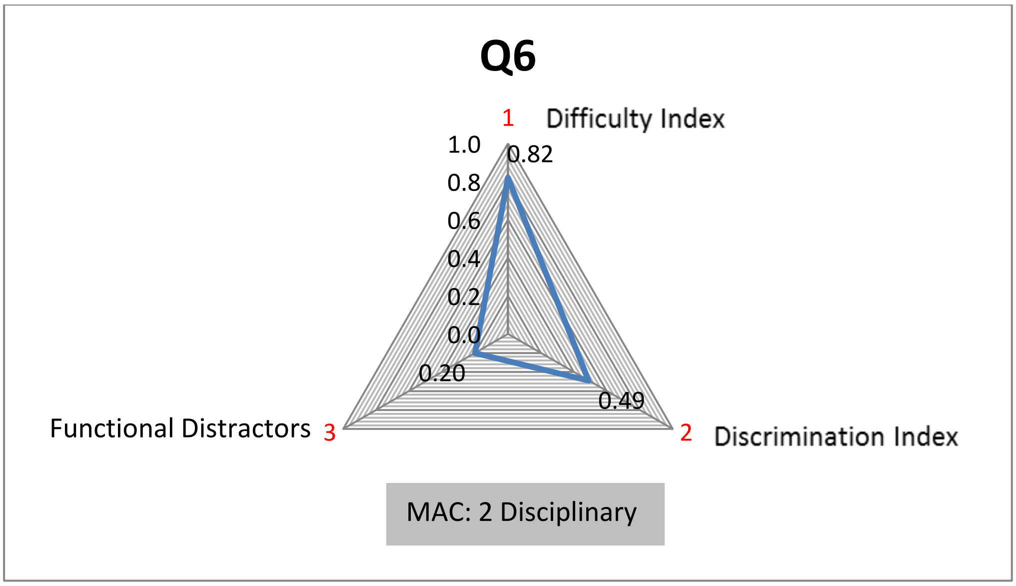
The question assesses on MAC Level 3 (conceptual), which is on the third cognitive level. The question can be regarded as easy with a difficulty index of $p = 0.71$. The discrimination is good but only 40% of the distractors are functional. Interestingly almost the same question was asked in Semester test 1 of the same year with minor differences. Here the students found the question more challenging ($p = 0.70$) and it had a weaker discrimination index of DI = 0.34 and more functional distractors (60%), which shows no significant improvement in the students.

Conclusion: The question is acceptable.

Q6:

If $\lvert x-3\rvert = \lvert 2x+1\rvert$, then		
(A) $x=\frac{1}{3}$ or $x=\frac{2}{3}$	.(B) $x=\frac{2}{3}$ or $x=-\frac{1}{4}$	(C) $x=\frac{2}{3}$ or $x=-4$
(D) $x=\frac{2}{3}$ or $x=-\frac{2}{3}$	(E) $x=\frac{1}{3}$ or $x=\frac{3}{4}$	(F) None of these

Question 3, Semester test 1, 2016



Q6	**Value**	**Comment**
Topic		Absolute value
Math Assessment Component	2	Disciplinary
Difficulty Index	0.82	Easy
Discrimination Index	DI = 0.49	Excellent
Distractor efficiency	0.20	20% Functional

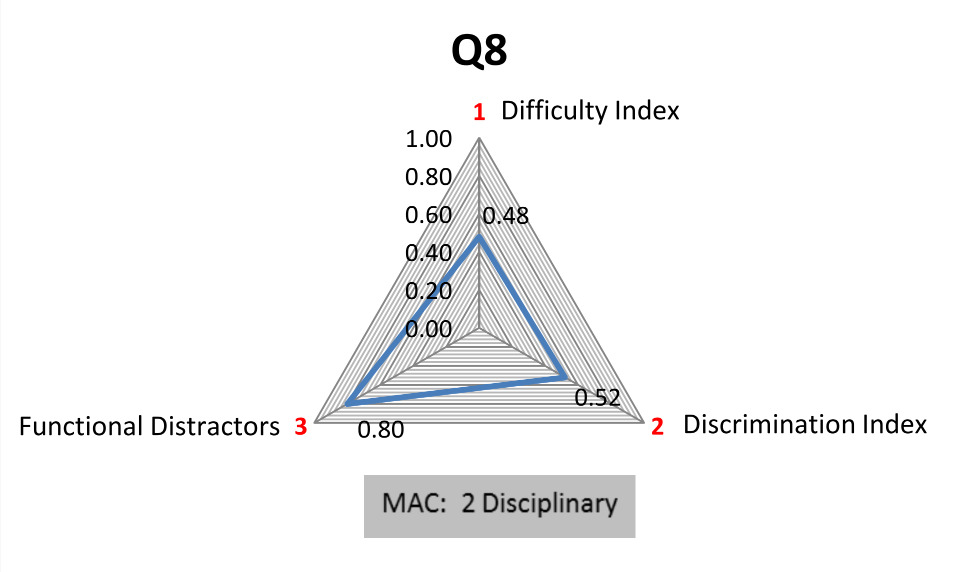
Although the question can be regarded as easy with a difficulty index of $p = 0.82$, the discrimination remains excellent with a discrimination index of DI = 0.49. Only 20% of the distractors are functional. The question can be improved by using more complicated absolute value functions.

Conclusion: The question is acceptable but can be made more challenging.

Q8:

The solution of $\frac{\ln(x-2)}{x^2+1} \leq 0$ is		
(A) $x \in (2,3]$	(B) $x \in (-\infty, 2)$	(C) $x \in (0,2]$
(D) $x \in (-\infty, 3]$	(E) $x \in (-\infty, 3)$	(F) None of these

Question 8, Semester test 1, 2017



Q8	**Value**	**Comment**
Topic		Logarithm inequality
Math Assessment Component	2	Disciplinary
Difficulty Index	0.48	Moderately difficult
Discrimination Index	DI = 0.52	Excellent
Distractor efficiency	0.80	80% Functional

The question can be regarded as acceptable since it has an excellent discrimination index of DI = 0.52. Upon that the question has 80% functional distractors. From the indices it seems that the weaker students found the moderately difficult question challenging.

Conclusion: The question is acceptable with well-balanced indices.

4.5.4 Exponential and logarithmic functions

Table 4.6 presents all the questions that are related to the exponential and logarithmic functions topic.

Table 4.6. Items related to exponential and logarithmic functions

No	ITEM	Question	Difficulty	Discrimination
1	ST1 2015	Q1	0.95	0.24
2	ST1 2015	Q10	0.32	0.43
3	ST1 2015	Q12	0.68	0.36
4	ST1 2015	Q13	0.69	0.39
5	ST1 2015	Q15	0.42	0.47
6	ST1 2016	Q2	0.77	0.49
7	ST1 2016	Q5	0.85	0.48
8	ST1 2016	Q9	0.59	0.48
9	ST1 2016	Q10	0.64	0.52
10	Exam 2016	Q2	0.27	0.48
11	ST1 2017	Q2	0.21	0.30
12	ST1 2017	Q5	0.22	0.38
13	ST1 2017	Q11	0.77	0.45
14	Exam 2017	Q3	0.77	0.46

Figure 4.5 presents a scatter plot of all the questions that are related to the exponential and logarithmic function topic.

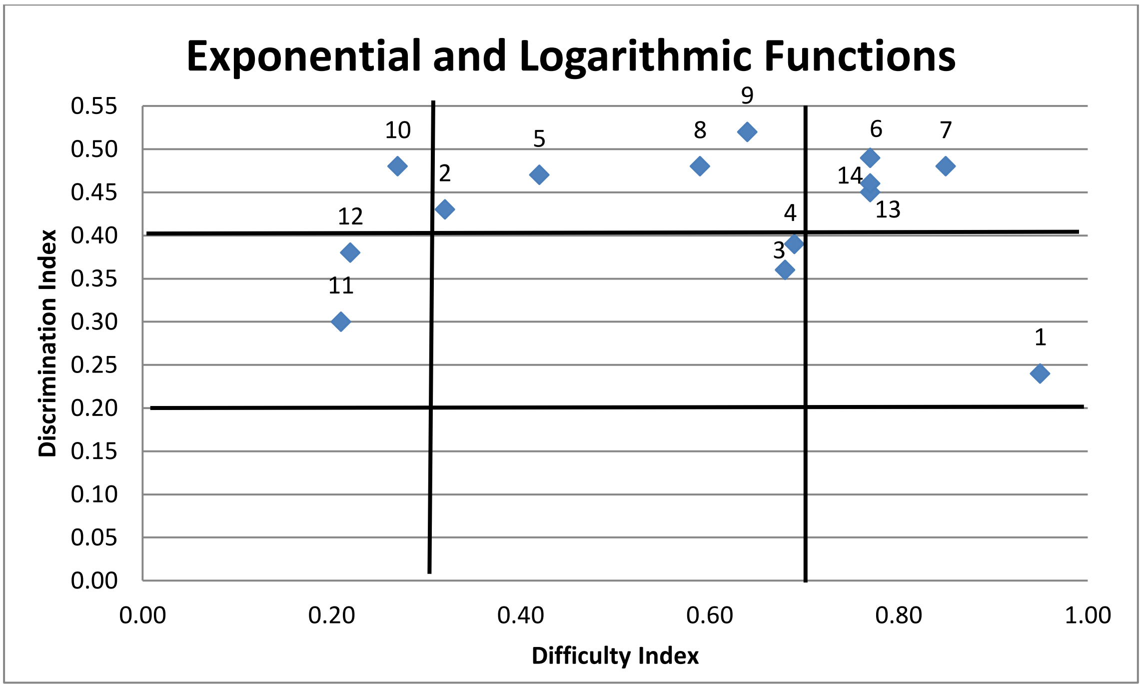


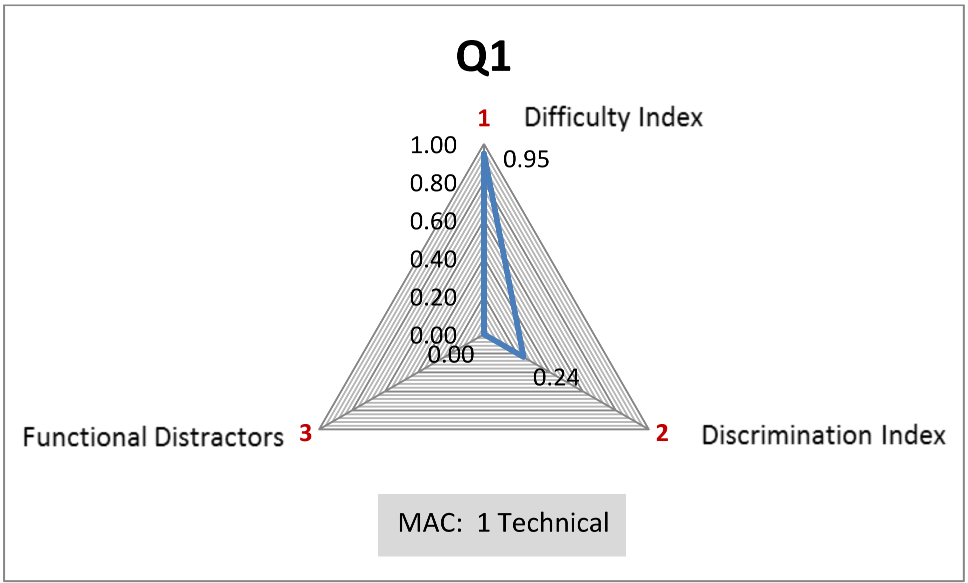
Figure 4.5. Scatter plot of all questions related to exponential and logarithmic functions

Questions 1, 7, 9, 11 and 12 are now discussed. The scatter plot in Figure 4.5 shows these questions to be problematic or commendable.

Q1:

If $\frac{(e^{3x})^2 e^{2x}}{e^{5x}} = e^a$, then $a =$
(A) $3x$ (B) $9x^2 - 3x$ (C) $\frac{18}{5}x^2$ (D) $-3x$ (E) $9x^2$ (F) None of these

Question 1, Semester test 1, 2015



Q1	Value	Comment
Topic		Exponential & Log functions
Math Assessment Component	1	Technical
Difficulty Index	0.95	Easy
Discrimination Index	DI = 0.24	Fair
Distractor efficiency	0.00	No functional distractors

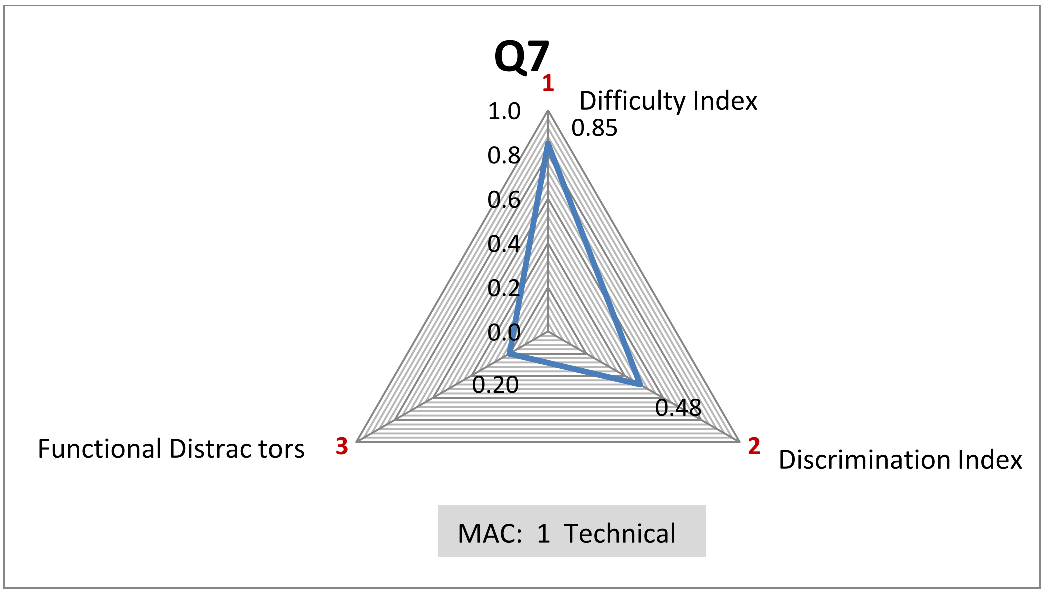
The question can be regarded as easy with a difficulty index of $p = 0.95$. The MAC level is 1 (technical), which is the lowest cognitive level. The question has no functional distractors, which show that the correct answer was evident to most students. The fair discrimination index goes hand in hand with the high difficulty index.

Conclusion: This question is flagged as problematic. The question is direct and easy and does not discriminate well between strong and weak students.

Q7:

$3\ln(e^{2x}(e^x)^3) + 2e^{\ln 1} =$
(A) $15x + 2$ (B) $15x$ (C) 4 (D) 1 (E) 0 (F) None of these

Question 5, Semester test 1, 2016



Q7	Value	Comment
Topic		Exponential & Log functions
Math Assessment Component	1	Technical
Difficulty Index	0.85	Easy
Discrimination Index	DI = 0.48	Excellent
Distractor efficiency	0.20	20% Functional

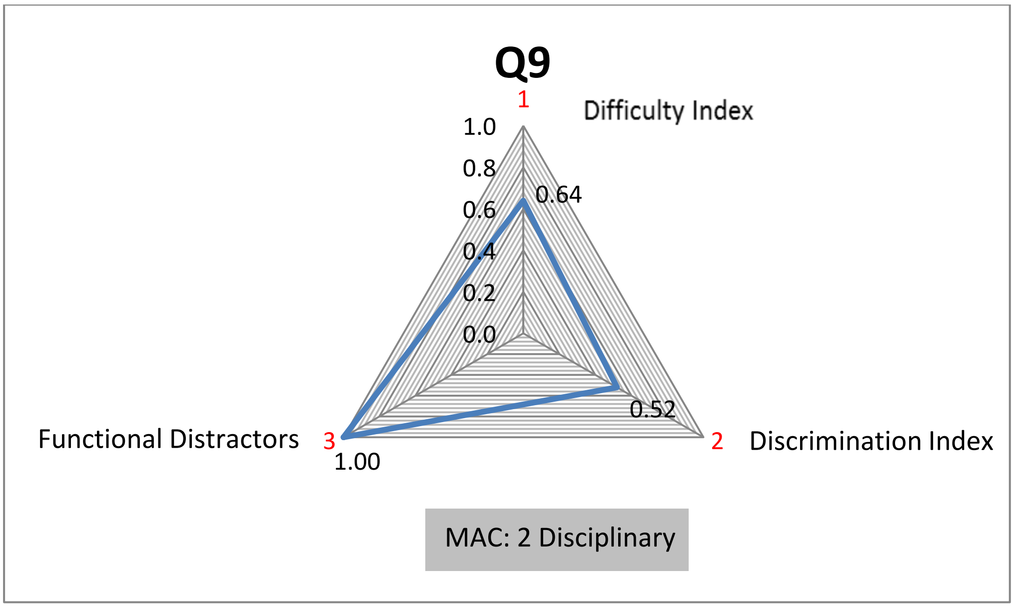
The question can be regarded as easy with a difficulty index of $p =$ 0.85. The question has only 20% functional distractors, which show that the correct answer was evident to most students. The discrimination index is excellent which makes this question a more acceptable question than the previous MAC Level 1 question.

Conclusion: The question is acceptable with excellent discrimination but the high difficulty index contributes to the 20% functional distractors.

Q9:

The domain of the function $f(x) = \frac{4}{x}\ln(4 - x^2)$ is
(A) $(-\infty, 0) \cup (0, \infty)$ (B) $x \leq \pm 2, x \neq 0$ (C) $(-2,0) \cup (0,2)$ (D) None of these

Question 10, Semester test 1, 2016



Q9	Value	Comment
Topic		Logarithmic functions
Math Assessment Component	2	Disciplinary
Difficulty Index	0.64	Moderately easy
Discrimination Index	DI = 0.52	Excellent
Distractor efficiency	1.00	All distractors functional

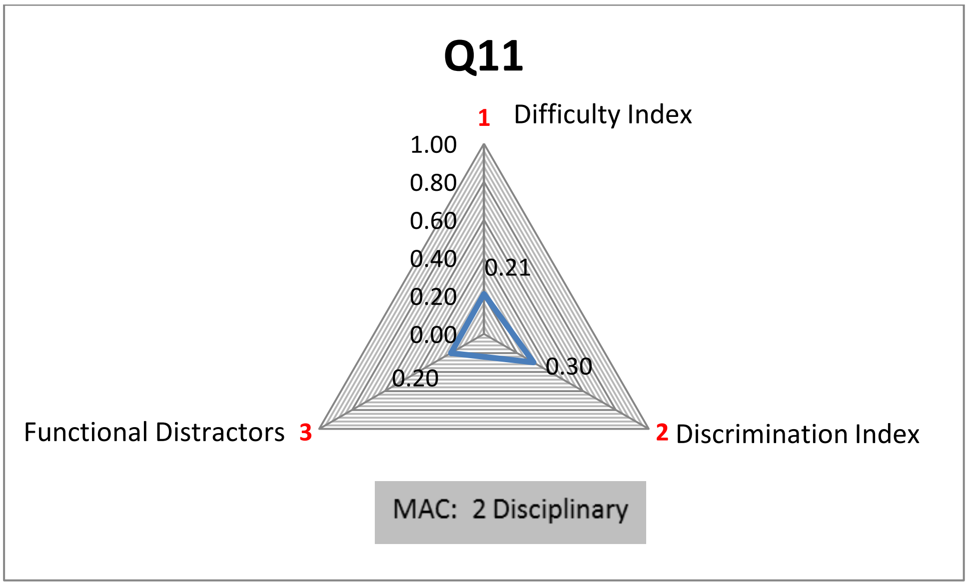
The question has an excellent discrimination (DI = 0.52) and a moderately easy difficulty index. All the distractors are functional.

Conclusion: An acceptable question with well-balanced indices and well-chosen functional distractors.

Q11:

If $2\ln x - \ln(x+2) = 0$, then
(A) $x = -1$ or $x = 2$ (B) $x = 0$ (C) $x = 1 \pm \sqrt{2}$
(D) $x = -1 + \sqrt{2}$ (E) $x = 2$ (F) None of these

Question 2, Semester test 1, 2017



Q11	**Value**	**Comment**
Topic		Exponential & Log functions
Math Assessment Component	2	Disciplinary
Difficulty Index	0.21	Difficult
Discrimination Index	DI = 0.30	Good
Distractor efficiency	0.20	20% Functional

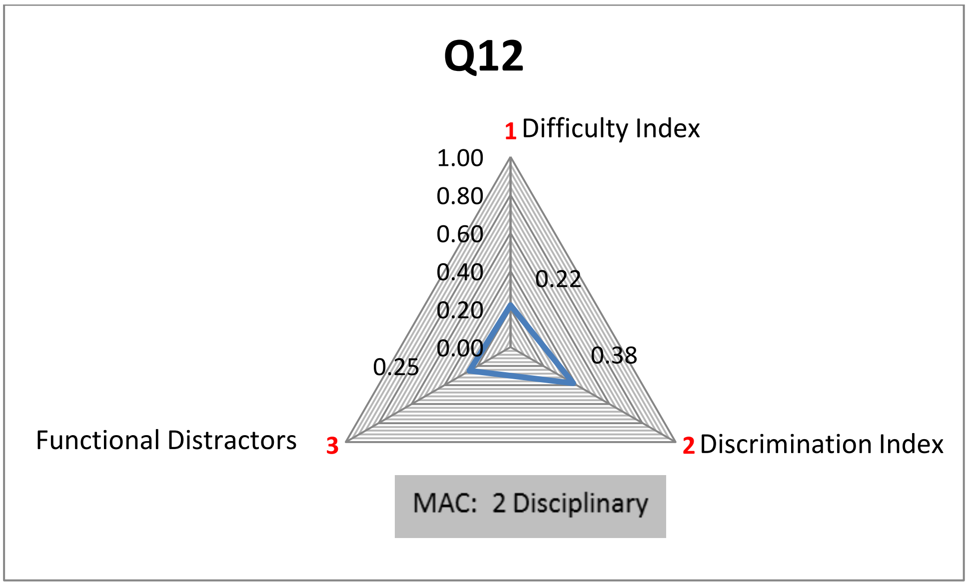
The question can be regarded as difficult with a difficulty index of $p = 0.21$. Only 20% of the distractors are functional. The discrimination is good between the stronger and weaker students. It seems that solving a natural logarithm is challenging. For Question 2 the majority of students (67%) chose the wrong option (A), whereas only 21% chose the key (option E). It is evident that the students forgot to check the domain of the equation since only answers greater than zero were valid.

Conclusion: The question is not problematic. It seems that the students forgot the properties of a natural logarithm.

Q12:

Given $y = e^{-x}$. The equation of the graph that results from shifting the function 2 units right is
(A) $e^{-x} - 2$ (B) e^{-x+2} (C) e^{-x-2} (D) $2e^{-x-2}$ (E) None of these

Question 5, Semester test 1, 2017



Q12	Value	Comment
Topic		Exponential functions
Math Assessment Component	2	Disciplinary
Difficulty Index	0.22	Difficult
Discrimination Index	DI = 0.38	Good
Distractor efficiency	0.25	25% Functional

The question can be regarded as difficult with a difficulty index of $p = 0.22$. Only 25% of the distractors are functional. The discrimination is good with a discrimination index of DI = 0.38. It seems that transforming an exponential function is challenging. For this question the majority of students (72%) chose the wrong option (C), whereas only 22% chose the key (option B). It is evident that the students did not substitute the x with an $x - 2$ in brackets resulting in a sign mistake: $-(x - 2) = -x + 2$.

Conclusion: The question is not problematic. A general mistake was made by the majority of students.

4.5.5 Limits and continuity

Table 4.7 presents all the questions that are related to the limits and continuity topic.

Table 4.7. Items related to limits and continuity

No	ITEM	Question	Difficulty	Discrimination
1	ST1 2015	Q17	0.58	0.42
2	ST1 2015	Q18	0.86	0.30
3	ST1 2015	Q19	0.53	0.40
4	ST2 2015	Q7	0.75	0.34
5	ST2 2015	Q10	0.64	0.50
6	Exam 2015	Q6	0.55	0.40
7	ST2 2016	Q3	0.68	0.42
8	ST2 2016	Q4	0.48	0.37
9	ST2 2016	Q5	0.57	0.43
10	ST2 2016	Q6	0.64	0.44
11	ST2 2016	Q7	0.51	0.42
12	ST2 2016	Q10	0.79	0.46
13	Exam 2016	Q6	0.58	0.43
14	ST2 2017	Q4	0.51	0.48
15	Exam 2017	Q2	0.64	0.39
16	Exam 2017	Q4	0.79	0.37

Figure 4.6 presents a scatter plot of all the questions that are related to the limits and continuity topic.

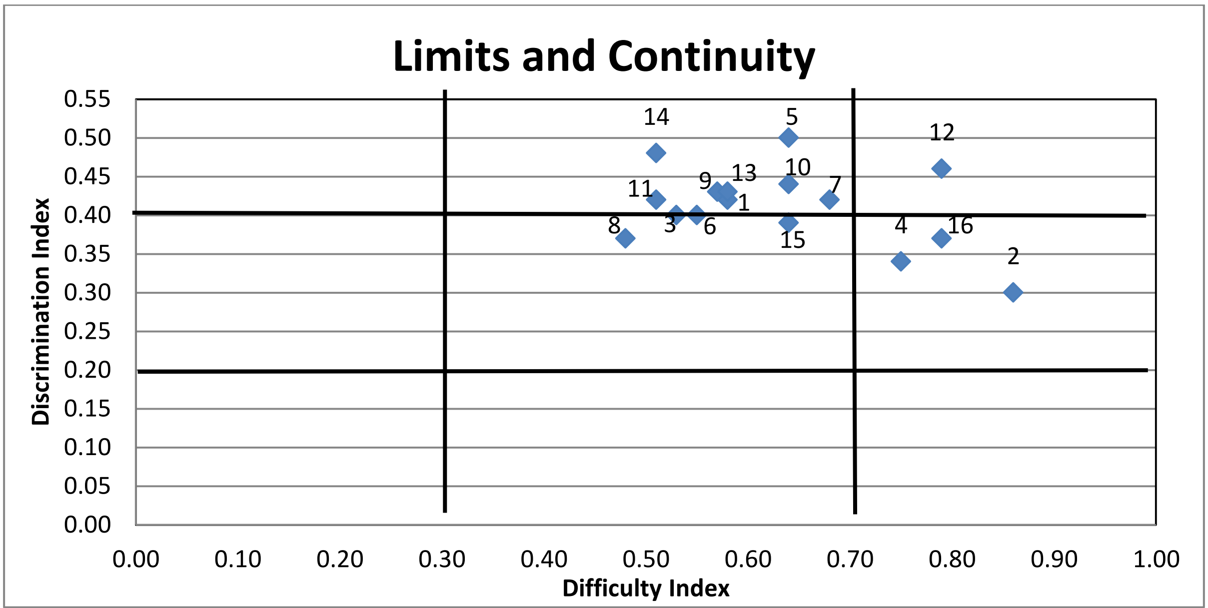


Figure 4.6. Scatter plot of all questions related to limits and continuity

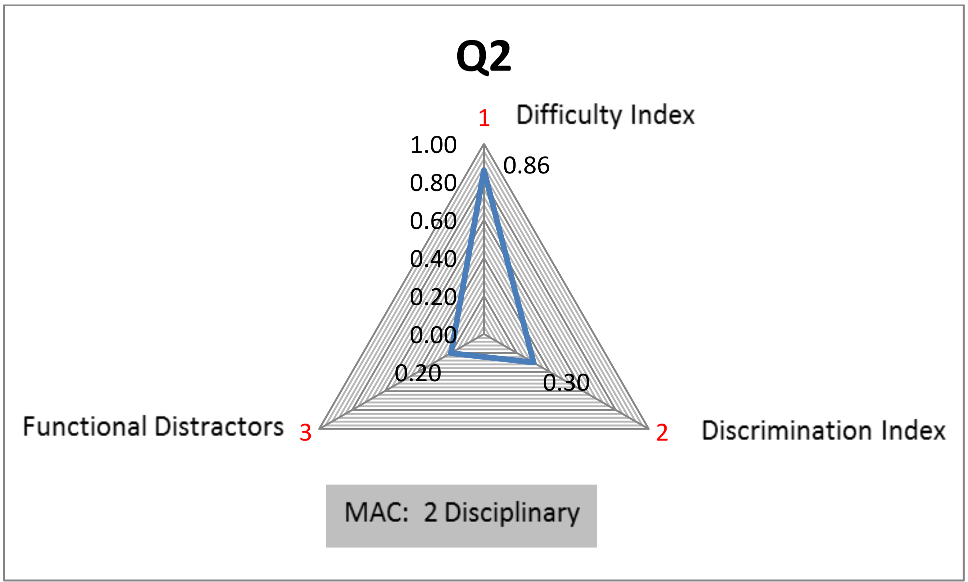
Questions 2, 5, 8, 12 and 14 are now discussed. The scatter plot in Figure 4.6 shows these questions to be problematic or commendable.

Q2:

$$\lim_{x \to a} \frac{x^2 - 2ax + a^2}{x^2 - a^2} =$$

(A) $\frac{1}{2a}$ (B) $-\frac{1}{2a}$ (C) $-\frac{1}{2a^2}$ (D) $\frac{1}{2a^2}$ (E) 0 (F) None of these

Question 18, Semester test 1, 2015



Q2	**Value**	**Comment**
Topic		Limits
Math Assessment Component	2	Disciplinary
Difficulty Index	0.86	Easy
Discrimination Index	DI = 0.30	Good
Distractor efficiency	0.20	20% Functional

The question can be regarded as easy with a difficulty index of $p = 0.86$. The difficulty goes hand in hand with the lower discrimination of DI = 0.30. The distractors are problematic since only 20% are functional.

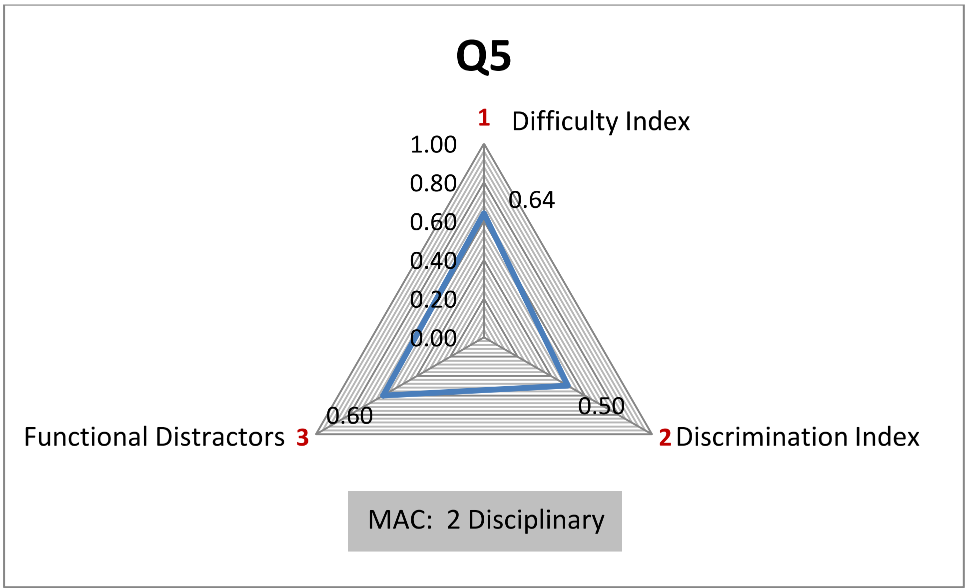
Conclusion: The question is too easy and the answer was evident to the majority of students. The question can be improved by complicating the expression.

Q5:

$$\lim_{t\to\infty} \frac{t - t\sqrt{t}}{2t^{3/2} + 3t - 5} =$$

(A) $-\frac{1}{2}$ (B) $-\frac{1}{3}$ (C) 0 (D) $-\frac{1}{5}$ (E) $-\frac{1}{6}$ (F) None of these

Question 10, Semester test 2, 2015



Q5	Value	Comment
Topic		Limits
Math Assessment Component	2	Disciplinary
Difficulty Index	0.64	Moderately easy
Discrimination Index	DI = 0.50	Excellent
Distractor efficiency	0.60	60% Functional

The question is similar to the previous question with the difference that the limit tends to infinity and not to a real number. The result is a higher difficulty index (p = 0.64), better discrimination (DI = 0.50) and 60% functional distractors.

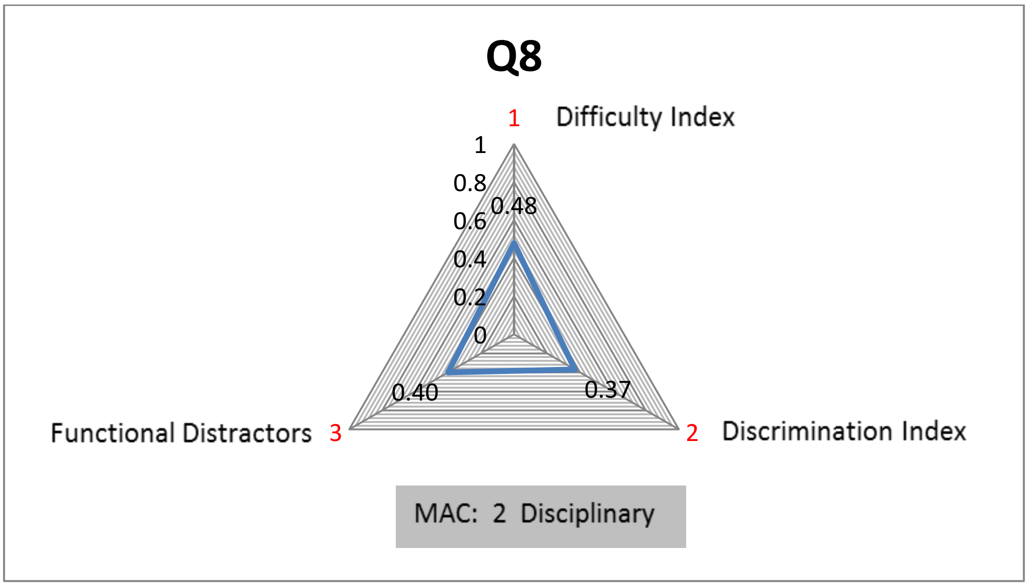
Conclusion: The question is acceptable with well-balanced indices and well-chosen functional distractors. It seems that the question is more difficult than the previous question but discriminates better.

Q8:

$$\lim_{x \to \infty} \tan \frac{6}{x} =$$

(A) 0 (B) ∞ (C) $-\frac{\pi}{2}$ (D) 1 (E) $\frac{\pi}{2}$ (F) None of these

Question 4, Semester test 2, 2016



Q8	**Value**	**Comment**
Topic		Limits & Trigonometry
Math Assessment Component	2	Disciplinary
Difficulty Index	0.48	Moderately difficult
Discrimination Index	DI = 0.37	Good
Distractor efficiency	0.40	40% Functional

The question assesses on MAC Level 2 (disciplinary) and combines two topics: limits and trigonometry. The question is moderately difficult but the discrimination is good between the stronger and weaker students. It seems that the periodic trigonometry function complicates the limit sum.

Conclusion: The question is acceptable.

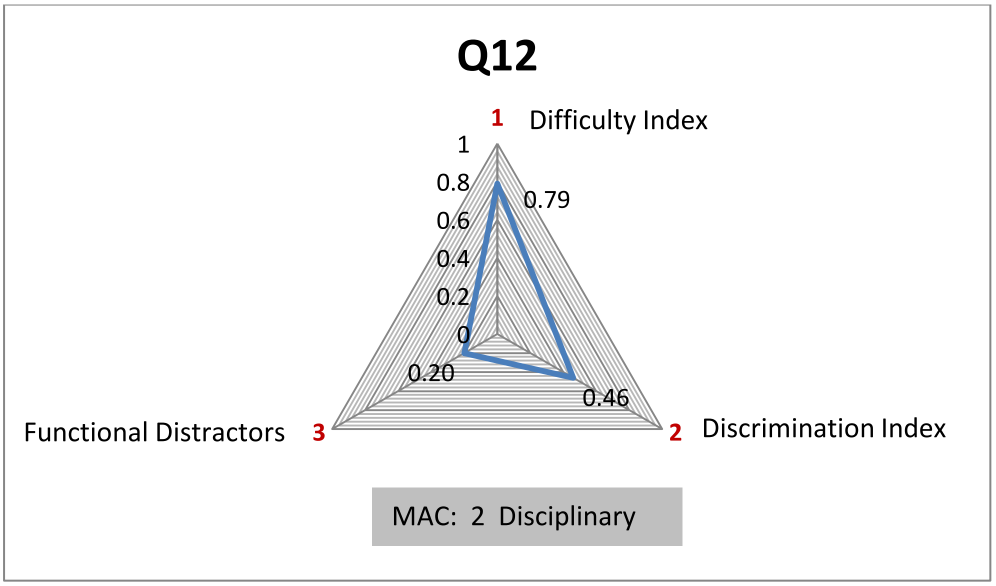
Q12:

$$\lim_{x \to -\infty} \frac{3x^2 + 27}{x^3 - 27} =$$

(A) $-\infty$ (B) ∞ (C) 0 (D) -9 (E) Does not exist

(F) None of these

Question 10, Semester test 2, 2016



Q12	**Value**	**Comment**
Topic		Limits
Math Assessment Component	2	Disciplinary
Difficulty Index	0.79	Easy
Discrimination Index	DI = 0.46	Excellent
Distractor efficiency	0.20	20% Functional

The question is easy with a difficulty index of $p = 0.79$. The question discriminates excellently between the stronger and weaker students (DI = 0.46) but only 20% of the distractors are functional.

