THE DEVELOPMENT OF A STUDY ORIENTATION QUESTIONNAIRE IN MATHEMATICS

by

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DECLARATION OF ORIGINALITY

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I hereby declare that this dissertation, "*THE DEVELOPMENT OF A STUDY ORIENTATION QUESTIONNAIRE IN MATHEMATICS,*" is submitted in accordance with the requirements for the degree Doctor Philosophiae at the University of Pretoria. I declare that this dissertation is my own original work and that it has not previously been submitted to any other institution of higher learning. All sources cited or quoted in this research paper are indicated and acknowledged in a comprehensive list of references.

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SUMMARY

THE DEVELOPMENT AND EVALUATION OF A STUDY ORIENTATION QUESTIONNAIRE IN MATHEMATICS

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The failure rate in mathematics at school is high, not only in South Africa, but also internationally. Furthermore, learners with an apparently high general ability or aptitude for mathematics sometimes underachieve in the subject, while some learners with an apparently low general intellectual ability or aptitude for mathematics sometimes achieve well in the subject. Little attention is nonetheless given to learners' study orientation in mathematics, in spite of the fact that research has indicated that school mathematics is one of the best predictors of success in tertiary studies.

An investigation into some epistemological approaches to the learning process in mathematics confirms that learners' achievement in mathematics is significantly affected by their study orientation in mathematics.

The primary aim of this thesis was the development and evaluation of a study orientation questionnaire (SOM) in mathematics. Data processing procedures especially referred to the following two primary aims with the study:

(a) Standardisation of the questionnaire.

Steps carried out to evaluate the questionnaire psychometrically, include factor and item analysis. In the case of Grade 8 and 9, three fields (Study habits in mathematics, Mathematics anxiety and Study attitudes in mathematics) were identified. A fourth

field, Locus of control regarding mathematics, was identified only in the case of learners in Grade 10 and 11. It was established that the SOM apparently has criterion related validity, as well as content and construct validity for the three language groups as a whole. Reliability coefficients for the SOM can in most cases be regarded as satisfactory.

(b) Comparative studies to determine the applicability of the SOM.

Analysis of variance techniques were used to determine where significant differences between groups (including grade-, mother tongue and sex groups) lay. Where MANOVAS showed significant differences, further investigation was carried out to determine in respect of which individual fields (single variables) groups differed significantly. By means of LSM it was determined which groups differed significantly in regard of the separate fields. Some of the findings include:

- ★ It seems that African language speakers in both grade groups are really trying to achieve in mathematics, but that their best efforts are not successful.
- ★ It would appear that girls' level of Mathematics anxiety drops in Grade 10 and 11.
- ★ Learners in Grade 10 and 11 show lower levels of Mathematics anxiety and more sufficient Study habits in mathematics than their counterparts in Grade 8 and 9.
- ★ Perhaps the most significant finding is the phenomenon that African language learners in Grade 10 and 11 achieved much worse in regard of Locus of control than Afrikaans and English-speaking learners. A number of factors probably contribute towards this state of affairs, including language problems, teachers who are underqualified, African language learners' less than optimal socio-economic status (SES) in general, a lack of facilities and text books and disruption which is still being experienced in many traditionally black schools. It is recommended that these matters are attended to in an effort to create circumstances for more sufficient achievement in mathematics by learners from all language groups.

Key words:

1. Study
2. Mathematics
3. Study habits in mathematics
4. Mathematics anxiety
5. Study attitudes towards mathematics
6. Locus of control regarding mathematics
7. Study orientation questionnaire
8. Cross-cultural
9. Study orientation in mathematics
10. Evaluation

CHAPTER 1

TITLE AND CLARIFICATION OF CONCEPTS, (PROVISIONAL) PROBLEM STATEMENT, AIM OF THE STUDY AND PROGRAMME ANNOUNCEMENT

1.1 GENERAL INTRODUCTION

Much attention is usually given to testing and evaluation from a psychological point of view. On the one hand the aim of testing is to compare learners' achievement, but on the other hand this offers psychologists[1] the opportunity of getting to know learners[2] better (Oosthuizen & Maree, 1993). In this respect Smit (1991) points out the difference between measurement and evaluation. Measurement provides the answer to the question "How much?" whereas evaluation answers the question "How well?" (Smit, 1990:13). Therefore psychological measurement has a broader meaning than measurement since psychologists make use of information that was obtained in various ways and from various sources in order to enable them to appraise values during evaluation (Owen & Chamberlain, 1995).

Learners' achievement in mathematics is usually related to their cognitive potential. In this respect the results of intelligence and aptitude tests are frequently regarded in isolation as the criterion of predicting learners' future achievement in mathematics. When learners do not achieve according to expectations it is frequently referred to as underachievement. Obviously this is an oversimplification of achievement and underachievement. Many other variables, apart from cognitive potential (as measured by standardised intelligence and aptitude tests) play a role in learners' ultimate achievement in any subject, but especially in their achievement in mathematics. These factors (that are fully discussed in Chapter 3 of this study) include capability, personality, interest, learners' background, culture and the quality of education. Boyd (1990:23) refers to this as follows:

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1 For the purpose of this study the term "psychologists" refers to psychologists as well as counsellors and (mathematics) teachers.

2 For the purpose of this study the term "learners" refers to school pupils (male and female) of all population groups, and also to students at tertiary institutions.

> *Some ... pupils are fortunate in living in homes where the adults are concerned with the full development of their children ... . But often the home environment is not meeting ... (their) emotional and intuitive needs ... and the present structure of our high schools is failing to provide the environment that will engender excitement in learning ... (leading to) personal uncertainties about themselves as human beings who are able to analyse their own learning styles.*

In order to put the learners' need for counselling in their study orientation in mathematics into perspective, the state of learners' achievement in mathematics in South Africa will receive attention.

1.1.1 The extent of inadequate achievement in mathematics in South Africa

Learners' mathematics marks do not merely determine whether they pass or fail. Such marks also affect factors such as possible admission to university study, the obtaining of bursaries or the finding of employment. In fact they influence the learner's whole life. Consequently it is not surprising that at national level there is concern about the high rate of attrition or dropout figures, as well as the poor achievement in mathematics; especially at secondary school level, but also at tertiary level. The results of the Third International Mathematics and Science Study (TIMMS) indicate that South African learners fared the worst of all 41 countries which completed the study (Howie, 1996). This study also indicated, *inter alia*, that South African learners do not have adequate problem-solving skills and that they are not able to construct their own answers.

Inadequate achievement in mathematics is also found in all population groups[3] in South Africa (Maree, 1995a). Brodie (1994) states that inadequate achievement in mathematics occurs more among black persons than among learners from other population groups, and that boys generally fare worse than girls. Furthermore she claims that a lower socio-economic standing is an important contributory cause of

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3 In this study the point of view is taken that any racial or ethnic classification of population groups is an artificial method of distinguishing among people. In addition it is reminiscent of the language of the apartheid era. However, it is done to highlight inequalities in the South African population in order to remedy the situation.

inadequate achievement in mathematics. The following figures illustrate that South African blacks' achievement in mathematics leaves much to be desired. Out of every 10 000 black learners who enter school in Grade 1:

★ 1 300 pass Grade 11;
★ only 270 go on to Grade 12;
★ of these only 113 pass;
★ 27 obtain matriculation exemption; and
★ 1 will obtain matriculation exemption in mathematics and chemistry (Christie, 1991).

To obtain a better perspective it should be stated that these figures were apparently obtained by comparing the number of pupils entering school in a particular year with the number of matriculants in the same year. This, Blankley (1994) points out, is a misleading estimate. During the past few years there has been a marked increase in the number of black learners entering primary school. A more reliable estimate can be obtained by comparing the Grade 1 learners in 1980 with the Grade 12 results in 1991. The ratio becomes 1:312, that is to say 32 out of every 10 000, which is still low.

The figures mentioned should furthermore be interpreted against the background of the figures provided in the following table:

TABLE 1.1: DISTRIBUTION OF SOUTH AFRICAN MATHEMATICS LEARNERS IN GRADE 12 (1993) PER MOTHER-TONGUE SPEAKING GROUP

SPEAKERS OF AFRICAN LANGUAGES					SPEAKERS OF AFRIKAANS AND ENGLISH			
	BOYS		GIRLS		BOYS		GIRLS	
Grade	N	%	N	%	N	%	N	%
Higher	36043	21,30	37573	16,60	11041	32,70	9613	28,83
Standard	15071	8,91	18607	8,22	13039	38,61	9905	29,71
Total (Grade 12)	169197	100	226321	100	33768	100	33342	100

From Table 1.1 it appears that girls from all population groups are less inclined to take mathematics up to Grade 12 than boys are. Table 1.2 indicates the relative pass figures per population group of the Grade 12 learners for mathematics (1993)[4].

TABLE 1.2: PERCENTAGE OF PASSES IN MATHEMATICS PER POPULATION GROUP IN 1993

	PERCENTAGE OF PASSES IN MATHEMATICS (1993) (NUMBER OF LEARNERS WHO PASS ÷ TOTAL NUMBER OF GRADE 12 LEARNERS			
Grade	Black	White	Coloured	Asian
Higher	1,58%	17,61%	3,30%	12,54%
Standard	3,08%	34,49%	33,76%	15,79%

From Table 1.2 it is clear that the percentages of passes are low throughout, but that the black learners' performance throughout is poorer than that of their fellow learners from other population groups.

Arnott, Kubeka, Rice and Hall (1997:12) confirm the above trends and in addition they point out that the national pass mark for mathematics in South Africa is the lowest of all subjects:

> *In 1995, of every 100 pupils enrolled in mathematics, 71 wrote exams, and only 33 pupils passed the subject ... . Mathematics and science matric results are lower than the national pass rates for other subjects .... . Science pass rates are (however) on average higher than mathematics pass rates.*

The challenges posed to twenty-first century learners will probably require them to have particular abilities, skills and qualifications especially in mathematics, the natural sciences and also in the technological field. That is one of the reasons why

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4 The 1993 figures are the most complete recent figures available (where this method is still used to distinguish between the learners) (Strauss, 1997).

researchers all over the world are striving to optimise learning in these fields (Howie, 1997).

1.1.2 Attempts to explain inadequate achievement in mathematics

Possible solutions to the problem of inadequate achievement in mathematics and an inadequate study orientation in mathematics by learners must be based on a learning theory or a combination of learning theories. Intervention strategies can only be planned on the grounds of an implicit learning-theoretical or epistemological basis. Furthermore, the various learning and cognitive styles will have to be taken into account during the search for solutions to this problem. *Learners from certain homes, with a certain background and history of development, do not necessarily learn in the same way as their fellow learners.*

In this study possible causes of the problem will be examined from a psychological perspective. The point of departure taken is that learners' achievement in mathematics can be improved significantly if learners with an inadequate study orientation in mathematics are assisted to optimise their study orientation. In this connection Taljaard and Prinsloo (1995:420) state the following with regard to the *Survey of Study Habits and Aptitudes* (SSHA):

> *The low correlation with the measurement of scholastic aptitude and close link with academic success make the SSHA suitable for inclusion in other scales in research on education.* (Translation)

From a psychological perspective it is therefore not of crucial importance that a high correlation between an adequate study orientation and intelligence or aptitude be found. On the other hand, a high correlation between an adequate study orientation in mathematics and academic success (achievement in mathematics) will make this type of scale suitable for inclusion in other scales for psychological measurement in education.

1.1.3 Psychological testing in mathematics

Murray, quoted by Madge (1981b:1) states the following about the nature and limitations of psychological testing in general:

> *The profession of psychology is much like living, which has been defined ... as "the art of drawing sufficient conclusions from insufficient premises". Sufficient premises are not to be found, and he who, lacking them, will not draw tentative conclusions, cannot advance.*

Strategies and techniques that identify learners' personal strengths and weaknesses should be used optimally by psychologists. This information should include the strengths and weaknesses regarding the cognitive, the affective, the conative and the psychomotor domains. As in the case of the *Survey of Study Habits and Aptitudes* (SSHA) (Taljaard & Prinsloo, 1995) the aim of incorporating tests and questionnaires in mathematics can be summarised as follows:

- ★ The identification of learners with a study orientation in mathematics that differs from that of learners scoring high marks.
- ★ To help learners with problems in mathematics.
- ★ To obtain insight into the causes of these problems.
- ★ To establish a basis for providing support to these learners to optimise their study orientation in mathematics so that they can do better in this subject.

All tests and questionnaires are diagnostic to a certain extent. However, there is a need for a specific questionnaire that can identify those study orientation problems that arise especially in the mathematics class. This questionnaire should not only be used when problems arise, but be integrated continuously to motivate also those learners with an adequate study orientation in mathematics so that they can attempt to optimise important feelings towards, and habits and attitudes relating to mathematics. It is important that the use of this questionnaire be followed up by discussions, but with the emphasis on individualisation.

1.2 THEORETICAL FOUNDATION FOR CERTAIN APPROACHES TO LEARNING MATHEMATICS

Nowadays especially the constructivist approach and the work of people like Ernest (1989a; 1989b), Jaworski (1988), Olivier (1989), Steffe, Cobb and Von Glasersfeld (1988) and Volmink (1990) who took the work of Piaget a step further are being given much attention. According to this approach **knowledge is acquired and it cannot be supplied or transferred**. In other words, teachers or textbooks cannot transfer knowledge to learners – they create knowledge themselves. Consequently the new draft syllabuses, for example, imply that problem-solving (problem centredness) should constitute the central focus of a study orientation in mathematics.

Apart from the fact that the ability to solve problems provides a reason for studying mathematics, it also supplies a context for the learning and doing of mathematics. The focus consequently shifts as follows:

- From learners as individuals who **do** something to learners as individuals who **think actively**;
- from mathematics as focused on concepts and skills to a focus on concepts, skills and **processes**;
- from the mastering of algorithmic skills to the development of algorithmic **thought**; and
- from the application of mathematics to solve problems to problem solving as an investigation method (Adler, 1992).

This approach emphasizes, *inter alia*, the following aspects of an adequate study orientation in mathematics: the importance of social interaction, working together in groups; problem solving, tendency to investigate and learner involvement in the mathematics classroom (Volmink, 1993).

1.3 STATEMENT OF THE PROBLEM

Mathematics obviously does not benefit from a traditional learning approach and traditional teaching style. Innovative thinking is first required before adequate insight and achievement can be attained in this subject. Furthermore, the development of a cognitive style in mathematics, to a far greater extent than is the case for any other subject, is particularly vulnerable to bad teaching (Freudenthal, 1980). Consequently if attempts to improve the situation are being considered, the proper place to start is with these matters in mind (Maree, 1995b).

Apart from questionnaires specifically aimed at obtaining background information on mathematics learners, it is also important to implement mathematics questionnaires from time to time. These questionnaires include those that identify knowledge gaps and the reasons therefore (these questionnaires concern learners' information, language and logic problems in mathematics). Although there are various ways that teachers can use to obtain information on the topics mentioned above, this aspect of the study orientation in mathematics is frequently neglected to the detriment of learners and their achievement in this subject. **Such questionnaires should, however, ideally be followed up by a specific questionnaire that concentrates on providing an opinion on learners' study orientation (of which cognitive style constitutes one facet).**

The foregoing can be summarised as follows: there is a particular need for a study orientation questionnaire in mathematics. This type of psychological test should be able to investigate other factors also rather than merely evaluating learners' cognitive ability. **The focus of this study is thus a cardinal aspect of the problems relating to inadequate achievement in mathematics: the root of the "problems" can probably, possibly be found outside the cognitive field.** The importance of a firm affective substructure as a basis for adequate cognitive achievement in mathematics should not be underestimated.

Learners' level of emotional functioning, their personality structure, motivation, their feelings towards mathematics, the way in which they relate to their teachers, the classroom atmosphere, their domestic circumstances and the teaching of the subject, in short, their **total study orientation in mathematics** plays a significant role in their ultimate achievement in this subject.

The primary problem that this study deals with centres on the fact that apart from a variety of intelligence, personality, interest, achievement, initial evaluation, performance, proficiency and diagnostic tests, there is at present no test to determine learners' study orientation, **specifically in mathematics**.

1.3.1 Aspects of learners' study orientation in mathematics that should be explored with the aid of a study orientation questionnaire

This study attempts to develop and evaluate an instrument to measure different categories of mathematical behaviour (Schoenfeld, 1985) including learners'

- ★ realisation of their need for specific mathematical knowledge as well as for general and specific discovery methods that can be used to solve problems;
- ★ cognitive style, including the way in which they process information in mathematics;
- ★ control relating to monitoring and decision making during the process of problem solving;
- ★ mathematical world view – on the self, on the nature of mathematics and on the learning of mathematics;
- ★ mathematics anxiety; and
- ★ study attitudes and dispositions: in short, their total study orientation in mathematics.

1.3.2 Aim of a study orientation questionnaire in mathematics

The aim of a study orientation questionnaire can be summarised as follows:

- ★ Identification. It should be possible by using the questionnaire, to identify learners whose study orientation in mathematics differs from the study orientation of learners who achieve well in mathematics.
- ★ Understanding. The results of the questionnaire should help psychologists better understand learners' with poor achievement in the subject.

- ★ Assistance rendering. Psychologists should be able to use these results to assist learners to optimise their study orientation in mathematics in order to realise their potential on a higher level.

1.3.3 The use of a study orientation questionnaire in mathematics

The following list indicates some possible uses of a study orientation questionnaire in mathematics:

- ★ As a diagnostic test. It should be possible to administer the questionnaire to learners, especially at the beginning of an academic year but also at any other time of the year, individually or to groups. Learners' scores should be examined to identify those that need assistance, remedial support and counselling.
- ★ Rendering assistance. The questionnaire should be able to provide mathematics teachers with a standardised means of systematically analysing important feelings, attitudes and uses relating to learners' academic orientation in mathematics. It should be comparatively easy to construct a profile of learners' study orientation in mathematics. An interpretation of learners' responses to the questionnaire and an analysis of shortcomings that could lead to poor achievement has to be made. An analysis of individual answers (particularly those that differ significantly from the answers that are usually given by good achievers in mathematics) can be a great help. Psychologists can be helped in their task when dealing with those aspects of the various fields of the questionnaire in which learners fare badly.
- ★ Study guidelines in mathematics. It should be possible to administer the questionnaire as a simple means of inculcating in pupils' minds certain basic principles of effective study in mathematics, as well as the important role that motivating factors play in academic success.
- ★ Research. The questionnaire should have the potential of a suitable measuring instrument to be included in other scales concerned with educational research.

1.4 DEFINITION OF TERMS

The following terms appear directly or by implication in the title of this study and require further elucidation: development, evaluation, study, orientation, study orientation, mathematics, learners and achievement.

1.4.1 Development

According to De Villiers, Smuts and Eksteen (1983) and Odendal (1981) the following meanings relate to the term: to make a plan; to sketch or form something; to compile something provisionally. For the purpose of this study the term "develop" indicates bringing about or compiling and ultimately evaluating the study orientation questionnaire in mathematics to which reference was made in paragraph 1.5.

1.4.2 Evaluation

According to Allen (1992) and De Villiers, *et al.*, (1983) the term "evaluation" can have the following meanings: determine the value; appraise; ascertain the numerical value for something. In this study evaluation indicates the collection and analysis of data on the study orientation questionnaire in mathematics with a view to determining its success or potential usage. The exact manner in which this evaluation will be done, is explained in detail in Chapter 5.

1.4.3 Study

The word "study" is etymologically and semantically related to and derived from the word *studeo* which can mean the following (Smith & Lockwood, 1987); to be keen; to take pains; to strive after something; to show an interest in something; to devote yourself to something; or to study something. Allen (1992) states the word can mean the following: to study something; to investigate or analyse; to devote yourself to study; to make an attempt to learn something; to go to extremes to obtain results; to try to control something.

1.4.4 Orientation

Schmeck, as quoted by Roos (1995:7), defines the term "learning orientation" as follows:

> *Orientation is regarded as the factor that summarises approaches, motives and styles and includes an element of study methods and attitudes.*
>
> (Translation)

Etymologically the term "orientation" indicates assumption of a point of view; determining an attitude; determining a position; the receipt of information on, or usage of orientation matters, personality dimensions (affective-cognitive-conative-psychomotor) as well as historicity (past-present-future) (Maree, 1986). It should consequently be possible to orient learners in a holistic way according to a study orientation questionnaire to determine their own position and to take a particular point of view with regard to relevant aspects of their study orientation in mathematics.

1.4.5 Study orientation

According to Taljaard and Prinsloo (1995:421) the term "study orientation" can be defined as "a joint measurement of the learner/student's study habits and attitudes".

Despite the importance of an adequate study orientation in mathematics as well as the pleasure that certain learners derive from the study of this subject, many learners have a negative attitude towards the subject once they leave school (Charles & Lester, 1984). The integration and follow-up of a student orientation questionnaire in mathematics can possibly contribute to solving the problem.

1.4.6 Mathematics

The metaphysical question "What is mathematics?" is related to the epistemological question: "What is meant when it is stated that people do or learn mathematics?" The latter question is discussed in Chapter 2, but at this stage the first question will be considered briefly.

Schoenfeld (1994) underscores agreeing with the view of Hoffman (1989) who defines the term "mathematics" as **the science of patterns**. Steen (1988:616), however, had already defined the term "mathematics" in 1988 as follows:

> *Mathematics is often defined as the science of space and number, as the discipline rooted in geometry and arithmetic. Although the diversity of modern mathematics has always exceeded this definition, it was not until the recent resonance of computers and mathematics that a more apt definition became fully evident. Mathematics is the science of patterns ... it begins with the search for pattern in data ... . Generalization leads to abstraction, to patterns in the mind. Theories emerge as patterns of patterns, and significance is measured by the degree to which patterns in one area link to patterns in other areas. Subtle patterns with the greatest explanatory power become the deepest results, forming the foundation for entire subdisciplines.*

Terblanche and Odendaal (1966:132) define the concept "mathematics" as *"The science that concerns itself with magnitudes and dimensions as independent data, geometry and algebra; mathematics; mathematica."* Gove (1976:1393) defines the concept as *"a science that deals with the relationship and symbolism of numbers and magnitudes and that includes quantitative operations and the solutions of quantitative problems."* According to Odendal and Schoonees (1979) mathematics is a science that investigates the properties of numbers and figures. Geometry, algebra and arithmetic are different subsections of mathematics.

Howson believes (1991:5) that mathematics can be defined according to different points of view. He mentions the following perspectives:

> *(maths is) an abstract structure with seemingly miraculous inter-relationships, (or) a collection of interesting and potentially useful results, methods and results, (or) an activity that relies upon the participant's ability to conjecture, prove, generalize, model, apply, define.*

The root "mathematics" (Gove, 1976; Terblanche & Odendaal, 1966) has been derived from the following words:

★	French	:	*mathematique*
★	Latin	:	*matematicus*
★	Greek	:	μαθεματικος (belonging to the sciences)
★	Greek	:	μαθεσομαι: "I will learn".
★	Latin	:	*tenere*: learn by investigating; make certain of; to have learnt to be very familiar with; to ask; to observe; to make an actual attempt.
★	Greek	:	μανθανω (Jones, 1968): to learn (especially through studying and hard practice); to obtain experience; to make a habit of something, or carry out an automatic action; to get used to; to observe; to notice; to take careful note of; to understand thoroughly; to obtain insight into something.
★	Greek	:	τό μαθημα (Jones, 1968) (something that has been learned; a lesson; learning; knowledge).
★	Greek	:	η μαΘησις (the learning action; an intense desire to learn; or to acquire knowledge).

To summarise it can be stated that an etymological-semantic analysis of the word "mathematics" shows that the subject cannot be mastered without pains being taken, without learning, experience, practising, insight, the will to learn, responsibility, self-discipline and perseverance (on an almost daily basis).

For the purpose of this thesis mathematics, with reference to the views of Steen and others, can be regarded as the science of patterns.

1.4.7 Learners

Currently the word "learner" is preferred to the term "pupil" although the two are regarded as synonyms. Although one should guard against the use of so-called "buzz-words", the trend is to use the word "learner" instead of "pupil" within contexts like that of this study (Grebe, 1997). The intention is to get away from the narrow view that learning mathematics is the privilege of some people only. Both words are derived from and related to different languages (Allen, 1992; De Villiers, *et al.*, 1983; Gove, 1976). The word "learner" can have the following meanings: persons who learn; persons preparing for a particular subject; persons who through lengthy and systematic study attain a high degree of expertise, skill and efficiency; persons who have (or ought to have) the following characteristics or attitudes: curiosity, perseverance, initiative, originality, creativity and integrity. These characteristics are precisely those that are regarded as essential for achievement in mathematics. For the purpose of this study learners refer to persons who are scholars, or are engaged in some or other form of tertiary study.

1.4.8 Achievement

The following meanings are attached to the word "achievement" (Gove 1976:16):

> *a result brought about by resolve, persistence and endeavour; performance by a student in a course; the quality and quantity of a student's work during a given period; the ability to perform, or the capacity to achieve a desired result; the manner of reacting to various stimuli.*

For the purpose of this study the word "achievement" indicates the learners' level of self-fulfilment in mathematics, as well as their ability to attain particular levels of achievement in mathematics through exertion and perseverance.

1.5 RESEARCH PROCEDURE

The following procedure will be followed in this investigation:

- ★ Chapter 2 contains an **evaluating** review of relevant literature on certain learning theories with regard to mathematics. The approach followed in this chapter is an eclectic one. An attempt will be made to indicate that no particular learning theory will be given special preference at the expense of others.

- ★ Chapter 3 provides a review of cognitive, affective, conative and psychomotor approaches to explain learners' inadequate study orientation in mathematics. The chapter concludes with certain models for dealing with learners' inadequate study orientation in mathematics.

★ Chapter 4 deals with a cross-cultural perspective on achievement problems in mathematics with regard to the measurement of a study orientation in this subject.

★ In Chapter 5 the research development and data processing procedure are explained. The primary hypotheses investigated in this study centre on the question whether the theoretical fields of the study orientation questionnaire in mathematics are confirmed by factorial validity. Furthermore an attempt is made to determine whether the different fields of the Study Orientation Questionnaire in Mathematics (SOM) differ statistically significantly for the various sex groups; whether the achievements of the various mother-tongue groups in the different fields of the SOM differ significantly; whether the achievements of all the grade groups differ statistically significantly in the various fields of the SOM; and whether there is a significant correlation between the achievement in the various fields of the SOM on the one hand, and the *Achievement Test in Mathematics* (Standard 7) as well as the *Diagnostic Tests in Mathematical Language* on the other hand.

★ Chapter 6 indicates the findings of the study.

★ Chapter 7 contains a brief summary and discussion of the study. Certain recommendations are made.

CHAPTER 2

PERSPECTIVE ON CERTAIN EPISTEMOLOGICAL POINTS OF DEPARTURE REGARDING LEARNING MATHEMATICS

2.1 INTRODUCTION

Stewart (1991:20) refers to the significance of an adequate study orientation in mathematics to do well in this subject:

> *The children who persisted in the face of failure tended to attribute failure to the lack of effort and motivation. These children were described as showing mastery-orientated behaviour, since for them failure was not insurmountable, but could be overcome by additional effort. However, for the learned helpless children, who attributed failure to things beyond their control, there was no point in increasing effort, and so they gave up in the face of failure ... following failure, learned-helpless children tended to abandon the correct problem-solving strategies they had previously used successfully, whereas the mastery-orientated children tended to develop more advanced and sophisticated strategies to try to solve the insoluble problems.*

Van Aardt and Van Wyk (1994:223) emphasise the detrimental effect of mathematics students' inadequate study orientation as follows:

> *There is general agreement that an increasing number of academically underprepared students are reading for university degrees, with the result that many fail to meet the academic demands ... recent evidence suggests that the use of effective learning and study strategies is an important factor in determining success at (school and) university level.*

Pollock and Wilkinson (1988) declare that the use of adequate study skills is probably the most important requirement for effective study.

Chapter 2 focuses on an **evaluative** synopsis of certain epistemological approaches to the study of mathematics to ascertain what constitutes the essential elements of an adequate study orientation in mathematics. The objective of this synopsis is to obtain perspective on the way learners learn mathematics. The approach to theory structuring followed in this chapter is an eclectic one; in other words, an attempt will be made to indicate that no particular learning theory should be given preference at the expense of others. Each learning theory represents a particular view of knowledge. In this study the point of view is taken that each theory is valid to a certain extent and of value when the very nature of an adequate study orientation in mathematics is considered.

Scientifically based learning theories reveal a historic development that to a high degree agrees with synchronous scientific and social values that were given preference during a particular period in a particular country. Although neither the particular values nor the climate in which most of the learning theories developed can be imposed directly on the South African situation, the generalised learning theories must necessarily have an effect on the local development of psychology as a science. These theories do not continue to develop in isolation but in interaction with one another. Finally these learning theories provide psychologists with the theoretical foundation for establishing a practice with the final objective an acceptable, applicable and suitable intervention in the interest of their clients.

The psychological foundations of research on the learning of mathematics are dealt with first.

2.2 PSYCHOLOGICAL FOUNDATIONS OF RESEARCH ON THE LEARNING OF MATHEMATICS

As far back as 1899 Binet identified three basic ways of research, namely the questionnaire method, observation, and experimentation (Kilpatrick, 1992). He currently receives the same degree of recognition for his contribution to the psychology of thinking as for constructing the first intelligence test.

2.2.1 Research on the nature of thinking

2.2.1.1 The measurement of intellectual ability

Initially Binet attempted to follow the trend of his time, namely to base his research on intelligence on the dimensions of skulls (Gould, 1981). Gall was the founder of phrenology (assessing the state of cognition according to the structure of the skull) whereas Broca refined this science as the study of skull measurement. Möbius's research on the skulls of prominent mathematicians convinced him that there was a relation between aptitude for mathematics and the shape and circumference of the skull. Binet, however, carefully investigated this phenomenon but concluded that there was no connection between physical dimensions and cognition. He had enough insight to realise (and to state) that a researcher had to set certain intellectual tasks for clients (for clients to display their cognitive skill) in order to obtain any indication of the state of their cognition.

Galton in his turn tried to apply Darwin's theory of evolution to the study of psychology. He was especially interested in proving, by using a number of physiological and psychological tests, that his theory of intelligence was (mainly) the result of heredity. Consequently he can be regarded as the one who initiated the science of mental testing. His successors (Burt, Pearson and Spearman) continued his work, but his tests were limited in the sense that they involved only limited aspects of the responses, namely reaction time, associations, as well as sensory discrimination. Kilpatrick (1992:8) states the following in this regard:

Had Binet's ideas about intelligence testing – the use of scores for diagnosis rather than ranking; the rejection of an innate, fixed quality known as "intelligence" – been preserved as his tests migrated across the Atlantic, "we would have been spared a major misuse of science in our century" ... Instead, American psychologists such as Henry Goddard, Lewis Terman, and Robert Yerkes developed out of Binet's tests a hereditarian theory of IQ that not only had some disastrous effects in its consequences for social policy ... but also colored the views of a generation of American psychologists in mathematics education about the prospects for improving mathematical abilities.

2.2.1.2 The study of mental development

Binet developed his intelligence scale with a particular purpose in mind. **He wanted to identify those learners whose achievement provided indications of special intervention strategies.** In other words, he was less interested in the **label** that a particular score provided than in the **assistance** that could be given to learners with particular scores. Piaget, a student of Binet, was particularly interested in the procedures that learners followed in order to arrive at their answers in mathematics – particularly the wrong answers (Flavell, 1963).

Hall, who worked with Wundt, was convinced that child development followed the same pattern as human evolution. He believed that there was not much sense in stimulating intellectual development in learners. Hall particularly emphasised the significance of learners' interests and their need for motivation.

2.2.1.3 Stimulation of productive thinking

Külpe broke away from the views of Wundt, namely that:

one could study the structure of consciousness through introspection (Kilpatrick, 1992:8).

The Würtzburg School believed that abstract thinking did not frequently accompany ideation and researchers consequently should not study thinking according to thinking **content** but according to thinking **functions**. Prompted by this form of thinking Wertheimer founded gestalt psychology. Although this line of thinking primarily focused on perception, it also acknowledged the processes of creativity, productive thinking and problem solving. Selz, whose work on problem solving influenced psychologists significantly, was also involved in this school of thought. The gestalt psychologists' work on thinking and reasoning influenced mathematicians' views on these matters, especially when behaviouristic theories were receiving the most attention.

To summarise, it appeared that early research on thinking usually followed this pattern: observation of individual or group differences under "typical" circumstances in the hope of improving achievement by providing suggestions or guidelines aimed at optimising learners' achievement. The focus was usually on facilitating changes over a longer period.

Criticism of research based on thought processes is usually that the test procedures used, for example correlation analyses, regression analysis and factor analysis are too comprehensively statistical in nature and that it is also assumed that ratios are linear and effects "additive". Apart from this, early research practically ignored cultural influences on thinking. The willingness of researchers in this field to investigate natural relationships as a potential source of hypotheses that have to be tested is, however, a positive contribution worth mentioning.

2.3 APPROACHES TO THE LEARNING OF MATHEMATICS (ARITHMETIC) IN THE TWENTIETH CENTURY

Cross (1981) points out that the study of learning processes dates back to the eighties of the previous century on account of the work of researchers like Ebbinghaus, Dewey, Thorndike, Watson and Levin. The work of Thorndike will be discussed briefly.

2.3.1 Mechanical drillwork versus acquisition of meaning

From 1900 to approximately 1920 the most general method of learning mathematics was the drill and practice method of Thorndike (Grossnickle, Reckzeh, Perry & Ganoe, 1983). The latter's views in this connection can be closely linked to those of Pavlov (in connection with conditioning) and Skinner (in connection with radical behaviourism) (Kilpatrick, 1992). An attempt was made to inculcate arithmetical skills and abilities in learners by making use of drillwork, arithmetic steps and memorising combinations. Learning was regarded as a form of relationship that could be strengthened by means of repeated drillwork. Learning theoreticians such as Brownell were strongly opposed to aspects of this approach stating, among other things, that not enough allowance was made for the acquisition of **insight**.

2.3.2 The social approach

This period lasted approximately from 1920 to 1935. In essence this approach boils down to trying to find possible applications of mathematics to true-life situations. The severest criticism levelled against this approach came from academics who seriously objected to the fact that the social utility value of the subject was regarded as the major criterion of the significance of the contents of mathematics, that the systematic study of mathematics was not given adequate attention, that insight into the true meaning of arithmetic was under-emphasized, and that the development of arithmetical skills and ability was neglected.

2.3.3 The meaningful approach

In broad outline this approach was followed from 1935 to early in the sixties. In brief it implied that the mathematical aspect of especially arithmetic received just as much recognition and attention as its social aspect (arithmetical knowledge and ability were to be used in everyday life situations) as well.

2.3.4 Direct versus incidental learning

In the period between the two world wars researchers like Dewey (DeVault & Kriewall, 1969) were in favour of incidental learning in mathematics. This means that mathematics is best mastered when other outcomes or objectives are set, such as problem-solving in other fields of study (for example the natural sciences). The study of mathematics as a separate subject was criticised.

2.3.5 The "New mathematics" of the sixties; discovery versus exposition

Directly after the Second World War in the midst of increasing industrialisation, technological progress and competition with the then Soviet Union, mathematics as a school subject began to take up a special place in school syllabuses. The launching of the Soviet Union's Sputnik in 1957 brought about a turning-point in the approach to learning mathematics. In the United States of America (USA) in particular a feverish attempt was made to optimise learners' achievement in mathematics and to update curricula in mathematics, physics and chemistry. In 1958 the so-called School Study Mathematics Group at Yale University initiated the biggest and best financed effort yet to improve mathematics at school. This led to the inception of new school curricula that were known as "New Mathematics". At this stage the emphasis was on **discovering** and mastering the **structure** of mathematics. Learning theorists such as Bruner (Orton, 1987) regarded mathematics as a **process** that learners had to **experience**, and not as a **product**.

Various factors led to the "New" or "Modern mathematics" making room for another far-reaching change of direction: the so-called Back-to-the-basics movement.

2.3.5.1 Anti-New mathematics forces

In the long run it appeared that "New mathematics" did not really have the desired effect of attaining the envisaged objectives. Grossnickle, *et al., (1983:5)* support this statement with the following quotation from the journal *Time* (Help! ... 1980:59):

> *[New maths is nothing more than] ... a faddish theory [that] swept through the profession, changing standards, techniques, procedures [that were introduced] without adequate try out, and poorly understood by teachers and parents [with the result of] lowered basic skills and test scores in elementary mathematics.*

The first landing on the moon in 1969 (Neil Armstrong) finally laid the ghost of Sputnik. The forces primarily responsible for the development of "Modern mathematics" were followed by various social, professional and technological pressure forces. Among other things changes to the following were insisted on:

- ★ The mathematical content being taught; and
- ★ the way in which learners should learn the mathematical content.

2.3.5.2 Research on cognitive development

Researchers such as Piaget and Brownell indicated that learners under the age of 7 could not understand the heavily emphasized structure of mathematics, and that learners up to at least the age of eleven had an intense desire for experience with concrete objects in order to constitute their world of thinking.

2.3.5.3 Poor academic achievement of school-leavers

It became all the more apparent that school-leavers did not really have the necessary arithmetical and reading skills when they applied for employment. Moreover, research showed that the following factors made a significant contribution to the undesirable state of affairs (Grossnickle, *et al.*, 1983):

- ★ Poor discipline
- ★ Limited time for reinforcement
- ★ Too little homework
- ★ Too much emphasis on socialising.

These and other reviews gave rise to a reversion to the abovementioned Back-to-the-basics movement.

2.3.6 The Back-to-the-basics movement

Various topics introduced in the sixties to emphasize the structural aspects of mathematics were now omitted from the syllabus. A return, as it were, was made to the three R's: Reading, (A)Rithmetic and (W)Riting. Arithmetic skills received special emphasis whereas important mathematical application ability and problem-solving skills were left in abeyance.

When the results of these new syllabuses were evaluated in the mid- and late seventies, it was found that learners' mathematical ability, their mathematical insight and their mathematical application and problem-solving skill had deteriorated considerably. In other words, a return to the **basics** did not significantly promote the learning of mathematics. Since mechanical drillwork was overemphasised, frequently without the necessary insight, learners were not given enough opportunity of practising with concrete material, of problem-solving in mathematics or of dealing with real-life problems.

Kriel (1990:335) puts the corresponding situation in South Africa into perspective as follows:

> *The recent history of mathematics syllabus development at school level in the RSA shows that subjects at one stage were included in (or removed from) the syllabus only to be removed again (respectively included) later on as a result of shortcomings that came to light during full-scale implementation.*
>
> (Translation)

2.3.7 Aim of mathematics teaching since the eighties

Grossnickle, *et al.,* (1983) set the following objectives, among others, for optimising learners' study orientation in mathematics:

- ★ Stakeholders should not only know and understand the subject content. They should understand learners and the way in which they learn and understand mathematics. They should therefore be familiar with psychological principles and be able to apply them to the level of mathematics teaching that relates to the learners' level of mental development.
- ★ Problem-solving strategies should be given first priority.
- ★ Clearly defined objectives constitute the basis of a comprehensive, balanced approach to the learning of mathematics. This implies that learners should develop, among other things, the following (Maree, 1994):
  - An ability to think quantitatively with regard to problem-solving situations.
  - A functional knowledge of the **language** and **structure**[1] **of mathematics including the ability to estimate, approach and probe the reasonableness of the results of problem-solving.**
  - Sensitivity to a wide variety of quantitative situations in the community, as well as the ability to apply mathematics in everyday situations.
  - An intelligent mastery of arithmetical skills and abilities. This implies that learners should have some insight into the reasons why they perform certain mechanical functions.
  - An evaluation of the use and importance of mathematics in modern society.
  - A favourable attitude towards learning and discovery with regard to mathematics.

The preceding argumentation brings the following research questions to the fore at this stage:

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[1] The technical language of mathematics includes the following: a) everyday words with special definitions for use in mathematics (example the **power** of a number), b) the technical mathematical words (like **sinus**), c) number indications (like the number 2 that can be expressed in many ways; it is essential for learners who want to master mathematics to understand this), d) the shorthand symbols $(=, \equiv, \sum)$ and e) language and communication structures (Cartesian level, algebra). These technical expressions should be carefully taught to pupils (Cangelosi, 1996).

What are the essential aspects of an optimal study orientation in mathematics? How do children learn mathematics? Are their mental processes the same as those of adults? Can mathematics, for example, be analysed to such a simplified level that it reaches a point where any child can understand it (Skinner)? Or are the developmental psychologists more correct in their assumption that learners are developing beings and do not learn in the same way as adults; that they first have to undergo particular developmental stages before they are ready for certain content in mathematics? To answer these questions attention will be given briefly to certain learning theories that are regarded as representative of and relevant to this study.

2.4 BEHAVIOURISM[2]: THE LEARNING OF MATHEMATICS AS THE ACQUISITION OF ARITHMETIC AND ARITHMETICAL SKILL

Mathematics can, inter alia, be defined as the learning of arithmetic rules and skills that include addition, subtraction, multiplication and division. This is traditionally known as arithmetic. In terms of arithmetical skills it is useful to make a distinction between the following terms:

- ★ Numerical facts including tables (addition, subtraction, multiplication and division) that have already been learned at primary school level.
- ★ Algorithms that refer to more complex procedures and skills, including a skill like long division where one multiple term is divided by another multiple term.

Each step is important; certain procedures **must** be mastered and certain numerical facts should be thoroughly known before proceeding to the use of numerical facts and algorithms in problem solving, known as word problems or story problems (Resnick & Ford, 1981). Word problems in reality mean that learners should be able to interpret words, “translate” them into a mathematical calculation and then apply suitable problem-solving procedures.

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[2] Behaviourism: A school of thought that attempts to explain human behaviour fairly completely in terms of reactions to external stimuli. The point of departure is that people can be manipulated in optimal circumstances to act in any desired manner.

Subsequently certain learning theories will be examined as examples to indicate how this approach works in practice. Initially the learning theory of Thorndike is examined. Thorndike can in a certain sense be regarded as the founder of the psychology of mathematics and as a representative of the **behaviouristic** school of thought.

2.4.1 Introduction

Olivier summarizes the behaviouristic point of view as follows:

> *Behaviourism therefore assumes that pupils learn what they are taught, or at least some subset of what they are taught, because it is assumed knowledge can be transferred intact from one person to another.*

The learner, in other words, is regarded as an empty vessel, a *tabula rasa*. Mistakes and misconceptions in mathematics are regarded as flaws in a computer – if undesired, they can merely be erased or one can overwrite them. Gagné (1983:15) describes this as follows:

> *The effects of incorrect rules of computation, as exhibited in faulty performance, can most readily be overcome by deliberate teaching of correct rules. My interpretation of previous psychological research on "unlearning" is that it is a matter of extinction. This means that teachers would best ignore the incorrect performances and set about as directly as possible teaching the rules for correct ones.*

2.4.2 Thorndike's behaviouristic theory of learning

As a psychologist Thorndike had a good grounding in the tradition of experimenting in a laboratory. However, he was equally interested in "translating" his findings in the laboratory into concrete guidelines for the classroom. Thorndike is probably best known for his statement in connection with the so-called "law of effect", an earlier version of the theory known as "the principles of reinforcement" (Resnick & Ford, 1981). This theory was not developed within the more complex mathematical context, but within the context of simple laboratory experiments with cats, dogs, monkeys and chickens. The experiment most closely associated with his theory is the following: He places a cat in a small wooden box. The terrified animal scratches and claws until it accidentally opens the latch. If this procedure is repeated, it takes the cat less and less time to open the latch (it is only the scratching which opens the lock that is repeatedly rewarded). According to Thorndike the cat did not learn or acquire insight into opening the box, but the reward for escaping linked the experimental situation to the specific response that made the escape possible. This led to the following conclusion (Thorndike 1913:4):

> *When a modifiable connection between a situation and a response is made and is accompanied or followed by a satisfying state of affairs, that connection's strength is increased: When made and accompanied or followed by an annoying state of affairs, its strength is decreased.*

Thorndike (1922:xi) states the following on human learning:

> *The aims of elementary education, when fully defined will be found to be the production of changes in human nature represented by an almost countless list of connections or bonds whereby the pupil thinks or feels or acts in certain ways in response to the situations the school has organized and is influenced to think and feel and act similarly to similar situations when life outside of school confronts him with them.*

As was the case with other psychologists of his time (the so-called *connectionists*[3] or *associationists)* he argued that all behaviour can be broken down into two simple components, namely **stimuli** (circumstances beyond people's control) and **responses** (actions that people carry out in reaction to those external situations). The more frequently a stimulus response pair is carried out, the stronger the correction becomes. This means that reinforcement that is rewarded or strengthened, is an important way in which people learn. Thorndike regards the learning of mathematics as the connection between separate elements. He suggests the following procedure: firstly the designated connections that have to be made must be

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[3] See paragraph 2.5.6.

analysed. When these connections (bonds) have been selected, they have to be formed and reinforced: **this is where drillwork and reinforcement come into play.** Important connections are reinforced more regularly and less important ones less frequently. From this it appears that Thorndike regards the learning of mathematics as the connection between separate elements. Thorndike emphasizes this matter as follows (Resnick & Ford: 1981:13-4):

> *As a first step, one would have to select the bonds to be formed. Naturally, any carefully constructed arithmetic curriculum, with or without benefit of psychological analysis, would divide the subject matter up into broadly defined topics.*

Drillwork and repetition are central to Thorndike's theory of learning and the teaching of arithmetic is frequently done by means of drillwork. Although Thorndike emphasizes that drillwork should be presented in an interesting manner and be verified with concrete objects and although his learning theory implies that drillwork is the main method of learning arithmetic, the conclusion must not be drawn that he regards drillwork as the only or exclusive way of learning.

Not all psychologists in Thorndike's time accepted his theories without question. Brownell was one of his critics.

2.4.3 Brownell's theory of learning

Brownell's first point of criticism of the aforementioned theory of learning was that it did not take into account qualitative differences in the arithmetical skill and ability of learners. When he tested a group of primary school learners' arithmetical skills he found that they had arrived at their answers in different ways. Some learners counted on their fingers, others used existing combinations to create new ones (6 + 6 = 12, therefore 6 + 7 has to be 13) and still others answered immediately, but it was the wrong answer which indicated that they had simply guessed. This led him to state that Thorndike's mechanical drillwork merely got learners to work faster and better with regard to the "immature" methods that they had discovered themselves, and not with regard to the type of direct recall of knowledge which adults have.

Secondly Brownell believes that the drill method provides a distorted view of the learning objective. Brownell is convinced that the criterion of arithmetical ability lies in learners' ability to think quantitatively, and not in learners' being able to respond 100% accurately to a list of problems (Resnick & Ford, 1981). It is of paramount importance to Brownell that a learner should learn:

> *until he understands something of the reason why 7 and 5 is 12; until he can demonstrate to himself and to others that 7 and 5 is 12; until he is so thoroughly convinced that 12 is the right answer for 7 + 5 that he can give it as the answer with assurance of correctness; and until he can use the combination in an intelligent manner - in a word, until the combination possesses meaning for him* (Brownell, 1928:198).

Brownell emphasizes the fact that learners should master certain mathematical principles and patterns. He also propagates the idea of generalisation in mathematics. Learners should apply their knowledge to new problems in each phase. They are allowed to count and to apply other problem-solving methods until they can comfortably change over to automation (the automatic way of knowledge recall).

Brownell's theory has been extended and refined by researchers who believe that especially generalisation in mathematics rather than mere drillwork, or a combination of both strategies, produces the best results (Bell, 1978).

For the sake of true insight into the learning of mathematics a theory of learning should be able to explain the phenomenon of transfer. In other words, it has to provide reasons why the learning of uncomplicated work makes it possible to learn more complex complicated work. Gagné was one of the well-known exponents of this theory.

2.4.4 Gagné's neo-behaviouristic (cumulative) theory of learning

Gagné (1976:3) defines learning as follows:

> *Learning is change in human disposition or capability, which persists over a period of time, and which is not simply ascribable to processes of growth.*

For Gagné (1985) the result that is associated with learning is particularly meaningful. He identifies the following five categories with regard to learners' study orientation in mathematics and achievement in this subject:

- ★ Intellectual ability (including the learning of the alphabet and terms and concepts relating to this);
- ★ verbal information that indicates that a learner is capable of putting data into his or her own words and then telling others about the data;
- ★ cognitive strategies;
- ★ attitudes; and
- ★ motor skills.

Gagné's theory implies that skills in mathematics are analysed according to systematized subskills or learning hierarchies. As far as Gagné is concerned any form of learning commences with a task analysis. What has to be learned? The skill has to be put specifically and behaviouristically into words. Copeland (1948:5) describes this process as follows:

> *It can be conceived as a terminal behavior and placed at the top of what will become a pyramid-like network.*

Suppose the skill to be discussed is "problem-solving". Learners first have to know certain principles. To know and understand these principles, certain concepts have to be mastered first. These concepts in turn set the preconditions that specific associations or facts should be mastered first. According to Shulman (1974) any such analysis ends essentially in operant conditioning. Gagné (1983:11) explains the term "task analysis" as follows:

> *A task analysis of the performance expected of a mathematics student would, I think, reveal three major phases.*

These three phases are:

- ★ *"Translating verbally described situations into mathematics"* (Gagné, 1983:11). For him the verbal statements are merely representations of concrete situations.
- ★ *"Validating the solution"* (Gagné, 1983:12). It is important to Gagné that learners are deliberately taught to test their answers in as many ways as possible.
- ★ *"The central computation phase"* (Gagné, 1983:12). To Gagné arithmetical skill is a completely concrete attribute.

Gagné classifies all learning operations from uncomplicated *signal learning* to more complicated problem-solving. When particular preconditions have been met, the learners are tested to determine what they know and what they still have to learn. Programmed learning is a logical consequence of this type of approach (Gagné, 1983).

2.4.4.1 Gagné: Synthesis

In spite of the strict behaviouristic nature of Gagné's epistemological views he recognised the value of learners' own responsibility, positive attitude, own will and creativity as aspects of their study orientation in mathematics and took these into account in practice. Among other things he emphasises the importance of insight into the hierarchical nature of mathematics; this emphasises that rules, definitions and principles should first be actively mastered before attempting to master higher order insights. The value of a problem-solving approach should also be recognised. Gagné emphasises that learners should learn to make use of concepts, rules and definitions in order to obtain insight into the structure of mathematics. It is for this reason that learners have to be guided so that they implement cognitive strategies in mathematics (in other words strategies in order to be able to concentrate, to think and to remember).

2.4.5 The radical learning theory of Skinner

The learning theory of Skinner closely corresponds with the view of Thorndike, Brownell and Gagné. Skinner's point of departure (Du Toit, 1986) is Thorndike's view that behaviour that provides satisfaction is promoted whereas conduct that leads to frustration is discouraged. The term **operant conditioning** requires elucidation (Skinner, 1974).

2.4.5.1 Operant conditioning

This means that the result of behaviour in turn determines behaviour. Behaviour that improves the environment is encouraged. Operant conditioning differs from mere responses that are regarded as reflexive, non-voluntary and uncontrolled. Operant conditioning is not determined by stimuli but by the effect of the subsequent behaviour. Not the stimulus but the result of the behaviour determines the reinforcement. Since reinforcement is regarded as the result of behaviour, it is implied that behaviour is determined by the results of that behaviour. The following types of reinforcement can be distinguished:

- ★ Continuous reinforcement. Behaviour is reinforced when it occurs.
- ★ Reinforcement at intervals. Behaviour is rewarded at intervals even though this may not have the desired effect.
- ★ Calculated reinforcement. A fixed number of actions are required before behaviour is rewarded.
- ★ Superstition. When learners accidentally carry out an operation that leads to the correct answer, they will probably be inclined to repeat this operation.
- ★ Positive reinforcement, negative reinforcement and punishment. Anything that encourages particular behaviour is described as positive reinforcement. Negative reinforcement means that unpleasant or negative consequences are changed or avoided by particular behaviour. However, the unpleasant conse-

quences cannot be avoided; they are experienced as punishment. Skinner believes that positive reinforcement is the most effective method to discourage undesirable behaviour.

★ Extinction. When behaviour that has previously been rewarded is no longer rewarded, the behaviour is diminished and falls into disuse. This implies that behaviour can be changed by withholding reinforcement.

2.4.6 Connectionism

As has already been mentioned, Thorndike based his theory on experiments with animals (for example dogs and cats) that were put into different problem situations. Initially the solution to the problems was found accidentally (by trial and error) but mistakes became fewer as the particular situation was repeated until the animal was able to find the solution "automatically". Thorndike called himself a connectionist (on account of the foundation on which his theory was based). He declared that learning primarily took place in terms of the forming of associations (also known as connectionism). This means that as a result of repetition animals made fewer and fewer mistakes (made better associations or connections) until they could solve a particular problem without making any errors. Thorndike regarded habit formation as an important aspect of learning mathematics; this depends on the strength of the associations that are formed. Thorndike formulated the following three laws of connection or the formation of associations:

★ The law of usage. The connection or association that has been formed, is strengthened by the repetition of the problem solution. Reinforcement and repetition are therefore conditions for effective learning.

★ The law of effect. The degree to which a connection or association is liked or disliked, strengthens or weakens the connection. Moreover, reward has a stronger effect than punishment.

★ The law of readiness. When animals are ready to do something, they enjoy carrying out the task. In such conditions animals exert themselves more and the reinforcement is more effective. However, if animals are not ready this results in dislike which in turn limits the learning effect.

These epistemological assumptions gained a considerable degree of acceptance especially because researchers realised that Pavlov's reflex theory could not explain the extensive response instability satisfactorily (Swenson, 1980). Pavlov (Hilgard & Bower, 1975) became famous on account of his experiments with salivating dogs.

2.4.6.1 Behaviourism: Synthesis

Behaviourism as epistemology is based mainly on the study of outwardly noticeable behaviour. An adequate study orientation in mathematics requires, among other things, repetition and (rapid and applicable) reinforcement of acceptable responses. Furthermore, the formation of series of applicable and correct associations is an important aspect of the learning of mathematics and functional practising of basic knowledge in mathematics form an important feature of study orientation in mathematics. An important objection to this epistemology is the fact that inner experience is not adequately taken into account as well as the fact that human behaviour is reduced too linearly to the level of the stimulus response; moreover, the importance of the learner's normative decision-making in this study is underdeveloped.

2.5 SOME COGNITIVE DEVELOPMENTAL AND LEARNING THEORIES RELATING TO THE LEARNING OF MATHEMATICS

Thus far the main focus has been on learning theories that regard mathematics as the acquisition of arithmetical skills. Apart from this learners have to acquire certain mathematical concepts in order to keep errors in mathematics to a minimum. Put differently, it is important that learners acquire insight into the concepts that are based on numerical facts and algorithms. They should learn to apply their conceptual and procedural knowledge and insight into mathematics when engaged in problem-solving strategies flexibily and correctly. The non-arithmetical aspects of mathematics also require time and attention; in other words, arithmetic skills and sound arithmetical knowledge cannot be regarded as the only criterion of achievement in mathematics. The ability to create concepts should be seen as an equally important criterion of mathematics in order to limit problems in this subject.

The problem of meaningful mathematics, as already indicated, was appreciated in Thorndike's time. Early attempts to give meaning to mathematics amounted to integrating arithmetic skills with practical, everyday true-life problems. Nevertheless mere rote learning occupied a very important position in learners' study orientation in mathematics until deep into the seventies, and even enjoyed preference in spite of researchers' honest attempts to promote meaningful conceptual learning of mathematics. Many questions, however, were never answered satisfactorily by the aforementioned learning theoreticians. These include the following: How do learners understand mathematical concepts? How do they use these concepts? How do they learn these concepts? How are these concepts best taught? What is the relationship between acquiring certain mathematical concepts and problem-solving skills? On what knowledge structures are these concepts based? Is enough attention being given to optimising learners' study orientation to enable them to master these concepts?

The launching of Sputnik[4] changed the situation dramatically. Schools were put under pressure to produce products that could meet the demands of the space age technology rapidly. A period of re-evaluation dawned and the need for meaningful mathematics was pointed out as a point of criticism. Psychologists began to propagate a conceptual rather than an arithmetical approach towards the mastering of mathematics. Significant learning would not only be based on applying the relevant arithmetical skills with regard to true-to-life problems. Significant learning would particularly depend on whether these processes could be integrated into the totality of knowledge of mathematics.

At the same time new developments in psychology took place. The field of cognitive psychology had been broached, partly encouraged by the work of gestalt psychologists such as Köhler, Koffka, Ausubel and Wertheimer.