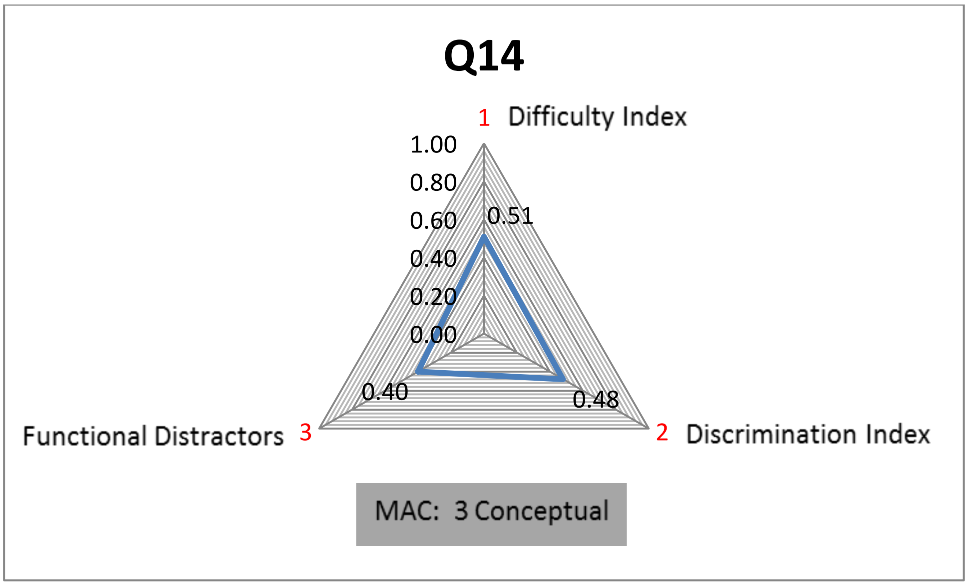
Conclusion: The question is acceptable with well-balanced indices. The distractors are evident because of the high difficulty index.

Q14:

$$\lim_{\theta \to 0} \frac{2\tan\theta}{2\theta + \sin 3\theta} =$$

(A) 0 (B) 5θ (C) $\frac{2}{3}$ (D) $\frac{5\theta}{2}$ (E) $\frac{2}{5}$ (F) None of these

Question 4, Semester test 2, 2017



Q14	**Value**	**Comment**
Theme		Limits & Trigonometry
Math Assessment Component	3	Conceptual
Difficulty Index	0.51	Moderately easy
Discrimination Index	DI = 0.48	Excellent
Distractor efficiency	0.40	40% Functional

The question assesses on MAC Level 3 (conceptual), which is the third cognitive level. The question combines trigonometry with limits. Students have to recognise the limit as a special limit since the students are not familiar with the Rule of L'Hospital yet. Only 18% of students chose option A. It seems zero (0) was substituted into the expression resulting in zero divided by zero which is not equal to zero. The question discriminates excellently between the weaker and stronger students. Only 40% of the distractors are functional.

Conclusion: The question is challenging but acceptable.

4.5.6 Derivatives

Table 4.8 presents all the questions that are related to the derivatives topic.

Table 4.8. Items related to derivatives

No	ITEM	Question	Difficulty	Discrimination
1	ST2 2015	Q2	0.49	0.51
2	ST2 2015	Q4	0.43	0.45
3	ST2 2015	Q9	0.67	0.45
4	Exam 2015	Q2	0.62	0.49
5	Exam 2015	Q3	0.56	0.44
6	Exam 2016	Q5	0.79	0.38
7	ST2 2017	Q1	0.78	0.40
8	ST2 2017	Q5	0.95	0.23
9	ST2 2017	Q6	0.60	0.43
10	ST2 2017	Q7	0.84	0.46

Figure 4.7 presents a scatter plot of all the questions that are related to the derivatives topic.

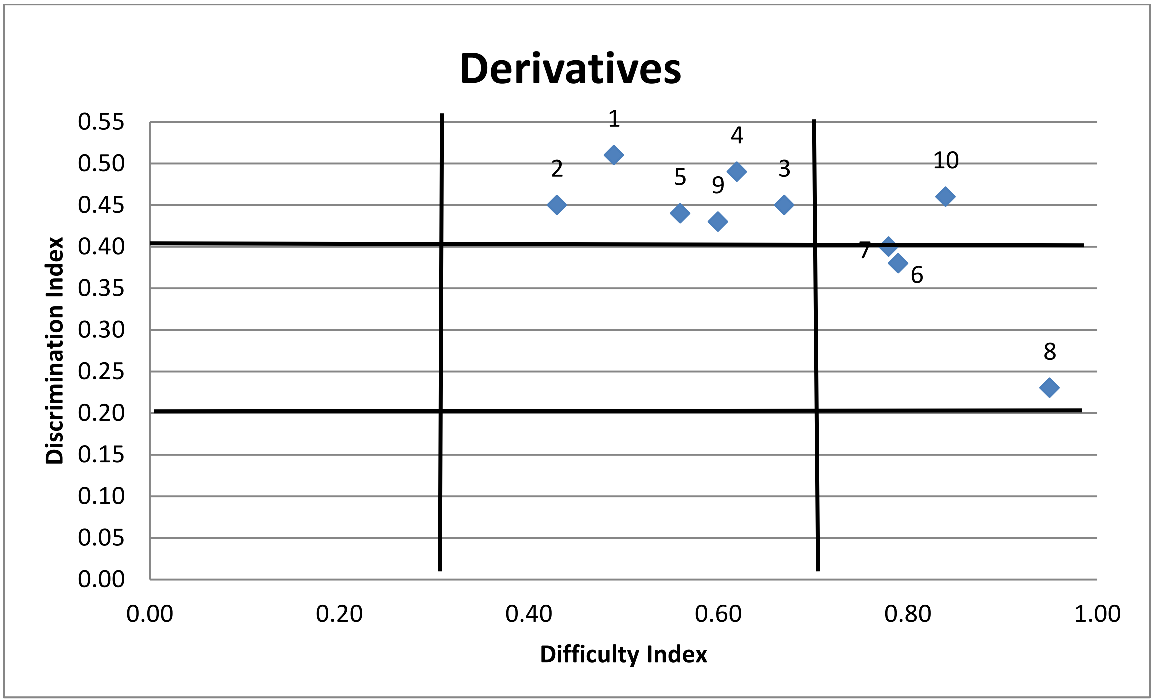


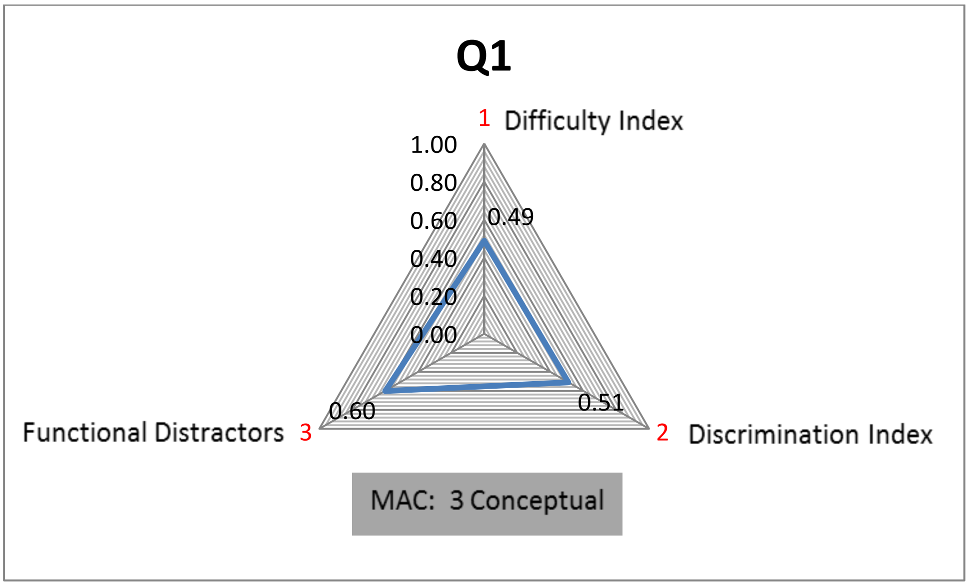
Figure 4.7. Scatter plot of all questions related to derivatives

Questions 1, 2, 8 and 10 are now discussed. The scatter plot in Figure 4.7 shows these questions to be problematic or commendable.

Q1:

If $S(x) = \ln x$, $g(3) = 4$ and $g'(3) = 5$, determine $(Sog)'(3)$.
(A) $\frac{5}{4}$ (B) $\frac{5}{3}$ (C) $4\ln 3$ (D) $5\ln 3$ (E) $\frac{20}{3}$ (F) None of these

Question 2, Semester test 2, 2015



Q1	**Value**	**Comment**
Topic		Derivatives
Math Assessment Component	3	Conceptual
Difficulty Index	0.49	Moderately difficult
Discrimination Index	DI = 0.51	Excellent
Distractor efficiency	0.60	60% Functional

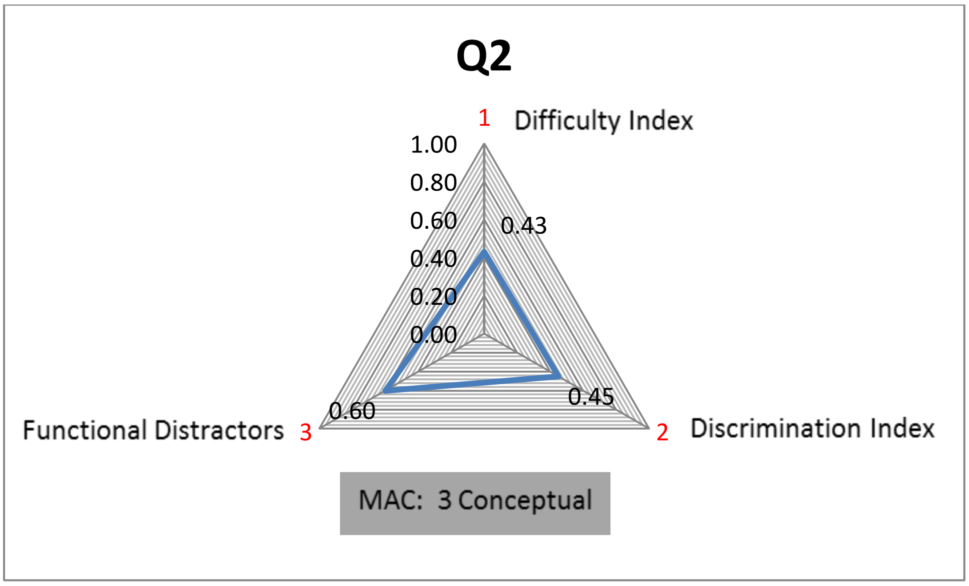
The question assesses on MAC Level 3 (conceptual) since it involves different concepts of differentiation. The question is moderately difficult with $p = 0.49$ and discriminates excellently with an index of DI = 0.51. The distractors are also well chosen with 60% functionality.

Conclusion: The question is acceptable.

Q2:

Given $f(x) = \begin{cases} x+2, & x \le 3 \\ 4x-7, & x > 3 \end{cases}$ Which of the following statements is true? $I.\ \ \lim_{x\to 3} f(x)$ exists $II.\ f$ is continuous at $x = 3$ $III.f$ is differentiable at $x = 3$ (A) Only I (B) Only II (C) Only III (D) I and II only (E) I and III only (F) I, II and III

Question 4, Semester test 2, 2015



Q2	Value	Comment
Topic		Limits, Continuity & Derivatives
Math Assessment Component	3	Conceptual
Difficulty Index	0.43	Moderately difficult
Discrimination Index	DI = 0.45	Excellent
Distractor efficiency	0.60	60% Functional

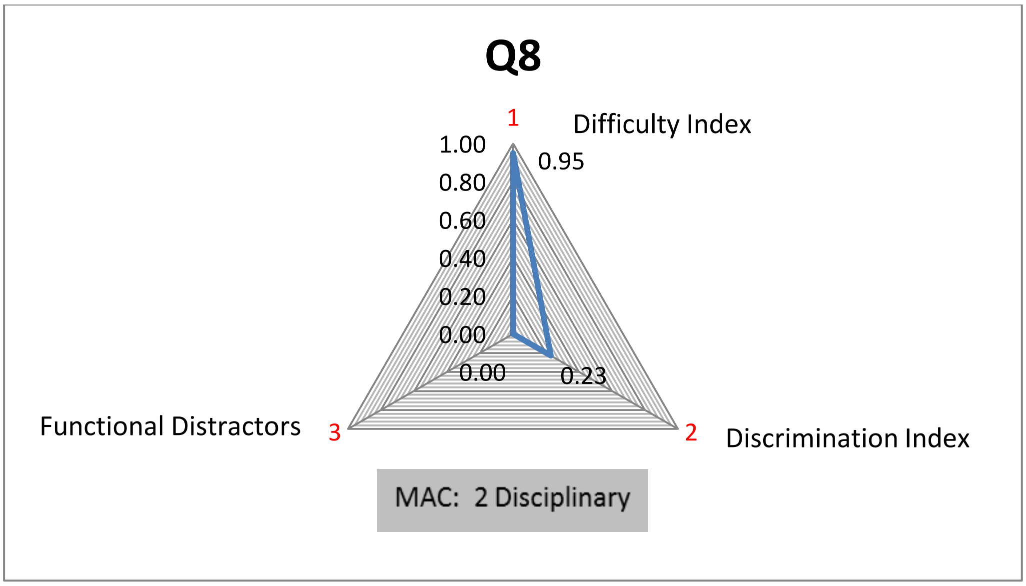
The question combines different topics of the module, which include limits, continuity and derivatives. The MAC level is 3 (conceptual). The question is moderately difficult but discriminates excellently with an index of DI = 0.45. The percentage functional distractors is 60%.

Conclusion: The question can be regarded as an acceptable MCQ.

Q8:

If $f(x) = \sinh(x^2 - 3x)$ then $f'(x) =$		
(A) $(2x - 3)\cosh(x^2 - 3x)$	(B) $(3 - 2x)\sinh(x^2 - 3x)$	(C) $\sinh(x^2 - 3x)$
(D) $(3 - 2x)\cosh(x^2 - 3x)$	(E) $-\sinh(2x - 3)$	(F) None of these

Question 5, Semester test 2, 2017



Q8	Value	Comment
Topic		Derivatives & Trigonometry
Math Assessment Component	2	Disciplinary
Difficulty Index	0.95	Easy
Discrimination Index	DI = 0.23	Fair
Distractor efficiency	0.00	No functional distractors

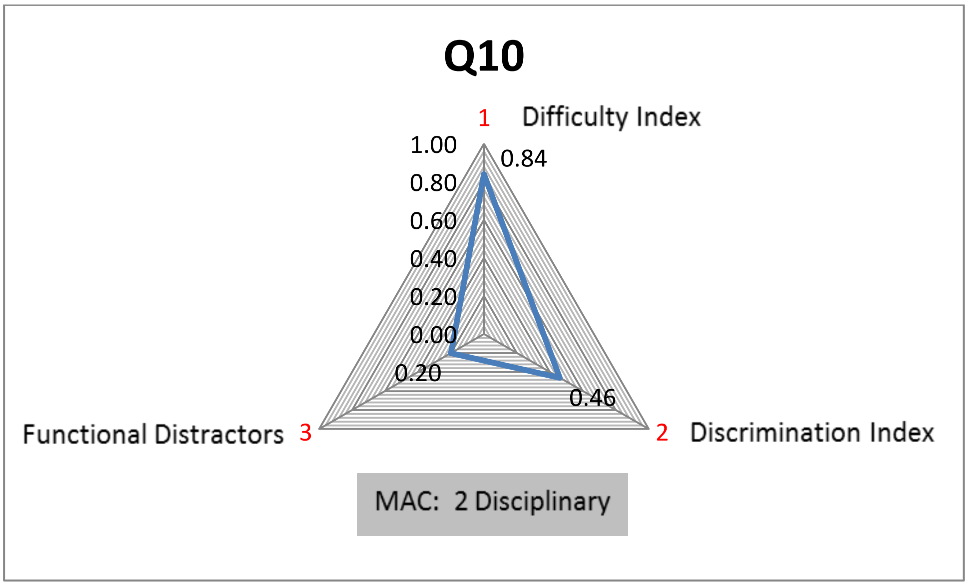
The question can be regarded as easy with a fair discrimination. The correct answer was evident; therefore, there is a decrease in the discrimination between stronger and weaker students. It goes hand in hand with the 0% functional distractors since the majority of students knew the correct answer.

Conclusion: The question is flagged as problematic because the question is too easy and direct.

Q10:

If $f(x) = x\ln x - 1$ then $f''(x) =$ (A) $\ln(x-1)$ (B) $-\frac{1}{x^2}$ (C) $-\frac{1}{x}$ (D) $\ln x - 1$ (E) $\frac{1}{x}$ (F) None of these

Question 7, Semester test 2, 2017



Q10	Value	Comment
Topic		Derivatives & Logarithms
Math Assessment Component	2	Disciplinary
Difficulty Index	0.84	Easy
Discrimination Index	DI = 0.46	Excellent
Distractor efficiency	0.20	20% Functional

The question combines logarithms and differentiation with a MAC Level 2 (disciplinary). The question can be regarded as easy with $p = 0.84$. The question discriminates excellently between the stronger and weaker students but the 20% functional distractors are problematic.

Conclusion: The question is acceptable.

4.5.7 Applications of differentiation

Table 4.9 presents all the questions that are related to the applications of differentiation topic.

Table 4.9. Items related to applications of differentiation

No	ITEM	Question	Difficulty	Discrimination
1	ST2 2015	Q1	0.60	0.48
2	ST2 2015	Q3	0.32	0.36
3	ST2 2015	Q5	0.67	0.41
4	ST2 2015	Q6	0.54	0.39
5	ST2 2015	Q8	0.69	0.45
6	Exam 2015	Q4	0.61	0.44
7	ST2 2016	Q8	0.73	0.38
8	ST2 2016	Q9	0.88	0.28
9	Exam 2016	Q4	0.58	0.47
10	Exam 2016	Q7	0.93	0.27
11	Exam 2016	Q8	0.41	0.39
12	Exam 2016	Q9	0.54	0.43
13	ST2 2017	Q2	0.57	0.52
14	ST2 2017	Q8	0.69	0.49
15	ST2 2017	Q9	0.36	0.39
16	ST2 2017	Q10	0.31	0.44
17	ST2 2017	Q11	0.64	0.53
18	ST2 2017	Q12	0.58	0.46
19	Exam 2017	Q8	0.60	0.30

Figure 4.8 presents a scatter plot of all the questions that are related to the applications of differentiation topic.

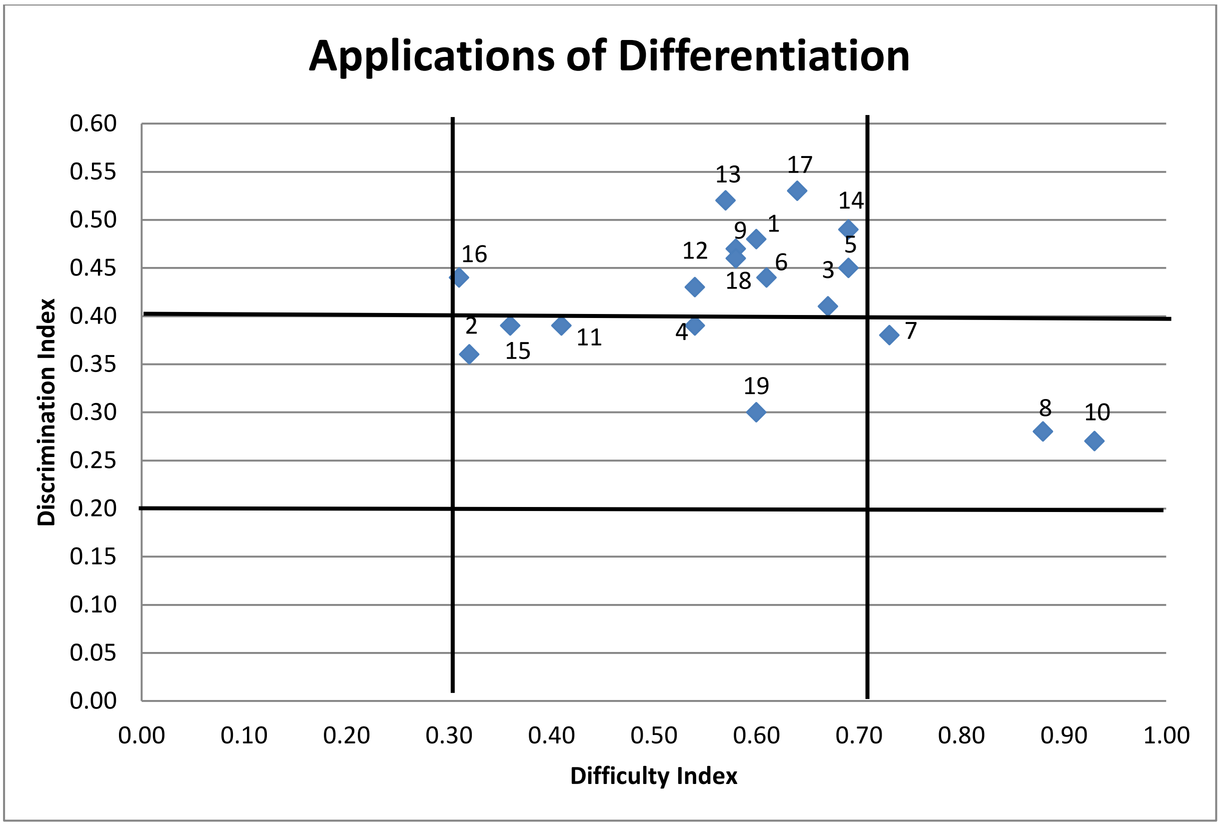


Figure 4.8. Scatter plot of all questions related to applications of differentiation

Questions 8, 10, 16, 17 and 19 are now discussed. The scatter plot in Figure 4.8 shows these questions to be problematic or commendable.

Q8:

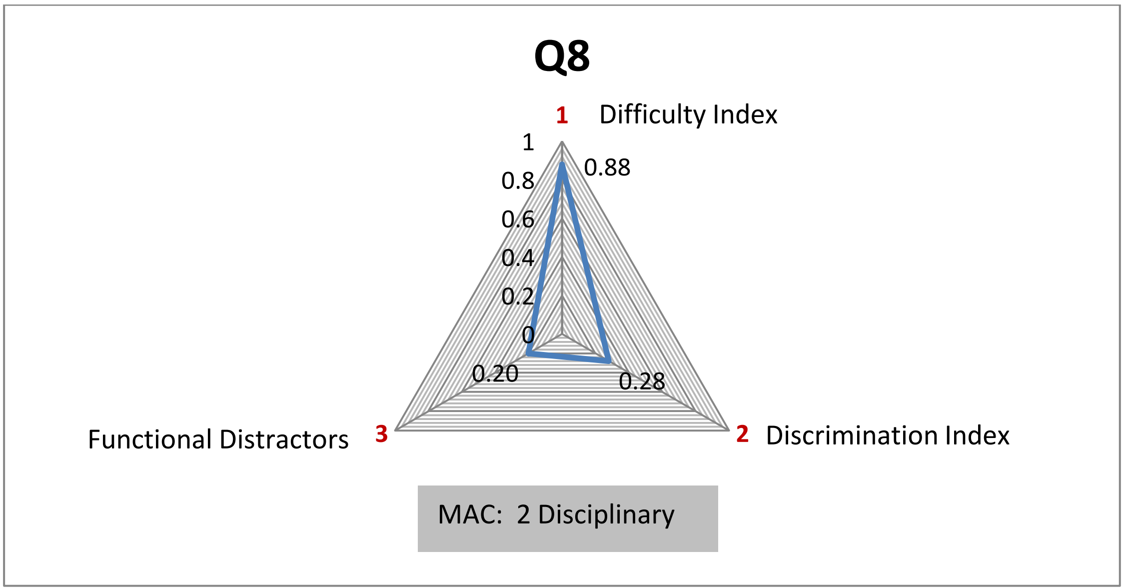
Given that $f(1) = -1$ and $f'(1) - 4$

What is the equation of the tangent line to f in the point where $x = 1$?

(A) $y = 4x - 3$ (B) $y = 4x - 5$ (C) $y = -x + 5$

(D) $y = -4x + 5$ (E) $y = -x - 3$ (F) None of these

Question 9, Semester test 2, 2016



Q8	**Value**	**Comment**
Topic		Applications of differentiation
Math Assessment Component	2	Disciplinary
Difficulty Index	0.88	Easy
Discrimination Index	DI = 0.28	Fair
Distractor efficiency	0.20	20% Functional

The question can be regarded as easy with a difficulty index of $p =$ 0.88. The question discriminates fairly with a discrimination index of DI = 0.28. Only 20% of the distractors were functional, which proves that the answer was evident to most students.

Conclusion: This question is flagged as problematic because it is too easy and direct.

Q10:

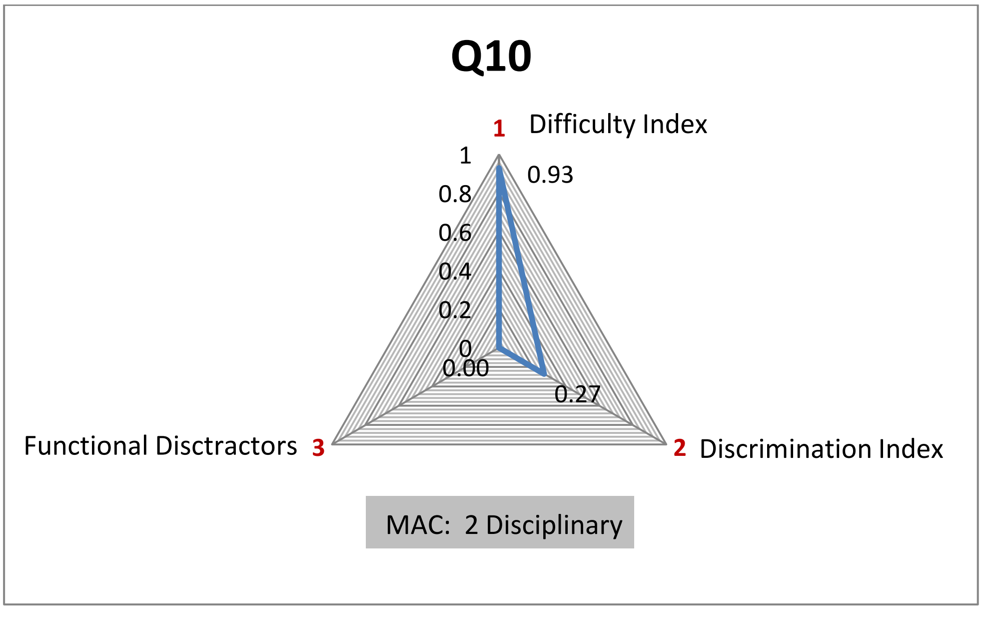
Consider the function $f(x) = \frac{x^2+2}{x^2-4}$ with $f'(x) = \frac{-12x}{(x^2-4)^2}$ and $f''(x) = \frac{12(3x^2+4)}{(x^2-4)^3}$

The equation(s) of the vertical asymptote(s) of f is(are) given by

(A) $x = 0$ (B) $x = 2$; $x = -2$ (C) $x = -2$

(D) $x = 0\,; x = 2$ (E) f has no vertical asymptotes (F) None of these

Question 7, exam, 2016



Q10	Value	Comment
Topic		Applications of differentiation
Math Assessment Component	2	Disciplinary
Difficulty Index	0.93	Easy
Discrimination Index	DI = 0.27	Fair
Distractor efficiency	0.00	No functional distractors

The question can be regarded as easy with a difficulty index of p = 0.93. The question does not discriminate well with a fair discrimination index. None of the distractors were functional, which proves that the question was too easy and the answer was evident to most of the students.

Conclusion: The question is flagged as problematic since the question was too easy and direct.

Q16:

Which of the following statements is/are always true?

I. If $f'(c) = 0$ then $f(x)$ has a maximum or minimum value at $x = c$

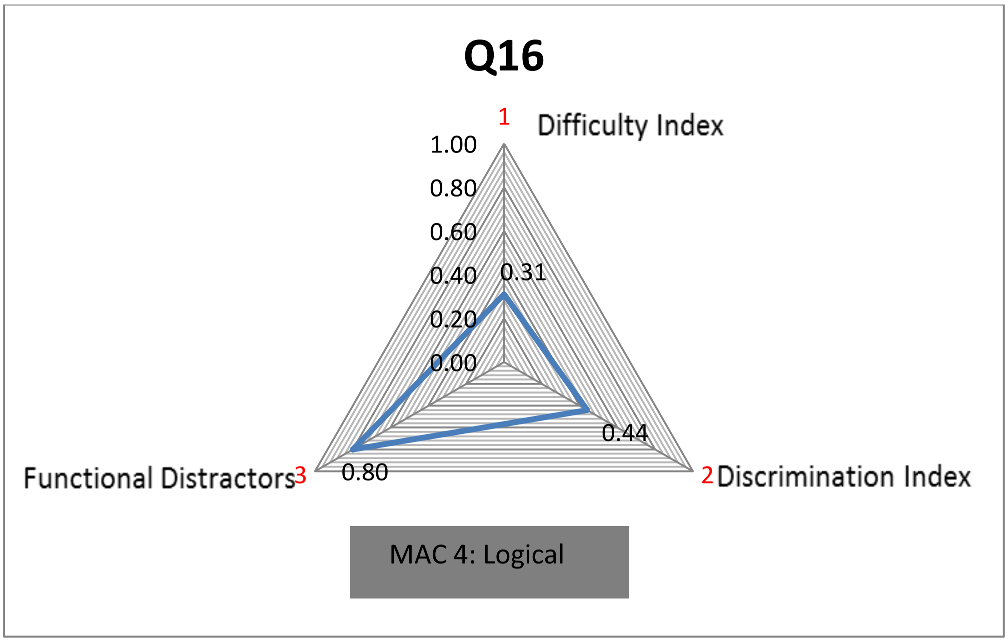
II. If $f'(x) = g'(x)$ for all x in the interval I then $f(x) = g(x)$ on I

III. If $f(x)$ is differentiable on the open interval (a, b) and c is a local maximum of f in (a, b) then $f'(x) = 0$

(A) Only I (B) Only II (C) Only III (D) I and II only

(E) I and III only (F) I, II and III

Question 10, Semester test 2, 2017



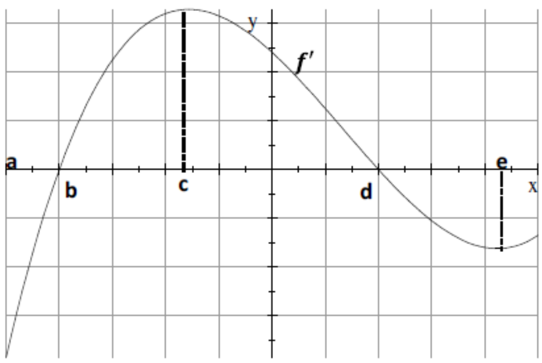
Q16	Value	Comment
Topic		Applications of differentiation
Math Assessment Component	4	Logical
Difficulty Index	0.31	Moderately difficult
Discrimination Index	DI = 0.44	Excellent
Distractor efficiency	0.80	80% Functional

The question assesses on MAC Level 4 (logical). The question has a difficulty index of $p = 0.31$ because different concepts of differentiation, which include several theorems, are involved. The discrimination is excellent and 80% of the distractors are functional. For this question the majority of students (38%) chose the wrong option (E) and 31% of the students chose the correct answer (option C). Statement III was the only correct statement. The majority of students wrongly included statement I as always true.

Conclusion: The question is acceptable and proves that MCQs can also assess on a higher cognitive level.

Q17:

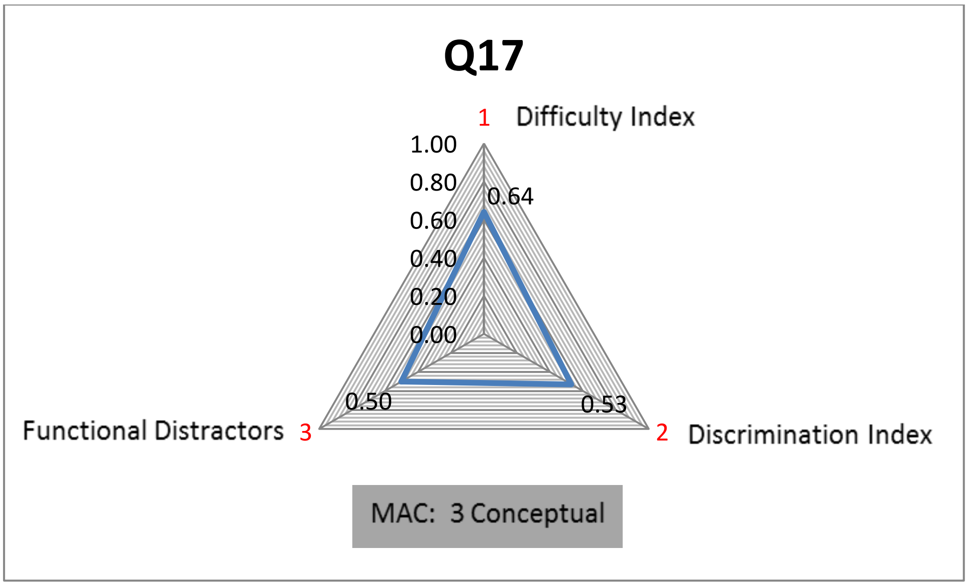
The graph of the first derivative f' of a function f is given.



The function f has a local minimum at

(A) a (B) b (C) d (D) e (F) None of these

Question 11, Semester test 2, 2017



Q17	Value	Comment
Topic		Applications of differentiation
Math Assessment Component	3	Conceptual
Difficulty Index	0.64	Moderately easy
Discrimination Index	DI = 0.53	Excellent
Distractor efficiency	0.50	50% Functional

The question discriminates excellently with a discrimination index of DI = 0.53 and 50% of the distractors are functional. The question is also moderately easy with an index of $p = 0.64$ and assesses on MAC Level 3 (conceptual).

Conclusion: The question is acceptable.

Q19:

Consider a function f with the following properties:

- f is continuous and differentiable on $[0,5]$
- f has critical numbers $x = 2$ and $x = 4$
- The following table gives function values of f:

x	0	2	4	5
$f(x)$	2	1	2	6

Select the correct statement. The function has

(A) a local maximum at $x = 2$ and an inflection point at $x = 4$.

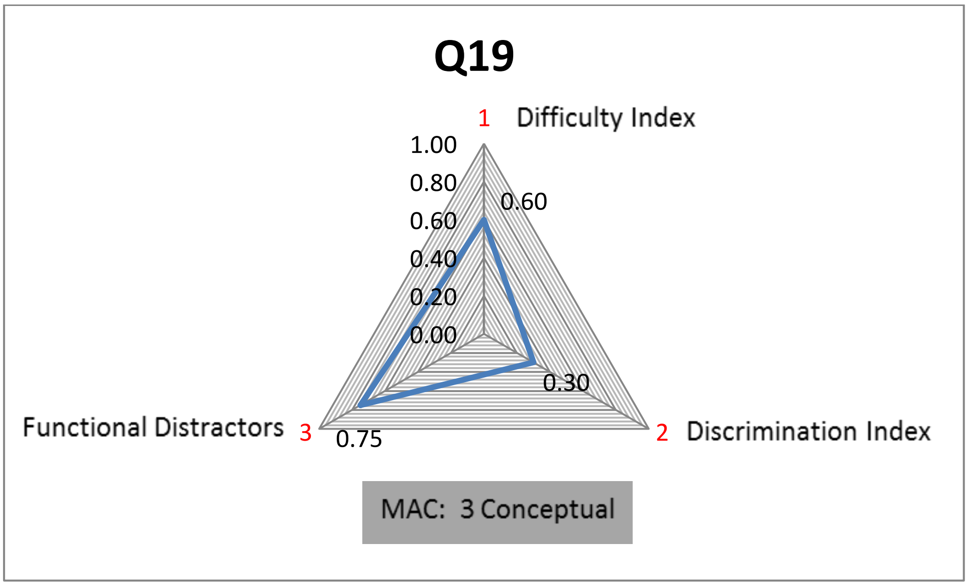
(B) a local minimum at $x = 2$ and a local maximum at $x = 4$.

(C) a local minimum at $x = 2$ and an inflection point at $x = 4$.

(D) local minima at both $x = 2$ and $x = 4$.

(E) Too little information is given to determine the extremes.

Question 8, exam, 2017



Q19	Value	Comment
Topic		Applications of Differentiation
Math Assessment Component	3	Conceptual
Difficulty Index	0.60	Moderately easy
Discrimination Index	DI = 0.30	Good
Distractor efficiency	0.75	75% Functional

The question has 75% functional distractors and a difficulty index of $p = 0.60$, which makes the question moderately easy. The discrimination is also good (DI = 0.30). The question assesses on the third cognitive MAC level (conceptual) because different aspects of applications of differentiation are involved.

Conclusion: The question can be regarded as acceptable.

4.5.8 Integration

Table 4.10 presents all the questions that are related to the integration topic.

Table 4.10. Items related to integration

No	ITEM	Question	Difficulty	Discrimination
1	Exam 2015	Q7	0.35	0.35
2	Exam 2015	Q8	0.80	0.40
3	Exam 2015	Q9	0.62	0.41
4	Exam 2016	Q3	0.68	0.44
5	Exam 2017	Q5	0.86	0.40
6	Exam 2017	Q6	0.79	0.50
7	Exam 2017	Q7	0.75	0.49

Figure 4.9 presents a scatter plot of all the questions that are related to the integration topic.

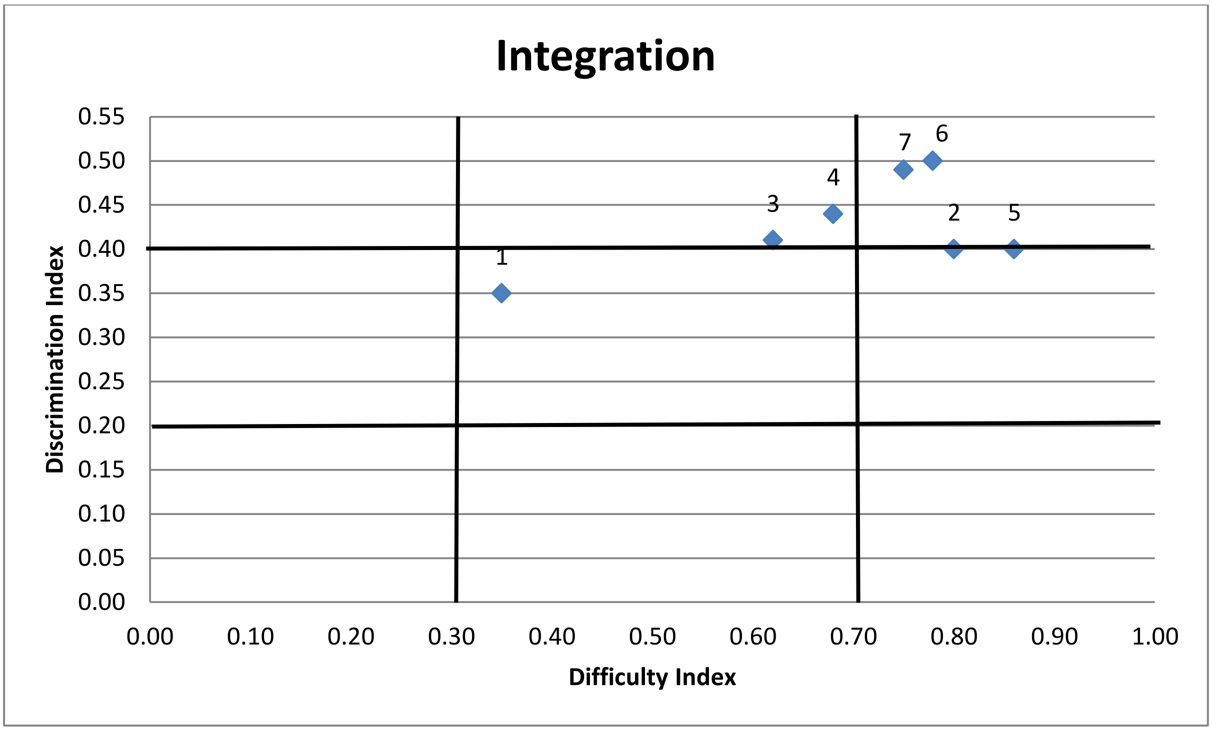


Figure 4.9. Scatter plot of all questions related to integration

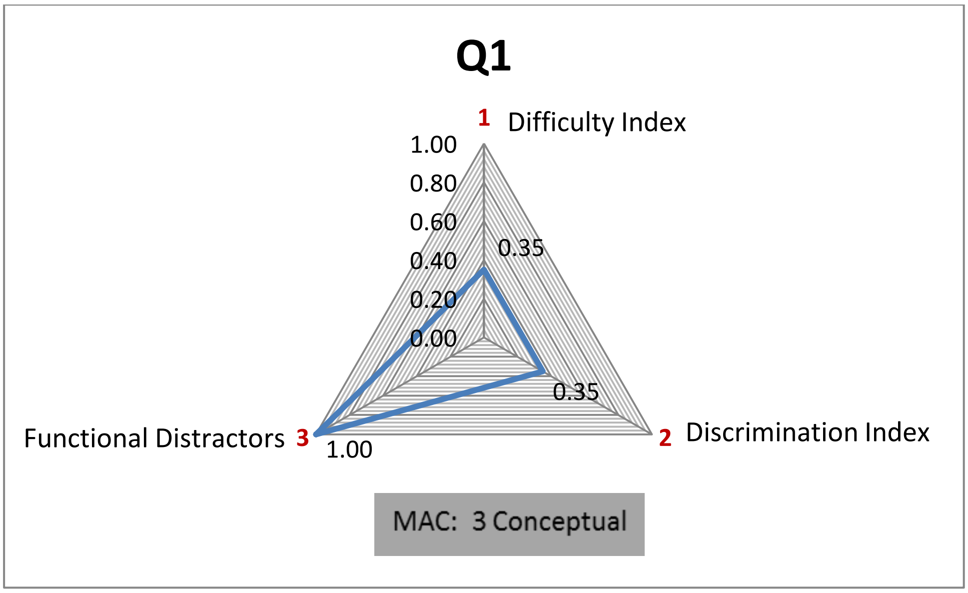
Questions 1, 2, 5 and 6 are now discussed. The scatter plot in Figure 4.9 shows these questions to be problematic or commendable.

Q1:

$$\int_0^2 \left(1+\sqrt{4-x^2}\right) dx \;=$$

(A) $2+\pi$ (B) $2+2\pi$ (C) $4+4\pi$ (D) $4+6\pi$ (E) None of these

Question 7, exam, 2015



Q1	**Value**	**Comment**
Topic		Integration
Math Assessment Component	3	Conceptual
Difficulty Index	0.35	Moderately difficult
Discrimination Index	DI = 0.35	Good
Distractor efficiency	1.00	All distractors functional

The question is moderately difficult with a difficulty index of $p =$ 0.35 and the discrimination is good with DI = 0.35. All the distractors are functional and the question assesses on MAC Level 3 (conceptual).

Conclusion: The question can be regarded as acceptable with well-balanced indices.

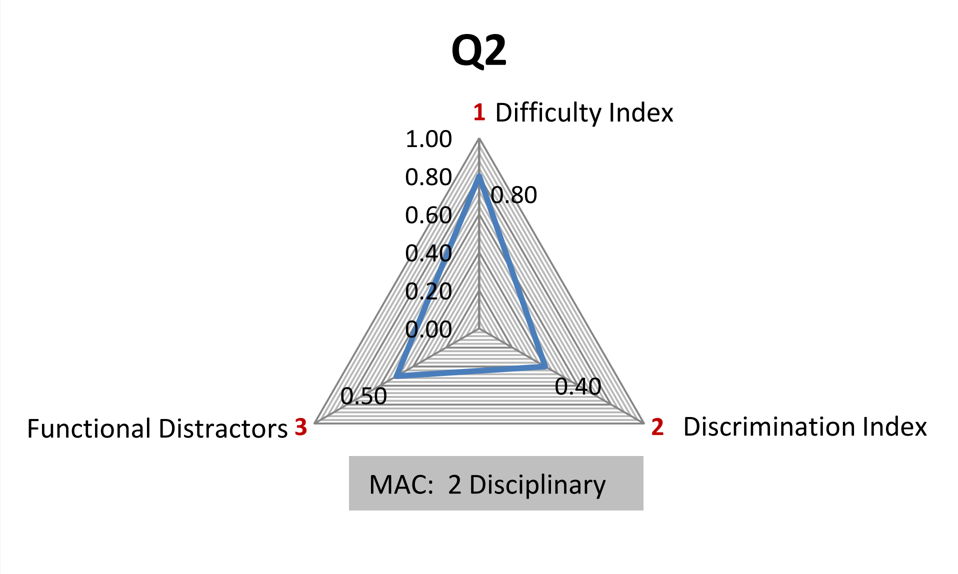
Q2:

$$\int \left(\frac{2}{1+3x^2}\right) dx \ =$$

(A) $\frac{2}{\sqrt{3}}\tan^{-1}(\sqrt{3}x) + C$ (B) $2\ln\sqrt[3]{\frac{1+3x^2}{4}} + C$ (C) $\frac{2}{\sqrt{3}}\tan^{-1}(3x) + C$

(D) $\frac{2}{(1+\sqrt{3}x^2)^2} + C$ (E) None of these

Question 8, exam, 2015



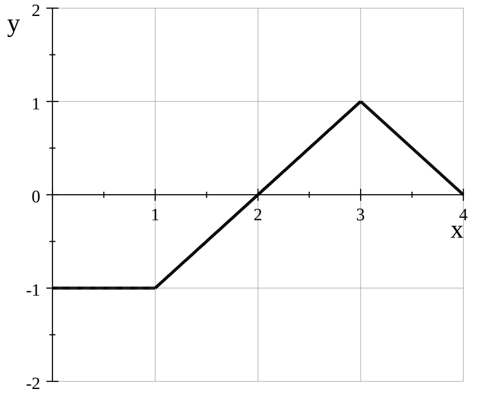
Q2	**Value**	**Comment**
Topic		Integration
Math Assessment Component	2	Disciplinary
Difficulty Index	0.80	Easy
Discrimination Index	DI = 0.40	Excellent
Distractor efficiency	0.50	50% Functional

The question is an example of a question that can be regarded as easy with a difficulty index of $p =$ 0.80. The question discriminates excellently between the stronger and weaker students and 50% of the distractors are functional.

Conclusion: The question can be regarded as acceptable.

Q5:

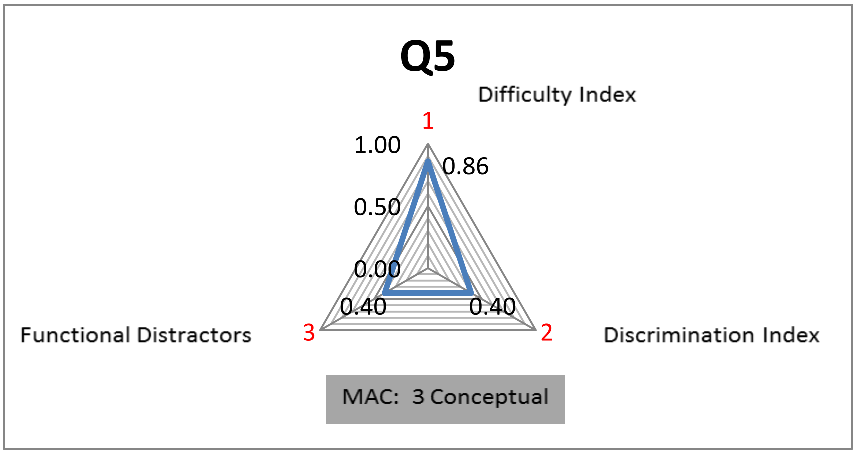
The graph of a piecewise defined function f on $0 \leq x \leq 4$ is given.



Determine $\int_0^4 f(x)dx$

(A) -0.5 (B) -1.5 (C) 2.5 (D) 0.5 (E) 3 (F) None of these

Question 5, exam, 2017



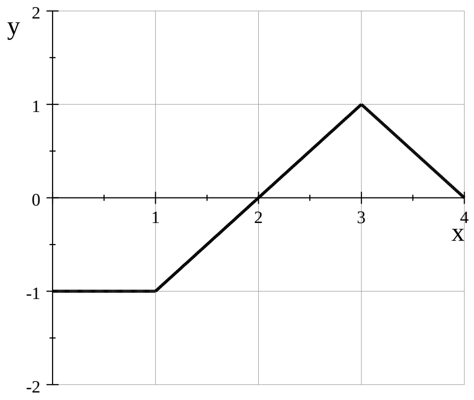
Q5	**Value**	**Comment**
Topic		Integration
Math Assessment Component	3	Conceptual
Difficulty Index	0.86	Easy
Discrimination Index	DI = 0.40	Excellent
Distractor efficiency	0.40	40% Functional

The question can be regarded as easy with a difficulty index of $p =$ 0.86. The question discriminates excellently between the stronger and weaker students and 40% of the distractors are functional. The question assesses on MAC Level 3 (conceptual).

Conclusion: The question can be regarded as acceptable.

Q6:

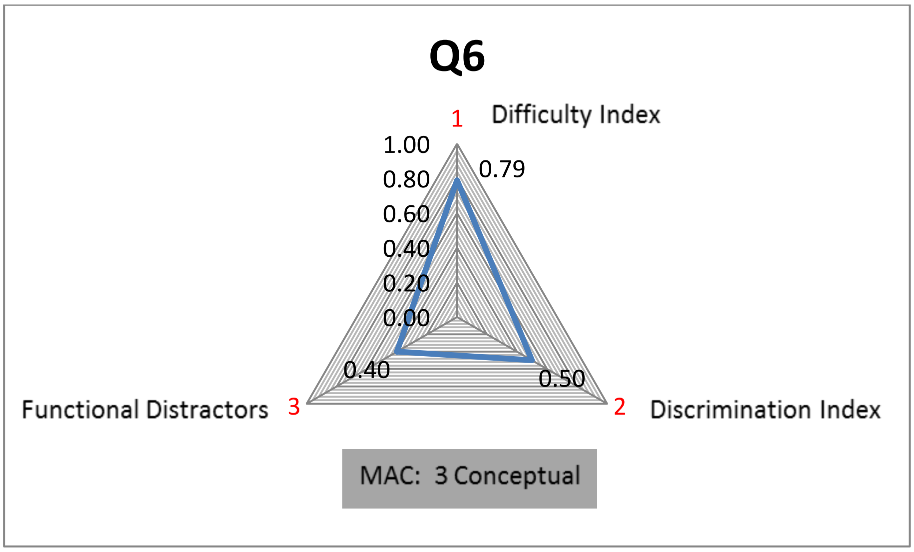
The graph of a piecewise defined function f on $0 \leq x \leq 4$ is given.



The area enclosed between the curve and the x-axis on $[0,4]$ is given by

(A) -0.5 (B) -1.5 (C) 2.5 (D) 0.5 (E) 3 (F) None of these

Question 6, exam, 2017



Q6	Value	Comment
Topic		Integration
Math Assessment Component	3	Conceptual
Difficulty Index	0.79	Easy
Discrimination Index	DI = 0.50	Excellent
Distractor efficiency	0.40	40% Functional

This question has a Level 3 (conceptual) assessment component and a difficulty index of $p = 0.79$. The question discriminates excellently between the stronger and weaker students and 40% of the distractors are functional.

Conclusion: The question can be regarded as acceptable.

4.5.9 Vector algebra

Table 4.11 presents all the questions that are related to the vector algebra topic.

Table 4.11. Items related to vector algebra

No	ITEM	Question	Difficulty	Discrimination
1	ST1 2015	Q20	0.93	0.25
2	Exam 2015	Q10	0.64	0.42
3	Exam 2016	Q10	0.57	0.52
4	Exam 2017	Q9	0.73	0.48
5	Exam 2017	Q10	0.64	0.46

Figure 4.10 presents a scatter plot of all the questions that are related to the vector algebra topic.

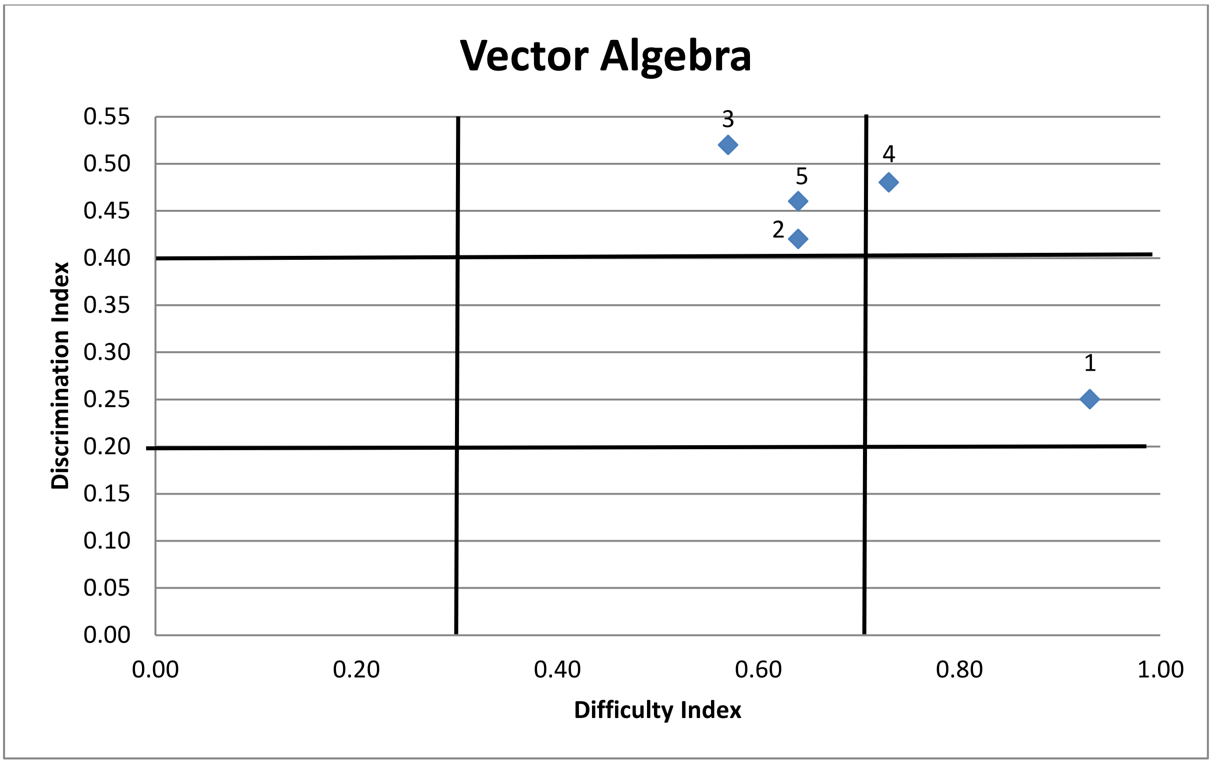


Figure 4.10. Scatter plot of all questions related to vector algebra

Questions 1, 3 and 4 are now discussed. The scatter plot in Figure 4.10 shows these questions to be problematic or commendable.

Q1:

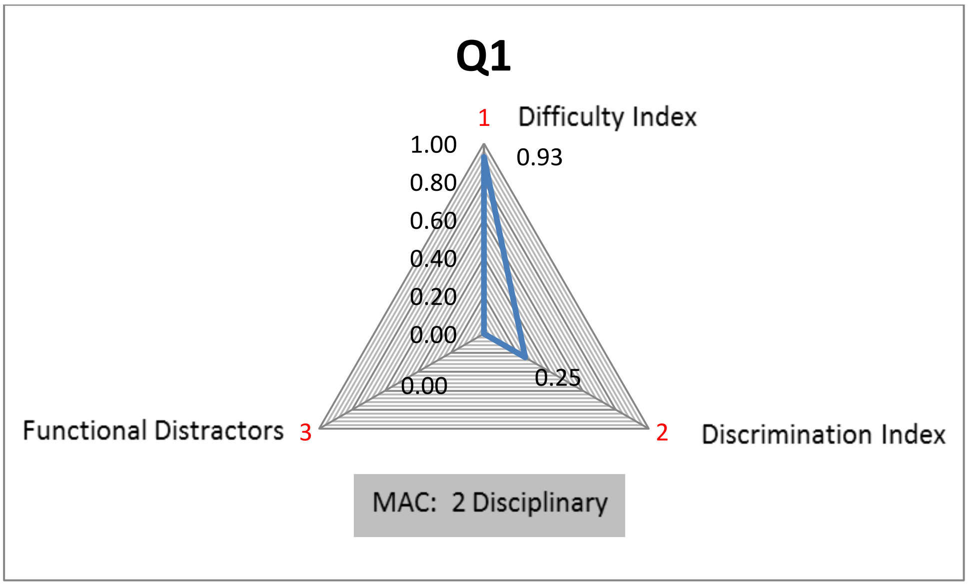
Given: $\bar{a} = < 1, 2, -3 >$ and $\bar{b} = < 4, 0, 7 >$.

Expressed as standard vectors, $2\bar{a} + 3\bar{b} =$

(A) $8\bar{\imath} - \bar{\jmath} + 4\bar{k}$ (B) $8\bar{\imath} + \bar{\jmath} + 4\bar{k}$ (C) $8\bar{\imath} - \bar{\jmath} + 415$

(D) $84 + 14\bar{\jmath} + 15\bar{k}$ (E) $14\bar{\imath} + 4\bar{\jmath} + 15\bar{k}$ (F) None of these

Question 20, Semester test 1, 2015



Q1	Value	Comment
Topic		Vector algebra
Math Assessment Component	2	Disciplinary
Difficulty Index	0.93	Easy
Discrimination Index	DI = 0.25	Fair
Distractor efficiency	0.00	No functional distractors

The question can be regarded as easy with a difficulty index of $p =$ 0.93. The question has no functional distractors, which show that the correct answer was evident to most students. The fair discrimination index (DI = 0.25) goes hand in hand with the high difficulty index. The MAC level was 2 (disciplinary).

Conclusion: This question is flagged as problematic. It is direct and easy and does not discriminate well between strong and weak students. No distractors are functional.

Q3:

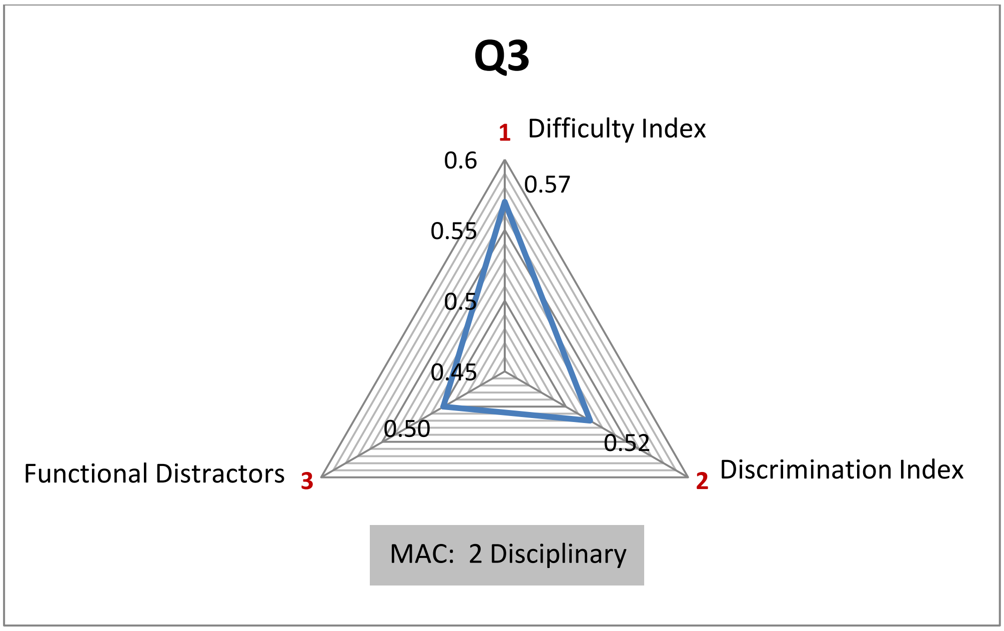
Given $|\bar{a}| = 3$ and $|\bar{b}| = \sqrt{6}$ and the angle between $\bar{a}$ and $\bar{b}$ is $\frac{\pi}{4}$.

Determine $\bar{a}.\bar{b}$

(A) $\frac{3\sqrt{6}}{2}$ (B) $\frac{9\sqrt{3}}{2}$ (C) $3\sqrt{3}$ (D) $\frac{1}{6\sqrt{3}}$ (E) $\frac{2\sqrt{3}}{9}$

(F) $\frac{1}{3\sqrt{6}}$ (G) None of these

Question 10, exam, 2016



Q3	Value	Comment
Topic		Vector algebra
Math Assessment Component	2	Disciplinary
Difficulty Index	0.57	Moderately easy
Discrimination Index	DI = 0.52	Excellent
Distractor efficiency	0.50	50% Functional

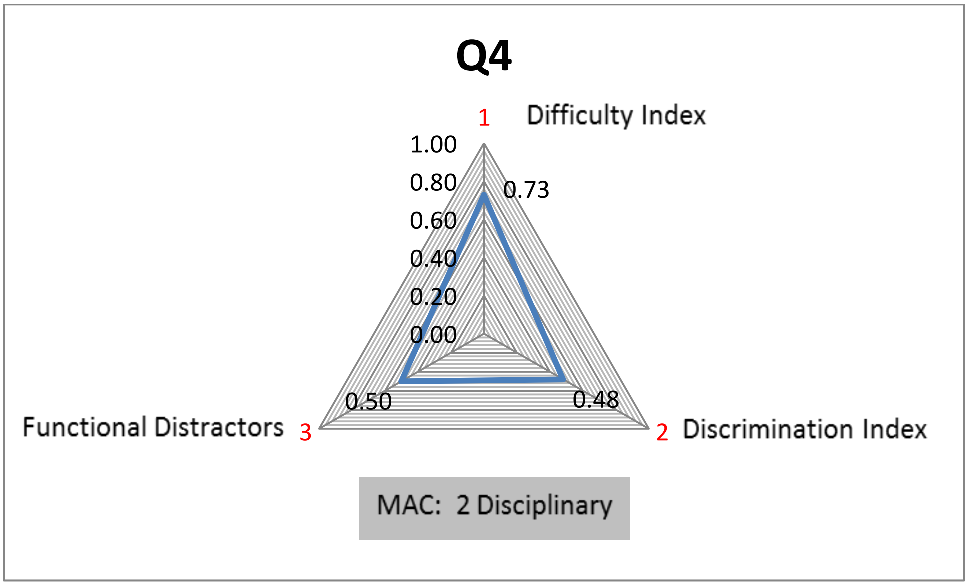
The question can be regarded as acceptable with excellent discrimination (DI = 0.52) and a moderately easy difficulty index. The distractors are also well chosen with 50% functionality.

Conclusion: The question is acceptable and all indices are well balanced.

Q4:

The distance from the point $(2, 1, 3)$ to the xy-plane is
(A) 3 (B) $\sqrt{13}$ (C) $\sqrt{5}$ (D) 2 (E) None of these

Question 9, exam, 2017



Q4	**Value**	**Comment**
Topic		Vector algebra
Math Assessment Component	2	Disciplinary
Difficulty Index	0.73	Easy
Discrimination Index	DI = 0.48	Excellent
Distractor efficiency	0.50	50% Functional

The question has a difficulty index of $p = 0.73$, which make it an easy question. The discrimination is excellent with DI = 0.48. The distractors are also 50% functional.

Conclusion: The question can be regarded as acceptable with well-balanced indices.

CHAPTER 5 SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

The final chapter of the study gives a discussion of the findings and a summary of the results so that conclusions can be drawn. The primary and secondary research questions are answered and reflections are provided on the conceptual framework and success of the parameters that were utilised to answer the research questions. Further reflections are given on the assessment model in higher education. Limitations of the study are mentioned and implications for further research are suggested. Finally some recommendations are made regarding assessing in higher education.

5.2 Reflections on the assessment model in higher education

The increase in student numbers worldwide is also cognisable in South Africa. The large numbers of students entering the higher education system has led to an excessive grading and administrative load on staff. The student numbers have necessarily impacted on the Department of Mathematics and Applied Mathematics at the University of Pretoria. The department is understaffed in terms of lecturing staff. Solutions had to be found to reduce excessive grading. It became imperative to find alternative forms of assessment.

The decision to make use of MCQs in tests and examinations, which could be graded by means of computer software, contributed considerably in reducing the grading work. Grading is done in less time and timeous feedback can be given to students to enhance the learning process.

The quality and cognitive levels of the MCQs had to be investigated. To determine the quality of the questions, an item analysis on the MCQs had to be done by utilising specific assessment parameters. These assessment parameters were the discrimination index, the difficulty index and the distractor efficiency. Although other assessment parameters are available, the decision was taken to use the abovementioned parameters. The reason for using the indices of difficulty and discrimination is because these values are available as part of the statistics when the university computer laboratory returns the test results. The distractor efficiency can easily be determined after receiving the statistical report from the computer laboratory.

The item analysis also included determining the cognitive level of each MCQ. The cognitive levels were determined by using Table 2.9, as well as the expertise of four staff members from the Department of Mathematics and Applied Mathematics, to categorise the questions accordingly.

5.3 Summary of the main findings

The secondary analysis that was executed in this study involved item analyses of all the MCQs under investigation. The item analysis showed that the majority of MCQs used in the first-year engineering calculus module are of a high quality. The MAC levels showed that the investigated MCQs utilised both higher and lower cognitive levels, though the vast majority of MCQs utilised lower cognitive levels. The item analysis showed that the MCQs discriminate well in general between the high-scoring and low-scoring students and the questions are generally of a reasonable difficulty level. For the majority of MCQs most distractors were functional, which shows that the questions were well written and the distractors were well chosen.

5.4 Addressing the research questions

- **Primary research question**

  How do the multiple choice questions in mathematics, as posed to first-year engineering students at the University of Pretoria, comply with the principles of good assessment for determining quality?

To answer the primary research question, two secondary research questions will be addressed first:

- **Secondary Research Question 1**

  Which levels of cognitive demand are addressed in multiple choice questions of the sample under investigation?

- **Secondary Research Question 2**

  How do the multiple choice questions measure with respect to the discrimination index, difficulty index and distractor efficiency?

5.4.1 Addressing Secondary Research Question 1

A summary of the findings with regard to the MAC levels and the total number of MCQs for each MAC level is shown in Table 5.1:

Table 5.1. Summary of the MACs for all MCQs under investigation

Mathematics assessment component level	Number of MCQs per level
Level 1 Technical	4
Level 2 Disciplinary	69
Level 3 Conceptual	32
Level 4 Logical	2
Level 5 Modelling	0
Level 6 Problem solving	0
Level 7 Consolidation	0
TOTAL	**107**

The mathematics assessment component (MAC) determines the cognitive level of each MCQ. Level 1 (technical) is on the lowest level of cognitive assessment, which gradually increases to Level 7 (consolidation), the highest level of the cognitive assessment segment. Although the majority of questions assess on MAC Level 2 (disciplinary) (69 out of 107) many questions assess on MAC Level 3 (conceptual) (32 out of 107) and very few questions assess on MAC Level 4 (logical) (two out of 107). Only four questions assess on MAC Level 1 (technical). The technical level is the lowest cognitive level for assessing, which involves manipulations and basic calculations.

The finding that the majority of questions (64%) assess on MAC Level 2 can be explained by the fact that the MCQs contribute only one or one and a half mark per question to the total of around 50 for the paper. More complicated concepts, assessed on higher cognitive levels, result in more marks for such questions. For MAC Level 2 (disciplinary), students need to recall formulae and apply it to the problem. Students also need to recall facts and knowledge and apply them to the question.

Around 30% of questions assess on MAC Level 3, the conceptual level. For MAC Level 3, students need to comprehend visual and graphical content from statements or graphs to reach conclusions. The interpretation of a graph, for example, is on a higher cognitive level than MAC Levels 1 and 2.

Only two questions assess on MAC Level 4 (logical) where students need to apply different theorems to solve the given problems. These MCQs are of a higher cognitive level and students need to have a solid knowledge and understanding of the interrelations between theorems and content.

No questions could be found that assess students' ability on MAC Level 5 (modelling) since translating words into mathematical symbols are covered in applied calculus and fall outside the first-year engineering mathematics scope.

No questions under investigation assess on MAC Level 6 (problem solving) or MAC Level 7(consolidation). It can be assumed that mathematics questions assessing on MAC Levels 6 and 7 are more suitable for the paper-based section of a test or exam in which students use logical, written steps to solve the problems. Since these questions involve written steps as a result of logical thinking and problem solving, such questions consume time and therefore earn more marks. Questions on MAC Levels 6 and 7 can therefore only be used as CRQs in the paper-based, written part of a test or examination.

5.4.2 Addressing Secondary Research Question 2

Scatter plots and radar plots were used to show how the MCQs measure with respect to the discrimination index, difficulty index and distractor efficiency. This is a novel and informative representation.

For scatter plots the difficulty index is presented on the x-axis and the discrimination index is presented on the y-axis. Figure 4.1 gives a summary of all 107 MCQs under investigation on one scatter plot. It seems that the vast majority of MCQs have a good or excellent discrimination index. Most questions have a difficulty index between 0.3 and 0.7, which is the preferred band. Many questions have a difficulty index even greater than 0.7. Some of these questions are problematic because they are too easy.

Each MCQ was presented on a radar plot: for each question all three indices were visually presented on a triangular-shaped diagram so they could be seen in relation to each other. The three scales ranged between 0 and 1. When all indices were well balanced the triangle had an equilateral shape, but when one or more indices were poorer or better, the triangle had a distorted shape. The benefit of a radar plot was that one could see at first glance if one or more of the indices were imbalanced or poor.

Table 5.2 summarises the outcome of all MCQs with regard to the discrimination index:

Table 5.2. Summary of the discrimination index

Discrimination index	Number of MCQs	Description
$DI < 0.2$	1	Poor discrimination
$0.2 \leq DI < 0.30$	6	Fair discrimination
$0.3 \leq DI < 0.40$	26	Good discrimination
$DI \geq 0.40$	74	Excellent discrimination

In Table 5.2 it is seen that the majority of MCQs (74 out of 107; 69.2%) discriminate excellently. Twenty-six out of 107 (24.3%) MCQs show good discrimination and 5.6% of the MCQs show fair discrimination. Only one MCQ has poor discrimination. With a total of 100 out of 107 (93.5%) MCQs discriminating either well or excellently the conclusion can be made that the questions are well constructed and provide a satisfactory outcome. Only seven out of 107 (6.5%) questions discriminate fairly or poorly. The difference in performance between the weaker and stronger students is evident.

Table 5.3 summarises the outcome of all MCQs with regard to the difficulty index:

Table 5.3. Summary of the difficulty index

Difficulty index	Number of MCQs	Description
$0 \leq p < 0.3$	3	Difficult question
$0.3 \leq p \leq 0.5$	20	Moderately difficult question
$0.5 < p < 0.7$	51	Moderately easy question
$p \geq 0.7$	33	Easy question

The difficulty indices range between $p = 0.21$ and $p = 0.95$. In Table 5.3 it is seen that the majority of MCQs (51 out of 107; 47.7%) are moderately easy and 33 out of 107 (30.8%) MCQs are considered easy. Twenty questions are moderately difficult and only three questions (2.8%) are regarded as difficult. Feedback will be given to the compilers of the MCQs with regard to the “easiness” of the questions under investigation.

The majority of MCQs have between 40% and 75% functional distractors. Table 5.4 summarises the outcome with regard to the distractors:

Table 5.4. Summary of the distractor efficiency

Functional distractors	Number of MCQs	% MCQs
All distractors are non-functional	5	4.7%
1	14	13.1%
2	28	26.2%
3	41	38.3%
4	10	9.3%
All distractors are functional	9	8.4%

A functional distractor is a distractor that is chosen by more than 5% of the examinees. Table 5.4 shows that five MCQs (4.7%) have no functional distractors. It seems that these questions were too easy and all the students found the key obvious. Nine questions (8.4%) have distractors that are all functional. It can be assumed that the students found these questions hard and all distractors were chosen to be the correct option. The majority of questions have either one (13.1%), two (26.2%) or three (38.3%) functional distractors. For these questions the distractors were well chosen and achieved the purpose for which they were selected.