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[4] See paragraph 2.4.5

2.5.1 The gestalt-psychological learning theory of Köhler

Köhler had the opportunity of observing a captured group of chimpanzees over a number of years. He was particularly interested in their efforts to solve everyday problems, for example the problem of obtaining food that was just beyond their reach. Whereas learning theoreticians such as Thorndike tried to explain the animals' problem-solving behaviour in terms of purposeful trial-and-error method, Köhler's (1930) observations convinced him that more global organising processes were at work during the problem-solving process. Behaviour is not always aimed at a direct objective; sometimes problem-solving strategies result in the envisaged objective being temporarily abandoned, while a round-about-method has to be followed to arrive at a solution ultimately.

Köhler aptly describes perceptive learning as the formation of a gestalt or whole consisting of separate parts. He explains that aspects of learning including the emotions, reproduction, attitudes, striving, thinking and deeds are distinguishable facets of a gestalt theory to the extent that these traits do not exist as isolated, independent elements but rather take on a particular meaning within the gestalt or are co-determined by it.

One implication of this statement is that it may not be **assumed** without further ado that learners who have mastered the separate parts of the content will arrive at a gestalt or insight into the whole. Learners have to be **guided** to perceive the **relation** between the parts and the whole and that which constitutes part of the solution. Learning achievement implies among other things that the whole of the learning operation has to be perceived, for example the integration of the visual view and the concrete presentation.

It is important to integrate not only these structures of the gestalt but also to determine the cognitive abilities that learners have. After that it should be determined how the reinforcement of these cognitive structures can be accomplished.

2.5.1.1 Köhler: Synthesis

Whereas the behaviouristic view implies that complex experiences have been constituted from the sum total of simple elements (by association united into a whole) the gestalt-psychological view among other things implies that individual elements take on a meaning and make sense only against the background and frame of reference of the whole. This whole is called the **gestalt**. These separate parts are therefore interdependent, and depend on the whole for the meaning of their integration. This epistemological view can be described as dynamic and totalitarian in contrast with the somewhat atomistic and mechanistic view of association psychology. The most significant contribution of Gestalt psychology is possibly that this epistemology acknowledged the true value of insight acquisition as a facet of an adequate study orientation in mathematics. In other words, this view implies that new insights are continuously being incorporated into the gestalt so that new problems (based on previous knowledge and insight) can be tackled with confidence. (Nowadays it is readily accepted that insight into the structure of the subject mathematics is a prerequisite for achievement in this subject.)

2.5.2 The gestalt-psychological (verbal) learning theory of Ausubel

In contrast with the behaviourists' views, the gestalt psychologists believe that learning is not essentially the same for people and animals. The inner cognitive structure's relation to the learning material is central in this respect. For Ausubel (1963) the discovery method of learning is not really important since he is convinced that cognitive development can occur effectively if the learner can conjure up a mental picture of the object. He nevertheless emphasises the importance of the learner being able to link new concepts to insights and concepts that have already been acquired (concepts and insight that already form part of the learner's frame of reference). For Ausubel the prerequisite for successful learning is individuals' existing cognitive structure and their precognition in the particular field of study. New content can only be presented to learners when they have mastered preceding content on which the new content is based. Ausubel (1963:230) expresses this as follows:

Hence new material in the sequence should never be introduced until all previous steps are thoroughly mastered.

The functioning of learners' inner cognitive structures in terms of learning content was studied in detail by Ausubel. He emphasises that true learning is only possible if the new learning content is integrated sensibly with learners' existing cognitive structures. In this way learners are able to classify and accommodate specific new concepts into more general and comprehensive concepts of a higher order. Content in mathematics is only potentially meaningful; it only acquires real meaning when learners find it meaningful (Ausubel, 1968:475):

Meaning can never be anything more than a personal phenomeno-logical product that emerges when potentially meaningful ideas are integrated within an individually unique cognitive structure.

The meaning of content is not situated in the symbols that the content represents, but in the individual himself or herself. Learners have to find meaning in their own frame of reference and then relate the new content to existing concepts. Then the new content is assimilated into the existing cognitive structure. In this regard Ausubel sees learning as two-dimensional. He distinguishes between the ways in which people learn (receptive learning and discovery or explanatory learning) and the ways in which people add new content to their frame of reference (meaningful learning but sometimes also meaningless learning). If content is merely memorised and not related to any existing knowledge structures there is a possibility of meaningless learning).

2.5.2.1 Ausubel: Synthesis

Despite the fact that Ausubel neglected the affective aspect of learning, generally seen, he made a worthwhile contribution to learning psychology. Ausubel's in-sistence that learners attach meaning to learning content underscores an important facet of an optimal study orientation in mathematics. His emphasis on new work being integrated with existing knowledge is generally accepted as an integral prerequisite for learning mathematics. The acquisition of an adequate study orientation in mathematics depends heavily on this principle. If learners have not thoroughly mastered foundation work, they cannot proceed to higher order assignments.

Bruner's learning theory with regard to intellectual functioning strongly directed the developmental efforts in the late sixties and seventies. He particularly emphasises the importance of learning the structure of mathematics.

2.5.3 The cognitive learning theory of Bruner

Bruner was particularly interested in **cognitive processes**. By this he means the following (Bruner, in Resnick & Ford, 1981:111):

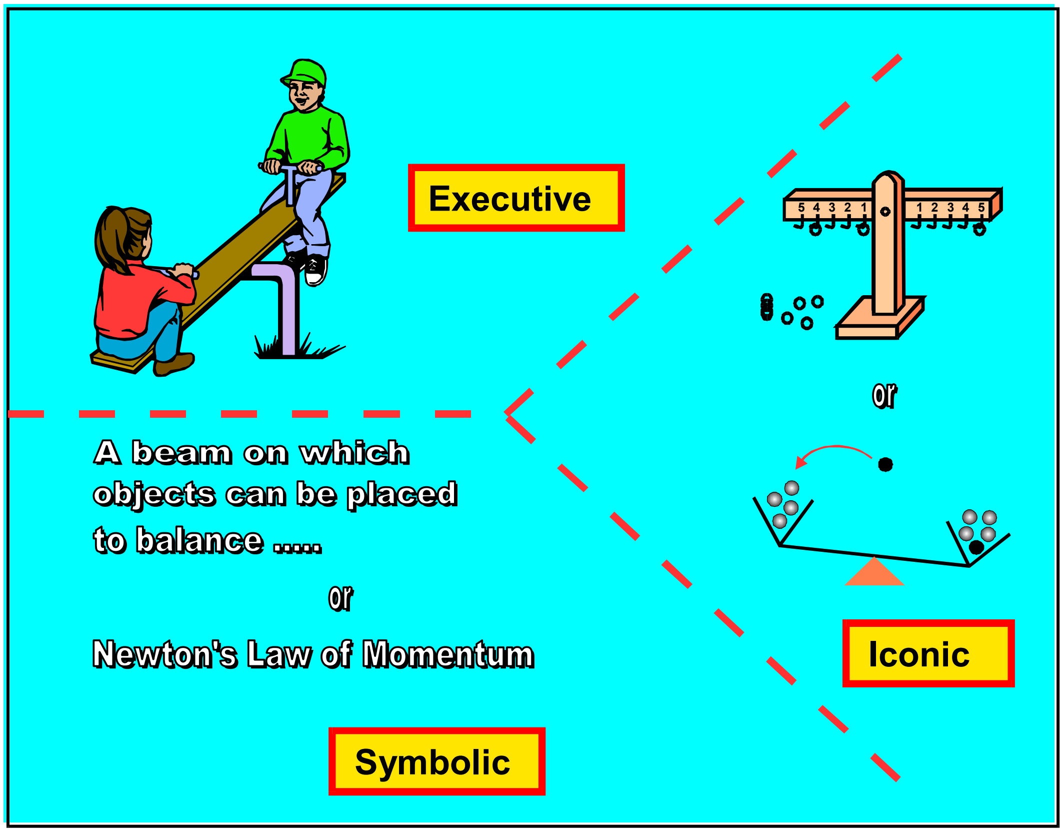
> *The means whereby organisms achieve, retain, and transform information.*

Bruner was particularly interested in the cognitive processes of learners, especially the ways in which learners mentally picture those concepts and ideas that they acquire. He comments as follows in this regard (Bruner, 1964a:2)

> *The most important thing about memory is not storage of past experience, but rather the retrieval of what is relevant in some usable form. This depends upon how past experience is coded and processed so that it may indeed be relevant and usable in the present when needed. The end product of such a system of coding and processing is what we may speak of as a representation.*

Bruner describes three modi of representation (1964b) that can be represented schematically as follows (Resnick & Ford, 1981):

FIGURE 2.1: BRUNER'S REPRESENTATION OF THE PRINCIPLES OF THE BALANCING BEAM



Adapted from Resnick & Ford (1981)

- ★ Executive or *enactive* representation. Here Bruner refers to the motor representation of something from the past (He regards this as the only way in which young children can remember things, but states that adults can also make use of this in particular circumstances.) Young children, for example, can shake their hand in imitation to indicate that they have lost a rattle. Adults' muscle systems can, for example, recall how to ride a bicycle even though a bicycle has not been ridden for many years.
- ★ Image forming or *iconic* representation. This is a step further than the merely concrete and physical towards the reality of mental representation. Adults giving directions to someone to reach a particular destination will, for example, provide a mental picture of the streets leading to that particular destination.
- ★ Symbolic representation. This method of representation is mainly made possible by a person's command of language. A symbol is a word or sign that

> represents something, but does not imitate it. For example, the number 6 does not resemble 6 objects that have been joined together! Symbols are used to refer to objects, events and ideas and the meaning of symbols is mainly shared because people **agree to share these meanings**.

These three modes develop in the order indicated, each depending on the thorough mastery of the other. Bruner links up closely with the work of Piaget. Whereas Piaget believes that one should first wait until learners are ready for specific content Bruner takes the opposite view (Bruner, 1966:44):

> *Any idea or problem or body of knowledge can be presented in a form simple enough so that any particular learner can understand it in a recognizable form.*

This has important implications in that there are ways in which even the most complex concepts can be presented so that learners of any age can master them on a level that is in line with their intellectual ability. (Bruner, for example, tried to teach quadratic equations and the characteristics of mathematical groups to learners in the lower grades.)

2.5.3.1 Bruner: Synthesis

The meaning that Bruner attaches to factors such as giving adequate attention to (concentration) and the acquisition of the necessary (practically oriented) precognition, is widely accepted. If learners have not concentrated or if they have inadequately mastered certain aspects of mathematics, or if the necessary precognition is lacking, the necessary knowledge cannot be withdrawn from the long-term memory. In such a case the short-term memory is overloaded with irrelevant or even faulty information (that is stored in the long-term memory).

In the language of Bruner an adequate study orientation in mathematics therefore implies that learners should spend enough time storing theoretical knowledge in their cognitive structures. Adequate information processing is only possible if content in mathematics relates easily to learners' development level. True understanding of work (which represents the highest level of information processing) can only occur when learners can relate and integrate incoming information with relevant pre-cognition, and this in turn leads to optimal retention of learning content in the long-term memory.

2.5.4 The field theory of Lewin

Lewin's theory can scarcely be regarded as a complete learning theory. Nevertheless he made a significant contribution with his formulation of the following insights (Lewin, 1951):

★ The individual's psychological environment and living space. Lewin believes that individuals find themselves in a psychological environment, surrounded by a non-psychological environment (moreover all people have their own unique life space). Man observes this environment in a unique way and interprets it in an equally unique manner. This life space originates as a result of man's unique physical and social environment. Obstructions in this field lead to tension systems. Action has to be taken in order to eliminate this tension. If learners' study orientation in mathematics is, for example, characterised by a tendency to avoid carrying out tasks, they should be punished (or threatened with punishment). In the process they are encouraged to carry out the tasks.

★ Need, tension and valency. Need causes tension or release of energy to increase in an individual's life space. Such a need can be physiological (hunger, thirst) but it can also be a higher order need (like the intention to complete a task, to study or to follow an occupation). Objects in people's field either attract or repel them. Objects attracting one have a negative valence and cause a power that forces one into movement, and vice versa (Aronstam, 1986). In terms of a study orientation in mathematics this implies that learners who have a negative valence towards the subject (irrespective of the reasons: poor teaching, poor motivation, inadequate study attitudes or habits, poor milieu or an inability to apply information properly) will lose interest and vice versa.

An author who worked closely with Bruner at the University of Harvard, is Dienes.

2.5.5 Dienes' cognitive theory of multiple embodiment

Dienes is of opinion that concepts should be presented to learners in so-called multiple embodiments. This implies that learners should be confronted with different kinds of material – and all the material should embody or represent the specific concept (Dienes, 1964). For Dienes mathematics involves the study of structures, the classification of structures, the sorting out of relationships between the structures and the categorising of relationships between the structures. He believes that learners can understand structures only when these concepts have been presented to them concretely and physically. Dienes defines the term "concept" as a mathematical structure. He differentiates between three different kinds of concept:

- ★ Pure mathematical concepts involve numerical classifications and relationships between numbers. Thus the concept *even number* is represented by 4, twelve and vi in spite of the fact that the representations differ.
- ★ Notational concepts are those characteristics of numbers that have a direct bearing on the way in which numbers are represented. Consequently 67 means six tens and seven ones.
- ★ Applied concepts indicate the application of pure and notational concepts to problem-solving in mathematics and related fields. When a learner, for example, makes the following mistakes:

  $3x + 9 = 15$

  $\Rightarrow$ $x + 9 = 15 - 3$,

  or: $p^3.p^4 = p^{12}$,

  it means that he or she is applying pure and notational concepts without understanding them properly. To avoid such mistakes and to learn mathematics adequately, Dienes believes that learners must first be able to do the following:

- ★ Analyse mathematical structures and their logical relationships;
- ★ abstract and classify common characteristics from different structures;

- ★ generalize class structures previously learnt by enlarging them to bigger classes; and
- ★ use previously learnt abstractions to construct more complex higher-order abstractions.

Dienes (Dienes & Golding, 1971) believes, as does Piaget, that mathematical concepts should be learnt in progressive stages of learners' development, namely:

- ★ Free play (unstructured and indirect);
- ★ *games* (more structured games). After learners have been introduced to representations in an unstructured manner, they will start to discover patterns and regularities themselves;
- ★ the search for common characteristics;
- ★ representation. After learners have observed the common elements of each example, they should obtain a single representation of the concept; a representation that includes all the commonalities. These commonalities can be diagrammatic, verbal or an inclusive example;
- ★ symbolisation; and
- ★ formalisation. After learners have acquired a concept and the related mathematical structures, they should be able to systematise the attributes of the concept more formally and to consider the consequences.

The similarity between the aforementioned stages and Piaget's stages of intellectual development is striking. The former is indeed based to a great extent on the latter.

Guilford's learning theory will subsequently be given some attention.

2.5.6 Guilford's model of the cognitive structure of the intellect

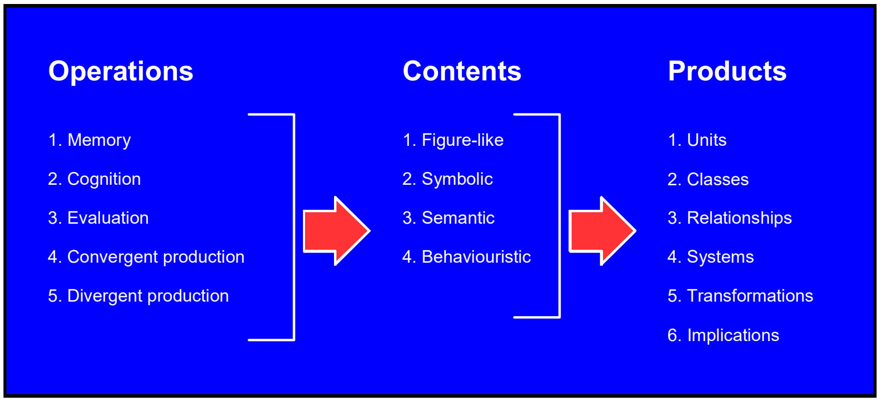
Whereas Piaget and others focused on the stages of intellectual development, Guilford developed a three-dimensional model containing 120 different types of intellectual aptitudes (Guilford, 1959). These 120 intellectual aptitudes include the majority of those variables that can be specified and quantified. By using this model Guilford and his colleagues attempted to analyse and structure the concept "general intelligence" according to a variety of specific mental aptitudes. The importance of their findings is that they verified the following fact: **Even particularly intelligent learners experience problems with certain aspects of mathematics whereas less intelligent learners may do surprisingly well in the execution of certain mental tasks.** In other words, it is important that psychologists realise that individual learners have a variety of strong and weak intellectual points. Tests have been developed to identify many of these factors in order to select suitable assignments by means of which learners can be helped to overcome achievement problems.

Guilford's model defines learning and intellectual development in terms of three variables:

- ★ Operations that indicate the gathering of mental processes used in the learning process (memory, cognition, evaluation, convergent and divergent production);
- ★ learning content that calls to mind figure content (triangles, parabolas) symbolic content (+, =), semantic content (words and ideas that recall a specific representation in the memory when they are used: tree, dog, cat) and behaviouristic content (the manifestations of persons' stimuli and responses). Figures, symbols, the spoken and written word (semantics) and behaviour combine to constitute all the distinguishable information in a person's environment; and
- ★ learning products that include the following: units (single symbol, figure, word), classes (collection units), relationships (connections between units and classes), systems (combinations of all three the above-mentioned), transformations (the process during which information is modified, reinterpreted and restructured) and implications (predictions of the consequences of interactions between all the above-mentioned).

On account of a lack of space this model cannot be discussed in general and its schematic representation will have to suffice.

FIGURE 2.2: GUILFORD'S ANALYSIS OF INTELLECTUAL ABILITY



Adapted from Bell (1978)

One repetitive theme in the learning theories discussed thus far is the view that mathematics learners' study orientation, ideally seen, should concentrate on obtaining insights into the **concepts** of mathematics – by either understanding the structure of the content or the interrelationship between the elements of a problem. Research on small children's arithmetical skills already brings the importance of conceptual insight into focus. Even the simplest addition and subtraction algorithms are rooted in conceptual insight into basic mathematical concepts. Researchers clearly show that even learners' errors are to a greater or lesser extent indicative of their insight into basic principles (Bell, 1978).

2.5.7 The cognitive learning theory of Vygotsky

Whereas developmental learning theoreticians are of the opinion that learning is brought about by cognitive, moral and social development (which means: experience learning, learning through concrete experience and through social interaction), Vygotsky (1962: 1978) believes that social growth is caused mainly by **social interaction**. According to this view the **relationship** between the role of the affect and the intellect is emphasized when simple but also complicated tasks are tackled. This view (Vygotsky, 1978) indicates that among other things problem-solving is caused by the **integration** of personal traits such as motivation, learners' ambitions, their cognitive strategies and the extent to which they implement meta-cognitive processes during problem-solving. Vygotsky (1962:8) states the following in this regard:

> *Their separation ... is a major weakness of traditional psychology since it makes the thought process appear as an autonomous flow of "thoughts thinking themselves", segregated from the fullness of life, from the personal needs and interests, the inclinations and impulses of the thinker.*

Piaget (1964; 1976) is of the opinion that insight into the basic structure of mathematics, as well as the ability to execute mathematical operations, is mastered by learners when they reconstruct their interactions within their physical, social and cultural environments. By this he means that mathematical development is the result of learners' self-regulating and autonomous reactions with their environments. Vygotsky's views (1981), however, differ from those of Piaget. He cannot accept that learning is subordinate to development. He also rejects the view that learning should be considered as the process of extending innate structures. The role of social interaction in Vygotsky's approach to learning should not be underestimated. He particularly emphasises the essential influence of learning on development. Vygotsky (in Wertsch & Toma, 1994:162) expresses this as follows:

> *Any function in the child's development appears twice, or on two planes ... the social plane, and ... the psychological plane. First it appears between people as an interpsychological category. This is equally true with regard to voluntary attention, logical memory, the formation of concepts, and the development of volition ... it goes without saying that internalization transforms the process itself and changes its structures and functions. Social relations or relations among people underlie all higher functions and their relationships.*

2.5.7.1 Vygotsky: Synthesis

Vygotsky is of the opinion that learning directs development rather than follows it. His notion concerning the zone of **proximal development** is widely accepted and respected. Kilpatrick (1992:9) states this as follows:

> *The difference in level of difficulty between problems that one can solve alone and those one could solve with the help of others is being used by researchers interested in the social mediation of cognitive change.*

Vygotsky's own definition of the zone of proximal development is as follows:

> *It is the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers (Vygotsky, 1978:86).*

Vygotsky's definition refers to those cognitive functions that are in a process of maturation, but have not yet matured at a particular stage or are still in an embryonic stage. Learners' attained development level indicates to Vygotsky cognitive development on a retrospective level whereas learners' zones of proximal development indicate their attainable or potentially attainable level. Vygotsky furthermore emphasizes the fact that cultural meanings should be mixed with personal meanings through a good education.

2.6 THE INFORMATION PROCESSING MODEL OF INSIGHT ACQUISITION

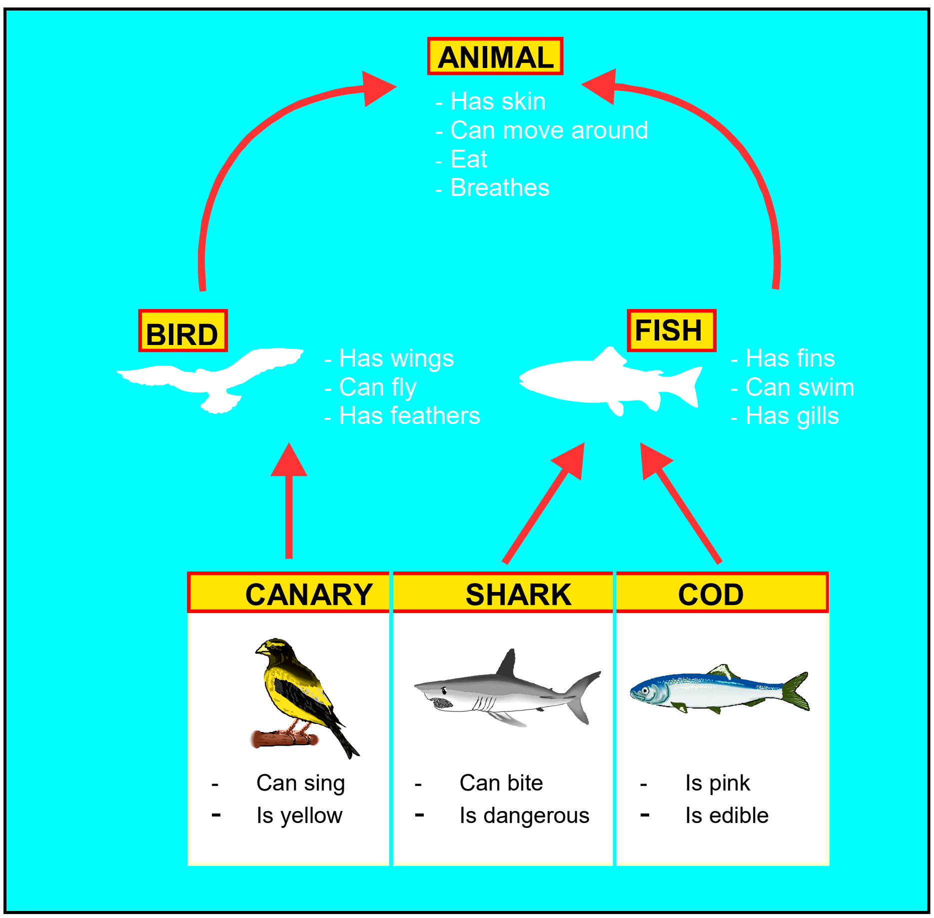
There are several information processing approaches to the learning process. McShane (1991:8) points out that *"communications theory ... the theory of computation ... artificial intelligence... and linguistics,"* among other things have led to the development of this theory. Case (1985), however, maintains that all the information processing models among others take the view that information transforming processes (the storage, processing and the potential for the recall of information) occurs in the field of human gnosis and that learners have a limited ability to process information.

A distinction is also made between what is called the **working memory** (where coded information is temporarily stored so that it can be immediately recalled and used) and the **long-term** or **semantic memory** (everything that individuals know, all knowledge that they have) in which everything permanent is stored. Cermak (1983:599) states the following:

> *The learning disabled students' slower speed of processing (is related) to the semantic content of the material, therefore leading to a diminished ability to store and retrieve information.*

Figure 2.3 illustrates the way in which knowledge, according to the theoreticians, is stored in the human brain (Resnick & Ford, 1981):

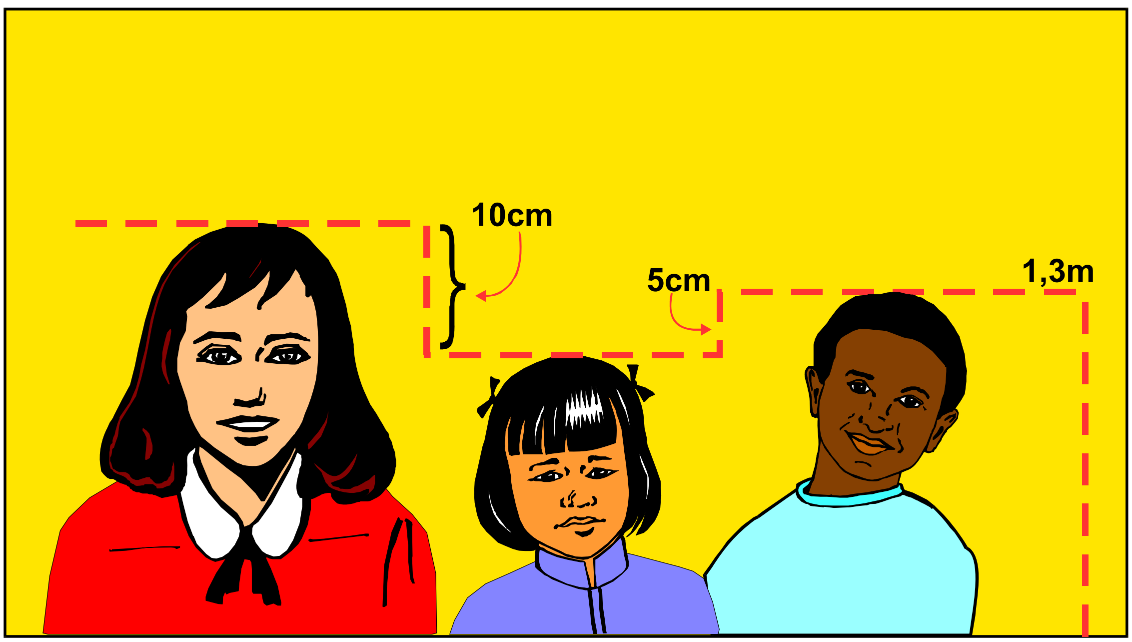
FIGURE 2.3: INFORMATION PROCESSING THEORETICIANS' REPRESENTATION OF THE WAY IN WHICH KNOWLEDGE CAN BE STORED IN THE HUMAN BRAIN



Adapted from Resnick & Ford (1981)

According to most general information processing theories all human knowledge is stored in a structured and organised way (a view which is closely related to the gestalt psychology's learning theories) in the form of specific knowledge structures. As in the case of Piaget (1952) and Bruner these theoreticians believe that the human mind is actively engaged and does not merely take in external associations. Goodstein (1981) believes that the presence of visual aids, *cue words*, vocabulary and semantic complexity are factors that play an important role in learners' ability to solve problems. The following figure illustrates the principle of "visualisation" (Resnick & Ford, 1981):

FIGURE 2.4: RESNICK AND FORD'S REPRESENTATION OF THE PRINCIPLE OF "VISUALISATION"



Adapted from Resnick & Ford (1981)

Ideas and concepts exist in a fixed relationship to one another, and learning comprises the construction of both new connections and relationships as well as the reception of new items of information. This means, among other things, that the primary aim of learning mathematics is the acquisition of **thoroughly structured**

knowledge of mathematics. There are three criteria for the thorough structuring of mathematical knowledge, namely:

- ★ Correspondence, defined by Resnick & Ford (1981:235) as "the match of one's mental picture with correct mathematical concepts";
- ★ integration, defined as "the degree of interrelatedness of concepts within a particular domain of mathematics" (Resnick & Ford, 1981:235); and
- ★ connectedness, indicating the extent to which knowledge in one particular knowledge domain of mathematics can be connected with knowledge in another domain.

The next question that has to be answered is: In what way can information processing knowledge and strategies be used to promote **problem-solving** and the acquisition of **problem-solving skills**?

2.6.1 Problem-solving by means of information processing

Bell (1978:119) puts the question of problem-solving within the context of information processing into perspective as follows:

> *Problem-solving is a higher order and more complex type of learning than rule-learning, and rule acquisition is prerequisite to problem-solving. Problem-solving involves selecting and chaining sets of rules in a manner unique to the learner which results in the establishment of a higher order set of rules which was previously unknown to the learner. Words like discovery and creativity are often associated with problem-solving. In rule-learning, the rule to be learned is known in a precise form by the teacher who structures activities for the student so that he or she will learn the rule in the form in which the teacher knows it and will apply it in the correct manner at the proper time. The rule exists outside the learner who attempts to internalize the existing rule. In problem-solving the learner attempts to select and use previously learned rules to formulate a solution to a novel (at least novel for the learner) problem.*

The information processing theoreticians, however, go somewhat further:

> *Stored subject-matter knowledge cannot solve problems. There must also be a mechanism to direct the mental search through the networks to retrieve information. And there has to be a mechanism for actively generating and testing new relations among concepts and structures when the needed information is not stored in exactly the form that seems to be required* (Resnick & Ford, 1981:236).

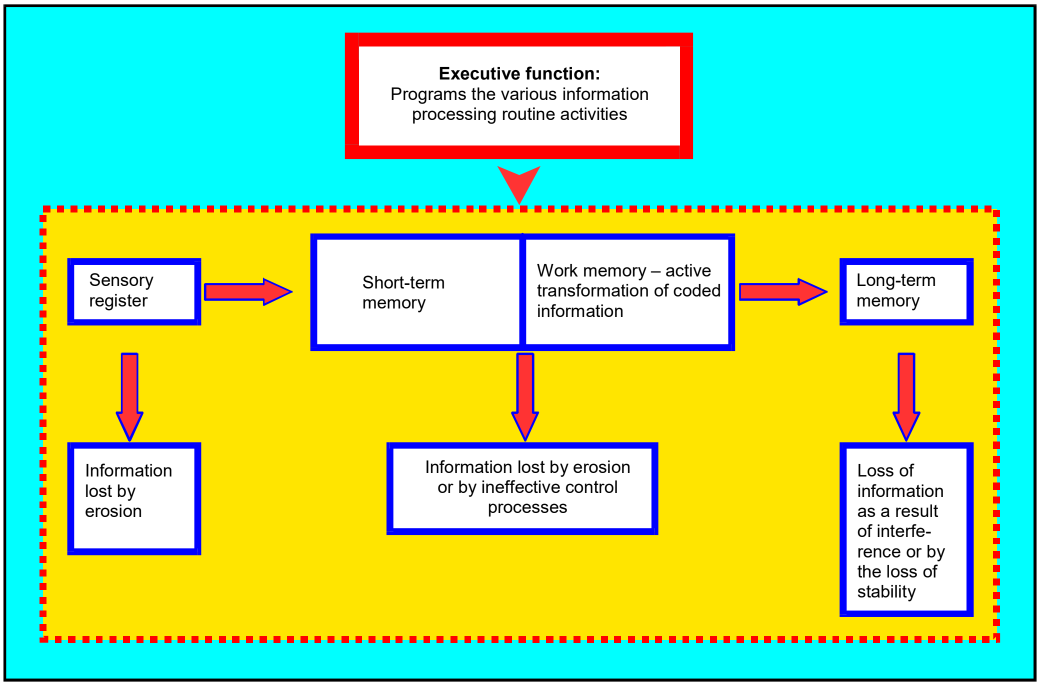
In other words, knowledge and information that have been internalised cannot solve problems. There is a need for a mechanism to direct the brain's search (through the networks of knowledge), as well as a need for a mechanism to create new relationships between concepts and structures when the required information is not available in the required form. According to information processing theories, apart from the knowledge structures mentioned, the human brain has a wide variety of problem-solving strategies available to it to help to interpret problems, to locate stored knowledge and procedures and to create new connections or relationships among the separately stored items. These strategies organise the thinking process and call on the various knowledge components to produce a plan to solve the problem.

Swanson (1987) refers to the potential use of the computer in serving the optimisation of learners' study orientation. He states that it has become common practice to use the concept of the personal computer as a model for explaining the way in which the human brain processes sensory information. In this regard Swanson (1987), for example, explains the following three components of information processing according to the working of the computer:

★ The structural component (like the computer's hardware) which defines the parameters within which information can be stored at a particular stage;
★ the controlling component (for example the computer's software) that describes the operations at various stages; and
★ the executive process that manages and monitors the learner's learning strategies. Information, as it is fed in, is simultaneously processed or transformed

as it flows through the various components of the system. Swanson (1987:3) represents the process as follows:

FIGURE 2.5: SWANSON'S REPRESENTATION OF THE WAY IN WHICH SUBJECT MATTER IS SYSTEMATIZED



Adapted from Swanson (1987)

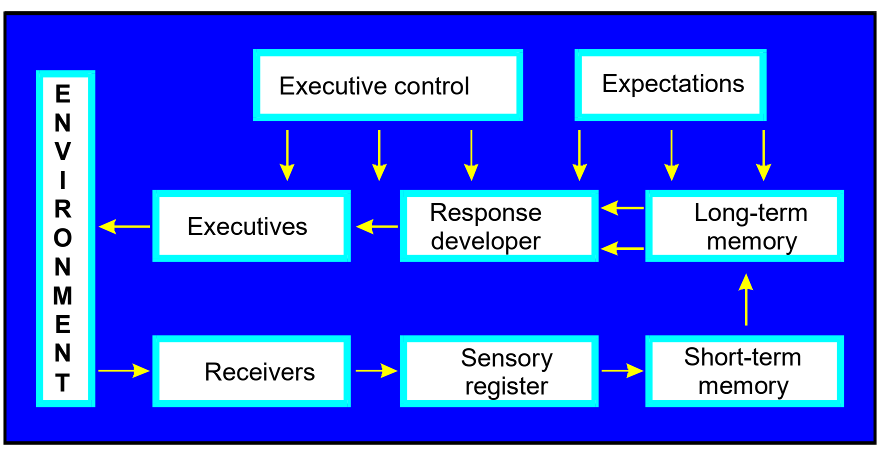
The aforementioned insights are summarised by Gagné (1983:8-10) as follows:

> *Cognitive learning theory proposes the following things about human learning:*
>
> 1. *The fundamental unit that is learned and stored in human memory is a semantic unit ... it is inherently meaningful.*
> 2. *... the physical stimulation that is delivered to the senses is transformed into nervous impulses, which are then best viewed as intricate masses of information ... [where] this dynamic complex undergoes several kinds of transformation ... sequential ... simultaneous or parallel.*

3. *The kinds of transformation that this information undergoes are called processes, and the main concern of modern cognitive theories is with what these processes are and how they work ... .*
4. *... a prominent part is played by ... "control processes", or "executive control processes" ... [which] are controlled by the learner ... [and are] the means he or she has of influencing the other processes of learning ... .*
5. *The processing that turns external stimulation into learned information may be said to be influenced by inputs from three sources:*
   a. *First, learning is affected by whatever organization or patterning is imposed on the external stimulus ... .*
   B. *Second, learning is influenced by the executive control processes available to, and used by, the learner ... .*
   c. *Third, learning is influenced by the contents of memory – in other words, by what has previously been learned.*

The abovementioned is represented by the following figure:

FIGURE 2.6: GAGNÉ'S REPRESENTATION OF THE WAY IN WHICH SUBJECT MATTER IS SYSTEMATISED



Adapted from Gagné (1983)

To summarise: Contemporary information processing learning theoreticians are of the opinion that the activities “learning” and “remembering” are caused by internal processes (that are influenced by the external organisation of stimuli). Executive management or control of these processes is brought about by learners as well as their memory content.

2.6.2 Information processing by means of metalearning

Lippert (1987:275) puts the aforementioned concepts into perspective as follows:

> *As scientific knowledge proliferates, information selection becomes more of a critical issue ... education still seems to presuppose an image of the student as a retainer of, rather than a processor of experience and information. We require students to memorize unintegrated bits of information rather than helping them refine and structure their knowledge by useful employment of it. We are more concerned with what answers are given than with how they are produced. Students therefore learn to solve problems by plugging given values into variables, and never adopt the conceptualization underlying the problem. As a result the principles, constraints and contextual issues inherent in the content are never really grasped - and thus forgotten within a short time. This shortcircuits not only retention, but also transfer.*

She defines **metacognition** as knowledge about knowledge and person functioning (Lippert, 1987) and points out that inadequate implementing of this strategic knowledge strategy seriously impedes problem-solving. Furthermore she points out that all learning is purposeful and that learners should consequently direct their activities and knowledge towards these objectives in order to keep learning problems to a minimum. Flavell and Wellman (1977) state that there are four classes of metacognitive knowledge:

★ Tasks, since knowledge of these tasks frequently affects achievement with regard to them;

- the self, including knowledge of the learners' idiosyncratic skills, strong and weak points;
- strategies (or knowledge of the differential value of potential problem-solving strategies); and
- interactions (knowledge of the mutual interaction among the aforementioned knowledge types influences cognitive achievement).

In conclusion Lippert (1987) points out the importance of learners' building up a **complete knowledge base** in mathematics. Such a knowledge base includes among other things the following factors that should be part of an adequate study orientation in mathematics:

- Critical thinking by means of the active implementation of analysis, synthesis and evaluation;
- that learners reflect on their own thinking (metalearning);
- that learners integrate the various knowledge domains;
- the stimulation of "conditional" thinking by not only thinking about the questions **what, when** and **how**, but also reflecting carefully on the **where**, **when** and **why**;
- that learners' decision-making ability should be stimulated in various contexts by making use of probabilities and a heuristic tendency, supported by qualitative and quantitative evidence;
- that learners discover relationships, patterns and correlations;
- that learners on the one hand try to solve problems **themselves** and on the other hand try to devise new problems (by making use of analogies); and
- that learners rather reason qualitatively instead of relying on so-called *number crunching*.

2.7 THE CONSTRUCTIVIST APPROACH TO LEARNING MATHEMATICS

2.7.1 The constructivist or developmental-procedural learning theory of Piaget

Piaget states the following in connection with children's original learning orientation (Piaget, 1980:26):

> *Within the space of a few years (the child) spontaneously reconstructs operations and basic structures of a logico-mathematical nature, without which he would be understanding nothing of what he will be taught in school ... He reinvents for himself, around his seventh year, the concepts of reversibility, transivity, recursion, reciprocity of relations, class inclusion, conservation of numerical sets, measurements, organization of spatial references.*

The following concepts occur repeatedly in Piaget's theories and they will be elucidated briefly:

★ The formation of cognitive structures. Piaget believes that cognitive structures consist of those activities and patterns of thinking by means of which learners systematize and plan their activities or learning actions.

★ Content. Learners obtain content through experience and action; through seeing, feeling, hearing, smelling and touching at a particular moment in their lives.

★ Schema. Schemata are clear units of physical or mental actions that are frequently repeated. Learners have schemata that are always developing, changing and becoming more complex. These schemata constitute the building blocks of learners' cognitive structures.

★ Functional non-variables or invariables. The way in which learners function during their lives remains constant. This occurs according to organisation (learners' innate tendency to co-ordinate structures and abilities) and adjustment (the process by which learners learn to handle their environment).

- ★ Equilibration and equilibrium. By means of assimilation certain concepts and experiences are integrated with existing concepts and experiences; certain changes in learners' existing cognitive structures are brought about through accommodation. Equilibrium indicates that assimilation and accommodation are in a state of balance, whereas equilibration indicates the process in which learners are continuously moving from one state of equilibrium to another more advanced or complex state.

- ★ Decentring. According to Piaget, in Inhelder and Chipman (1976) the gradually developing state of equilibrium between assimilation and accommodation is the result of successive decentrings. This indicates that learners are able to concentrate their attention, at a given time, on a particular matter (or aspect thereof).

- ★ Operations. Piaget regards operations as thinking processes that are carried out according to certain rules that are reversible. Operations have four characteristics. They can be carried out mentally or physically; they are actions with an inversion; they represent the retention of the invariable, although transformation is possible (7 may be 4 + 3 or 5 +2); and no operation is isolated since it always constitutes part of a larger structure or whole.

Piaget's view on learners' cognitive development stages is just as important as his views on the learning process.

2.7.1.1 Piaget's views on learners' cognitive developmental stages

Piaget classifies these stages of cognitive development as follows:

- ★ 0-2 years: the sensory-motor stage. At this stage children learn to control and co-ordinate their sensory-motor activities. Thus they learn to exercise control over their environment and to distinguish between their bodies and the environment. Children also acquire classification and permanence which can be regarded as the forerunner of conservation (Liebeck, 1984).

- 2-7 years: the pre-operational stage. From about 2-4 years (when language is rapidly developing) children are in the stage of pre-conceptual or symbolic thinking and from 4-7 years in the intuitive stage when they make more and more use of symbols. Since learners are so egocentric, they are not capable of decentring. The transition from pre-logical to logical thinking, or from the pre-operational to the operational stage, is tested in a simple way by determining whether the learners have conservation or invariance at their disposal. This indicates that certain aspects of a matter always remain constant whereas others change. Thus two sets of five figures are shown simultaneously to children and they are asked whether the two sets contain the same number of objects. Then the figures in one of the sets are spread out and the question repeated. Children in the pre-operational stage do not realise that the two sets still contain the same number of objects (Copeland, 1982; Piaget, 1977).

- 6/7-12 years: the concrete-operational stage. Piaget calls the period between learners' seventh year and their eleventh to twelfth year their concrete-operational developmental stage (Louw, Schoeman, Van Ede & Wait, 1996). Egocentrism diminishes and learners get prepared for understanding reversibility, classification and the systematisation of objects. At this stage it is important to provide children with enough concrete mathematical material as a basis for the development of mathematical ideas and concepts. While they are working with concrete objects, they gradually acquire the ability to discover underlying mathematical ideas or structures. Piaget (1973) regards the lack of concrete materials in learning environments as the basis for many learners' failure in mathematics. (Although he regards the age of 7 as the age at which children master *conservation of number*, he warns that this indication of the time, like all his other indications of a specific time, are relative.) (Lavatelli, 1974).

- 11/12 years up to adulthood: the formal operational stage. Learners in this stage can function on an abstract level of thinking, where they do not need to take refuge in the concrete, real world. Mathematics can now be learnt more formally. From an arbitrarily chosen point of departure learners can proceed by means of logical deductive steps to the abstract or symbolic level. This level of thinking is not peculiar to the primary school learner who still needs first-hand experiences with concrete material.

Researchers such as Chiappetta, McKinnon and Renner (in Gadanidis, 1994), however, point out that Piaget was overoptimistic in his estimate of 11/12 as the starting point of the formal operational stage and that their research shows that 50% of all learners, 16 years and older, still function on the concrete operational level. Copeland (1984) refers among other things to the study of Herron (1975) that shows that 50% of all entrants to the USA universities function on the concrete operational level whereas less than 25% operate on the formal operational level.

In conclusion: Piaget did not regard knowledge as a previously determined, unfolding inner process. According to him knowledge and intelligence do not originate in either the learner or in the environment but in the interaction between the two.

2.7.1.2 The learning and cognitive theory of Piaget: Synthesis

Certain concluding remarks will subsequently be made on Piaget's views on the theory of learning mathematics.

- ★ The relationship between the view of Piaget and gestalt psychology. Piaget's work reveals certain similarities with the work of the gestalt psychologists. He, however, describes the difference between the two approaches as follows: whereas the gestalt psychologist works with a structured system, he works with a structuring system.

- ★ Individualisation or socialisation? Piaget does not support individualised teaching. He aptly points out learners' inherent egocentric tendency and adds that concrete operational children are already able to assimilate various points of view and thereby bring their assessment of concepts more into line with reality. Social interaction has an important purpose in his opinion. A clash of views makes learners especially aware of other points of view that they must be reconciled with, and in this way learners are assisted to relinquish a state of egocentricity.

★ Aptitude for mathematics. Piaget is fairly outspoken concerning the question whether certain learners have an aptitude for the subject, are "good" at the subject and others are not (Piaget, 1971:44):

> *(Mathematics involves) a technical language comprising a very particular form of symbolism ... So the so-called aptitude for mathematics may very well be a function of the student's comprehension of that language itself, as opposed to that of the [mathematical] structures it describes ... Moreover, since everything is connected in an entirely deductive discipline [such as mathematics], failure or lack of comprehension of any single link in the chain of reasoning causes the student to be unable to understand what follows.*

With reference to the above he distances himself from the statement that certain learners are mathematically minded and others are not. Copeland's views (1982:16) link up with those of Piaget:

> *The central problem in mathematics teaching then becomes one of relating the particular logic sequence being taught to the psychological or intellectual structures available to the child.*

2.7.2 Modern constructivism

The **constructivist** point of view, as initiated by learning psychologists such as Piaget and Skemp (1982), can briefly be summarised as follows: knowledge cannot be transferred from one person to another by means of a computer. Learners participate actively in the learning process (Van Glasersfeld, 1991). **Assimilation** refers to the process when new but still recognisable ideas are encountered and these can be directly linked to existing knowledge structures. In this way existing schemata are extended and broadened. **Accommodation** refers to the process when new ideas differ from existing knowledge structures. In such a case there may be knowledge structures that are relevant, but not completely adequate. Then a need is created for existing knowledge structures to be reconstructed and reorganised. An existing knowledge structure is **not** removed, but continues to exist as a component of new structures. Consequently when a new idea is understood, it means that it has been incorporated into a relevant existing schema (Gadanides, 1994; Olivier, 1989). Consequently knowledge schemata or structures constantly change. Researchers such as Skemp (1971), however, emphasize that the creation of new concepts depends on learners' first **consolidating** concepts that have been mastered earlier on. **Verbalising** concepts that have to be consolidated was of great importance to Skemp (1971).

The world-wide problem of inadequate achievement in mathematics can, among other things, be ascribed to the overemphasis of the absolute objective and structural nature of mathematics. This point of view gave rise to various product-oriented learning approaches (Ernst, 1989).

In contrast to this the supporters of the constructivist approach believe that operationalizing the points of departure in the constructivist learning approach makes mathematics more accessible to and understandable for learners. Marsh (1993:145) refers specifically to the situation in South Africa and his views link up with those of Ernst:

> *Perhaps social constructivism with its group-based, process oriented, problem solving and investigatory approach can serve to make the subject more accessible, more user-friendly and palatable to the average pupil.*

Von Glasersfeld (1991:31) defines constructivism as follows:

> *Constructivism ... asserts two main principles whose application has far-reaching consequences for the study of cognitive development and learning as well as for the practise of teaching, psychotherapy, and interpersonal management in general. The two principles are: (a)* ***knowledge is not passively received but actively built up*** *by the cognizing subject; (b) the function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality.*

Von Glasersfeld (1991) furthermore points out the radical difference between a constructivist approach in which learning strategies are aimed at understanding problems and subject content *(teaching)* and a behaviouristic approach in which the main aim is the repetition and drill of certain fixed patterns or methods *(training)*. Volmink (1993:33-34) elucidates this as follows:

> *Loosely defined, constructivism is a theory about how we construct our knowledge as active participants rather than receive knowledge as passive recipients. One of the perspectives that a constructivist paradigm provides, is a strong commitment to encourage students to realise that they live in a world constituted by their own experience and that they therefore should take charge of their own learning experiences.*

Jaworski (1988) states that **radical constructivism** has two basic learning theoretical points of departure. In the first place supporters of this learning theory accept that knowledge should be actively constructed by learners. Learners cannot “receive” knowledge in a passive way from a teacher or from the environment. In the second place these learning theoreticians believe that the acquisition of knowledge or *coming to know* (Jaworski, 1988) is an adaptation process during which learners reorganise their experiential world. Learners do not discover an independent world that exists outside their mental world each time. **Social constructivism** emphasizes the significance of communication and the construction of “divided meanings”. Linguistic communication is of cardinal importance here. Learners are encouraged to talk to or communicate with their friends, to listen to them and negotiate meaning with them without being afraid that they will be regarded as “stupid” or wrong (Brodie, 1994).

2.7.2.1 Modern constructivism: Synthesis

The constructivist approach to learning puts the emphasis on learners and their activities. With the help of teaching media such as the *activity sheet* or **facilitating page** (Gadanidis, 1994) as well as the *mind map* or thinking card learners are given the opportunity to speculate, discover and justify. Learners listen to their friends and are given the opportunity to internalise group processes (for example to communicate the importance of knowledge and ideas clearly and understandably, to make room for different perspectives, to justify one's own perspectives and to evaluate the quality of one's own knowledge). The focus in the learning situation moves to discovery learning, metalearning and problem solving. Learning content should now link up with learners' frame of reference and motivate learners to **want** to do mathematics. Learning content should activate learners towards an adequate study orientation in mathematics.

Volmink (1993:34) points out that modern constructivism regards small well-equipped classrooms as a given, but that:

> *In South Africa ... large classes will be the norm rather than the exception. We need to ask therefore, what adjustments need to be made to the constructivist approaches to teaching and learning, in order to make it more effective and appropriate within our context.*

Psychologists throughout the world find that some learners have problems with their involvement and participation in the learning process as well as in understanding it. Problem-solving in mathematics is indeed one of the aspects of the subject that learners experience the most problems with. De Corte (1995:2) justifies this view as follows:

> *There is at present substantial research evidence showing that many students in today's schools do not, or at least not sufficiently master the knowledge and skills underlying skilled learning and problem solving.*

Learners in general reveal a certain shortsightedness when they are confronted with word problems in mathematics. Most learners experience problems with the practical interpretation of solutions to mathematical problems. De Corte (1995:3) refers to a study in the USA in which learners had to try to solve the following problem:

A defence force bus is capable of transporting 36 soldiers. If 1128 soldiers have to be transported to a training field, how many buses will the defence force need? (Translation)

The answer to the division sum is 31 remainder12. Seventy per cent of the learners could solve the problem but only 23% could interpret the answer correctly and come to the conclusion that 32 buses were required. This finding is similar to local experience in this regard and it once again emphasizes the importance of reality orientation as a facet of learners' study orientation in mathematics. The learning of mathematics should especially concentrate on preparing learners to hold their own as adults. Learners who try to solve their problems themselves and who create their own methods generally do not experience problems in interpreting their results. However, learners who slavishly follow specific methods more frequently have problems in interpreting their results.

Maree (1995a) in conclusion emphasises that learners cannot learn completely on their own but they can nevertheless direct the learning process to a certain extent. He (Maree, 1995b:68) defines the role of constructivism within the complex learning process as follows:

The problem-solving approach, problem-centred learning, (social) constructivism, learner involvement during which learners discover their own algorithms or standard strategies to solve problems, "construct" or form, are most acceptable, [but then] in combination with other approaches.

(Translation)

2.8 PROBLEM-CENTRING

The implementation of the problem-centred approach to the learning of mathematics in South Africa has led to different reactions. It is, however, clear that there are no final answers to the questions regarding the success of the new approach. Maree (1995b:66) furthermore points out the following:

> *In spite of the absence of substantial data to support these statements, the quotations illustrate the ignorance concerning precisely what the new approach to teaching and learning mathematics involves, what the role of the parent should (not) be, confusion and frustration and fear that the lessons history has taught us, have not been taken to heart.* (Translation)

The implementation of the problem-centred approach was not without growing pains. Maree states that although this approach has already been successfully applied in practice, there is no formal proof that the positive objectives of this approach have been supported or refuted. Maree (1995b:70) emphasizes the divergent views on the problem-centred approach as follows:

> *Whereas many mathematicians swear by this approach, there are also many lecturers in mathematics who have serious objections to this approach.*
>
> (Translation)

This approach is subsequently discussed to put this matter into perspective.

2.8.1 What is the problem-centred approach to the learning of mathematics?

The problem-centred approach is a teaching approach that especially relates to the learning theoretical aspect of the (social) constructivist points of departure. It does not necessarily imply or mean that "new mathematics" is now learnt by the learners. The change can be found particularly in **the approach to learning mathematics**. These changes in approach did not occur overnight but are based on years of research. The problems that learners experience with mathematics stimulated this research worldwide. After a study of problem-centred learning Norman and Schmidt (1992:557) came to the following conclusions:

> *Learning in a problem-based format may foster, over periods up to several years, increased retention of knowledge;*
> *some preliminary evidence suggests that problem-based learning curricula may enhance both transfer of concepts to new problems and integration of basic science concepts into (clinical) problems;*
> *problem-based learning enhances intrinsic interest in the subject matter; and*
> *problem-based learning appears to enhance self-directed learning skills, and this enhancement may be maintained.*

In countries such as the USA, the United Kingdom, the Netherlands and Brazil continuous extensive research has been done concerning the most suitable approach to learning mathematics. The research unit for mathematics teaching at the University of Stellenbosch, in co-operation with the teachers and officials of the Cape Education Department, introduced this new problem-centred approach in a few primary schools (Pythagoras, 1995).

After further research the new syllabus that endorses the problem-centred approach was implemented in Grade 4 in 1992. One of the most important changes affected learners' number concept. Learners were then permitted to work with larger numbers than had been traditionally prescribed. Primary school learners with an above-average number concept were, for example, allowed to experiment with bigger numbers. **Learning methods** and a **study orientation** based on traditional concepts and prescriptions like adding with units and tens columns also disappeared from the syllabus. Learners were encouraged to look for solutions themselves as part of a new problem-directed/solving approach.

> *The new approach emphasises that learners in a mathematics study orientation should concentrate on mastering the limited, technical language of mathematics adequately. An inadequate mastery of this is potentially*

> *very destructive in terms of optimising learners' problem-solving ability in mathematics* (Maree, 1995c). (Translation)

Maree (1995b:69) points out the following advantages of the problem-centred approach:

> *The child (thus) reaches a solution in a significant perceptive manner. The meaning of operations and problem-solving strategies is discovered by true problem-solving, which is considerably more than "sums with words" ... mathematics is thus regarded as a process; a* ***structuring way of thinking****. When someone prescribes to a child how to think, that child is deprived of the opportunity of forming his own thought patterns, thinking structures and own way of thinking; precisely those instruments that give meaning to the world.* (Translation)

In reaction to the already mentioned criticism of this approach Maree (1995b) states that this approach is not empirically comparable with the traditional approach. The main reason for this is that different objectives are pursued and consequently these are not measurable on the same level. Further advantages of the new approach that Maree highlights, are that the learners still learn their tables, certain rules and principles. However, they do this within a problem-solving context and do not learn them merely as meaningless jingles. The new approach also makes more provision for problem-solving as an aspect of learners' study orientation in mathematics, as well as for accommodating different learning styles. Learners are not merely "trained". Social interaction, group work, problem-solving and maximum learner involvement are strongly advocated. Murray, Olivier and Human (1993:193) state the following about this approach:

> *In a problem-centered learning approach compatible with a constructivist view of knowledge and learning, social interaction among students and attempts by students to make sense of their own and others' constructions lead to the development of shared meanings and to individual students' constructions of increasingly sophisticated concepts and procedures.*

2.8.2 Problem-solving in mathematics

Volmink (1993:32) illustrates the inadequate knowledge frequently found in learners who have not been exposed to the problem-centred approach by means of the following problem:

> ***There are 20 sheep and 16 goats. How old is the shepherd?*** *If experience elsewhere in the world is anything to go by, there is much greater than even chance that most students will attempt an answer to this "problem" and that their answers would be remarkably similar ... The cumulative experience of students has led them to adopt the view that mathematics is the necessary outcome of meaningless things.*

Traditional problem-solving in mathematics focused especially on knowledge of language, quantitative knowledge and arithmetical skills. Mayer (1982:68-82) added to this by saying that the ability to master problem-solving in mathematics among other things depended on four types of knowledge, namely:

- ★ linguistic and factual knowledge;
- ★ algorithmic knowledge;
- ★ schematic knowledge; and
- ★ strategic knowledge.

Pólya (1946; 1957) and Schoenfeld (1985) distinguish the following five phases in their well-known problem-solving strategy:

- ★ Understanding the problem;
- ★ developing a plan to solve the problem;
- ★ transformation of the problem into a routine assignment;
- ★ executing the plan; and
- ★ verification of the solution.