5.4.3 A summary of the problematic MCQs

Eight out of the 107 MCQs under investigation were flagged as problematic. These questions have one or more of the following defects:

- The question is too easy: When questions are too easy and the answers too obvious, a high difficulty index ($p > 0.8$) can be expected. The result is poor discrimination between stronger and weaker students.
- The question has no functional distractors: When questions have no functional distractors the quality of the question is jeopardised and all students will choose the correct option.
- The question has a low discrimination index: When questions have a low discrimination index (DI < 0.2) it implies that no clear distinction can be made between knowledgeable and less knowledgeable students.

With five MCQs under investigation, the majority students chose the wrong option instead of the key, which, for these questions, may point to misconceptions and lack of knowledge. The five MCQs are:

- Question 8 from ST1 2015 in which students had to determine the value of $\operatorname{cosec}\left(\frac{47\pi}{2}\right)$. The majority said the correct answer is 1, but the correct answer is -1. Students did not realise that for $x = \frac{47\pi}{2}$ the value of $\operatorname{cosec} x$ is negative.
- For Question 2 from ST1 2017 students had to solve the logarithmic question $2\ln x - \ln(x+2) = 0$. The majority of students said the correct answer is $x = -1 \text{ or } x = 2$, but they did not check the domain of the logarithmic equation, which allows only solutions of $x > 0$, which makes $x = 2$ the only solution.
- In Question 5 from ST1 2017 students had to transform the exponential equation $y = e^{-x}$ units to the right. The majority said the solution is

$y = e^{-x-2}$ but they made a sign mistake because $-(x-2)$ gives $-x+2$.

- Question 7 of ST1 2017: If $f: A \rightarrow B$ and f^{-1} is the inverse of f then $(f o f^{-1})(x) = x$ for x an element of a domain that the students had to determine. It seems the students did not understand where the domain is to be found for a function f and the inverse f^{-1}.
- Question 10 from ST2 2017 engages different theorems regarding applications of differentiation in one question. The question was:
  Which of the following statements is/are always true?
  I If $f'(c) = 0$ then $f(x)$ has a maximum or minimum value at $x = c$.
  II If $f'(x) = g'(x)$ for all x in the interval I, then $f(x) = g(x)$ on I.
  III If $f(x)$ is differentiable on the open interval (a, b) and c is a local maximum of f in (a, b) then $f'(c) = 0$. It seems the students got confused when selecting between the three given statements. A lack of knowledge resulted in them selecting the wrong option.

5.4.4 A summary of the commendable MCQs

The vast majority of MCQs were commendable. Ninety-nine out of the 107 MCQs under investigation were well constructed and clear. All the MCQs were of average difficulty, had good or excellent discrimination and had a few functional distractors. The following findings can be supplied to confirm the comment:

- The indices were well balanced with each other.
- The functional distractors were well chosen.
- The question discriminated excellently irrespective of a lower difficulty index.
- The question discriminated excellently between the stronger and weaker students.
- The question assessed on a higher MAC level.
- Apart from the high difficulty index (i.e. very easy items) the question discriminated excellently.

5.5 Limitations of the study

The semester tests and examinations used in this study were conducted with tertiary students in their first year of study at the University of Pretoria, enrolled in the engineering calculus module WTW 158. The study could be extended to other tertiary institutions as well as to other calculus modules. The study could also be extended to courses beyond the first-year level.

In the study the indices of difficulty and discrimination and the distractor efficiency were utilised to determine the quality of the MCQs under investigation. In the literature other assessment parameters are mentioned that could have been used successfully to determine the quality of MCQs, such as the confidence index and the measures of dispersion and tendency in the interest of determining how the data was spread at aggregated level.

The study could be extended to include constructed response questions (CRQs), which are usually part of a second section of a test or exam and to which students write their answers in full steps on paper, to determine the quality of CRQs. In this way the tests could have been viewed from a more complete perspective and any correlations between the MCQs and constructed response items could have been established.

5.6 Recommendations for further research

In this study three parameters were identified to measure the quality of multiple choice questions: discrimination index, difficulty index and distractor efficiency. Further work needs to be carried out to investigate whether more contributing measuring criteria can be identified to measure the overall quality of a good multiple choice question.

It is recommended that the use of Rasch analysis to determine item–person fit can be used in further research. The Rasch model is a more precise and moral technique that can be used to comment on group ability (Rasch, 1980). The Rasch model provides an alternative to the classical test theory route that has been used for the purposes of this study. The Rasch model is the only item response theory model in which the total score across items characterises a person’s ability. A minimum of parameters is needed and only one parameter that corresponds to each category of a question item is needed. The RUMM2020 approach is the recommended software (Rasch-analysis, 2017).

5.7 Feedback

Staff members at the Department of Mathematics and Applied Mathematics can benefit from this research study when creating new MCQs. The quality of the questions can be tested by executing an item analysis to determine the MAC level of the questions and to determine the indices of discrimination and difficulty. The examples discussed in this study can provide some guidance as to what makes items exemplary and what possible shortcomings they could have. Unfortunately the distractor functionality can only be seen after a test is written.

5.8 Conclusions

The use of MCQs in tests and examinations in higher education contributes greatly to the reducing of excessive grading work. The ever-increasing numbers of students entering the higher education system made it imperative to use different forms of formative and summative assessment. The use of clickers in class is a quick form of formative assessment where students answer MCQs by using a clicker. For summative assessments the use of MCQs proves to be a successful method for assessment since the results are valid, reliable and time-saving, reducing human errors and promoting timeous feedback, which is an important part of learning.

The research showed that the multiple choice questions in mathematics, as posed to first-year engineering students at the University of Pretoria, comply with the principles of good assessment. In addition, the lecturers involved in setting MCQs should ensure that more questions assess on higher cognitive levels, instead of only assessing on lower levels, as is currently the case. The MCQ assessment format does not have to be restricted to lower cognitive tasks requiring only a surface approach. MCQs must also assess on Level 6 (problem solving) and Level 7 (consolidation) for a deeper approach to learning. Although MAC Levels 6 and 7 are more often assessed in the paper-based, written part of tests and examinations at the department, MAC Levels 6 and 7 can successfully be incorporated in MCQs.

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APPENDICES

Appendix A: Mathematics assessment component (MAC) taxonomy and cognitive skills

The following tables present the MAC level assigned to each question in the semester tests and exams of 2015, 2016 and 2017 by each of the four lecturers, as well as each question's final MAC level based on the majority decision.

Table A1. Semester test 1, 2015

Question	Assessor 1	Assessor 2	Assessor 3	Assessor 4	FINAL MAC
1	1	1	1	1	1
2	2	2	2	3	2
3	2	2	2	3	2
4	3	3	2	3	3
5	2	2	1	1	1
6	2	2	2	3	2
7	2	2	1	2	2
8	2	2	1	2	2
9	3	3	2	3	3
10	2	2	2	1	2
11	2	2	3	3	2
12	3	3	1	1	3
13	2	2	1	3	2
14	2	2	1	1	1
15	3	3	2	4	3
16	2	2	2	3	2
17	2	2	2	2	2
18	2	2	2	1	2
19	3	3	2	3	3
20	2	2	1	3	2

Table A2. Semester test 2, 2015

Question	Assessor 1	Assessor 2	Assessor 3	Assessor 4	FINAL MAC
1	3	3	2	2	3
2	3	3	3	3	3
3	2	2	3	3	2
4	3	3	3	3	3
5	3	3	3	3	4
6	3	3	3	3	3
7	2	2	3	3	2
8	3	3	3	3	3
9	2	2	2	2	2
10	3	3	2	2	2

Table A3. Exam, 2015

Question	Assessor 1	Assessor 2	Assessor 3	Assessor 4	FINAL MAC
1	2	2	3	3	2
2	3	3	3	3	3
3	3	3	2	1	2
4	3	3	3	3	3
5	3	3	2	1	3
6	3	3	2	1	2
7	3	3	2	1	3
8	2	2	2	1	2
9	2	2	3	3	2
10	2	2	2	1	2

Table A4. Semester test 1, 2016

Question	Assessor 1	Assessor 2	Assessor 3	Assessor 4	FINAL MAC
1	3	3	2	3	3
2	2	2	2	1	2
3	2	2	2	3	2
4	3	3	3	3	3
5	1	1	1	1	1
6	2	2	1	2	2
7	2	2	1	2	2
8	3	3	3	4	3
9	3	3	3	3	3
10	2	2	3	2	2
11	2	2	2	3	2
12	2	2	2	1	2
13	2	2	2	3	2

Table A5. Semester test 2, 2016

Question	Assessor 1	Assessor 2	Assessor 3	Assessor 4	FINAL MAC
1	2	2	2	2	2
2	2	2	2	3	2
3	3	3	2	1	2
4	3	3	1	2	2
5	3	3	3	3	3
6	2	2	2	2	2
7	3	3	2	1	3
8	2	2	3	3	2
9	2	2	2	3	2
10	2	2	1	1	2

Table A6. Exam, 2016

Question	Assessor 1	Assessor 2	Assessor 3	Assessor 4	FINAL MAC
1	2	2	2	3	2
2	2	2	2	2	2
3	3	3	3	3	3
4	3	3	3	3	3
5	3	3	2	1	2
6	3	3	2	1	2
7	2	2	2	2	2
8	3	3	2	2	2
9	3	3	2	2	2
10	2	2	2	1	2

Table A7. Semester test 1, 2017

Question	Assessor 1	Assessor 2	Assessor 3	Assessor 4	FINAL MAC
1	2	2	2	3	2
2	2	2	2	1	2
3	2	2	1	2	2
4	2	2	2	3	2
5	2	1	2	3	2
6	2	2	2	1	2
7	3	2	2	3	3
8	2	2	2	3	2
9	2	2	2	1	2
10	2	3	2	3	2
11	2	1	2	3	2
12	2	2	2	3	2

Table A8. Semester test 2, 2017

Question	Assessor 1	Assessor 2	Assessor 3	Assessor 4	FINAL MAC
1	3	3	2	3	3
2	2	2	2	1	2
3	2	2	2	2	2
4	3	2	3	1	3
5	2	2	1	1	2
6	2	2	3	3	2
7	2	2	1	1	2
8	2	2	3	3	2
9	2	2	2	2	2
10	4	4	3	1	4
11	3	3	3	3	3
12	3	3	3	6	3

Table A9. Exam, 2017

Question	Assessor 1	Assessor 2	Assessor 3	Assessor 4	FINAL MAC
1	2	3	2	3	3
2	3	2	2	3	3
3	2	2	2	2	2
4	2	2	2	3	2
5	2	3	3	3	3
6	2	3	3	3	3
7	2	2	2	1	2
8	4	3	3	3	3
9	1	2	2	3	2
10	2	2	2	1	2

Appendix B: Difficulty and discrimination tables for all tests and exams

The following tables present the indices of difficulty and discrimination for the semester tests and exams of 2015, 2016 and 2017.

Table B1. Semester test 1, 2015

YEAR	TEST	QUESTION	Difficulty Index	Discrimination Index
2015	**ST1**	**1**	0.95	0.24
		2	0.84	0.28
		3	0.74	0.38
		4	0.70	0.34
		5	0.88	0.25
		6	0.84	0.38
		7	0.66	0.35
		8	0.35	0.40
		9	0.55	0.38
		10	0.32	0.43
		11	0.64	0.19
		12	0.68	0.36
		13	0.69	0.39
		14	0.62	0.40
		15	0.42	0.47
		16	0.38	0.45
		17	0.58	0.42
		18	0.86	0.30
		19	0.53	0.40
		20	0.93	0.25

Table B2. Semester test 2, 2015

YEAR	TEST	QUESTION	Difficulty Index	Discrimination Index
2015	**ST2**	**1**	0.60	0.48
		2	0.49	0.51
		3	0.32	0.36
		4	0.43	0.45
		5	0.67	0.41
		6	0.54	0.39
		7	0.75	0.34
		8	0.69	0.45
		9	0.67	0.45
		10	0.64	0.50

Table B3. Exam, 2015

YEAR	TEST	QUESTION	Difficulty Index	Discrimination Index
2015	**EXAM**	**1**	0.39	0.41
		2	0.62	0.49
		3	0.56	0.44
		4	0.61	0.44
		5	0.71	0.36
		6	0.55	0.40
		7	0.35	0.35
		8	0.80	0.40
		9	0.62	0.41
		10	0.64	0.42

Table B4. Semester test 1, 2016

YEAR	TEST	QUESTION	Difficulty Index	Discrimination Index
2016	**ST1**	**1**	0.74	0.42
		2	0.77	0.49
		3	0.82	0.49
		4	0.40	0.39
		5	0.85	0.48
		6	0.47	0.48
		7	0.60	0.50
		8	0.54	0.44
		9	0.59	0.48
		10	0.64	0.52
		11	0.63	0.49
		12	0.54	0.48
		13	0.79	0.49

Table B5. Semester test 2, 2016

YEAR	TEST	QUESTION	Difficulty Index	Discrimination Index
2016	**ST2**	**1**	0.39	0.39
		2	0.48	0.51
		3	0.68	0.42
		4	0.48	0.37
		5	0.57	0.43
		6	0.64	0.44
		7	0.51	0.42
		8	0.73	0.38
		9	0.88	0.28
		10	0.79	0.46

Table B6. Exam, 2016

YEAR	TEST	QUESTION	Difficulty Index	Discrimination Index
2016	**EXAM**	**1**	0.43	0.47
		2	0.27	0.48
		3	0.68	0.44
		4	0.58	0.47
		5	0.79	0.38
		6	0.58	0.43
		7	0.93	0.27
		8	0.41	0.39
		9	0.54	0.43
		10	0.57	0.52

Table B7. Semester test 1, 2017

YEAR	TEST	QUESTION	Difficulty Index	Discrimination Index
2017	**ST1**	**1**	0.59	0.43
		2	**0.21**	**0.30**
		3	0.54	0.40
		4	0.61	0.48
		5	**0.22**	**0.38**
		6	0.70	0.49
		7	**0.36**	**0.41**
		8	0.48	0.52
		9	0.76	0.46
		10	0.50	0.41
		11	0.77	0.45
		12	0.69	0.39

Questions 2, 5, and 7, as indicated by the shaded blocks above, are questions where the majority of students chose the wrong distractor.

Table B8. Semester test 2, 2017

YEAR	TEST	QUESTION	Difficulty Index	Discrimination Index
2017	**ST2**	**1**	0.78	0.40
		2	0.57	0.52
		3	0.51	0.41
		4	0.51	0.48
		5	0.95	0.23
		6	0.60	0.43
		7	0.84	0.46
		8	0.69	0.49
		9	0.36	0.39
		10	**0.31**	**0.44**
		11	0.64	0.53
		12	0.58	0.46

Question 10, as indicated by a shaded block above, is a question where the majority of students chose the wrong distractor.

Table B9. Exam, 2017

YEAR	TEST	QUESTION	Difficulty Index	Discrimination Index
2017	**EXAM**	**1**	0.62	0.48
		2	0.64	0.39
		3	0.77	0.46
		4	0.79	0.37
		5	0.86	0.40
		6	0.79	0.50
		7	0.75	0.49
		8	0.60	0.30
		9	0.73	0.48
		10	0.64	0.46

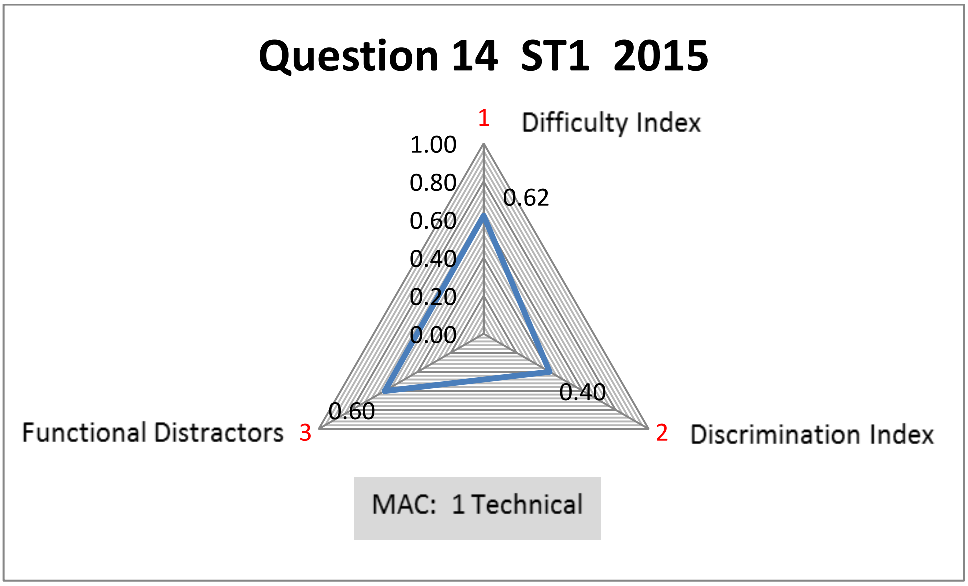
Appendix C: Scatter plots of all multiple choice questions that are *__not__* included in Chapter 4

The questions are divided into nine topics.

1. Functions

Question 14, Semester test 1, 2015

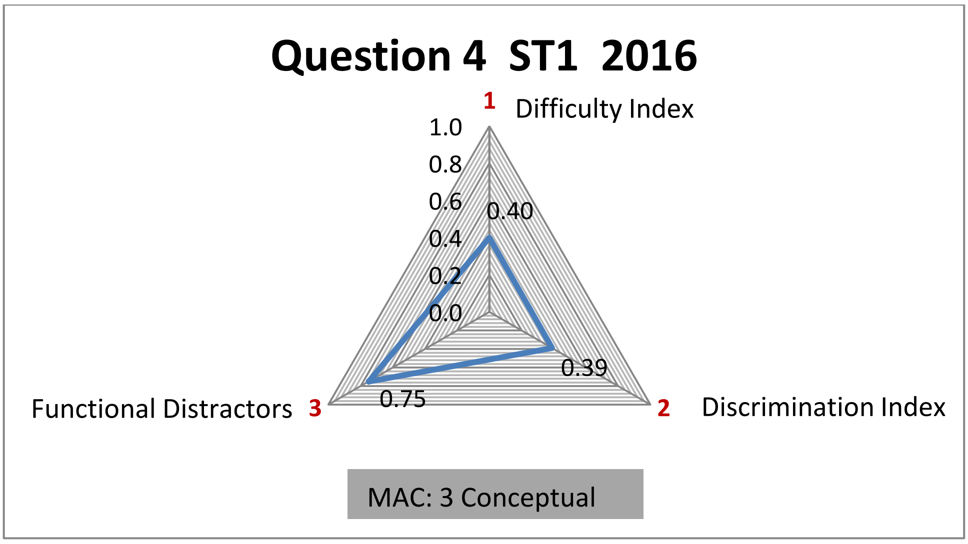
The inverse function to $f(x) = \frac{x+1}{x+2}$ is $f^{-1}(x) =$
(A) $\frac{x-1}{x-2}$ (B) $\frac{2x-1}{x-2}$ (C) $\frac{x-1}{2x-1}$
(D) $\frac{1-2x}{x-1}$ (E) $\frac{1-2x}{x-2}$ (F) None of these



Question 14 ST1 2015	Value	Comment
Topic		Functions
Math Assessment Component	1	Technical
Difficulty Index	0.62	Moderately easy
Discrimination Index	DI = 0.40	Excellent
Distractor efficiency	0.60	
Question 14 discriminates excellently (DI = 0.4) and has 60% functional distractors. The question is moderately easy.		
Conclusion: This question is acceptable as the indices are well balanced. The formulation of the question is clear and the distractors are well chosen.		

Question 4, Semester test 1, 2016

Let $f(x) = \begin{cases} 2, & \lvert x \rvert \le 1 \\ 1 - x, & \lvert x \rvert > 1 \end{cases}$
For the piecewise defined function, which statement is true?
(A) The domain of f is $[0, \infty)$ and the range is R
(B) The domain of f is R and the range is $(-\infty, 0) \cup [2, \infty)$
(C) The domain of f is R and the range is $R\backslash(0,2]$
(D) The domain of f is $[0, \infty)$ and the range is $(-\infty. 2]$
(E) None of these

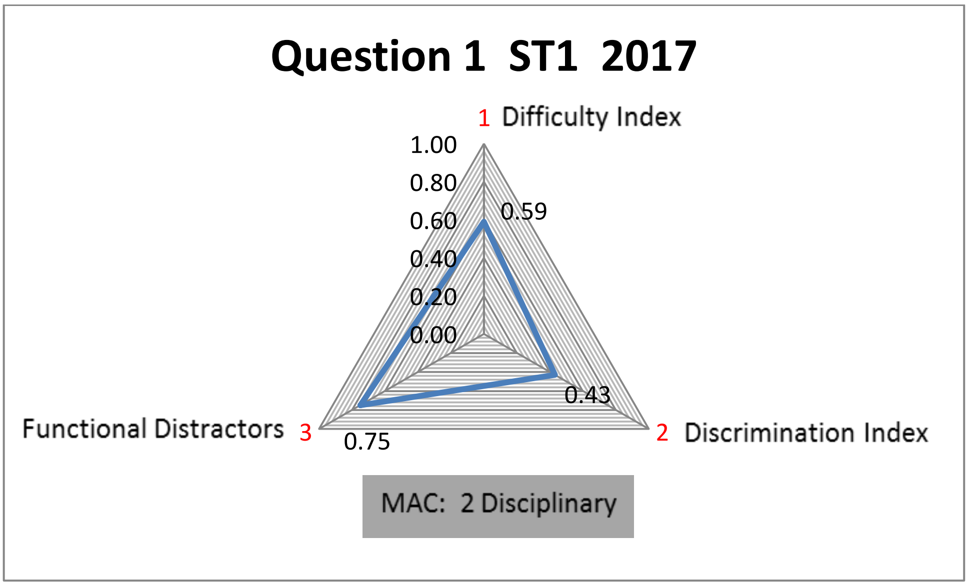


Question 4 ST1 2016	Value	Comment
Topic		Functions
Math Assessment Component	3	Conceptual
Difficulty Index	0.40	Moderately difficult
Discrimination Index	DI = 0.39	Good
Distractor efficiency	0.75	

This question is moderately difficult and shows good discrimination. The high percentage of functional distractors (75%) corresponds to the level of difficulty of the question. The question tests on a conceptual level (MAC is on Level 3). Conclusion: This question is acceptable and offers an example of a moderately difficult question of which the indices are well balanced. The distractors are well chosen.

Question 1, Semester test 1, 2017

Determine the domain of the function $f(x) = \frac{\lvert x \rvert}{\sqrt{2+x-x^2}}$
(A) $(-1,2)$ (B) $(-\infty,-1) \cup (2,\infty)$ (C) $(-2,1)$
(D) $(-\infty,-2) \cup (1,\infty)$ (E) None of these

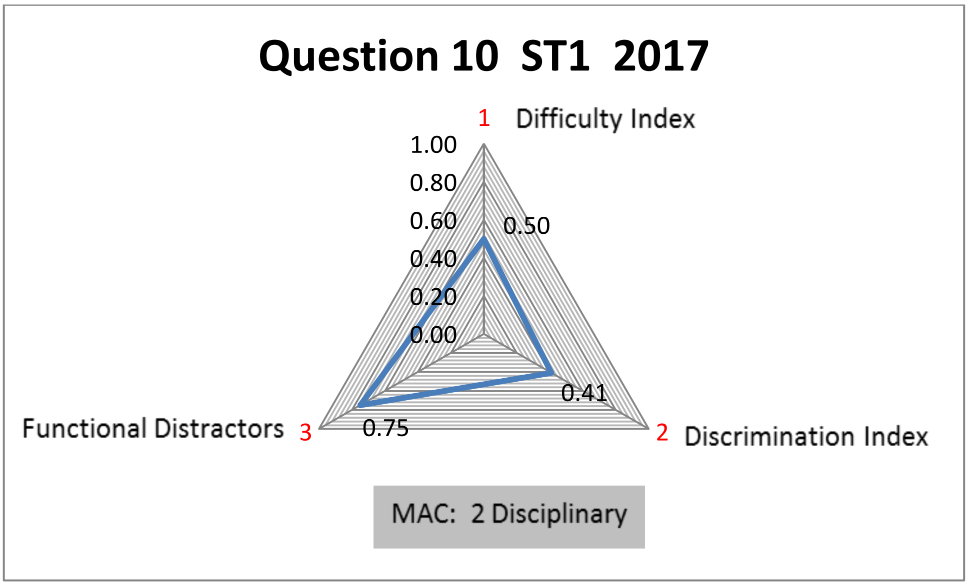


Question 1 ST1 2017	Value	Comment
Topic		Functions
Math Assessment Component	2	Disciplinary
Difficulty Index	0.59	Moderately easy
Discrimination Index	DI = 0.43	Excellent
Distractor efficiency	0.75	

Question 1 can be regarded as a good MCQ since it has 75% functional distractors and discriminates excellently regardless of the fact that the question is moderate to easy. The academically weaker students found the question challenging because of the root with a quadratic equation as a denominator for determining the domain.

Question 10, Semester test 1, 2017

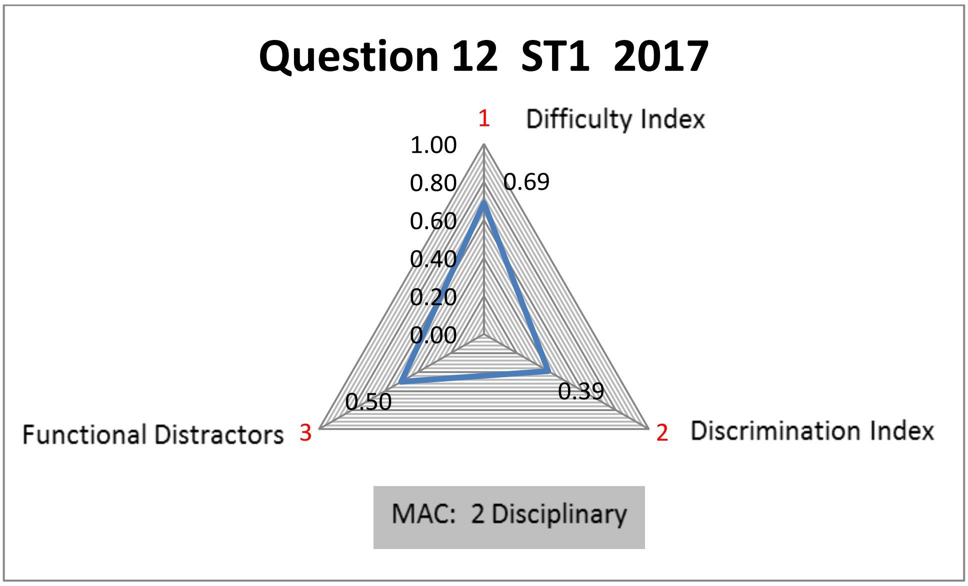
Which one of the following is true for the function $f(x) = \frac{x-2}{x^2+x-6}$?
(A) f has a vertical asymptote $x = 2$
(B) f has a vertical asymptote $x = -3$
(C) f has vertical asymptotes $x = 2$ and $x = -3$
(D) f has no vertical asymptotes
(E) None of these



Question 10 ST1 2017	Value	Comment
Topic		Functions
Math Assessment Component	2	Disciplinary
Difficulty Index	0.50	Moderately difficult
Discrimination Index	DI = 0.41	Excellent
Distractor efficiency	0.75	
Question 10 can be regarded as a good question since the discrimination is excellent, the difficulty is moderate and the number of functional distractors is acceptable.		

Question 12, Semester test 1, 2017

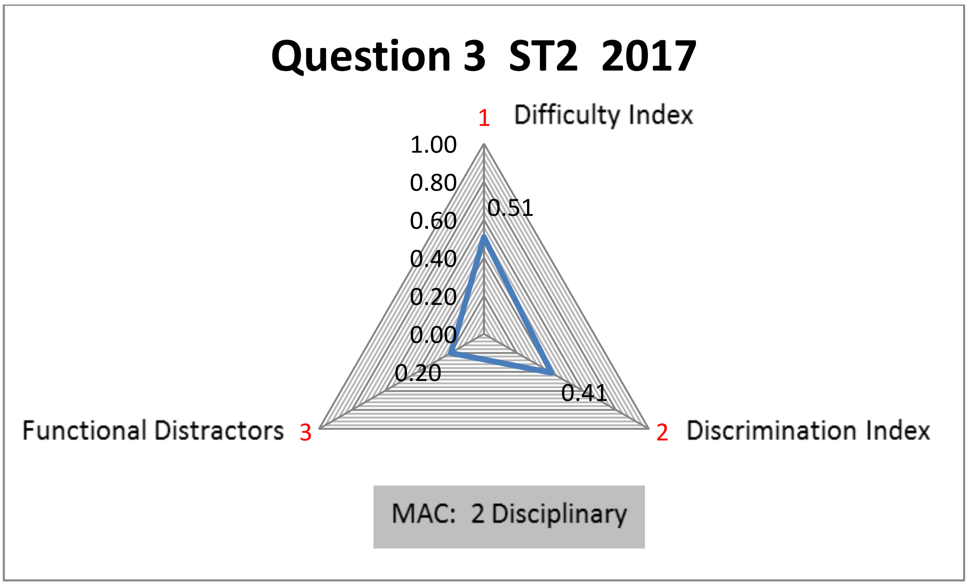
The average rate of change of $f(x) = x^2 + 3$ over the interval $[1,3]$ is				
(A) 2	(B) 4	(C) 6	(D) 8	(E) None of these



Question 12 ST1 2017	Value	Comment
Topic		Functions
Math Assessment Component	2	Disciplinary
Difficulty Index	0.69	Moderately easy
Discrimination Index	DI = 0.39	Good
Distractor efficiency	0.50	

Question 3, Semester test 2, 2017

Given: $(x) = \frac{x^2-4}{x^2-5x+6}$. The equation(s) of all vertical asymptotes is/are		
(A) $x = 3\,,\ x = 2$	(B) $y = 3$	(C) $y = 2\,,\ y = 3$
(D) $x = 3$	(E) $x = 2$	(F) None of these



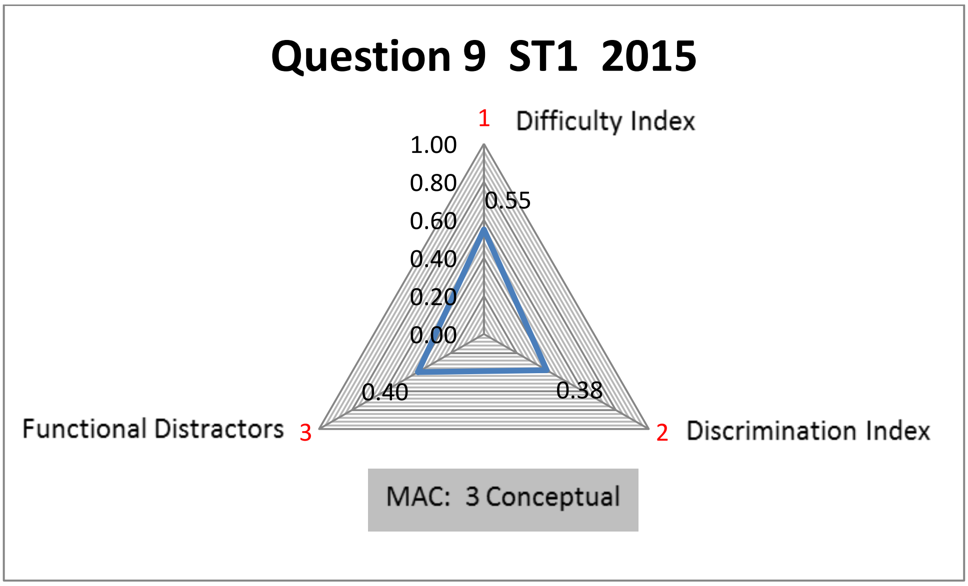
Question 3 ST2 2017	**Value**	**Comment**
Topic		Functions
Math Assessment Component	2	Disciplinary
Difficulty Index	0.51	Moderately easy
Discrimination Index	DI = 0.41	Excellent
Distractor efficiency	0.20	

Question 3 is of moderate difficulty and discriminates excellently.

2. Trigonometry

Question 9, Semester test 1, 2015

For which value(s) of x is $\tan(\tan^{-1}x) = x$?		
(A) $0 \leq x \leq \pi$	(B) $-1 \leq x \leq 1$	(C) $-\pi \leq x \leq \pi$
(D) $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$	(E) $x \in R$	(F) None of these

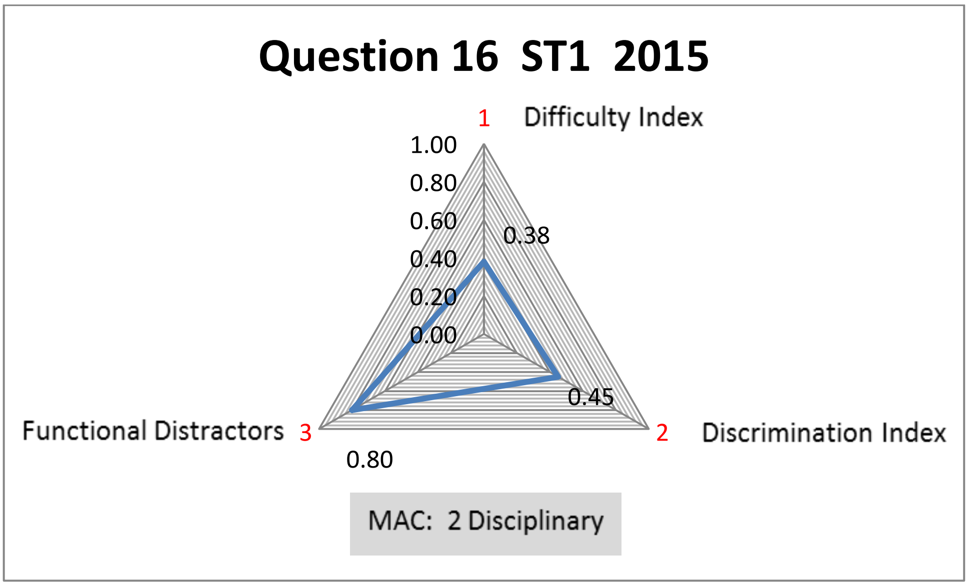


Question 9 ST1 2015	Value	Comment
Topic		Trigonometry
Math Assessment Component	3	Conceptual
Difficulty Index	0.55	Moderately easy
Discrimination Index	DI = 0.38	Good
Distractor efficiency	0.40	

Question 9 is a conceptual question since students need to visualise the graphical representation of the inverse tan graph. The question is of moderate difficulty and discriminates well. The 40% functional distractors are not acceptable, though.

Question 16, Semester test 1, 2015

For which value(s) of x is $y = \sin^{-1}(2x-1)$ defined?
(A) $0 \leq x \leq 1$ (B) $-1 \leq x \leq 1$ (C) $-\frac{\pi}{4}+\frac{1}{2} \leq x \leq \frac{\pi}{4}-\frac{1}{2}$
(D) $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ (E) $x \in R$ (F) None of these

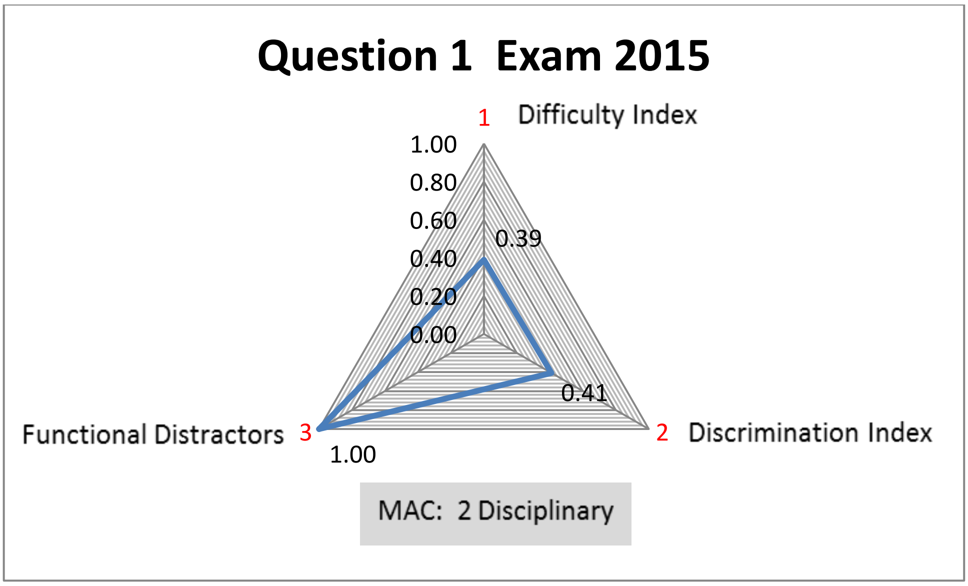


Question 16 ST1 2015	Value	Comment
Topic		Trigonometry
Math Assessment Component	2	Disciplinary
Difficulty Index	0.38	Moderately difficult
Discrimination Index	DI = 0.45	Excellent
Distractor efficiency	0.80	

For Question 16 four out of the five distractors were functional. This is in line with the excellent discrimination between the academically stronger and weaker students. The students found the question to be moderate to difficult since they had to consider the graph of the inverse sin-function and the transformation of half a unit to the right.

Question 1, exam, 2015

The domain of $f(x) = \cos^{-1}(2x - 1)$ is given by
(A) $(-1,1)$ (B) $[-1,1]$ (C) $[0,1]$
(D) $[\frac{1}{2}, \frac{\pi+1}{2}]$ (E) $(-\infty, \infty)$ (F) None of these

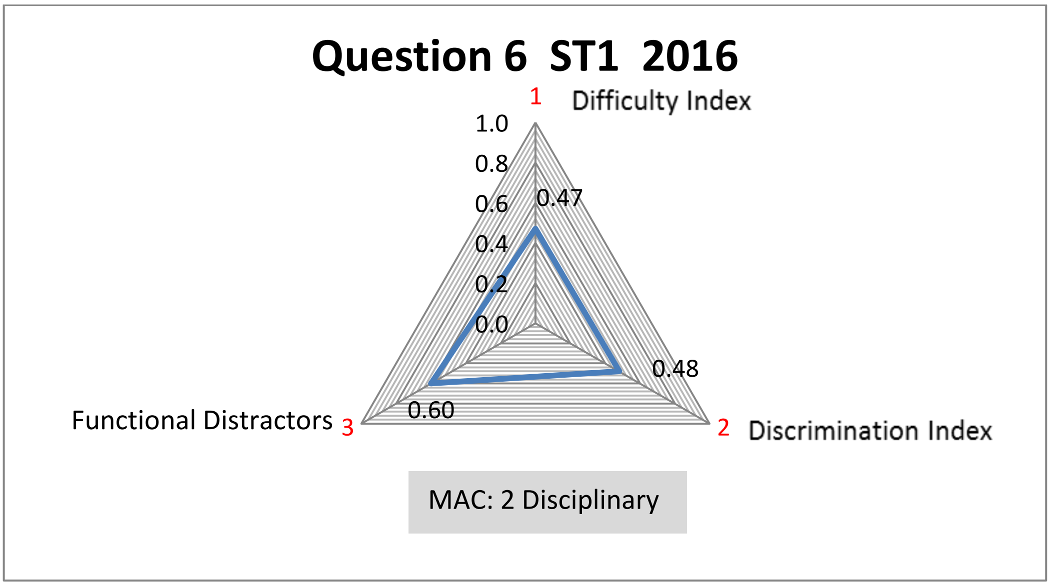


Question 1 Exam 2015	Value	Comment
Topic		Trigonometry
Math Assessment Component	2	Disciplinary
Difficulty Index	0.39	Moderately difficult
Discrimination Index	DI = 0.41	Excellent
Distractor efficiency	1.00	

Question 1 is similar to Question 16 of Semester test 1 of the same year. All the distractors are functional and the discrimination is excellent. The question is moderate to difficult and the MAC level is 2. This question can be regarded as a good question.

Question 6, Semester test 1, 2016

$$\tan\left(-\frac{17\pi}{6}\right) =$$
(A) $-\sqrt{3}$ (B) -1 (C) $\frac{1}{\sqrt{3}}$ (D) $\sqrt{3}$ (E) $-\frac{1}{\sqrt{3}}$
(F) None of these



Question 6 ST1 2016	Value	Comment
Topic		Trigonometry
Math Assessment Component	2	Disciplinary
Difficulty Index	0.47	Moderately difficult
Discrimination Index	DI = 0.48	Excellent
Distractor efficiency	0.60	

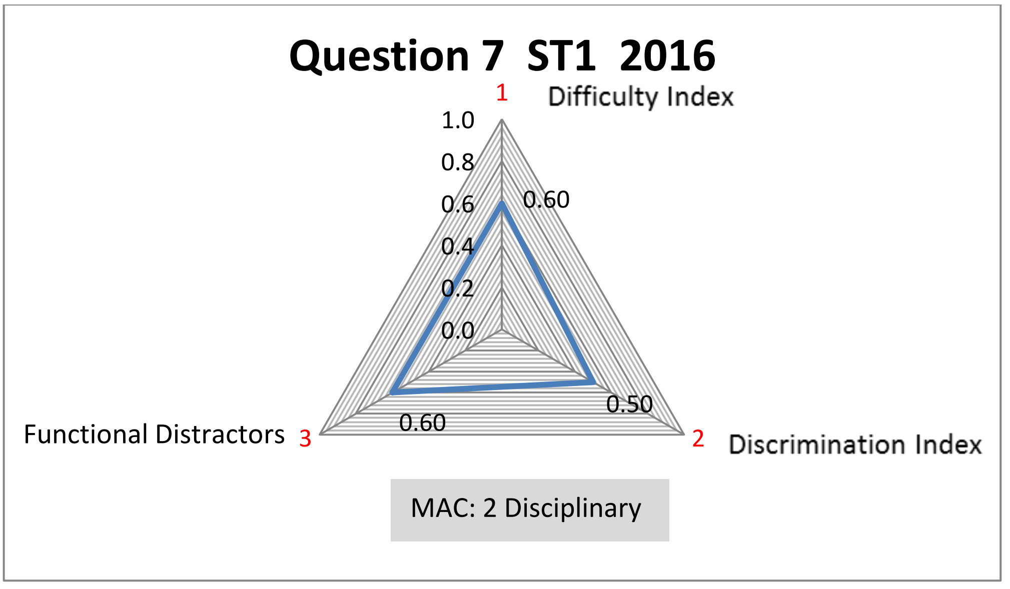
Question 6 can be regarded as a good MCQ since the difficulty index is moderate to difficult, the discrimination is excellent and the 60% functional distractors are acceptable.

Question 7, Semester test 1, 2016

$$\sec\left(\frac{14\pi}{7}\right) =$$

(A) 1 (B) -1 (C) 0 (D) $\frac{\sqrt{3}}{2}$ (E) $-\frac{2}{\sqrt{2}}$

(F) None of these

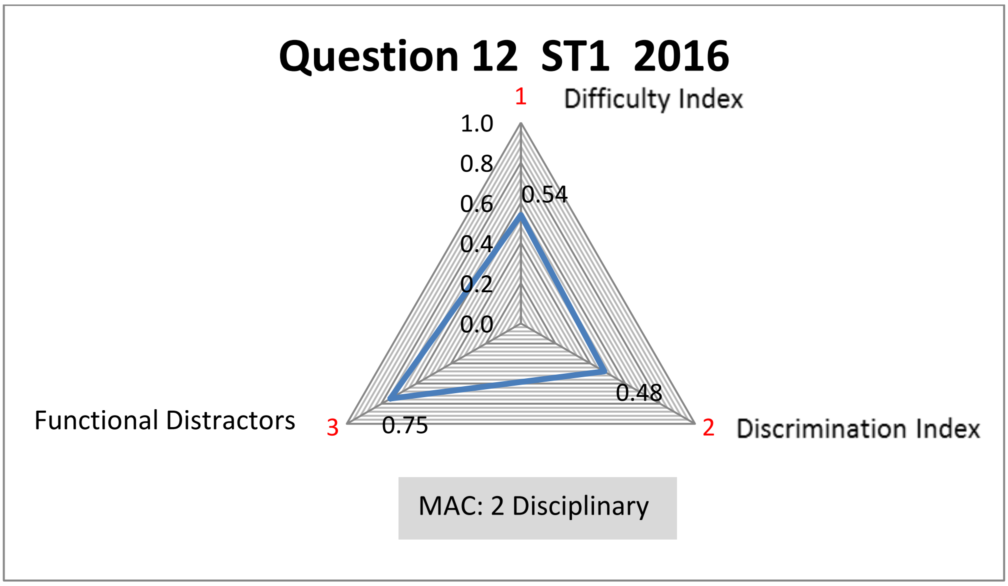


Question 7 ST1 2016	Value	Comment
Topic		Trigonometry
Math Assessment Component	2	Disciplinary
Difficulty Index	0.60	Moderately easy
Discrimination Index	DI = 0.50	Excellent
Distractor efficiency	0.60	

Question 6 can be regarded as a good MCQ because although the difficulty index is moderate to easy, the discrimination is excellent and the 60% functional distractors are acceptable.

Question 12, Semester test 1, 2016

The solution of $2\sin x + 1 = 0$ on $[-2\pi, 2\pi]$ is	
(A) $x \in \left\{\frac{5\pi}{6}, \frac{7\pi}{6}, -\frac{5\pi}{6}, -\frac{7\pi}{6}\right\}$	(B) $x \in \pm\frac{7\pi}{6}$ or $x \in \pm\frac{11\pi}{6}$
(C) $x \in \left\{\frac{7\pi}{6}, \frac{11\pi}{6}, -\frac{\pi}{6}, -\frac{5\pi}{6}\right\}$	(D) $x = \frac{7\pi}{6} + 2k\pi$ or $x = \frac{11\pi}{6} + 2k\pi$
(E) None of these	

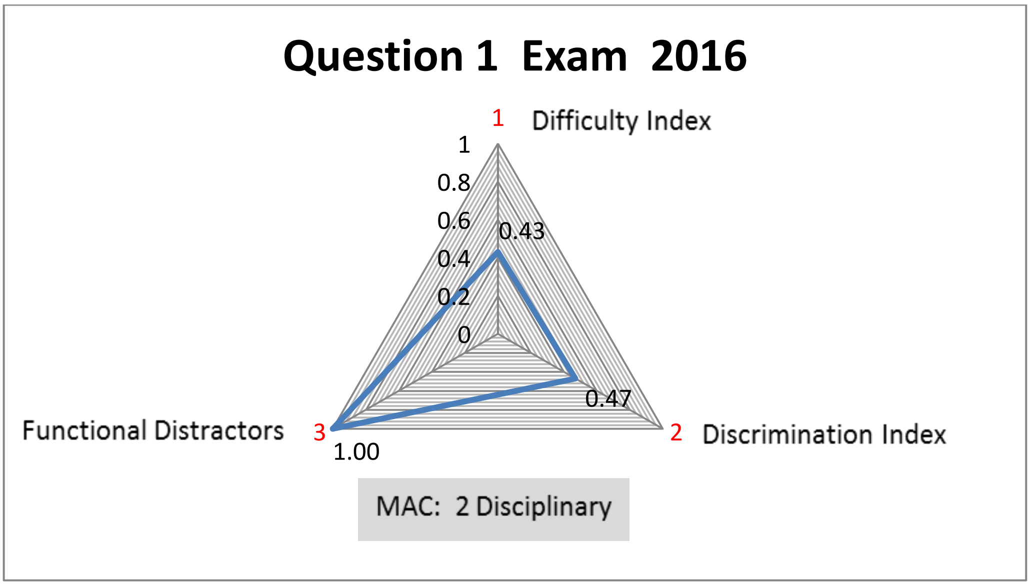


Question 12 ST1 2016	Value	Comment
Topic		Trigonometry
Math Assessment Component	2	Disciplinary
Difficulty Index	0.54	Moderately easy
Discrimination Index	DI = 0.48	Excellent
Distractor efficiency	0.75	

Question 6 can be regarded as a good MCQ since the difficulty index is moderate to easy, the discrimination is excellent and the 75% functional distractors are acceptable.

Question 1, exam, 2016

The domain of $f(x) = \cos^{-1}(2x - 1)$ is given by (A) $(-1,1)$ (B) $[-1,1]$ (C) $[0,1]$ (D) $[\frac{1}{2}, \frac{\pi+1}{2}]$ (E) $(-\infty, \infty)$ (F) None of these



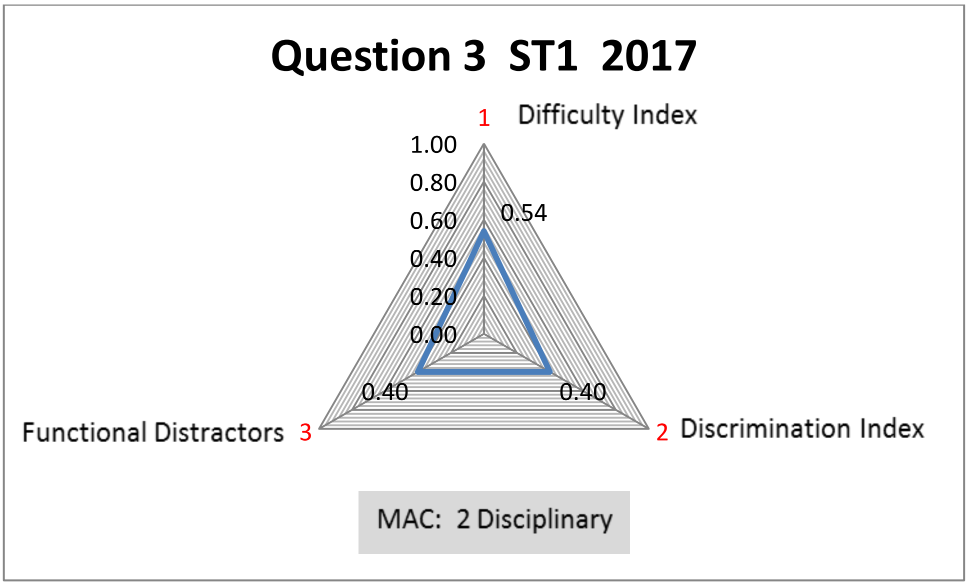
Question 1 Exam 2016	Value	Comment
Topic		Trigonometry
Math Assessment Component	2	Disciplinary
Difficulty Index	0.43	Moderately difficult
Discrimination Index	DI = 0.47	Excellent
Distractor efficiency	1.00	

Compare this Question 1 with Question 1, exam of 2016. It is the same question. The students found the question easier in 2016, the question discriminated better and all distractors are still functional. The question is also similar to Question 16 of Semester test 1 of 2016. All the distractors are functional and the discrimination is excellent. The question is moderate to difficult and the MAC level is 2. Question 1 can be regarded as a good question.

Question 3, semester test 1, 2017

$$\operatorname{cosec}\left(\frac{39\pi}{2}\right) =$$

(A) $\frac{\sqrt{3}}{2}$ (B) -1 (C) $-\sqrt{2}$ (D) 1 (E) 0 (F) None of these

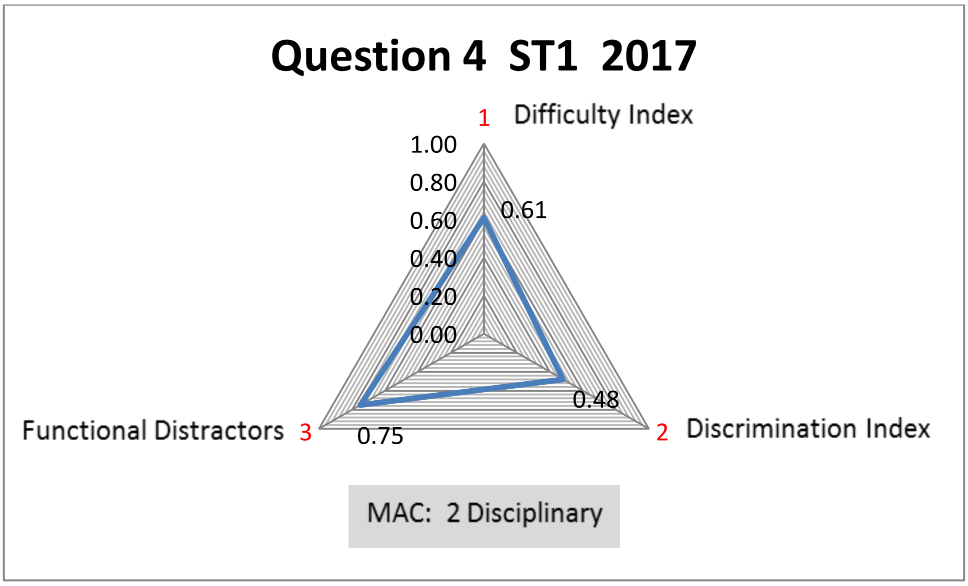


Question 3 ST1 2017	Value	Comment
Topic		Trigonometry
Math Assessment Component	2	Disciplinary
Difficulty Index	0.54	Moderately easy
Discrimination Index	DI = 0.40	Excellent
Distractor efficiency	0.40	

Question 4, Semester test 1, 2017

$$\cos^{-1}\left(-\frac{1}{2}\right) =$$

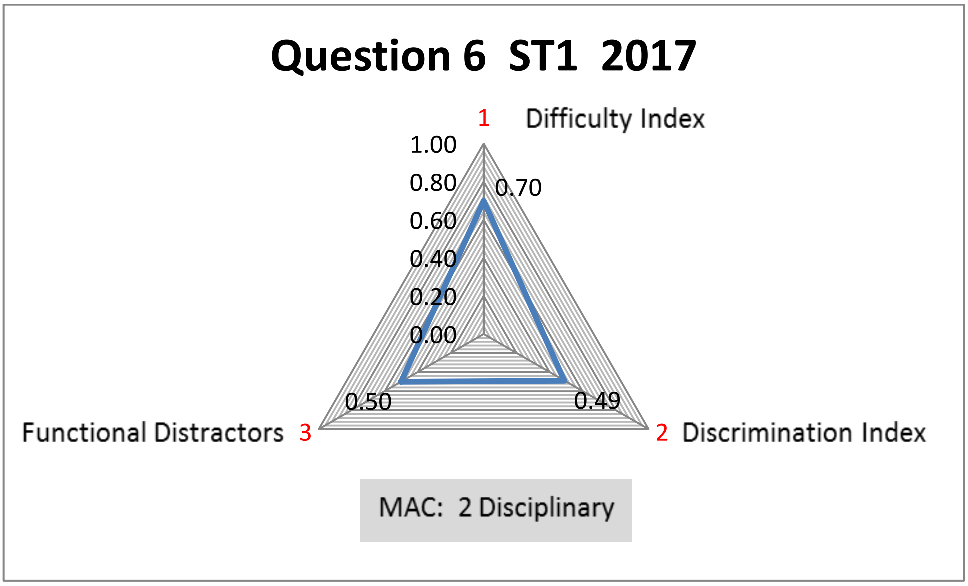
(A) $-\frac{\pi}{6}$ (B) $-\frac{\pi}{3}$ (C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$ (E) None of these



Question 4 ST1 2017	**Value**	**Comment**
Topic		Trigonometry
Math Assessment Component	2	Disciplinary
Difficulty Index	0.61	Moderately easy
Discrimination Index	DI = 0.48	Excellent
Distractor efficiency	0.75	

Question 6, Semester test 1, 2017

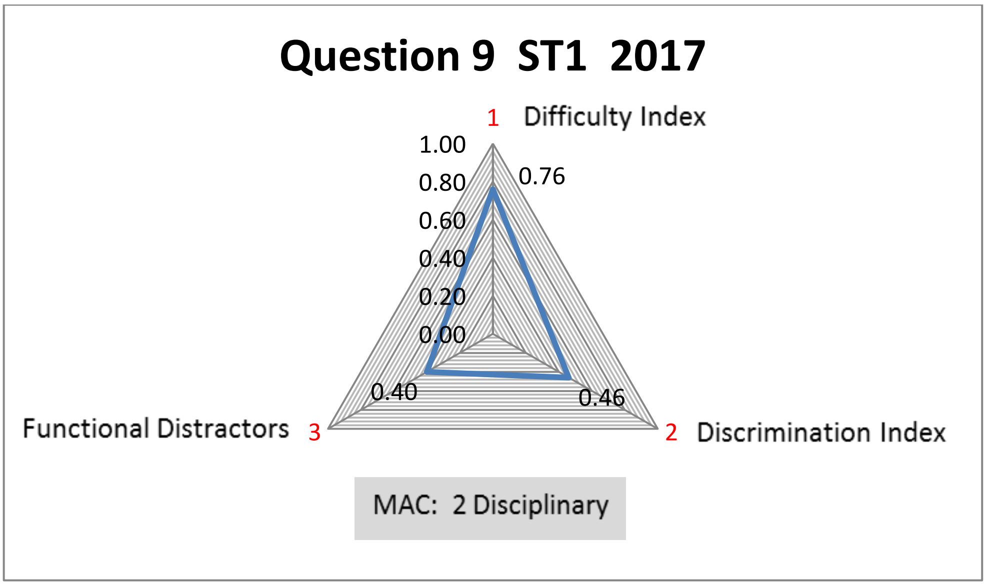
If $\sin\theta = -\frac{1}{2}$ and $0 \le \theta < 2\pi$, then $\theta =$		
(A) $\frac{\pi}{3}$ or $\frac{2\pi}{3}$	(B) $-\frac{\pi}{6}$ or $\frac{5\pi}{6}$	(C) $\frac{7\pi}{6} + 2k\pi$ or $\frac{11\pi}{6} + 2\mathrm{k}\pi, \mathrm{k} \in \mathrm{Z}$
(D) $\frac{7\pi}{6}$ or $\frac{11\pi}{6}$	(E) None of these	



Question 6 ST1 2017	Value	Comment
Topic		Trigonometry
Math Assessment Component	2	Disciplinary
Difficulty Index	0.70	Easy
Discrimination Index	DI = 0.49	Excellent
Distractor efficiency	0.50	

Question 9, Semester test 1, 2017

The solution of $\sin x + \cos x = 0$ for $x \in [0,2\pi)$ is		
(A) $x \in 0$	(B) $x = \frac{\pi}{2}$	(C) $x = \frac{3\pi}{4}$ or $x = \frac{7\pi}{4}$
(D) $x = \frac{\pi}{4}$ or $x = -\frac{3\pi}{4}$	(E) $x = \frac{\pi}{4}$ or $x = \frac{5\pi}{4}$	(F) None of these



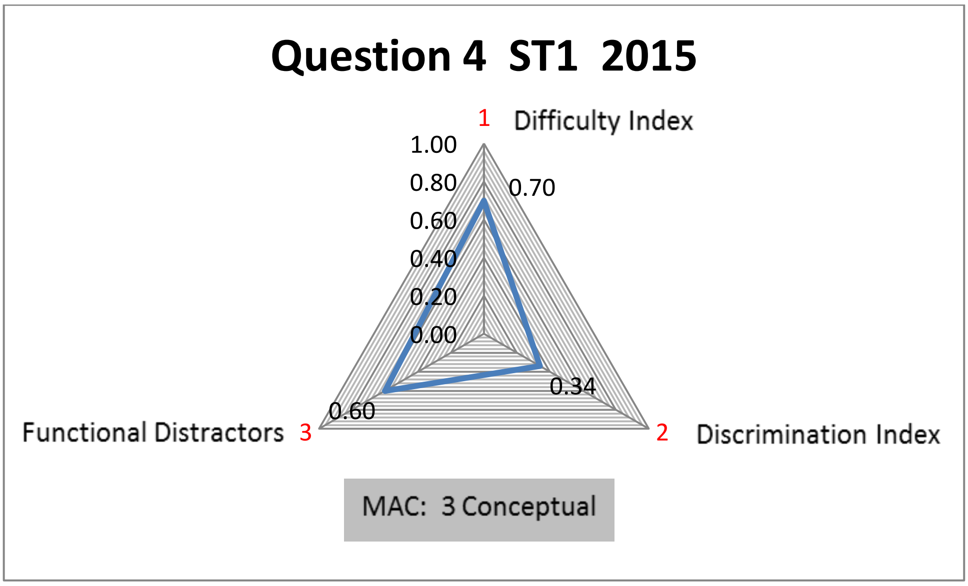
Question 9 ST1 2017	Value	Comment
Topic		Trigonometry
Math Assessment Component	2	Disciplinary
Difficulty Index	0.76	Easy
Discrimination Index	DI = 0.46	Excellent
Distractor efficiency	0.40	

3. Absolute value and inequalities

Question 4, Semester test 1, 2015

If $\frac{x-2}{|x-3|} \geq 0$, then

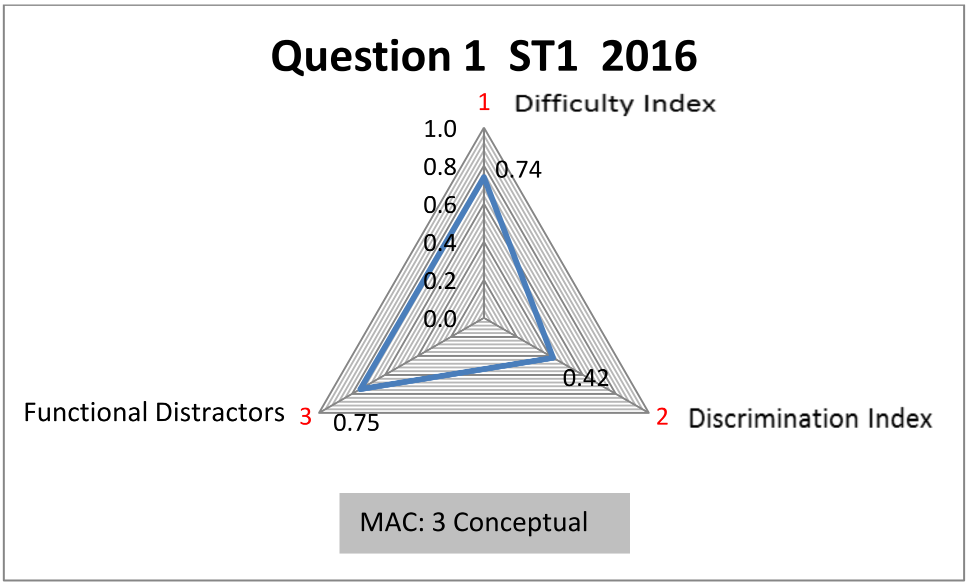
(A) $x \in [2, \infty)$	(B) $x \in (2,3) \cup (3, \infty)$	(C) $x \in [2,3) \cup (3, \infty)$
(D) $x \in (-\infty, 3)$	(E) $x \in (-\infty, 2) \cup (2,3)$	(F) None of these



Question 4 ST1 2015	Value	Comment
Topic		Absolute value & inequality
Math Assessment Component	3	Conceptual
Difficulty Index	0.70	Easy
Discrimination Index	DI = 0.34	Good
Distractor efficiency	0.60	

Question 1, Semester test 1, 2016

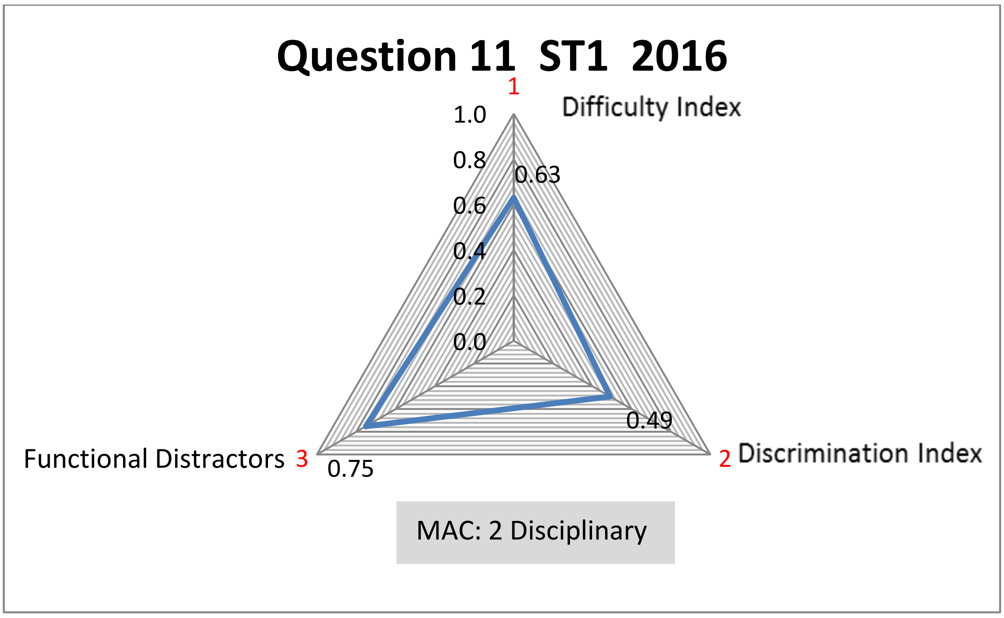
If $\lvert x^2+1 \rvert \geq 2$ then
(A) $x \in [-1,1]$ (B) $x \in [-2,2]$ (C) $x \geq 1$ and $x \leq -1$
(D) $x \in (-\infty,-1] \cup [1,\infty)$ (E) None of these



Question 1 ST1 2016	Value	Comment
Topic		Absolute value & inequality
Math Assessment Component	3	Conceptual
Difficulty Index	0.74	Easy
Discrimination Index	DI = 0.42	Excellent
Distractor efficiency	0.75	

Question 11, Semester test 1, 2016

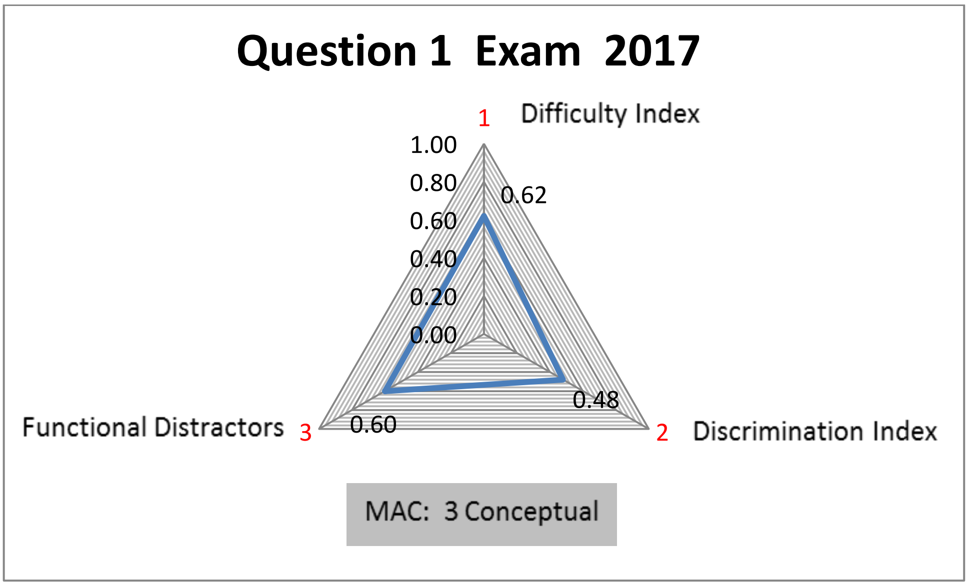
The solution of $\frac{x^2-4x+3}{x} > 0$ is
(A) $x \in (0,1) \cup (3,\infty)$ (B) $x \in (-\infty,1) \cup (3,\infty)$ (C) $x \in (-\infty,0) \cup (1,3)$
(D) $x \in (-4,-2)$ (E) None of these



Question 11 ST1 2016	Value	Comment
Topic		Inequality
Math Assessment Component	2	Disciplinary
Difficulty Index	0.63	Moderately easy
Discrimination Index	DI = 0.49	Excellent
Distractor efficiency	0.75	

Question 1, exam, 2017

The solution of $\frac{\lvert x-5\rvert}{x+1} > 0$ is
(A) $x < -1$ (B) $x < -1$ or $x > 5$ (C) $-1 < x < 5$ (D) $x > -1$ and $x \neq 5$
(E) $x > 5$ (F) None of these

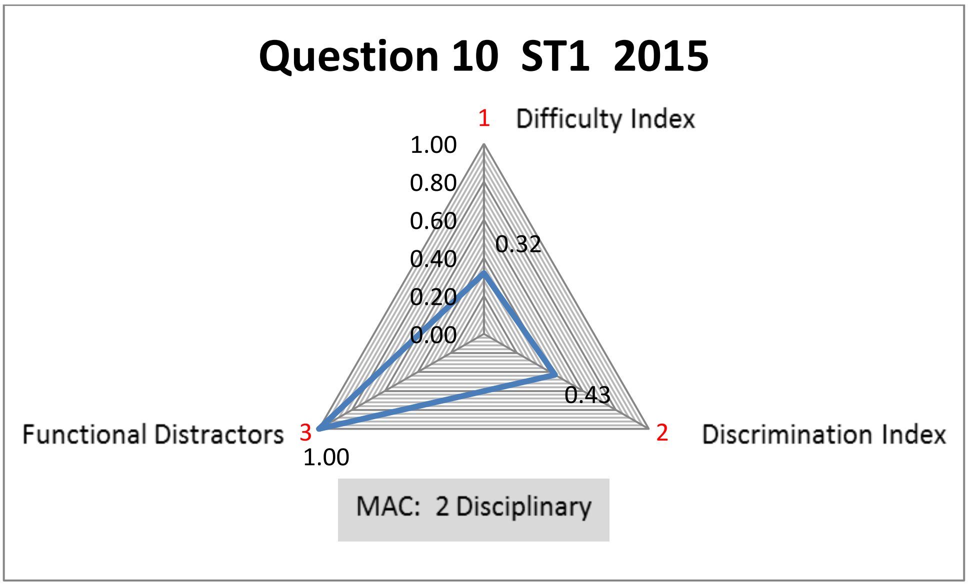


Question 1 Exam 2017	Value	Comment
Topic		Absolute value & inequality
Math Assessment Component	3	Conceptual
Difficulty Index	0.62	Moderately easy
Discrimination Index	DI = 0.48	Excellent
Distractor efficiency	0.60	

4. Exponential and logarithmic functions

Question 10, Semester test 1, 2015

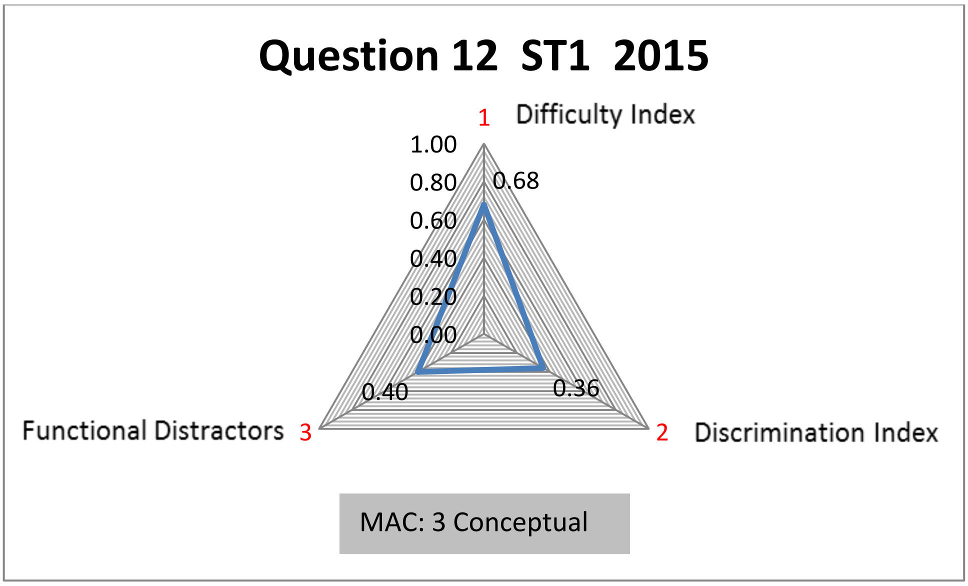
If $\ln x + \ln(x+2) = 0$ then		
(A) $x = 0$ or $x = -2$	(B) $x = -1$	(C) $x = -1 \pm \sqrt{2}$
(D) $x = -1 + \sqrt{2}$	(E) $x = 1$	(F) None of these



Question 10 ST1 2015	Value	Comment
Topic		Logarithmic functions
Math Assessment Component	2	Disciplinary
Difficulty Index	0.32	Moderately difficult
Discrimination Index	DI = 0.43	Excellent
Distractor efficiency	1.00	

Question 12, Semester test 1, 2015

If $xe^{-x} + 2e^{-x} = 0$, then		
(A) $x = -2$	(B) $x = -2$ or $x = 0$	(C) $x = 0$ or $x = 1$
(D) $x = 0$	(E) $x = -2$ or $x = 1$	(F) None of these



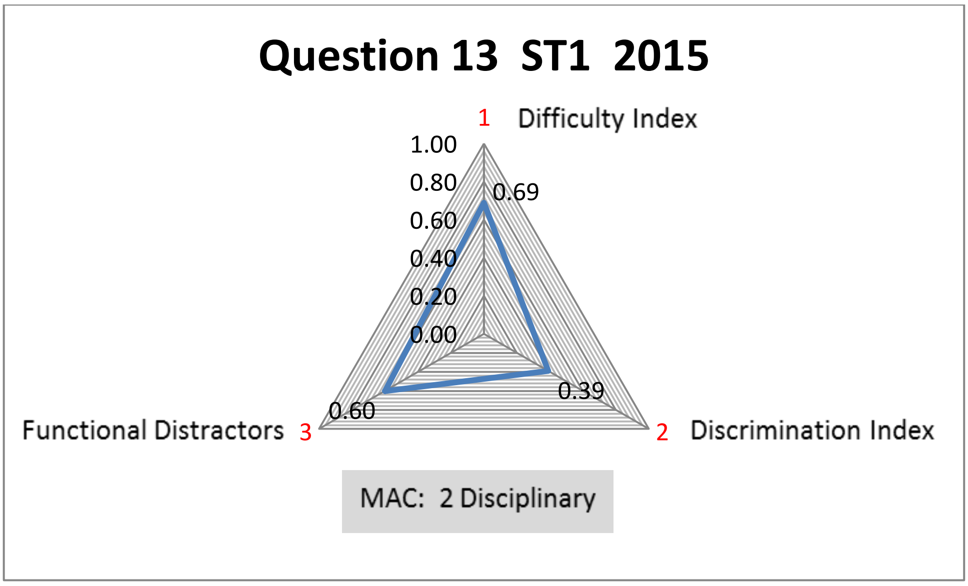
Question 12 ST1 2015	Value	Comment
Topic		Exponential functions
Math Assessment Component	3	Conceptual
Difficulty Index	0.68	Moderately easy
Discrimination Index	DI = 0.36	Good
Distractor efficiency	0.40	