The most important stage of the problem-solving process is probably the first, namely becoming aware of the problem. Since a situation which one person experiences as problematic is not necessarily a problem for another person, it is difficult to determine when a problem will stimulate or challenge a learner to participate.

Bell (1978:311) stresses the importance of problem-solving as follows:

> *Problem solving is an appropriate and important activity in school mathematics because the learning objectives which are met by solving problems and learning general problem solving procedures are of significant importance in our society.*

Maker (1993:69) defines problem-solving as follows:

> *(The ability to) solve problems in the most acceptable way and to reach the most acceptable solution(s).*

Schoenfeld (1992) aptly declares that the term "problem-solving" is used in a broad sense, from routine exercises to the level on which mathematics is done professionally. Halmos (1980) states that problem-solving is **the core of mathematics**. Costello (1991:1) defines problem-solving as:

> *The kind of insight into a problem which provides a strategy leading to its solution.*

Psychologists have recently associated successful problem-solving in mathematics with adequate and applicable "managing skills" or "metacognitive ability" (Schoenfeld, 1992). Flavell (1985:104) defines metacognition as follows:

> *(It is the capability of) monitoring and evaluating one's current capabilities, knowledge, or cognitive activity that takes as its object, or regulates, any aspect of any cognitive enterprise.*

According to this problem-solving ability in mathematics can be regarded as an example of a "cognitive undertaking" that requires active participation and involvement by the learner. This implies among other things that mathematics learners should continually adapt to task requirements through the suitable selection, application and evaluation of problem-solving strategies. Flavell (1985) describes this as the interaction of personal, task and strategy variables.

2.8.3 Discovery learning

Shulman (1970:53) defines discovery learning as follows:

> *a roller-coaster ride of successive disequilibria and equilibria terminating in the attainment or discovery of a desired cognitive state.*

Shuell (1992) maintains that (more or less) active participation or active cognitive learning material processing by learners is essential to the learning process if learners wish to acquire knowledge and skills adequately. The degree of guidance that occurs during discovery learning has always been the subject of an intense debate (Gadanides, 1994). De Corte (1995:8) prefers learning that occurs through *"appropriate intervention and guidance"* whereas Slavin (1994:47) issues the following warning:

> *Teachers should be available as resources, but should not become the authorities who enforce correct answers.*

Put in other words, the view is that learners continuously have a need (or at any time might experience a need) for facilitators who are qualified and prepared to help them professionally and with empathy when necessary. The term "facilitator" implies that teachers will regularly stand back so that learners will be able to communicate with one another, to express their own ideas and to form concepts on their own (Vygotsky, 1986).

2.8.4 Metalearning

A concept aimed at developing active and independent learning as part of learners' study orientation, is the principle "metalearning". The term "metalearning" originated in psychology through two other terms, namely cognition and meta-cognition. Flavell introduced the term "metacognition" in 1970. He describes it as follows (Flavell, 1976:98):

> *"Metacognition" refers to one's knowledge concerning one's own cognitive processes and products or anything related to them e.g. the learning-relevant properties of information or data. For example I am engaging in metacognition ... if I notice that I am having more trouble learning A than B; if it strikes me that I should double-check C before accepting it as a fact.*

Metacognition and effective learning go together. The term "metalearning" that originated from this, reveals a number of interfaces with the points of departure for information processing. Metalearning is the activity of learners who are aware of their learning actions, and who plan, execute, monitor and evaluate them themselves (Biggs & Telfer, 1987). Metalearning, according to Nisbet and Shucksmith (1986:vii), depends on learners developing a *"'seventh sense', an awareness of one's mental processes ... Cultivating this seventh sense should be the prime aims of the curriculum".*

Slabbert (1988:107) defines metalearning as follows:

> *Metalearning involves higher-order learning actions or the control actions of learning; for example planning, monitoring and evaluating. These higher order learning actions exercise control over the lower order learning actions or the executive actions of learning that make up the learning process as such. Metalearning manages and thus controls the learning process.*
>
> (Translation)

Metalearning encourages learners to take part more actively in mathematics, in the learning process and to **determine** their own learning activities rather than to **wait** passively for instructions (Nisbet & Schucksmith, 1986; Schoenfeld, 1985; 1992; 1994).

The concept "**emotional intelligence"** is also related to learners' study orientation, their problem-solving and metalearning. Goleman (1996:36) sees this relation as follows:

> *Emotional life is a domain that, as surely as math or reading, can be handled with greater or lesser skill and requires its unique set of competencies. And how adept a person is at those is crucial to understanding why one person thrives in life while another, of equal intellect, dead-ends: emotional aptitude is a meta-ability, determining how well we can use whatever other skills we have, including raw intellect.*

Goleman furthermore points out that it is important for learners to "manage" their emotions in such a way that the learners are not at the mercy of emotions such as depression, anxiety and irritation, as well as poor inter- or intra-personal relationships and inadequate motivation. This implies furthermore that learners should acquire the insight that it is necessary, under certain circumstances, to postpone certain activities (which they find more pleasant) until the more important activities (such as finalising the more difficult work in mathematics) have first been completed satisfactorily. This also means that the phenomenon of **mathematics anxiety** can be particularly destructive and can promote inadequate achievement in mathematics. To conclude this section on learning theories co-operative learning will be scrutinised.

2.8.5 Co-operative learning

Although co-operative learning implies that learners work together in small groups, it is not the only principle involving group work. Davidson (1990:1) defines co-operative learning as follows:

> *Cooperative learning involves more than just putting students together in small groups and giving them a task. It also involves very careful thought and attention to various aspects of the group process.*

Co-operative learning then takes place when a number of learners work together in a small group for the purpose of learning. Slabbert (1993) reflects upon certain characteristics and requirements of co-operative learning:

- ★ **Group size:** two to five learners (four learners are considered ideal).
- ★ **Positive interdependency:** to improve their chances of successful learning, the group's members should be mutually dependent on one another.
- ★ **Promotional interaction:** learners should help, support and motivate one another during their problem-solving efforts.
- ★ **Co-operative skills:** learners should learn and apply interpersonal and small group skills such as leadership, decision-making, communication, respect, acknowledgement and conflict handling.
- ★ **Individual involvement:** each learner should be actively involved in the learning process. The group's success is measured by to the success of each individual.
- ★ **Evaluation:** regular evaluation of the functioning of each group is necessary. Contributions and conduct that promote or are harmful to learning within the group should be pointed out.

Park (1995:42) appreciates the significance of social activities as an aspect of co-operative learning as follows:

> *The constructivist learning theoreticians are in agreement that the construction of knowledge is promoted through a social learning environment*

> *in which the learner interacts with others in contrast to an individual or isolated learning environment. This emphasis of learning as a social activity is especially confirmed by the works of Bruner and Vygotsky.*
>
> (Translation)

Sapon-Shevin and Schniedewind (1994:2) are of the opinion that co-operative learning has enormous potential to help learners attain better achievement:

> *Both educational research literature and the more popular press abound with examples of the power and potential of cooperative learning to improve academic achievement, teach social skills and build classroom community.*

Davidson (1990) summarizes the reason for supporting co-operative learning as follows:

- ★ All learners are provided with opportunities for success and learners help one another to achieve a common goal.
- ★ Learners are provided with a social support network; they exchange, among other things, ideas and feelings.
- ★ Learners are exposed to various methods of problem-solving and they question each others' ideas and solutions.
- ★ By explaining concepts to fellow-learners, learners understand them better themselves and they acquire communication and conversation skills.
- ★ Learners learn by considering and discussing problems together.
- ★ Opportunities for creative thinking and the solving of complex problems are created.
- ★ Groups can tackle complex problems that are beyond the ability of individual learners.

If these principles are operationalised in learners' study orientation in mathematics, they ought to make a significant contribution to optimising their achievement in this subject.

2.9 RATIONALE FOR THE DEVELOPMENT OF A STUDY ORIENTATION QUESTIONNAIRE IN MATHEMATICS: THREE APPROACHES

2.9.1 Introduction

To sum up at this stage three main approaches to the learning-theoretical and consequently the study orientation approach in mathematics can be distinguished: a) the traditional curricular model, (b) the information processing model and c) the constructivist model.

2.9.2 The traditional model

This model is based on a behaviouristic approach to learners' study orientation in mathematics. Learners' mistakes in mathematics are seen as the result of their unsatisfactory exposure to curricular units. This implies that learners' mistakes in mathematics are the result of a less than adequate study orientation in mathematics, and that learners were unable to master the preconditions for specific assignments and study units. Remedial instruction is based on attempts to narrow and close the gaps in the learners' knowledge of mathematics by concentrating on doing many additional examples. Textbooks usually follow this approach. Work is dealt with step by step. Remedial instruction is generally the same for all learners since learning mathematics is regarded as a constant advance from simple to more complex work from one unit to the next. Wachsmuth and Lorenz (1987:44) describe this as follows:

> *The theoretical and practical aim of this model is thus the organization of an optimal path through mathematical content.*

2.9.3 The information processing model

The approach followed by this model is that subject content knowledge structures have to be created or constructed by the students **themselves**. Learners are regarded as systems that absorb information and then process it. An optimal study orientation aims at a change in their knowledge base and this is achieved by learners' active participation in learning situations. Better achievement in mathematics is promoted, *inter alia,* by the following:

> *concepts are (not) taught before individuals have developed the necessary cognitive structures to accommodate them* (Castle, 1992:228).

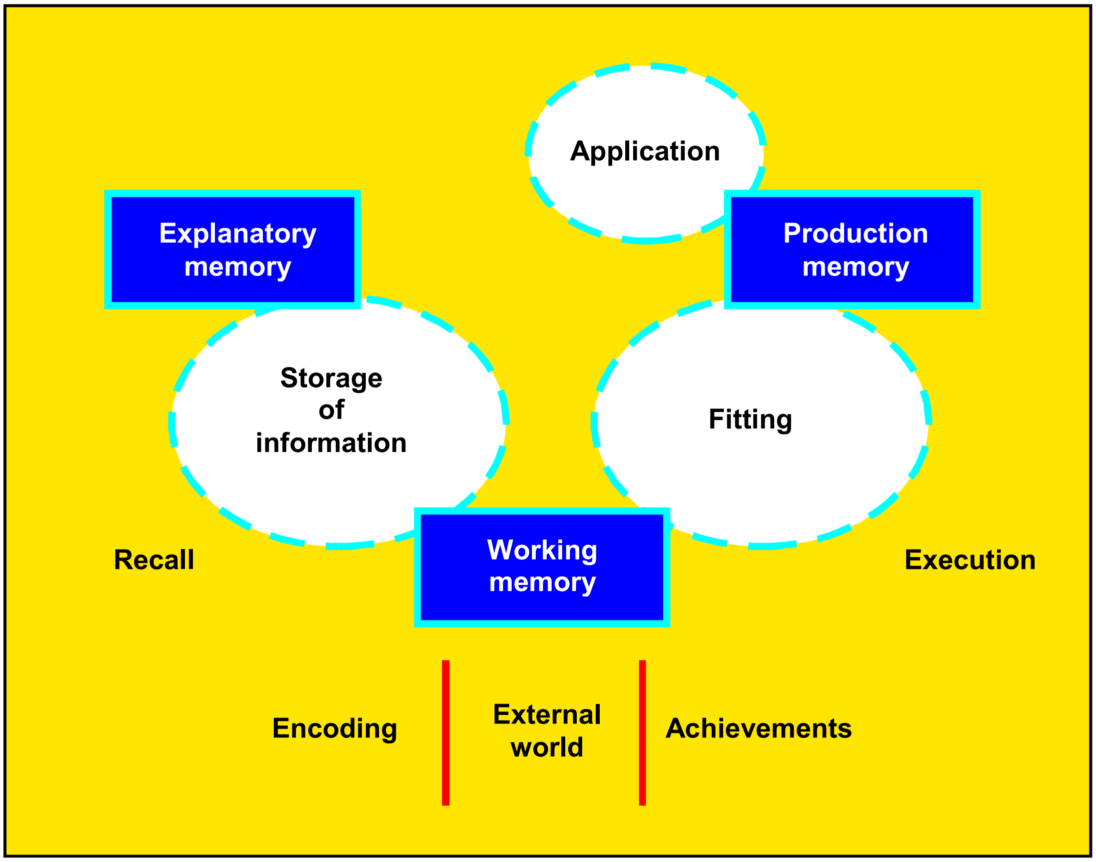
The difference between this approach and the behaviouristic one lies in the fact that information processing theoreticians mention a detailed analysis of problem-solving processes in which specific assumptions concerning aspects of the mental processes are made. The way in which knowledge is stored in the memory is, for example, considered to be of critical importance. Learners' problem-solving skill depends to a great extent on their insight into problem-solving strategies, that in turn are supported by the quality of learners' mental representational ability and organisation of knowledge.

The marked relationship between information processing theories and computer science is elucidated by Montague (1990:11-12) as follows:

> *The computer is a natural vehicle not only for a mechanical simulation of human information processing, but also for understanding what has come to be known as the construction of knowledge.*

Montague conceptualizes the different memory functions as follows:

FIGURE 2.7: MONTAGUE'S CONCEPTUALISATION OF THE DIFFERENT MEMORY FUNCTIONS



Adapted from Montague (1990)

By means of encoding, knowledge outside the working memory is brought in, whereas achievement implies that assignments outside the working memory are converted into behaviour. Montague (1990:12) aptly remarks that:

> *New productions are learned from studying the history of application of existing productions. Thus, in a sense, (this) theory of procedural learning is one of learning by doing.*

2.9.4 Constructivism and a problem-centred approach to the learning of mathematics

Problem-solving, discovery as well as **social interaction** are central to this approach and the focus falls on learners' ability to acquire knowledge structures **actively themselves**.

With reference to what has already been stated in this research on problem-solving in mathematics Murray's (1992:10-11) view on this perspective is given here:

> *Children respond deeply and seriously to word problems that make sense to them. Children need writing and scribbling materials to help them think about a problem. Drawing the problem situation is a natural and commendable thinking aid, and far more popular than using counters to pack out the problem. Concepts and skills develop naturally over a period of time. There is no rush. Late developers frequently construct the strongest and most dependable concepts. Misconceptions and mistakes are best "treated" by discussion among children (under the teacher's chairmanship if necessary) but not by the teacher directly interfering and explaining.*

2.9.5 Perspective

In conclusion it is helpful to note that certain researchers in the USA are increasingly inclined to make a plea for a (partial) return to "the basics" in mathematics (Adler, 1992). In other words, these researchers feel strongly that learners should still master the four basic operations, that they must know their tables and that practice must form an integral part of learners' study orientation in mathematics. However, it is quite clear that the last word on which approach has the most advantages and the least disadvantages has not yet been spoken.

The preceding theories are also viewed from a phenomenological and humanistic-existential-oriented perspective, among other things, in the light of the fact that it has repeatedly been mentioned that some of these learning theories are based on the results of research on animals.

2.10 THE SIGNIFICANCE OF A PHENOMENOLOGICAL AND HUMANISTIC-EXISTENTIAL APPROACH TO THE LEARNING PHENOMENON

From this point of view human beings are not linearly considered as measurable beings and merely the sum total of their characteristics. People are not atomised or molecularized entities, but rather distinguishable though not separable units (Phares, 1992). The view in this case is that man cannot be understood quantitatively by means of measurement (cannot be measured and understood). Emphasis is placed on the qualitative, the being, origin, the meaningful existence and destination of man. Whereas the “pure” naturalist sometimes apparently absolutises or reduces man to a restrictee as a result of heredity and environment, this approach regards hereditary traits and environmental matters as possibilities that can be realised by mental exertion. In terms of a study orientation in mathematics this view implies, *inter alia,* that learners are still in a position to choose whether they wish to optimise their study orientation in mathematics in order to do better in the subject. Learners have their own responsibility, are free to choose and are not subject to fate or circumstances beyond their control.

2.11 CONTEXTUALISATION

Adler (1992:29) gives perspective to the different but also fluctuating accents in the learning theoretical points of departure in mathematics:

> *In addition to ... epistemological debates within the constructivist movement, it is interesting to note that since the Cockcroft Report, the UK's move to a National Curriculum ... has been argued as a shift “back to the basics” with a renewed emphasis on testable skills ... and as a reflection of confusion in the UK of over utilitarian and creative aspects of mathematics ... We need also be aware that there are new programmes*

> *which run counter to constructivist principles and assume the acquisition of hierarchical skills and content knowledge as the necessary grounding on which numeracy ... develops and which also claim success. In other words debates on the ... learning of mathematics are alive and well.*

An acceptable definition of the way in which learners learn mathematics can be determined by giving preference to one or more learning theories. According to this (these) theory (theories) the development and evaluation of a study orientation questionnaire in mathematics can then be commenced. Not one of the preceding theories can be regarded as adequate or comprehensive but:

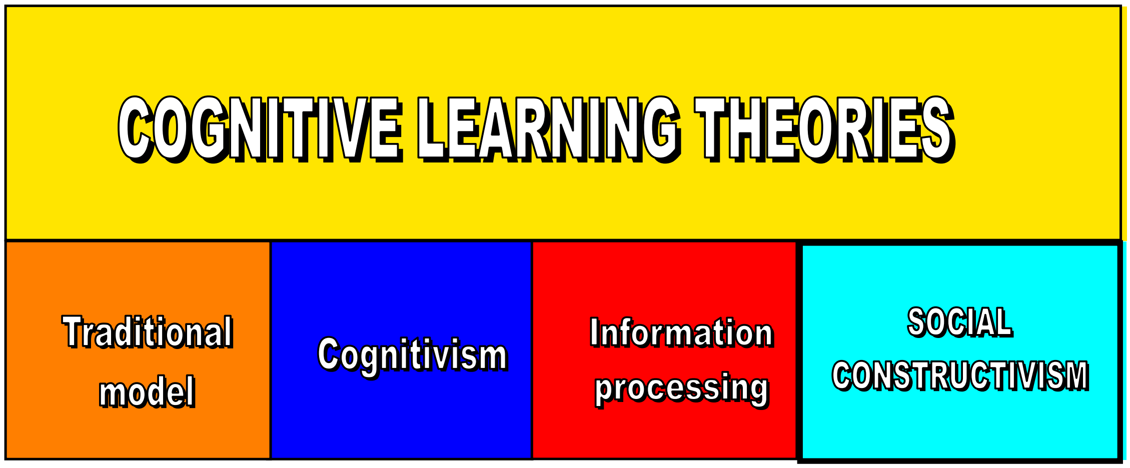
> *This does not mean that (any specific view) is here regarded as "correct". It merely illustrates that one's observations tend to be directed by one's theory* (Fourie, 1991:166).

Some of these theories may well be more complete and comprehensive than others but each theory can claim to represent the truth to a certain extent in spite of the particular theory's potentially biased emphasis of particular aspects of the way in which learners learn mathematics. This means that each theory in particular circumstances and for a particular objective can be regarded as part of a particular framework for research and practice improvement (Maas, 1980).

Small (1990) describes the study of cognitive learning theories as the study of learners' command of knowledge, of the way in which this knowledge is organised, as well as of the processes at learners' disposal to use this knowledge in everyday cognitive processes such as giving attention, learning, remembering, understanding, comprehending and solving problems. In this chapter an attempt was made to investigate those aspects of certain learning theories with regard to mathematics, including matters such as the nature of human cognition and human development.

The various theoretical approaches can be schematically represented as follows:

FIGURE 2.8: SCHEMATIC SUMMARY OF CERTAIN LEARNING-THEORETICAL APPROACHES



The traditional model is based mainly on **behaviourism**. This approach which concentrates mainly on the study of outwardly observable behaviour, particularly emphasises the value of repetition and (rapid and applicable) reinforcement of acceptable responses, of the formation of a series of applicable and correct associations, and of the functional practising of basic knowledge. A learner is regarded as an empty vessel: a *tabula rasa*. Errors and misconceptions in mathematics are regarded as erroneous conceptions in a computer. If information is undesirable, it can simply be deleted or overwritten. Some critics of this epistemology argue that inner experience is not adequately taken into account and that human behaviour is reduced too linearly to the level of stimulus-response whereas the role of learners' normative decision-making is not sufficiently taken into account. A strict behaviouristic point of departure assumes, *inter alia,* that learners learn that which they are taught (or at least a part thereof). It is assumed that knowledge can be transferred intact from one person to another – a view that is not accepted without further ado in this study. In this study the view is taken that learners can and should generate their own knowledge structures. The point of view is also taken that optimal learning occurs through self-discovery, self-work, problem-centring (a problem-solving mindedness) and social interaction, in conjunction with other factors such as a willingness to study hard, a realisation of the value of practice and motivation and the creation of optimal learning conditions by facilitators.

In conclusion reference is made to Verster (1987:51) who criticises some of the standard research methods of behaviouristic theoreticians as follows:

> *Who ever had a random sample of rats or monkeys? Yet even our most rigorous scientists generalize freely about the behaviours of rats and monkeys based on the few they happened to have available.*

Criticism that can possibly be levelled against the **information processing model** is that the model is excessively mechanical. Small (1990) maintains that this model is too dependent on the view that the human brain is a complicated cognitive system that can be compared with a digital computer. Meyer and Van Ede (1996) point out that information processing theoreticians are not able to indicate what changes take place during information processing. Furthermore, the model does not make adequate provision for responsible decision-making, critical thinking and the ability to think creatively that are especially necessary if learners want to meet the challenges of the twenty-first century with confidence. Terms such as the following: artificial intelligence, defined by Plug, Meyer, Louw and Gouws (1993) as the computer simulation of tasks that normally require human cognitive ability; sensory register; short-term and long-term memory; central processing; a response system; the input process during which information from the learners' environment is fed in, encoded and moves into the memory (where the central processor, that handles the setting of objectives and plans to carry out these objectives) after which the response system manages output, all go to make up the language of the computer. Many of these concepts have to be superimposed on the process of human learning. Human learning for the purpose of this study can be regarded as a process during which thinking in particular has to take place; a period when information has to be critically evaluated; during which social interaction should take place and responsibility towards the environment and the welfare of others has to be demonstrated, in contrast to the information processing theoreticians' overemphasis of the **mechanical** aspect of human learning. This model does not really operationalize the realisation adequately that problem-solving contexts cannot exist in isolation.

Modern constructivism implies, *inter alia,* that the focus has moved to discovery learning; learning that should occur in realistic contexts; true-life problem statements; the value of discussions; the scrutiny of problems from different perspectives; meta-learning and problem-solving. This view basically represents the view that learners have to find sense and meaning during the learning process (during which the emphasis is placed on practical as well as group activities but with the emphasis on personal participation and the individual expression of personal constructs) during which a variety of ideas and experiences are interpreted in a personal way. Based on their own experiences learners construct a personal view of the world that they are experiencing. Learning content is then planned to link up with learners' frame-work of reference and to motivate learners to **want** to do mathematics. The value of **reflecting on thinking** (the implementation of metacognitive learning strategies) with regard to making room for other (even clashing) points of view as well as the fact that learners construct knowledge in their own personal way, is strongly emphasised.

Whereas modern constructivism accepts small, well-equipped classrooms as a given, it is a given that this situation cannot realistically be realised in South Africa.

Other points of criticism of the constructivist point of view are that some of the epistemological assumptions concerning the value of the learning styles and strategies of this approach have not properly been tested empirically in South Africa.

Furthermore, in this study the view is taken that the constructivist point of view of learning being a process during which learners discover, construct or form their own algorithms or standard strategies to solve problems, is acceptable as **one** approach, **one** way of discovering mathematical truth **in combination with other approaches**. The constructivist theoreticians' emphasis of the problem-solving approach, problem-centred learning, (social) constructivism, learner involvement, social inter-action and (large and small-scale) group work is nevertheless strongly supported. Attention has shifted from the facilitator to the learners who to a much greater extent will be held responsible for their own learning achievements. Facilitators' main task is to organise the learning activities in such a way that learning occurs optimally and that the ideal of lifelong learning is promoted. Certain other points of view on the learning of mathematics supported in this study are the following:

- ★ The ability to learn is not inherited genetically, but transferred from generation to generation. Learners do not, for instance, have to recreate the whole mathematics syllabus; they learn this in their cultural milieu with the aid of parents, brothers, sisters, friends, the radio, television, the computer and books.

- ★ Learners do NOT learn completely on their own. There is always a balance between personal knowledge and cultural inheritance. Learners absorb the cultural inheritance into their own frame of reference (assimilation) and then adapt their own knowledge to the cultural inheritance (accommodation). In the same breath:

- ★ Learning is for more than mere instruction. Just as dependent as learners are on culture for information and guidance (for instance they can only learn to count if the names of the numbers are taught to them) just as certain it is that they direct the learning process to a certain extent themselves. They decide on what interests them, when they want to learn, and when they wish to ask for information. There is always a degree of tension between learners' own contribution and that which is transferred through culture, between those aspects of culture which they will incorporate into their frame of reference and the extent to which they will adapt themselves to culture.

- ★ Discovery or creation in the mathematics class does not need to or should not **just** take place in a logical-deductive manner. Classroom discussions (also group discussions), own activities, (class) discussions and self-work make a contribution to the construction of new mathematics.

The various approaches referred to in this study are not easily **empirically** comparable for the simple reason that they (to a greater or lesser extent) strive to achieve various **aims**, and their points of departure are found in various learning-theoretical and philosophical points of view. The problem is that various approaches to the learning of mathematics can only empirically-significantly be compared with one another if they strive after more or less the same aims. In such a case it would perhaps be possible to construct suitable tests to evaluate the extent to which the set aims have been realised, and in this way come to a conclusion about which one is the "best". However, in cases where aims strived after (frequently completely) are widely divergent, such a comparison is simply not possible. In such a case comparison will involve a theory evaluation and comparison of the respective aims and it will be influenced subjectively by the researcher's own theoretical points of view. On the other hand it is both possible and useful to do research on whether the proposed aims of both approaches are attainable (and to what extent).

A final and conclusive theory on the learning of mathematics has not yet been formulated and will probably not be formulated. In the meantime the serious researcher has to make use of one or more of the existing learning theories. For Hall and Lindsay (in Maas, 1980:8) a theory is not an aim on its own, but what is of importance is its utilitarian value in terms of how effectively the particular theory or theories can put into operation representations that can be verified with regard to related occurrences. In other words, there is always a continuous search in progress for theories that can serve as a frame of reference in the case of specific problems.

CHAPTER 3

DEFINITION OF THE CONCEPT "STUDY ORIENTATION" AND A REVIEW OF SOME FACTORS THAT CAN INFLUENCE LEARNERS' STUDY ORIENTATION IN MATHEMATICS

3.1 INTRODUCTION

The rate of failure in mathematics at school and at tertiary institutions is high; not only in South Africa but also in other countries (Blankley, 1994; Christie, 1991; Cockcroft, 1982; Nongxa, 1996). According to Smit (1996), basic training in the natural sciences, and especially in mathematics, is a *sine qua non* for successful instruction in by far the most professional fields. Furthermore he indicates that training in the natural sciences at (school and) university level is in a state of crisis and that the situation is deteriorating further. For example, the percentage of learners graduating annually in the natural sciences has declined from 24% in 1988 to 21,8% in 1993. With reference to this it also appears as if learners' interest in mathematics is not always optimal. Arnott, *et al.,* (1997:11) point out that the number of learners taking mathematics is still declining steadily.

> *Enrolment in mathematics ... at the standard level is dropping further from its historically low base.*

Goldenberg (1989) points out the importance of a discovering tendency in learners as an aspect of adequate study orientation in mathematics and he expresses concern about teachers' inability to motivate learners towards a more adequate study orientation in mathematics (Goldenberg, 1989:170-171):

> *(mathematics) can be the most freeing of subjects ... It is a game whose players frequently use the words elegant and beauty, and whose beauties are both visual and intellectual. Yet we show little or none of this to our students.*

It is a well-known fact that learners with an apparently high level of general intelligence or an aptitude for mathematics sometimes achieve poor results in the subject. On the other hand it is also known that learners with an apparently low level of intelligence or poor aptitude for mathematics sometimes do well in the subject. Why do learners avoid mathematics or under-achieve in the subject? Several reasons can be given for this state of affairs. It is also important to note that usually, for various reasons, little attention is given to learners' **study orientation** in mathematics (Maree, 1995b). This is unacceptable, since, as Visser puts it (1989:212):

> *Research has shown that achievement in school mathematics is one of the best predictors of success in tertiary studies.*

According to Gannon and Ginsburg (1985:405) by far most learners should be able to master mathematics at school level. They also point out the following:

> *Failure does not necessarily indicate that correct learning cannot take place, only that it has not.*

Ginsburg (1977:110) furthermore states

> *Children make mistakes because they use faulty rules ... The faulty rules have sensible origins. Children's mistaken procedures are in fact good rules badly applied or distorted to some degree,*

Glencross and Fridjhon (1989:36) show that it is important to seek **error patterns**:

> *If one is to attempt to find reasonable explanations for mathematical behaviour, then it seems sensible to begin by looking for systematic* ***patterns*** *in that behaviour.*

If one looks at the problems only on an *ad hoc* basis, there is the danger of concentrating on a short-sighted dealing with the symptoms only. Radatz (1979:170) comments as follows on the phenomenon of learning problems in mathematics:

> *It is quite often difficult to make a sharp separation among the possible causes of a given error because there is such a close interaction among causes. The same problem can give rise to errors from different sources.*

What results in problems for one learner will possibly not cause problems for another. Similarly a problem that leaves one learner without motivation, could particularly motivate another learner. Likewise it is sometimes not possible to distinguish between specific problems in mathematics.

Subsequently the construct "study orientation" will be defined. Thereafter some factors influencing learners' study orientation in mathematics will be investigated. The chapter is concluded with some models explaining inadequate achievement in mathematics.

3.2 STUDY ORIENTATION IN MATHEMATICS

The item pool for the envisaged study orientation questionnaire, ideally seen, should summarise the essential aspects of the construct; among other things to help ensure content validity. This implies that these essential factors have to be thoroughly highlighted beforehand.

3.2.1 What is study orientation in mathematics?

Du Toit (1970:23) who defined the *SSHA* for South African conditions refers to the concept "study" and defines it as follows:

> *Relatively protracted application to a topic or problem for the purpose of learning about the topic, solving the problem, or memorizing part or all of the presented material.*

He emphasises that here there is a clearly defined possibility of acquired behaviour that should be measured in some way or other with a view to optimising learners' study orientation. Biggs (1987) uses the term "learning approach" and thereby refers to the perceptible behaviour of a specific person in a particular situation. This behaviour is primarily moulded by a person's motive and strategy. Entwistle and Ramsden (1983) prefer the term "learning orientation", thereby referring to the consistency of a learner's approach to learning at school and university. Schmeck (1988) uses the term "orientation" to refer to the factor that summarises approaches, motives and styles and includes study methods and attitudes. This definition will be accepted for the purpose of this study.

3.2.2 The role of learners' study orientation in their mathematics achievement

Viewed holistically, learners' study orientation in mathematics probably significantly influences their problem-solving ability and their ultimate achievement in the subject. Reynolds and Wahlberg (1992:157) emphasise the fact that there is close interaction between the various aspects of learners' study orientation and their problem-solving ability in mathematics, as follows:

> *Explanatory factors operate in a complex network of direct, indirect, and mediating effects ... changing one factor simultaneously affects another.*

Several researchers have already indicated that there is a statistically significant relation between various aspects of a study orientation in mathematics, including anxiety, motivation, attitudes towards mathematics, the use of effective (metacognitive) learning strategies in mathematics, effective time management, concentration, the will to do well in mathematics, parent expectation as well as the social, physical and experienced milieu of mathematics learning in general (Cobb, Wood, Yackel & Perlwitz, 1992; Corno, 1992; Reynolds & Wahlberg, 1992; Van Aardt & Van Wyk, 1994; Visser, 1989; Wong, 1992).

3.3 HOW DO STUDY ORIENTATION PROBLEMS IN MATHEMATICS ORIGINATE

According to Denvir's (1984:18-19) research on the causes of study orientation problems in mathematics, the following general summary of possible causes for such problems (potential problem areas) have come to light:

- ★ *The pupils are culturally deprived. Their language is limited and they get very little encouragement and support at home.*
- ★ *They cannot grasp relationships.*
- ★ *Low intelligence.*
- ★ *Many have had insufficient practical experience in infancy.*
- ★ *Too fast a teaching pace in the early years ... later on, teachers with limited knowledge of the early stages of mathematics and of the subject's development.*
- ★ *Pupils get hang-ups about the subject because they keep getting work marked wrong. Then they either switch off or mess around.*

It is unlikely that it can always be stated with certainty what each learner's problems are exactly and also what the causes of such problems are. Nevertheless certain general factors can be singled out, and the aforementioned author suggests the following classification (Denvir, 1984:19):

- ★ Physical, physiological or sensory problems
- ★ Emotional or behavioural problems
- ★ Physical causes such as fatigue, drugs, general poor health
- ★ Attitude problems: anxiety, poor motivation
- ★ Inadequate teaching
- ★ Repeated changing of teachers, lack of continuity
- ★ General inability to grasp concepts quickly
- ★ Cultural differences; language of teaching is not learners' home language
- ★ Impoverished home background
- ★ Inability to express themselves orally
- ★ Poor reading ability

- ★ Gaps in the education and learning process, absence from school, repeated transfers from one school to another
- ★ Immaturity, late development, youngest in the particular class group
- ★ Poor self-image which leads to poor self-confidence

Study orientation problems and inadequate achievement in mathematics are caused by several factors. Nevertheless each learner's problems manifest themselves in an idiosyncratic and probably unique manner.

3.3.1 Attempts to classify errors in mathematics

Radatz (1979, 1980), a strong supporter of the information processing learning theory, presents one potential model for the classification of errors in mathematics. He describes five mechanisms that produce errors in the whole spectrum of mathematics:

- ★ Errors attributed to language problems
- ★ Errors attributed to inadequate mastery of basic mathematical skills, facts and concepts
- ★ Errors attributed to problems in processing information on their own position in space
- ★ Errors attributed to inapplicable associations or exaggerated rigid thinking
- ★ Errors attributed to the application of irrelevant rules or strategies

Movshovitz-Hadar, Zaslavsky and Inbar (1987) classify errors in mathematics into the following six sections:

- ★ Data used incorrectly
- ★ Language interpreted incorrectly
- ★ Logically invalid inferences, deductions or conclusions
- ★ Distortion of theories or theorems
- ★ Unverified solutions
- ★ Technical errors

With the aforementioned summarising models as a frame of reference, a more detailed examination of certain approaches to the origin of study orientation problems in mathematics will be made.

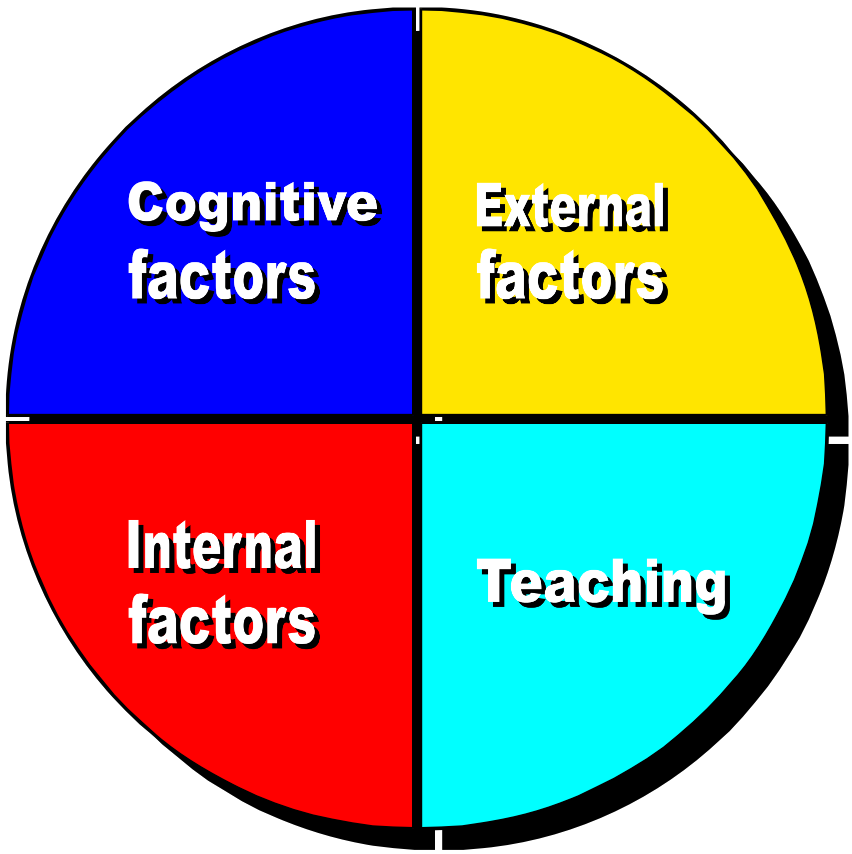
3.4 REVIEW AND CLASSIFICATION OF CERTAIN FACTORS THAT CAN INFLUENCE LEARNERS' STUDY ORIENTATION IN MATHEMATICS

From the preceding introduction and by taking into account the review of certain epistemological views on the learning of mathematics (Chapter 2) it is possible in theory to group together those factors that influence learners' study orientation in mathematics in various ways. For the purpose of this study the following classification can be made[1]:

(i) Cognitive factors;
(ii) external factors;
(iii) internal or intrapsychic factors; and
(iv) teaching.

[1] Compare the summarising model in this connection: Chapter 4, paragraph 4.6.

FIGURE 3.1: CLASSIFICATION OF SOME POSSIBLE FACTORS THAT COULD INFLUENCE LEARNERS' STUDY ORIENTATION IN MATHEMATICS



The focus will firstly be on some cognitive factors.

3.4.1 Cognitive factors

Gage and Berliner (1992) define cognition as all the ways in which people think, as well as the cognitive strategies that are used to facilitate learning and thinking. Wood (1989) defines the term "cognition" as the collection of skills involved in the learning process, the term "metacognition" as thinking about thinking, and the term "epistemic cognition" as the knowledge that all human knowledge is limited.

Intelligence, which co-determines the quality and content of human thinking, will be discussed first.

3.4.1.1 Intelligence

Phares (1992:182) classifies theories relating to intelligence into three main categories, namely:

- ★ Definitions that emphasise learners' adaptation to their environment;
- ★ definitions that focus on the learner's learning ability; and
- ★ definitions that emphasise abstract thinking ability, as well as the ability to use a wide range of symbols, concepts, as well as verbal and numerical symbols.

This classification is very similar to that of Van den Berg (1995), according to whom the following themes play a prominent role in the definition of intelligence:

- ★ The ability to adapt to new situations;
- ★ the ability to learn;
- ★ the ability to handle abstract relationships and symbols; and
- ★ the ability to solve new and divergent problems.

Naglieri and Reardon (1993:128) define the concept "intelligence" according to an information processing perspective as:

> *One's ability to attend, process information, and utilize those processes to solve problems.*

Gardner (1983:60-61) emphasizes the importance of problem-solving with the following remark:

> *(a definition of human intelligence) must entail a set of skills of problem solving enabling the individual to resolve genuine problems or difficulties that he or she encounters and, when appropriate, to create an effective product – and must also entail the potential for finding or creating problems – thereby laying the groundwork for the acquisition of new knowledge.*

Van Eeden (1991) indicates that:

> *Intelligence or any inflection thereof ... developed academic potential (is implied).* (Translation)

Her view links up with the hypothesis in this study on the relationship between intelligence, an adequate study orientation in mathematics and achievement in mathematics. This means that there is a significant relationship between achievement in mathematics on the one hand and an adequate study orientation in mathematics on the other hand. Intelligence tests possibly reflect to a greater extent that which learners have already learnt rather than accurately predict precisely what they can learn.

3.4.1.2 Brain dominance

It appears that the learners' left hemisphere of the brain controls their *verbal, numerical* and *logical* functions (the abstract of symbolic representations where the symbols do not have to have any physical similarity to the objects that they represent. The right hemisphere controls *spatial, visual, perceptual, intuitive* and *imagination* functions (including creative skills and emotions like sadness for instance); in other words representations that are isomorphic to reality (Conners, 1990; Corballis, 1980; Kolb, 1984).

Consequently when learners think, read, write and listen they use their left hemisphere. Learners with a dominant right hemisphere should be able to see, feel, taste, imagine and manipulate. According to Kolb (1984), it especially means that the two methods by means of which mathematics can be mastered, namely the concrete and the abstract ones, are of the same value and also complement one another. This view is in sharp contrast to earlier views that concrete experience-oriented learning is inferior compared to abstract reasoning.

3.4.1.3 Creativity

Theories and definitions of creativity are frequently based on problem-solving, discovery and *"bringing something into being that has newness and value"* (Maker, 1993:69). Malherbe (1991) surmises that the ability to think creatively is one of the most accurate predictors of achievement in engineering. So too is the ability to do well in geometry – in his view a consequence or indication of latent creative ability. Strauss (1983), however, believes that creativity is the ability to notice new dimensions within a convergence of circumstances and to process them into something new.

Woodrow (1984:7) asks the following question about the relation between creativity and achievement in mathematics:

> *Could not mathematics be taught so as to encourage creativity ... intuition, expressiveness and extraversion? We often choose to teach mathematics in a manner which makes these characteristics disadvantageous in the mathematics classroom even though at later stages of mathematical education they may become valuable attributes.*

3.4.1.4 Critical thinking

Critical thinking is one of the cornerstones of the new national South African curriculum (Curriculum 2005) (NDE, 1997). At the same time it is regarded as one of the eight essential outcomes and is one step further than creative thinking (Ellis, 1997). Ellis describes the construct "critical thinking" as the sorting out of clashing points of view, the weighing up of evidence in favour of various standpoints or views and the sacrificing of personal prejudice in order to take up a personal point of view. This implies ongoing discussion and practice and it can be described as a process rather than a product.

3.4.1.5 The limited, technical language of mathematics

Sharma (1981:61-71) describes mathematics as a *bona fide* second language with its own alphabetical symbols, vocabulary, syntax, grammar and literature. Although problem-solving is the subject in mathematics most frequently associated with a good **vocabulary, reading** and **command of language,** all mathematical thinking and achievement (even the simplest arithmetical skills) are in reality to a great extent dependent on the adequate mastery of the language of mathematics (Kosc, 1981). Rothman and Cohen (1989:133) wonder quite justifiably:

> *Yet where ... is the language of math specifically taught? Few seem to realize that proficiency in math, both for computation and problem solving, means learning its language, which constitutes one complex component of a symbolic-communicative function.*

3.4.1.6 Space or laterality

Psychological tests bring to light that certain learners do not know how to indicate positions in space or are unable to determine the exact meanings of the following: close, far, three-dimensional, fluctuating, corresponding, parallel as well as up, down, front, back, left and right. Furthermore these tests also confirm the suspicion that certain learners struggle to determine their place or to distinguish a figure in a particular background – a skill which, *inter alia,* is important in trigonometry.

Brown, in Rothman and Cohen (1989:133) state the following with regard to the function of the symbolic language of mathematics:

> *Mathematics may be regarded as a symbolic language whose practical function is to express quantitative and spatial relationships.*

If learners are unable to integrate spatial relations, quantitative reasoning and the language of mathematics, their achievement in mathematics could be adversely affected.

3.4.1.7 Cognitive style

Researchers alternate the use of the terms learning style and cognitive style and usually refer to the same concept. Ellis (1997) points out that theoreticians involved with this subject usually focus on different combinations of the construct "**observation**" (either in a concrete, personal, sensory or intuitive way, or in an abstract, analytical or intellectual way) and on **"processing"** (experiment actively and **do**, or reflectingly observe and ponder over matters). Keefe and Monk (1990:1) define the concept "learning style" as:

> *The composite of characteristic cognitive, affective, and physiological factors that serve as relatively stable indicators of how a learner perceives, interacts with, and responds to the learning environment. It is demonstrated in that pattern of behaviour and performance by which an individual approaches educational experiences. Its basis lies in the structure of neural organization and personality which both molds and is molded by human development and the learning experiences of home, school and society. This definition incorporates broad categories – cognitive, affective, and physiological – but learning style itself is a gestalt. It is a complexus of related characteristics in which the whole is greater than its parts. Learning style combines internal and external operations that are derived from the individual's neurobiology, personality, and development that are reflected in learner behaviour. Learning style represents both inherited characteristics and environmental influences.*

In other words learning style or cognitive style is the way in which a person reacts to stimuli from a learning environment or context. Kolb (1981) also regards cognitive style as the result of hereditary factors, previous life experience and the appeal or demands from the present environment. Personality factors, including responsibility, sociability, perseverance, self-discipline, motivation, volition and Locus of control (McCarthy, 1980) all fall under the affective factors. Cognitive factors are encoding, decoding, information processing as well as the storing and withdrawing of information (Gagné, 1985; Kirby, 1979). The physiological or environment-related domain refers to sensory observation and environment-related factors (Barbe & Swassing, 1979; Jenkins, Letteri & Rosenlund, 1990).

(i) Learning conception

Bloom (1976) and Trollip (1991) describe the construct "learning" in tabular form as follows:

TABLE 3.1: BLOOM AND TROLLIP'S DESCRIPTION OF LEVELS OF INSIGHT

LEVEL OF CATEGORY	ACTION/VERBS OR DEFINITION
1. Knowledge	Knowledge of specificities, of ways and means of relating to these specificities, and of abstractions and universal aspects of a specific field. Characterised by words such as name, identify, join, define, select and describe
2. Comprehension	Conversion, interpretation and extrapolation of knowledge. Characterised by action verbs such as classify, explain, transform, summarise and predict
3. Application	Demonstration of comprehension. Characterised by action verbs such as calculate, dissolve and arrange
4. Analysis	Analysis of elements, relationships and organisational principles. Characterised by action verbs such as differentiate, put in diagram form, estimate, arrange, deduce and subdivide
5. Synthesis	Production of unique communication means, of a plan for a proposed system of operations and deduction of a set of abstract relationships. Characterised by action verbs such as create, combine, formulate, construct and design
6. Evaluation	Assess in terms of internal evidence and external criteria. Characterised by action verbs such as assess, criticise, discriminate, compare, conclude that, justify and deduce

(Bloom, 1976; Trollip, 1991)

Rossum, in Entwistle (1988) sees the learning concept in diagram form as follows:

TABLE 3.2: LEARNING LEVELS, ACCORDING TO ROSSUM AND ENTWISTLE

CATEGORY	DEFINITION
1. Acquirement of knowledge	Provisional, not sharply defined, conception of learning
2. Memorising	Storage of information with a view to reproduction
3. Use of knowledge	Discovering or becoming aware that learning involves more than mere memorising of facts; that knowledge and skills are useful
4. Retrieval of meaning	The realisation that learning (can) lead to relationship imprinting on both micro- and macro-level (between subject fields, but also between subject fields and the general reality)
5. Interpretation and comprehension	Acquiring of the insight that learning is a method of obtaining knowledge; something that could lead to reality being understood
6. Self-actualisation	Learning as self-realisation or personal growth

(Rossum & Entwistle, 1988)

To summarise it appears that several authors agree that ultimately learning gives special attention to making mental content function and to integrate such mental content and functions, with, as final objective, self-realisation, personal growth and the shaping of the individual (Roos, 1995).

(ii) Learning approaches: the difference between the surface and the in-depth approach

The Swiss researchers Marton (1975), Marton, Hounsell and Entwistle (1984), Martin and Säljö (1984), as well as Svensson (1976), did pioneering work in the field of learning approaches. In contrast with most researchers of their time they indicated that the quantitative results of learning, namely the number of facts and ideas that have been **memorised** and **remembered**, are subordinate to the **comprehension** of that which was read. These researchers distinguished between the surface and the in-depth approach whereas Biggs (1988) also referred to the achievement approach.

(a) The surface approach

The memorising of facts and ideas in order to try and remember facts, with very poor comprehension and less knowledge of detail.

(b) The in-depth approach

The studying of subject matter in order to relate new ideas to previous knowledge and personal experience. Persons making use of this learning approach, are more actively involved in the subject matter, understand it and are more able to remember the detail for a longer period than those using the surface approach. This approach leans heavily on a positive affective orientation and intrinsic interest in a specific task as well as on expectation of benefiting from the outcome of the task. This leads to strategies' being developed to discover the intrinsic meaning of the task, that the task be considered according to one's own experience and that new material be integrated with existing knowledge structures, after which theory formation and hypothesis statement can follow.

(c) The achievement approach

Learners with an achievement approach are affectively minded to prove how outstandingly they achieve when compared with other learners. This leads them to try to obtain the best marks. Consequently this method can be regarded as supplementary to the previous approaches. According to Biggs (1988) an in-depth achievement approach is a characteristic of good achievers.

The preceding three concepts can be summarised as follows (Biggs, 1987; 1988):

TABLE 3.3: MOTIVE AND STRATEGY AS ASPECTS OF LEARNING APPROACHES

APPROACH	MOTIVE	STRATEGY
Surface	Extrinsic and instrumental. Aim: Achieve success without too much input. Motivation depends on negative (test anxiety) and positive (obtain good marks) reinforcement	Factual reproduction of information
In-depth	Intrinsic, directed towards the satisfaction of interest, acquisition of insight and extension of knowledge	Task involvement, forming of relationships and con-textualisation of subject-matter by reading more widely. Leads to satisfactory results
Achievement	Compete with others, as well as reinforcement of learners' own ego (that to a great extent depends on obtaining good marks, irrespective of whether subject matter is interesting or not)	Optimal organisation of (study) time and work space typify outstanding learners

(Biggs, 1987; 1988)

(iii) Field dependence versus field independence

The bipolar field dependence field independence style has probably been researched the most thoroughly of all the learning styles. Kolb (1984), as well as researchers such as Witkin, Moore, Goodenough and Cox (1977) refer to this construct as a tendency for persons to organise their experiences in an analytical or global fashion. A field-independent learner, while confronted with a number of irrelevant but nevertheless challenging stimuli, is able to focus on one particular stimulus, whereas a field-dependent learner is unable to do this. Field-independent learners are able to overcome the whole or the structure of a given field or are able to reconstruct this, whereas field-dependent learners accept a field as it is and have difficulty in distinguishing its particular parts within the whole context. It would appear that learners who grew up in an autonomous, prestige-oriented climate, tend to be field-independent in contrast with learners who grew up in an overprotected climate and who tend more to be field dependent (Owen, 1995).

3.4.1.8 Information processing errors as cognitive style problems in learners' homework

Authors such as Bickhard (1980), Campbell and Bickhard (1986) and Schutz (1994) particularly emphasise that the constructivist approach to learners' study orientation implies that learners *should* make mistakes and that they *should* learn from them. They maintain that human learning is a constructive or creative process; one that includes the making of mistakes, but also diagnosing and correcting the mistakes.

McKeachie, quoted by Pintrich and Garcia (1994:121) states the following in connection with general learning and thinking strategies:

> *Students should continue to learn and use their learning in more effective problem solving for the rest of their lives. When one takes life-long learning and thinking as the major goal of education, knowledge becomes a means rather than an end, and other formerly implicit goals become more explicit.*

The ideal to strive after is for an adequate study orientation in mathematics to create a foundation for facilitating lifelong learning.

Pintrich and Garcia's (1994:113) research led them to the following conclusion:

> *Students who use more deep-processing strategies like elaboration and organization are more likely to do better in the course in terms of grades on assignments, exams, and papers, as well as overall course grade. In addition, students who attempt to control their cognition and behaviour*

through the use of planning, monitoring, and regulating strategies also do better on these academic performance measures.

Thus the focus is not just on personality types such as introversion – extroversion, field-dependence – field independence and the Myers-Briggs profiles. The assumption is made that an adequate study orientation in mathematics will be evinced through active, constructive (creative) learners using cognitive as well as metacognitive learning strategies when learning mathematics. The assumption furthermore is that these cognitive and metacognitive learning strategies do not represent "personality" traits, but rather that learners can acquire, master or learn to control these cognitive and metacognitive learning strategies.

Subsequently the focus is on certain "errors" that learners make in mathematics. These types of "error" are in reality information processing errors that usually have a negative effect on achievement in mathematics, but *can* (and should) be diagnosed.

- ★ Linearisation (Davis & McKnight, 1979). Under linearisation these authors understand errors such as the following:

  $(a+b)^2 = a^2 + b^2$, and:

  $\sin(x+y) = \sin x + \sin y$

- ★ Cancellation errors, such as the following:

  (m + n – p)/(m + t) = (n – p)/t

- ★ Zero product principle (Glencross & Fridjhon, 1989). This includes the following type of error in:

  $(x-2)(x-3)=8:$

  $\Rightarrow \quad x-2=8 \quad$ or $\quad x-3=8,$

  $\Rightarrow \quad x=10, \quad$ or $\quad x=11$

- ★ Over-generalisation (Matz, 1980). Learners put the equation (x – a)(x – b) = 0 equal to the equation (x – a)(x – b) = c and forget the critical value of the 0.

- ★ Problems with word sums (like, for instance, incorrect application of variables in word problems that lead to comparisons) (Glencross & Fridjhon, 1989). The problem "Write an equation to represent the following statement (use the letters x and y): There are six times as many students than professors at this university. Use S for the number of students and P for the number of professors" was answered incorrectly by 37% of a group of first-year students in Rosnick's (1981) investigation group. When the ratio of 6:1 was changed to 4:5 more than 73% of the students had the answer incorrect. The ratio is expressed as 6S = P which indicates, *inter alia,* that many students used the letters S and P (abbreviations for Students and Professors) as abbreviations for the units (6 litres = 6 ℓ) instead of regarding them as symbols.

- ★ Application of rules without the necessary insight.
- ★ Problems with inequalities. In the problem: Solve x if $3x^2 - \times \geq 2$, the learners are often uncertain whether the solution is of the form $a \leq x \leq b$ or of the form $x \leq a$ or $x \geq b$.
- ★ Generalisation before operations (Davis, 1983). This includes mistakes like the following:

$$\begin{array}{r} 233 \\ -\underline{178} \\ 145 \end{array}$$

- ★ Slips (mistakes that do not appear systematically and that can be ascribed to incorrect processing of information, usually as a result of carelessness) and inaccuracies (errors made in a systematic way as a result of erroneous planning). Errors are either symptoms of underlying erroneous principles in the conceptual structures or misconceptions.
- ★ Displacement of assignment (questions are sometimes read or interpreted incorrectly; consciously or subconsciously. Thus $(x-y)(y-x)$ in the lower grades is sometimes replaced with $(x-y)(x-y)$ (a better known type of problem).
- ★ Inability to abstract or to represent concepts and principles. Symbols such as 2, ¾ and 5,345 are literal representations of numbers, whereas symbols such as a x/y and n = 2m – 4 are variables where letters represent unknown quantities. It is important for learners to first get to know a specific concept or principle by means of concrete representations and then gradually move to symbolic representations (Schminke, Maertens & Arnold, 1978). Learners are given the opportunity to learn on a conceptual and problem-solving level when they are allowed to work with objects, to speak about what they observe and are allowed to write down the observed relationships.

- ★ Inability to move between different levels of thinking. Learners sometimes struggle to substitute simpler problems for the more complicated ones. (How much is x more than y? should, for example, be converted to an example like: How much more is 12 than 5?)
- ★ Clumsy or careless language usage and writing. Learners quite often do not know what elementary concepts such as “factor”, “expression”, “equation”, “counter”, “square” or “quadrant” mean, or they fall into verbalism (the use of symbols without the ability to name them). They are often able to recognise and use the symbols in familiar situations, but struggle when the symbols appear in an unknown context.
- ★ Circular reasoning. Learners use (especially in geometry) precisely the theorem that they still have to prove, in their reasoning – in other words they accept exactly that which they have to prove.
- ★ Carelessness.
- ★ Copy other learners’ work.
- ★ Mark incorrect work right.

3.4.1.9 Self-directed behaviour

McKeachie, Pintrich and Lin’s research (1985) emphasized the joint influence of cognition and motivation on self-directed behaviour. When the relationship or interaction between cognition and motivation is considered, it becomes possible, *inter alia,* to identify learners’ aims (Schutz, 1994). Pintrich and Schrauben (1992) in this connection distinguish between intrinsically oriented aims (mastery, challenges, acquisition of knowledge or curiosity) and extrinsically oriented aims (pass or obtain grades, rewards or the approval of others). Schutz’s (1994) investigation indicated the following rank order of interdependent areas of aims among learners:

1. Occupation (continuous progress in a particular occupation and enjoying the work).
2. Family (marriage and being happy).
3. Education (a degree or obtaining high marks).
4. Travel and adventure.
5. Personal welfare (be happy or obtain self-knowledge).

6. Physical comfort.
7. Power and riches.
8. Giving social aid (becoming a leader or helping other people).
9. Friendship.
10. Religion (helping others to develop their religion or promoting religion).

He points out that these objectives arose from the interaction between biological influences (hereditary factors and evolutionary factors, including genetical factors and life cycle processes such as growth, maturation and physical decline), eco-cultural influences (environmental and cultural influences exercised by friends, family members, the community and the state) and a person's current level of subjective consciousness (ultimate objectives, values or core objectives) (Ford, 1992; Winell, 1987). Human objectives can thus be described as cognitive representations of that which persons would like to see happen, as well as those things they would like to avoid in the future. Human behaviour is directed towards and regulated by its relationships with these objectives. An adequate study orientation in mathematics can thus be seen as a form of human behaviour directed towards and regulated by learners' ultimate objectives including learners' ideal to realise their personal potential optimally.

3.4.1.10 Strategic learning

According to Weinstein (1994), strategic learners have five categories of knowledge at their disposal:

- ★ Knowledge of themselves as learners;
- ★ knowledge of various kinds of academic assignments;
- ★ knowledge of tactical strategies to obtain new knowledge, to integrate it, apply it and to reflect upon it;
- ★ applicable precognition; and
- ★ knowledge in connection with current and future contexts in which this knowledge should be useful.

Strategic learners must **want** to learn further, should be able to monitor their own progress and should know how to do self-evaluation or self-testing to determine whether they have attained their learning objectives (Brown, 1987; Flavell, 1979). In addition to this they should be able to implement metacognitive awareness and management strategies so that they can control their own learning process. They should also be able to follow a systematic approach (Weinstein, 1994). Such a systematic method of approach includes the following: setting an objective, making a plan to achieve this objective, choosing and using specific strategies to achieve such objectives, monitoring their own progress and plan, methods or even modifying the original plan (if necessary) and evaluating what has been done to decide whether these strategies are suitable for future use. The ideal is that learners acquire a repertoire of strategies they can fall back on automatically in the future (Anderson, 1990b).

Questionnaires such as the *SSHA* (Du Toit, 1981) the *Learning and Study Strategies Inventory (LASSI)* (Weinstein, Palmer & Schulte, 1987) and the *Motivated Strategies for Learning Questionnaire (MSLQ)* (Pintrich, Smith, Garcia & McKeachie, 1991) have specially been developed to measure, *inter alia,* the following aspects of strategic learning: attitudes, habits, lecturer approval, acceptance of education, motivation, time management, anxiety, concentration, information processing, selection of main ideas, self-testing and test strategies. The envisaged study orientation questionnaire in mathematics will consequently focus on measuring a meaningful combination of these factors as far as mathematics is concerned.

3.4.2 External factors

Under this heading unforeseen or unexpected stimuli can be found that occur frequently without warning but are nevertheless so compelling that learners' study orientation in mathematics can be influenced by them.

3.4.2.1 Pathological primary education situation

Poor home background, poverty, marital discord between parents as well as the absence of one parent are examples of a poor domestic background that could lead to mathematics problems. Claassen (1989) and Van Eeden (1991) relate in this connection the differences found existing between the aptitude and intelligence levels of Afrikaans- and English-speaking learners to the higher socio-economic status of the English-medium group.

3.4.2.2 Changing schools and teachers

When learners change their schools and get new teachers, this leads to a lack of continuity and potentially contributes to study orientation problems in mathematics.

3.4.2.3 Expectation of achievement in mathematics

The following attitudes are sometimes found:

- ★ Learner A has a high IQ; if she or he does well accordingly in mathematics, A is merely lazy.
- ★ Learner B has a low IQ; if she or he does well in mathematics, B is over-achieving; if she or he does badly, then B is just stupid.

Seen from a psychological perspective, this implies that learners' parents, in particular the expectation of the father, can have a significant influence on their study orientation (Maree, 1990). If parents set no or a very low ideal for their children as learners, learners sometimes accept that expectation as criterion for their achievement.

The matter involving other people's expectations of a learner's achievement in mathematics in a changing South Africa, and the changeover from a policy of apartheid to a democratic system in schools, is potentially significant. Hannan (1988:28) poses the following questions (that could also possibly be posed with reference to South Africa) with regard to the situation in Britain:

> *Does the teaching of maths in our schools make assumptions about culture? Do white middle class pupils do well because they are inevitably predisposed to do so given the intrinsic nature of mathematical learning or are we teaching white-middle-class-maths which effectively excludes others not in possession of their variety of cultural capital? Or, perhaps more convincingly, do teachers make assumptions about mathematical abilities which favour white middle class pupils and thus make their success something of a self-fulfilling prophecy?*

3.4.3 Internal or intrapsychic factors

The first problem which is given attention in this section is the phenomenon of maths anxiety.

3.4.3.1 Maths anxiety

The term "maths anxiety" is not really correct since anxiety is free-flowing and not directed towards anything in particular. It would be more correct to refer to maths fear or a maths phobia but the term "maths anxiety" has become so commonplace that it is used to refer to learners' negative attitudes towards mathematics or their fear of the subject. Visser (1988:38) defines maths anxiety as follows:

> *Maths anxiety may be defined as an irrational and impedimental dread of mathematics. The term is used to describe the panic, helplessness, mental paralysis and disorganization that arise among some individuals when they are required to solve a problem of mathematical nature.*

According to this author the phenomenon could occur at any time in a learner's career and usually it does not disappear spontaneously. Visser (1998:38-39) surmises that maths anxiety can be caused by the interaction of different factors. Thus learners at primary school level can handle most mathematics problems merely by memorising rules and strategies without obtaining real insight into the subject. In the secondary school and at tertiary level learners cannot get away with this and it is often too late to learn problem-solving strategies. Other factors include the fact that mathematics, other than most other subjects, does not offer learners the opportunity to start each year from scratch. Each day, week, month and year's work is based on the work of the previous period. Castle (1992:228) states the following in this regard:

> *It has been argued that unsatisfactory past experience of learning mathematics at school, rather than any lack of aptitude in mathematics, causes "maths anxiety".*

Maths anxiety influences learner's attitudes unfavourably whereas attitudes in particular constitute an important part of learners' study orientation. Wong (1992:33) expresses the view that:

> *Attitudes, again, play a crucial role in the learning of mathematics. When attitudes are used as predictors of achievement in mathematics, significant positive correlations are usually found ... Positive attitudes also have a strong influence on student motivation ... and the intention to learn.*

Strauss (1990) suggests the following strategies in order to handle the problem:

- ★ Learners should learn to think for themselves.
- ★ Learners should learn to understand fully the mechanisms and strategies in mathematics and not just merely apply them mechanically. The use of as many technological aids as possible is indispensable.
- ★ Correct concept formation is important. Learners' incorrect answers should not merely be considered as wrong, but they should be analysed to determine **why** wrong answers have been given.

Strauss suggests, *inter alia,* the use of text books in particular mathematics tests since this could contribute to the meaningful understanding of the subject.

Sharma (1979) believes that the vocabulary of mathematics should be taught just like the vocabulary of each other field of knowledge. Just as learners have to learn the alphabet before they can begin to read, they should first learn the symbols and vocabulary of quantitative reasoning before starting with mathematics. Sharma is of the opinion that this can contribute to reducing learners' maths anxiety.

3.4.3.2 Self-image and self-confidence

Covington and Roberts (1994) regard the need for acceptance as the highest human need. Academic achievement (as well as experiencing being able or unable to achieve) determines to a great extent whether learners will be accepted (or perceive that they are accepted) or not. According to these authors, the learners' aim-related disposition can be distinguished as follows:

(i) Avoidance of failure

These learners, doubting their ability, enter the domain of mathematics and are usually anxious and frightened that they will be proved to be incompetent. Their self-doubt makes them spend too much time on efforts to obtain relief from their anxiety, *inter alia,* by refusing to take responsibility for success or failure (for example, they blame teachers and the particular nature of the subject for anticipated failure) and by under-estimating the importance of academic achievement. These learners reveal on the one hand the classic signs of skill deficit (anxiety arises since learners realise they are unprepared and because of this they might fail) and on the other hand they reveal retrievals deficits (an inability to recall knowledge that has been acquired previously).

(ii) Over-strivers

In such cases the learners' study anxiety and self-doubt lead to intensive study directedness, but their learning strategies are ineffective. Fear of anxiety leads to superficial learning that offers less resistance to the forgetting of information, especially on account of test anxiety (retrieval anxiety). Hence the phenomenon of learners "striking a blank".

(iii) Acceptance of failure

Such learners have ceased to establish a sense of self-value through achievement, or of retaining such a sense. They are passive and accept their fate. They do not achieve poorly because they are unable to recall knowledge (retrieval deficit), as in the case of the over-strivers, but rather (as in the case of avoidance of failure) on account of the inability to acquire knowledge in the first place.

(iv) Achievement-oriented learners

These learners set learning objectives for themselves, just beyond their (present) reach but still attainable if they are prepared to work hard enough.

Anderson (1990a:266) states the following with regard to the meaning of self-image in learners' study orientation in mathematics:

> *A person's self-concept is influenced by what others, especially significant others, think of that person ... self-concept is resistant to change.*

When self-confidence as an aspect of learners' study orientation declines, its effect is very clearly seen in terms of achievement in mathematics. Bloom (1976) explains that no learner is only cognitively involved in the learning process. Factors relating to feelings, for example interest, motivation, attitudes and self-concept play an important role in the learning of any content. Burns' (1979) study also indicates that the correlation between learners' self-image and academic achievement is statistically positively significant.

3.4.3.3 Interest

As far as learners' interest (or lack of interest) in mathematics is concerned, the following factors are of importance (Maree, 1992; Taljaard & Prinsloo, 1995):

- ★ There is a significant positive correlation between mathematics ability and a learner's interest in the subject.
- ★ Feeling plays an important role in learners' interest in mathematics. This implies that learners will probably enjoy the subject more if they achieve well in the subject, and vice versa.

3.4.3.4 Character, motivation and perseverance

Learners who do not have a positive study attitude, who do not realise the importance of hard work in mathematics or the particular role of mastering (new) information and its dependence on previous knowledge, could struggle to do well in mathematics (Emenalo & Okpara, 1990). These authors particularly emphasize the importance of certain study methods in mathematics. At this stage it is important to point out the new approach to learning mathematics (problem-solving, co-operative learning, constructivism) does not mean that learners no longer have to work hard in mathematics. Engelbrecht (1997) emphasises that learners should discover aspects of mathematics but should also work very hard in the subject and "not discover the wheel again". Alper, Fendel, Fraser and Resek (1995:632) state this as follows:

> *Once students have invented a process, they need to practice using it so that it does not need to be totally rediscovered every time it is needed.*

Grossnickel, *et al.,* (1983:18) define motivation as follows:

> *Motivation is an emotional state that provides the driving force to cause an individual to learn and make the effort to achieve.*

In other words, if the learner's emotional and affective disposition are unstable, study-oriented problems in mathematics can be expected. The behaviouristic point of view is that motivation is an external matter. A **reward** should be offered in the form of a star or some or other kind of approval or praise. The constructivist point of view is that motivation is an internal matter in the sense that it is more important for learners to realise the **meaning** of their efforts relating to this subject. If they regard mathematics as being important for their future, they ought to improve their study orientation in it accordingly.

Corno's (1992:72) criticism is that the term "motivation" does not go far enough:

> *The question of volition should be addressed as well ... volition connotes a kind of diligence that goes beyond simple interest or goal directedness ... "sheer willpower" means industrious, conscientious, disciplined – all stronger personal characteristics than "motivated" ... Recently, scientific psychology ... has ... rejected the historical connection between failures to apply volitional resources and weakness of moral character ... Where motivation denotes commitment, volition denotes follow-through.*

This author describes the exercising of willpower as a strategic activity that learners implement to direct and manage their own behaviour, as well as that of others, with a view to achieving a particular goal.

3.4.3.5 Locus of control

Cognitive explanations of motivation (attribution theories) are in reality efforts to explain the reasons for success or failure (Woolfolk, 1993). Various researchers have done research on these and related subjects.

Pedersen, Draguns, Lonner and Trimble (1989:315) distinguish between the constructs "internal" and "external Locus of control" as follows:

> *By virtue of the nature of the social environment that fosters achievement and autonomy, (some learners) can be characterized as "internals" – that*

> *is, they perceive reinforcement accruing from the results of their own efforts since they control the environment. Where reinforcement is random or capricious and in general not under one's control, an "external" person may believe that fate, luck, or chance is the rule of thumb in life.*

The aforementioned authors based their view of the Locus of control on Rotter's view (1954). He refers to persons with an internal Locus of control as those who believe that they **themselves** are responsible for their own fate. Such people work willingly in situations where skills and effort can lead to success. Other people display an external Locus of control. These people believe that persons and forces beyond their control are in control of success or failure in their lives. Such people prefer work environments in which luck determines success or failure.

Locus of control can be directly influenced by the actions of others. Phares (1976) identifies the following three potential causes of the internal-external Locus of control phenomenon:

- Family circumstances: "warm" protective families help learners to develop an internal Locus of control.
- Consistent experiences: inconsistent exercising of authority lets children think that the world is evil and unpredictable and this promotes the perception of an external Locus of control.
- Social circumstances: those who have little access to power or to the opportunity for growth in the financial or personal field, acquire an external Locus of control conviction.

Seligman (1975) is of the opinion that **learned helplessness** is the result of the perception that people have only slight or no control over their situation whereas Abramson, Seligman and Teasdale (1978) hypothesise that the inferences or attributes that persons put forward for the suspected absence of control are critical determinants of learned helplessness.

Weiner (1979) too states that learners classify the reasons for their success or failure mainly into three categories, namely: **Locus** (localising the causes of success or failure) as **internal** (within the learner) or **external** (outside the learner) **stability** (the question of whether the cause is stable or if it can change) as well as **responsibility** (whether the learner can control the cause or not). Weiner (1984) believes that an internal or external Locus of control is closely related to a learner's self-image. If success or failure can be ascribed to internal factors, success should lead to heightened motivation whereas failure might harm a learner's self-image. Stability, however, is closely related to future expectation. If learners believe that success (or failure) can be ascribed to stable factors such as a test's degree of difficulty, they will expect to pass (or fail) in the future in difficult tests. If, however, they ascribe their test results to unstable factors such as moods or luck, then they expect or hope for changes when they are confronted with similar tasks in the future.

Recent research by the Human Sciences Research Council (HSRC) (Howie, 1997) has brought to light that most South African mathematics learners (who did the worst in the TIMMS study) believe that "luck" played a definite role in their ultimate achievement in mathematics, in contrast with learners in a country like Singapore (that fared the best in the TIMMS study) who were convinced that hard work was more probably responsible for their achievement in mathematics.

The responsibility dimension is closely linked with emotions such as rage, gratefulness, sympathy or shame. If learners fail in something about which they feel they should have succeeded in, then they will feel ashamed or guilty about it. If they succeed, then they will feel proud of it. When learners fail to solve "uncontrollable" problems, they will experience anger towards the person or body in control, whereas success will be accompanied by feelings of happiness or gratitude.

Parsons, Meece, Adler and Kaczala (1982:430) state the following about an adequate study orientation to help learners to combat acquired helplessness:

> *Learned helplessness has also been defined in terms of perceived control of one's success and failures. Perceived control can be assessed by*

> *looking at the use of such attributions as immediate effort, consistent effort, and skill or knowledge.*

These authors are of the opinion that an adequate study orientation indicates, *inter alia,* a perception of "being in control" of the assigned task.

3.4.3.6 Insecurity

When learners lose their feeling of security mathematics is usually the first subject in which their achievement declines (Maree, 1992). When learners are assessed by psychological tests, the results confirm this statement. Parents who get divorced, marital discord, deaths, absent parents and overprotection are examples of factors that can deprive learners of their security with potentially destructive consequences. The result is frequently that learners are deprived of their preparedness to "dare". These factors give rise to learners' losing their security, and their affective stability which in turn affects their study orientation adversely (Maree, 1992).

3.4.3.7 Attention and concentration

When learners do mathematics, they have to concentrate. When they merely listen to a number of facts and try to remember them, then they pay attention. In other words in order to concentrate they have to be **active**. In order to pay attention they have to be receptive though passive. The concepts "active" and "passive" are two key concepts with regard to mathematics achievement. If learners cannot concentrate, it gives rise to study orientation problems. Mathematics requires that learners should write, think, do, make representations and sketches and also use their imagination and fantasy. In other words, they have to be **active**. Problem solutions cannot merely be "thought out". As far as possible learners should use their whole brain and all their senses during the study process in mathematics. Bloom and Broder, quoted in Witkowski (1988:165), draw the following comparison in this regard:

> *The major difference between the successful and unsuccessful problem solvers in their extent of thought about the problem was in the degree to*

which their approach to the problem might be characterized as active or passive.

3.4.3.8 Hesitant behaviour

Madge and Van der Westhuizen are of the opinion that the following facets of hesitant behaviour can lead to study orientation problems in mathematics:

- ★ Excessive reserve;
- ★ anxiety; and
- ★ an excessive need for self-protection that implies that the learner is inclined to defensive action. Learners who regard questions as threatening, or as a potential source of a threat because they are so frightened of being mocked that they are unwilling to expose themselves to such questions, may experience problems to attain optimal achievement in mathematics.

3.4.3.9 Physical problems

A problem like deafness or poor vision does not necessarily have to lead to problems but if the physical problem is not properly handled it could contribute to learners' developing learning problems in mathematics.

3.4.3.10 Poor health

Low blood-sugar, general fatigue, deficient diet (Conners, 1990) and lack of energy are examples of physical problems that could lead to secondary problems, in this case learning problems, in mathematics.

3.4.3.11 Developmental problems

Learners who develop or mature later than their class group or who are younger or smaller than their companions, sometimes experience study orientation problems in mathematics.

3.4.3.12 Emotional problems

Morgan, Deese and Deese (1981:104) advise learners as follows in this regard:

> *If you don't understand, don't let it go by. Don't put it down to being "dumb in math" or "having no head for science". The chances are that you missed learning something that is essential – just being home sick a couple of weeks while in the seventh grade could have done it.*

Learners who experience emotional problems can react in one of two ways to the unfavourable circumstances. The circumstances might motivate them to strive for good achievement so that they can escape from their negative circumstances depending on the supporting structures that are at their disposal. Such learners, however, can develop a lower aspiration and achievement of self-realisation level than that which they are capable of (Maree, 1992).

3.4.3.13 Attitudes

Passow and Schiff (1989:5) emphasize indirectly an important aspect of an adequate study orientation in mathematics in the following words:

> *We must sensitize gifted children and youth to the major problems our world societies face – among them, poverty, famine, war and nuclear annihilation, racial/tribal conflict, depletion of natural resources, environmental pollution, cultural conflict, personal and communal health, genetic changes, population growth, quality of life ... to devote their lives to building bridges of understanding.*

In other words, an adequate study orientation in mathematics should be based on, *inter alia,* the ideal of optimal self-actualisation; not only in the service of the individual, but especially in the service of the more lofty social ideals such as the upliftment of mankind in general.

3.4.3.14 Gender-related differences in achievement

Macleod's (1995) research led her to conclude that there is a general perception that girls are endowed with less natural aptitude than boys, that they are regarded as needy, hardworking, 'good' and empathic, in contrast to boys who are seen as competitive, full of self-confidence, self-assertive and endowed with a natural talent for mathematics. Related to this there is a general perception that mathematics is regarded as a subject in which competition, self-confidence and natural talent are important. Thus the conclusion can readily be drawn (Macleod, 1995) that girls find themselves in a double-bind position. They are encouraged to take mathematics and to do well in the subject but at the same time the unspoken perception exists that mathematics is not really a subject for girls (Tartre & Fennema, 1995). Research by the HSRC (1997) confirms that the natural sciences are still a "male" domain and that the position of women did not improve in the period 1985 to 1994.

Fennema and Hart (1994) point out intensive research, on such subjects, with the following trends coming to the fore:

- ★ Gender-related differences with regard to achievement in mathematics may well be on the decline, but these differences exist especially concerning –
  - the learning of more advanced mathematics;
  - views on mathematics; and
  - mathematics-related career choices.
- ★ Gender-related differences concerning mathematics are influenced by –
  - socio-economic status and ethnicity; and
  - learners' schools and teachers.
- ★ Teachers are inclined to structure their classrooms in such a way that boys benefit from this at the expense of the girls.
- ★ Intervention strategies can be designed and applied to rectify the situation.

3.4.3.15 Help-seeking behaviour

Learners are sometimes afraid to look for help when their achievement is inadequate. For these learners the implication is that they are looking for assistance since they acknowledge failure. These learners avoid seeking help and they would rather deny their own study orientation and achievement problems than run the risk of having their self-image harmed in the eyes of their companions – something that is painful to such learners' ego potential. Pollock and Wilkinson (1988) nevertheless believe that it is frequently necessary for learners to seek help so that they can improve their chances of success.

3.4.4 Teaching problems

3.4.4.1 The difference between learning and achievement problems

To begin with, a brief comparison will be drawn between the two mentioned classes of study-orientation problems in mathematics.

(i) Learning problems

In this case the learning process is adequate but emotional problems handicap the learner. A learner with a traumatic home background is, for example, vulnerable and potentially a candidate for study orientation problems in mathematics.

(ii) Achievement problems

Other than in the case of learning problems, learners with achievement problems experience few problems in learning the work. Some or other factor, however, interferes with the way in which the learners reproduce that which has indeed been learnt. A learner with an achievement problem is thus someone who masters mathematical concepts within one context, but does not succeed in reproducing them within another context. The excessively perfectionistic learner who wants to do everything right and as a result of this experiences problems, illustrates this phenomenon.

Gannon and Ginsburg (1985) come to the conclusion that the following factors could adversely affect achievement in mathematics:

- ★ Cognitive style;
- ★ the particular nature of the task;
- ★ test anxiety; and
- ★ variables that relate to the learner's mental condition, including fatigue and boredom.

The same cause can affect different learners in different ways. Some learners might experience emotional problems and be unable to learn as a result whereas others may also experience emotional problems, but as a result be unable to reproduce their knowledge.

3.4.4.2 Problem-solving and problem-centring

VanderStoep and Seifert (1994:34) ask the following question in connection with problem-solving skills:

> *Are people's cognitive skills limited to the contexts in which they were acquired, or can they be used in a variety of situations?*

These authors (1994:43) later surmise that inference of the problem-solving process probably occurs only in the case of mathematics:

> *The generality of the problem-solving process – identification, access, application – is not known. It is possible that mathematical problem solving is the only content domain in which these particular cognitive operations occur in this fashion.*

A problem-centred approach to a study orientation in mathematics has above all the optimising of problem-solving behaviour in mathematics as its objective (NCSM, 1977; Cockcroft, 1982). This approach implies that learners, ideally, are still learning certain rules and principles in mathematics. The cardinal difference is, however, that they are doing this within **problem-solving contexts** instead of merely **memorising** these rules, theorems and principles.

With reference to what already has been stated on the subject in Chapter 2, at this stage it can only be confirmed that the problem-centred approach as a basis for a study orientation in mathematics is founded on the following principles:

- ★ The application of problem-solving as a medium of learning.
- ★ Learners develop their problem-solving skills through coming across problem situations.
- ★ Learners learn to evaluate and explain their own efforts.
- ★ Learners acquire the ability to think logically.
- ★ Learners develop a healthy attitude of daring and independence.
- ★ Learners learn to check their own hypotheses and results.
- ★ Learners, on account of their own involvement, develop a positive self-image in mathematics.

3.4.4.3 Social interaction

Cobb, *et al.,* (1992:485) state the following on the significance of social interaction in mathematics.

> *An emphasis on social interaction brings with it the notion that mathematical learning is a process of enculturation in which students come to be able to participate increasingly in the mathematical practises institutionalized by the wider society ... Learning opportunities can then be seen to arise for students as they and the teacher interactively constitute taken-as-shared mathematical interpretations and understandings ... students (should) have frequent opportunities to discuss, critique, explain ... when necessary, justify their interpretations and solutions ... work collaboratively in small groups and ... participate in whole-class discussions of their problems, interpretations, and solutions.*

Their view is that the debate should not be about the question whether learners construct their own ways to acquire knowledge, but rather whether the metaphor of active construction of knowledge is a useful and ethically suitable way to describe certain internal structures in learners.

3.4.4.4 Co-operative learning

Co-operative learning can be described as an approach in which small groups of learners work together as a team to solve a problem, to complete a task or achieve a common aim. Group members should realise they are members of a team and that the group's success or failure will be shared by all the members. To obtain the group's aim members will have to communicate with each other with regard to the problem and also help one another. The NCTM (1989:79) describes the principles of co-operative learning in mathematics as follows:

> *Small groups provide a forum in which students ask questions, discuss ideas, make mistakes, learn to [make provision for] others' ideas, offer constructive criticism, and summarize their discoveries in writing.*

By studying mathematics in this manner learners at the same time acquire important life skills. The social side of group work is pleasant, learners make new friends, they learn to respect their differences (in aptitude and other personal traits) and divergent opinions. The learner who helps others, experiences the joy of giving. Learners who know that they can count on others for assistance, do not experience the anxiety that learners frequently feel when they do not understand mathematics. Consequently co-operative learning provides intrinsic motivation to master mathematics.

3.4.4.5 Discussion

(i) The value of discussion in mathematics

Discussion is probably the most basic form for learning mathematics. The Cockcroft Report (Britain) regards discussion for example as the main reason for learning mathematics (Brissenden, 1989:3):

> *Because it is a powerful means of communication.*

Skemp (1982) also emphasizes how important it is that learners should, in good time or otherwise, talk about mathematics before they begin to read mathematics.

Why are conversations, discussions and communication so important in the mathematics class?

(a) Command of language

Mathematics terminology should be introduced in all discussions with learners and also controlled so that this terminology can be **used** optimally, because:

> *Mathematics does not grow through a monotonous increase in a number of indubitably established theorems but through the incessant improvement of guesses by speculation and criticism, by the logic of proofs and refutation* (Lakatos, 1976:5).

(b) The meaning of conversation in the development of insight

Brissenden (1989), in following the views of Skemp, declares that the aim of conversation in the mathematics class is both “relational insight” (insight into the reasons why rules work) as well as “logical insight” (the ability to explain to others) instead of mere “instrumental insight” (the use of rules without knowing why these rules work).

(c) Conversation as a way of developing social skills

If learners are able to hold their own in the mathematics class they can, outside the class, more easily function as member or as mouthpiece of a group, by arguing for and against something, and produce or deal with criticism.

(d) Conversation as a means of evaluating

Diagnostic discussion is probably much better than any other medium of evaluation in mathematics. With the aid of conversation learners' insight and progress during group discussions can be evaluated in a continuous and detailed manner. In addition this makes immediate and flexible feedback possible. The quality of learners' conversations in mathematics can be evaluated in the same way.

(e) The nature of class discussions

Brissenden (1989:12) defines the class conversation in mathematics as follows:

> *Pupils ... meet together to solve a common problem, or achieve a common goal, by sharing goals and modifying their opinions, ideas and understanding.*

(f) Opinion polls in the mathematics class (Schminke, *et al.*, 1978)

Psychologists have a wide range of standardised tests and questionnaires at their disposal by means of which learners' study orientation (and related factors) can be evaluated. Few teachers are psychologists (and therefore permitted to use psychological tests to evaluate study orientation problems with the help of sophisticated tests and questionnaires. Study orientation problems in mathematics can nevertheless be explored at this level if teachers from time to time make use of informal questionnaires in order to determine learners' study orientation in mathematics. Such questionnaires could include the following:

- ★ Learner preference questionnaires. Learners can simply be asked to provide a list of their school subjects and allocate a 1 to the subject they like best, a 2 to the subject liked second best, etc.
- ★ Making notes on observations. Systematic observation and taking down notes on the behaviour of individual mathematics learners frequently bring to light relevant information on their attitudes towards the subject and the reasons for their achievement problems in mathematics.
- ★ Formal attitude tests. Learners can for instance be requested to indicate their attitudes towards mathematics according to a semantic differential scale.

3.4.4.6 Technical errors

Movshovitz-Hadar, Inbar and Zaslavsky (1986) point out the inhibiting effect of "technical problems" on learners' achievements in mathematics. These resultant problems include the following:

- ★ Artless or careless printing of examination papers and tests;
- ★ misleading figures;
- ★ inadequate design of tests and examination papers; and
- ★ ambiguous phrasing.

This chapter will be concluded by briefly focussing on certain psychological models explaining inadequate achievement in mathematics.

3.5 PSYCHOLOGICAL MODELS[2] EXPLAINING STUDY ORIENTATION AND ACHIEVEMENT PROBLEMS IN MATHEMATICS

In their search for explanations of inadequate achievements in mathematics and the factors that constitute the foundation for these inadequate achievements, researchers focus on already known theories or they create new theories. These theories and explanatory models are not mutually exclusive, and the implementation of new theories does not mean that early theories are "obsolete" or "wrong".

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[2] A model can be described as a particular view of a specific set of assumptions concerning a particular matter; something that researchers can apply to make scientific observations (Plug, Meyer, Louw & Gouws, 1993).

One theory does not necessarily cancel another and weaker theories do not necessarily mean “nothing” (Carson & Butcher, 1992; Shaffer, 1996; Theron & Louw, 1995; Thompson & Rudolp, 1992).

The first model to be considered is the developmental model.

3.5.1 The developmental model

According to this model learners first have to undergo certain developmental stages before they are ready or able to understand various mathematical concepts and principles. According to them drillwork and repeated practice in mathematics will not necessarily lead to “perfect mathematics”. Accelerated learning is not recommended although it is possible. Self-learning and the discovery of mathematics and mathematical concepts are recommended as a requirement for the adequate learning of mathematics.

3.5.2 The behaviouristic model

This model interprets learning problems in mathematics as a matter of acquired behaviour. Skinner, in Copeland (1984), expresses the principle as follows: if you provide the right circumstances you will be able to get people to do anything you require. Study orientation problems in mathematics will be regarded as “poor” behaviour. This “poor” behaviour can be wiped out through a process of deconditioning. An inadequate study orientation in mathematics therefore does not imply the absence of higher order thinking processes, but the repeated application and practice of incorrect or poor behaviour in the mathematics class. Learners’ mistakes have to be “unlearned” by being taught the correct principles for these learners instead of carrying them out through error analyses and follow-ups.

3.5.3 The medical model

This model regards problem behaviour as the result of chemical and organical dysfunctioning. It emphasises the potentially causal role of medical factors as the source of poor mathematical achievement. Heredity plays a role in causing achievement problems in mathematics in the sense that both the weaker genes as well as a tendency towards particular medical defects are sometimes inherited by learners. Learners' medical problems should be treated with suitable medication.

3.5.4 The psycho-analytic model

Here the emphasis is especially on the unconscious or symbolic factors in the causation of pathology (Hughes, 1983). The phenomenon of mathematics anxiety (Morgan, *et al.*, 1981; Visser, 1989) is put forward by supporters of this approach as indicative of the contributory role that unconscious or symbolic factors can play in the origin of study orientation problems.

3.5.5 The cultural model

Supporters of this model reason that it is not correct to accept that mathematics is culture-free, merely because it is regarded as a universal and international language (Woodrow, 1984). Factors such as the following: mathematics is taught mostly in English – learners speak their mother-tongue at home, but switch to English as learning medium at school and university (Christie, 1989); there are particular cultural differences present between the groups, but these factors are not always taken into account in the learning situation; the fact that some mathematics textbooks are prejudiced and discriminate against certain racial groups and women (Hudson, 1987) are considered to be important (Steen, 1987).

3.5.6 The curricular model

The emphasis here is, *inter alia,* that syllabuses in mathematics are usually 10 years old:

> *Math texts are usually 10 years out of date; yet, math is a rapidly growing field* (Steen, 1987:39).

It is also argued that there is not much connection between commerce, industry and mathematics. Welch (1988), however, believes that mathematics should be studied in a more practical, but not necessarily simpler way.

3.5.7 The statistical model

An inadequate study orientation in mathematics is regarded as a standard deviation from the accepted norm. If one looks at the normal distribution of the population, it can be expected that there may be a significant number of learners who might experience (even serious) study orientation problems in mathematics.

3.5.8 The social model

This model emphasises the attenuated social development of the learner with study orientation problems in mathematics. Factors such as a poor educational milieu, poor language development, domestic instability, inadequate parental guidance and poor adjustment to the learner's environment can lead to achievement problems in mathematics (Grossnickle, *et al.*, 1983). Unless these problems can be solved there cannot be any possibility of remediating the study orientation problems in mathematics.

3.5.9 The transactional model

This model points out that mathematics achievement, like all human behaviour, is relative to, and dependent on, significant others. Study orientation problems in mathematics are regarded as being primarily the result of a disturbance in learners' relationships and communications with other significant others. Thus a father who encourages his children in an inadequate manner, a mother who shows lack of interest in her children's mathematics and their achievement in this subject, and families in which the value of mathematics is underestimated, are examples of transactions that have an inhibiting effect on study orientation in mathematics.

3.5.10 The moral model

Study orientation problems in mathematics are seen as a deviation from the ethical and moral standards of a particular community. Laziness, inadequate motivation and undisciplined behaviour are relevant examples.

3.5.11 Dyscalculia as model

Reference is being made here to certain learners' apparent inability to master mathematics. Giordano (1987:70) defines this model as follows:

> *Dyscalculia is the peculiar inability to learn mathematics, an inability that cannot necessarily be predicted on the basis of an individual's success in nonmathematical subjects.*

3.5.12 Dyspedagogia as model

Gannon and Ginsburg (1985:411) criticise the training of teachers in this connection as follows:

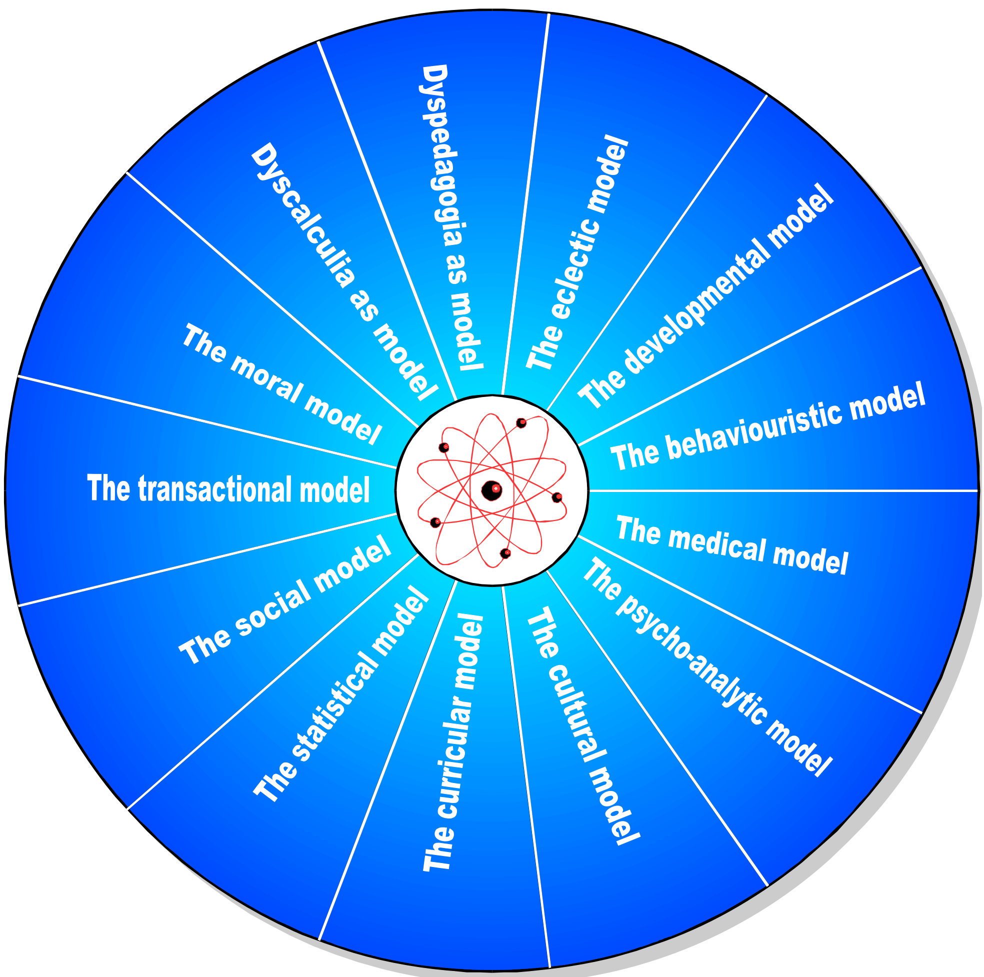
> *Some learning problems are really teaching inadequacies ... Teachers in training usually study little mathematics and are seldom introduced to research on children's mathematical thinking.*

3.5.13 The eclectic model

This model shows that (a combination) of certain of the aforementioned models has significance in the phenomenon of an inadequate study orientation in mathematics. A multidimensional approach to optimising learners' mathematics achievement is suggested.

The preceding models are schematically represented by Figure 3.2.

FIGURE 3.2: SCHEMATIC REPRESENTATION OF SOME POSSIBLE MODELS FOR EXPLAINING STUDY ORIENTATION AND ACHIEVEMENT PROBLEMS IN MATHEMATICS



3.6 SYNTHESIS

In Chapter 3 the emphasis was on the "construct study orientation" and a review of relevant literature was obtained on certain factors that influence study orientation and achievement in mathematics. The following summarising remarks can be made:

- ★ Each learner has his or her own lifestyle and consequently also his or her own cognitive style in mathematics
- ★ Each learner's inadequate study orientation and achievement in mathematics will have to be approached in a unique way to make possible the identification of that learner's idiosyncratic achievement and study orientation problems in mathematics.

The preceding argumentation confirms the existence of the need for a standardised questionnaire to measure aspects of learners' study orientation in mathematics. Such a questionnaire should offer psychologists the opportunity to obtain more information about their learners than mere information on their cognitive achievement in mathematics. It is particularly important to investigate continuously factors other than the mere evaluation of objectives that are intended to measure cognitive progress in mathematics. The focus of this questionnaire should be a cardinal aspect of the potential problems relating to the learning of mathematics: the root of the problems mentioned is **also** found, possibly **especially**, **outside** the cognitive domain. The importance of a firm affective foundation as a necessary supporting structure for adequate cognitive achievement in mathematics, can scarcely be overestimated. Learners' emotions, their habits in and attitudes towards mathematics, the way in which they process their mathematical information, their problem-solving conduct (problem-solving mindedness and ability in mathematics), social factors such as learners' study milieu (social, physical and experienced milieu) have to be explored. In other words their feeling towards mathematics, the way in which they experience their teachers, the class atmosphere, their domestic circumstances and how they experience the teaching of the subject, play a significant role in their ultimate study orientation in mathematics. **These factors will have to be taken into**

account thoroughly when establishing an item pool for the envisaged study orientation questionnaire in mathematics.

The significance of a holistic approach to handling achievement problems in mathematics is also discussed in this chapter. Naturally no **single** approach in isolation will be able to provide enough information for **all** learners' study orientation problems in mathematics. Each of the preceding models was developed in contexts other than the context they were used for in this study. The aim of this study is particularly to indicate that each of these mentioned approaches can be useful in handling various learners' achievement problems in mathematics.

Although it may appear that some of these models are practically irreconcilable, the view is taken that each model has potential significance for individual cases, depending on the nature of the individual case. Wachsmuth and Lorenz (1987:44) state the following in this connection:

> *Models ... are not to be termed right or wrong in the way that they describe a behavior, as they take different perspectives upon what questions are posed and answered concerning a given behavior. A debate between different diagnostic approaches does not refer to the consistency and validity of the models but to the question of whether a particular problem can be answered by a specific theory.*

For the purpose of this study the term "holistic" will be used to emphasise that study orientation and achievement problems in mathematics should not be considered in isolation, but at all times be assessed within the context of the person who is experiencing these problems. In this way a good understanding of study orientation and achievement problems in mathematics is always combined with humanitarian understanding of learners as unique individuals, or personal, circumstantial as well as relationship levels with regard to their psychobiological composition, their psychophysiological constitution and their intrapsychic functioning.

Irrespective of which models psychologists regard as primary, and which as secondary in the identification and explanation of study orientation problems in mathematics, the point of departure should always be that these models complement but do not contradict or oppose one another. These models should be regarded as supplementary, mutually enriching and equivalent.

CHAPTER 4

A CROSS-CULTURAL PERSPECTIVE ON ACHIEVEMENT PROBLEMS IN MATHEMATICS WITH REFERENCE TO THE MEASUREMENT OF A STUDY ORIENTATION IN MATHEMATICS

4.1 BACKGROUND

4.1.1 Introduction

There is cause for concern about the relevance and effectiveness of some of the accepted and established tests in South Africa when these tests are applied to members of minority groups. South African blacks have, for a considerable time, constituted such a "minority group", simply because the apartheid system relegated them to an inferior social and legal status, with little power or human dignity, on the grounds of their ethnic origins (Sibaya, Hlongwane & Makunga, 1996).

The potential impact of cultural forces/influences and of the acculturation process (including the influence of the environment on learners — such as the effect of urbanization on socio-cultural factors like the language they use), as well as the impact of lifestyle and levels of education should be considered when any measuring instrument is designed and standardized. This applies to South Africa in particular, where specifically the black population is being urbanized and black as well as other learners are increasingly being exposed to English as the language of instruction and communication. Curran (1988) cautions educators to remember that different socio-cultural groups (societies) value different types of skills and expertise, with the result that members of distinct social groups do not develop along uniform lines.

Ideally, quantitative measurement, observation of behaviour and a qualitative analysis of test results should be combined to enable psychologists to enter the phenomenological world of the testee. The approach, therefore, should be process-centred rather than strictly product-centred (Sibaya, *et al.*, 1996). Indeed,

> *understanding another culture is not a discrete process, but rather a continuous process* (Callahan, 1994:122).

4.1.2 Defining some concepts

4.1.2.1 Culture

According to Jahoda (1984) culture is probably the most elusive concept in the vocabulary of the social sciences. He (Jahoda, 1993) points out that cross-cultural psychology is characterized by positivism and empiricism and that it uses the construct "culture" as if it were an external phenomenon. Researchers (Jahoda, 1993) commonly try to minimize the importance of this concept in psychology, as they regard it as elusive. Poortinga (1990) describes culture as shared restrictions that limit the repertoire of behaviour that is available to the members of a certain socio-cultural group, in a way that differs from any other group. Schein (1993) defines culture as the sum total of what a particular group has learned as a group. This sum total of insights is usually embodied in a collection of shared, basic, underlying assumptions that no longer function at a conscious level, but are commonly accepted as representative of the world as this group perceives it. Retief (1992) adds that the concepts of "**systems of meanings"** and "**communication of meanings"** are essential and important aspects of the construct "**cultural systems"**. Linton's definition of the construct "culture" (1945) is, after many years, still useful, and is accepted for the purposes of this study. He refers to culture as the configuration of acquired behaviour, as well as the results of behaviour, of which the components and elements are shared and passed on by members of a particular society. From a post-modernist perspective one should remember that there are no absolute or "master" truths, and that behaviour is always viewed from a specific point of view, at a specific time and for a specific purpose. Human perceptions of factors such as culture, behaviour and the results of behaviour are, in other words, relative and subject to time and circumstances.

4.1.2.2 Cross-cultural

Plug, Meyer, Louw and Gouws (1993) define intercultural or cross-cultural research as research aimed at evaluating the impact of social and environmental influences on behaviour, by examining and comparing various cultures. Biesheuvel (1987) points out that although the construct "cross-cultural" may be the most crucial discipline in psychology, it has remained relatively vague and undefined. According to Cronbach and Drenth (in Biesheuvel, 1987) it is not only research involving two or more nations that can be referred to as cross-cultural research. Research may also be regarded as cross-cultural when two or more distinct groups within the same nation are tested.

4.1.2.3 Multicultural

Pederson (1991) defines the concept "multi-culturality" as a fourth theoretical premise, in addition to psychodynamic, behaviouristic and humanist structures, when human behaviour is described and explained. According to Pederson, the basic problem psychologists grapple with is describing behaviour in terms of a specific culture, and, at the same time, comparing it with similar behaviour in another culture or other cultures. When general and specific views are reconciled, a multicultural perspective is reached (Pederson 1984). Woodrow (1984) believes that mathematics may be regarded as a useful tool in promoting a multicultural society.

4.1.2.4 Socio-cultural

According to Ardila (1995) the term "socio-cultural" encompasses three (closely related) variables, namely education, culture and language. Shuttleworth-Jordan (1996) furthermore cautions that socio-cultural influences on **cognitive** measuring instruments also appertain to the design of **non-cognitive** measuring instruments. She believes that one should clearly distinguish between the following:

- ★ Racial diversity (ethnic factors that are more or less robust or constant) that on its own could account for differences between groups;
- ★ socio-cultural differences (that are constantly changing and are chiefly dependent on environmental influences, and that include factors such as socialization,

mother-tongue, the language currently being used, levels of education, socio-economic status and test sophistication). These differences are often associated with racial dissimilarities and are known to cause significant discrepancies in the performance of testees. Van den Berg (1992) warns that communication problems may arise when psychological and educational tests are conducted in multicultural communities in which more than one home language is spoken.

4.1.2.5 Ethnomathematics

Ethnomathematics is closely interwoven with multicultural learning. Statements on ethnomathematics and studies of it are often of a political nature. The term was coined by D'Ambrosio (1985:45), who defines the concept of "ethnomathematics" as

> *mathematics which is practised among identifiable cultural groups, such as national tribal societies, labour groups, children of a certain age bracket, professional classes, and so on.*

Vithal (1993) points out that all persons who are engaged in mathematics are, according to this definition, busy with "ethnic mathematics". Vithal (1993:275) summarizes D'Ambrosio's subsequent explanation of his original definition as follows:

> *In later papers, D'Ambrosio suggests that ethnomathematics be defined etymologically as the art or technique (**tics**) of understanding, explaining, learning about, coping with and managing reality (**mathema**) in different natural, social and cultural (**ethno**) environments.*

Problems with study orientation in mathematics in South Africa should be viewed against the background of a multicultural population. Advocates of *ethnomathematics* draw attention to the fact that, in mathematics, subject matter is predominantly westernized. A problem that is perceived to be practical and to the point by learners from one population group, may remain abstract, theoretical and irrelevant to learners belonging to another group.

Supporters of *ethnomathematics* believe that subject matter should be compiled with the needs of specific culture groups in mind. This implies that subject matter that is appropriate for one population group, will not necessarily meet the requirements of another. A number of authors, however, warn that this approach to the teaching of mathematics might result in the creation of a new kind of "apartheid" in the mathematics classroom (Brodie, 1994).

4.1.3 The need for cross-cultural tests

There is a need to find measuring instruments that will facilitate the creation of common psychological and educational programmes for the multiracial, multi-ethnic and multicultural communities of post-apartheid South Africa. Thembela (1991:1) puts it as follows:

> *As we move towards non-racial and non-discriminatory education systems, the idea of multiculturalism in education becomes very important if children from cultural minorities are not to be disadvantaged in the development of a sense of self-esteem and self-respect.*

Research into bias in testing is still in its initial stages in South Africa, as authorities until recently were of the opinion that there was little need to construct general tests suitable for all the different population groups. This shortcoming is criticised by Sibaya, *et al., (1996:108)*:

> *There is very limited scientific literature on standardized psychological tests or instruments used with minority groups and, in particular, with black people in the RSA.*

The changing socio-political situation in post-apartheid South Africa, however, has led to a growing tendency to replace separate tests for various population groups with common tests.

According to Claassen (1996) and Prinsloo (1996) the opinion has repeatedly been expressed that separate tests (or the same tests in different languages) are not only undesirable, but totally unacceptable to the majority of South Africans. This will evidently have a significant impact when cross-cultural research – particularly with regard to the selection of strategies to establish different levels of equivalence – is planned and carried out.

4.2 CULTURE AND PROBLEMS WITH MATHEMATICS ACHIEVEMENT

4.2.1 The situation at international level

Scholnick (1988:86-87) points out that people's cultural background can influence their orientation towards study and their achievement in mathematics in general. He states:

> *There are also strong biases about who can learn mathematics, and pervasive differences in learning skills ... . There is hot debate about whether there are genetic differences in mathematical capacities ... . Similarly, there may be cultural differences in the patterning of skills that reflect attitude and values about the role of mathematics in daily life ... . Although mathematics is not a Rorschach blot that every society and family within a society can interpret, nevertheless there may be fundamental differences in aspects of mathematics that different cultures may stress ... that may account in part for the difference in mathematics achievement.*

Garrison (1986) and Griffin (1990) suggest the following reasons for underachievement in mathematics among Afro-American students:

- Penurious socio-economic background;
- inadequate academic preparation;
- inability to adjust to a college or school;
- cultural disorientation with regard to academic competition and success;
- lack of institutional structures to assist students' personal development.

Marsh (1993:144) quotes McKnight on the situation in the USA:

In school mathematics the United States is an underachieving nation, and our curriculum is helping to create a nation of underachievers.

Jones and Minor (1981) add a list of factors that contribute to the situation being perpetuated: inadequate financial support, low expectations regarding the ability of black students to do well – specifically in mathematics – and a lack of aggressive and well-organized programmes to enlist, select and retain students. Anderson (1990a:266) agrees:

Behaviors that result from students operating under perceptions of low achievement and even lower expectations gravely alter the progress of those students. Thus, the significant others must become more sensitive to the effects of their behavior on students' performance, especially in mathematics.

Johnson (1984) states that blacks in America underachieve gravely in mathematics, a situation for which he blames culture-related factors such as the following:

(a) an absence of role models; (b) a lack of significant others, such as parents, who show an interest in mathematical achievement; (c) a failure to receive positive career counselling; (d) a view of mathematics as a subject appropriate for white males; (e) an inability to see the usefulness and relevance of mathematics to their lives, both present and future; and, of course, (f) a lack of success in previous mathematics courses. These factors are related to one another and are rooted in centuries of institutionalized racism that perpetuated unequal education for black people.

The National Council of Teachers of Mathematics (NCTM, 1991:144) states the following regarding the impact of cultural factors on learners' study orientation in mathematics:

The preservice and continuing education of teachers of mathematics should provide multiple perspectives on students as learners of mathematics *by developing teachers' knowledge of the influence of students' linguistic, ethnic, racial, and socio-economic backgrounds and gender on learning mathematics.*

The Cockcroft report (Cockcroft, 1982) draws attention to the extent of the problem in Britain.

4.2.2 The situation in South Africa

After an extensive statistical investigation Molepo (1997) found that culture-related factors play a significant role in causing the inadequate performance of black learners in mathematics. He has, *inter alia*, found that a statistically significant percentage of **rural black parents** prefer their children not to show an adequate study orientation in mathematics. These parents are especially afraid that achievement in mathematics may lead to their children becoming "tsotsis" or "too clever, and therefore criminals".

With reference to what already has been stated on this topic in Chapter 1, the following table (Blankley, 1994) is provided to indicate the extent to which the achievement profiles of various South African population groups differ in mathematics and physical science.

TABLE 4.1: ESTIMATE OF THE RELATIVE PROGRESS MADE BY SOUTH AFRICAN LEARNERS IN NATURAL SCIENCES IN SOUTH AFRICAN SCHOOLS

Population group → ↓ Number of persons who:	Coloured	Indian	White	Black
enter school for the first time	45,9	6,2	5,1	312
pass matric	7,0	3,5	3,7	30,9
obtain matriculation exemption	1,8	1,7	1,6	6,4
obtain exemption in mathematics and physical science	1,0	1,0	1,0	1,0
% of those entering school who will pass matric with exemption in mathematics and physical science	2,18	16,12	19,61	0,32

Table 4.1 indicates that the performance of black learners in physical science and mathematics at the end of matric is considerably poorer than that of learners belonging to other population groups.

4.2.3 Some hypotheses to explain inadequate achievement in mathematics specifically among black learners

The following are some of the probable reasons for black learners' inadequate achievement in mathematics:

- ★ Teacher training may prove to be unsatisfactory;
- ★ the culture of apartheid that, for many years, prevailed in mathematics classrooms – "at a time when South Africa was trying to prove the rest of the world wrong" (Smit, 1992:4); and
- ★ the phenomenon that mathematics did not seem to flourish under traditional styles of learning and teaching.

Hypotheses offering explanations for this phenomenon can be divided into three main groups:

Hypothesis One: Black learners of mathematics are incapable of competing on a level with whites. Black culture is "inferior" to white culture. Jung (in Masson, 1988) considers Blacks to be 1 200 years behind Westerners in terms of development. He voices his concern about Afro-Americans living among whites, as follows:

> *living together with these barbaric races exerts a suggestive effect on the laboriously tamed instinct of the white race and tends to pull it down.*

He furthermore attempts to justify this racist point of view by referring to the South African situation:

> *The Dutch, who were at the time of their colonizing a developed and civilized people, dropped to a much lower level because of their contact with the savage races* (Masson, 1988:115).

Mjoli (1987:8) states that

> *the main cause of this poverty and underdevelopment lies in some cultural factors which militate against creativity, productivity and the like ... some people will say ... that I am talking nonsense and that the real cause ... is oppression by whites ... (however) ... if our culture had been as highly developed as Western Civilization, we would never have been oppressed by the whites in the first place.*

In this study, these and similar views are regarded as racist, totally unacceptable and not based on scientific research. There is no scientific proof to support the view taken by exponents of this hypothesis (Kamin, 1995). Neither are there any scientific grounds for the view that the cultural background of blacks undermines their **potential** for creativity, productivity and achievement in mathematics.

Hypothesis Two: Inadequate performance in mathematics by black learners may be regarded as an example of acquired helplessness, as a consequence of the cumulative effect of various factors.

Gobodo (1990:95) explains this hypothesis as follows:

> *Black people have been historically silenced, and as a coping or survival measure, they have learned to be submissive. It would thus be logical to talk about black people's "learned helplessness" rather than begin to blame the victim's culture.*

In this study the view is taken that South African blacks have been disenfranchised for a long time, have been deprived of the right to be taught mathematics in their own language (up to Grade 12 level), to study at universities of their choice, to receive treatment and education worthy of human beings; as well as the right to compete for jobs on an equal footing with whites. This has, to a significant extent, contributed to the disempowerment of blacks in general, and specifically to the unsatisfactory level they presently reach in mathematics.

Hypothesis Three: Blacks are not inherently "inferior" to whites. They are capable of performing as well as their counterparts from other population groups if they are given equal opportunities for self-actualization. Supporters of this hypothesis argue that blacks perform poorly in mathematics because historically, educationally and economically they have been disadvantaged. They are of the opinion that there are clearly identifiable reasons for the problem - reasons such as the historical set-backs blacks suffered and racial discrimination - that have cumulatively contributed to the under-achievement of blacks in mathematics.

Castle (1992) speculates that factors such as the following have led to the gloomy situation: the lack of schools for black learners, poorly qualified teachers, a chronic shortage of books and other materials, the lack of compulsory education for blacks, social dissent by black learners (boycotting school, for instance), the destruction of school property by blacks, as well as the presence of the security forces in the townships. Arnott *et al.* (1997), as well as Carstens, Du Plessis and Vorster (1986) agree.

They hypothesise that the extent of the problem of poor performance in mathematics in black schools could be ascribed to, *inter alia*, a lack of suitably and adequately qualified black teachers of mathematics, and the resulting poor quality of teaching. A mere 11% of all black teachers possess a degree, and only a small minority of these teachers took mathematics as one of their subjects (Carstens, *et al.*, 1986). Howie (1997) furthermore mentions the extremely poor family background and general socio-economic environment that blacks have to contend with:

> *[circumstances] so poor that they can scarcely be imagined by first-world researchers. Survival is often given priority over education* (Howie, 1997:52).

She mentions contributing factors such as poor conditions in black schools; the negative impact of peer pressure on black learners (who are often uncomfortable if they perform better than their friends); the fact that girls are not encouraged to enter mathematics-related careers; the fact that speakers of African languages have to learn mathematics through medium of English; irrelevant syllabi; the lack of encouragement to blacks, for the past forty years, to perform well in mathematics; and teaching styles: *"The general approach is 'chalk-and-talk' rather than hands-on"* (Howie, 1997:55).

In this study **hypothesis three** is endorsed. The view is taken that a combination of the factors mentioned in the paragraphs on hypotheses two and three above, provides a satisfactory explanation for South Africa's black learners' inadequate performance in mathematics.

4.3 CULTURE-RELATED LINGUISTIC PROBLEMS

4.3.1 Introduction

Mathe (1991:40) indicates the following potential study orientation problems when learners learn mathematics through medium of a second language.