Question 13, Semester test 1, 2015

If $\ln a = r$, $\ln b = s$ and $\ln c = t$, then $\ln\left(b\sqrt{\frac{a}{c^2}}\right) =$

(A) $2r + 2s - \frac{1}{2}t$ (B) $r + s + \frac{1}{2}t$ (C) $\frac{1}{2}(r + s) - t$

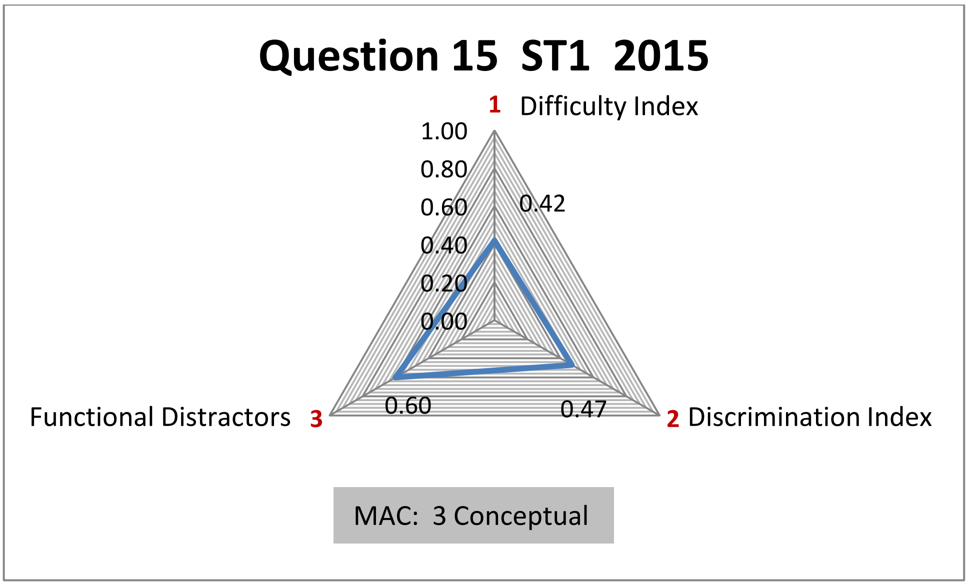
(D) $\frac{1}{2}r + s - t$ (E) $r + \frac{1}{2}s + t$ (F) None of these



Question 13 ST1 2015	Value	Comment
Topic		Logarithmic function
Math Assessment Component	2	Disciplinary
Difficulty Index	0.69	Moderately easy
Discrimination Index	DI = 0.39	Good
Distractor efficiency	0.60	

Question 15, Semester test 1, 2015

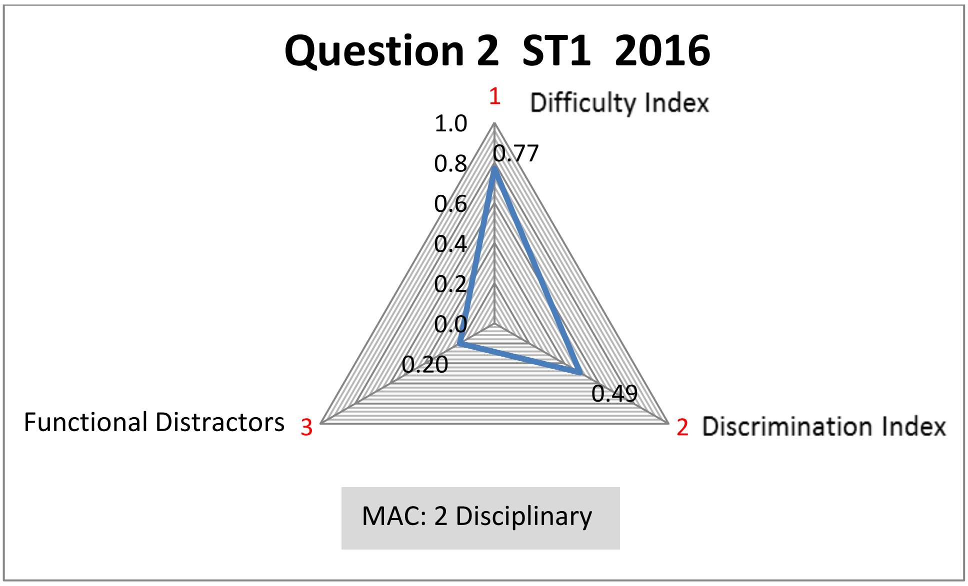
The domain of the function $f(x) = \frac{1}{1+\ln(x+1)}$ is		
(A) $\left(-1+\frac{1}{e}, \infty\right)$	(B) $(-1, \infty)$	(C) $\left(-1, -1+\frac{1}{e}\right) \cup \left(-1+\frac{1}{e}, \infty\right)$
(D) $x \in \left(-1, -1+\frac{1}{e}\right)$	(E) $x \in R, x \neq -1+\frac{1}{e}$	(F) None of these



Question 15 ST1 2015	Value	Comment
Topic		Logarithmic function
Math Assessment Component	3	Conceptual
Difficulty Index	0.42	Moderately difficult
Discrimination Index	DI = 0.47	Excellent
Distractor efficiency	0.60	

Question 2, Semester test 1, 2016

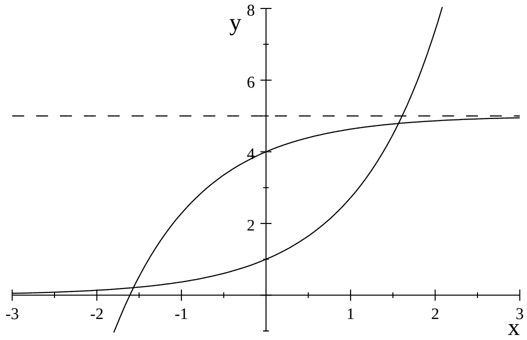
If $\lvert \ln x - 1 \rvert = 1$ then		
(A) $x = 1$ or $x = e - 1$	(B) $x = 1$ or $x = e^2$	(C) $x = 1$ or $x = e$
(D) $x = 0$ or $x = e - 1$	(E) $x = 0$	(F) None of these



Question 2 ST1 2016	Value	Comment
Topic		Logarithms & absolute value
Math Assessment Component	2	Disciplinary
Difficulty Index	0.77	Easy
Discrimination Index	DI = 0.49	Excellent
Distractor efficiency	0.20	

Question 9, Semester test 1, 2016

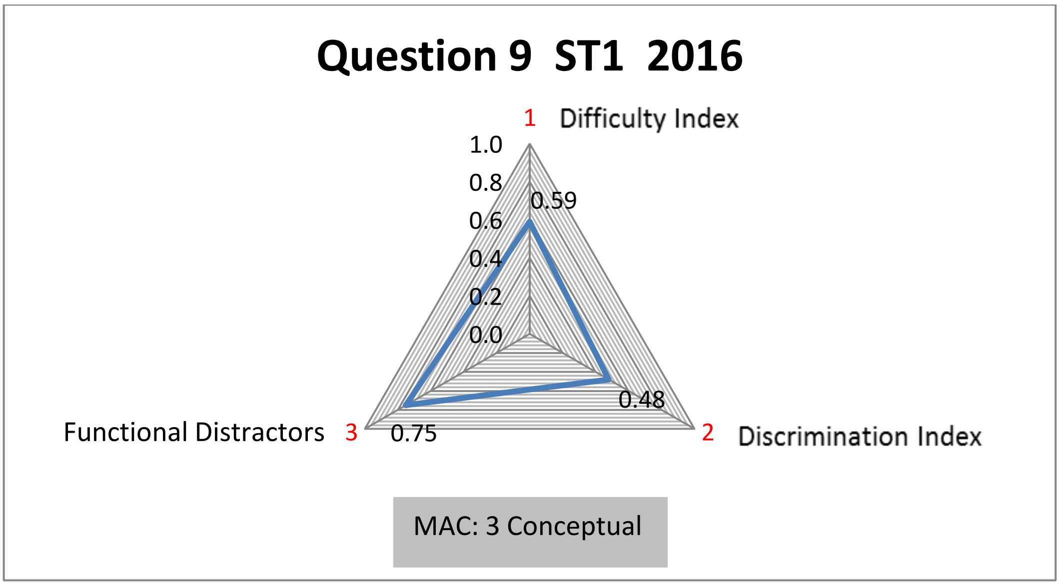
The function $f(x) = e^x$ and the transformed function $g(x)$ are shown in the sketch.



The function $g(x)$ is given by

(A) $g(x) = 5 - e^x$ (B) $g(x) = e^{-x} + 5$ (C) $g(x) = -e^x + 5$

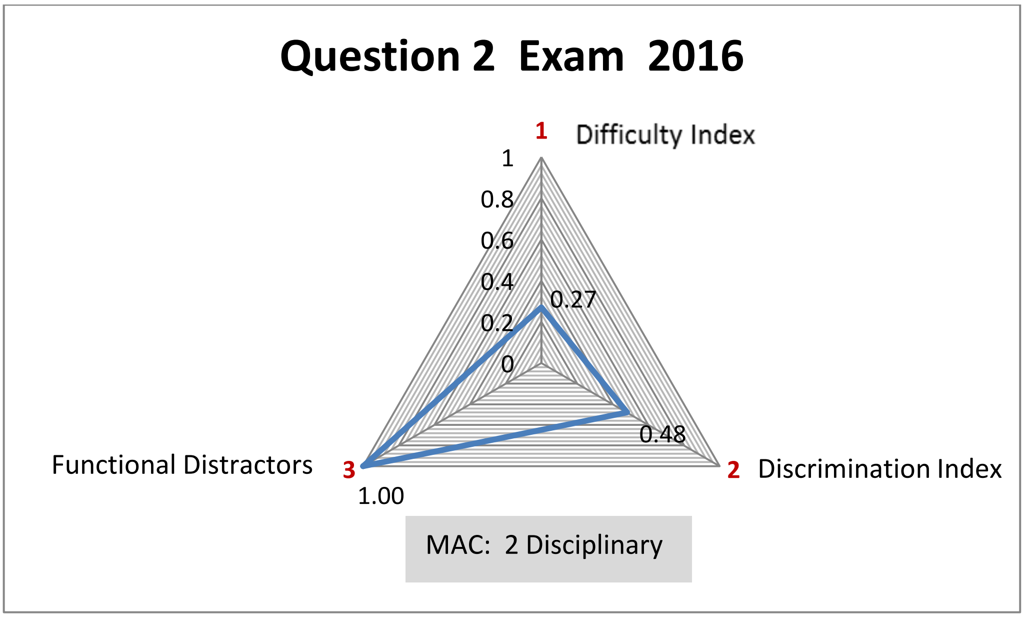
(D) $g(x) = 5 - e^{-x}$ (E) None of these



Question 9 ST1 2016	Value	Comment
Topic		Exponential functions
Math Assessment Component	3	Conceptual
Difficulty Index	0.59	Moderately easy
Discrimination Index	DI = 0.48	Excellent
Distractor efficiency	0.75	

Question 2, exam, 2016

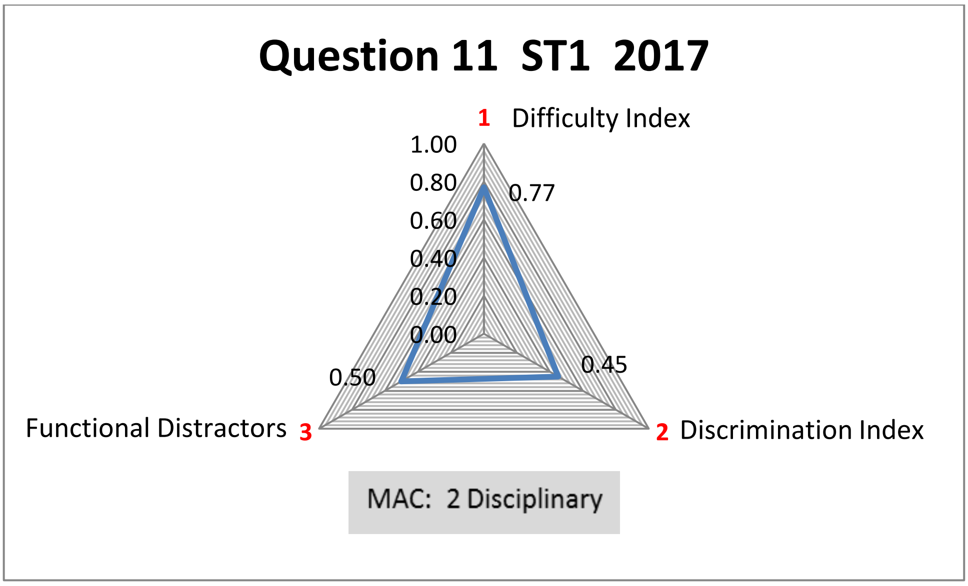
$\cosh(\ln 3) =$					
(A) $\frac{1}{3}$	(B) $\frac{1}{2}$	(C) $\frac{2}{3}$	(D) $\frac{4}{3}$	(E) $\frac{5}{3}$	(F) None of these



Question 2 Exam 2016	Value	Comment
Topic		Logarithmic function & trigonometry
Math Assessment Component	2	Disciplinary
Difficulty Index	0.27	Difficult
Discrimination Index	DI = 0.48	Excellent
Distractor efficiency	1.00	

Question 11, Semester test 1, 2017

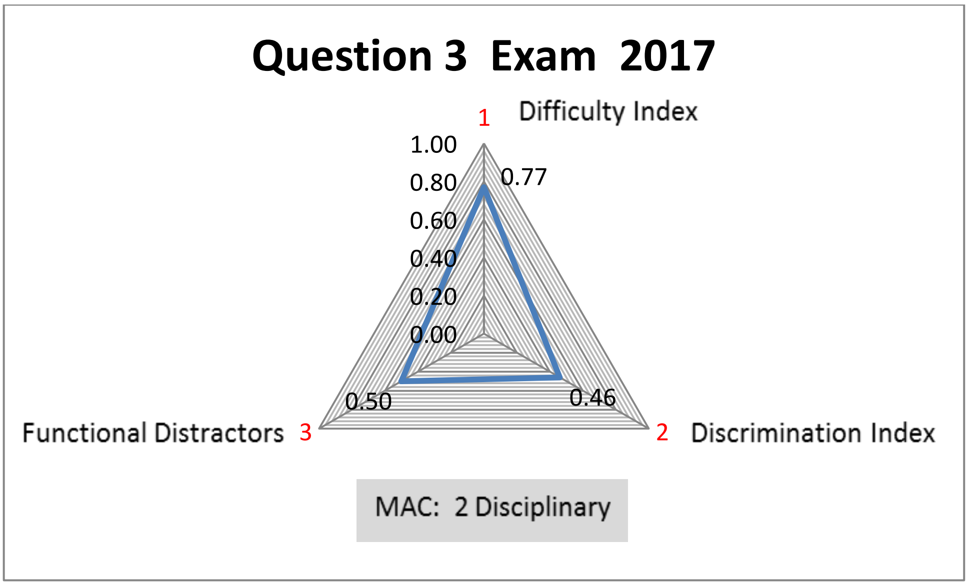
The solution of $\ln(\ln(x)) = 1$ is				
(A) $x = 0$	(B) $x = 1$	(C) $x = e^e$	(D) $x = e$	(E) None of these



Question 11 ST1 2017	Value	Comment
Topic		Logarithmic function
Math Assessment Component	2	Disciplinary
Difficulty Index	0.77	Easy
Discrimination Index	DI = 0.45	Excellent
Distractor efficiency	0.50	

Question 3, exam, 2017

The inverse function of $f(x) = e^x, x \in [0,1]$ is		
(A) $y = e^{-x}, x \in [0,1]$	(B) $y = \ln x, x \in [0,1]$	(C) $y = e^{\frac{1}{x}}, x \in [1, e]$
(D) $y = \ln x, x \in [1, e]$	(E) None of these	

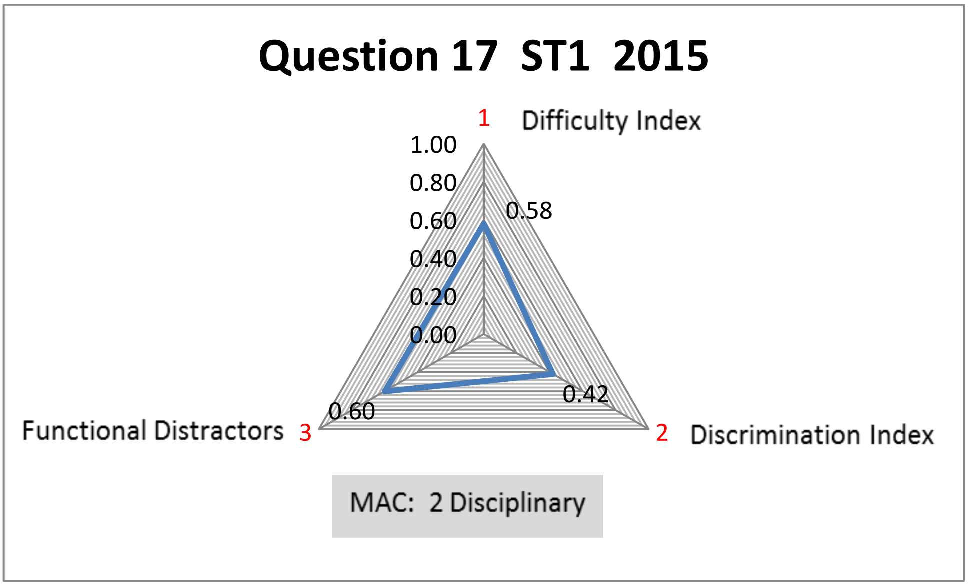


Question 3 Exam 2017	**Value**	**Comment**
Topic		Logarithmic function
Math Assessment Component	2	Disciplinary
Difficulty Index	0.77	Easy
Discrimination Index	DI = 0.46	Excellent
Distractor efficiency	0.50	

5. Limits and continuity

Question 17, Semester test 1, 2015

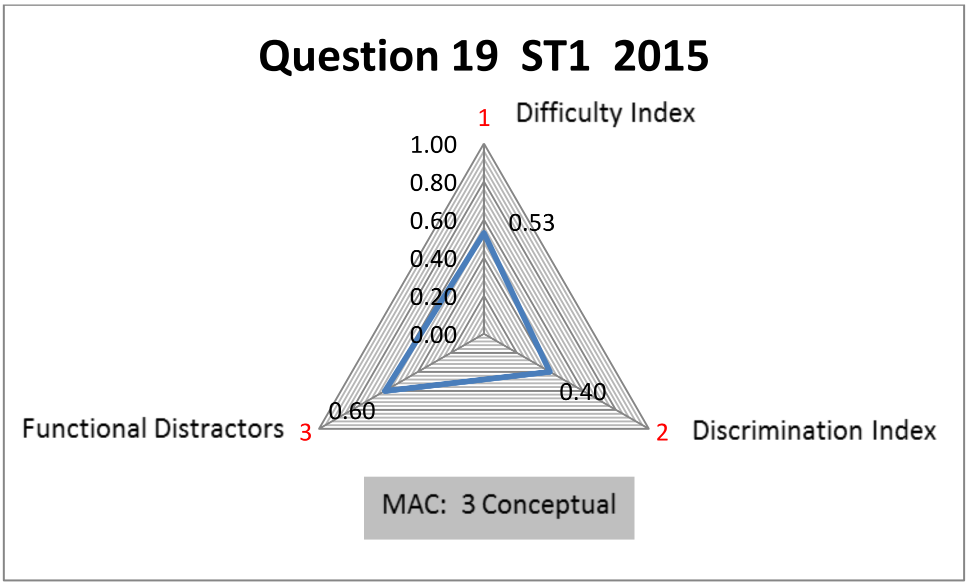
$$\lim_{x \to 5} \operatorname{cosec}\left(\frac{\pi x}{4}\right) =$$					
(A) 1	(B) -1	(C) $-\sqrt{2}$	(D) $-\frac{1}{\sqrt{2}}$	(E) $\sqrt{2}$	(F) None of these



Question 17 ST1 2015	Value	Comment
Topic		Limits & trigonometry
Math Assessment Component	2	Disciplinary
Difficulty Index	0.58	Moderately easy
Discrimination Index	DI = 0.42	Excellent
Distractor efficiency	0.60	

Question 19, Semester test 1, 2015

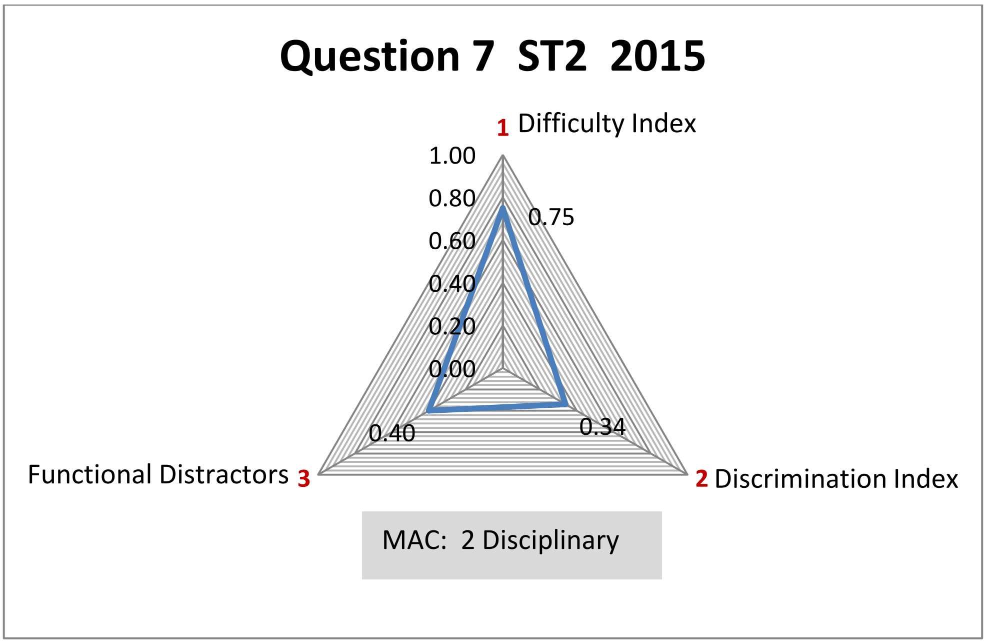
$$\lim_{x \to 0} \frac{\sin 2x \tan x}{x^2} =$$					
(A) 0	(B) 1	(C) 2	(D) π	(E) -1	(F) None of these



Question 19 ST1 2015	Value	Comment
Topic		Limits & Trigonometry
Math Assessment Component	3	Conceptual
Difficulty Index	0.53	Moderately easy
Discrimination Index	DI = 0.40	Excellent
Distractor efficiency	0.60	

Question 7, Semester test 2, 2015

The horizontal line $y = 3$ is an asymptote for the graph of the function f for $x \geq 0$. Which of the following statements <u>must</u> be true?
(A) $f(0) = 3$ (B) $f(x) \neq 3$ for all x (C) $f(3)$ is undefined
(D) $\lim_{x\to 3} f(x) = \infty$ (E) $\lim_{x\to\infty} f(x) = 3$ (F) None of these

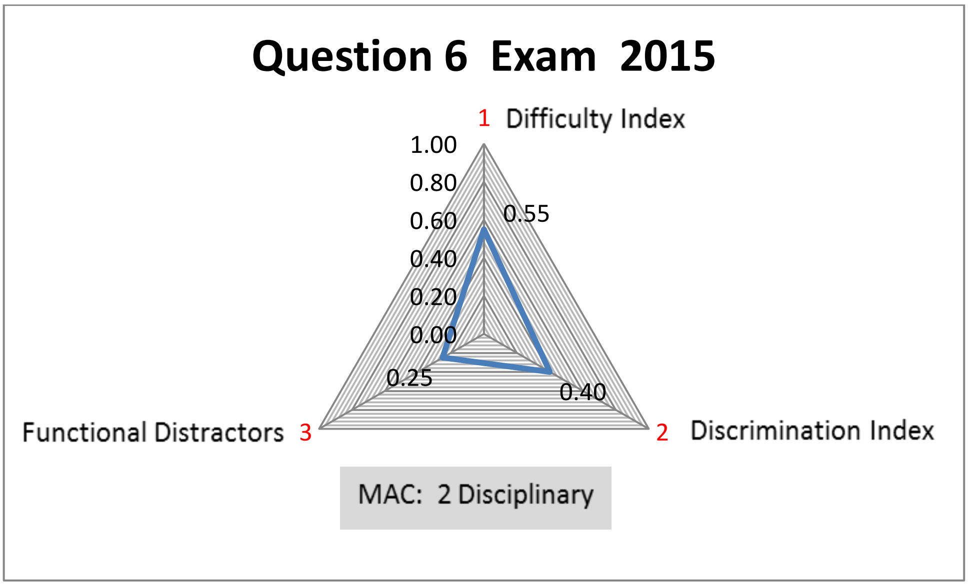


Question 7 ST2 2015	**Value**	**Comment**
Topic		Limits & continuity
Math Assessment Component	2	Disciplinary
Difficulty Index	0.75	Easy
Discrimination Index	DI = 0.34	Good
Distractor efficiency	0.40	

Question 6, exam, 2015

$$\lim_{t \to 1^+} \left(\frac{3}{t-1} - \frac{5}{t^2-1} \right) =$$

(A) 0 (B) $-\infty$ (C) ∞ (D) -1 (E) None of these



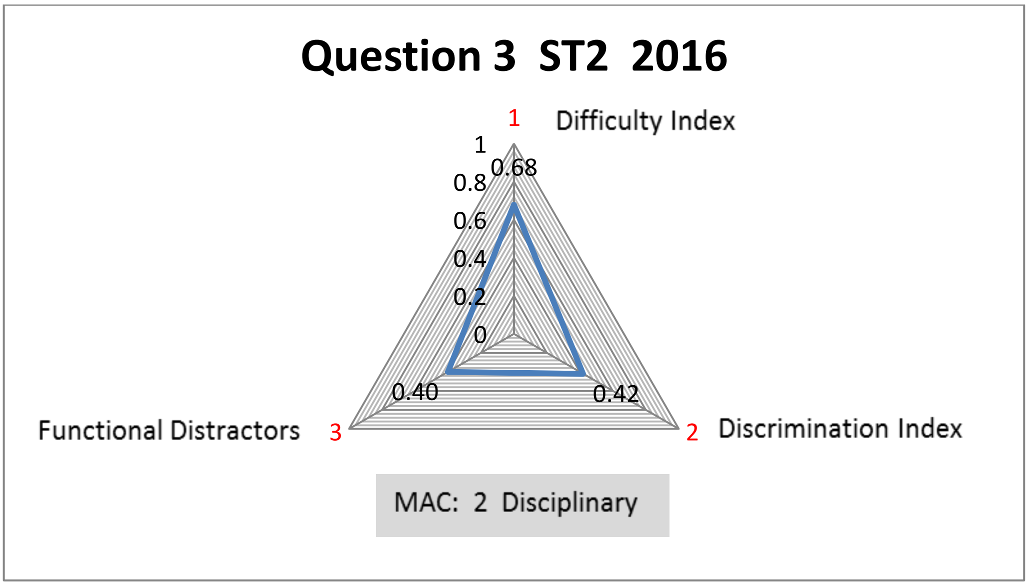
Question 6 Exam 2015	Value	Comment
Topic		Limits
Math Assessment Component	2	Disciplinary
Difficulty Index	0.55	Moderately easy
Discrimination Index	DI = 0.40	Excellent
Distractor efficiency	0.25	

Question 3, Semester test 2, 2016

$$\lim_{x \to -1} \frac{x^2 + 2x + 1}{x\sin(x+1)} =$$

(A) 0 (B) ∞ (C) -1 (D) 1 (E) Does not exist

(F) None of these



Question 3 ST2 2016	Value	Comment
Topic		Limits & trigonometry
Math Assessment Component	2	Disciplinary
Difficulty Index	0.68	Moderately easy
Discrimination Index	DI = 0.42	Excellent
Distractor efficiency	0.40	

Question 5, Semester test 2, 2016

Let $f(x) = \begin{cases} x^2 - 1, & x \neq 1 \\ 4, & x = 1 \end{cases}$

Which of the following statements is/are true?

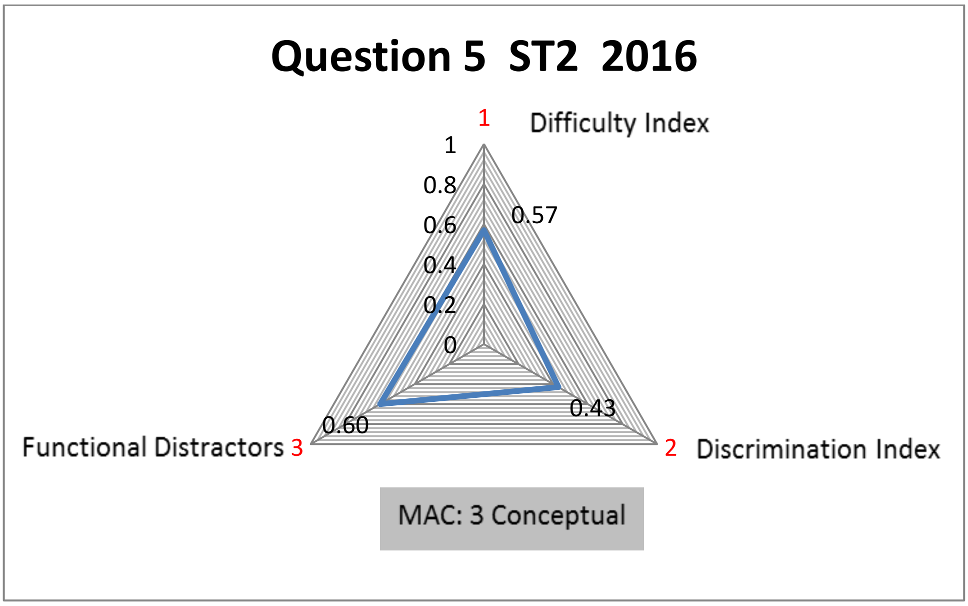
$I.$ $\lim_{x \to 1} f(x)$ exists

$II.$ $f(1)$ exists

$III.$ f is continuous at $x = 1$

(A) Only I (B) Only II (C) Only III (D) I and II

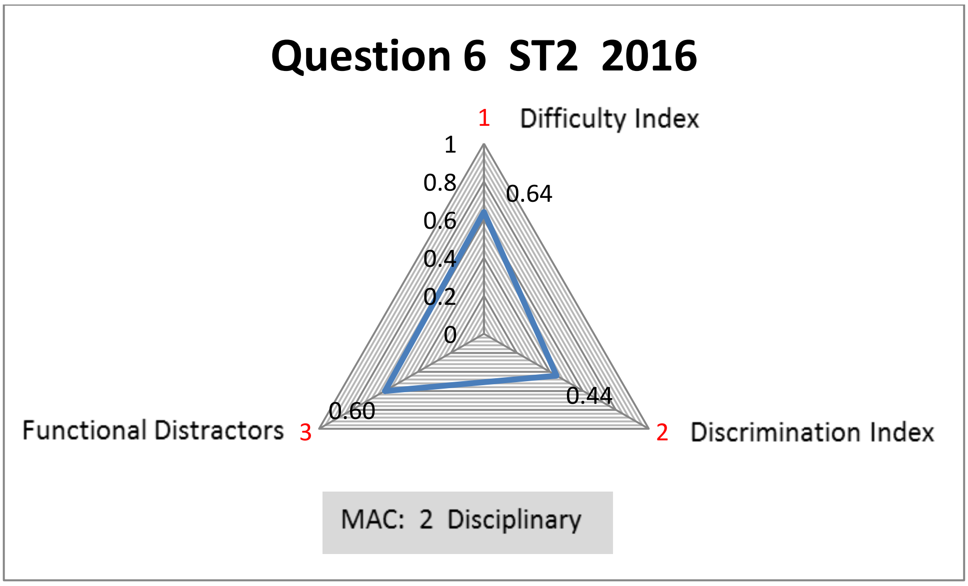
(E) All of them (F) None of these



Question 5 ST2 2017	Value	Comment
Topic		Limits & continuity
Math Assessment Component	3	Conceptual
Difficulty Index	0.57	Moderately easy
Discrimination Index	DI = 0.43	Excellent
Distractor efficiency	0.60	

Question 6, Semester test 2, 2016

The horizontal asymptotes (if any) of $f(x) = \frac{ax^3}{b+cx+dx^2}$ is given by
(A) $y = \frac{a}{b}$ (B) $y = 0$ (C) $y = \frac{a}{d}$ (D) $y = a$
(E) There are no horizontal asymptotes (F) None of these

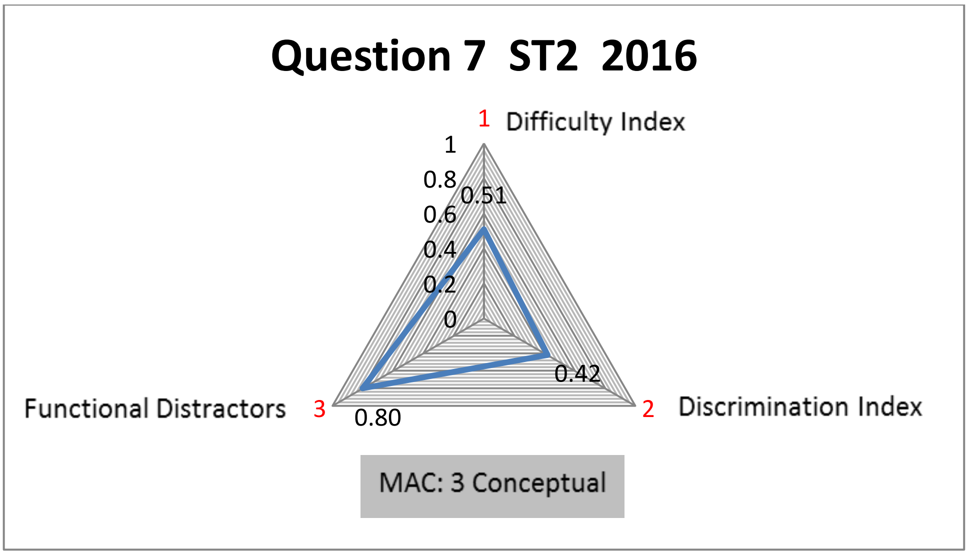


Question 6 ST2 2016	Value	Comment
Topic		Limits
Math Assessment Component	2	Disciplinary
Difficulty Index	0.64	Moderately easy
Discrimination Index	DI = 0.44	Excellent
Distractor efficiency	0.60	

Question 7, Semester test 2, 2016

$$\lim_{\theta \to 0} \frac{\sin 3\theta}{\theta + \tan \theta} =$$

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) 1 (E) Does not exist (F) None of these

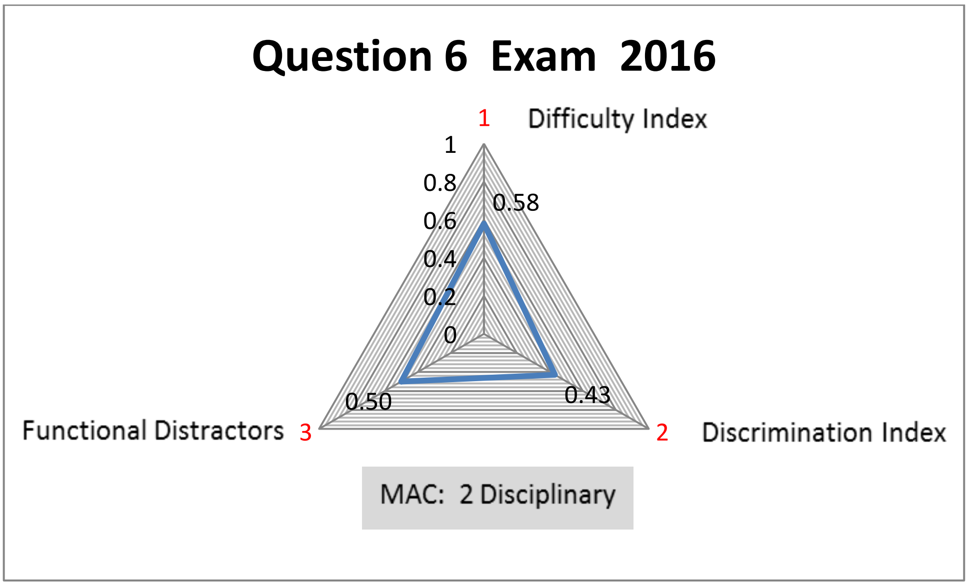


Question 7 ST2 2016	Value	Comment
Topic		Limits & trigonometry
Math Assessment Component	3	Conceptual
Difficulty Index	0.51	Moderately easy
Discrimination Index	DI = 0.42	Excellent
Distractor efficiency	0.80	