> *It seems to me that a central problem in (learning) mathematics remains that pupils do not understand the teacher's subject related language as the*

teacher intends ... . A child, in particular, is often ignorant of the distinctive non-vernacular meanings of apparently familiar words.

4.3.2 Teaching in a language other than the mother-tongue

There is usually a connection between study orientation problems in mathematics in a multicultural society and the language of instruction. Learning mathematical jargon in a multicultural society is a complex matter. Presmeg (1989:19) declares:

> *there are at least two types of language-related difficulties in learning mathematics (in multicultural classrooms) ... type A is caused by lack of fluency in the language of instruction, ... type B arises when the thought processes assumed by the teacher or curriculum developer are not those of the learner.*

Viewed from a linguistic perspective it is understandable that learners will experience difficulties with study orientation in mathematics. When one considers the fact that South Africa has eleven official languages; and that learners often speak their mother-tongue at home, are taught in a second language (a regional language) during their first four school years, and then have to switch to English (an international language), it becomes clear why South Africa currently has problems in connection with mathematics (Christie, 1989). Mathe (1991:49) puts it this way:

> *For the majority of ... children English learnt in the classroom lacks sustaining environment outside the school.*

Misconceptions that start in the primary school, often continue on secondary and tertiary level. To this the discrepancy between everyday English and mathematical English may be added. Du Toit (1987:14) states:

> *getting the meaning from the verbal problem in mathematics is a difficulty because of the differences between Ordinary English and Mathematical English.*

According to Hannan (1988) mathematics is usually regarded as a field of study with its own universal and international language, with neutral symbols and a preoccupation with logic. Consequently, it is argued, racism, sexism or any other hidden prejudice cannot possibly cause individual learners to develop problems with study orientation in mathematics. This, however, does not reflect reality. Even though mathematics is a subject that employs neutral symbols, learners may experience culture-related difficulties. An assumption that sometimes causes problems in schools is that white middle-class learners are able to do better in mathematics than their black counterparts because of the intrinsic nature of the subject. The truth, however, possibly lies in the tendency of teachers to teach black learners the same mathematics as that designed for white middle-class learners. Because the cultural background of black learners is different, they feel left out. Hannan (1988) and Woodrow (1984) conclude that teachers often have preconceived ideas about the ability of different learners to perform well in mathematics[1] – which turns achievement in mathematics into a self-fulfilling prophecy. Furthermore, references to the male sex predominate in textbooks (as in: *"A father gives his son R50 000 for his eighteenth birthday. Calculate ... .")*

The language of instruction, even when instruction is through medium of English, is often inappropriate for speakers of a different English dialect, or who speak non-standard English or an African language (Fynn, 1989).

The names that occur in mathematics textbooks often do not represent the names of (particularly black) working class learners. Activities referred to in textbooks reflect mainly the lifestyle of whites (for example: *"A swimming pool of 20 square metres is to be built in a smallish garden ..."*). This may easily lead black learners to believe that mathematics is not meant for them, not about them, and that they do not belong in the mathematics classroom. Michau (as quoted by Castle, 1992) furthermore contends that switching from a black learner's first language to English as the language of instruction, may complicate conceptual learning in mathematics.

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1 Learners are black → they are unable to do mathematics, or the opposite: They are white → they should be able to do mathematics.

The situation in Australia – where a large number of learners study mathematics through medium of English, their second language – is very similar to the one in South Africa. MacGregor describes the linguistic difficulties that inhibit achievement in mathematics as follows:

> *For (these) students, who form the majority in many schools, the main barrier to enjoyment and success in mathematics is their lack of proficiency in English.*

MacGregor (1993:31) also mentions the following culture-related difficulty that has a negative impact on the study orientation of learners in mathematics:

> *The cultural ethics of some immigrant groups prevent students from asking the teacher questions and even from answering questions posed to the class.*

4.4 CULTURE-RELATED NON-LINGUISTIC PROBLEMS

4.4.1 Introduction

There is adequate evidence that different cultures emphasise different aspects of cognitive style, language, visualization and mathematics, and that this either facilitates or impedes the learning of certain mathematical concepts. Woodrow (1984:6) gives the following examples to substantiate this statement:

> *The existence or non-existence of distinctive words such as "numeral", "digit", "number" helps or hinders the development of differentiated concepts.*

Mitchelmore (1980) likewise draws attention to the problems that may arise in the mathematics classroom as a result of the emphasis a specific culture lays or does not lay on visualization. Castle (1992:220) mentions culture-based problems with study orientation in mathematics and their impact on achievement.

He says:

> *The school system for black students emphasises rote learning, strict discipline, excessive corporal punishment, and one-way communication (teachers to learners). Students are discouraged from asking questions and using their initiative or imagination. Their role is passive, their opportunities for problem-solving and decision-making are limited. Popular access to mathematical knowledge and skills is limited by these deficiencies in the school system.*

MacDonald as quoted by Castle (1992:222) alleges that "*rote learning is the norm in black schools because of its deep roots in cultures with a strong oral tradition*". Oral tradition implies, *inter alia*, that communication is from the "top" down, with few questions from the "bottom". The focus is on listening, memorising and recalling information, with the result that cognitive processes of a higher order such as strategic planning, hypothesis testing and evaluating results are not developed. Learners' culture-based experiences of education and learning impede proper study orientation in mathematics and lead to the conviction that mathematics is a symbolic, abstract and irrelevant subject. Mathematics, furthermore, is exceptionally vulnerable to poor instruction and inadequate study orientation (Freudenthal, 1980).

To sum up: mathematics is commonly presented at school and university level in a way that strongly encourages traits such as reticence, conformation to rules and sophisticated language usage. Hudson (1987:34-35) comments as follows:

> *In examining certain general characteristics of particular groups it can be seen how mathematics discriminates against certain personality traits which may in turn be strongly culturally influenced.*

A proper study of mathematics should indeed be built on characteristics such as creativity, group coherence, intuition, imagination, forthrightness and the ability to express oneself freely. Such traits and attitudes can promote better performance in mathematics, particularly with a view to the later stages of the learners' careers.

4.4.2 Different thought processes

4.4.2.1 Visualization in multicultural mathematics classrooms

Research (Presmeg, 1989) has shown that visualization (presenting information by means of sketches or pictures) can assist a learner's study orientation in mathematics. Dawe (1983) points out that the need for representation and visualization is even greater when the medium of instruction is not the learner's mother-tongue, as the global perspectives that visualization can bring about, may make it easier to deal with and overcome some of the study orientation difficulties associated with mathematics.

4.4.2.2 The role of different thinking processes in causing intergroup discrepancies in tests

When the thinking processes of psychologists do not correspond with those of learners, or when learners from a certain linguistic and cultural society do not participate in the development of subject matter, subtler culture-related problems regarding study orientation and achievement promptly arise (Berry, 1985). Berry (1985) is of the opinion that meaningful learning depends on the subject matter being in harmony with the learners' natural cognitive styles. Presmeg (1989:19) uses the situation in Botswana to illustrate the above:

> *An important British curriculum did not resonate with the local culture which was characterised by a concern with small numbers, a rich vocabulary for individual cattle, a taboo on precise enumeration of cattle, and a concern with the here and now which made hypothetical thinking difficult. Visualization is likely to be an integral part of such a modified curriculum.*

Carson and Butcher (1992) believe that research concerning cross-cultural measurement is harassed by factors such as differences in language and thought processes. They also mention "prevailing political and cultural climates" as factors that impede objective investigation. Cultural relativists moreover argue that arriving at conclusions from test results is meaningless, because prevailing norms, standards by which behaviour is judged to be "normal" or acceptable and values, as well as demands for survival differ so radically from culture to culture (Ullmann & Krasner, 1975).

4.4.3 Differences in the development of cognitive style

As was explained in Chapter 3, the specific cognitive style of testees could contribute towards discrepancies in the achievement of the groups and subgroups that are tested. Valverde (1984), for example, regards cognitive style as the single factor that has the most significant impact on the achievement of Spanish learners of mathematics. He states this as follows (Valverde, 1984:127):

> *In my view, mathematics education has been organized to favor the field-independent rather than the field-dependent child; that is, open-ended discovery rather than definite outcomes, individualization of instruction rather than group learning, and competitive more than cooperative activities.*

Barker (1995) believes that girls reach adolescence between the ages of 11 and 13, and boys between 13 and 17 years of age. At this stage learners begin to display a more flexible, subtler cognitive style. Piaget (in Lavatelli, 1974:158), however, warns firmly against rigid time slots for human development:

> *So, it's essentially relative to a statistical convention. Secondly, it is relative to the society in which one is working ... in certain societies ... we have found a systematic delay of three to four years. Consequently the age at which ... problems are solved is only relative to the society in question ... the mean chronological age is variable.*

Shaffer (1996) emphasises that learners who display acquired helplessness are not necessarily the ones with the least aptitude, and that even highly gifted learners could easily fall victim to acquired helplessness. Weiner and Dweck (Shaffer, 1996) are of the opinion that Locus of control (external or internal) is to a large extent determined by aspects of the learners' cognitive style such as mastery orientation (a tendency to persevere with tasks because learners believe that they are highly capable and that past failures could be overcome by hard work) or acquired helplessness orientation (a tendency to give up after failure, because these failures are blamed on the limited capabilities of the person, and the belief that little can be done about it). Discouraging conditions include a poor family background and socio-economic status (SES); a home language that is not English; a home in which book and school learning is not valued; domestic, social and economic difficulties that impede the development of a stable and supportive environment; parents who are unable to act as teachers for their children; poverty; a lack of toys and books as well as a sense of alienation from the dominant culture. These are the typical problems that have, for so long, confronted young Black learners in particular (Owen, 1995). These difficulties intensify the previously mentioned achievement problems in mathematics, and probably lead to the learners' perception that the Locus of control is external.

4.4.4 The limited value of intelligence testing

Circumstances completely beyond the control of black learners – the political dispensation for instance, and social and psychological factors that should be ascribed to the environment and not, in the first place, to cognitive differences – cause black learners to enter any mathematics classroom intellectually handicapped. To label certain learners "unintelligent" or "average" and then to base expectations regarding their ability to achieve in mathematics on such a faulty assumption is an injustice to learners and often causes gross errors of judgment. Phares (1992:186-7) puts it as follows:

> *It does seem apparent that grades in school are related to a host of variables: motivation, teacher expectation, cultural background, attitudes of parents.*

Maker (1993:76) concludes:

> *Students who have been taught to solve simple problems, and to view them within the narrow context of a single academic discipline, are likely to grow up as adults who view world problems with a similarly narrow focus.*

Castle (1992:228) furthermore emphasises the importance of clearly distinguishing between "*aptitude (for mathematics) and attitude (towards mathematics)*".

Referring to the limited value of intelligence testing, Jordaan (1991:12) declares:

> *Intelligence tests provide but a limited estimation of intellectual ability - limited, because most tests measure only the kind of ability that is regarded as important in Western society. This in itself invalidates a direct comparison of the intellectual capacities of different races ... . Intelligence tests are by nature achievement tests. The result depends on knowledge previously gained through formal life experience, and intellectual skills acquired through good instruction.* (Translation)

Jordaan (1996:8) also points out that intelligence tests (IQ tests) were often regarded as the most important criterion for the assessment of learners' academic worth and ability and that a preoccupation with IQ scores has led

> *to many faulty counselling practices; restrictive subject and training options, sexist ideas about the abilities of women; racist statements about the intellectual capacity of various nations and other instances of human potential being disparaged.* (Translation)

It is not denied that a number denoting a learner's intelligence could be an indication of his ability, but:

> *Any behavior is complexly determined by many variables other than just general or specific intelligence. It is probably true to say that IQ's correlate best with success in school. It could be argued, then, that intelligence tests measure this rather than "intelligence"* (Phares, 1992:187).

Kamin (1995:85-6) adds to this view:

> *The socio-economic status of one's parents cannot in any immediate sense 'cause' one's IQ to be high or low ... but income and the other components*

> *of an index can serve as rough indicators of the rearing environment to which a child has been exposed... . And extensive practise at reading and calculating does affect, very directly, one's IQ score.*

It is true that genetic factors do limit a person's intellectual potential, and that environmental factors will co-determine how close the person will come to realizing that potential, but there is no evidence that the genetically determined limitations of one population group differ from those of another group (Jordaan, 1994).

Piaget (Copeland, 1984) shares this view. He says that although genetic factors do play a role in the development of intelligence, they remain little more than "open factors". They alone cannot realize any particular potential. Genetic factors alone cannot account for what happens at various stages of development. According to Piaget (Lavatelli, 1974) intelligence tests at best measure **achievement**, and achievement will vary according to the nature of the **social environment in which learners have developed.**

4.4.5 SES (Socio-economic status)

Plug, Meyer, Louw and Gouws (1993) define SES as a person's position in a particular community. This is jointly determined by factors such as financial status, social status and profession. Kagitchibasi and Berry (1989) add that potential intergroup differences in SES should be acknowledged before any meaningful cross-cultural comparisons can be drawn. SES is co-determined by factors that could inhibit education, including poorly educated and unambitious parents; inadequate motivation; poverty; disease; poor educational facilities; poor adjustment or maladjustment; insufficient stimulation and a lack of possessions (books, furniture, pictures, television, radios, cars). Having such goods does indeed contribute significantly to the shaping and polishing of a learner's behaviour and thought patterns. The family background and the environment that shape learners, vary. Some learners grow up in wealthy homes, others come from needy homes. Their ethnic and cultural background differs. One culture differs from another regarding the learners' motivation to perform well academically, their interests, and the value their parents attach to learning. Learners from a stimulating environment draw from a wealth of experience and often learn fairly easily. But the reverse is also true - learners from needy homes (non-stimulating environments) may lag behind. Because of their limited experience, they struggle and learn more slowly. Castle (1992: 220) sums it all up:

> *A great deal has been written about the effects of apartheid policies ... . Education is often seen as a key instrument in this process of oppression ... . Lack of schools ... poorly qualified teachers, chronic shortages of books and equipment ... all contributed to the large number of black South Africans who have never been to school ... . It is worth noting that there is a particular dissatisfaction with mathematics ... within the black school system.*

4.5 ENCULTURATION AND ACCULTURATION

Behaviour is affected not only by the culture in which learners develop (enculturation), but also by the cultures they come into contact with.[2] For many years whites have been regarded as the dominant group in South Africa, and other population groups as the acculturating ones. The culture of the whites was perceived to be "better". Other groups could therefore profit by trying to adjust to it, it was thought (Claassen, 1989). Stabb and Harris (1995) regard the acculturation process as potentially the most significant factor in promoting our understanding of the influence of westernization on black students in the USA. According to these authors acculturation is, *inter alia*, the process through which non-white students adopt the attitudes, behaviour patterns and mannerisms of the dominant Euro-American culture, and relinquish the behaviour that is typical of their own culture. Other authors (Berry, 1980a; Gordon, 1978) regard acculturation as a dynamic process through which the customs and values of a culture are passed on and adopted within a group or across cultural borders. Figure 4.1 represents Berry's model for explaining the behaviour of individuals.

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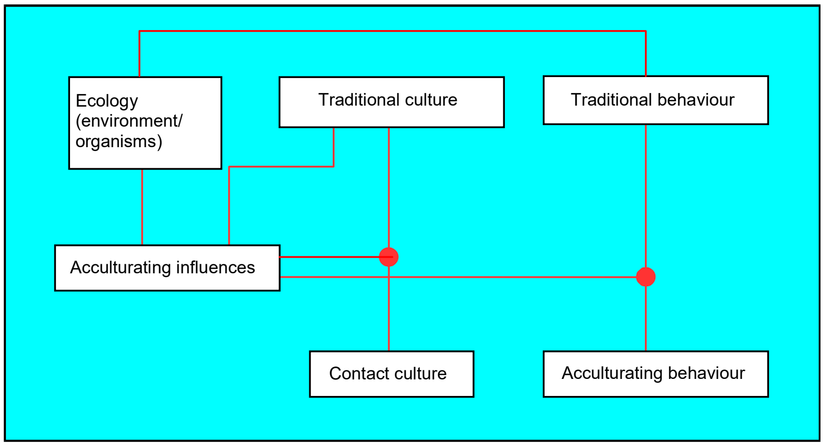
[2] See Table 4.1.

FIGURE 4.1: BERRY'S ECOLOGICAL-CULTURAL-BEHAVIOURISTIC MODEL FOR THE EXPLANATION OF THE BEHAVIOUR OF INDIVIDUALS



Adapted from Berry (1980a; 1980b)

In order to satisfy their primary needs people have to interact with their physical environment. Extensive differences in physical environments lead to various economic alternatives for fulfilling these needs. The interaction between organisms and their environment results in **traditional behaviour** (the connection between context and behaviour) that is influenced by acculturating forces. The word **ecological** refers to the interaction between human organisms and their habitat. **Cultural** refers to behaviour patterns that groups of organisms have in common, and **behaviouristic** denotes the resulting behaviour. **Acculturating influences** (mainly the processes of urbanization and education) bring about intensive contact with technologically dominant societies. A **contact culture** is created (certain cultures do not only adjust to their habitat but also to acculturating forces). This results in **acculturating behaviour** (shifts in behaviour from previous levels) and **acculturating stress**. This is new – and fairly pathological. Berry uses this model not only to establish a link across the four curves (Figure 4.2) but also to indicate vertical relationships. He attempts to find evidence of the **internal validity** of experiments and conclusions (by means of horizontal links) as well as the **external validity** (vertical links indicating context).

Acculturation in South Africa is not only **moving** towards westernization. It has, according to research such as that conducted by the Bureau for Market Research at the University of South Africa (Groenewald, 1996) **already taken place** to such a significant extent, particularly among urban blacks, that any evaluation or explanation of potential cultural differences between black, white and brown in South Africa should not be assessed without taking cognizance of this. Van der Reis (1996) confirms this tendency and expresses the opinion that South Africa's black youth have reached a crossroads in the process of acculturation: they are faced with many bewildering and contradictory situations. He believes, however, that these young people are able to handle the situation (Van der Reis, 1996:1):

> *However, they appear to be dealing with this situation by embracing a mixture of traditional African and Western values.*

The importance of understanding the process of acculturation of learners becomes apparent when tests are developed for **specific groups**, within a **specific cultural context or contexts.** Such factors should be taken into account when attempts are made to determine whether tests are appropriate or inappropriate for individuals belonging to specific sub-groups.

4.6 A POSSIBLE MODEL INDICATING THE MANNER IN WHICH LINGUISTIC AS WELL AS NON-LINGUISTIC FACTORS AFFECT STUDY ORIENTATION IN SOUTH AFRICA

Cocking and Chipman (1988) propose that the study orientation problems learners experience should be viewed from the following frame of reference:

- ★ Poverty (poverty and low socio-economic status);
- ★ language (inadequate command of language);
- ★ culture-related factors (values, the support of significant others, motivation);
- ★ cognitive capability patterns (pace of learning, cognitive styles that differ from one culture to the other). They propose the following comprehensive model to represent the impact of linguistic and non-linguistic factors on study orientation in mathematics:

TABLE 4.2: FACTORS AFFECTING LEARNERS' STUDY ORIENTATION IN MATHEMATICS

Most important influences on the study orientation of learners in mathematics		
1.	Learners' initial characteristics	Knowledge of mathematics and language
2.	Opportunities to learn	School Opportunities for tertiary education
3.	Learners' levels of motivation	Anticipated reward Attitude towards mathematics System of values adhered to by significant others and specific culture groups

Adapted from Cocking and Chipman, 1988

The developmental status of learners and their linguistic ability to receive information and avail themselves of opportunities to learn, are considered. The model may be extended in terms of input (**to** learners) and output or level of mastery (the learner's **achievement**).

TABLE 4.3: INPUT AND OUTPUT (OR LEVEL OF MASTERY) IN MATHEMATICS

Input	Cognitive ability patterns	Mathematical concepts Language proficiency Reading proficiency Strategic learning ability
	Learning opportunities	Time spent studying Quality of instruction Appropriate language Support of significant others
	Motivation to become involved	Cultural values Anticipation of reward Motivational nature of interaction This includes the nature of feed-back and whether it is culturally suitable.
Evaluation of level of mastery (output)	Evaluation of measurement	Sensitivity to learners' development level Cultural equity Availability of psychological services and suitable measuring instruments.
	Language of test	Formulation of items (instructions may cause problems) Classes or categories of questions: Are they representative of the construct to be measured?
	Variations in achievement level	Comparing achievement in mathematics, performance in other measurements with achievement in study orientation questionnaires.

Adapted from Cocking & Chipman, 1988

Poverty, language differences and cognitive abilities become inputs. The cultural environment represents an aspect of learning opportunities, either at the cultural home or support level, or at the level of instruction. Schools in environmentally disadvantaged societies have more unfavourable teacher/learner ratios, poorly trained teachers (or even teachers who are unable to speak the learners' mother-tongue) and hardly any access to psychological support structures. The ultimate goal is to establish a research model that will, in a holistic manner, take the different classes of variables into account – rather than focusing on clusters or groups of variables, grouped together on the grounds of assumptions about poverty, language, cognitive capability patterns and culture.

4.7 CULTURE, MEASUREMENT AND BIAS IN PSYCHOLOGY

Harris (1995) points out that background factors and personality traits are often used as potentially measurable predictors of students' experiences, but that the significance of cultural values is sometimes ignored. Defining students' needs, measuring instruments and data procedures rest mainly on the assumption that these aspects are culture-free.

The influence of culture on measurement is generally difficult to detect, but researchers nevertheless agree that culture could be a source of bias in tests – cognitive tests in particular (Berry, 1984; Retief, 1988).

When Binet and Simon introduced their earliest intelligence test, it was clear that socio-cultural factors such as class and culture might play a role in test performance. Owen (1995:86) explains as follows:

> *During the 1960s it was realized that the situation in the USA called for urgent attention. It was a matter of concern that the achievement of blacks and members of other minority groups (the culturally disadvantaged) was generally lower than that of white Americans, in a wide variety of tests including those measuring intelligence, scholastic aptitude and performance.* (Translation)

Since the seventies the concepts of test bias and fairness have come strongly to the fore. The move towards respect for the cultures of ethnic minorities, and an understanding and recognition of their worth, is known as **cultural pluralism**. This has given rise to the concept of antiracist mathematics – a multicultural, strictly antiracist approach to all facets of mathematics (Ernest, 1991). In mathematical statistics the word "bias" refers to a systematic under- or overestimation of a population parameter using statistics based on a random sample of the relevant population. When "bias" is used in connection with psychometric testing, however, it refers to systematic errors in determining the prediction or construct validity of the test scores of individuals belonging to the norm group (Jensen, 1980).

4.8 COMPARABILITY AND EQUIVALENCE

Poortinga (1983) and Verster (1987) define the construct "comparability" as a series of statistical conditions that have to be complied with (**after** data have been collected) to determine whether valid conclusions may be drawn from intergroup differences (in terms of a common scale) that may be apparent from test scores. According to Hui and Triandis (1985:133)

> *precision and meaningfulness of comparison are two basic desiderata that, very often, cannot be maximised at the same time in cross-cultural research.*

To be able to compare the responses and differences in behaviour of testees in a meaningful manner, we need a common denominator on the basis of which comparisons could be made. According to Claassen (1989) and Hui and Triandis (1985) a basic assumption in all cross-cultural measurement is that humanity displays a psychic equivalence. This means that all forms of human behaviour have certain basic characteristics in common, characteristics that are to some degree measurable - regardless of the extent of cultural differences. These authors distinguish five levels of equivalence:

(i) Cross-cultural conceptual equivalence (when a construct could be discussed meaningfully in different cultures). Conceptual equivalence is closely related to functional equivalence.

(ii) Functional equivalence (when persons from different cultures display similar behaviour to achieve equivalent objectives, and the causes and consequences of the behaviour are similar). Sears (1961) mentions three requirements for conceptual or functional equivalence:

- ★ A universal learning situation;
- ★ an identifiable objective; and
- ★ the requirement that the relation between cause and consequence should be the same in all cultures.

(iii) Construct operationalizing equivalence (operationalizing implies the transition from theory to measurement) – when a construct is operationalized using the same procedure or instrument in different cultures.

(iv) Item equivalence. (The instruments should be identical at item level – each item of a psychological test should mean the same to persons from different cultures.) Verster (1987) is of the opinion that (iii) and (iv) may be grouped together as metric equivalence – when concepts may be measured in the same measuring units across various cultures.

(v) Scale equivalence (when it can be shown that a specific construct is measured on the same scale across cultures). A numerical value on the scale should indicate the same degree, intensity or size of the construct, regardless of the population group the testee is a member of.

Quantitative comparisons across cultures can only be meaningful if it can be shown that the measuring instruments are equivalent with regard to all the above-mentioned aspects (Hui & Triandis 1985).

4.9 STRATEGIES FOR CROSS-CULTURAL MEASUREMENT

According to Huysamen (1996) there are three ways to deal with the challenge of compiling norm tables in a heterogeneous society, namely:

- ★ Using a stratified norm group and compiling norm tables for the composite group. The danger does exist that the results may not be typical of any of the groups.
- ★ Compiling tests that are culture-fair or culture-free. This is done by trying to eliminate items that are biased or more familiar to one group than to another. Huysamen makes it clear that few researchers accept this approach as they fear that the search for common items may end up in a collection of items that are so unrepresentative of the relevant universum that they are practically useless.
- ★ Different norm tables could be compiled for different subgroups within the universum. The general population should evidently be divided into homogeneous subgroups (such as age, educational level and cultural background).

Claassen and Schepers (1990:294) quote Goodenough in connection with the prevention of bias in test items:

> *We must be sure that the test items from which the total trait is to be judged are representative and valid samples of the ability in question as it is displayed within the particular culture with which we are concerned.*

If potentially biased items are retained in a specific test, test users should be informed of this and the diagnostic value of such items should be reassessed when test results are finalized.

We shall now focus on potential strategies that might be employed in an effort to establish some satisfactory cross-cultural measurement (Claassen, 1989; Cronbach 1990; Hui & Triandis 1985; Owen, 1995).

4.9.1 Direct comparison

This is the most popular (and intuitive) method of cross-cultural comparison. The measuring instrument is applied simultaneously to persons from different cultures. If the mother-tongue of the groups differ, the instrument has to be translated. The basic assumption is that the construct that is to be measured is present in all the cultures concerned and can be operationalized equivalently. Scale equivalence is assumed when statistical tests are implemented.

4.9.2 The ethnographic (emic) method

It is assumed that it is possible to describe behaviour in a culture accurately without allowing external factors to affect the description significantly (as used in anthropology). Certain behaviour traits are systematically defined within their natural ecological and cultural contexts. This approach is qualified as emic. Behaviour is thus described and classified in a **qualitative manner**. Retief (1987:47) states the following:

> *The conception of culture as a system of meanings has been implicit or explicit in the majority of research traditions in anthropology, and has been associated with a more qualitative methodological emphasis. In practice, this normally consists of a detailed description of the behaviours, customs, and activities of other cultural groups, from which inferences about cultural values and rules are drawn.*

4.9.3 Regression method

Poortinga (1975) suggests that scale equivalence could be ascertained by determining whether the regression parameters of the criteria or constructs that are to be predicted and regarding which inferences are to be drawn, are the same for the different population groups that are to be studied. Two sets of test scores that have been arrived at by means of a measuring instrument, ought, in a similar manner, to correspond to an external criterion. This method is relatively economical and simple. Discrepancies in the reliability of the scales for different population groups, however, as well as varying degrees to which specific behaviour traits are present in test populations, may lead to fluctuations in regression parameters and sound false alarms. Furthermore, a criterion that is biased in terms of other criteria will also lead to differences in parameters.

4.9.4 Item response theory (IRT)

Lord's item response theory (1977) rests on two premises. The first is that a testee's response to an item can be predicted or explained by means of a set of factors called latent traits or abilities – latent, because they are not discernible as they cannot be measured directly. These latent traits explain the connection between the different items of a test. The second premise is that the relation between an individual's achievement in an item and the set of traits underlying the achievement could be described by a monotonically increasing function known as the item-characteristic curve. This technique enables psychologists to overcome the difficulty of finding a relevant and non-biased criterion the measuring instrument may be judged by. Major disadvantages of the IRT are that large samples (n > 1000) are required, that computer software for the application of IRT has not been fully developed, and that the assumption of one-dimensionality has to be satisfied if item parameters are to be estimated accurately. Other types of equivalence at a more abstract level, such as the operationalization and conceptualization of constructs, are presupposed, as IRT is not concerned with these.

4.9.5 Response pattern method

This method deals mainly with item equivalence and does not indicate scale equivalence. The basic assumption is that the item response pattern of persons of the same ability and belonging to the same culture group will, to a large extent, be in agreement. Significant differences, however, could be expected to occur between different culture groups. Angoff (1982) suggests that the difficulty values of items should be converted to delta values. The discrepancies between delta values could then be examined. It is reasoned that any discrepancies in the relative degree of difficulty indicates that persons from one culture may find certain items easier or more difficult than persons from another culture. In other words the items do not measure the same attribute in two different cultures. This method could be extended to measure personality traits other than intelligence by arranging the average number of times an item was selected by members of the population group correspondingly. There is, however, no generally acceptable, objective standard by means of which one could determine whether correlation coefficients are high enough. Differences in discrimination values could also influence differences in sequence.

4.9.6 Translation techniques

Unsatisfactory item equivalence (as indicated by IRT or other methods) could be corrected by improving the translation. Methods such as translating back into the original language, applying items to a bilingual group, a committee approach and experimental pre-testing could be used. The common aim is to administer the same test in different languages to various groups, retaining the same ideas across linguistic borders (Brislin, 1986). Translation is, of course, fallible. There is always the possibility of non-equivalence at a more abstract level such as conceptual equivalence. Potential problem areas include variations in social desirability and motivation levels as well as poor test administration, which could negatively affect measuring instruments (that, at other levels, do facilitate accurate comparison). Oakland (1977) points out that the language used in any test should enable all testees to understand clearly what is expected from them so that they may be able to respond freely and confidently. Otherwise, the test could be biased. When, in addition, a test is conducted in a second language, it should be made very clear to the testees what exactly they are expected to do, lest the test fail to measure what it is intended to measure. This may, in other words, reduce the validity of the test.

4.9.7 Internal structural congruence

This method is used to investigate the cross-cultural equivalence of a construct. Its internal structure is examined by means of investigative and confirmatory factor analyses. It is reasoned that a construct that remains the same across different cultures will display the same internal structures or components (ordered in a similar way) in all of these cultures (Cudeck & Claassen 1983). Comparing internal structures across cultural borders presupposes a clear theory that is at least founded on the culture in which the study was initiated (Hui & Triandis 1985). Equivalence of internal structure is, furthermore, not the only precondition for cross-cultural comparison. Neither does it provide conclusive proof of scale equivalence.

Retief (1987) explains that certain preventative measures should be taken when attempting to achieve factorial equivalence. These measures include adequate translation, comparison of item analyses, correlation of each item with the full scale, item correlations, factor analyses and the determination of the correlation between scale scores and those of other variables.

Kline (1983) believes that this type of metric equivalence not only makes a meaningful comparison of various cultures possible, but also guarantees conceptual equivalence, as it is highly unlikely that variables with different meanings will have the same factorial patterns.

4.9.8 The combined etic-emic approach

Whereas **emic** refers to culture-specific measures, **etic** refers to cross-cultural matters (derived from a phonemic-phonetic analysis in linguistics, in which grammatical rules are explained in order to distinguish between general and specific aspects) (Pederson, 1991). This combined etic-emic approach is used in an effort to reconcile the two approaches. Davidson, Jaccard, Triandis, Morales and Diaz-Guerrero (1976:2) define the procedure as follows:

> *Initially, the researcher identifies an etic construct that appears to have universal status. Secondly, emic ways of measuring this construct are developed and validated. Finally the emically defined construct can be used in making cross-cultural comparisons.*

The items that are used should, at least partially, extend beyond cultural borders. If they do not, the measuring instrument would lack item and scale equivalence and direct comparisons would become practically impossible. That measuring instrument could then no longer be claimed to measure the same construct in different cultures.

4.9.9 Determining validity through the nomological network

This strategy is used to determine whether a construct in one culture is embedded in the same network of constructs as it is in another culture. It is based on the rationale that if a construct has the same meaning cross-culturally, it will display the same empirical relations between constructs within each of the cultures. (Variables that measure a construct in one culture will, in other cultures, be similarly related to other variables). This manner of establishing validity presupposes that the constructs that are used as external criteria will be equivalent cross-culturally. If the networks appear to correspond, measuring instruments that are used should not only be cross-culturally appropriate, but also conceptually and functionally equivalent. If this is not the case, it will be difficult to determine which of the constructs and criteria are not equivalent cross-culturally. Finally, the percentage correlation that is ascribed to factors such as researcher bias and common method variance should not be confused with signs of correlation in the validity network (Hui & Triandis 1985).

4.10 A MODEL FOR CROSS-CULTURAL MEASUREMENT

4.10.1 Culture-free measuring instruments

Culture-free measuring instruments may be described as measuring instruments that have been constructed specifically to minimize the effect of irrelevant culture-related influences on the achievement of testees (Shaffer, 1996). Helms (1992) is of the opinion that intergroup differences in (particularly) cognitive tests are mainly a result of testing procedures. Anastasi (1988) contends that intergroup differences in test achievement cannot be attributed to biased testing only. Zigler and Finn-Stevenson (1992) believe that intergroup discrepancies should rather be ascribed to motivational factors.

Owen (1987:334) refers to bias in respect of construct validity and proposes the following criteria by which to evaluate the similarity of constructs in different groups:

- ★ Similar test validity;
- ★ similar rank order correlations of item difficulty values; and
- ★ factorial similarity.

He furthermore claims:

> *Abnormality in the behaviour of an item could be due to* *true* *bias, or* *seeming* *bias, or a combination of both. The reason why previous researchers have been unable to find a notable number of generalizable principles in connection with bias may be that there are fewer differences between groups than are generally supposed (Owen 1987:342).* (Translation)

He adds that the challenge regarding bias in psychometric testing is to find methods to determine which discrepancies in the test performance of groups represent a true difference between the groups, and which ones are merely the result of bias, such as in the case of a poorly phrased item. He adds that true bias can generally be ascribed to language problems (unintelligible words and concepts in items where language is not directly at issue), as well as to distractors that are too inviting, item formats and contents that are foreign to the world in which the testees live; and knowledge and ideas that are usually acquired informally (where there is evidence that the basic general knowledge the groups possess is not comparable). Seeming bias refers mainly to a testee's behaviour – which means that the cause of bias lies beyond the field of the item. Factors that may lead to seeming bias include selective attention to the information supplied in an item, deviation from the facts given, lack of a logical disposition of mind, as well as a lack of specific knowledge that is usually acquired formally – that is, at school.

4.10.2 Cronbach's view of the cross-cultural aspects of measurement

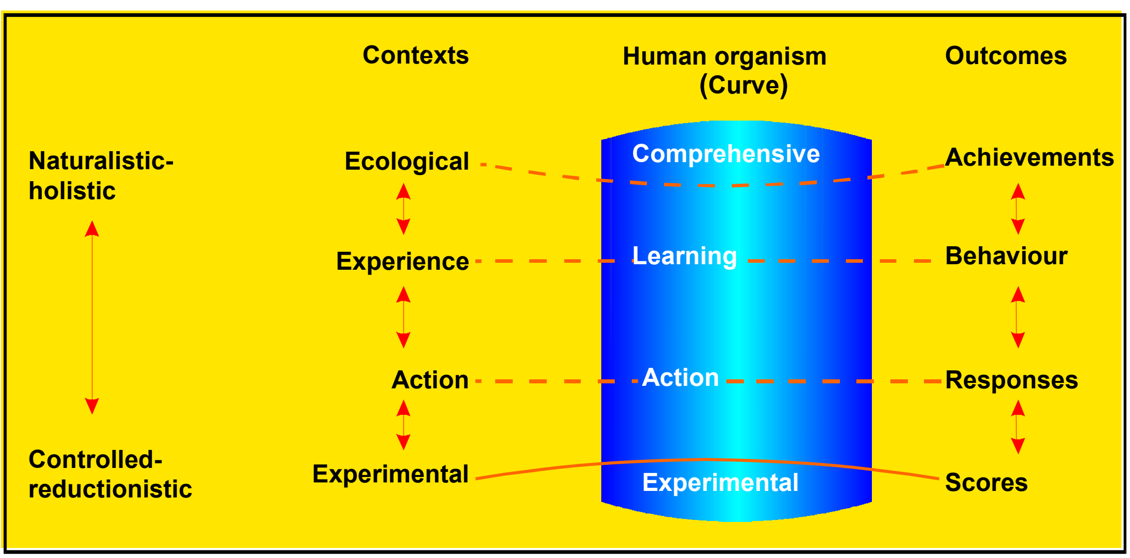
Cronbach (1990) points out the complexity of socio-cultural factors that should be considered during measurement. He emphasises the importance of sensitivity towards the problems generally encountered when groups that have traditionally been disadvantaged educationally, environmentally or culturally, are measured. Persons who compile and administer tests should, however, guard against overemphasising these problems. At the same time the glacier effect of ever-changing socio-cultural conditions should be borne in mind in order to minimize invalid inferences. According to Cronbach (1990) faulty inferences generally occur when researchers assume that socio-cultural situations are static rather than dynamic. Cronbach (1990) summarises certain aspects of the acculturation process (aspects that probably also apply to the situation in South Africa) as follows:

- ★ Discrepancies in the means obtained by different race groups lose much of their significance when samples are compared on the basis of socio-economic factors such as the parents' income, educational level or profession.
- ★ Differences in achievement that are ascribed to cultural variations are not static.
- ★ In retrospect, criticism that tests are culturally biased often seems to have been premature. The discrepancies in the performance of blacks and whites often prove to have been exaggerated.
- ★ There is, however, good reason to object to the **unqualified** use of mainstream psychological tests to assess unsophisticated testees from rural areas.

4.10.3 Berry's framework for the structuring of aspects of culture

Retief (1988) regards Berry's (1980a; 1980b) generalizability model (Figure 4.2) for structuring and operationalizing aspects of culture as suitable for the development of a working model of culture for the contextualization of variables in cross-cultural research.

FIGURE 4.2: BERRY'S MULTI-LEVEL CURVE MODEL FOR THE GENERALIZABILITY OF (HUMAN) BEHAVIOUR ACROSS DIFFERENT CONTEXTS



Adapted from Berry (1980a; 1980b)

Berry believes that when psychological measurement was developed, exaggerated emphasis was laid on a reductionist and experimental approach. Behaviour was measured without attention to functional or authentic contexts. According to the researcher Barker (1965) society does not consist merely of discrete sets of variables, but rather of meaningful contexts in which variables could be grouped together.

Berry (1980a; 1980b; 1983) and Verster (1987) distinguish between constructs such as "environment" (the context in which organisms and their behaviour are studied) and "ecology" (the relations of organisms to the physical surroundings in which they function). Any ecological analysis should account not only for environment, organisms and relations, but also for behaviour, as a fourth category. For Berry (1980a; 1980b) the ecological context includes Brunswik's cultural habitat, Lewin's physical world and Barker's perceptual world. This context includes both the context of experience (the pattern of repeated experiences that form the basis of learning) and the context of action (the more limited group of environmental variables that lead more directly to specific behaviour). The experimental context could figure within or without the above three contexts.

The fourth environmental context represents those environmental aspects that are structured by researchers in such a way that they elicit certain classes of responses. The extent to which the experimental context is situated in the first three contexts, determines the ecological validity of a particular experiment or task. This implies that experimental procedures that are alien to particular cultures will not further valid conclusions about behaviour in those cultures.

Berry links the four environmental contexts to four corresponding classes of behaviour outcomes. Contexts and outcomes are connected at every human level.

The arrow on the far left represents a continuum of contexts and outputs, from naturalistic to reductionalistic (that are more controlled). **Achievement** refers to behavioural patterns; and **behaviour** to general conduct (including skills, traits and attitudes). **Response** refers to performance elicited by instant stimulation or experience. The term **scores** indicates specific behaviour that is measured or reported during psychological experiments or tests. An experiment that is ecologically valid will probably also be valid with respect to behaviour. The links between elements across the model are indicated by curved lines. The main curve indicates the global life situation organisms live and perform in. The learning curve indicates the relation of recurrent independent variables in people's environment to their typical behaviour. The action curve indicates more specific actions as functions of instant or present experiences. The experimental curve is limited to the laboratory or refers to controlled experiments where tasks are varied systematically and where scores are studied.

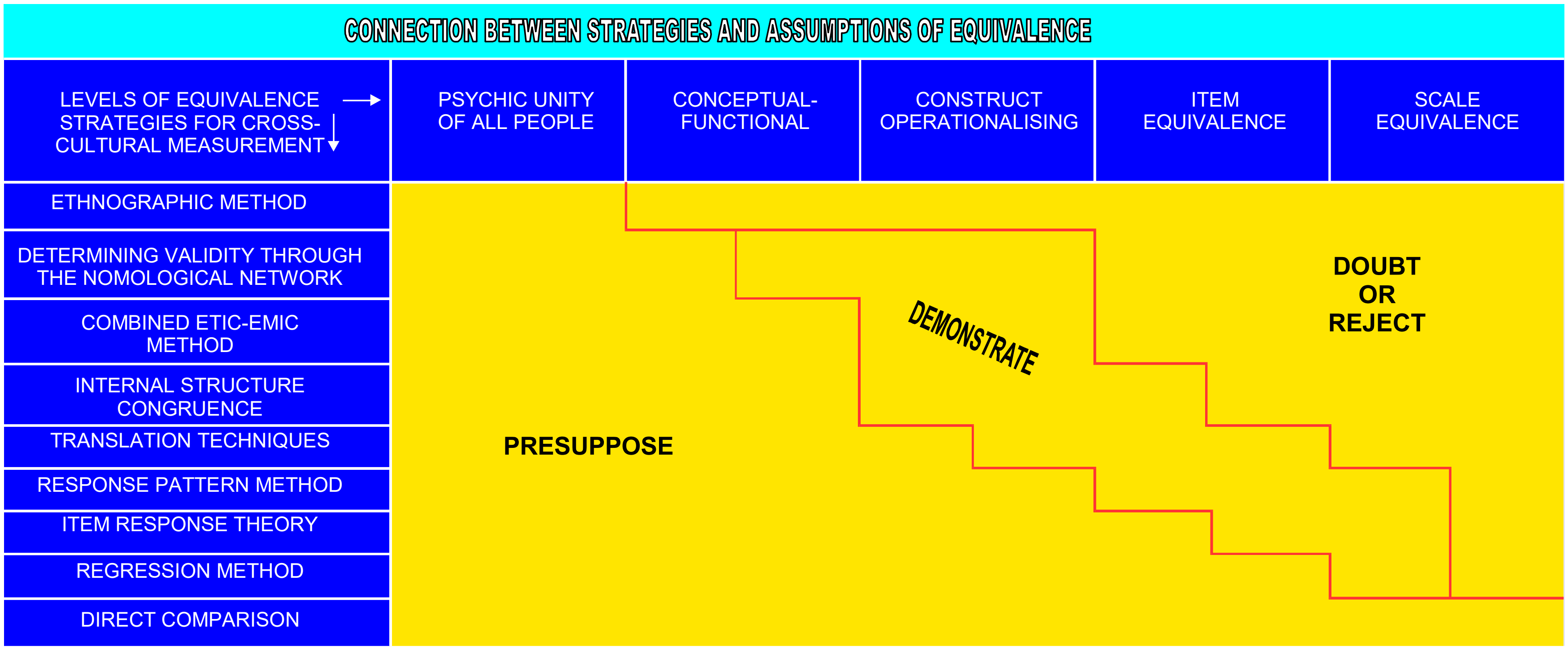
Berry (1983) is of the opinion that experimental psychology does not succeed in rising along the vertical dimension in order to reach valid conclusions regarding causal relations at the two central levels. Cross-cultural psychology has, once again, not succeeded in moving down the vertical dimension in order to specify which variables in the contexts of experience, action or experimentation are responsible for any specific behaviour or performance across different cultures. Cross-cultural psychology has, according to Berry, been unable to avoid the pitfall of working in specific contexts.

To summarise: the concepts **context** and **environment** should not be limited to any one level of analysis. Any comprehensive study of cross-cultural variations in behaviour will have to consider extensive systems as well as particular situations (as elements of a context) in order to indicate connections between behaviour and environment at all levels. This means that the relations of behavioural outcomes to environmental contexts should be investigated systemically at all levels of analysis.

4.10.4 Hui and Triandis's model for cross-cultural measurement

Figure 4.3 represents a model for cross-cultural measurement as proposed by Hui and Triandis (1985).

FIGURE 4.3: HUI AND TRIANDIS'S REPRESENTATION OF THE CONNECTION BETWEEN STRATEGIES FOR CROSS-CULTURAL MEASUREMENT AND ASSUMPTIONS OF EQUIVALENCE



Adapted from Hui & Triandis (1985)

Some of the above-mentioned strategies imply only the simplest and most common assumptions, whereas others imply more abstract assumptions. In the top column the types of equivalence are arranged from more abstract to more concrete. In the left-hand column a number of strategies that are commonly used in cross-cultural measurement are given. These techniques do not fulfil the same function, neither are they merely interchangeable. A certain level of equivalence is presupposed whatever strategy or technique is to be used, and each strategy can only be applied to certain types of equivalence.

The centre part of the model has been divided into three parts that represent the concepts of "presuppose", "demonstrate" and "doubt". Although the psychic unity of man is, as a rule, accepted or presumed, all other forms of equivalence should preferably be indicated.

4.11 CONCLUSION

In this chapter an effort was made to put problems concerning the measurement of achievement and study orientation in mathematics in some cross-cultural perspective. Dealing with the culture-related causes of inadequate performance in mathematics will probably require a holistic approach which should include ending all forms of discrimination, the psychological, social, educational and economic upliftment of **all** the inhabitants of South Africa; equal rights for everyone and the creation of a just material and financial dispensation in South Africa; as well as political stability.

It is impossible to provide **complete** answers to all the questions posed on culture-related issues such as achievement in mathematics, the study orientation of learners and strategies for cross-cultural psychological measurement. We are concerned not only with experimental designs and empirical data, but also with a comparison of – and the conflict between – theoretical premises. Freudenthal (1980:42-44) states in this connection:

> *There may be things I cannot prove with scientific force; but I refuse to obtain them surreptitiously by pseudo-science. I intend to present them with reasonable arguments as consequences of a reasonable faith ... . I have*

> *not proved that what I aspire to is better, as little as one really knows whether teaching is more effective without beating - possibly it is not better. I am advocating another method because I believe in it, because I believe in the right of the learning child to be treated as a human being. This is my view on education; defending it I call philosophy; but do not ask me for scientific proofs.*

Referring to the *16-PF-Personality Questionnaire*, and specifically to factor G (opportunism - low superego versus diligence - strong superego) Cattell, Eber and Tatsuoka (1970:90) put the issue of cross-cultural measurement in mathematics into perspective:

> *So far as the cultural modifications are concerned a core not unlike the Ten Commandments is found as a common denominator.*

In other words: in matters such as these the focus should not be on differences only, but certainly also on the existence of universal traits, occurring across cultural borders.

In Chapter 4 strategies for providing scientific grounds for accepting or rejecting hypotheses on equivalence were dealt with. A pitfall to avoid, according to Hui and Triandis (1985), is for researchers to use their favourite technique to indicate equivalence and then to make all kinds of comparisons. It should furthermore be borne in mind that equivalence at all relevant levels cannot be indicated in isolation by a single technique or strategy.

In conclusion three schools of thought, concerned with the consideration and explanation of the impact of culture on psychological measurement, may be mentioned:

- ★ Those researchers who contend that cultural differences between groups are so deep-seated and that cross-cultural measurement poses such high risks that the development and evaluation of culturally less biased tests become virtually impossible;
- ★ those who believe that cross-cultural differences are largely exaggerated; and

- ★ the researchers who believe that, whereas cultural differences between groups are potentially important, there should also be enough leeway to consider the similarities that exist. The differences that may in fact be present, should not be allowed to hamper honest efforts to develop "culturally less biased" tests. Cultural differences will, at any rate, have to be interpreted against a certain ever-changing framework. **In this study, the third view is supported.**

CHAPTER 5

METHOD OF INVESTIGATION

5.1 PROBLEM STATEMENT AND MOTIVATION FOR THE INVESTIGATION

The argumentation in Chapter 3 and 4 highlighted the following facets or aspects of study orientation in mathematics:

- ★ The forming of basic concepts in mathematics is important. This acquisition of concepts is an essential prerequisite for learning the more advanced work in mathematics.
- ★ Learners display particular study attitudes towards mathematics. These attitudes include motivation and expectations relating to the subject, and they influence other learners' interest in mathematics (Stewart, 1991). Learners' self-concept, the nature of mathematics and the learning of mathematics fall under the aforementioned factors.
- ★ When subject matter in mathematics does not relate to learners' knowledge and thinking level, it leads to frustration that inhibits motivation to do well in mathematics.
- ★ Learners' affective disposition influences their disposition towards the subject. A phenomenon like **mathematics anxiety** (which comes to the fore in the form of aimless, repetitive behaviour) will probably influence their interest in mathematics negatively. If mathematics content does not make sense to learners, it creates anxiety and uncertainty.
- ★ When mathematics (especially in the early stages) is presented in a form that is too abstract or theoretical without learners' being adequately exposed to enough concrete material, this leads to incomplete initial concept formation.
- ★ Learners' **study habits** in mathematics are, *inter alia,* more important in terms of inculcating important mathematical insights in the learners. The use of acquired, consistent effective study methods (including test and self-test strategies, the carrying out of assignments in mathematics as well as the consistent and thorough inculcation of basic concepts, constitutes an important part of

learners' study orientation in mathematics (Pintrich & Johnson, 1990). **Study attitudes** towards mathematics thus lead to particular study habits: *"The most consistent relationships occur between study methods and motivation"* (Pollock & Wilkinson, 1988:80).

- ★ Learners' **problem-solving disposition** (that could include aspects such as problem-centring, co-operative learning and the implementation of metacognitive learning strategies) has a potentially significant influence on their ultimate achievement in mathematics.
- ★ Learners' **study milieu** (social, physical and experienced milieu) constitute an integral part of their study orientation. Learners indeed come from different homes and have different backgrounds. They differ with regard to ethnic and cultural backgrounds; motivation differs from culture to culture, as do learners' interests and the value that parents attach to achievement in mathematics. Reynolds and Wahlberg (1992:156), for example, come to the following conclusion: *"Home environment [has a] pervasive effect on later achievement [in Mathematics]."* Learners from non-stimulating environments frequently lag behind, display a lesser degree of willingness to dare and are frequently slower learners than learners from less restricted environments.
- ★ Learners believe that success or failure lies outside or within their control (external or internal **Locus of control**). The way in which learners for example experience their teachers, in all probability exercises a significant influence on their attitude towards the subject.
- ★ The way in which learners process information in mathematics, co-determines to a significant extent their achievements in the subject. **Information processing** includes critical thinking as well as general and specific comprehension, learning, summarising and learning strategies. These strategies can be used to solve problems in mathematics and frequently provide a measure of the extent to which learners really understand mathematics.
- ★ When concept formation has not been completed, that is when transfer of learning is inadequate, problem-solving in mathematics is inhibited. In such cases learners do not easily see how concepts relate to one another which constitutes an important aspect of the problem-solving process. In such cases learners will use theorems and formulas without considering whether they are applicable to a particular situation or not.

- ★ Seen holistically, learners' total **study orientation** in mathematics probably influences their problem-solving ability and their ultimate achievement in the subject significantly.

The preceding arguments underline the need for a measuring instrument to measure study orientation in mathematics.

5.2 AIM OF THE INVESTIGATION

Various methods are used to evaluate learners' study orientation in mathematics. These include observation, the interview method, and the perusal of scripts, test and examination answers. The questionnaire method is seldom used. Consequently there is a need for a questionnaire with good psychometric qualities; one that takes relatively little time to complete, that produces reliable results and that can easily be applied to large groups of learners.

Madge and Van der Walt (1995) refer to different types of test interpretation in general. The *Study Orientation Questionnaire in Mathematics (SOM)* has been developed with a view to promoting some of these factors of test interpretation, particularly in mathematics.

(i) The questionnaire should in the first place provide information on various aspects of learners' study orientation in mathematics.

(ii) Accurate analysis of the questionnaire should make it possible for psychologists to find explanations for the phenomenon that certain learners display an adequate study orientation in mathematics and other learners a less adequate orientation.

(iii) The hypothesis should be confirmed that in the *SOM*, as in the case of the *SSHA*, there is a significant relationship between achievement in the particular questionnaire and academic achievement in mathematics in spite of the fact that the usefulness of this type of questionnaire is limited by its dependence on honest answers by learners.

(iv) The questionnaire should provide an overall picture, not only to enable psychologists to evaluate learners' study orientation, but also to establish guidelines to optimise learners' achievement in mathematics.

In a nutshell: mathematising, defined by Volmink (1993:34) as:

> *A process which finds its origins in an active interaction with our world when we act purposefully and with awareness towards the achievement of certain goals ... a) understanding of the physical world and acting on it ... b) understanding and transforming the socio-political realities which impact on our lives ... c) creating new ideas, new perspectives, insights, images and symbols,*

should, in a significant way, benefit from the development, evaluation and implementation of a study orientation questionnaire in mathematics that will have meaning for all mother-tongue-speaking groups in South Africa.

The main aim of this study is consequently to develop a study orientation questionnaire in mathematics. The use of such a measuring instrument should therefore contribute to optimising learners' problem-solving ability and achievement in mathematics.

A second aim of this study is to determine the applicability of the SOM. Sub-aims of this study consequently include a comparison of the achievements of the various grade, mother-tongue and gender groups. To do this, statistical procedures (MANOVA, ANOVA and post hoc comparisons – comparison of the means) will be carried out on the different variables (**as measured by the fields[1] of the final questionnaire**) to analyse differences (Steyn, Smit, Du Toit & Strasheim 1995). Achievement in the following fields[2] for the purpose of this study will be regarded as dependent variables:

★ Study habits in mathematics;

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1 When there is an initial reference to the theory of factor analysis, the term "factors" is used in this study. In all other discussions, in line with accepted conventions, reference is made to the fields of the SOM.

2 See Chapter 6, paragraph 6.2 for a description of the fields.

- ★ mathematics anxiety;
- ★ study attitudes towards mathematics; and
- ★ Locus of control with reference to mathematics (only Grade 10 and 11).

As independent variables:

- ★ grade;
- ★ mother-tongue; and
- ★ gender groups.

In other words the following will be investigated: differences in the achievement of the various mother-tongue, gender and grade groups with regard to the different fields in the final questionnaire, namely Study habits in mathematics, Mathematics anxiety, Study attitudes towards mathematics, as well as Locus of control with regard to mathematics.

A further aim of this study, by using regression analysis, is to determine the joint and the individual contributions of the different grade, mother-tongue[3] and gender groups towards achievement in the fields of the final questionnaire.

The results of the study will be used to make recommendations on aspects of implementing the SOM as part of a comprehensive strategy for optimising learners' mathematics achievement.

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[3] The term "mother-tongue groups" in this study refers to three distinguishable groups, namely:

a) African language speaking persons who completed the questionnaire in English.

b) English-speaking persons who completed the questionnaire in English; and

c) Afrikaans-speaking persons who completed the questionnaire in Afrikaans.

5.3 RESEARCH DESIGN AND PROCEDURE: THE DRAWING UP AND STANDARDISATION OF THE STUDY ORIENTATION QUESTIONNAIRE IN MATHEMATICS (SOM)

5.3.1 General: Administering the provisional questionnaire[4] for item analysis and selection

The questionnaire was administered by psychologists of the education departments and teacher counsellors of schools according to standard instructions. The provisional questionnaire was administered at most schools in August and September 1994. However, in certain schools testing only took place at the end of the first term of 1995.

5.3.2 Planning and drawing of the samples

Schepers (1992) emphasises that every psychometric test has to be administered to a random and representative sample with a view to item analysis.

For the purpose of this investigation the population is defined as all learners who took mathematics in Grade 8, 9, 10 and 11 in public high schools of the then education departments of Gazankulu, KwaZulu, Lebowa, Venda, Bophuthatswana, Transkei, Ciskei, the House of Representatives, House of Delegates, the House of Assembly and the Department of Education and Training.

The idea was to draw three independent samples, namely a sample of Grade 8 and 9 learners, a sample of Grade 10 and 11 learners, and a sample of Grade 9 learners. The latter sample would be used for determining the predictive validity of the questionnaire. The testers were also responsible for drawing the samples at the schools. The testers were given instructions on how to draw the required number of learners.

[4] See paragraph 5.3.4 for a description of the way in which the questionnaire was drawn up, and also for its particular structure.

Sample size was planned according to the HSRC's education database for 1991. The database was established by making use of the data that the former education departments had given to the HSRC. The data consist of the names of schools together with their controlling departments, the entries for mathematics, the medium of teaching, etc.

The population was divided into strata of part populations to ensure that each important part of the population had been adequately represented in the sample. The following strata were taken into account: control (education departments), medium of education (Afrikaans or English) and area (town or country).

5.3.3 General discussion on the samples

The sampling was carried out in two stages (Guy, Edgley, Arafat & Allen, 1987; Rea & Parker, 1992; Robson, 1995). First of all a certain number of schools (20) with selection probability equal to the size of the strata and schools were chosen. Then a specific number of learners were chosen systematically from each selected school. The method of systematic sampling implies that from the first k sample units (learners) one is chosen randomly and then each k-th successive sample unit till the required number of sample units has been chosen. Alphabetical name lists or class registers of learners in the particular grades were used for the selection of learners in a school.

The intention was to choose 20 schools for each of the three samples. Thirty learners per grade had to be tested at each school irrespective of whether it was a big or small school. The latter brought about an equalising of the total selection probability of all the sample units in that big schools' sample units that had a bigger chance of being selected, now had a smaller chance of being selected than the sample units of the smaller schools.

It was necessary to do separate item analysis for Afrikaans, English and African language-speaking learners to determine whether the items were suitable for all the groups. The number of learners receiving education in their mother-tongue, namely in English or Afrikaans was proportionately for fewer than the number of learners who did not receive education in their mother-tongue, namely the African language speaking learners. Consequently it was decided to select a number of additional schools for the House of Representatives, the House of Delegates and the Assembly. Approximately twice as many schools from these education departments were chosen than were required. The total number of schools that were ultimately chosen, was thus 26 in the case of Grade 8 and 9) and 27 (in the case of Grade 10 and 11).

Tables 5.1 and 5.2 indicate the school population and the realised samples for Grades 8 and 9, and Grades 10 and 11. Since in some education departments more learners were deliberately included in the samples, the distribution of the number of learners that were tested deviates from the distribution of the school population of the education departments[5]. As a result of the deviations in the realised samples it was not possible to choose the learners exactly in the same proportion as those in the different education departments. The Department of Education and Training is underrepresented in the case of the Grade 8 and 9 and the Grade 10 and 11 samples by 10% and 3% whereas the self-governing territories and Bophuthatswana were slightly overrepresented. The deviations regarding the other education departments are small (about 2%). The samples used for the determination of norms can consequently be regarded as representative of the target population and the norm tables as applicable to this population.

TABLE 5.1: FREQUENCIES IN TERMS OF MOTHER-TONGUE AND GRADE GROUP DISTRIBUTION

Language Group (Grade)	Frequencies	%
8/9 Afr	494	16,4
8/9 Eng	231	7,7
8/9 African languages	1016	33,7
Total Grade 8/9	1741	57,8
10/11 Afr	393	13
10/11 Eng	418	13,9
10/11 African languages	461	15,3
Total Grade 10/11	1272	42,2
	3013	100

5 See Column 3 of Tables 5.1 and 5.2

TABLE 5.2: SCHOOL POPULATION DISTRIBUTION AND THE REALISED SAMPLES FOR GRADE 8 AND 9 (ACCORDING TO EDUCATION DEPARTMENT)

Education departments	% in school population	Number of learners tested			
		Original sample		Proportional sample	
		N	%	N	%
House of Delegates	2,8	59	3,6	53	4,3
Bophuthatswana	6,2	149	8,5	111	9,5
Education and Training	24,7	173	9,9	173	14,0
Self-governing territories	37,0	498	28,6	498	40,1
Transkei and Ciskei	9,1	90	5,2	90	7,3
Venda	2,7	94	5,3	74	5,9
House of Representatives	8,1	354	20,3	106	8,5
House of Assembly	9,4	324	18,6	129	10,4
TOTAL	100	1741	100	1241	100

TABLE 5.3: SCHOOL POPULATION DISTRIBUTION AND THE REALISED SAMPLES FOR GRADE 10 AND 11 (ACCORDING TO EDUCATION DEPARTMENT)

Education departments	% in school population	Number of learners tested			
		Original sample		Proportional sample	
		N	%	N	%
House of Delegates	8,1	210	16,6	83	4,3
Bophuthatswana	7,4	60	4,7	60	9,5
Education and Training	16,0	110	8,6	110	14,0
Self-governing territories	27,2	173	13,6	173	40,1
Transkei and Ciskei	3,7	55	4,3	50	7,3
Venda	2,3	52	4,1	41	5,9
House of Representatives	3,2	88	6,9	35	8,5
House of Assembly	32,1	524	4,2	262	10,4
TOTAL	100	1272	100	814	100

TABLE 5.4: SCHOOL POPULATION DISTRIBUTION AND THE REALISED SAMPLES FOR GRADE 8 AND 9 (ACCORDING TO MOTHER-TONGUE DISTRIBUTION)

Language group	% in school population	Number of learners tested	Proportional sample	% in proportional sample
Learners not tested in their mother-tongue	79,8	1004	953	76,8
Learners tested in their mother-tongue	20,2	737	288	23,2
TOTAL	100	1741	1241	100

TABLE 5.5: SCHOOL POPULATION DISTRIBUTION AND THE REALISED SAMPLES FOR GRADE 10 AND 11 (ACCORDING TO MOTHER-TONGUE DISTRIBUTION)

Language group	% in school population	Number of learners tested	Proportional sample	% in proportional sample
Learners not tested in their mother-tongue	56,6	450	434	53,3
Learners tested in their mother-tongue	43,4	822	380	46,7
TOTAL	100	1272	814	100

All the data of the learners who were tested were used for item analysis purposes. The full proportional sample was used to determine the norm tables for the questionnaire.

5.3.4 Data collection: Structure of the questionnaire

The SOM was not based on one specific theory. As explained in Chapter 2 and 3 the theoretical points of departure of problem-centred learning were cornerstones in establishing the item pool. The theoretical background study carried out in Chapter 2 and 3 brought to light the essential aspects of study orientation in mathematics (seen from a multi-dimensional perspective). The following additional sources influenced the choice of items and structure of the SOM.

(i) *The Summary of Study Habits and Attitudes (SSHA)* (Du Toit, 1981).
(ii) *The Learning and Study Strategies Inventory (LASSI)* (Weinstein, *et al.*, 1987).
(iii) *The Motivated Strategies for Learning Questionnaire (MSLQ)* (Pintrich, *et al.*, 1991).
(iv) *Informal Study Orientation Questionnaires in Mathematics* (Oosthuizen & Maree, 1993; Schminke, *et al.*, 1978).
(v) *The Learning Style Profile* (Keefe & Monk, 1989).

An analysis of the questionnaires indicates that the content of all the above-mentioned questionnaires can to a certain extent be reconciled with several principles of the problem-centred approach to the learning of mathematics.

Certain additional factors that have to be taken into account in establishing a final item pool include:

★ The teaching and learning situation in mathematics, particularly in the South African situation;
★ the content of the items and the words used in them had to be at such a level that they would be able to be understood by all testees; and
★ some of the testees had little or no experience of the problem-centred learning approach.

Initially it was thought that the questionnaire could consist of seven different fields. As explained in paragraph 5.1 of Chapter 5, a literature study brought the following seven potential fields to light:

a) Study habits in mathematics
b) Mathematics anxiety
c) Study attitudes towards mathematics
d) Problem-solving disposition towards mathematics
e) Study milieu in mathematics
f) Locus of control with regard to mathematics
g) Information processing in mathematics. Taken together these seven fields would constitute a learner's study orientation in mathematics.

5.3.4.1 Assessment of the items by experts

The provisional questionnaire, in the case of Grade 8 and 9 learners, consisted of 150 statements, but in the case of Grade 10 and 11 learners, it comprised 165 statements. The statement relates to how individuals feel or relate towards aspects of their achievement in mathematics. Testees are placed in various hypothetical situations in which they have to choose from various alternatives the one that reflects their feeling or their probable action. Each statement has to be responded to according to a five-point scale, namely rarely, sometimes, frequently, generally or almost always. Some assessments are favourable towards some of the statements and others are not.

The questionnaire was submitted to a committee of experts at the HSRC for an assessment of the statements. In assessing the statements, attention was given to lucidity, uniqueness, unambiguity, use of words with precise meanings and the equivalence of Afrikaans and English statements.

Attention was also given to the position of the statements in particular fields. Statements which the committee thought did not belong to the particular field in which they were placed, were either changed or repositioned in a more applicable field. After this the questionnaire was sent for comment to various mathematicians or the staff of universities (Dr G.F. du Toit: the University of the Orange Free State; Prof. P.E.J.M. Laridon: the University of the Witwatersrand; Dr Al Olivier: the University of Stellenbosch; Prof. J. Strauss: the Rand Afrikaans University; Dr D.C.J. Wessels: the University of South Africa). The questionnaire was adapted further according to the comments received.

5.3.4.2 Administration of the preliminary questionnaire on a group of testees

The preliminary questionnaire was first administered to a group of 60 Grade 8 learners in a black school in order to bring to light possibly obscure instructions and items. Testees were requested to circle the numbers of the items they did not understand and also to underline the phrases and words they did not comprehend. According to the testees' reactions towards the items a number of them were reformulated.

5.4 DATA PROCESSING PROCEDURES

5.4.1 Hypotheses

The main hypothesis investigated in this study focuses on the justification of the theoretical framework of the SOM. The subhypotheses centre in general on the relationships between the achievements of

(i) the various mother-tongue groups (African language, English- and Afrikaans-speaking persons;
(ii) the different grade groups (Grade 8 and 9, and grade 10 and 11); and
(iii) the two gender groups (boys and girls)

in the various fields (independent variables) of the SOM.

Achievement in the various fields of the SOM functions as dependent variables, whereas grade, mother-tongue and gender function as independent variables.

5.4.1.1 Main research hypothesis and statistical procedure to test the hypothesis

The main research hypothesis investigated in this study, is the following:

The theoretical fields of the SOM are confirmed by factor validity.

The testing of the main hypothesis is based particularly on:

★ factor analysis; and
★ item analysis.

5.4.1.2 First subhypotheses and statistical procedures to test the hypothesis

(i) The first group of subhypotheses investigated are the following:

★ The achievements of the various gender groups in the various fields of the SOM differ statistically significantly from one another.
★ The achievements of different mother-tongue groups in the various fields of the SOM differ statistically significantly from one another.
★ The achievements of the combined grade groups in the different fields of the SOM differ statistically significantly from one another.

(ii) The techniques used to test these subhypotheses are based mainly on analysis of variance (multiple as well as single variable analysis of variance) and *post hoc* comparisons. In this case the Least Squares Means technique (LSM) was used (Hays, 1994; Howell, 1992; Kirk, 1982).

5.4.1.3 Second subhypotheses and statistical procedures to test the hypotheses

(i) The second group of subhypotheses investigated is the following:

There is a significant relationship between achievements in the fields of the SOM (Study habits in mathematics, Mathematics anxiety, Study attitudes towards mathematics, as well as total scores) on the one hand and achievement in the Achieve-

ment test in mathematics (Standard 7) and the Diagnostic tests in mathematical language on the other hand.

(ii) Techniques of correlation (Pearson's correlation coefficient and regression analysis) (Howell, 1992; Huysamen, 1996; Sincich, 1993) were used to test the aforementioned hypothesis.

5.4.2 Variables

The variables used for the purpose of this investigation are the following:

5.4.2.1 Dependent variables

These include the following:

(i) Grade 8 and 9 learners' achievement in the various fields of the SOM, namely:

★ Study habits in mathematics
★ Mathematics anxiety
★ Study attitudes towards mathematics

(ii) Grade 10 and 11 learners' achievement in the various fields of the SOM, namely:

★ Study habits in mathematics
★ Mathematics anxiety
★ Study attitudes towards mathematics
★ Locus of control

(iii) Achievement in:

★ *The Achievement test in mathematics (Standard 7)*

★ *The Diagnostic tests in mathematical language*

5.4.2.2 Independent variables

These include the following:

- ★ grade;
- ★ mother-tongue; and
- ★ language groups.

5.4.3 Standardising of the SOM

In this case different procedures were followed to evaluate the SOM psychometrically. In the first place the questionnaire had to have validity.

5.4.3.1 Validity

Schepers (1992) emphasises that each psychometric test should be theoretically well grounded to meet the requirements of content validity. A suitable item format should be used, while the language of all the items should have been thoroughly revised. Several steps were taken to determine the content validity of the SOM.

(i) General

Huysamen (1980) and Van den Berg (1995) point out that information on criterion-related validity, content validity and construct validity is necessary to determine whether a particular learning instrument is suitable for the purpose it is used for.

The validity of a measuring instrument can be defined as the extent to which it serves the stated purpose or the extent to which the test scores reach their objectives (Huysamen, 1996; Van den Berg, 1995). Consequently the evaluation of a measuring instrument's validity always occurs in relation to that instrument's specific use. Since a measuring instrument is usually constructed for different purposes, its validity for each of the possible purposes it might be used for, has to be determined. The measuring instrument could have a high degree of validity for one function, but low validity for another. Consequently it is not correct to refer to **the** validity of a test.

Madge (1981a) points out that it is more correct to talk of determining the validity of the conclusions (or use) drawn from the measuring instrument's scores rather than the validity determination as such, whereas Huysamen (1996:33) also states that it is more correct to refer to the validity of a test for "a particular application thereof".

(ii) Content validity

Although content validity mainly concerns achievement tests, it is also applicable to other cognitive and non-cognitive tests. In the case of non-cognitive tests, for example personality, interest and study orientation questionnaires, various hypothetical situations are placed before testees from which they have to choose the one which agrees with their actions or preferences. The content validity of such a measuring instrument will be determined by the extent to which the situations mentioned in the test are representative of the universe of such situations that are being considered (Huysamen, 1978). Content validity relates to the content validity of a measuring instrument and is not expressed in terms of a quantitative index. It is based on the logical analysis, by experts, of the content and aims of the measuring instrument.

The following was done to ensure the content validity of the SOM:

★ A comprehensive study of relevant literature on the subject was undertaken.
★ The wording and positioning of the items in the fields were checked by various experts.
★ The item field correlations were evaluated.

Cronbach (1971:457) explains the phenomenon that high item test correlations do not necessarily ensure content ability as follows:

> *Low item correlations do not necessarily imply failure of the test content to fit the definition. Indeed, if the heterogeneous, consistently high intercorrelations imply inadequate sampling ... when the test constructor routinely discards the items whose intercorrelations with the total score for the pool are low, he risks making the tests less representative of the defined universe.*

Items theoretically can produce high correlations with one another (in a particular field), yet measure another construct (that “accidentally” has a high correlation with a particular field). This brings item validity to the fore, and a thorough analysis of the individual items’ content will indicate whether items still belong in particular fields or not.

(iii) Secondly steps were taken to determine the construct validity of the SOM.

Construct validity is concerned with the extent to which the measuring instrument measures the theoretical construct(s) that it is supposed to measure. Consequently in order to determine construct validity it is necessary that the constructs the measuring instrument is supposed to measure, are identified and clearly defined.

As far as the construct validity of this questionnaire is concerned, the main focus here is on the internal structure of the questionnaire, namely on the mutual relationship between the items (homogeneity coefficients) and between the fields (factor analysis).

Huysamen (1980:106) states the following in connection with factor analysis:

> *When a factor analysis is carried out to investigate construct validity, a confirming factor analysis rather than an exploratory one is required. Construct validity, which is investigated by means of factor analysis methods, is usually referred to as factorial validity.* (Translation)

(a) Factor analysis

Factor analysis is usually used as the technique to determine construct validity. By using this technique a small number of theoretical constructs or factors are identified that are responsible for the correlations between a large number of variables (Huysamen, 1980). A factor is mentioned when a group of variables reveal a number of similarities for some reason or other. Correlation techniques are used to identify these related variables. A factor can thus be regarded as the end product of a group of variables that have a particular attribute in common (Roos, 1995). To find out whether a group of variables have something in common it is necessary to measure the nature of the correlations between each pair of variables. Child (1970:8) describes factor analysis as:

> *The dual task of simplification based on a mathematical model, followed by an evaluation based on a psychological model, which would add meaning relevant to his (the behavioral scientist's) purposes.*

Carson and O'Dell (1978:27-28) summarise the primary aim of factor analysis as follows:

> *In short, the basic problem is that of deciding what the precise, smaller number of factors will be that will be required to account for the larger number of variables or factors ... there is no definitive way to solve this problem, but there are many approximations or guesses that are used to make the decision. Then, once one has determined how few factors one can get away with, the remaining problem is that of the relationship of the smaller number of factors to the larger number of descriptors. These two problems - (1) finding out how few factors are needed to account for the larger number of variables, and (2) finding the relationship of the larger number of variables to the smaller number of factors - are the two basic problems that must be solved in any factor analysis.*

The purpose of factor analysis is thus twofold (Schepers, 1992). On the one hand the aim is to determine the factor structure of tests, in other words to determine the underlying constructs of the tests so that more information in connection with these tests can be obtained. On the other hand the aim of factor analysis is the description of testees according to certain factor scores.

Child also emphasises the following important aspects of factor analysis (1990:2;3;7):

> *When a group of variables has, for some reason, a great deal in common a factor may be said to exist. These related variables are discovered using the technique of correlation ... . We are now more cautious in ascribing*

> *cause-effect relationships between variables ... . The important difference between exploratory and confirmatory analysis is that in the former one is trying to discover structure in the variables used, whilst in the latter one chooses variables to confirm a predetermined structure ... . (however) The distinction between testing and creating hypotheses in factor analysis is not very sharp.*

To determine the construct validity of the SOM, both the SAS computer system (SAS, 1990) and the BMDP4M computer program (Dixon, Brown, Engelman & Jennrich, 1993) were used.

The first factor analysis was done with a SAS computer system (SAS PROC FACTOR) with a *varimax* rotation carried out on all 150 items (Grade 8 and 9) and 165 items (Grade 10 and 11) of the SOM. Main factor analysis with *varimax* rotation was carried out on all the items. A *Scree* plot indicated that there were probably respectively three or four factors (Grade 8 and 9) and four or five factors (Grade 10 and 11) present. The number of factors was determined with the aid of Kaiser's criterion (Child, 1990). According to this criterion only factors with eigenvalues bigger than one are regarded as common factors. Applying the *varimax* rotation was thus focused on providing a *Scree* plot which could help to withdraw the number of fields, or to determine how many fields there were.

Five further successive factor analyses were carried out on the items relating to the three fields (in the case of Grade 8 and 9) and four fields (Grade 10 and 11) as identified by the first factor analysis by using the BMDP4M computer program with the direct *quartimin* rotation method (Cureton & D'Agostino, 1983). The reason for this is that this rotation is most suited for the data analysis that is expected in the sense that it probably contributes to identify the fields uniquely (Browne, 1992).

After the conclusion of each of the successive factor analyses that were made, certain items were associated with certain fields. Certain items were subsequently left out and the remaining items subjected to a next factor analysis. This process of "factor refinement" (refinement of the identified fields) was thus repeated until the fields could be uniquely identified.

Throughout it was attempted to determine which items of the SOM produced high loadings in respect of the fields (constructs) that had to be measured. In deciding whether an item should be left out, factor loadings (loading of items on the fields) were considered, as well as the content of the items. The criteria used for the interpretation of the factor matrix, is Child's (1990) arbitrary criterion of 0,30 with regard to factor loadings: *"this is quite a rigorous level"* (Child, 1990:39). In an attempt to retain as many items as possible, it was decided to retain those items with loadings in the region of 0,30 and to allow a small variation. Items that loaded on more than one construct, were left out when the loading was more or less the same for each of the two constructs. In cases where a particular item did indeed load on two or more fields, but the item loaded more strongly on one of the fields, the item was included in that field.

5.4.3.2 Item analysis

Item analysis was carried out per field for the final 90[6] items (Grade 10 and 11) and 77 items (Grade 8 and 9), with the aid of ITEMAN™ ver 3.50 (ITEMAN™, 1993) (corrected[7] item discrimination values are given in Chapter 6). In the study under discussion the views of authors Huysamen (1996) and Owen (1995), who point out that items with discrimination values below 0,20 should preferably not be included in a test, are applied.

Item analysis was carried out per total group as well as on the six groups formed by language and grade. Item field correlations were used to determine whether items indeed belonged to particular fields.

The intercorrelations of the different fields were calculated for each of the two grade groups, as well as for the grade groups in general.

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6 See Chapter 6, paragraph 6.2.2.1.

7 The use of corrected discrimination values amounts to a "purer" method in the sense that a particular item is removed from a particular test when that item's **corrected** discrimination value is calculated. It thus provided a more accurate indication of the item's correlation with a particular test (Owen, 1995).

5.4.3.3 Reliability

The reliability of a psychological test can generally be described as the extent to which it measures consistently, whatever it measures (Owen, 1995). In particular, reliability refers to the consistency of the scores obtained by the same individuals on the same or on different test occasions with the same or with different sets of equivalent items. The reliability of a test indicates how much confidence can be placed in the particular score of a test. It is thus necessary that the level of reliability of a test or questionnaire be known.

Owen (1995) emphasises that a test's particular aim plays a decisive role in evaluating the acceptability of that test's reliability coefficient. According to Nunnally (1978) and Owen (1995) a test instrument with a reliability coefficient of approximately 0,60 to 0,65 can produce useful information, provided the test results are handled with the necessary care and expertise. In the case of personality tests it is not always possible to construct tests that are as reliable as test compilers would like them to be. Huysamen (1996:30) states that even coefficients of 0,65 may be acceptable *"if decisions on groups are required".*

In the case of the SOM there is a choice of five answer possibilities. In cases like these reliability is estimated with the aid of Cronbach's coefficient alpha (α) (Howell, 1992; Sincich, 1993). In this case the reliability of the adjusted fields was determined with the Cronbach alpha coefficient. Reliability coefficients were determined for the questionnaire in general for Grade 8 and 9, and for Grade 10 and 11 separately, as well as for the language groups separately.

5.4.3.4 Item bias

Certain researchers are of the opinion that culture plays a smaller role in several tests than is generally expected (Cronbach, 1990; Owen, 1987). Kline (1983:340) believes the following in this regard.

> *If the factor loadings of tests have been shown to be similar across cultures, then all the objections to tests which have been made by cross-*

> *cultural researchers are silenced. For identity rules out error and thus questions of conceptual or metric equivalence become irrelevant, as do all other potential sources of error in cross-cultural measurement ... . Nevertheless, ... we do not mean that all cross-cultural testers have to do is to factor their tests in all their cultures.*

In the case of the SOM bias in items concerning language, gender, race and socio-economic milieu had to be limited. Several steps were taken to handle the complex matter of item bias, for instance translating items back into the original language, the committee approach and experimental pre-testing, as suggested by Brislin (1986) and Oakland (1977). The aim was to present the SOM in both Afrikaans and English to the different mother-tongue groups, with the retention of the same ideas across linguistic boundaries.

The cross-cultural equivalence of the SOM was explored further by investigating its internal structure with the aid of investigative factor analyses, comparison of item analyses and the calculation of reliabilities and correlations (Cudeck & Claassen, 1983; Kline, 1983; Retief, 1988).

5.4.3.5 Norm tables

Raw scores can be converted to various types of derived scores or norm points. In the case of this questionnaire percentile ranks were used as norm points because they are generally used in the interpretation of psychological tests or questionnaires and are easy to interpret. The norm points were determined by converting the distributions of the raw scores for the various fields and for the whole questionnaire into cumulative proportions and using them to read off the percentiles. Percentile ranks were determined with the aid of the cumulative percentages (Ghiselli, Campbell & Zedeck, 1981; Howell, 1992; Huysamen, 1996).

A percentile rank indicates an individual's relative position or rank in the norm group according to the percentage of individuals who obtained lower scores than the individual in question. If a Grade 9 testee's raw total score in the questionnaire for example corresponds to a percentile rank of 74, it means that this learner obtained a higher score than 74 per cent of the learners in the norm group. In other words a percentile rank of 74 indicates that 74 per cent of the norm group obtained lower scores than this. Percentile ranks range from 1 to 100.

The most important advantages of percentile ranks is that they are easily interpretable, that a single glance indicates the relative position of an individual in the norm group, and that the percentile rank is not dependent on assumptions concerning the distribution of the attribute or the typical behaviour pattern of the population. An important disadvantage of the percentile rank is that it is a rank order scale and consequently not suitable for the calculation of statistics such as averages and standard deviations.

The percentile ranks for the SOM are based on the results of the proportional sample obtained from administering the questionnaire as described in paragraph 5.3.

Since the sample for the Grade 8 and 9 learners represents all Grade 8 and 9 learners at high school and the sample for the Grade 10 and 11 learners represents only learners in these grades that take mathematics, separate norm tables were determined for the two grade groups. Norm tables are provided for the different fields separately, and for the whole questionnaire. The norm tables for Grade 8 and 9 learners appear in Table 6.14 and for Grade 10 and 11 learners in Table 6.15. The first and last columns indicate the percentile ranks and the other columns in the raw scores.

The differences between the means of the different subpopulations were generally very small and could usually be explained in terms of environmental variables. One set of norm tables was provided for learners in Grade 8 and 9 on the one hand and one set for learners in Grade 10 and 11 on the other hand.

5.4.4 Comparative studies to determine the suitability of the SOM

5.4.4.1 Descriptive statistics

For each of the three independent variables the following were determined by means of the standard SAS procedure:

(i) Arithmetic means
(ii) Standard deviation
(iii) Skewness
(iv) Kurtosis

These descriptive measures appear in table form in Chapter 6.

5.4.4.2 Analysis of variance

Analysis of variance was done to investigate the various variables' arithmetic means further. Analysis of variance is used to investigate the relationship between the variables, while the effect of nuisance variables is controlled statistically. Du Toit (1985:261) puts this as follows:

> *With the F test all the means together are tested, in one operation, for the presence of possible significant differences.* (Translation)

When carrying out the one-way analysis of variance, only those variables were used that together discriminated significantly between the various language, gender and grade groups.

Analysis of variance techniques in this case were used to determine where significant differences between languages, gender and grade groups could be found. Firstly a MANOVA (multiple analysis of variance) was carried out to determine whether the groups differed significantly from one another with regard to three fields (Grade 8 and 9) or four fields (Grade 10 and 11 jointly; that is to say, how do the groups differ overall with regard to study orientation. Wilks' Lambda, Pillai's Trace, Hotelling-Lawley's Trace and Roy's Biggest Root are used as criteria (Howell, 1992; SAS, 1990). Where the MANOVA statistically indicated significant differences, further investigation was carried out in an effort to determine in terms of which individual fields (single variables) groups differed significantly. By means of LSM it was determined which groups differed significantly from one another with respect to the separate fields.

Where F values were significant at the 5% level, *post hoc* comparisons were used (in this case the LSM technique) to determine between which groups' means the differences were significant.

The analyses were done with the aid of the GLM procedure of the SAS computer system and the 5% level of significance was used for purposes of interpretation.

5.4.4.3 *Post hoc* comparisons

Hurlburt (1994:281) describes *post hoc* tests as:

> *Hypothesis tests performed after a significant ANOVA to explore which means or combinations of means differ from each other.*

Post hoc comparisons (comparison of means) that will be determined by LSM, follow a general F test of the differences between the means of three or four more variables (Hays, 1994). Hays (1994:454) justifies the use of the *post hoc* comparisons as follows:

> *Even though tests for planned comparisons form a useful technique in experimentation, it is far more common for the experimenter to have no special questions to begin with. Initial concern is to establish only that some real effects or group differences do exist in the data. Given a significant overall F test, the task is then to explore the data to find the source of these effects and to try to explain their meaning. In particular, when comparisons are suggested by the data themselves, these are called "posterior" or "post hoc" comparisons.*

When a significant F test score has thus been obtained, in other words when the F test indicates that there are significant differences within the collection of means, the data have to be investigated to find the source of these effects, as well as to explain the meaning thereof. When comparisons are suggested by the data, they are called *post hoc* comparisons.

5.4.4.4 Criterion-related validity: Pearson correlations

Criterion-related validity indicates the accuracy with which the scores obtained by means of a measuring instrument, predict scores in a criterion (Madge, 1982). Two types of validity can be distinguished in this category, namely simultaneous and predictive validity. Both refer to the relationship between test scores and a specific variable and the accuracy with which the scores that were obtained in the tests predict the relative position of the individual in relation to the variable. The correlation between the test scores and the scores obtained from a relevant criterion of the variable concerned, are calculated. This correlation coefficient or validity coefficient can be regarded as a statistical index of the validity of the test. Nunnally (1978) believes that it is unrealistic to expect exceptionally high correlation coefficients. Coefficients of 0,20 and higher can be regarded as significant (Anastasi, 1976).

The criteria that measuring instruments predict, also indicate the nature of the construct(s) that the instrument measures. The data of the criterion-related validity studies can thus also provide relevant information for evaluating construct validity. The next paragraph will provide information on the criterion-related validity of the questionnaire.

The simultaneous validity of the SOM refers to the extent to which the scale distinguishes between learners who differ in their academic behaviour in mathematics. On account of practical considerations the *Achievement test in mathematics* (Standard 7), as well as the *Diagnostic tests in mathematical language,* was administered to Grade 9 learners and used as a criterion for determining simultaneous validity[8].

(i) *Diagnostic tests in mathematical language* (Barnard, 1990)

The aim of these tests is to establish diagnostic aids in basic mathematics according to which shortcomings or handicaps regarding knowledge and comprehension of mathematical terminology can be determined. The basic assumption is that there are certain terms that all learners must know and understand. Without this framework of reference no progress can be made in mathematics.

(ii) *Achievement test in mathematics (Standard 7)* (De Kock, 1993)

This test comprises 30 multiple-choice items (questions) in mathematics that were taken from the National Item Bank for Mathematics that is maintained by the HSRC. These items were thus standardised on the general population of South Africa since use was made, in an experimental phase, of a representative sample of the country's population. This test aims at testing the general level of knowledge and comprehension of mathematics in Grade 9 and it can consequently be regarded as an achievement test in mathematics at Grade 9 level. It was attempted to construct the test in such a way that the content thereof would be representative of the Core syllabus for mathematics: Grade 9 as it was applicable in 1993 (still in use in 1997) as far as this was possible with a limited number of items (30). Efforts were made to make use of items with a discrimination value higher than 0,20.

In Chapter 6 the Pearson correlations between achievements in the fields Study habits in mathematics, Mathematics anxiety, Study attitudes towards mathematics and the *Achievement test in mathematics (Standard 7)* as well as the *Diagnostic tests in mathematical language* are indicated. Pearson's product-moment correlation coefficient was calculated with the aid of the SAS computer system (PROC CORR).

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[8] Only the African and Afrikaans-speaking learners' results will be reported since the sample of English learners, on account of circumstances beyond the control of the researcher, was too small to allow significant conclusions to be drawn.

5.4.4.5 Regression analysis

On calculating a single correlation coefficient one variable is correlated with another variable. Multiple regression analysis is an extension of this (Hurlburt, 1994; Robson, 1995). With the aid of this technique a multiple correlation coefficient is calculated between one measure (dependent variable) and two or more psychological predictors (independent variables). In other words, the joint and separate contributions of two or more independent variables to the dependent variable are determined. Multiple regression as it were “explains” the variation in the dependent variable by determining the relative contributions of two or more independent variables. In this case the fields Study habits in mathematics, Mathematics anxiety and Study attitudes towards mathematics were used as independent variables (predictors) and achievement in the *Achievement test in mathematics (Standard 7)* as well as achievement in the *Diagnostic tests in mathematical language* as the dependent variables.

5.5 SUMMARY

In an attempt to test the generally posed hypotheses, namely to test whether the theoretical fields of the SOM were confirmed by factor validity; to determine the suitability of the SOM; and to determine the joint and individual contributions of the different grade, mother-tongue and gender groups to achievement in the fields of the final questionnaire, the following procedure was followed:

(i) Firstly, the samples were selected.
(ii) Secondly, the way in which the SOM items were generated, was discussed.
(iii) Thirdly, the research hypotheses and the nature of the variables were discussed.
(iv) The steps taken to determine the validity of the SOM were explained next.
(v) A discussion of the nature of the item analysis (with the aid of ITEMAN™,) per total group, as well as the six groups constituted by language and grade group, comes next.
(vi) The reliability of the various fields of the SOM were determined next.
(vii) The possibility of item bias was subsequently investigated.

(viii) Norm tables for each of the various fields were calculated, and then for the questionnaire as a whole. Norm points were determined by converting the distributions of the raw scores and the questionnaire as a whole into cumulative proportions and then using these cumulative proportions to read off the percentiles. With the aid of the cumulative percentages the percentile ranks were determined.

(ix) The descriptive statistics (in Chapter 6) were explained. This includes means, standard deviations, skewness and kurtosis per:

- ★ grade;
- ★ grade and gender;
- ★ gender;
- ★ grade and language; and
- ★ language group separately.

(x) Analyses of variance was undertaken of the variables that differed significantly between the groups.

(xi) *Post-hoc* comparisons (comparison of means) were calculated to find the source of the differences and to explain their significance.

(xii) Pearson correlations were calculated to determine the criterion-related validity of the SOM.

(xiii) Lastly, regression analyses were carried out to determine the joint and separate contributions of the independent variables to the variables.

CHAPTER 6

RESULTS AND DISCUSSION

6.1 INTRODUCTION

The results of the study are reflected in this chapter and provisionally interpreted. The discussion will be structured as follows:

- ★ In the first place the results of the final factor analysis will be indicated, as well as a brief description of, and rationale for each field of the SOM.
- ★ Thereafter the results of the final item analysis will be provided. The intercorrelations between the various fields will be indicated and discussed.
- ★ Then follows a discussion of the reliabilities of the various fields of the SOM.
- ★ Next the possibility of item bias is briefly investigated.
- ★ A discussion of the norm tables comes next.
- ★ The means, standard deviations, skewness and kurtosis are subsequently reproduced.
- ★ The results of the analysis of variance that were carried out as well as the results of the *post hoc* comparisons, are discussed next.
- ★ Aspects of the SOM's criterion-related validity are subsequently discussed.

6.2 DATA PROCESSING: STANDARDISATION OF THE SOM

6.2.1 Determining the SOM's construct validity: Factor analysis

The results of the final factor analysis appear on the pages that follow. After this a short discussion of, and a rationale for each field of the SOM follows.

TABLE 6.1: FINAL FACTOR ANALYSIS OF THE SOM: GRADE 8 AND 9

ROTATED FACTOR ANALYSES:
GRADE 8 AND 9

Item number: Original questionnaire	Item number in list of corrected discrimination values (See Table 6.3)	Factor 1: Habits	Factor 2: Anxiety	Factor 3: Attitudes
81	67	0,291	0,077	**0,480**
111	73	0,250	0,066	**0,453**
121	68	0,267	0,077	**0,474**
32	75	0,286	0,052	**0,413**
52	74	0,173	0,124	**0,489**
92	1	**0,389**	0,023	0,000
102	2	**0,444**	-0,014	0,188
112	76	0,233	-0,051	**0,269**
122	77	0,198	0,058	**0,314**
33	39	0,078	**0,468**	-0,144
63	40	0,101	**0,501**	-0,036
73	41	0,088	**0,475**	-0,090
83	42	-0,015	**0,535**	0,001
93	43	-0,052	**0,506**	0,184
103	44	0,038	**0,409**	0,096
113	45	-0,077	**0,553**	0,144
123	46	0,053	**0,478**	0,030
133	47	0,073	**0,466**	-0,168
143	48	-0,077	**0,535**	0,053
24	69	0,236	0,019	**0,285**

Item number: Original questionnaire	Item number in list of corrected discrimination values (See Table 6.3)	Factor 1: Habits	Factor 2: Anxiety	Factor 3: Attitudes
64	3	**0,425**	0,026	0,096
74	4	**0,482**	0,161	0,202
134	49	0,209	**0,416**	-0,247
144	5	**0,428**	0,015	0,150
5	6	**0,477**	0,143	0,069
15	70	0,165	0,092	**0,417**
25	71	0,086	-0,048	**0,504**
35	7	**0,507**	0,150	0,059
65	50	0,222	**0,458**	-0,019
85	8	**0,429**	0,008	-0,076
105	72	0,261	-0,028	**0,390**
125	51	0,108	**0,473**	-0,048
135	9	**0,640**	-0,013	-0,137
145	10	**0,363**	0,052	0,318
6	11	**0,448**	0,076	-0,002
16	52	0,043	**0,409**	-0,077
26	53	-0,018	**0,398**	0,049
56	12	**0,505**	-0,044	0,074
86	13	**0,385**	0,089	0,213
126	14	**0,367**	0,003	0,223
136	15	**0,451**	-0,053	-0,031
7	16	**0,383**	0,137	0,163
17	17	**0,481**	0,199	0,087
47	18	**0,462**	0,031	-0,106

Item number: Original questionnaire	Item number in list of corrected discrimination values (See Table 6.3)	Factor 1: Habits	Factor 2: Anxiety	Factor 3: Attitudes
57	19	**0,574**	0,061	-0,012
67	20	**0,519**	-0,007	0,016
87	21	**0,489**	0,102	0,173
97	54	0,032	**0,453**	-0,096
107	22	**0,441**	0,121	0,125
127	23	**0,518**	-0,087	-0,209
147	24	**0,465**	0,097	-0,010
28	25	**0,517**	-0,028	-0,181
38	26	**0,511**	0,099	0,032
48	27	**0,391**	0,082	0,159
58	28	**0,426**	0,010	0,060
68	29	**0,646**	-0,031	-0,160
88	30	**0,435**	0,100	0,082
108	55	0,024	**0,387**	-0,157
118	56	-0,060	**0,411**	0,219
138	31	**0,371**	0,073	0,132
148	32	**0,458**	-0,081	-0,089
9	57	0,188	**0,369**	-0,216
19	33	**0,402**	-0,093	-0,062
49	35	**0,344**	-0,088	0,106
69	36	**0,598**	-0,066	-0,133
79	58	0,199	**0,409**	-0,117
89	59	-0,113	**0,437**	0,166
109	37	**0,295**	-0,093	0,135

Item number: Original questionnaire	Item number in list of corrected discrimination values (See Table 6.3)	Factor 1: Habits	Factor 2: Anxiety	Factor 3: Attitudes
139	38	**0,373**	-0,083	0,107
149	34	**0,391**	-0,069	0,167
10	60	-0,068	**0,425**	0,123
20	61	0,107	**0,500**	-0,077
70	62	-0,173	**0,376**	0,054
90	63	-0,106	**0,523**	0,181
110	64	-0,050	**0,432**	0,062
130	65	-0,178	**0,407**	0,054
140	66	-0,154	**0,512**	0,150
Eigenvalues		12,486	6,033	2,882
Percentage variation in the vector space that is explained		15,27	6,85	2,81

The eigenvalues of the main factors are all bigger than 1 and these factors can thus be interpreted.

TABLE 6.2: FINAL FACTOR ANALYSIS OF THE SOM: GRADE 10 AND 11

ROTATED FACTOR ANALYSES:
GRADE 10 AND 11

Item number: Original questionnaire	Item number in list of corrected discrimination values (See Table 6.4)	Factor 1: Habits	Factor 2: Anxiety	Factor 3: Attitudes	Factor 4: Locus of control
11	78	0,122	0,074	0,029	**0,423**
31	79	0,009	0,269	-0,023	**0,402**
51	80	0,052	-0,113	0,187	**0,292**
81	67	-0,011	0,019	**0,687**	-0,080
111	73	-0,095	0,002	**0,725**	-0,061
121	68	-0,134	0,021	**0,807**	-0,057
32	75	0,222	-0,001	**0,481**	-0,120
42	82	0,023	0,037	0,194	**0,484**
52	74	0,071	0,145	**0,325**	0,277
72	83	0,028	0,042	0,182	**0,292**
92	1	**0,377**	0,110	0,069	-0,081
102	2	**0,411**	-0,030	0,236	-0,076
112	76	0,108	-0,022	**0,323**	0,028
122	77	0,210	-0,017	**0,258**	0,094
33	39	0,036	**0,484**	0,044	-0,314
63	40	0,042	**0,563**	-0,012	-0,087
73	41	0,056	**0,490**	-0,015	-0,075
83	42	-0,018	**0,552**	0,042	-0,006
93	43	0,033	**0,417**	0,052	0,130
103	44	0,061	**0,304**	0,088	0,146
113	45	0,016	**0,540**	0,109	0,075

Item number: Original questionnaire	Item number in list of corrected discrimination values (See Table 6.4)	Factor 1: Habits	Factor 2: Anxiety	Factor 3: Attitudes	Factor 4: Locus of control
123	46	-0,019	**0,479**	0,063	-0,049
133	47	0,036	**0,539**	-0,054	-0,175
143	48	-0,013	**0,543**	0,054	0,084
24	69	0,091	0,073	**0,434**	-0,008
64	3	**0,300**	0,193	0,079	-0,043
74	4	**0,502**	0,161	0,065	0,102
84	84	0,051	-0,084	-0,072	**0,311**
104	81	0,154	0,049	0,207	**0,314**
124	85	0,008	-0,160	-0,011	**0,444**
134	49	0,218	**0,400**	0,007	-0,336
144	5	**0,395**	-0,012	0,146	-0,023
5	6	**0,476**	0,179	0,059	-0,027
15	70	0,019	0,096	**0,524**	0,146
25	71	-0,010	0,029	**0,474**	0,181
35	7	**0,496**	0,224	0,042	0,030
65	50	0,240	**0,484**	0,112	-0,058
85	8	**0,302**	0,191	0,011	-0,115
105	72	0,168	-0,058	**0,585**	-0,225
115	86	0,030	0,248	-0,024	**0,401**
125	51	0,171	**0,456**	0,044	-0,077
135	9	**0,453**	0,030	0,249	-0,383
145	10	**0,296**	-0,055	0,254	0,091
6	11	**0,476**	0,070	-0,043	0,021
16	52	-0,002	**0,389**	0,004	-0,050

Item number: Original questionnaire	Item number in list of corrected discrimination values (See Table 6.4)	Factor 1: Habits	Factor 2: Anxiety	Factor 3: Attitudes	Factor 4: Locus of control
26	53	0,056	**0,381**	-0,010	0,164
56	12	**0,480**	0,002	0,147	-0,090
76	87	-0,091	0,040	-0,018	**0,341**
86	13	**0,406**	0,067	0,047	0,209
126	14	**0,420**	0,069	0,039	0,190
136	15	**0,454**	-0,073	0,142	-0,220
7	16	**0,383**	0,158	0,062	0,140
17	17	**0,514**	0,155	-0,019	0,012
47	18	**0,493**	-0,097	-0,106	-0,030
57	19	**0,640**	0,057	0,008	-0,108
67	20	**0,484**	-0,041	0,035	-0,004
87	21	**0,573**	0,094	0,038	0,140
97	54	0,039	**0,447**	-0,043	-0,038
107	22	**0,599**	0,109	-0,075	0,133
127	23	**0,377**	0,020	0,133	-0,413
147	24	**0,436**	0,181	0,010	-0,039
28	25	**0,481**	-0,060	0,038	-0,189
38	26	**0,514**	0,102	-0,068	0,101
48	27	**0,385**	0,104	-0,005	0,161
58	28	**0,510**	-0,021	-0,021	0,081
68	29	**0,587**	-0,034	0,065	-0,311
88	30	**0,571**	0,037	0,031	0,077
108	55	-0,037	**0,360**	-0,073	-0,049
118	56	0,157	**0,344**	-0,002	0,211

Item number: Original questionnaire	Item number in list of corrected discrimination values (See Table 6.4)	Factor 1: Habits	Factor 2: Anxiety	Factor 3: Attitudes	Factor 4: Locus of control
138	31	**0,449**	0,051	0,080	0,211
148	32	**0,467**	-0,077	-0,092	-0,056
9	57	0,173	**0,414**	-0,024	-0,252
19	33	**0,354**	-0,042	0,124	-0,085
49	35	**0,400**	-0,027	0,009	0,108
69	36	**0,509**	-0,087	0,108	-0,158
79	58	0,060	**0,410**	0,199	-0,259
89	59	-0,001	**0,356**	0,012	0,156
99	88	0,149	-0,080	0,013	**0,446**
109	37	**0,338**	-0,009	0,133	0,068
139	38	0,408	-0,040	0,117	0,052
149	34	**0,398**	-0,123	0,082	0,080
10	60	**-0,008**	**0,403**	0,154	0,176
20	61	**0,070**	**0,590**	-0,005	-0,167
30	89	0,001	0,201	0,022	**0,319**
70	62	-0,046	**0,417**	0,045	0,038
90	63	-0,071	**0,391**	-0,021	0,338
100	90	-0,230	0,140	-0,094	**0,261**
110	64	0,004	**0,415**	0,020	0,108
130	65	-0,047	**0,336**	-0,095	0,210
140	66	-0,077	**0,473**	0,027	0,207
Eigenvalues		14,391	5,997	4,087	2,865
Percentage variation in the vector space that is explained		15,19	5,87	3,69	2,34

The eigenvalues of the main factors are all bigger than 1 and these factors can thus be interpreted.

★ Interpretation of the factor matrix

It is clear from Tables 6.1 and 6.2 that all 90 chosen items belong to the four fields.

The first factor analysis was carried out on all 150 items (Grade 8 and 9) and 165 items (Grade 10 and 11) of the SOM. The second factor analysis was carried out on 141 items (Grade 10 and 11) and 127 items (Grade 8 and 9), the third factor analysis on 126 items (Grade 10 and 11) and 116 items (Grade 8 and 9). A fourth factor analysis was then carried out on 100 items (Grade 10 and 11) and 85 items (Grade 8 and 9). The main problem at this stage was that some items that belonged to a particular field for logical reasons, indeed, in the case of one grade group, loaded on the "right" field but in the case of the other grade group loaded less strongly on the particular field than for another grade group. Several items were consequently again left out and the fifth factor analysis was carried out on 92 items (Grade 10 and 11) and 79 items (Grade 8 and 9). After this the final factor analysis was carried out on 90 items (Grade 10 and 11) and 77 items (Grade 8 and 9).

Initially the possibility of four or five fields for Grade 8 and 9 (five or six in the case of Grade 10 and 11) was investigated. **The six factor analyses, however, indicated that three fields could be distinguished in the case of Grade 8 and 9, and four fields in the case of Grade 10 and 11.** In the case of Grade 8 and 9 the fields **Study habits in mathematics** (38 items), **Mathematics anxiety** (28 items) and **Study attitudes towards mathematics** (11 items) could be distinguished. In the case of Grade 10 and 11 a fourth field was identified, namely **Locus of control (with respect to mathematics)** (13 items). Certain aspects of **Study milieu** also appeared to belong to this field.

Aspects of **problem-solving disposition** as well as **information processing** appeared to belong to the field **Study habits** in the final questionnaire. **Mathematics anxiety** came clearly to the fore whereas certain aspects of **Study milieu** ultimately seemed to belong to the field **Study attitudes** in the final version of the SOM.

The final version of the SOM thus consisted of 77 items (Grade 8 and 9) and 90 items (Grade 10 and 11)[1].

★ *The main research hypothesis investigated in this case is the following:*

The theoretical fields of the SOM were confirmed by factor validity.

It appeared that limited support for this hypothesis could be found. Initially, the possibility of four or five fields for Grade 8 and 9 (five and six in the case of Grade 10 and 11) was investigated, but the six factor analyses indicated that only three fields could be distinguished for Grade 8 and 9, and 4 fields in the case of Grade 10 and 11.

6.2.1.1 Brief description of and rationale for each field of the SOM

(i) Study habits in mathematics (SH)

Description and rationale: this field consists of 38 questions and includes the following:

(a) The use of acquired, consistent, effective study methods and habits (for example the planning of one's time and preparation, doing previous tests and examination questions, working out more than just known problems, as well as following up problems in mathematics). This includes a willingness not only to **obtain insight** into certain aspects of mathematics, but also to **memorise** thoroughly theorems, rules and definitions after insight has been obtained into the underlying theorems, rules and definitions, as well as carrying out specific assignments in mathematics.

(b) The extent to which learners carry out instructions and assignments in mathematics, keep their homework up to date, keep up with mathematics and avoid wasting time.

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[1] The final version of the SOM is attached in the form of an appendix. The original questionnaire is obtainable on request.

(c) **Manifesting** the willingness to do mathematics consistently despite the fact that more attractive and "nicer" activities are available. This field thus indicates the extent to which study attitudes towards mathematics manifest themselves in certain study habits in mathematics.

(d) **Problem-solving conduct in mathematics**

Metacognitive learning strategies in mathematics include **planning, self-monitoring, self-evaluation, self-regulation** and **decision-making** during the process of problem-solving in mathematics. This can be described as "thinking about thinking" in mathematics (**when learners try to determine what subsections of mathematics they do not understand**). This includes strategies such as looking for patterns and relationships in mathematics, the continuous testing, estimating and approximating of answers, carrying out Pólya's four steps during problem-solving, abandoning strategies when they are not successful in favour of trying out alternative strategies, and the consistent search for a holistic structure between (even apparently diverse) aspects of the subject.

Implementing these strategies helps learners to **generalise** in mathematics (**inference**). Maker (1993:76) for example came to the following conclusion:

> *Effective problem solving processes will enable educators to prepare all children to meet the challenges they face as adults.*

These learning strategies thrive in a learning environment where preference is given to a problem-centred approach and the co-operative tackling of mathematics problems, and where socialising (social interaction) in the mathematics class has been carried out adequately. Learners should actively participate in acquiring the language of mathematics and enculturisation should take place in the classroom so that certain forms of expression, terms and explanations become acceptable in that particular classroom, that is to say become part of the classroom culture. In other words, a culture where learners acquire the insight that it has formative value to discuss relevant concepts with friends and teachers, to explain this to friends, parents and others and have enough insight to look for possible applications of mathematics in real life.

(e) General and specific learning, summarising and reading strategies, critical thinking and comprehension strategies (for example the optimal use of sketches, tables, diagrams) in mathematics. This field provides a measure of the extent to which learners really understand mathematics. When inadequate concept formation has occurred in mathematics, this frequently becomes apparent from actions such as the following: unsuitable rendering of proof, exaggerated technical errors (erroneous calculations), erroneous allocation of values to unknowns, erroneous assumptions and erroneous allocation of attributes. In such cases learners struggle to distinguish between that which is "given" and that which is "asked for" in mathematics assignments. This makes problem-solving difficult or impossible since the transfer of learning has not been adequate. Learners do not succeed in realising what concepts relate to one another, do not understand and know work in such cases thoroughly, are frequently careless, and under such circumstances use theorems and formulas without considering whether they are applicable to the specific situation or not.

(ii) Mathematics anxiety (MA)[2]

Description and rationale: this field consists of 28 questions. Panic, anxiety and worry manifest themselves, for example, in the form of aimless, repetitive behaviour (for instance chewing nails, excessive sweating, playing with objects, excessive need to visit the toilet, deletion of correct answers and an inability to speak clearly). Learners' motivation in mathematics is negatively influenced when they are emotionally upset. When learners have not mastered the limited, technical language of mathematics, this contributes to mathematics anxiety (Visser, 1988). Emotional instability in the mathematics class (when learners are for example frightened to discuss their problems with their teachers or even to ask questions) inhibit learners' attitude of daring in mathematics and handicap their cognitive functioning. Self-confidence can to a certain extent be regarded as the opposite of this field; in other words an antipole on this scale.

2 Although it is more correct to talk of a learner's mathematics phobia or negative attitude towards mathematics, the term "mathematics" anxiety has become generally accepted by the people and consequently used in this study.

(iii) Study attitudes towards mathematics (SA)

Description and rationale: this field consists of 11 questions and refers to feelings (subjective, but also objective experiences), practical-mindedness and attitudes (towards mathematics and aspects of mathematics) that consistently arise and influence learners' motivation and interest in mathematics. This includes: learners' "mathematical world view" regarding the **self**, the **nature** of mathematics and the **nature of the learning of mathematics**. Learners' study attitudes can be regarded as the driving force of their Study habits in mathematics. Attitudes include particular factors, for example the enjoyment of the subject, self-confidence, perceptions regarding the usefulness of the subject and the challenge that it offers.

(iv) Locus of control with regard to mathematics (LC)

Description and rationale: this field consists of 13 questions. Mathematics learners come from different environments and have different backgrounds. Learners from non-stimulating environments frequently have backlogs, struggle and are slower learners as a result of more limited experiences. The factors involved include frustration, restrictive home conditions, non-stimulating learning and study environments, physical problems (for example poor vision or hearing), reading problems, names and lifestyle in word problems that do not come from learners' field of experience, and language problems. These include the typical problems brought about by second language teaching, language background that is restrictive, inadequate comprehension of the technical language of mathematics and milieu disadvantages.

(v) Study orientation in mathematics (SOM)

Generally viewed, the SOM provides a summary of the aforementioned factors and also a measure of learners' study orientation in mathematics. After this questionnaire has been administered, it should always be followed by an assignment-directed interview. Wachsmuth and Lorenz (1987:43) state the following in this connection:

The diagnosis of student errors is relevant only with respect to the remediation the teacher can give.

6.2.2 Item analysis

6.2.2.1 Introduction

Item analysis was carried out per field on the final 90 items (Grade 10 and 11) and 77 items (Grade 8 and 9) to determine the merit of the final items of the SOM. Table 6.3 and 6.4 indicate the corrected discrimination values of the various items. As stated in Chapter 5, the corrected discrimination values ≥ 0,30 (for the total group) are regarded as good values whereas the corrected discrimination values ≥ 0,20 (in the case of analysis of the items of the language groups separately) for the purpose of this study are regarded as good values.

The results of the final items analysis will be indicated next.