Question 6, exam, 2016

$$\lim_{t \to 1^+} \left(\frac{3}{t-1} - \frac{5}{t^2-1} \right) =$$

(A) 0 (B) $-\infty$ (C) ∞ (D) -1 (E) None of these

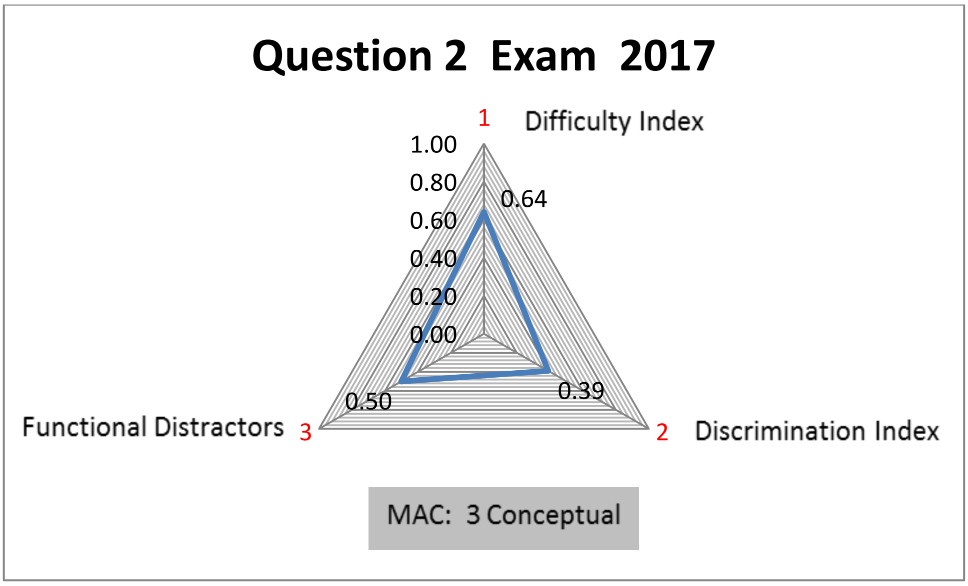


Question 6 Exam 2016	Value	Comment
Topic		Limits
Math Assessment Component	2	Disciplinary
Difficulty Index	0.58	Moderately easy
Discrimination Index	DI = 0.43	Excellent
Distractor efficiency	0.50	

Question 2, exam, 2017

$$\lim_{t \to 1^-} \frac{4}{t^2 - 1} =$$

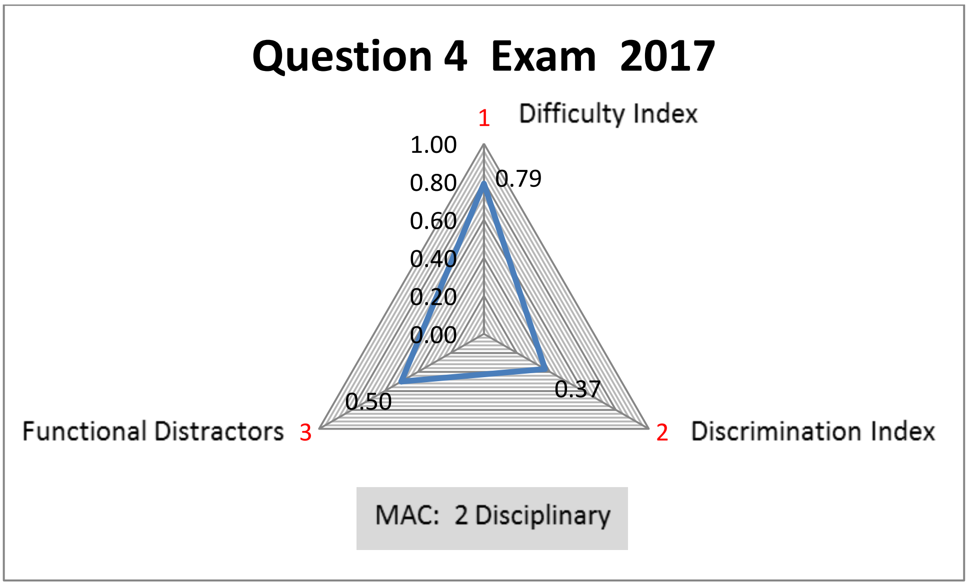
(A) 0 (B) $-\infty$ (C) ∞ (D) -1 (E) None of these



Question 2 Exam 2017	Value	Comment
Topic		Limits
Math Assessment Component	3	Conceptual
Difficulty Index	0.64	Moderately easy
Discrimination Index	DI = 0.39	Good
Distractor efficiency	0.50	

Question 4, exam, 2017

For the function $f(x) = \frac{2x+1}{2x^2-x-1}$, which one of the following is true?
(A) f has a horizontal asymptote $y = 0$
(B) f has a horizontal asymptote $y = 1$
(C) f has a horizontal asymptote $y = -\frac{1}{2}$
(D) f has no horizontal asymptotes

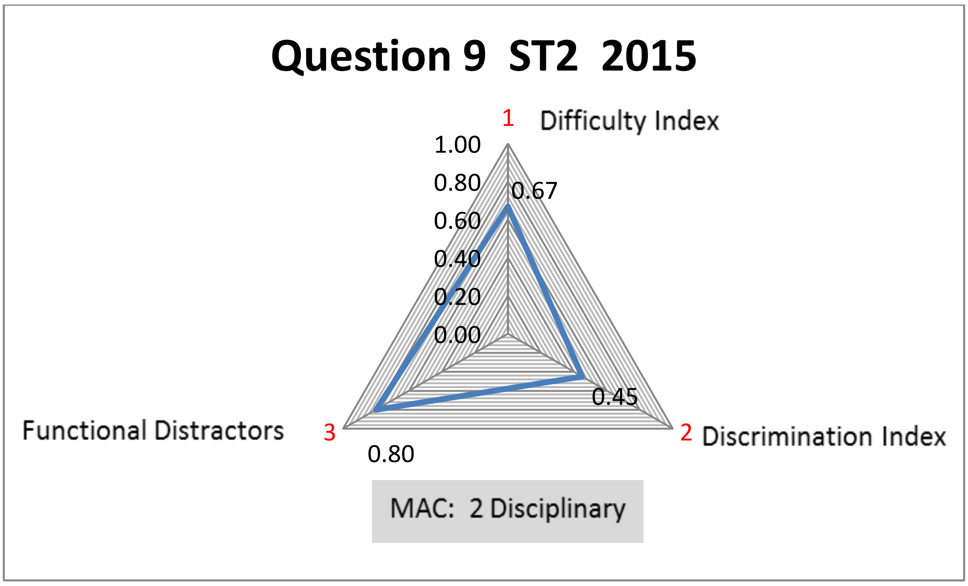


Question 4 Exam 2017	Value	Comment
Topic		Limits
Math Assessment Component	2	Disciplinary
Difficulty Index	0.79	Easy
Discrimination Index	DI = 0.37	Good
Distractor efficiency	0.50	

6. Derivatives

Question 9, Semester test 2, 2015

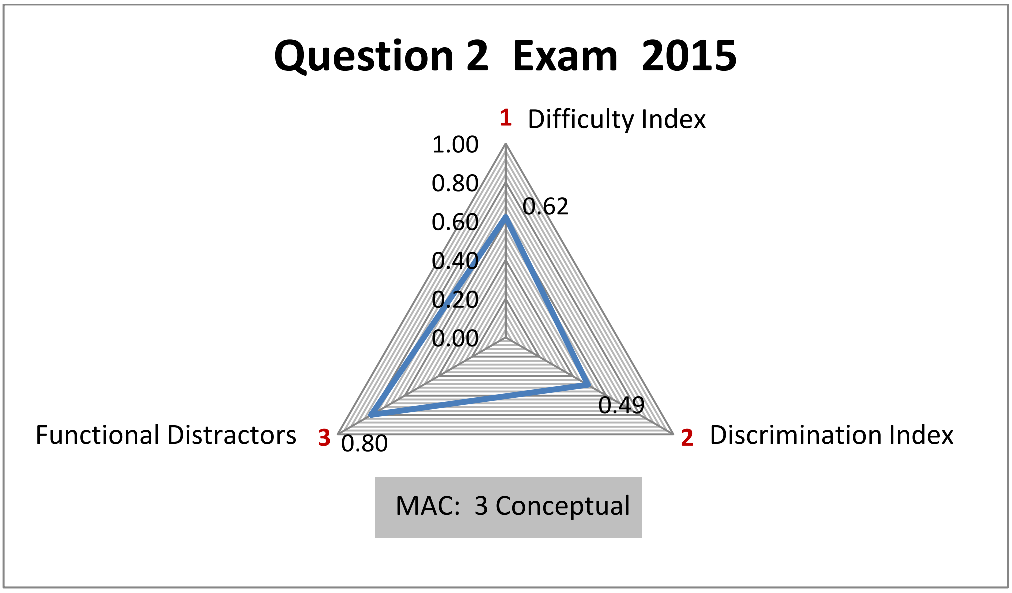
Given: $f(3)=3,\ f'(3)=2,\ g(3)=-4$ and $g'(3)=3$. Find $\left(\frac{f}{g}\right)'(3)=$
(A) $\frac{1}{16}$ (B) $\frac{1}{9}$ (C) $-\frac{17}{9}$ (D) $-\frac{17}{16}$ (E) $\frac{17}{16}$ (F) None of these



Question 9 ST2 2015	Value	Comment
Topic		Derivatives
Math Assessment Component	2	Disciplinary
Difficulty Index	0.67	Moderately easy
Discrimination Index	DI = 0.45	Excellent
Distractor efficiency	0.80	

Question 2, exam, 2015

Let f be a function defined by $f(x) = \begin{cases} px - x^2, & x \leq 1 \\ x^2 + x + k, & x > 1 \end{cases}$ For which values of p and k will f be continuous and differentiable at $x = 1$? (A) $k = 5$ and $p = 2$ (B) $k = 4$ and $p = 2$ (C) $k = 2$ and $p = 4$ (D) $k = 2$ and $p = 5$ (E) $k = -2$ and $p = 2$ (F) None of these



Question 2 Exam 2015	Value	Comment
Topic		Continuity & Derivatives
Math Assessment Component	3	Conceptual
Difficulty Index	0.62	Moderately easy
Discrimination Index	DI = 0.49	Excellent
Distractor efficiency	0.80	

The question utilises theorems of continuity and differentiability and therefore has a conceptual assessment component. Although the question was moderate to easy, the question discriminates well where the weaker students chose different options, other than the correct option of D, giving the question 60% functional distractors. The MCQ can be regarded as good.

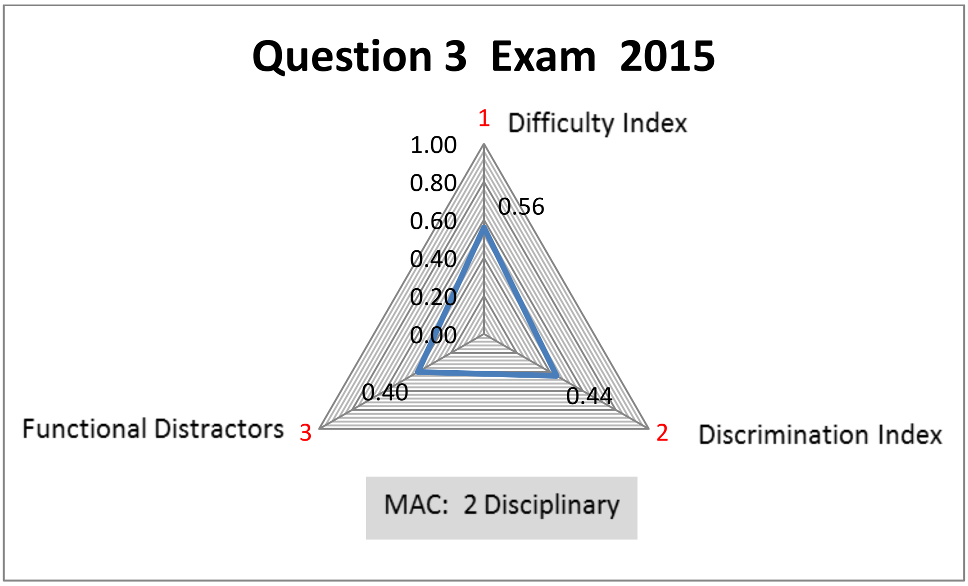
Question 3, exam, 2015

Assume that f is a differentiable function on the interval $[-1,3]$ and define g by $g(x) = f(f(x))$. Use the table below to find $g'(1)$.

x	0.0	0.5	1.0	1.5	2.0	2.5
$f(x)$	1.7	1.8	2.0	2.4	3.1	4.4
$f'(x)$	1.0	1.5	2.0	2.5	3.0	4.0

(A) $g'(1) = 1.5$ (B) $g'(1) = 2$ (C) $g'(1) = 6$

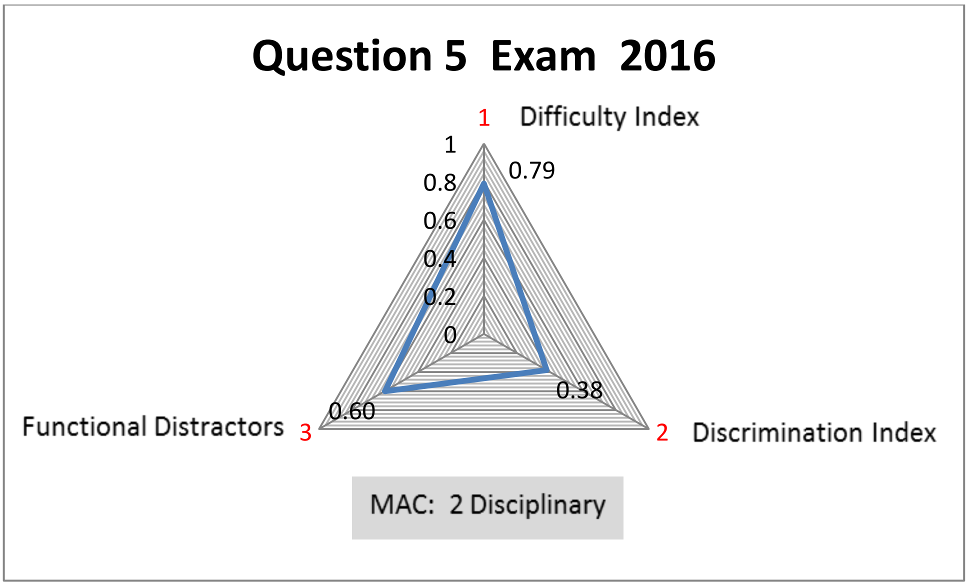
(D) $g'(1) = 7.5$ (E) $g'(1) = 10$ (F) None of these



Question 3 Exam 2015	**Value**	**Comment**
Topic		Derivatives
Math Assessment Component	2	Disciplinary
Difficulty Index	0.56	Moderately easy
Discrimination Index	DI = 0.44	Excellent
Distractor efficiency	0.40	

Question 5, exam, 2016

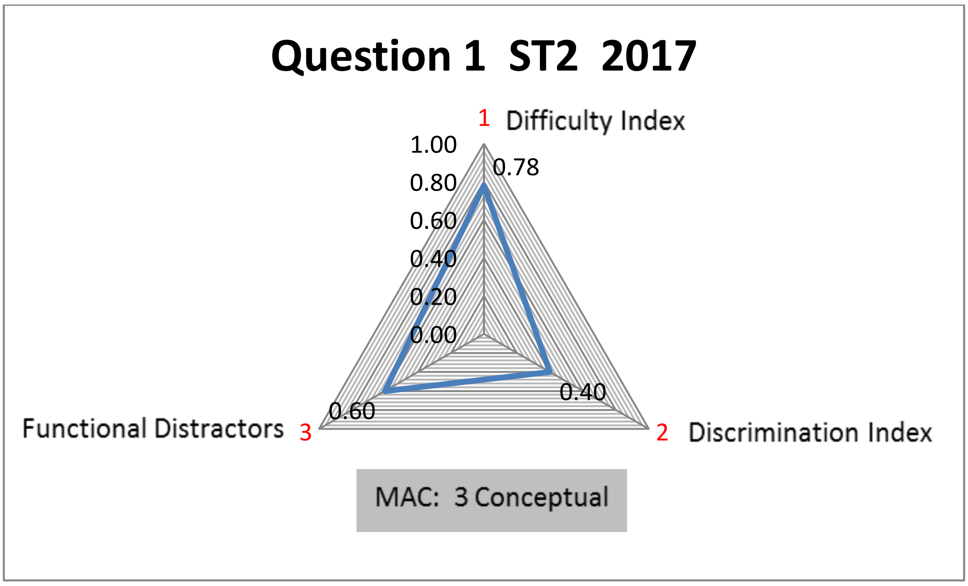
If $f(x) = \sin^{-1}(x^2 + x) + 5^x$ then $f'(0) =$					
(A) $5 \ln 5$	(B) $\frac{\pi}{2} + \ln 5$	(C) 2	(D) $1 + \ln 5$	(E) 1	(F) None of these



Question 5 Exam 2016	**Value**	**Comment**
Topic		Derivatives & trigonometry
Math Assessment Component	2	Disciplinary
Difficulty Index	0.79	Easy
Discrimination Index	DI = 0.38	Good
Distractor efficiency	0.60	

Question 1, Semester test 2, 2017

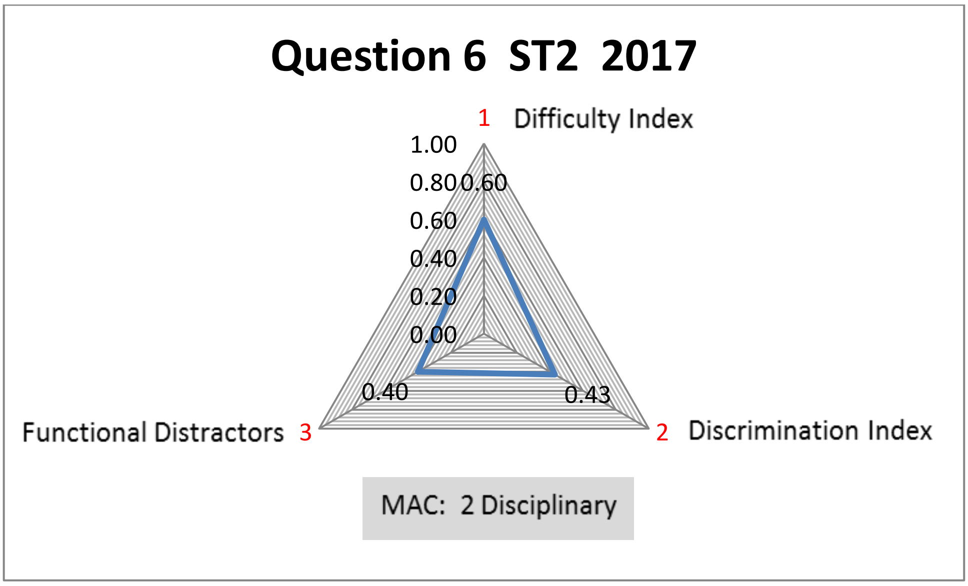
The given limit represents the derivative of some function f at some number a. $$\lim_{h \to 0} \frac{(1+h)^8 - 1}{h}$$ The function f and the number a is given by
(A) $f(x) = x^9, a = 0$ (B) $f(x) = x^8 - x, a = 0$ (C) $f(x) = x^8, a = 1$
(D) $f(x) = x^8 - 1, a = 1$ (E) $f(x) = x^7, a = 1$ (F) None of these



Question 1 ST2 2017	Value	Comment
Topic		Derivatives
Math Assessment Component	3	Conceptual
Difficulty Index	0.78	Easy
Discrimination Index	DI = 0.40	Excellent
Distractor efficiency	0.60	

Question 6, Semester test 2, 2017

Suppose that: $f(2) = 3,\ f'(2) = 4,\ g(2) = -1$ and $g'(2) = 2$. Find $(fg)'(2)$
(A) 2 (B) -2 (C) 8 (D) 3 (E) $\frac{9}{2}$ (F) None of these

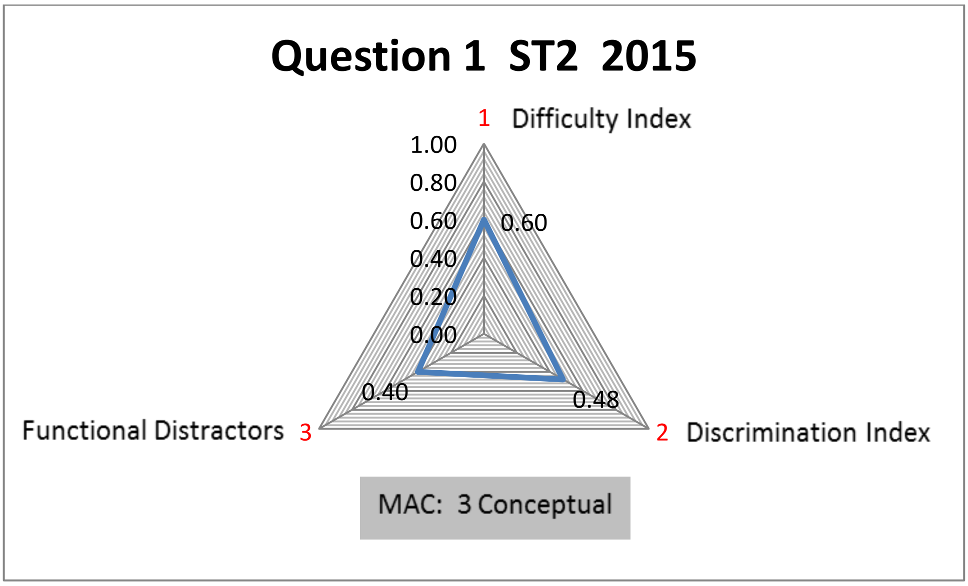


Question 6 ST2 2017	Value	Comment
Topic		Derivatives
Math Assessment Component	2	Disciplinary
Difficulty Index	0.60	Moderately easy
Discrimination Index	DI = 0.43	Excellent
Distractor efficiency	0.40	

7. Applications of differentiation

Question 1, Semester test 2, 2015

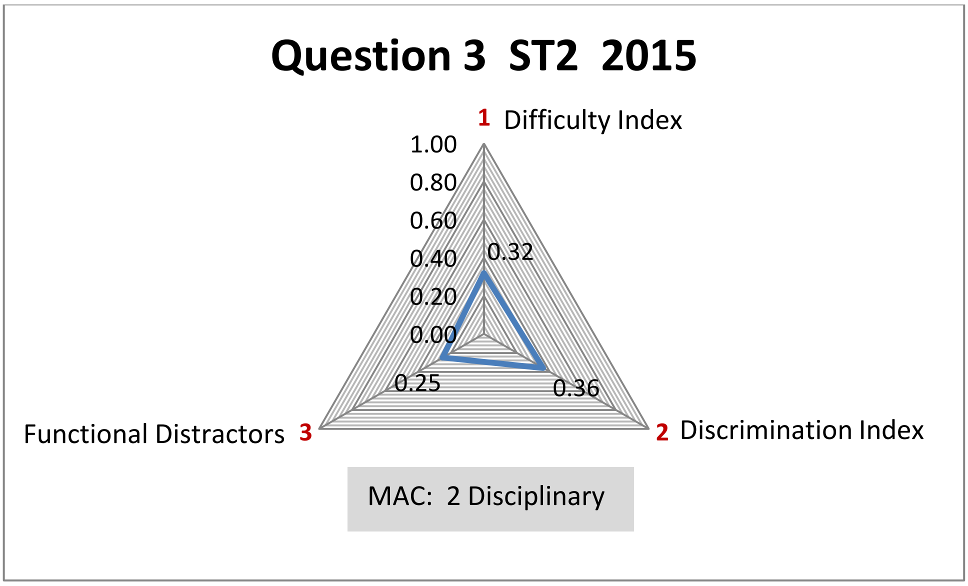
If it is given that the function $f(x) = xe^{ax}$ has a critical number at $x = 3$, then $a =$
(A) 3 (B) $\frac{1}{3}$ (C) $-\frac{1}{3}$ (D) $-\frac{1}{2}$ (E) a does not exist
(F) None of these



Question 1 ST2 2015	Value	Comment
Topic		Applications of differentiation
Math Assessment Component	3	Conceptual
Difficulty Index	0.60	Moderately easy
Discrimination Index	DI = 0.48	Excellent
Distractor efficiency	0.40	

Question 3, Semester test 2, 2015

Choose the statement that completes the following definition:
A function f is decreasing on an interval $I = [a, b]$ if:
(A) $f' < 0$ on the interval I
(B) $f(x_2) < f(x_1)$ for any $x_1, x_2 \in I$ where $x_2, < x_1$
(C) $f(x_2) < f(x_1)$ for any $x_1, x_2 \in I$ where $x_2, > x_1$
(D) $f(b) < f(a)$
(E) None of these

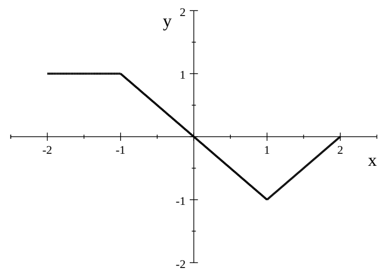


Question 3 ST2 2015	Value	Comment
Topic		Applications of differentiation
Math Assessment Component	2	Disciplinary
Difficulty Index	0.32	Moderately difficult
Discrimination Index	DI = 0.36	Good
Distractor efficiency	0.25	

The question is moderately difficult since only 32% of the students chose the correct option (C). It can be assumed that the weaker students found the question challenging, hence the 60% functional distractors. Although the question has a challenging nature because of all the facts incorporated in the options, the discrimination of D = 0.36 is good. This can be regarded as a good MCQ.

Question 5, Semester test 2, 2015

Given the graph of f' the derivative of a function f.

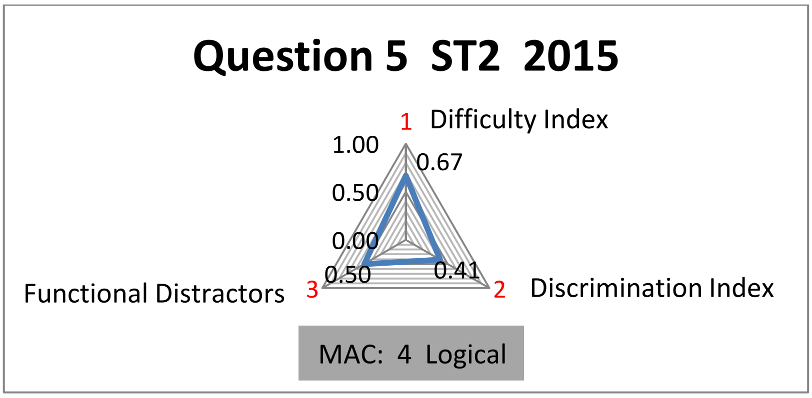


Which of the following statements is true about f?

(A) f is not differentiable at $x = -1$ and $x = 1$

(B) f is decreasing on $-1 \leq x \leq 1$……(C) f is increasing on $1 \leq x \leq 2$

(D) f has a local minimum at $x = 0$…..(E) f has a local maximum at $x = 0$

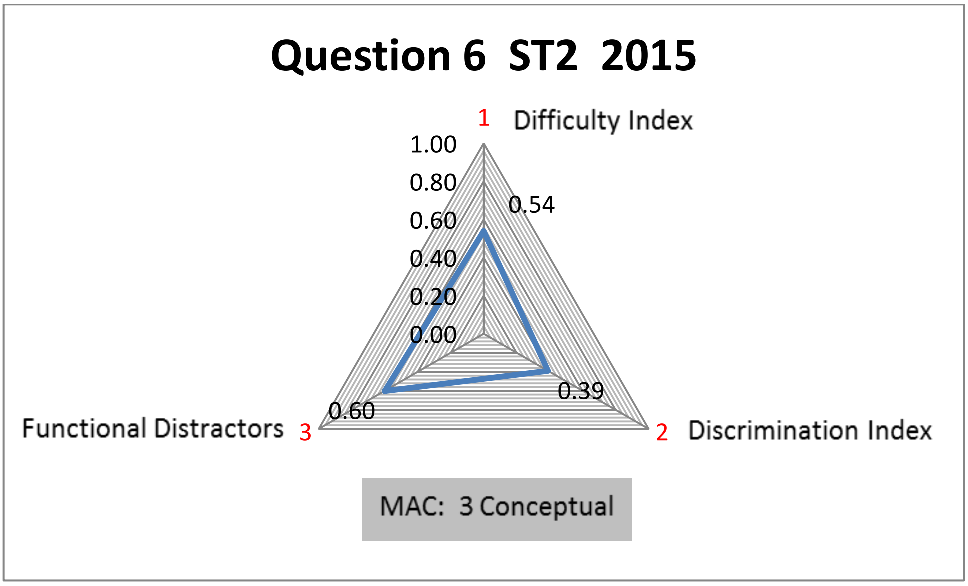


Question 5 ST2 2015	Value	Comment
Topic		Applications of differentiation
Math Assessment Component	4	Logical
Difficulty Index	0.67	Moderately easy
Discrimination Index	DI = 0.41	Excellent
Distractor efficiency	0.50	

The MAC of the question is Level 4 (logical) which implicates that this MCQ utilises theorems regarding how the derivative affects the shape of a graph. Level 4 is on a higher cognitive level therefore making 60% of the distractors functional. Nevertheless, the students found the question to be moderate to easy since 67% of them chose the correct option (D). This question can be regarded as a good MCQ since the discrimination is good (DI = 0.41).

Question 6, Semester test 2, 2015

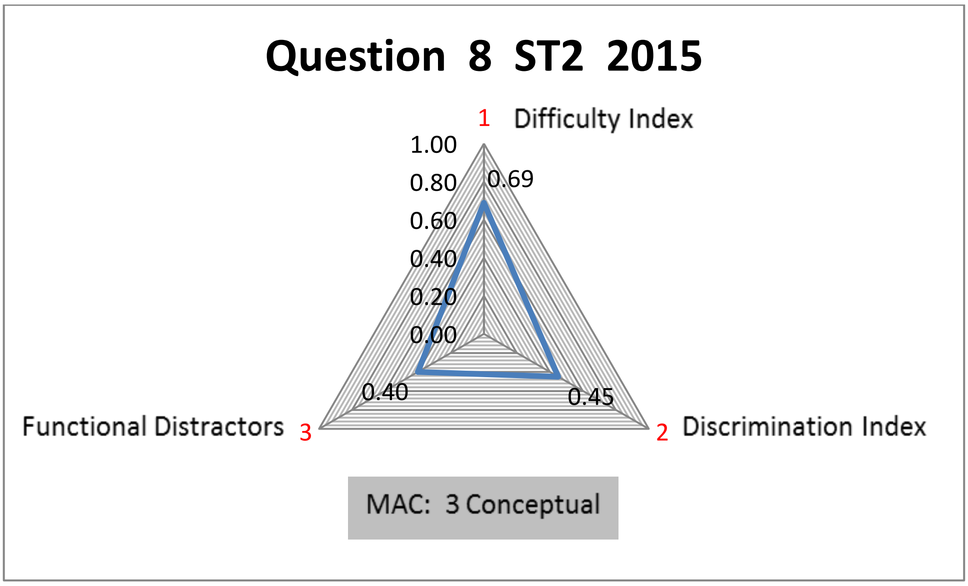
Let f be a function with derivative function $f'(x) = x - \frac{4}{x}$. On which of the following intervals is f increasing?
(A) $(-\infty, 0) \cup (0, \infty)$ (B) $(-\infty, 0) \cup (0,2)$ (C) $(-2,0) \cup (2, \infty)$
(D) $(-2,2) \cup (2, \infty)$ (E) $(-\infty, -2) \cup (0,2)$ (F) None of these



Question 6 ST2 2015	Value	Comment
Topic		Applications of differentiation
Math Assessment Component	3	Conceptual
Difficulty Index	0.54	Moderately easy
Discrimination Index	DI = 0.39	Good
Distractor efficiency	0.60	

Question 8, Semester test 2, 2015

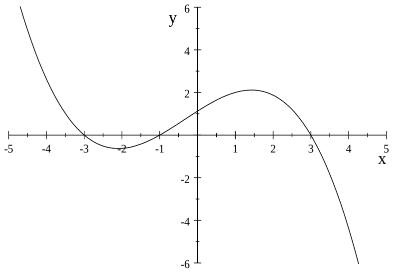
The Mean Value Theorem does not apply to $f(x) = \|x-3\|$ on $[1,4]$ because:	
(A) f is not continuous on $[1,4]$	(B) f is not differentiable on $(1,4)$
(C) $f(1) \neq f(4)$	(D) $f(1) > f(4)$
(E) $f(1) < f(4)$	(F) None of these



Question 8 ST2 2015	**Value**	**Comment**
Topic		Applications of differentiation
Math Assessment Component	3	Conceptual
Difficulty Index	0.69	Moderately easy
Discrimination Index	DI = 0.45	Excellent
Distractor efficiency	0.40	

Question 4, exam, 2015

The graph of the derivative f' of a function f is shown, with local extreme values at $x = -2$ and $x = 1.5$



Which of the following is true about the original function f ?

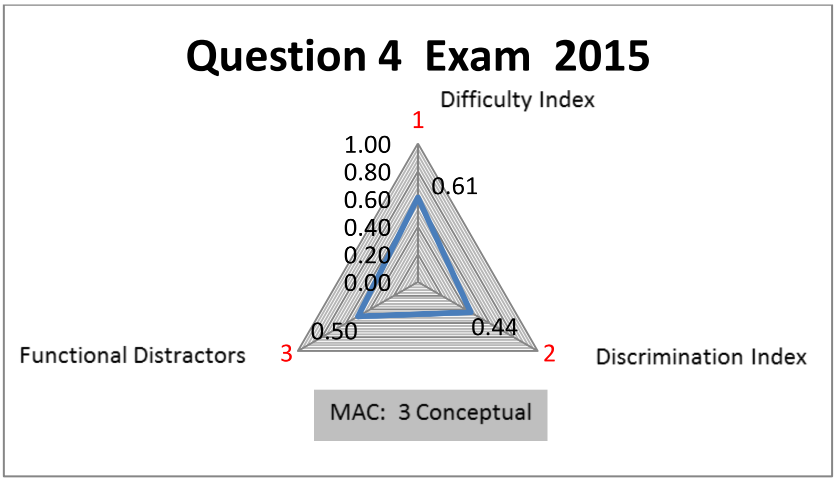
$I.$ f is increasing on the interval $(-2, 1.5)$

$II.$ f is continuous at $x = 0$

$III.$ f has an inflection point at $x = -2$

(A) Only I (B) Only II (C) Only III (D) II and III $only$

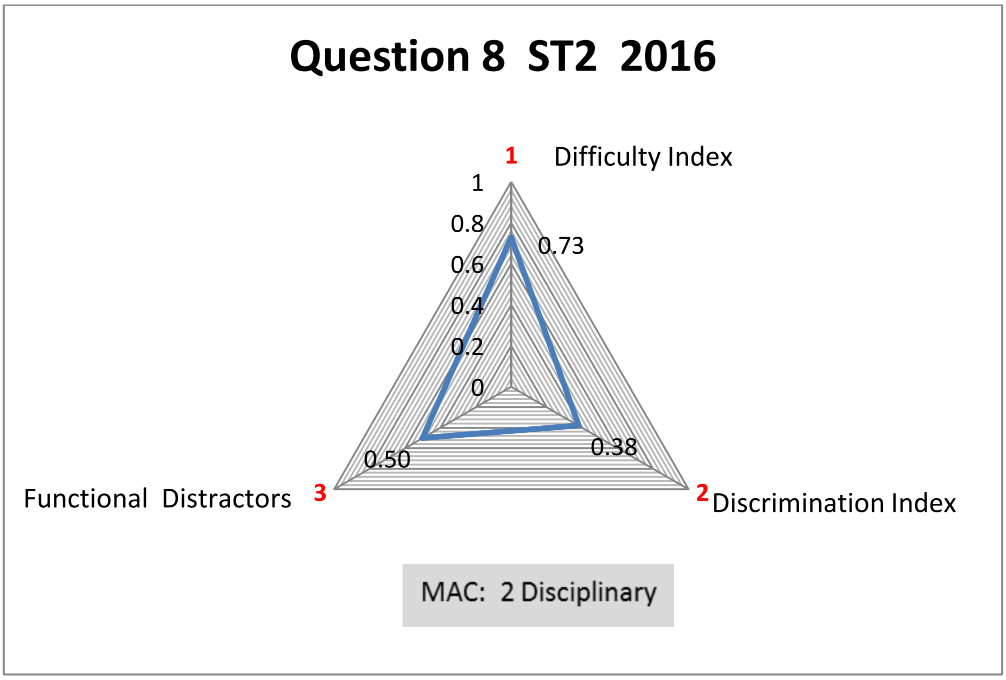
(E) I, II and III



Question 4 Exam 2015	Value	Comment
Topic		Applications of differentiation
Math Assessment Component	3	Conceptual
Difficulty Index	0.61	Moderately easy
Discrimination Index	DI = 0.44	Excellent
Distractor efficiency	0.50	

Question 8, Semester test 2, 2016

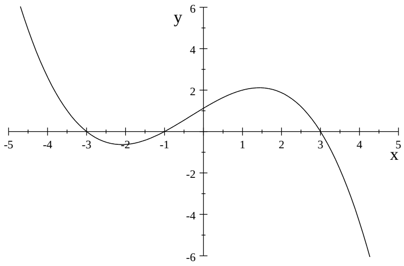
Given the function $f(x) = e^x - 2x$. For which value(s) of x will the function have a horizontal tangent?
(A) 2 (B) e^2 (C) $\ln 2$ (D) 2.5 (E) None of these



Question 8 ST2 2016	Value	Comment
Topic		Applications of differentiation
Math Assessment Component	2	Disciplinary
Difficulty Index	0.73	Easy
Discrimination Index	DI = 0.38	Good
Distractor efficiency	0.50	

Question 4, exam, 2016

<table>
<tr><td>
The graph of the derivative f' of a function f is shown, with local extreme values at $x = -2$ and $x = 1.5$



Which of the following is true about the original function f ?

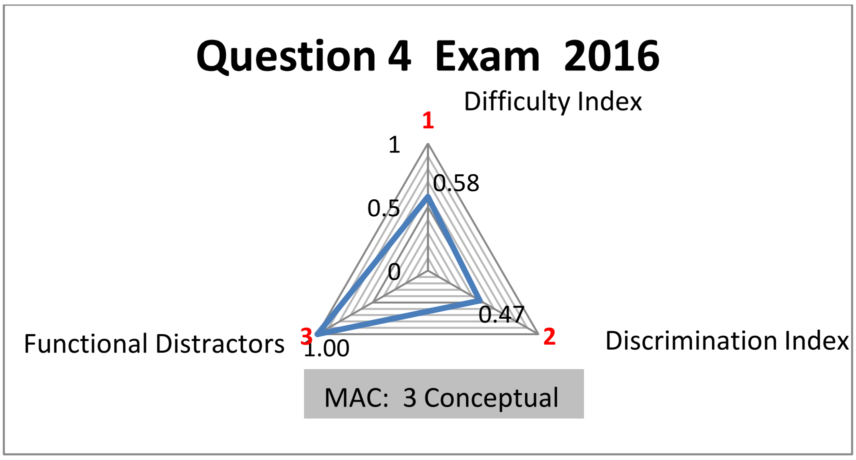
$I.$ f is increasing on the interval $(-2, 1.5)$

$II.$ f has a local maximum at $x = -3$

$III.$ f has an inflection point at $x = -2$

(A) Only I (B) Only II (C) Only III (D) II and III $only$

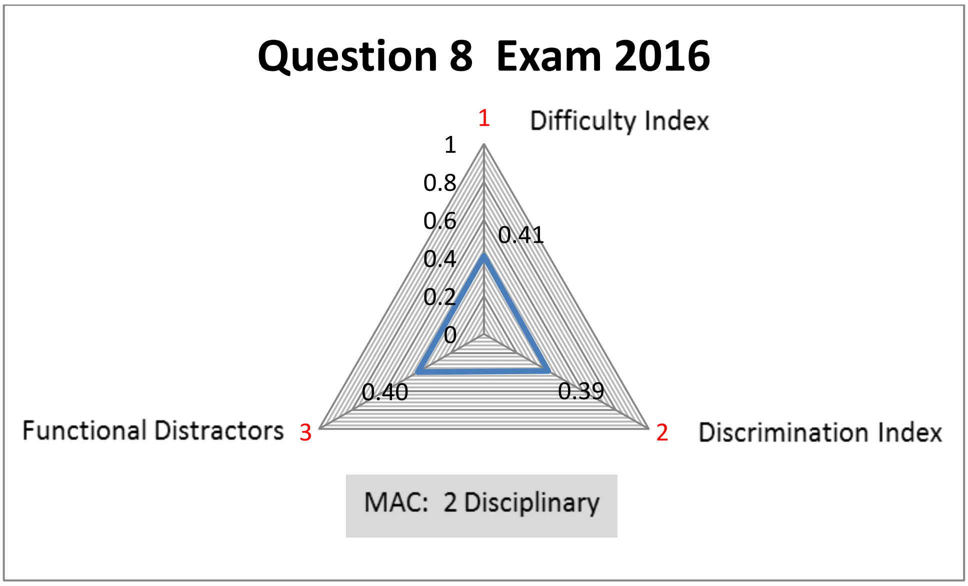
(E) I, II and III
</td></tr>
</table>



Question 4 Exam 2016	**Value**	**Comment**
Topic		Applications of differentiation
Math Assessment Component	3	Conceptual
Difficulty Index	0.58	Moderately easy
Discrimination Index	DI = 0.47	Excellent
Distractor efficiency	1.00	

Question 8, exam, 2016

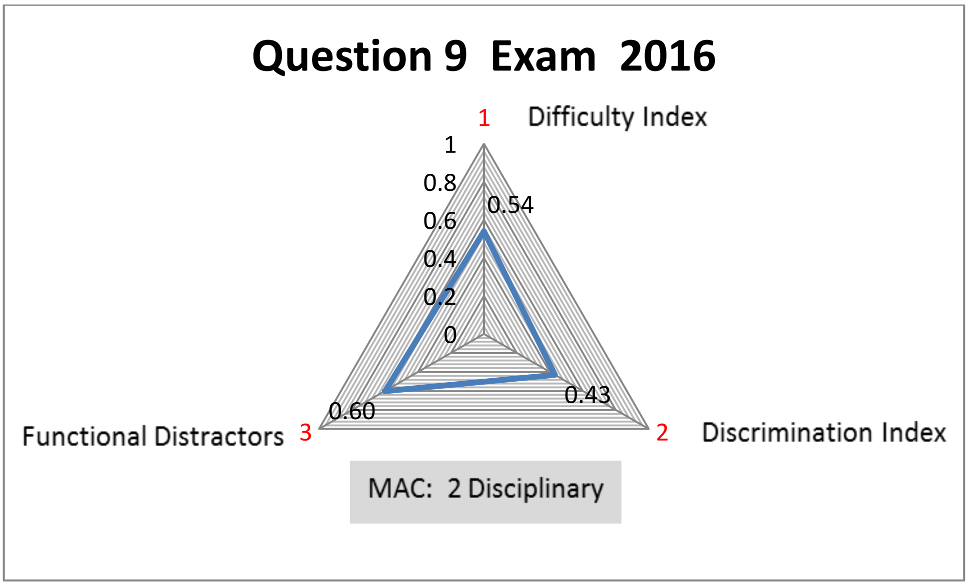
Consider the function $f(x)=\frac{x^2+2}{x^2-4}$ with $f'(x)=\frac{-12x}{(x^2-4)^2}$ and $f''(x)=\frac{12(3x^2+4)}{(x^2-4)^3}$
The interval(s) on which the function f is increasing is
(A) $(-\infty,0)$ (B) $(-\infty,-2)\cup(-2,0)$ (C) $(-\infty,-2)\cup(2,\infty)$
(D) R (E) $(-2,0)\cup(0,2)$ (F) None of these



Question 8 Exam 2016	Value	Comment
Topic		Applications of differentiation
Math Assessment Component	2	Disciplinary
Difficulty Index	0.41	Moderately difficult
Discrimination Index	DI = 0.39	Good
Distractor efficiency	0.40	

Question 9, exam, 2016

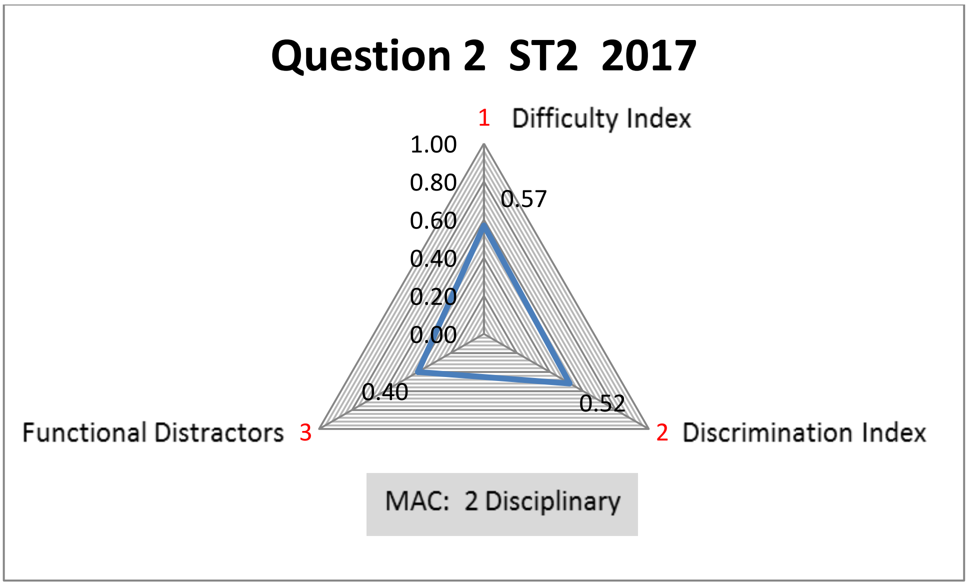
Consider the function $f(x) = \frac{x^2+2}{x^2-4}$ with $f'(x) = \frac{-12x}{(x^2-4)^2}$ and $f''(x) = \frac{12(3x^2+4)}{(x^2-4)^3}$ The interval(s) on which the function f is concave down is (A) $(-2,0)$ (B) $(-\infty,-2)$ (C) $(-\infty,-2) \cup (2,\infty)$ (D) $(-2,2)$ (E) $(-2,0) \cup (0,2)$ (F) None of these



Question 9 Exam 2016	Value	Comment
Topic		Applications of differentiation
Math Assessment Component	2	Disciplinary
Difficulty Index	0.54	Moderately easy
Discrimination Index	DI = 0.43	Excellent
Distractor efficiency	0.60	

Question 2, Semester test 2, 2017

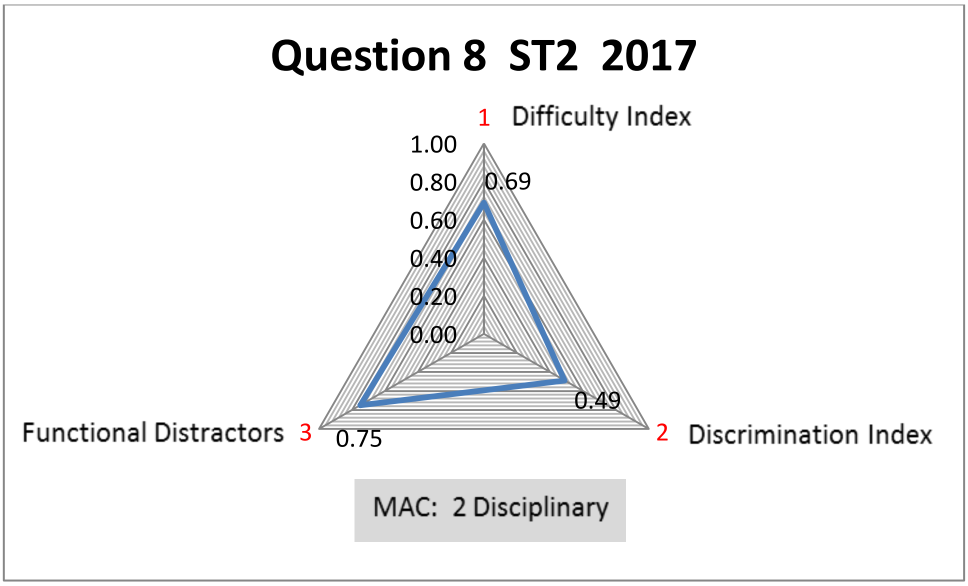
The slope of the tangent line to the graph of $f(x) = \sin 2x$ at the point $x = \frac{\pi}{2}$ is					
(A) $-\frac{1}{\sqrt{2}}$	(B) -2	(C) 2	(D) 0	(E) $-\frac{\sqrt{3}}{2}$	(F) None of these



Question 2 ST2 2017	Value	Comment
Topic		Applications of differentiation
Math Assessment Component	2	Disciplinary
Difficulty Index	0.57	Moderately easy
Discrimination Index	DI = 0.52	Excellent
Distractor efficiency	0.40	

Question 8, Semester test 2, 2017

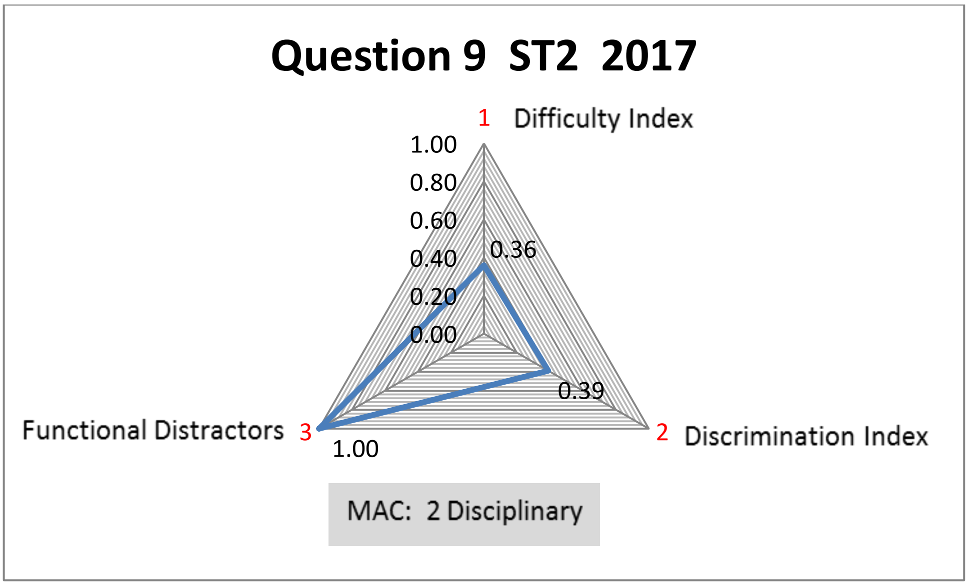
The point on the curve $y = \frac{e^x}{x+2}$ at which the tangent is horizontal is
(A) $(2,0)$ (B) $(1,\frac{e}{2})$ (C) $(-1,\frac{1}{e})$ (D) $(0,1)$
(E) None of these



Question 8 ST2 2017	Value	Comment
Topic		Applications of differentiation
Math Assessment Component	2	Disciplinary
Difficulty Index	0.69	Moderately easy
Discrimination Index	DI = 0.49	Excellent
Distractor efficiency	0.75	

Question 9, Semester test 2, 2017

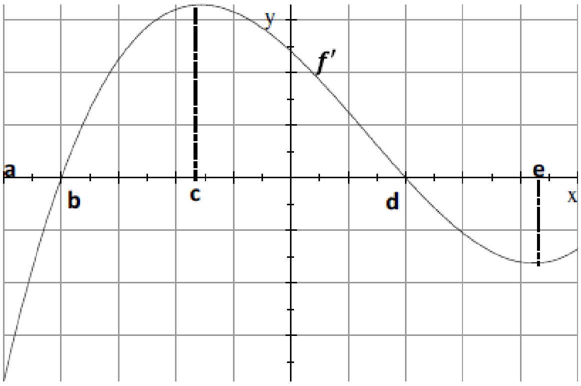
The value(s) of c that satisfy the Mean Value Theorem for the function $f(x) = x - x^3$ on the interval $[0,2]$ is given by
(A) $\frac{2}{\sqrt{3}}$ and $-\frac{2}{\sqrt{3}}$ (B) $\frac{\sqrt{3}}{2}$ and $-\frac{\sqrt{3}}{2}$ (C) $\frac{4}{3}$ (D) $\frac{2}{\sqrt{3}}$ (E) None of these



Question 9 ST2 2017	Value	Comment
Topic		Applications of differentiation
Math Assessment Component	2	Disciplinary
Difficulty Index	0.36	Moderately difficult
Discrimination Index	DI = 0.39	Good
Distractor efficiency	1.00	

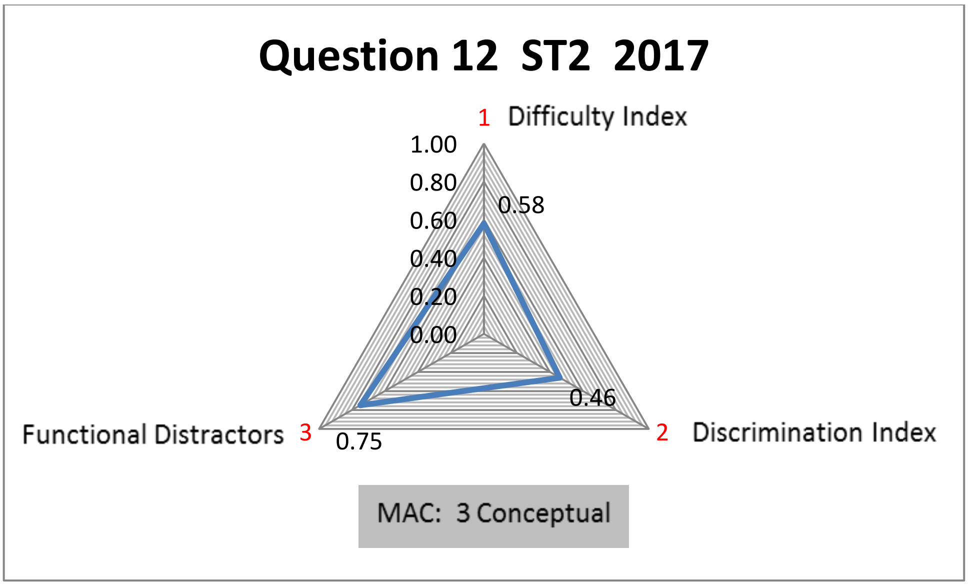
Question 12, Semester test 2, 2017

The graph of the first derivative f' of a function f is given.



The function f is concave down on the interval

(A) (a, c) (B) (b, c) (C) (b, d) (D) (c, e) (F) None of these

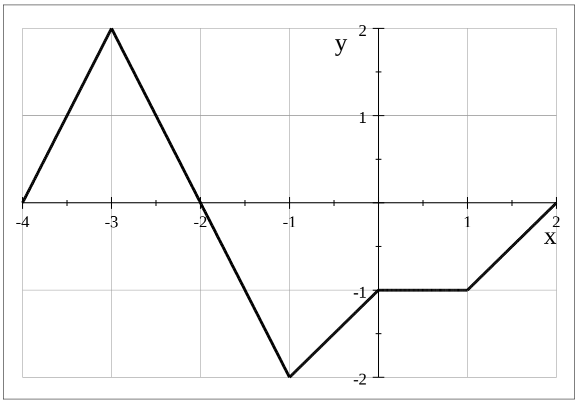


Question 12 ST2 2017	Value	Comment
Topic		Applications of differentiation
Math Assessment Component	3	Conceptual
Difficulty Index	0.58	Moderately easy
Discrimination Index	DI = 0.46	Excellent
Distractor efficiency	0.75	

8. Integration

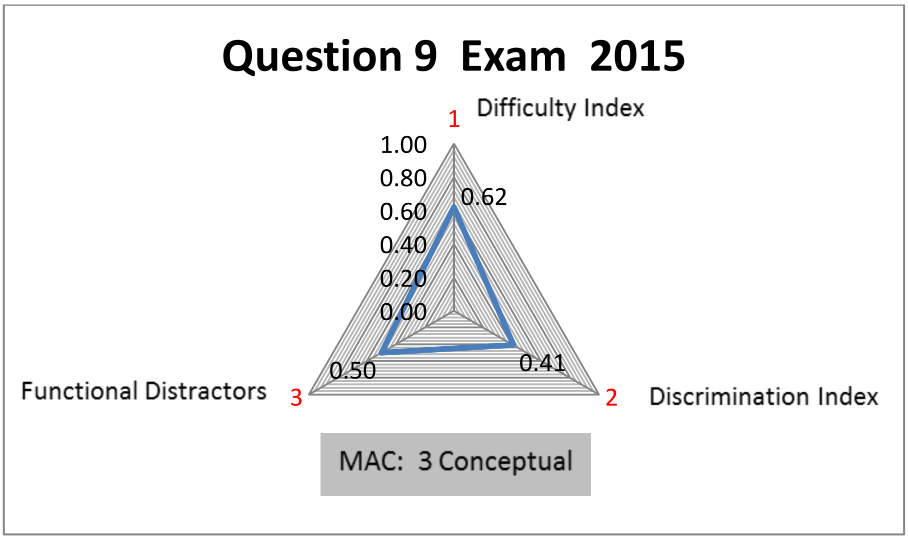
Question 9, exam, 2015

The graph of the function f on the interval $[-4,4]$ is shown below.



$$\int_{-4}^{2} f(x)dx =$$

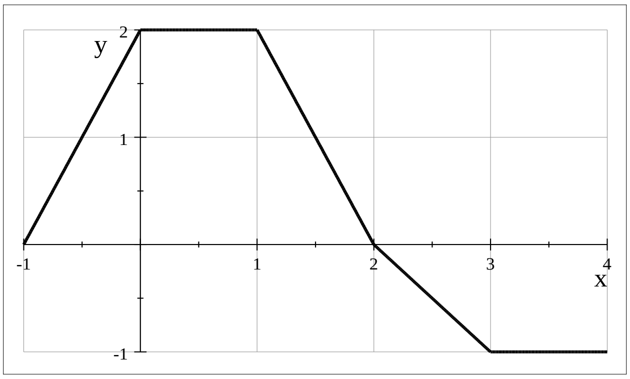
(A) 1 (B) -1 (C) 2 (D) -2 (E) 4 (F) 6 (G) None of these



Question 9 Exam 2015	Value	Comment
Topic		Integration
Math Assessment Component	3	Conceptual
Difficulty Index	0.62	Moderately easy
Discrimination Index	DI = 0.41	Excellent
Distractor efficiency	0.50	

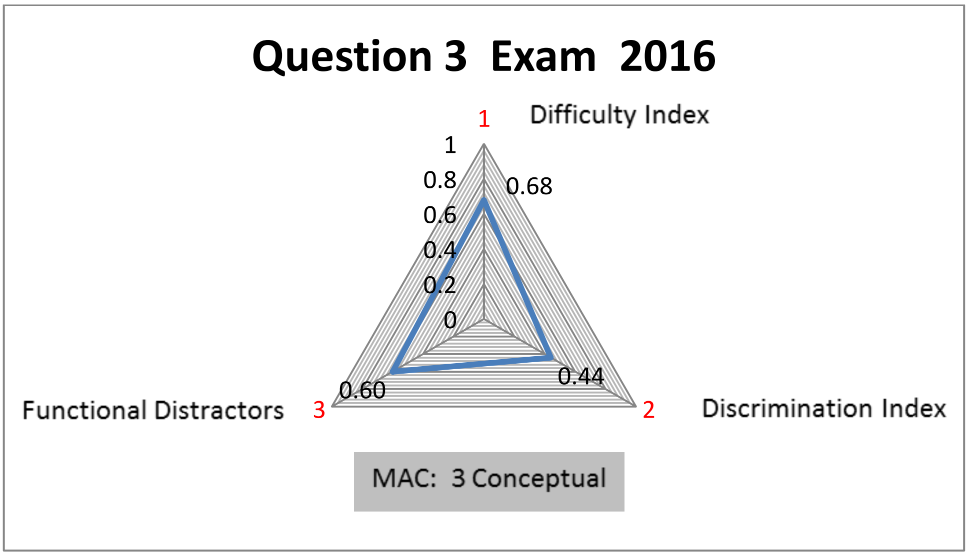
Question 3, exam, 2016

The graph of a piecewise defined function f , for $-1 \leq x \leq 4$ is given.



$$\int_{-1}^{4} f(x)dx =$$

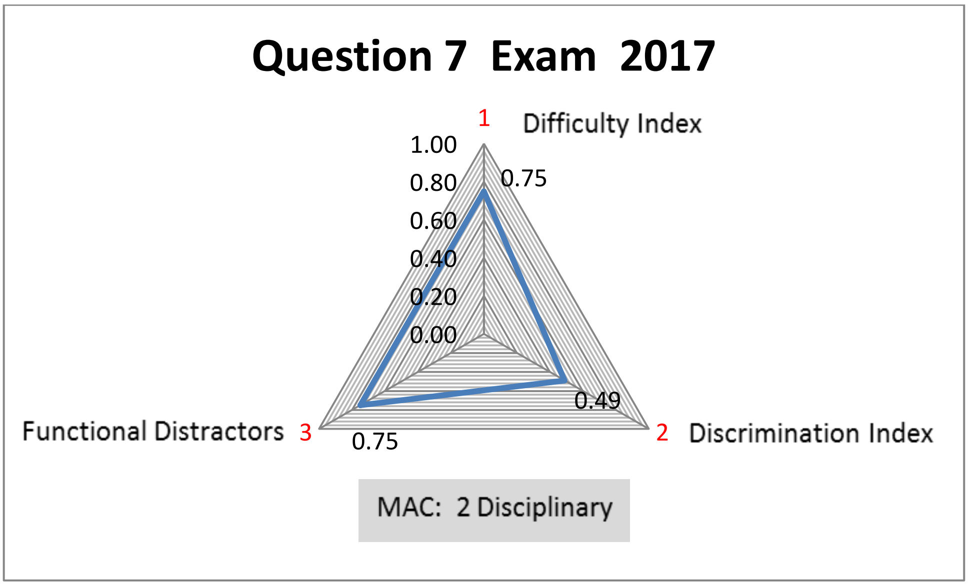
(A) 1 (B) 2.5 (C) 4 (D) 5.5 (E) 8 (F) None of these



Question 3 Exam 2016	Value	Comment
Topic		Integration
Math Assessment Component	3	Conceptual
Difficulty Index	0.68	Moderately easy
Discrimination Index	DI = 0.44	Excellent
Distractor efficiency	0.60	

Question 7, exam, 2017

The Riemann sum approximation of $\int_0^4 x^2 dx$ using the left-hand sum with four subintervals is (A) 30 (B) 14 (C) 10 (D) 25 (E) None of these



Question 7 Exam 2016	Value	Comment
Topic		Integration
Math Assessment Component	2	Disciplinary
Difficulty Index	0.75	Easy
Discrimination Index	DI = 0.49	Excellent
Distractor efficiency	0.75	

9. Vector algebra

Question 10, exam, 2015

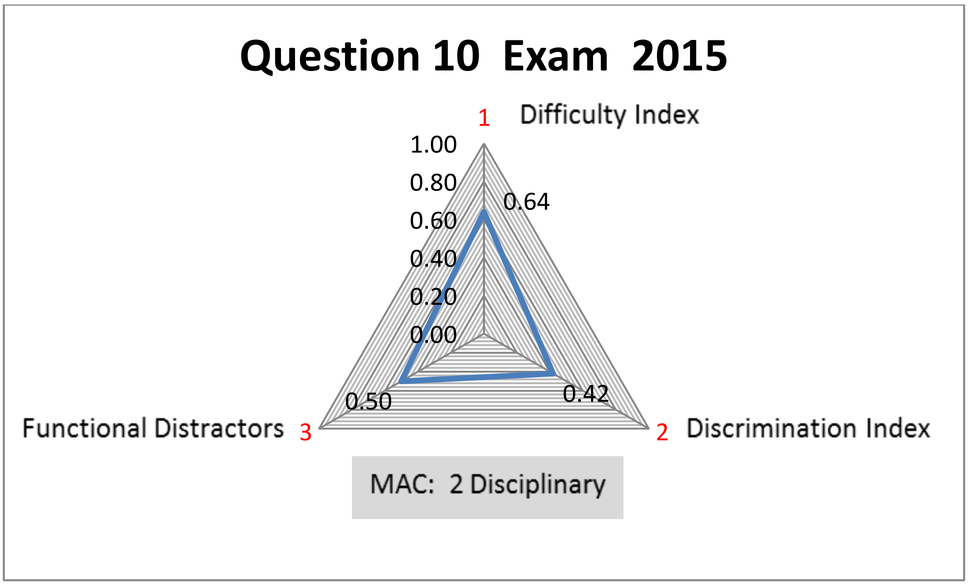
Given $|\bar{a}| = 3$ and $|\bar{b}| = \sqrt{6}$ and the angle between $\bar{a}$ and $\bar{b}$ is $\frac{\pi}{4}$.

Determine $\bar{a}.\bar{b}$.

(A) $\frac{3\sqrt{6}}{2}$ (B) $\frac{9\sqrt{3}}{2}$ (C) $3\sqrt{3}$ (D) $\frac{1}{6\sqrt{3}}$ (E) $\frac{2\sqrt{3}}{9}$

(F) $\frac{1}{3\sqrt{6}}$ (G) None of these

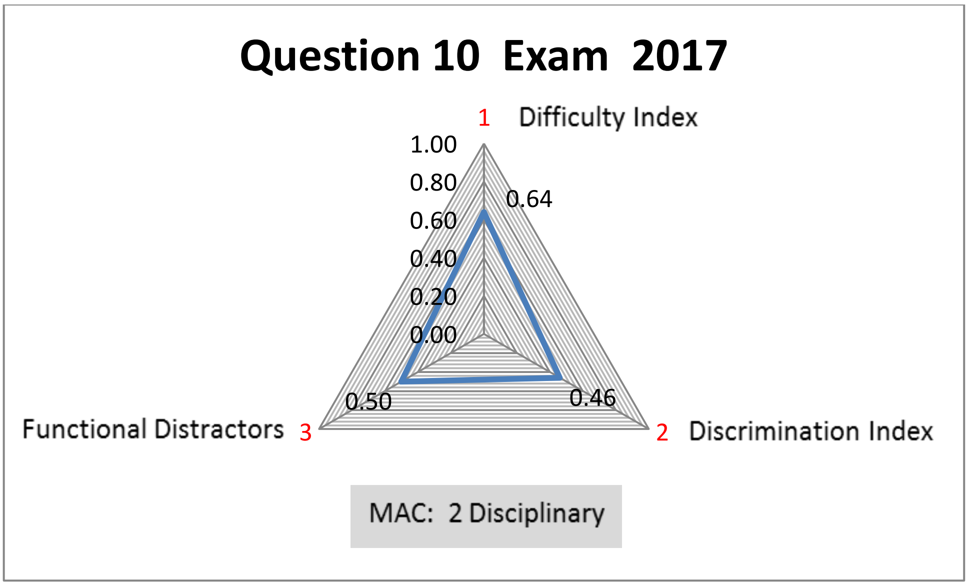
.



Question 10 Exam 2015	Value	Comment
Topic		Vector algebra
Math Assessment Component	2	Disciplinary
Difficulty Index	0.64	Moderately easy
Discrimination Index	DI = 0.42	Excellent
Distractor efficiency	0.50	

Question 10, exam, 2017

Given $\lvert\bar{a}\rvert = 3$, $\lvert\bar{b}\rvert = \sqrt{6}$ and the angle between $\bar{a}$ and $\bar{b}$ is $\frac{\pi}{4}$. Determine $\bar{a}\cdot\bar{b}$.
(A) $\frac{3\sqrt{6}}{2}$ (B) $\frac{9\sqrt{2}}{2}$ (C) $3\sqrt{3}$ (D) $\frac{1}{6\sqrt{3}}$ (E) $\frac{2\sqrt{3}}{9}$
(F) $\frac{1}{3\sqrt{6}}$ (G) None of these



Question 10 Exam 2017	Value	Comment
Topic		Vector algebra
Math Assessment Component	2	Disciplinary
Difficulty Index	0.64	Moderately easy
Discrimination Index	DI = 0.46	Excellent
Distractor efficiency	0.50	

Appendix D: Ethical clearance letter of approval

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education
Fakulteit Opvoedkunde
Lefapha la Thuto

Ethics Committee
31 January 2017

Dear Mr G Btits

REFERENCE: SM 15 /10/02

Your application was carefully considered by the Faculty of Education Ethics Committee and the final decision of the Ethics Committee is:

Your application is approved.

This letter serves as notification that you may continue with your fieldwork. Should any changes to the study occur after approval was given, it is your responsibility to notify the Ethics Committee immediately.

Please note that you will have to fulfil the conditions specified in this letter from the Faculty of Education Research Ethics Committee. The conditions include:

1) The ethics approval is conditional on the research being conducted as stipulated by the details of all documents submitted to the Committee. In the event that a further need arises to change who the investigators are, the methods or any other aspect, such changes must be submitted as an Amendment (Section E) for approval by the Committee.
   - Any amendments to this approved protocol need to be submitted to the Ethics Committee for review prior to data collection. Non-compliance implies that the Committee's approval is null and void.
   - Final data collection protocols and supporting evidence (e.g.: questionnaires, interview schedules, observation schedules) have to be submitted to the Ethics Committee before they are used for data collection.

2) The researcher should please note that this decision covers the entire research process, until completion of the study report, and not only the days that data will be collected.
3) Should your research be conducted in schools, please note that you have to submit proof of how you adhered to the Department of Basic Education (DBE) policy for research.

4) The Ethics Committee of the Faculty of Education does not accept any liability for research misconduct, of whatsoever nature, committed by the researcher(s) in the implementation of the approved protocol.

Please note that this is not a clearance certificate.
Upon completion of your research, you need to submit the following documentation to the Ethics Committee:

- **Integrated Declaration Form (Form D08),**
- **Initial Ethics Approval letter and,**
- **Approval of Title.**

On receipt of the above-mentioned documents you will be issued a clearance certificate. Please quote the reference number **SM 15 /10/02** in any communication with the Ethics Committee.

Best wishes

Prof Liesel Ebersöhn
Chair: Ethics Committee
Faculty of Education

Appendix E: Permission letter from the Deputy Dean of the Faculty of Engineering, Built Environment and Information Technology

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

2 November 2016

Prof S Maharaj
Dean of Faculty of Engineering and the Built Environment
University of Pretoria

Dear Prof Maharaj

Re: Permission to conduct a research project in which first year engineering students are involved.

I am currently enrolled for an MEd degree in the Faculty of Education. I am also a lecturer in the Department of Mathematics and Applied Mathematics.

Permission is required by the Ethics Committee of the Faculty of Education to conduct the proposed research for my masters dissertation as it involves engineering students. In short, the study involves analysing the responses of first year students to the multiple choice questions posed in the tests and examination in the calculus module WTW 158 in order to determine the quality of the questions posed. Anonymity of students is guaranteed at all times. The proposed title of the dissertation is: *An investigation into the quality of Provided Response Questions in first year Engineering Mathematics.*

Dr Surette van Staden of the Department of Mathematics, Science and Technology in the Faculty of Education is my supervisor and Prof Ansie Harding from the Department of Mathematics and Applied Mathematics of the Faculty of Natural and Agricultural Sciences is my co-supervisor.

More detail regarding the research project:

The University of Pretoria has experienced a significant increase in student numbers in recent years. Simultaneously increased pressure has been experienced across the academic spectrum on producing research output. The increase in student numbers and well as increase in research pressure has impacted on the workload of academic staff.

The decision was taken to make more use of provided response questions (multiple choice questions) in semester tests and examinations that could be graded by means of computer software. It has become common practice to include multiple choice questions in all tests and examinations.

Although using multiple choice questions for assessment has become common practice, this practice has been the subject of criticism. Such criticism stems from the fact that the multiple choice questions are home grown, designed by staff members who, in most cases, have had little formal training in setting such questions. The aim of the research project is to investigate the quality of the multiple choice questions, for which a framework will be developed.

All student names and student numbers will be omitted to keep their anonymity. Only their performances will be used in determining the quality of each question.

The outcome of the study would be of benefit not only to the department itself but to a wider community.

I attach the research proposal.

May I request your urgent attention in this matter as the Ethics Committee has a final review for 2016 on 9 November.

Kind regards

Gideon Brits

Gidcon.brits@up.ac.za

012 420 5707 / 012 842 3656

Supported.

2016.11.21

Deputy Dean Research (EBIT