TABLE 6.3: CORRECTED DISCRIMINATION VALUES OF THE ITEMS FOR THE DIFFERENT LANGUAGE GROUPS WITH REGARD TO THE FINAL VERSION OF THE SOM: GRADE 8 AND 9

CORRECTED DISCRIMINATION VALUES OF THE ITEMS IN THE SOM: *GRADE 8 AND 9*

Number	Original questionnaire number	African languages	English	Afrikaans	Total group
1	92	0,34	0,39	0,52	0,40
2	102	0,47	0,52	0,51	0,49
3	64	0,38	0,59	0,58	0,46
4	74	0,55	0,66	0,68	0,59
5	144	0,49	0,56	0,49	0,50

Number	Original questionnaire number	African languages	English	Afrikaans	Total group
6	5	0,48	0,61	0,58	0,53
7	35	0,50	0,64	0,62	0,56
8	85	0,41	0,34	0,40	0,41
9	135	0,52	0,55	0,59	0,56
10	145	0,52	0,52	0,51	0,49
11	6	0,36	0,53	0,62	0,47
12	56	0,48	0,51	0,56	0,52
13	86	0,49	0,57	0,61	0,51
14	126	0,47	0,42	0,58	0,47
15	136	0,36	0,52	0,51	0,44
16	7	0,44	0,57	0,57	0,48
17	17	0,50	0,66	0,64	0,56
18	47	0,41	0,53	0,47	0,45
19	57	0,49	0,65	0,62	0,56
20	67	0,47	0,48	0,60	0,52
21	87	0,51	0,72	0,67	0,57
22	107	0,47	0,60	0,63	0,53
23	127	0,37	0,46	0,42	0,42
24	147	0,44	0,56	0,59	0,50
25	28	0,40	0,37	0,53	0,46
26	38	0,50	0,57	0,58	0,54
27	48	0,50	0,46	0,53	0,48
28	58	0,39	0,53	0,56	0,46
29	68	0,50	0,59	0,63	0,57
30	88	0,40	0,62	0,63	0,49

Number	Original questionnaire number	African languages	English	Afrikaans	Total group
31	138	0,39	0,64	0,59	0,46
32	148	0,39	0,48	0,41	0,42
33	19	0,32	0,41	0,45	0,38
34	149	0,41	0,46	0,52	0,45
35	49	0,37	0,45	0,48	0,40
36	69	0,48	0,54	0,55	0,53
37	109	0,31	0,66	0,34	0,36
38	139	0,36	0,51	0,49	0,41
39	33	0,38	0,66	0,60	0,47
40	63	0,38	0,68	0,66	0,51
41	73	0,40	0,62	0,61	0,48
42	83	0,40	0,66	0,66	0,52
43	93	0,48	0,57	0,57	0,52
44	103	0,44	0,46	0,52	0,46
45	113	0,50	0,63	0,70	0,57
46	123	0,48	0,57	0,56	0,51
47	133	0,38	0,64	0,61	0,47
48	143	0,50	0,56	0,61	0,54
49	134	0,47	0,49	0,51	0,44
50	65	0,47	0,52	0,59	0,51
51	125	0,43	0,58	0,63	0,49
52	16	0,40	0,41	0,48	0,42
53	26	0,33	0,35	0,56	0,42
54	97	0,43	0,44	0,55	0,46
55	108	0,36	0,36	0,50	0,39

Number	Original questionnaire number	African languages	English	Afrikaans	Total group
56	118	0,42	0,39	0,52	0,45
57	9	0,35	0,58	0,53	0,40
58	79	0,51	0,63	0,39	0,46
59	89	0,43	0,48	0,48	0,46
60	10	0,43	0,41	0,45	0,44
61	20	0,47	0,53	0,63	0,52
62	70	0,32	0,46	0,40	0,37
63	90	0,54	0,42	0,57	0,55
64	110	0,41	0,47	0,56	0,46
65	130	0,34	0,45	0,44	0,40
66	140	0,45	0,49	0,61	0,51
67	81	0,63	0,81	0,68	0,67
68	121	0,65	0,76	0,73	0,69
69	24	0,49	0,62	0,54	0,51
70	15	0,53	0,68	0,62	0,58
71	25	0,51	0,70	0,56	0,57
72	105	0,55	0,68	0,56	0,55
73	111	0,65	0,69	0,68	0,67
74	52	0,58	0,52	0,55	0,58
75	32	0,59	0,67	0,60	0,59
76	112	0,42	0,47	0,48	0,44
77	122	0,46	0,48	0,46	0,47

TABLE 6.4: CORRECTED DISCRIMINATION VALUES OF THE ITEMS FOR THE DIFFERENT LANGUAGE GROUPS WITH REGARD TO THE FINAL VERSION OF THE SOM: GRADE 10 AND 11

CORRECTED DISCRIMINATION VALUES OF THE ITEMS IN THE SOM: *GRADE 10 AND 11*

Number	Original questionnaire number	African languages	English	Afrikaans	Total group
1	92	0,45	0,45	0,55	0,49
2	102	0,39	0,54	0,53	0,53
3	64	0,35	0,45	0,53	0,43
4	74	0,53	0,58	0,64	0,58
5	144	0,41	0,52	0,52	0,50
6	5	0,39	0,59	0,59	0,56
7	35	0,52	0,59	0,59	0,58
8	85	0,45	0,33	0,42	0,42
9	135	0,52	0,63	0,48	0,61
10	145	0,43	0,27	0,55	0,43
11	6	0,40	0,51	0,51	0,49
12	56	0,44	0,58	0,62	0,57
13	86	0,42	0,55	0,54	0,47
14	126	0,45	0,46	0,61	0,48
15	136	0,44	0,53	0,41	0,53
16	7	0,46	0,53	0,56	0,48
17	17	0,50	0,56	0,61	0,56
18	47	0,37	0,46	0,38	0,43
19	57	0,62	0,61	0,62	0,66

Number	Original questionnaire number	African languages	English	Afrikaans	Total group
20	67	0,39	0,51	0,55	0,50
21	87	0,49	0,64	0,69	0,61
22	107	0,53	0,63	0,59	0,57
23	127	0,40	0,52	0,29	0,49
24	147	0,47	0,53	0,55	0,54
25	28	0,45	0,48	0,38	0,50
26	38	0,46	0,50	0,55	0,50
27	48	0,45	0,41	0,53	0,41
28	58	0,37	0,57	0,56	0,51
29	68	0,48	0,60	0,58	0,61
30	88	0,58	0,62	0,65	0,61
31	138	0,38	0,61	0,64	0,51
32	148	0,41	0,47	0,33	0,43
33	19	0,28	0,45	0,50	0,43
34	149	0,36	0,44	0,45	0,42
35	49	0,36	0,52	0,52	0,42
36	69	0,40	0,53	0,54	0,55
37	109	0,32	0,57	0,42	0,42
38	139	0,34	0,57	0,56	0,49
39	33	0,31	0,59	0,58	0,48
40	63	0,41	0,68	0,66	0,58
41	73	0,37	0,54	0,69	0,52
42	83	0,49	0,59	0,61	0,56
43	93	0,47	0,51	0,45	0,47
44	103	0,39	0,39	0,37	0,39

Number	Original questionnaire number	African languages	English	Afrikaans	Total group
45	113	0,50	0,66	0,65	0,59
46	123	0,44	0,57	0,52	0,51
47	133	0,47	0,59	0,63	0,54
48	143	0,56	0,58	0,61	0,57
49	134	0,48	0,52	0,52	0,48
50	65	0,52	0,66	0,64	0,60
51	125	0,46	0,54	0,61	0,53
52	16	0,40	0,44	0,42	0,42
53	26	0,31	0,51	0,52	0,44
54	97	0,44	0,22	0,65	0,45
55	108	0,38	0,40	0,34	0,36
56	118	0,43	0,48	0,46	0,44
57	9	0,35	0,52	0,60	0,47
58	79	0,51	0,60	0,34	0,47
59	89	0,47	0,31	0,44	0,40
60	10	0,37	0,44	0,56	0,46
61	20	0,51	0,61	0,69	0,60
62	70	0,38	0,49	0,51	0,45
63	90	0,55	0,32	0,38	0,41
64	110	0,44	0,48	0,49	0,46
65	130	0,33	0,40	0,43	0,36
66	140	0,46	0,48	0,60	0,50
67	81	0,51	0,72	0,75	0,68
68	121	0,57	0,76	0,76	0,71
69	24	0,52	0,67	0,56	0,58

Number	Original questionnaire number	African languages	English	Afrikaans	Total group
70	15	0,57	0,71	0,71	0,63
71	25	0,49	0,72	0,69	0,59
72	105	0,52	0,72	0,71	0,67
73	111	0,52	0,68	0,69	0,65
74	52	0,45	0,55	0,55	0,46
75	32	0,53	0,72	0,62	0,65
76	112	0,38	0,49	0,50	0,46
77	122	0,41	0,47	0,37	0,42
78	11	0,45	0,40	0,48	0,50
79	31	0,50	0,32	0,52	0,51
80	51	0,24	0,49	0,50	0,39
81	104	0,48	0,39	0,51	0,40
82	42	0,52	0,55	0,44	0,58
83	72	0,32	0,45	0,37	0,42
84	84	0,31	0,33	0,32	0,38
85	124	0,33	0,36	0,38	0,47
86	115	0,51	0,38	0,40	0,50
87	76	0,29	0,30	0,41	0,44
88	99	0,25	0,51	0,61	0,49
89	30	0,39	0,42	0,40	0,45
90	100	0,21	0,30	0,23	0,38

From the tables it appears that the corrected discrimination values for the different mother-tongue groups throughout are ≥ 0,20. The corrected discrimination values of the total group throughout are ≥ 0,30.

6.2.2.2 Intercorrelations between the fields

The intercorrelations of the fields for Grade 8 and 9 learners and for Grade 10 and 11 learners are indicated in Tables 6.5 to 6.12.

TABLE 6.5: INTERCORRELATIONS OF THE FIELDS FOR GRADE 8 AND 9 LEARNERS TOGETHER (N = 1740)[3]

Fields	1	2	3
1			
2	0,266		
3	0,612	0,286	

TABLE 6.6: INTERCORRELATIONS OF THE FIELDS FOR GRADE 8 AND 9 (AFRICAN LANGUAGES; N = 1016)

Fields	1	2	3
1			
2	0,247		
3	0,703	0,228	

TABLE 6.7: INTERCORRELATIONS OF THE FIELDS FOR GRADE 8 AND 9 (ENGLISH, N = 231)

Fields	1	2	3
1			
2	0,342		
3	0,559	0,262	

3 Since learners in certain cases neglected to indicate their grade, language group or gender, it sometimes appears as if the value of **N** is incorrect or inconsistent.

TABLE 6.8: INTERCORRELATIONS OF THE FIELDS FOR GRADE 8 AND 9 (AFRIKAANS, N = 493)

Fields	1	2	3
1			
2	0,377		
3	0,663	0,360	

Since the items of each field were drawn up to measure a particular facet or aspect of study orientation in mathematics, the correlations between the various fields should generally be low. From Tables 6.5 to 6.8 it appears that a fair to high relationship occurs between Fields 1 and 3. The intercorrelations of the fields for Grade 8 and 9 learners vary from 0,228 to 0,703.

TABLE 6.9: INTERCORRELATIONS OF THE FIELDS FOR GRADE 10 AND 11 LEARNERS TOGETHER (N = 1262)

Fields	1	2	3	4
1				
2	0,410			
3	0,571	0,299		
4	0,107	0,245	0,212	

TABLE 6.10: INTERCORRELATIONS OF THE FIELDS FOR GRADE 10 AND 11 (AFRICAN LANGUAGES; N = 451)

Fields	1	2	3	4
1				
2	0,404			
3	0,404	0,285		
4	0,251	0,296	0,396	

TABLE 6.11: INTERCORRELATIONS OF THE FIELDS FOR GRADE 10 AND 11 (ENGLISH; N = 418)

Fields	1	2	3	4
1				
2	0,400			
3	0,632	0,305		
4	0,456	0,301	0,480	

TABLE 6.12: INTERCORRELATIONS OF THE FIELDS FOR GRADE 10 AND 11 (AFRIKAANS; N = 393)

Fields	1	2	3	4
1				
2	0,508			
3	0,522	0,323		
4	0,461	0,322	0,350	

From the intercorrelation matrix in Tables 6.9 to 6.12 it can be seen that certain fields' correlations were reasonably high. Intercorrelations between the various scales varied between 0,107 and 0,632. Fairly high intercorrelations indicate that the scales are not completely independent. On the other hand the high correlation coefficients indicate that the fields measure a common underlying factor. A thorough examination of the results, however, indicates that there are no items with a higher correlation with any other field than the field in which it is included. When working from the theoretical model, it should be kept in mind that certain fields have a high correlation.

The intercorrelations of the fields for the Grade 10 and 11 learners show the same trend as that for Grade 8 and 9 learners. There is once again a high intercorrelation between Fields 1 and 3.

(i) Intercorrelation of Field 1 (Study habits) and 3 (Study attitudes)

The high correlation between Fields 1 and 3 can be ascribed to, among other things, the fact that learners' study attitudes towards mathematics are reflected in their study habits in mathematics, whereas adequate study habits indicate adequate study attitudes towards the subject (Corno, 1992; Du Toit, 1970). Visser, for example, indicates that there is a significant relationship between the mathematics achievement of girls in Grade 9 and 11 and their motivation (Visser, 1989). Visser (1989:213) goes further by stating that girls at the end of Grade 9 rather make decisions according to affective, social and attitude-related considerations about whether to take or not to take mathematics in Grade 10:

> *Students generally do not take into account their intellectual capabilities when they make this decision.*

6.2.3 Reliabilities

The reliability of the adapted fields was determined by the Cronbach alpha coefficient. Reliability coefficients were determined for the questionnaire as a whole for Grade 8 and 9, and for Grade 10 and 11 separately. The reliability coefficients for the grade groups appear separately in Table 6.13, and those for the individual language groups in Table 6.14.

6.2.3.1 Grade groups separately

TABLE 6.13: RELIABILITY COEFFICIENTS FOR THE QUESTIONNAIRE AS A WHOLE FOR GRADE 8 AND 9, AND GRADE 10 AND 11 SEPARATELY

Fields	r_{tt} Grade 8 and 9 (N = 1740)	r_{tt} Grade 10 and 11 (N = 1262)
1	0,918	0,924
2	0,889	0,890
3	0,816	0,829
4		0,692
Total	0,931	0,934

6.2.3.2 Language groups separately

TABLE 6.14: RELIABILITY COEFFICIENTS FOR THE VARIOUS FIELDS FOR GRADE 8 AND 9, AS WELL AS GRADE 10 AND 11, TOGETHER, ACCORDING TO LANGUAGE GROUPS

r_{tt} Fields	Grade 8 and 9 (N = 1740)			Grade 10 and 11 (N = 1262)		
	r_{tt} African languages (N = 1016)	r_{tt} English (N = 231)	r_{tt} Afrikaans (N = 493)	r_{tt} African languages (N = 451)	r_{tt} English (N = 418)	r_{tt} Afrikaans (N = 393)
1	0,900	0,934	0,940	0,887	0,931	0,933
2	0,866	0,901	0,923	0,851	0,905	0,921
3	0,793	0,861	0,832	0,712	0,879	0,857
4				0,623	0,783	0,676
Total	0,919	0,943	0,952	0,907	0,947	0,951

Most reliability coefficients are in the order of 0,70 to 0,90. For the questionnaire as a whole the reliability coefficients vary from 0,623 to 0,952. The reliability coefficients can be regarded as highly satisfactory for the purpose for which this questionnaire will be used.

6.2.4 Item bias

In this study the view was taken that the mere fact that the mean test scores differed for two or more of the groups, did not necessarily indicate bias of the test with regard to the variables that formed the groups. Bias regarding the language, gender and educational level was limited by careful item selection. The matter of cultural bias was, however, too complex to be avoided by item selection.

An analysis of the SOM's test reliabilities reveals satisfactory agreement in test reliabilities, as well as factorial similarity (in this case, there is not really a case for item difficulty values and for this reason it appears that construct bias in this case is not a significant problem).

In the case below, one set of norm tables was provided for all learners in Grade 8 and 9 and one set of norm tables for learners in Grade 10 and 11, irrespective of the ethnic group, language or gender. **From an accurate analysis of the discrimination values of the final items it appears that not one of the chosen items favoured one population group more significantly than another; it appears that none of the observed group differences in the test means was significantly different from what psychologists would tend to expect, and that none of the chosen items would be understood in a radically different way by another population group. These items produce invaluable information on the situation as it occurs in certain schools.**

6.2.5 Norm tables

6.2.5.1 Introduction

The differences found between the means (see paragraph 6.3) of the various subpopulations, were relatively small and could usually be explained in terms of the environmental variables. One set of norm tables is provided for learners in Grade 8 and 9 on the one hand and one for learners in Grade 10 and 11 on the other hand. The norm tables appear in Table 6.15 and 6.16.

6.2.5.2 Use of the norm tables

Norm tables are provided separately for the various fields and for the whole questionnaire. The norm tables for Grade 8 and 9 learners appear in Table 6.15 and for Grade 10 and 11 learners in Table 6.16. The first and last columns indicate the percentile ranks and the other columns the raw scores. Only approximately each fifth percentile norm appears in the tables. The use of more percentile points has been deliberately avoided because this would make the percentile rank and the percentiles on the scales appear to be excessively precise. In cases where a learners' raw scores cannot be converted to a percentile rank directly by means of inspection of the tables, use should be made of interpolation to calculate the percentile rank, as explained in the following example: Suppose a learner in Grade 9 obtains a raw score of 94 in Field 1 (Study habits in mathematics). The percentile rank for 93 is 65. His/her percentile rank is thus: $65 + (^1/_3 \times 5 \approx 2) = 65 + 2 = 67$.

TABLE 6.15: PERCENTILE RANKS FOR GRADE 8 AND 9 LEARNERS TOGETHER

Percentile rank	Raw scores				Percentile rank
	Field 1 Habits	Field 2 Anxiety	Field 3 Attitudes	Total	
99,9	151	111	44	302	99,9
99	136	105	43	272	99
97	127	103	42	260	97
95	124	100	41	251	95
90	115	96	39	237	90
85	109	93	37	229	85
80	104	90	36	218	80
75	101	87	34	211	75
70	96	84	32	203	70
65	93	81	31	196	65
60	89	78	30	191	60
55	86	76	28	184	55
50	83	73	27	178	50
45	79	71	25	173	45
40	76	68	24	168	40
35	72	65	23	163	35
30	69	62	22	158	30
25	65	60	20	153	25
20	60	57	19	148	20
15	55	53	17	141	15
10	48	49	14	132	10
5	38	41	11	117	5
3	29	37	9	104	3
1	19	27	5	86	1

TABLE 6.16: PERCENTILE RANKS FOR GRADE 10 AND 11 LEARNERS TOGETHER

Percentile rank	Raw scores					Percentile rank
	Field 1 Habits	Field 2 Anxiety	Field 3 Attitudes	Field 4 Locus of control	Total	
99,9	150	112	44	52	340	99,9
99	137	108	43	51	323	99
97	129	105	42	50	308	97
95	125	103	41	49	299	95
90	116	99	40	48	285	90
85	109	97	38	47	275	85
80	104	94	37	46	265	80
75	100	92	36	45	259	75
70	96	90	35	44	253	70
65	93	88	33	43	247	65
60	89	86	32	42	242	60
55	85	84	31	41	236	55
50	82	81	30	40	232	50
45	79	79	28	39	226	45
40	75	76	27	38	220	40
35	72	74	26	37	215	35
30	69	71	24	36	208	30
25	65	68	23	35	202	25
20	61	65	21	33	193	20
15	56	62	18	31	185	15
10	49	56	16	29	174	10
5	39	49	13	26	155	5
3	34	44	11	23	145	3
1	20	36	6	19	121	1

6.3 DATA PROCESSING: COMPARATIVE STUDIES TO DETERMINE THE APPLICABILITY OF THE SOM

6.3.1 Means, standard deviations, skewness and kurtosis

6.3.1.1 Means and standard deviations for Grade 8 and 9, and Grade 10 and 11 separately

TABLE 6.17: MEANS ($\bar{X}$) AND STANDARD DEVIATION (S) FOR GRADE 8 AND 9, AND GRADE 10 AND 11 SEPARATELY

Fields	Grade 8 and 9 (N = 1741)		Grade 10 and 11 (N = 1262)	
	$\bar{X}$	S	$\bar{X}$	S
1	81,92	25,87	82,15	25,46
2[4]	72,53	18,23	79,29	16,62
3	26,94	9,21	28,85	8,88
4			39,42	7,41
Total	181,39	41,21	229,70	43,11

4 A high score in the field Mathematics anxiety in the SOM indicates the **absence** of anxiety, whereas a low score indicates the **presence** thereof.

6.3.1.2 Means and standard deviations for gender groups separately

TABLE 6.18: MEANS ($\overline{X}$) AND STANDARD DEVIATIONS (S) FOR GENDER AND DEGREE GROUPS SEPARATELY

Fields[5]	Grade 8 and 9			
	Girls (N = 931)		Boys (N = 798)	
	$\overline{X}$	S	$\overline{X}$	S
1	82,39	25.90	81,31	25,79
2	71,36	18,53	73,85	17,73
3	26,32	9,36	27,67	8,966
Total	180,07	41,89	182,84	40,35
	Grade 10 and 11			
	Girls (N = 648)		Boys (N = 607)	
	$\overline{X}$	S	$\overline{X}$	S
1	84,50	24,09	79,56	26,60
2	79,67	16,10	78,88	17,16
3	28,15	8,91	29,59	8,78
4	40,10	7,29	38,71	7,47
Total	232,42	41,10	226,74	44,87

[5] Field 1: Study habits in mathematics (SH)
Field 2: Mathematics anxiety (MA)
Field 3: Study attitudes towards mathematics (SA)
Field 4: Locus of control with regard to mathematics (LC)

TABLE 6.19: MEANS ($\overline{X}$) FOR GENDER GROUPS SEPARATELY

Fields	Boys (N = 1405)		Girls (N = 1579)	
	$\overline{X}$	S	$\overline{X}$	S
1	80,56	26,15	83,26	25,18
2	76,03	17,66	74,77	18,04
3	28,50	8,93	27,07	9,22
4	38,71	7,47	40,10	7,23
Total	201,87	47,61	201,57	48,89

6.3.1.3 Means and standard deviations for language groups separately

TABLE 6.20: MEANS ($\overline{X}$) FOR LANGUAGE AND GRADE GROUPS SEPARATELY

Fields	Grade 8 and 9		
	African languages (N = 1016)	English (N = 231)	Afrikaans (N = 494)
	$\overline{X}$	$\overline{X}$	$\overline{X}$
1	85,83	73,16	77,95
2	70,69	79,08	73,27
3	25,38	28,94	29,20
Total	181,91	181,19	180,41
Fields	**Grade 10 and 11**		
	African languages (N = 451)	English (N = 418)	Afrikaans (N = 393)
	$\overline{X}$	$\overline{X}$	$\overline{X}$
1	93,88	74,46	76,86
2	79,47	80,66	77,62
3	31,32	27,64	27,29
4	33,83	42,19	42,88
Total	238,51	224,95	224,65

TABLE 6.21: MEANS ($\overline{X}$) FOR LANGUAGE GROUPS SEPARATELY

Fields	African languages	English	Afrikaans
	$\overline{X}$	$\overline{X}$	$\overline{X}$
1	88,31	74,00	77,46
2	73,39	80,10	75,20
3	27,21	28,10	28,35
4	33,84	42,19	42,88
Total	199,31	209,37	200,03

6.3.1.4 Skewness and kurtosis

TABLE 6.22: SKEWNESS AND KURTOSIS WITH REGARD TO THE DISTRIBUTION OF THE VARIABLES FOR GRADE 8 AND 9, AND GRADE 10 AND 11 SEPARATELY

Grade group	Variable	N	Skewness	Kurtosis
8 and 9	Habits[6]	1741	-0,17	-0,24
	Anxiety		-0,30	-0,30
	Attitudes		-0,24	-0,60
	Total		0,04	-0,27
10 and 11	Habits	1262	-0,09	-0,23
	Anxiety		-0,60	0,17
	Attitudes		-0,45	-0,47
	Locus		-0,72	0,14
	Total		-0,23	0,10

6 For the sake of brevity the names of the fields in the tables have been abbreviated as follows: Habits, Anxiety, Attitudes and Locus.

TABLE 6.23: SKEWNESS AND KURTOSIS WITH REGARD TO THE DISTRIBUTION OF THE VARIABLES FOR GRADE 8 AND 9, GENDERS SEPARATE

Gender	Variable	N	Skewness	Kurtosis
Boys	Habits	798	-0,23	-0,08
	Anxiety		-0,45	-0,01
	Attitudes		-0,36	-0,43
	Total		-0,10	-0,26
Girls	Habits	931	-0,11	-0,38
	Anxiety		-0,18	-0,45
	Attitudes		-0,13	-0,69
	Total		0,18	-0,24

TABLE 6.24: SKEWNESS AND KURTOSIS WITH REGARD TO THE DISTRIBUTION OF THE VARIABLES FOR GRADE 10 AND 11, GENDERS SEPARATE

Gender	Variable	N	Skewness	Kurtosis
Boys	Habits	607	-0,04	-0,26
	Anxiety		-0,68	0,28
	Attitudes		-0,67	-0,04
	Locus		-0,69	0,19
	Total		-0,27	0,19
Girls	Habits	648	-0,09	-0,26
	Anxiety		-0,51	0,02
	Attitudes		-0,24	-0,74
	Locus		-0,77	0,15
	Total		-0,15	-0,12

TABLE 6.25: SKEWNESS AND KURTOSIS WITH REGARD TO THE DISTRIBUTION OF THE VARIABLES FOR THE TOTAL GROUP, GENDERS SEPARATE

Gender	Variable	N	Skewness	Kurtosis
Boys	Habits	1405	-0,15	-0,17
	Anxiety		-0,54	0,06
	Attitudes		-0,49	-0,32
	Locus		-0,69	0,19
	Total		0,01	-0,20
Girls	Habits	1579	-0,11	-0,32
	Anxiety		-0,34	-0,34
	Attitudes		-0,19	-0,71
	Locus		0,18	-0,24
	Total		0,05	-0,45

TABLE 6.26: SKEWNESS AND KURTOSIS WITH REGARD TO THE DISTRIBUTION OF THE VARIABLES FOR THE GRADE 8 AND 9 GROUP, LANGUAGES SEPARATE

Language group	Variable	N	Skewness	Kurtosis
African	Habits	1016	-0,20	0,03
	Anxiety		-0,12	-0,23
	Attitudes		-0,08	-0,54
	Total		0,20	-0,22
English	Habits	231	0,01	-0,57
	Anxiety		-0,63	-0,11
	Attitudes		-0,51	-0,52
	Locus		-0,06	-0,59
Afrikaans	Habits	494	-0,08	-0,45
	Anxiety		-0,51	-0,30
	Attitudes		-0,46	-0,41
	Locus		-0,13	-0,36

TABLE 6:27: SKEWNESS AND KURTOSIS WITH REGARD TO THE DISTRIBUTION OF THE VARIABLES FOR THE GRADE 10 AND 11 GROUP, LANGUAGES SEPARATE

Language group	Variable	N	Skewness	Kurtosis
African	Habits	451	-0,19	-0,13
	Anxiety		-0,39	-0,35
	Attitudes		-0,37	-0,38
	Locus		-0,21	-0,29
	Total		-0,10	-0,37
English	Habits	418	0,09	-0,10
	Anxiety		-0,69	0,45
	Attitudes		-0,34	-0,77
	Locus		-0,97	0,92
	Total		-0,11	0,00
Afrikaans	Habits	393	-0,05	-0,10
	Anxiety		-0,66	0,21
	Attitudes		-0,36	-0,65
	Locus		-1,54	5,04
	Total		-0,132	-0,33

TABLE 6.28: SKEWNESS AND KURTOSIS WITH REGARD TO THE DISTRIBUTION OF THE VARIABLES FOR THE TOTAL GROUP, LANGUAGES SEPARATE

Language group	Variable	N	Skewness	Kurtosis
African	Habits	1467	-0,22	0,01
	Anxiety		-0,22	-0,31
	Attitudes		-0,24	-0,51
	Locus		-0,21	-0,29
	Total		0,20	-0,43
English	Habits	649	0,06	-0,27
	Anxiety		-0,67	0,23
	Attitudes		-0,40	-0,70
	Locus		-0,97	0,92
	Total		-0,04	-0,24
Afrikaans	Habits	887	-0,06	-0,31
	Anxiety		-0,59	-0,09
	Attitudes		-0,42	-0,53
	Locus		-1,54	5,04
	Total		-0,14	-0,24

It is accepted that for a normal distribution the skewness coefficient = 0 and the kurtosis coefficient = 3 (Spiegel, 1961; Stuart & Ord, 1987). From the data it appears that some of the variables do not meet the requirements of a normal distribution. In certain of the statistical procedures that were followed, a normal distribution would have been a requirement. In such cases the results have to be interpreted with caution (Crowther, 1997; Sincich, 1993).

In the preceding paragraphs the arithmetic means are indicated without comment. In the paragraphs that follow analysis of variance is carried out to investigate the different variables' means further. *Post hoc* comparisons (in this case the Least Squares Means technique discussed in Chapter 5) are used to determine between which groups' means the differences are statistically significant.

6.3.2 Analysis of variance and *post hoc* comparisons

The results of the analysis of variance and *post hoc* comparisons appear in the following tables, after which the results are discussed. This analysis of variance was carried out on the following dependent variables: Study habits in mathematics, Mathematics anxiety, Study attitudes towards mathematics as well as Locus of control relating to mathematics. Grade, language and gender functioned as independent variables.

TABLE 6.29: MULTIPLE ANALYSIS OF VARIANCE (MANOVA) CARRIED OUT ON GRADE 8 AND 9 WITH HABITS, ANXIETY, ATTITUDES AND TOTAL AS DEPENDENT VARIABLES AND LANGUAGE AND GENDER AS INDEPENDENT VARIABLES

Statistics	Value	F value	Degrees of freedom in numerator	Degrees of freedom in denominator	P value
LANGUAGE EFFECT					
Wilks's Lambda	0,78934201	72,0696	6	3444	0,0001*[7]
Pillai's trace	0,21271678	68,3553	6	3446	0,0001*
Hotelling-Lawley's trace	0,26426972	75,8014	6	3442	0,0001*
Roy's maximum root	0,25400112	145,8813	3	1723	0,0001*
GENDER EFFECT					
Wilks's Lambda	0,98235435	10,3105	3	1722	0,0001*
Pillai's trace	0,01764565	10,3105	3	1722	0,0001*
Hotelling-Lawley's trace	0,01796261	10,3105	3	1722	0,0001*
Roy's maximum root	0,01796261	10,3105	3	1722	0,0001*

[7]* Indicates results significant at 5% level of significance.

TABLE 6.30: ANALYSIS OF VARIANCE CARRIED OUT ON GRADE 8 AND 9 WITH HABITS, ANXIETY, ATTITUDES AND TOTALS AS DEPENDENT VARIABLES AND LANGUAGE AND GENDER AS INDEPENDENT VARIABLES

Variation sources	Degrees of freedom	Sum of squares[1]	Mean sum of squares	F value	P value
HABITS					
Language and gender	3	41397,45	13799,15	21,39	0,0001*
Error	1724	1112210,59	645,13		
Corrected Total	1727	1153608,04			
Language	2	40895,26	20447,63	31,70	0,0001*
Gender	1	392,19	392,19	0,61	0,4357
ANXIETY					
Language and gender	3	15992,74	5330,91	16,52	0,0001*
Error	1724	556331,11	322,70		
Corrected Total	1724	572323,85			
Language	2	13313,93	6656,96	20,63	0,0001*
Gender	1	2460,06	2460,06	7,62	0,0058*
ATTITUDES					
Language and gender	3	6607,97	2202,66	27,23	0,0001*
Error	1724	139436,70	80,88		
Corrected Total	1724	146044,67			
Language	2	5829,19	2914,59	36,04	0,0001*
Gender	1	752,63	752,63	9,31	0,0023*

[1] Type 3 sum of squares is used in the analysis of variance to indicate the partial effect of the variables separately. Type 1 sum of the squares reproduces the marginal effect.

Variation sources	Degrees of freedom	Sum of squares[1]	Mean sum of squares	F value	P value
TOTALS					
Language and gender	3	4164,65	1388,22	0,82	0,4840
Error	1724	2926759,93	1697,66		
Corrected Total	1724	2930924,58			
Language	2	886,59	443,30	0,26	0,7702
Gender	1	3275,20	3275,20	1,93	0,1650

TABLE 6.31: *POST HOC* COMPARISON WITH THE AID OF THE LEAST SQUARES MEANS BETWEEN LANGUAGE AND GENDER (GRADE 8 AND 9) WITH HABITS, ANXIETY, ATTITUDES AND TOTALS AS DEPENDENT VARIABLES

Independent variables: Language and gender	Variable: Least squares means	Exceedance probabilities concerning language and gender		
		African	English	Afrikaans
LANGUAGE	Habits			
African	85,80			
English	73,15	0,0001*		
Afrikaans	77,87	0,0001*	0,0201*	
LANGUAGE	Anxiety			
African	70,78			
English	79,11	0,0001*		
Afrikaans	73,26	0,0123*	0,0001*	
LANGUAGE	Attitudes			
African	25,44			
English	28,95	0,0001*		
Afrikaans	29,26	0,0001*	0,6682	

Independent variables: Language and gender	Variable: Least squares means	Exceedance probabilities concerning language and gender		
		African	English	Afrikaans
LANGUAGE	Total			
African	182,02			
English	181,22	0,7883		
Afrikaans	180,40	0,4738	0,8035	
GENDER	Habits	Male	Female	
Male	78,46			
Female	79,42	0,4357		
GENDER	Anxiety			
Male	75,58			
Female	73,19	0,0058*		
GENDER	Attitudes			
Male	28,55			
Female	27,22	0,0023*		
GENDER	Total			
Male	182,59			
Female	179,83	0,1650		

The results (summarised in Table 6.31) indicate, *inter alia*, the following:

★ In the case of the Grade 8 and 9 groups the means of the African language-speaking persons for all three fields differ significantly statistically from the means of the Afrikaans and English-speaking persons. The means of the Afrikaans-speaking persons, in the case of study habits and mathematics anxiety, differ statistically significantly from the means of the English-speaking persons. The African language speaking persons' study habits in mathematics are consequently more adequate than those of the Afrikaans and English-speaking persons. However, African language speaking persons experience higher anxiety levels and less adequate study attitudes towards mathematics than the other two mother-tongue groups.

It would appear that African language speaking persons in Grade 8 and 9 have a more positive attitude towards mathematics and display more adequate study habits than their English and Afrikaans-speaking counterparts. This is in agreement with the following finding of Møller (1994:44):

> *Township youth take their education and after-class assignments very seriously.*

Her study confirms the surmise that learners "spontaneously" take precautions (for example working at school in the afternoon together with friends) in an effort to counteract the harmful effect of milieu disadvantage, but that these measures do not really have the desired effect (1994:44):

> *the compensation strategies of most pupils from poor quality home environments are not working well.*

In this connection, African language speaking persons reveal higher levels of mathematics anxiety as well as a less adequate study attitude towards mathematics.

★ With regard to gender there were statistically significant differences regarding the fields anxiety and attitudes. Boys in Grade 8 and 9 showed lower anxiety levels in mathematics and more adequate study attitudes towards mathematics than the girls. This finding correlates with that of Visser (1989:213), who found that:

> *The attitudes of females become more negative in the period between Std 5 and Std 7. They become more anxious about their mathematics studies ... Their interest in the subject wanes.*

For a variety of reasons an unacceptably high number of girls give up taking mathematics at the end of Grade 9 (Costello, 1991; Maker, 1993). Those who do take the subject, have a study orientation in mathematics that is obviously more adequate than the study orientation of those who do not take the subject. In the Grade 10 and 11 group an opposite trend to the one for Grade 8 and 9 was perceived.

TABLE 6.32: MULTIPLE ANALYSIS OF VARIANCE (MANOVA) CARRIED OUT ON GRADE 10 AND 11 WITH HABITS, ANXIETY, ATTITUDES, LOCUS OF CONTROL AND TOTALS AS DEPENDENT VARIABLES AND LANGUAGE AND GENDER AS INDEPENDENT VARIABLES

Statistics	Value	F value	Degrees of freedom in numerator	Degrees of freedom in denominator	P value
LANGUAGE EFFECT					
Wilks's Lambda	0,49181532	132,8909	8	2496	0,0001*
Pillai's trace	0,51688402	108,8229	8	2498	0,0001*
Hotelling-Lawley's trace	1,01559536	158,3059	8	2494	0,0001*
Roy's maximum root	0,99786937	311,5847	4	1249	0,0001*
GENDER EFFECT					
Wilks's Lambda	0,93170150	22,8712	4	1248	0,0001*
Pillai's trace	0,06829850	22,8712	4	1248	0,0001*
Hotelling-Lawley's trace	0,07330513	22,8712	4	1248	0,0001*
Roy's maximum root	0,07330513	22,8712	4	1248	0,0001*

TABLE 6.33: ANALYSIS OF VARIANCE CARRIED OUT ON GRADE 10 AND 11 WITH HABITS, ANXIETY, ATTITUDES, LOCUS OF CONTROL AND TOTALS AS DEPENDENT VARIABLES AND LANGUAGE AND GENDER AS INDEPENDENT VARIABLES

Variation sources	Degrees of freedom	Sum of squares	Mean sum of squares	F value	P value
HABITS					
Language and gender	3	102564,34	34188,11	60,31	0,0001*
Error	1251	709162,49	566,88		
Corrected Total	1254	811726,83			
Language	2	94930,94	47465,47	83,73	0,0001*
Gender	1	5236,11	5236,11	9,24	0,0024*
ANXIETY					
Language and gender	3	2005,17	668,39	2,43	0,0638
Error	1251	344308,09	275,23		
Corrected Total	1254	346313,26			
Language	2	1810,48	905,24	3,29	0,0376
Gender	1	138,35	138,35	0,50	0,4785
ATTITUDES					
Language and gender	3	4975,30	1658,43	22,13	0,0001*
Error	1251	93761,94	74,95		
Corrected Total	1254	98737,24			
Language	2	4323,73	2161,87	28,84	0,0001*
Gender	1	855,16	855,16	11,41	0,0008*
LOCUS OF CONTROL					
Language and gender	3	23081,19	7693,73	210,54	0,0001*
Error	1251	45715,44	36,54		
Corrected Total	1254	68796,63			
Language	2	22478,01	11239,01	307,55	0,0001*
Gender	1	1094,17	1094,17	29,94	0,0001*

Variation sources	Degrees of freedom	Sum of squares	Mean sum of squares	F value	P value
TOTAL					
Language and gender	3	61044,36	20348,12	11,25	0,0001*
Error	1251	2261853,00	1808,04		
Corrected Total	1254	2322897,36			
Language	2	50972,93	25486,47	14,10	0,0001*
Gender	1	7736,67	7736,67	4,28	0,0388*

TABLE 6.34: *POST HOC* COMPARISONS WITH THE AID OF THE LEAST SQUARES MEANS BETWEEN LANGUAGE AND GENDER (GRADE 10 AND 11) WITH HABITS, ANXIETY, ATTITUDES, LOCUS OF CONTROL AND TOTALS AS DEPENDENT VARIABLES

Independent variables: Language and gender	Variable: Least squares means	Exceedance probabilities regarding language and gender		
		African	English	Afrikaans
LANGUAGE	Habits			
African	93,67			
English	74,36	0,0001*		
Afrikaans	76,95	0,0001*	0,1225	
LANGUAGE	Anxiety			
African	79,43			
English	80,64	0,2844		
Afrikaans	77,65	0,1232	0,0108*	
LANGUAGE	Attitudes			
African	31,37			
English	27,68	0,0001*		
Afrikaans	27,30	0,0001*	0,5287	
LANGUAGE	Locus of Control			
African	33,71			
English	42,15	0,0001*		
Afrikaans	42,95	0,0001*	0,0588	

Independent variables: Language and gender	Variable: Least squares means	Exceedance probabilities regarding language and gender		
		African	English	Afrikaans
LANGUAGE	Total			
African	238,17			
English	224,83	0,0001*		
Afrikaans	224,86	0,0001*	0,9926	
GENDER	Habits	Male	Female	
Male	79,61			
Female	83,71	0,0024*		
GENDER	Anxiety			
Male	78,91			
Female	79,57	0,4785		
GENDER	Attitudes			
Male	29,61			
Female	27,95	0,0008*		
GENDER	Locus of Control			
Male	38,67			
Female	40,54	0,0001*		
GENDER	Total			
Male	226,80			
Female	231,78	0,0388*		

The results (summarised in Table 6.34) indicate, *inter alia*, the following:

★ Also in the case of Grade 10 and 11 African language speaking learners reveal that they have more adequate study habits than their Afrikaans and English counterparts.

★ Afrikaans-speaking learners' means for anxiety differ statistically from those for English-speaking learners. English-speaking learners have significantly lower anxiety levels than Afrikaans-speaking learners, who have the highest anxiety levels of all three groups.

- ★ With regard to Study attitudes it appears that African language speakers in Grade 10 and 11 have significantly more adequate study attitudes than their Afrikaans and English fellow-pupils.
- ★ The most significant finding of the *post hoc* comparisons, however, is that the achievement of African language speaking learners in Grade 10 and 11 is much poorer in the field Locus of control than that of Afrikaans and English-speaking learners. A variety of factors probably contribute to this, including language problems, teachers who are underqualified and less optimal SES under African-speaking persons in general. Møller's (1994:43) view is in accordance with the preceding views:

  The most striking finding to emerge from the study is that the poor quality home environment provides little support for homework activities ... Furthermore, poor school and home environments tend to go hand in hand.

  These findings are in agreement with the findings of Haladyna, Shaugnessy and Shaugnessy (1983), Hanna, Kündiger and Larouche (1991), Kaisner-Messmer (1993) and Wong (1992) who point out the positive correlation between unfavourable background factors, external Locus of control and inadequate achievement in mathematics.

- ★ The girls' means in Grade 10 and 11 differ statistically significantly from those of the boys in these grades (except in the field Anxiety). Girls who take mathematics after Grade 9 are clearly more inclined to feel that they have control over the situation in the mathematics class.

TABLE 6.35: MULTIPLE ANALYSIS OF VARIANCE (MANOVA) CARRIED OUT ON THE TWO GRADE GROUPS WITH HABITS, ANXIETY, ATTITUDES AND TOTALS AS DEPENDENT VARIABLES AND GRADE GROUPS AS INDEPENDENT VARIABLES

Statistics	Value	F value	Degrees of freedom in numerator	Degrees of freedom in denominator	P value
Wilks's Lambda	0,05652058	12506,98	4	2997	0,0001*
Pillai's trace	0,94347942	12506,98	4	2997	0,0001*
Hotelling-Lawley's trace	16,6926719	12506,98	4	2997	0,0001*
Roy's maximum root	16,6926719	12506,98	4	2997	0,0001*

TABLE 6.36: ONE-WAY ANALYSIS OF VARIANCE (MANOVA) CARRIED OUT ON THE TWO GRADE GROUPS TOGETHER WITH HABITS, ANXIETY, ATTITUDES AND TOTALS AS DEPENDENT VARIABLES AND GRADE GROUPS AS INDEPENDENT VARIABLES

Variation sources	Degrees of freedom	Sum of squares	Mean sum of squares	F value	P value
HABITS					
Grade groups	1	38,28	38,28	0,06	0,8097
Error	3000	1981227,13	660,41		
Corrected Total	3001	1981265,41			
ANXIETY					
Grade groups	1	33358,88	33358,88	108,10	0,0001*
Error	3000	925807,52	308,60		
Corrected Total	3001	959166,40			
ATTITUDES					
Grade groups	1	2678,54	2678,54	32,53	0,0001*
Error	3000	246985,75	82,83		
Corrected Total	3001	249664,29			

TABLE 6.37: *POST HOC* COMPARISON WITH THE AID OF THE LEAST SQUARES MEANS BETWEEN THE TOTAL GRADE GROUPS WITH HABITS, ANXIETY, ATTITUDES AND TOTALS AS DEPENDENT VARIABLES

Independent variables: Grade groups	Variable: Least squares means	Exceedance probabilities with regard to grade groups	
		8 and 9	10 and 11
Grade group	Habits		
8 and 9	81,92		
10 and 11	82,15	0,8097	
Grade group	Anxiety		
8 and 9	72,53		
10 and 11	79,29	0,0001*	
Grade group	Attitudes		
8 and 9	26,94		
10 and 11	28,85	0,0001*	

The results (summarised in Table 6.37) indicate, *inter alia*, the following:

★ Learners in Grade 10 and 11 have lower anxiety levels than their fellow-learners in Grade 8 and 9. This is understandable, since many learners who are "frightened" of the subject or whose achievement is inadequate in mathematics, discontinue studying the subject at the end of Grade 9.

★ Learners in Grade 10 and 11 have more adequate study attitudes than their counterparts in Grade 8 and 9.

TALBE 6.38: MULTIPLE ANALYSIS OF VARIANCE (MANOVA) CARRIED OUT ON THE TWO GRADE GROUPS (MALE) WITH HABITS, ANXIETY, ATTITUDES AND TOTALS AS DEPENDENT VARIABLES AND GENDER AS INDEPENDENT VARIABLE

Statistics	Value	F value	Degrees of freedom in numerator	Degrees of freedom in denominator	P value
Wilks's Lambda	0,95655532	2121,01	3	1401	0,0001*
Pillai's trace	0,04344468	2121,01	3	1401	0,0001*
Hotelling-Lawley's trace	0,04541785	2121,01	3	1401	0,0001*
Roy's maximum root	0,04541785	2121,01	3	1401	0,0001*

TABLE 6.39: ONE-WAY ANALYSIS OF VARIANCE CARRIED OUT ON THE TWO GRADE GROUPS (MALE) JOINTLY WITH HABITS, ANXIETY AND ATTITUDES AS DEPENDENT VARIABLES AND GENDER AS INDEPENDENT VARIABLE

Variation sources	Degrees of freedom	Sum of squares	Mean sum of squares	F value	P value
HABITS					
Grade groups	1	1057,64	1057,64	1,55	0,2137
Error	1403	958665,11	683,30		
Corrected Total	1404	959722,75			
ANXIETY					
Grade groups	1	8727,20	8727,20	28,55	0,0001*
Error	1403	428903,77	305,71		
Corrected Total	1404	437630,97			
ATTITUDES					
Grade groups	1	1276,10	1276,10	16,18	0,0001*
Error	1403	110623,15	78,85		
Corrected Total	1404	111899,25			

TABLE 6.40: *POST HOC* COMPARISON WITH THE AID OF THE LEAST SQUARES MEANS BETWEEN GENDERS (TOTAL MALE GROUP) WITH HABITS, ANXIETY, ATTITUDES AND TOTALS AS INDEPENDENT VARIABLES

Independent variables: Language and gender	Variable: Least squares means	Exceedance probabilities with regard to language and gender	
TOTAL GROUP (MALE)	Habits	Grade 8 and 9	Grade 10 and 11
Grade 8 and 9	81,31		
Grade 10 and 11	79,56	0,2137	
TOTAL GROUP (MALE)	Anxiety	Grade 8 and 9	Grade 10 and 11
Grade 8 and 9	73,85		
Grade 10 and 11	78,88	0,0001*	
TOTAL GROUP (MALE)	Attitudes	Grade 8 and 9	Grade 10 and 11
Grade 8 and 9	27,67		
Grade 10 and 11	29,59	0,0001*	

The results (summarised in Table 6.40) indicate, *inter alia*, the following:

★ The male group of Grade 8 and 9 learners have statistically significantly higher and less adequate study attitudes towards mathematics than their fellow learners in Grade 10 and 11.

TABLE 6.41: MULTIPLE ANALYSIS OF VARIANCE (MANOVA) CARRIED OUT ON THE TWO GRADE GROUPS (FEMALE) WITH HABITS, ANXIETY, ATTITUDES AND TOTALS AS DEPENDENT VARIABLES AND GENDER AS INDEPENDENT VARIABLE

Statistics	Value	F value	Degrees of freedom in numerator	Degrees of freedom in denominator	P value
Wilks's Lambda	0,94382933	31,2248	3	1574	0,0001*
Pillai's trace	0,05617067	31,2248	3	1574	0,0001*
Hotelling-Lawley's trace	0,05951359	31,2248	3	1574	0,0001*
Roy's maximum root	0,05951359	31,2248	3	1574	0,001*

TABLE 6.42: ONE-WAY ANALYSIS OF VARIANCE CARRIED OUT ON THE TWO GRADE GROUPS (FEMALE) JOINTLY WITH HABITS AS DEPENDENT VARIABLE AND GENDER AS INDEPENDENT VARIABLE

Variation sources	Degrees of freedom	Sum of squares	Mean sum of squares	F value	P value
HABITS					
Grade groups	1	1687,86	1687,86	2,66	0,1028
Error	1576	998534,17	633,59		
Corrected Total	1577	1000222,03			
ANXIETY					
Grade groups	1	26416,58	26416,58	85,51	0,0001*
Error	1576	486859,83	308,92		
Corrected Total	1577	513276,41			
ATTITUDES					
Grade groups	1	1277,07	1277,07	15,16	0,0001*
Error	1576	132728,40	84,21		
Corrected Total	1577	134005,47			

TABLE 6.43: *POST HOC* COMPARISON WITH THE AID OF THE LEAST SQUARES MEANS BETWEEN GENDERS (TOTAL FEMALE GROUP) WITH HABITS, ANXIETY, ATTITUDES AND TOTALS AS DEPENDENT VARIABLES

Independent variables: Language and gender	Variable: Least squares means	Exceedance probabilities with regard to language and gender	
TOTAL GROUP (FEMALE)	Habits	Grade 8 and 9	Grade 10 and 11
Grade 8 and 9	82,39		
Grade 10 and 11	84,49	0,1028	
TOTAL GROUP (FEMALE)	Anxiety	Grade 8 and 9	Grade 10 and 11
Grade 8 and 9	71,36		
Grade 10 and 11	79,67	0,0001*	
TOTAL GROUP (FEMALE)	Attitudes	Grade 8 and 9	Grade 10 and 11
Grade 8 and 9	26,32		
Grade 10 and 11	28,15	0,0001*	

The results (summarised in Table 6.43) indicate, *inter alia*, the following:

- ★ *The female group of Grade 8 and 9 learners have statistically significantly higher anxiety levels and less adequate study attitudes towards mathematics than their fellow learners in Grade 10 and 11.*
- ★ *The first group of subhypotheses investigated in this section is the following:*
  - ▪ *The achievements in the various fields of the SOM for the grade groups together differ statistically significantly from one another.*
  - ▪ *The achievements in the various fields of the SOM of the different mother-tongue-groups differ statistically significantly from one another.*
- ★ *The achievements in the different fields of the SOM of the various gender groups differ statistically significantly from one another.*

From the results it is clear that these hypotheses are supported. The analysis of variance indicates that there are practically throughout statistically significant differences between the mean achievements for both grade groups (8 and 9 on the one hand and 10 and 11 on the other) and the language and the gender groups. Post hoc comparisons indicate exactly where the differences between the study group lie, as indicated where applicable.

6.3.3 Criterion-related validity

Tables 6.44 to 6.48 contain Pearson correlations (discussed in paragraph 5.4.4.4) whereas the results of the multiple regression analysis appear in Table 6.4.9.

6.3.3.1 Simultaneous validity: Pearson correlations

TABLE 6.44: PEARSON CORRELATIONS OF FIELDS WITH STANDARDISED MATHEMATICS TESTS FOR GRADE 9 LEARNERS (TOGETHER; N = 1072)

Fields → Tests ↓	Habits	Anxiety	Attitudes	Total
Achievement test in mathematics (Standard 7) (N = 472)	0,13*	0,46*	0,32*	0,36*
Diagnostic tests in mathematical language (N = 470)	0,10	0,45*	0,33*	0,33*

TABLE 6.45: PEARSON CORRELATIONS OF FIELDS WITH STANDARDISED MATHEMATICS TESTS FOR GRADE 9 LEARNERS (MALE; N = 469)

Fields → Tests ↓	Habits	Anxiety	Attitudes	Total
Achievement test in mathematics (Standard 7) (N = 208)	0,01	0,40*	0,25*	0,25*
Diagnostic tests in mathematical language (N = 209)	0,06	0,37*	0,27*	0,19*

TABLE 6.46: PEARSON CORRELATIONS OF FIELDS WITH STANDARDISED MATHEMATICS TESTS FOR GRADE 9 LEARNERS (FEMALE; N = 594)

Fields → Tests ↓	Habits	Anxiety	Attitudes	Total
Achievement test in mathematics (Standard 7) (N = 262)	0,22*	0,50*	0,37*	0,43*
Diagnostic tests in mathematical language (N = 259)	0,19*	0,51*	0,37*	0,42*

TABLE 6.47: PEARSON CORRELATIONS OF FIELDS WITH STANDARDISED MATHEMATICS TESTS FOR GRADE 9 LEARNERS (AFRIKAANS; N = 355)

Fields → Tests ↓	Habits	Anxiety	Attitudes	Total
Achievement test in mathematics (Standard 7) (N = 201)	0,28*	0,50*	0,28*	0,42*
Diagnostic tests in mathematical language (N = 198)	0,32*	0,48*	0,30*	0,44*

TABLE 6.48: PEARSON CORRELATIONS OF FIELDS WITH STANDARDISED MATHEMATICS TESTS FOR GRADE 9 LEARNERS (AFRICAN; N = 717)

Fields → Tests ↓	Habits	Anxiety	Attitudes	Total
Achievement test in mathematics (Standard 7) (N = 271)	0,21*	0,36*	0,34*	0,39*
Diagnostic tests in mathematical language (N = 272)	0,24*	0,43*	0,37*	0,45*

From Tables 6.44 to 6.48 it appears that, except in certain cases, the correlations are significant at the 5% level. This means that most of the scores of the individual fields, as well as of the total fields, correlate positively with the criterion tests, namely the *Achievement test in mathematics (Standard 7)* and the *Diagnostic tests in mathematical language.*

6.3.3.2 Predictive validity: Multiple regression analysis

TABLE 6.49: MULTIPLE REGRESSION ANALYSIS WITH HABITS (X_1), ANXIETY (X_2), AND ATTITUDES (X_3) AS INDEPENDENT VARIABLES AND *ACHIEVEMENT IN THE ACHIEVEMENT TEST IN MATHEMATICS (STANDARD 7)* (MATHS 7) AND THE *DIAGNOSTIC TESTS IN MATHEMATICAL LANGUAGE* (DIAG) AS DEPENDENT VARIABLES (Y)

DEPENDENT VARIABLE	GROUP	R	R^2	PREDICTIVE OR REGRESSION EQUATIONS WITH REGRESSION COEFFICIENTS
Maths 7	Total	0,5054	0,2554	$y = 1,17 - 0,04x_1^* + 0,12x_2^* + 0,19x_3^*$
Diag	Total	0,5263	0,2770	$y = 5,47 - 0,20x_1^* + 0,44x_2^* + 0,80x_3^*$
Maths 7	African	0,4623	0,2137	$y = 1,92 - 0,00x_1 + 0,07x_2^* + 0,13x_3^*$
	Afrikaans	0,4983	0,2483	$y = 3,59^* - 0,01x_1 + 0,14x_2^* + 0,02x_3$
Diag	African	0,5290	0,2798	$y = -4,43 - 0,01x_1 + 0,33x_2^* + 0,54x_3^*$
	Afrikaans	0,4896	0,2397	$y = 26,02^* + 0,04x_1 + 0,35x_2^* + 0,11x_3$
Maths 7	Male	0,4753	0,2259	$y = 2,79 - 0,05x_1^* + 0,11x_2^* + 0,20x_3^*$
	Female	0,5355	0,2868	$y = 0,29 - 0,03x_1 + 0,13x_2^* + 0,19x_3^*$
Diag	Male	0,5198	0,2702	$y = 14,46^* - 0,30x_1^* + 0,38x_2^* + 0,93x_3^*$
	Female	0,5494	0,3018	$y = 0,01 - 0,12x_1^* + 0,47x_2^* + 0,72x_3^*$

From Table 6.49 it appears that the fields Study habits, Mathematics anxiety and Study attitudes practically throughout were significant predictors (at the 5% level) of achievement in the *Achievement test in mathematics (Standard 7)* as well as of achievement in the *Diagnostic tests in mathematical language*.

★ *The second group of subhypotheses investigated in this section, consists of the following:*

- There is a significant relationship between achievement in the fields of the SOM (Study habits in mathematics, Mathematics anxiety, Study attitudes towards mathematics and Total scores) on the one hand and the *Achievement test in mathematics (Standard 7)* and the *Diagnostic tests in mathematics* on the other hand.

Only limited support for these hypotheses could be found. Most scores of the individual fields, as well as the total scores, correlate significantly positively with the criterion tests, namely the Achievement test in mathematics (Standard 7) and the Diagnostic tests in mathematical language.

6.4 SUMMARY

The results of the investigation can be summarised as follows:

(i) Evaluation of the SOM

★ The SOM has content validity.

★ Several steps were taken to determine the construct validity of the SOM for the three population groups **jointly**. These include a comprehensive literature study, ensuring that the most important facets of a SOM were taken into account in the different fields, checking of items and placing them in fields by different experts, as well as factor and item analysis.

★ The reliability coefficients for the SOM can be considered to be highly satisfactory for the purpose for which this questionnaire will be used.

(ii) Analysis of variance and *post hoc* comparisons

★ Statistically significant differences were found by means of multiple analysis of variance (MANOVA) between the means of the two grade groups with regard to the various fields in general. Subsequently one-way analysis of variance (ANOVA) was carried out. Where F values were significant at the 1%

level, the LSM technique (Least Squares Means) was used to determine between which groups' means the differences were statistically significant.

★ The analysis of variance indicates that practically throughout there were statistically significant differences between the mean achievements of both grade groups (8 and 9 on the one hand and 10 and 11 on the other) and the language and the gender groups. *Post hoc* comparisons furthermore indicate exactly where the differences between the study groups lie. Some of the findings are the following:

- The African language speaking persons' (Grade 8 and 9) Study habits in mathematics are more adequate than those of Afrikaans and English-speaking persons.
- African language speaking persons (Grade 8 and 9) have consistently higher mathematics anxiety levels as well as less adequate study habits in mathematics.
- With regard to gender, in Grade 8 and 9 there are statistically significant differences with regard to the fields Anxiety and Attitudes. Boys in Grade 8 and 9 have lower anxiety levels in mathematics and more adequate Study attitudes towards mathematics than the girls.

→ In the case of Grade 10 and 11 learners, too, African language speaking learners have more adequate study habits than their Afrikaans and English counterparts.

→ Afrikaans-speaking learners' means for anxiety differs statistically significantly from that of English-speaking learners.

→ English-speaking learners have significantly lower anxiety levels than Afrikaans-speaking learners who have the highest anxiety levels of the three groups.

→ With regard to study attitudes, it is apparent that the African language speakers in Grade 10 and 11 have significantly more adequate study attitudes than their Afrikaans and English fellow-learners.

→ The most significant finding of the *post hoc* comparisons, however, is that African language speaking learners in Grade 10 and 11 do much worse in the field Locus of control than Afrikaans and English-speaking learners.

→ The means of the girls in Grade 10 and 11 for the fields Study habits in mathematics, as well as study attitudes towards mathematics differ statistically significantly from those of the boys in these grades. It would appear that girls in these grades have more adequate study habits, whereas their study attitudes in mathematics are still less adequate than those of boys.

- Learners in Grade 10 and 11 have significantly lower anxiety levels than their fellow learners in Grade 8 and 9.
- Learners in Grade 10 and 11 have more adequate study attitudes towards mathematics than their counterparts in Grade 8 and 9.
- The anxiety levels of boys as well as girls in Grade 10 and 11 are statistically significantly lower than those of their fellow learners in Grade 8 and 9, whereas both the boys and the girls in Grade 10 and 11 have more adequate study attitudes than their counterparts in Grade 8 and 9.

★ Most of the scores for the individual fields, as well as the total scores, correlate significantly positively with the criterion tests, namely the *Achievement test in mathematics (Standard 7)* and the *Diagnostic tests in mathematical language*. The trend has been confirmed by the Pearson correlations and the application of the statistical technique multiple regression analysis. The SOM clearly has criterion-related validity.

In Chapter 7 the results will be put into perspective and recommendations will be made.

CHAPTER 7

SUMMARY AND RECOMMENDATIONS

7.1 INTRODUCTION

7.1.1 The need for counselling with regard to learners' study orientation in mathematics

In Chapter 1 and 4 of this study it was pointed out that the failure rate in mathematics at school is high, not only in South Africa, but also internationally (Maree, 1995b). The most recent research by the HSRC suggests that South Africa is even further behind than the most negative estimates indicate (Howie, 1996). Edwards (Concern ..., 1997) goes even further:

> *We know that the standard of teaching in science and mathematics in black schools is hopeless. This is being overcome by bridging programmes for matrics.* (Translation)

Furthermore it is a well-known fact that learners with an apparently high general aptitude for mathematics sometimes underachieve in the subject. On the other hand some learners with an apparently low general aptitude for mathematics do well in this subject. In this connection it is important to note that, for various reasons, little attention is given to learners' **study orientation in mathematics**. In spite of this, research indicates that achievement in school mathematics is one of the best predictors of tertiary success (Visser, 1989). Pollock and Wilkinson (1988:80) regard this potential connection as follows:

> *The evidence suggests that academic achievement of students differs significantly on strategies and motives but not on abilities.*

7.1.2 Summary of the results of the literature study

An investigation of the most important learning theoretical approaches to the learning process in mathematics that was carried out in Chapter 2, 3 and 4 has brought, in summarised form, *inter alia,* the following facets or aspects of a study orientation in mathematics to light:

- ★ Understanding basic concepts in mathematics is a precondition for learning more advanced work in mathematics.
- ★ Learners' study attitudes towards mathematics include factors such as motivation and expectations with regard to the subject. These factors influence, among other things, learners' interest in mathematics.
- ★ When subject matter in mathematics does not link up with learners' knowledge and level of thought, it leads to frustration that inhibits the motivation to do well in mathematics.
- ★ Learners' affective disposition influences their attitude towards the subject.
- ★ Learners' study habits in mathematics are, *inter alia,* more important in terms of their reinforcement of important insights in the subject.
- ★ Learners' problem-solving behaviour (that could include factors such as problem-centring, co-operative learning, the implementing of metacognitive learning strategies) exercise a potentially significant influence on their ultimate achievement in mathematics.
- ★ Learners' study milieu (social, physical and experienced milieu) constitutes an integral part of their study orientation. Learners come from different homes, have different backgrounds and differ in their ethnic and cultural backgrounds. Motivation differs from culture to culture, as do learners' interests and the premium that parents put on achievement in mathematics. Learners from less stimulating environments frequently lag behind, reveal less daring, attitudes and are frequently slower learners than those from less restricted environments.
- ★ The way in which learners experience their teachers, probably influences their attitude towards the subject significantly.
- ★ The way in which learners assimilate information in mathematics (including critical thinking, general and specific comprehension, learning, summary and reading strategies), co-determines their ability to solve problems in mathema-

tics and frequently provides a measure of the extent to which learners really understand mathematics; and

★ seen holistically, learners' total study orientation in mathematics probably significantly influences their problem-solving ability and ultimate achievement in the subject.

7.1.3 Problem-centring

In Chapter 2 the principles of a problem-centring approach to a study orientation in mathematics were discussed. Confidence in the merits of this approach constitutes one of the points of departure of the SOM. This approach especially envisages the optimising of problem-solving behaviour in mathematics (Cockcroft, 1982; NCSM, 1977). The focus thus moves –

★ from the learner as someone who **does** something, to the learner as someone who **thinks actively**;

★ from mathematics as focused on concepts and skills to a focus on concepts, skills and **processes**; and

★ to social interaction, working together in groups, an investigative attitude and learner involvement in the mathematics classroom.

The following is emphasised:

★ A functional knowledge of the **language** and **structure** of mathematics, including the ability to estimate, approximate and to gauge the fairness of the results of problem-solving.

★ An intelligent mastering of arithmetical skills and abilities. By this is meant that learners should also have insight into the reasons for carrying out certain mechanical operations.

★ A valuation of the use and importance of mathematics in modern society.

★ A healthy positive attitude towards learning and discovery with regard to mathematics.

7.1.4 What is study orientation in mathematics?

Chapter 3 indicates that study orientation refers to the patterns in learners' study approach at school and university. It contains a combination of styles and motives and includes, *inter alia,* approaches, attitudes, adjustments, motives, habits and problem-solving conduct (Entwistle & Ramsden, 1983). Here acquired behaviour is involved which should be able to be measured in some way or other with a view to optimising learners' study orientation.

7.1.5 The role of study orientation in achievement in mathematics

The use of a standardised questionnaire to measure learners' study orientation in mathematics offers psychologists[1] the opportunity of obtaining more information about learners than just information on their cognitive achievement in mathematics. Various researchers have already shown that there is a statistically significant relationship between aspects of learners' study orientation in mathematics (including anxiety, motivation, attitudes towards mathematics, the use of effective (metacognitive) learning strategies in mathematics, effective time management, concentration, the will to do well in mathematics, parental expectations as well as the social, physical and experienced milieu of mathematics learning) on the one hand, and achievement in mathematics on the other hand.

7.1.6 Techniques for measuring study orientation in mathematics

Various methods are used to evaluate learners' study orientation in mathematics. These include observation, interviews, the assessing of scripts, testing and examination. The primary aim of this study is to develop a study orientation questionnaire in mathematics.

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[1] For the purpose of Chapter 7 the term "psychologists" is further on linked to both psychologists as well as counsellors and mathematics teachers.

7.1.7 Considerations in developing the SOM

The following were taken into account in developing the SOM:

(i) The content has to be meaningful to the testee.
(ii) The questionnaire should have diagnostic value.
(iii) Bias in items with regard to language, gender, race and socio-economic milieu should be limited.
(iv) Allocation of marks should be objective.
(v) There should be joint norm tables for learners of all population groups.

7.1.8 Rationale (description of the SOM's fields)

In the case of Grade 8 and 9, three fields (Study habits in mathematics, Mathematics anxiety, and Study attitudes) could ultimately be distinguished, whereas, in the case of Grade 10 and 11 a fourth field (Locus of control) could also be identified. The four fields will be described next.

(i) Study habits in mathematics (SH)

Description and rationale: this field consists of 38 questions. These include:

(a) Making use of acquired, consistent, effective study habits and methods (planning the use of one's time and preparation, writing previous tests and examinations, working out more than just the known problems and following up problems in mathematics).

(b) The extent to which learners carry out instructions and do assignments in mathematics punctually, see that their homework is done regularly, keep up in mathematics and do not waste time.

(c) The willingness to do mathematics consistently in spite of the fact that other activities (that are more "enjoyable" for the learner) could have been performed instead of mathematics.

(d) Problem-solving behaviour in mathematics. **Metacognitive learning strategies** in mathematics include **planning**, **self-monitoring**, **self-evaluation**, **self-regulation** and **decision-making** during the process of problem-solving in mathematics. This can be described as **"thinking about thinking" (as, for example, when learners try to find out which subsections of mathematics they do not understand)**. Using these strategies helps learners to **generalise** in mathematics **(to infer).** These learning strategies work well in a learning environment in which preference is given to a problem-centred solution approach where mathematics problems are tackled co-operatively and where socialising (social interaction) in the mathematics class is actively actualized. Learners should participate actively in acquiring the language of mathematics and certain activities should be encultured in the classroom so that certain expressions, terms and/or explanations in the particular classroom become acceptable; that is to say, they become part of the classroom culture.

(e) General and specific learning, summarising and reading strategies, critical thinking and comprehension strategies (for example the optimal use of sketches, tables, diagrams). This field provides a measure of the extent to which learners really understand mathematics. When concept formation in mathematics has taken place inadequately, it frequently becomes apparent from the following acts: inappropriate proving techniques, excessive technical errors (incorrect calculations), incorrect allocation of values to unknowns, incorrect assumptions and incorrect allocation of attributes.

(ii) Mathematics anxiety (MA)

Description and rationale: this field consists of 28 questions. Panic, anxiety and worry become noticeable in the form of aimless repetitive conduct (like chewing nails, excessive sweating, playing with objects, excessive need to visit the toilet, deletion of correct answers and an inability to speak clearly). Learners' motivation in mathematics is negatively influenced when they become emotionally upset. When learners have not mastered the limited, technical language of mathematics adequately, this contributes to rising levels of mathematics anxiety.

(iii) Study attitudes towards mathematics (SA)

Description and rationale: this field consists of 11 questions and refers to feelings (subjective, but also objective experiences) predispositions and attitudes (towards mathematics and aspects of mathematics that consistently manifest themselves), and learners' motivation and expectations of, as well as interest in mathematics. These include learners' "mathematical world view" on the **self**, the **nature** of mathematics and the **nature of the learning** of mathematics.

(iv) Locus of control with regard to mathematics (LC)

This field consists of 13 questions and includes the following: mathematics learners come from different environments and have different backgrounds. Learners from non-stimulating backgrounds frequently fall behind, struggle and are slower learners as a result of limited experiences. Frustration, restrictive home circumstances, non-stimulating learning and study environments, physical problems such as poor vision or hearing, reading problems, names and lifestyles in word problems that do not come from the learners' field of experience and language problems (including the typical problems brought about by second language education, language background that is restrictive and milieu disadvantages) confuse and undermine achievement in mathematics.

(v) Study orientation in mathematics (SOM)

Viewed generally, the SOM provides a summary of the aforementioned factors and is also a measure of learners' study orientation in mathematics. The whole questionnaire consists of 77 questions for Grade 8 and 9, and 90 questions in the case of Grade 10 and 11.

The administering of this questionnaire should at all times be followed up by a task-directed interview. It should also be kept in mind that learners' study problems in mathematics should not be seen linearly as mere learning problems but rather as teaching and learning problems. This emphasises the fact that study orientation and achievement problems in mathematics should not be seen in isolation, but at all

times be seen within the context of the person experiencing these problems. Thus a good understanding of study orientation and achievement problems in mathematics should always be combined with a humanitarian understanding of learners as unique individuals, on personal, environmental as well as relationship levels with regard to their psycho-biological composition, their psycho-physiological constitution and their intrapsychic functioning.

7.2 FINDINGS AND IMPLICATIONS OF THIS INVESTIGATION

Chapter 5 contains information on the development and procedure of the research. Chapter 6 provides the results of the study. The data processing procedures relate especially to the three primary aims of the study. These aims will be discussed next.

7.2.1 Standardisation of the SOM

In this case certain steps were taken to evaluate the SOM psychometrically. These include the undertaking of a relevant literature study; ensuring that the most important facets of the SOM in the different fields were taken into account; checking the items and having them placed in the fields by different experts; factor analysis and item analysis. It was found that the SOM had content and construct validity for the three population groups **jointly**. Furthermore, the reliability coefficients of the SOM in most cases could be regarded as highly satisfactory for the purpose for which the questionnaire would be used.

7.2.2 Comparative studies to determine the suitability of the SOM

Analysis of variance techniques was used to determine where significant differences existed between the groups (including grade, mother-tongue and gender groups). Firstly a MANOVA was carried out to determine whether groups differed significantly from one another with regard to the three fields (Grade 8 and 9) or the four fields (Grade 10 and 11) jointly; in other words, how the groups differed with regard to study orientation was determined. In cases where the MANOVA indicated significant differences, further investigation was conducted to try to determine with regard to which individual fields (single variables) the groups differed significantly.

With the aid of LSM it was determined which groups differed significantly with regard to the separate fields.

Some of these findings will subsequently be critically assessed against the frame of reference of a review of the way in which these results link up with previous results as well as with relevant theoretical considerations.

7.2.2.1 Comparisons between the language groups

It would appear that African language speaking learners taking mathematics in Grade 8 and 9, but also in Grade 10 and 11, are generally more positive towards mathematics and reveal more adequate study habits than their English and Afrikaans-speaking counterparts. This is in agreement with the findings of Møller (1994). Her study confirms the surmise that these learners "spontaneously" take other measures (for example to work with friends in the afternoons at school) in an effort to counter the harmful effect of milieu disadvantages.

Afrikaans-speaking learners in Grade 8 and 9 reveal mathematics anxiety that is statistically significantly higher than that of the English-speaking learners, whereas African language speaking learners experience the highest anxiety levels and have less adequate study attitudes towards mathematics than the other two mother-tongue groups. The fact that African language speaking learners fare so much worse in mathematics (Blankley, 1994; Christie, 1991; Maree 1995b) has to be read in conjunction with this. The considerable backlog that African language speaking learners have compared with their English and Afrikaans-speaking counterparts is probably directly related to the fact that conditions in the traditionally black schools are less optimal than conditions in the traditionally white schools (Jansen, 1996). The milieu disadvantage of the black learners contributes to the fact that, on cognitive level, they develop later than their white counterparts. This view can be related to that of Piaget (quoted by Lavatelli, 1974) as discussed in Chapter 2 and 4. It appears that the depressing factors in traditionally black areas (including schools) impede these learners' cognitive development in mathematics to such an extent that African language speaking learners have a backlog of a year or more in cognitive development when compared with learners from other language groups. It should be emphasised that this backlog is not deemed to be the result of "genetic differences" between the various mother-tongue groups, but should rather be ascribed to the negative effect of milieu impairment as explained in Chapter 3 and 4.

Brodie (1994:5) points out that "many *[authors] speak of the alienation and anxiety that most pupils experience while learning mathematics at school".* These authors ascribe this, *inter alia,* to the existence of racism and sexism in schools as well as to teacher perceptions that black learners and girls are hardworking and "not naturally bright" (Brodie, 1994:13).

The fact that Afrikaans-speaking learners reveal considerably higher anxiety levels than their counterparts from the other language groups can probably also be ascribed, *inter alia,* to language problems. Stumpf (Concern ... , 1997) states this as follows:

> *Afrikaans learners did very well. The reason is clear: Afrikaans is the first language of these learners, whereas English is the second language of most learners in English-medium schools. Many of the (African language speaking) learners in English medium schools did not even understand the context of the questions properly*. (Translation)

Chapter 3 and 4 of this study indicate to what extent inadequate comprehension or mastery of the limited technical language of mathematics, as well as non-mother-tongue education can give rise to learning problems in mathematics.

English-speaking learners in Grade 10 and 11 likewise have significantly lower anxiety levels than Afrikaans-speaking learners. This finding relates to Van Eeden's (1991) who, as described in Chapter 4, found that English-speaking learners do better than their Afrikaans-speaking counterparts in the *SSAIS-R (The Senior South African Individual Scale – Revised)*. Claassen (1987) mentions the higher socio-economic status of the English-speaking as reason for this type of phenomenon. As explained in Chapter 4, the fact that English-speaking parents attach more meaning to the development of personality traits such as creativity, an exploring disposition, independence and extroversion probably contributes to this phenomenon.

African language speaking learners in Grade 8 and 9 have statistically significant higher anxiety levels than their counterparts in other language groups. The situation changes marginally in Grade 10 and 11, where Afrikaans-speaking learners' anxiety levels differ statistically significantly from those of the English-speaking group. The phenomenon that Afrikaans-speaking learners in Grade 10 and 11 have anxiety levels that are somewhat higher than those of the African language speaking learners (although not statistically significantly) has to be considered together with the fact that a particularly high percentage of African language speaking learners give up taking mathematics at the end of Grade 9 (Blankley, 1994; Christie, 1991; Maree, 1995b). The extent of the problem is apparent from the figures in the following table:

TABLE 7.1: DISTRIBUTION OF SOUTH AFRICAN MATHEMATICS LEARNERS IN GRADE 12 ACCORDING TO MOTHER-TONGUE-SPEAKING GROUP (1993) (STRAUSS, 1997)

AFRICAN LANGUAGE SPEAKING					AFRIKAANS- AND ENGLISH-SPEAKING			
	Boys		Girls		Boys		Girls	
Grade/total	N	%	N	%	N	%	N	%
Higher	36043	21,30	37573	16,60	11041	32,70	9613	28,83
Standard	15071	8,91	18607	8,22	13039	38,61	9905	29,71
Total Grade 12)	169197	100	226321	100	33768	100	33342	100

From the data in Table 7.1 it is clear that more boys than girls with Afrikaans and English as mother-tongue take mathematics. A considerably lower percentage of African language speaking boys than Afrikaans and English-speaking boys take mathematics up to Grade 12, whereas the situation is even worse as far as African language speaking girls are concerned (Strauss, 1997).

African language speaking learners see themselves as forced into a situation which they cannot change. In spite of their best efforts circumstances are simply too depressing and supporting structures are practically non-existent. They just cannot handle mathematics and rather let it alone – in most cases without the actual problems being properly dealt with. This usually occurs without taking learners' true potential into account and without any attempt to provide adequate information to these learners or the potential harmful consequences of a potentially short-sighted decision (Molepo, 1997).

The most significant finding that is apparent from these *post hoc* equations is that African language speaking learners in Grade 10 and 11 do so much worse in the field Locus of control than the Afrikaans and English-speaking learners. A variety of factors probably contribute to this state of affairs, including language problems, underqualified teachers, the shortage of women teachers in mathematics and the less optimal SES among African language speaking learners in general. The TIMMS report (Howie, 1996) confirms the assumption that there is a strong link between depressing domestic circumstances, milieu disadvantage and an inadequate study orientation as well as underachievement in mathematics. These findings are also in agreement with those of researchers such as Wong (1992), Haladyna, *et al.,* (1983), Hanna, *et al.,* (1991) and Kaisner-Messmer (1993) who point out the positive correlation between unfavourable background factors, external Locus of control and inadequate achievement in mathematics.

Research by Schoenfeld (1988; 1994) confirms that poor achievers in mathematics are frequently of opinion that luck is an important factor influencing achievement in mathematics. The results of recent research by the HSRC (Howie, 1997) agree with this finding. By far the highest percentage of South African learners (who did the worst in the TIMMS study) are convinced that factors beyond their control, for example "luck" are determining factors for good achievement in mathematics. Learners in a country like Singapore (that did best in the TIMMS study), believe that factors within their control (like hard work) are responsible for their doing so well in mathematics. Herskovitz and Gefferth (1992) state the following on the psychological implications of the experiencing of an external or internal Locus of control:

> *Internal locus of control, or responsibility for one's achievement, is a prerequisite for a person to be able to attribute personal successes and/or failures first of all to his/her own efforts, abilities or the lack of them (versus chance, luck, other person); without which (call it task commitment or*

> *achievement motivation) it is inconceivable for a person to maintain achievement.*

To interpret this quotation within the present South African context it has to considered against the frame of reference that has so far been sketched in this chapter. This implies, *inter alia,* that attempts to bridge or do away with the present unsatisfactory situation (with regard to achievement in mathematics in South Africa) can only meet with success if the learning situation or milieu of all learners, but particularly the situation of African language speaking learners, is drastically improved.

7.2.2.2 Comparisons between the gender groups

When comparisons are made between the gender groups, it should be borne in mind that fewer boys (46%) than girls (54%) are currently enrolled in schools in South Africa (Arnott, *et al.,* 1997). As far as gender is concerned, the present study revealed statistically significant differences with regard to the fields Mathematics anxiety and Study attitudes. Boys in Grade 8 and 9 have lower anxiety levels in mathematics and more adequate study attitudes towards the subject than girls. This finding correlates with that of Visser (1989:213) who found that

> *the attitudes of females become more negative in the period between Std 5 and Std 7. They become more anxious about their mathematics studies ... . Their interest in the subject wanes.*

Arnott, *et al*., (1997:12) support similar views and they state that

> *proportionally fewer girls than boys are likely to enrol [in mathematics] ... . Boys are more likely to pass matric mathematics and science than girls.*

Mwamwenda (1994) is of the opinion that academic underachievement by girls is generally caused by girls being more inclined to experience test anxiety than boys. Sibaya and Sibaya (1997) believe that in certain circumstances girls are regarded as shy, withdrawn and less skilled in terms of social interaction than boys, and consequently the questions put to boys in mathematics classes are more difficult than those put to girls.

Relatively large numbers of girls from all the mother-tongue groups give up mathematics at the end of Grade 9[2] (Strauss, 1997) for a variety of reasons (Costello, 1991; Maker, 1993). Those who do take the subject evidently have lower mathematics anxiety levels, have more adequate study attitudes towards mathematics and more adequate study habits in mathematics. In other words, girls in Grade 10 and 11 have a more adequate study orientation in mathematics than their male counterparts.

The findings of this study are apparently in agreement with those from previous research and indicate that girls in Grade 8 and 9 give up mathematics at the end of Grade 9 (Fennema & Hart, 1994; Visser, 1989). This is done on account of a variety of considerations (for instance that they are afraid of the subject, that they are not motivated to take it and that they are more easily convinced than boys that they cannot do well in mathematics). It appears as if these considerations have little to do with whether the girls or their parents are convinced that girls are cognitively capable of good achievement in the subject. Visser (1989:213) states the following in this connection:

> *A study of the factors which influence the decision to continue with mathematics yielded surprising results. Students (females in particular) generally do not take into account their intellectual capabilities when they make this decision. They are strongly influenced by social, attitudinal and emotional considerations, particularly their perception of the usefulness of mathematics, and self-confidence and motivation in the subject.*

Leder (1987) and Macleod (1995) agree with this view, but go further and indicate that gender differences, as far as mathematics is concerned, can be explained according to different models. These models include heredity factors, fear of success, failure orientation, environmental factors, differences in values, external Locus

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[2] See Table 7.1, paragraph 7.2.2.1

of control, sociological factors as well as learner-teacher-interaction patterns. Macleod (1995) discusses the gender-related differences in achievement in mathematics, especially in a post-structural framework and confirms the existence of several prevailing perceptions, for example that boys have a more natural talent for mathematics and that the intrinsic, absolute nature of mathematics ("a person can either do it or you cannot") makes the subject more suitable for boys than for girls. This occurs in spite of the fact that the perceptions of gender-related differences have linguistic rather than intrinsic value.

Such research results again underline the need for adequate counselling with regard to these matters concerning the school. Researchers such as Fennema and Hart (1994) point out that these matters can and should be rectified. Psychologists will have to help learners in a more professional way to decide whether they are going to take mathematics after Grade 9 or not. This decision will have to be taken in a responsible way by taking into account learners' total situation, including their cognitive, interest, personality, scholastic and normative profile. In short, learners' total future career choice profile will have to be taken into consideration in a holistic way when decisions in this connection have to be made. Short-sighted choices can seriously impede learners' career choices and prevent learners from adequately fulfilling their potential in a future occupation. Arnott, *et al.,* (1997:37) refer to the girls' position with regard to taking mathematics as follows:

> *Girls need to see the career options that will be open to them with qualifications in these subjects.*

It furthermore becomes apparent that girls in Grade 10 and 11 are more inclined to feel that they exercise control over the situation in the mathematics class than boys in these grades. This could possibly be related to the fact that so many girls give up their study of mathematics at the end of Grade 9, possibly because of, among other things, their high anxiety levels in mathematics. Those learners who take mathematics probably include persons with acceptable anxiety levels in mathematics. The study milieu of these girls is obviously more optimal or supporting than the study milieu of those that give up mathematics at the end of Grade 9. Visser's findings relate to this finding as follows:

> *In the case of Std 7 females ... the encouragement of both parents also correlated highly with the intention to continue with mathematics.*

The finding of the HSRC (1997) that women's position in the natural sciences from 1985 to 1994 remained relatively weak in comparison with that of their male counterparts, confirms the view in this study, namely that urgent attention should be given to the optimising of girls' study orientation in mathematics. The causes of the above situation should be given attention, namely that relatively few girls take mathematics up to Grade 12 (and also after this grade).

7.2.2.3 Comparisons between the grade groups

Learners in Grade 10 and 11 have lower anxiety levels than their counterparts in Grade 8 and 9. This is understandable, since many learners who are "afraid" of the subject or do badly in mathematics, give up the subject at the end of Grade 9. Other possible explanations for this are that the learners in Grade 10 and 11 are beginning to function on a more formal level of thinking and are consequently able to adopt a more distanced attitude towards mathematics-related problems, however, slight this may be.

It is illuminating to note that the field Locus of control could only be identified in the case of learners in Grade 10 and 11. One of the reasons for this is that learners in Grade 8 and 9 are compelled to take mathematics at school. Only in Grade 10, 11 and 12 can learners themselves decide (admittedly accompanied by significant others) whether they wish to take the subject or give it up. In other words, learners in Grade 10, 11 and 12 experience for the first time the idea that they can exercise control over the situation in mathematics. Piaget (1964, 1973, 1976) declares that adolescence is a period during which children especially move to the phase in which they are more able to carry out formal thinking operations. His view, however, that learners at the age of 12/13 enter the formal operational thinking stage, has already been regarded by several researchers as overoptimistic (Copeland, 1982). It would rather appear that learners in general enter this particular thinking stage later on, depending on several factors including their socio-economic status, cultural milieu and the extent to which their parents support them. Negative circumstances and milieu disadvantage in general have, as has already been pointed out in this study, a depressing effect on learners' movement towards a more formal, intellectual level of thinking.

Barker (1995) declares that girls enter puberty earlier than boys (in the case of the boys between the age of 13 and 17, in the case of girls between the ages of 11 and 13). Thompson and Rudolph (1992) are of the opinion that juveniles become adolescents between the ages of 12 and 18 years. During this phase juveniles develop a more flexible style, critical thinking ability and style of social interaction (Barker, 1995; Erikson, 1965; Thompson & Rudolph, 1992). **Based on the results of this study it appears that South African learners in general reach a stage of development between Grades 10 and 12 in which they start to show evidence of experiencing an external or internal Locus of control in the academic field (especially with regard to the situation in mathematics).** African language speaking learners in particular clearly indicate that they do not feel that they have control over their circumstances; that they feel they are "at the mercy of" negative circumstances beyond their control and are not themselves responsible for their poor achievement in mathematics; and that they are not capable of achievement in mathematics, in spite of their apparently really trying to master the subject.

Learners in Grade 10 and 11 display more adequate study attitudes towards mathematics as well as a more adequate study orientation in general towards mathematics than their counterparts in Grade 8 and 9. This can be ascribed, *inter alia,* to the fact that many learners whose study orientation in mathematics is inadequate, give up taking the subject at the end of Grade 9, and also ascribed to cognitive maturation that in any case occurs during the adolescent stage. However, those who do indeed take the subject display more adequate study habits in mathematics and more adequate study attitudes than those who have the study of this subject. Likewise, learners drop the subject because they display mathematics anxiety, and the levels of mathematics anxiety of those who do take the subject, are understandably lower in Grade 10 and 11. On the other hand the following should also be borne in mind: greater maturity, as well as one's own choice (internal Locus of control) will in all probability bring anxiety under control and thus lower the overt anxiety levels.

7.2.3 Determining the SOM's criterion-related validity by means of Pearson correlations and regression analysis

By using Pearson correlations and the statistical technique multiple regression analysis, statistically significant correlations almost throughout were found between the fields Study habits, Mathematics anxiety and Study attitudes, as well as the total score in the SOM on the one hand, and achievement in the criterion tests that were used; namely the *Achievement test in mathematics (Standard 7)* as well as the *Diagnostic tests in mathematical language* on the other hand. This probably indicates that learners' achievement in mathematics can be significantly improved when aspects of their study orientation in mathematics (for example motivation, attitudes and the absence of anxiety in the mathematics class) improve (Van Aardt & Van Wyk, 1994). The SOM thus has criterion-related validity.

7.3 RECOMMENDATIONS

The research results of this study are significant for the process of learning development; at school but also at tertiary institutions. Implementing the following recommendations can possibly contribute to optimising learners' and students' learning process in mathematics.

7.3.1 ☞ Optimising study habits in mathematics

The finding that African language speaking learners in both grade groups have study habits that are more optimal than those of the other two mother-tongue groups, is clearly not in agreement with the present achievement figure (pass mark) in mathematics in all grades. (African language speaking learners' pass rate in mathematics is still consistently lower than that of other mother-tongue speakers.)

It appears that factors such as a lack of facilities and textbooks, inadequate follow-up of work, inadequate follow-up of African language speaking learners' work, and the fact that they were often only able to utilize 100 out of 195 potential schooldays; in short, what Saunders (1996:18) calls:

contribute significantly to even these learners' dedicated efforts to master mathematics (on their own, as it were) not being crowned with success. Seen against this background and taking into account the disruption still being experienced in most schools (Matrics, 1996) the recommendation is thus made that these matters should be given serious attention in an attempt not only to optimise learners' study habits in mathematics, but especially to **create conditions** (see paragraph 7.3.5) under which effective study habits in mathematics can be established more consistently than at present and under which the obstacles mentioned (that seriously impede mathematics, in spite of learners' best intentions) can be removed.

In the case of the other two language groups there is still room for improvement in their study habits in mathematics.

7.3.2 ☞ Dealing with learners' mathematics anxiety levels

It is recommended that (with, *inter alia,* the help of the SOM) the factors that could potentially cause anxiety in the mathematics class be investigated on an on-going basis, and that these factors then be given urgent attention.

The analysis of individual answers (especially those that differ significantly from the answers of good achievers in mathematics) can, for example, be of great help here. By using such analysis, psychologists can offer therapy to those learners whose achievement is unfavourable or inadequate in the different sections of the different fields of the SOM. These forms of therapy include self-image therapy, therapy in techniques for handling mathematics anxiety, therapy in handling emotional problems relating to mathematics problems and the handling of learners' experiencing of the external Locus of control (including facilitating an internal Locus of control).

7.3.3 ☞ Facilitating more adequate study attitudes towards mathematics

African language speaking learners in Grade 8 and 9 have study attitudes towards mathematics that differ significantly from the study attitudes of both the other language groups (in the sense that they are less adequate). Seen against this background and within the context of the circumstances established in the previous paragraphs, it is not difficult to understand. The situation is once again reversed in Grade 10 and 11. This can probably be ascribed to the fact that many African language speaking learners give up mathematics after Grade 9 and the group that remain, have more adequate study attitudes towards mathematics. The phenomenon that girls in Grade 8 and 9 display less adequate study attitudes towards mathematics than boys, but that the situation is reversed in Grade 10 and 11, has already been discussed. Evidently many girls give up the subject mathematics at the end of Grade 9, whereas those who still take it, predominantly have more adequate study attitudes towards mathematics than their male counterparts.

Measures should be taken to optimise the study attitude towards mathematics of African language speaking learners of both sexes as well as of Afrikaans and English-speaking girls. This should be done in an attempt to convince bigger numbers of these learners to take mathematics in Grade 10 and 11, apart from the fact that continuous attention should be given to optimising the study attitudes of all learners. The motivating and advisory role of psychologists is of decisive importance here, whereas decision makers will have to make efforts continuously and in an innovative way deal with typical Third World problems such as overcrowded classrooms, too few schools, poorly trained teachers (in certain areas) unmotivated teachers, inadequate parent involvement (in certain areas) and relatively lower SES of African language speakers.

7.3.4 ☞ Locus of control in mathematics

From a cross-cultural research perspective the differences in the various mother-tongue speaking groups' responses with regard to the items in the field Locus of control do not necessarily indicate any serious "shortcomings" in the items themselves, or too high a degree of bias with regard to these items. Retief (1992:203) is probably the closest to the truth when he states this as follows:

> *an item that emerges from a traditional item analysis as "biased" may simply imply that it was endorsed (or interpreted) differently. It is as a consequence obvious that:*
>
> - *culturally related patterns causing such differences can be identified, if possible;*
> - *the reasons for such consistent differences can be identified, if the reasons for observed differential item endorsements are analysed.*

Certain further "cultural reasons for *endorsing* certain items differently" (Retief, 1992:203) for the differential ways in which items in this field were answered by the various population groups, will be discussed next.

Many learners (African language speaking, but also learners from other mother-tongue groups) come from non-stimulating environments and experience their teachers of mathematics as unapproachable (and inadequately trained). Frequently these learners have the feeling that they have little control over the situation in the mathematics class and outside it. This is not surprising, seen against the background of the phenomenon of Third World problems in these learners' classrooms, in that some of them can only find teachers at school on 50% of all schooldays, and on top of that, they can only attend school for two to three hours a day. When as many as 200 learners are squeezed into a class that is supposed to accommodate a maximum of 35 to 40 learners (Saunders, 1996) there is a strong possibility that their achievement in mathematics cannot be optimal. The experiencing of an external Locus of control is a logical result of such precarious circumstances and this, to a great extent, is responsible for learners' expression of anxiety and feeling of helplessness in the mathematics class. As has been repeatedly pointed out, the problem of learners receiving mathematics instruction in a second language contributes potentially significantly to the extent of the problem. Visser (1988:39) makes the following remark in this connection:

> *The nature of Mathematics itself, and specifically the "language" in which it is presented, are undoubtedly the most probable factors causing mathematics anxiety.*

Learners' physical, non-stimulating and unsupportive study environments indeed provide a measure of learners' **helplessness, anxiety and lack of control in mathematics;** It also suggests reasons why learners have this particular disposition towards mathematics. Whereas factors such as non-comprehension of the language of mathematics and (experienced) milieu disadvantages give rise to mathematics anxiety and undermine achievement in this subject, factors such as self-confidence in mathematics (that can be regarded as the antipole of mathematics anxiety), should result in more adequate study habits in mathematics and satisfactory problem-solving behaviour (Visser, 1989).

The prognosis for an improvement in the current situation in which so many learners' mathematics achievement is unsatisfactory, is probably good, providing that these factors (that can and should be rectified) are rectified. The factors that have been highlighted above will have to be dealt with as speedily and effectively as possible, otherwise the South African society will deteriorate into what Saunders (1996:19) calls:

> *one lost generation after another.*

7.3.5 ☞ Steps to overcome African language speaking learners' milieu disadvantage

Urgent steps have to be taken immediately to at least try to bridge all learners' milieu disadvantage, but especially that of African language speaking learners. This includes the following steps (Pretorius, 1996):

(i) Suitable (re-)training of teachers in all schools to prepare them for the curriculum, learning programme or curriculum changes that are coming. Arnott, *et al.,* (1997:37) describe the situation as it occurs in SA schools as follows:

> *More than 50% mathematics teachers and 68% science teachers have no formal training in these subjects.*

(ii) The development and implementation of programmes to improve the management of schools.

(iii) The creation of more peaceful school environments so that learners' school attendance and level of teaching can be improved.

(iv) The building of new schools to alleviate the need for overcrowded schools.

(v) The provision of essential basic facilities, including running water, electricity and toilets to schools – this study has repeatedly pointed out the connection between learners' achievement in mathematics and school facilities.

(vi) Apart from the need for better basic facilities, measures have to be taken to eliminate the shortage of textbooks that pupils are experiencing at schools.

(vii) Immediate attention has to be given to the situation in which mainly African language speaking learners are taught by teachers who are scarcely better qualified than the learners themselves. Learners in schools that were traditionally "white" receive their mathematics instruction from teachers who are predominantly outstandingly qualified in mathematics.

(viii) Only about 21% of the learners are taught mathematics in their mother-tongue. This promotes communication gaps and poor conceptualisation in this subject. Efforts should be made to develop all eleven official languages so that mathematics instruction can, at least until the end of Grade 7, be given in learners' mother-tongue. Thereafter, instruction should be in one official language (probably English). **At the same time care should be taken to ensure that learners who are given mathematics instruction in English, understand this language adequately.**

(ix) Local African language speaking experts should be empowered to help develop mathematics curricula and instruction and learning strategies that are closely linked to African language speaking learners' natural thinking patterns and that are embedded in **their own language and culture**. This also implies that the adequate training of African language speaking educationists/teachers with regard to both instruction and learning methods in mathematics, as mathematical curriculum design is of cardinal importance (Berry, 1985).

(x) **The inadequate (and disproportionate) provision of psychological services to learners needs attention. Kriegler (1993) maintains that the "privileged" sector of the South African population has First World psychological services at its disposal, whereas little provision has been made for the vast "non-privileged" part of the South African population.**

7.3.6 ☞ Use of the SOM to promote certain aspects of test interpretation in mathematics

(i) The questionnaire should be used in the first place to obtain information on the various aspects of learners' study orientation in mathematics.

(ii) Careful analysis of the questionnaire should help psychologists to better understand why some learners display an adequate and others a less adequate study orientation in mathematics.

(iii) Continued research should confirm the hypothesis that in the SOM, as in the case of the *SSHA*, there is a significant relationship between achievement in the particular questionnaire and academic achievement in mathematics, in spite of the fact that the usefulness of these questionnaires is dependent on honest answers by the learners.

(iv) In the light of the overall picture (as obtained by the SOM) psychologists should be able not only to evaluate learners' study orientation but also to establish guidelines for the optimisation of learners' study orientation and their achievement in mathematics.

7.3.7 ☞ Aim of the SOM

It is recommended that psychologists use the SOM for, *inter alia,* the following purposes:

(i) Identification. Learners whose study orientation in mathematics does not further their achievement in mathematics, can be identified by means of the questionnaire.

(ii) Understanding. The results of the SOM can help psychologists to understand those learners whose achievement in the subject is unfavourable.

(iii) Aid. It is accepted that psychologists can use those results to help learners to optimise their study orientation in mathematics so that they can realise their potential on a higher level.

7.3.8 ☞ Uses of the SOM

SOM can possibly play a significant role in facilitating the following aspects of learning potential optimising:

(i) As a diagnostic test. The SOM can be administered to a group of learners at the beginning of an academic year, or individually. Learners' scores can then be checked to identify those who need assistance, support, remedial instruction and advice. However, the questionnaire can be administered to individuals or groups of learners at any time of the year.

(ii) Therapy. The SOM provides psychologists with a standardised instrument to systematically analyse important background data, feelings, attitudes, habits and customs with regard to learners' academic orientation in mathematics. A profile of a learner's mathematical orientation can easily be drawn. An interpretation of learners' responses to the questionnaire and an analysis of what may potentially lead to poor achievement can be made.

(iii) Study guidelines in mathematics. The SOM can be used as an aid to inculcate in learners certain basic principles for the effective study of mathematics, as well as the significant role that study conditions, including motivational and background factors, can play in academic success.

(iv) Research. The satisfactory correlation with achievement and diagnostic tests in mathematics makes the SOM a suitable measuring instrument for inclusion with other scales in psychological research. Research indicates that the *SSHA* has a high predictive value with regard to academic achievement (Du Toit, 1970). There is a strong suspicion that the SOM will predict achievement in mathematics in similar vein. It is strongly recommended that research in this connection be continued and that the research findings be released. The SOM correlates further highly enough with achievement and diagnostic tests in mathematics to justify the deduction that this questionnaire can contribute to diagnosing factors that inhibit achievement in mathematics.

(v) Improvement of mathematics teachers' and instructors' teaching. Mathematics teachers and instructors can use the SOM in an effort to improve their own teaching methods. The questionnaire was particularly developed to investigate the idio-syncratic cognitive style of mathematics learners. By introducing learners to aspects of recent, contemporary learning styles in mathematics (that include personal involvement, group work and socialising, metacognitive learning, co-operative learning, discovery learning, question posing, self-work and problem centring) mathematics teachers and instructors can achieve more than when they focus only on perspectives that do not necessarily have the required effect (where the emphasis falls too strongly on mere teaching according to a behaviouristic learning approach).

(vi) Analysis of feelings, customs and attitudes as aspects of learners' academic work. The SOM provides psychologists with a standardised test to analyse important feelings, customs, and attitudes as aspects of learners' academic disposition. The test has primarily been developed for use in mathematics, but it has a broader significance in the sense that improvement in learners' mathematics achievement can potentially give rise to the optimising of aspects such as learners' self-image, as well as their achievement in related subjects (where insight into basic mathematical principles is a pre-condition for optimal achievement). When learners are given aid, it would pay psychologists to analyse learners' responses to individual test items. An analysis of those cases where learners' answers differ significantly from those answers usually given by good achievers in mathematics can be of great value. Psychologists should be able to use those aspects of the various fields of the SOM where the learner's achievement is unfavourable, in order to assist such a learner.

7.3.9 ☞ Further research

The SOM can be used, *inter alia,* to do continued research on the following:

(i) The influence of environmental factors (SES, spoken language) and gender on, for example, motivation and cognitive learning.

(ii) The effectiveness of intervention programmes that are based on the optimisation of cognitive learning and motivation strategies.

(iii) The relationship between cognitive abilities, academic achievements and teaching styles on the one hand and cognitive learning and motivation strategies on the other hand; and

(iv) the effectiveness of psychological therapy in cases where learners are unable to attain optimal achievement in mathematics.

7.3.10 ☞ Follow-up studies relating to the SOM

On account of practical considerations, all aspects of test validity were not completely covered during the standardisation of the SOM. Some aspects concerning test validity and reliability that should be investigated further, include the following:

(i) Further tryout of the SOM on students at universities, technikons and colleges. Students from all branches of study for which mathematics is a major or ancillary subject should be involved in such research, provided the sample of students represents the multicultural nature of the South African student society, that the potential differences between the student orientation of the two different gender groups be investigated and that students in different years of study be evaluated separately.

(ii) Test-retest reliability (administering the test to the same group of testees, with reasonable intervals between testings) should be given attention so that test reliability (accuracy and consistency of the SOM can be explored further.

(iii) Learners' marks in subjects such as mathematics, physics and chemistry as well as biology and the total score should, ideally seen, be obtained when the SOM is administered in order to explore simultaneous validity (an aspect of criterion-related validity).

(iv) In the light of the many changes taking place at all levels of South African society and the increasing acculturation (the rapid changes in the cultural patterns of all three language groups) that accompany them (Groenewald, 1996; Van der Reis, 1997) the SOM should be regularly administered to a sample of learners that represent the multicultural nature of the South African school and student population in order to take into account the effect of acculturation on learners' student orientation in mathematics in good time.

7.4 SUMMARY

George and Christiani (1990) summarise the main aim of counselling psychology as follows:

(i) Facilitating a change in behaviour.
(ii) Improving handling mechanisms.
(iii) The promotion of decision-making ability.
(iv) The improvement of relationships.
(v) Facilitating the client's potential. In terms of the SOM, this means that the use of this test can possibly help psychologists to:

★ change learners' negative or less than optimal mathematical conduct;
★ help learners' to deal with problems preventing optimal mathematical self-realisation;
★ optimise learners' decision-taking ability in a mathematical sense;
★ help learners optimise their relationship with themselves and their environment by means of a more optimal mathematics achievement; and

★ facilitate learners' problem-solving disposition in mathematics.

Phares (1992) is of the opinion that the role of theory includes helping psychologists to understand how clients acquire relatively stable behavioural traits (and sometimes to change them), as well as its helping psychologists to create appreciation for the way in which individuals give expression to these behavioural traits. The main function of personality theories is thus to understand people and their behavioural traits and to help predict their conduct. In terms of the SOM this means that the questionnaire can help psychologists to understand learners' idiosyncratic behaviour with regard to mathematics with a view to predicting and optimising their future behaviour (achievement) in mathematics.

Grasha and Kirschenbaum (1980) state that any form of adjustment is aimed at meeting the demands of the environment more adequately. The proper way to achieve this aim is:

> *to try to change ourselves and our environment* (Grasha & Kirschenbaum, 1980:9).

The recommendations made in this chapter are (separately and jointly) on the one hand aimed at creating a more adequate study orientation in the learners themselves, and on the other hand, at changing their learning environments so that more adequate learning opportunities will be facilitated for them. This should certainly be done with a view to more adequate self-fulfilment in mathematics, but especially with an eye on holistic self-fulfilment, and ultimately having a better chance of success in the career world and life in general.

APPENDIX A: STUDY ORIENTATION QUESTIONNAIRE IN MATHEMATICS (SOM)

STUDY ORIENTATION QUESTIONNAIRE IN MATHEMATICS (SOM

INSTRUCTIONS

The purpose of this questionnaire is to examine various aspects of your achievement in mathematics. If you answer all the statements on the following pages honestly and after careful consideration, it will enable you to determine what you can do to improve your achievement in mathematics. Please be completely honest. *Your answers will be regarded strictly confidential.*

Please mark your answers on a separate answer sheet. There are 77 statements for learners in Grade 7, 8 and 9 and 90 for learners in Grade 10, 11 and 12. Decide how you feel about each statement and then indicate your answer on the answer sheet. Choose one of the five possible answers: *Rarely, sometimes, frequently, generally or almost always.*

For example, if you feel that the statement is rarely true, *shade* the space marked R on your answer sheet. In marking your answer make sure that the number of the statement corresponds with the number on the answer sheet and that all the marks are very clear. Do not make any other marks on your answer sheet and erase completely any mark if you wish to change an answer.

Choose one of the 5 possible answers: rarely, sometimes, frequently, generally or almost always. The symbols used are the following:

R	:	Rarely	(0 to 15% of the time)
S	:	Sometimes	(16 to 35 % of the time)
F	:	Frequently	(36 to 65% of the time)
G	:	Generally	(66 to 85% of the time)
A	:	Almost always	(86 to 100% of the time)

You are expected to rate yourself n*ot as you think you <u>should</u> act or feel, and not as you think that <u>other people</u> perhaps act or feel; but as <u>you yourself are in the habit of doing or feeling</u>.* If you are unable to answer the statement because you have not had the actual experience, answer as you are most likely to act in such a situation. Remember that there are no “right” or “wrong” answers and there is no time limit for completing the questionnaire. Nevertheless work as quickly as you can without being careless. Please do not skip any questions.

Example: I work hard in mathematics.

Indicate your answer to each item on your answer sheet as follows:

1. If you rarely work hard in mathematics, shade R.
2. If you sometimes work hard in mathematics, shade S.
3. If you frequently work hard in mathematics, shade F.
4. If you generally work hard in mathematics, shade G.
5. If you almost always work hard in mathematics, shade A.

In this questionnaire simple and clear language usage is preferred. The emphasis is on the communication of ideas. It is hoped that the questions will be understood clearly by all the learners, including those who study mathematics through the medium of English as a second language.

R – RARELY	S – SOMETIMES	F – FREQUENTLY	G – GENERALLY	A – ALMOST ALWAYS

1. (68).[1] I plan my mathematics homework.

2. (9). When I write mathematics tests and examinations I frequently think that others will do better than I, because it seems that they find the papers easier than I do.

3. (5). I succeed in concentrating on mathematics (homework, work sheets) even though there are things that can distract my attention.

4. (6). I try to solve mathematics problems myself before I seek help.

5. (15). To me, mathematics is a useful subject.

6. (7). I make certain that I understand formulas and theorems (rules in mathematics) before I memorise them.

7. (17). I make certain that I understand previous work when I do revision in mathematics.

8. (10). My teacher uses unfamiliar words that confuse me.

9. (19). I try to establish relationships between the various aspects of, as well as the different sections in mathematics.

10. (24). I think the topics in mathematics are suitable (meaningful).

11. (28). I draw up tables, make sketches and draw diagrams when I prepare for mathematics tests and examinations.

12. (16). I lose marks in mathematics tests and examinations because I delete correct answers.

13. (35). I listen with the necessary attention to the mathematics teacher's explanation or instructions.

14. (25). I will find school mathematics useful in some way or another in life, even if I do not use it directly in my occupation.

[1] Number of item in original questionnaire.

R – RARELY	S – SOMETIMES	F – FREQUENTLY	G – GENERALLY	A – ALMOST ALWAYS

15. (38). My mathematics homework is neat and systematic.

16. (47). I enter the dates of my mathematics tests and examinations as well as my mathematics marks in my diary.

17. (48). I do my mathematics corrections (aftercare in mathematics).

18. (20). I am uncertain whether my work is correct, but am hesitant to ask my teacher questions.

19. (49). I quickly read through all the work in order to obtain an overall picture before I start learning.

20. (56). I try to find a logical structure in everything that I learn in mathematics.

21. (32). Achievement in mathematics is important to me because I feel that mathematics can help me make my world a better place.

22. (57). I make certain that I know how much time I need for revision for mathematics tests and examinations, and I plan my time accordingly.

23. (26). I have a problem in understanding certain words in mathematics.

24. (58). I work out previous mathematics tests and examination papers.

25. (33). I feel nervous when I write mathematics tests and examinations.

26. (64). It is easy for me to express myself well in mathematics tests and examinations.

27. (63). I panic when I write mathematics tests and examinations and can remember little.

28. (67). I try to identify possible test and examination questions when I prepare for mathematics tests and examinations

29. (69). I talk to my friends about the work, discuss mathematical terms and concepts with them.

R – RARELY	S – SOMETIMES	F – FREQUENTLY	G – GENERALLY	A – ALMOST ALWAYS

30. (65). As soon as I start doing mathematics, I become sleepy, tired or bored.

31. (52). I believe that I can do well in mathematics.

32. (74). When I come across a strange word or symbol in mathematics, I make sure that I understand it.

33. (70). Unhappiness or frustration prevents me from working as hard as I can in mathematics.

34. (85). I ask questions and make remarks during the mathematics lesson.

35. (73). I play nervously with my pen, a key, a ruler or something else when I have to do difficult sums.

36. (86). I read a longer problem repeatedly until I thoroughly understand what is going on.

37. (79). When I do not understand mathematics, it is because it is too difficult.

38. (81). Later on in my career I will be able to use the mathematics that I learn at school.

39. (87). I make certain that I follow up my test and examination questions in mathematics and understand why I made mistakes.

40. (83). I stutter and stammer when I have to answer a question in the mathematics class unexpectedly.

41. (89). I read slowly and therefore I cannot finish my mathematics tests and examinations.

42. (88). I try to carry out the following four steps in problem-solving in mathematics: See what is given and what is required; make a plan; carry it out and then test the plan.

43. (92). My friends ask me to help them with mathematics.

R – RARELY	S – SOMETIMES	F – FREQUENTLY	G – GENERALLY	A – ALMOST ALWAYS

44. (105). I think it is important to use mathematics to help to make the world a better place.

45. (102). It is important to me to do more in mathematics than my teacher expects of me.

46. (90). I do not see nor hear well in the mathematics class but I am hesitant to mention this to the teacher.

47. (107). I keep my mathematics homework up to date by doing each day's work properly.

48. (93). When my friends in the mathematics class talk about a particular sum or solution, I chew my finger nails, pencil or other objects.

49. (97). I hesitate to ask my teacher to explain to me mathematics which I do not understand, until I do understand.

50. (109). I stop when I am doing a long sum to make certain that I understand what I have already done.

51. (126). When I get completely stuck while trying to solve a problem in mathematics, I go back to the beginning of the problem.

52. (127). I work ahead in mathematics.

53. (111). I need mathematics for my future career.

54. (103). In the mathematics class I find that I want to leave the room (go to the toilet).

55. (135). I talk to my parents and friends about mathematics because the subject interests me and I try to convey my enthusiasm to them.

56. (108). I struggle with certain sums because I have not read the problem carefully.

57. (110). The examples and names that appear in mathematics textbooks are unfamiliar to me.

R – RARELY	S – SOMETIMES	F – FREQUENTLY	G – GENERALLY	A – ALMOST ALWAYS

58. (136). I use the headings and sub-headings of chapters to make sure how the different aspects of the subject relate to one another.

59. (113). When I do mathematics in the classroom, I get nervous.

60. (112). I think it can be expected of average learners to achieve in mathematics.

61. (118). I copy my mathematics homework from someone else and then pretend it is my own.

62. (138). I work out a variety of problems and not only certain problems when I prepare for mathematics tests and examinations.

63. (123). When my mathematics teacher asks me a question, I move my hands, body or feet.

64. (139). If I find that I do not understand a sum, I approach it from another angle or I read it in another way.

65. (125). After working for a little while I find that I cannot concentrate on mathematics any longer.

66. (130). Problems of a non-mathematical nature prevent me from doing as well as I can in mathematics.

67. (121). I will be able to use mathematics in my future career.

68. (144). I try to understand why the rules in mathematics work.

69. (133). When I do mathematics tests and examinations I become worried when I see how quickly other children work while I have to think very hard about my answers.

70. (134). I make careless mistakes in mathematics.

71. (145). I try to be interested in mathematics.

72. (147). I ask my teacher to explain work that I did incorrectly in the mathematics tests and examinations.

R – RARELY	S – SOMETIMES	F – FREQUENTLY	G – GENERALLY	A – ALMOST ALWAYS

73. (140). I am too frightened of my mathematics teacher to ask her/him any questions.

74. (148). I make certain that my geometry sketches are big and clear; I use colour pencils to make the sketches “speak” to me as it were.

75. (143). I am frightened to discuss my problems in mathematics with my teacher.

76. (122). My teacher expects me to do well in mathematics.

77. (149). As far as possible I look for a more simple form of a problem in an effort to solve it (to determine the relationship between the work I already know and new work in mathematics).

LEARNERS IN GRADES 8 AND 9 STOP HERE. HAVE YOU ANSWERED ALL THE QUESTIONS? CHECK AND SEE. WAIT UNTIL YOU ARE TOLD WHAT TO DO.

ONLY LEARNERS IN GRADE 10, 11 AND 12 SHOULD COMPLETE THIS SECTION.

78. (11). The responsibility to work hard in mathematics is my own.

79. (30). It is my teacher’s or my parents’ fault that I do not work hard in mathematics.

80. (31). I mark incorrect homework in mathematics correct.

81. (42). I am convinced that I can do well in mathematics provided I pay careful attention, work hard in the subject and spend enough time on it.

82. (51). It is important to me to do well in mathematics, although I am not particularly interested in it or do not find it especially interesting.

83. (72). My parents are positive about mathematics.

84. (76). Mathematics is a subject in which you merely work out problems; you do not have to learn certain parts of the subject.

R – RARELY	S – SOMETIMES	F – FREQUENTLY	G – GENERALLY	A – ALMOST ALWAYS

85. (84). I use a calculator when it can help me with my mathematics.

86. (99). Self-discipline is important if a person wants to do well in mathematics.

87. (100). I will do better in mathematics if my teacher does not talk about other things in the classroom.

88. (104). It is more important to know how to solve a mathematics problem than just to find the answer.

89. (115). Even though I know certain sums are wrong, I still mark them correct.

90. (124). In mathematics tests and examinations I first leave out the sums that I cannot do and go on with the sums that I can do.

STOP HERE. MAKE CERTAIN THAT YOU HAVE ANSWERED ALL THE QUESTIONS.

APPENDIX B: LETTER FROM THE HSRC GRANTING PERMISSION FOR THE SOM TO BE INCLUDED IN THIS THESIS

HSRC

RGN

Ref. 24/2/3/1
Name: T. Avenant
Tel. 012 327 4872

Prof. J.G. Maree
1300 Arcadia Street
HATFIELD
0083

Dear Prof. Maree

INCLUSION OF THE NEW STUDY ORIENTATION QUESTIONNAIRE IN MATHEMATICS IN YOUR THESIS

With reference to our telephone conversation it is hereby confirmed that the above-mentioned matter was also discussed with Dr. S.W.H. Engelbrecht of the Group: Education and Dr. H.S. van der Walt of the Group: Human Resources.

I take pleasure in confirming that there is no objection to including in your thesis the new questionnaire that you developed on the basis of further research (and of which a copy was submitted to the HSRC).

Best of luck with your research.

Kind regards

Yours sincerely

(Signed) T. Avenant

PROMARK
1997/04/04

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