SOME FUNDAMENTAL DEFINITIONS OF THE ELASTIC PARAMETERS FOR HOMOGENEOUS ISOTROPIC LINEAR ELASTIC MATERIALS IN PAVEMENT DESIGN AND ANALYSIS

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ABSTRACT

This paper is a brief introduction to some of the well known fundamental engineering properties of homogeneous isotropic linear elastic materials. It specifically shows the relationships between different elastic moduli. It covers some of the extensive (but not so well known) relationships between the various moduli of these materials and also illustrates the importance of parameters such as Poisson's Ratio. The aim of the paper is to summarise the fundamental elastic properties of the above types of materials, as well as their interrelationships and to use these as *background* to engineering in road modelling and simulation. An additional aim is to show that moduli measured under different conditions should probably be treated differently, such as creep speed measurements of elastic surface deflection basins carried out with the Benkelman Beam (BB) (or Road Surface Deflectometer (RSD)) associated with Heavy Vehicle Simulator (HVS) testing, and those measured under relatively short time impulse loading (i.e. seismic) by means of the Falling Weight Deflectometer (FWD).

1. INTRODUCTION

1.1 Background

Rehabilitation design and analysis of existing road structures require appropriate equipment for testing and for associated evaluation methodologies. Current state-of-the-art equipment includes apparatus such as the Falling Weight Deflectometer (FWD), Rolling Deflectometer (RD), Deflectographs, high speed deflectometers, Benkelman Beam (BB), Multi-Depth Deflectometers (MDD) etc. for the measurement of mainly elastic *(i.e. recoverable)* surface (and in-depth) deflection profiles of a road structure under pre-defined loading conditions. These deflection results are then typically used in back-calculation routines for multi-layered road pavement systems based on the theory of homogenous isotropic linear elasticity. As this theory is used as an approximation of reality and since most solutions are non-unique, there are many practical problems associated with the theoretical back-calculation technique. A major problem is the allocation of the appropriate input engineering parameters such as the *effective elastic moduli* and the associated *Poisson's Ratios* for the materials based on the theory of linear elasticity. This paper addresses some of the fundamental definitions of these *elastic* parameters, especially those associated with creep (slow speed) tyre loading conditions as opposed to those done at higher speed conditions (including impulse loading such as with the FWD). It is shown that Young's moduli under seismic impulse small-strain loading can theoretically be as much as 3.5 times greater than those done at slower speed (and higher strain) conditions on the same material, depending on the assumed Poisson's Ratio. Some practical guidelines are given here for the appropriate use of these parameters for the

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design and analysis of road pavement structures.

1.2 Scope and aim of paper

The *scope* of this paper is briefly to introduce some well referenced fundamental engineering properties of homogeneous isotropic linear elastic materials and specifically to show the relationships between different elastic moduli. It covers some extensive (but not so well known) relationships between the various moduli of these materials and also illustrates the importance of parameters such as Poisson's Ratio. The *aim* of the paper is to summarize the fundamental elastic properties of the above types of materials and their interrelationships and to use these as *background* to engineering in road modelling and simulation. An additional aim is to show that moduli measured under different conditions should probably be treated differently, such as high strain creep speed measurements of elastic surface deflection basins measured by the BB, RSD (mainly associated with HVS testing) and those measured under relatively short-time impulse loading (i.e. relatively small strain seismic loading) with the Portable Seismic Pavement Analyser (PSPA) and possibly with the Falling Weight Deflectometer (FWD).

Note: The main content of this paper was taken and adapted from http://en.wikipedia.org/wiki (Wiki, 2008) and is therefore not original but is intended for information sharing amongst pavement engineers and technicians.

2. LINEAR ELASTICITY

In *linear elasticity*, the Lamé parameters (named after *Gabriel Lamé*) consist of two parameters ø, also called Lamé's first parameter, and μ, the Shear modulus or Lamé's second parameter which, in homogenous, isotropic materials, satisfy Hooke's Law in 3 dimensions:

$$\sigma = \lambda\, tr(\varepsilon) I + 2\mu\varepsilon \qquad (1)$$

Where: σ is the stress, $tr(\varepsilon)$ the trace function of the strain matrix, I the identity matrix and ε the strain tensor. The first parameter ø has no physical interpretation, but serves to simplify the stiffness matrix in Hooke's Law above. Both parameters constitute a parameterization of the elastic moduli for linear isotropic (uniformity in all directions) homogeneous media and are thus related to the other elastic moduli. Hooke's Law is a two-parameter material model. For an isotropic material, the deformation of a material in the direction of one axis will produce a deformation *(i.e. elastic deflection)* of the material along the other axes in three dimensions. Thus it is possible to generalize Hooke's Law into three dimensions, given by Equation 2 below:

$$\begin{aligned}
\varepsilon_x &= \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] \\
\varepsilon_y &= \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] \qquad (2) \\
\varepsilon_z &= \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right]
\end{aligned}$$

Where: ε_x, ε_y and ε_z are *strain* in the direction of the *x*, *y* and *z* axes and σ_x, σ_y and σ_z are *stresses* in the direction of the *x*, *y* and *z* axes and ν is Poisson's Ratio (which, in the case of linear isotropic homogeneous materials, is the same in the *x*, *y* and *z* directions. Also the stress and strain responses are linearly related at any position in such a material (Nair and Chang, 1973)).

3. EXAMPLES OF VARIOUS MODULI TYPES AND FORMULATIONS

3.1 Bulk Modulus (*K*)

The bulk modulus (*K*) of a substance essentially measures the resistance of a substance to uniform compression. It is defined as the increase in pressure required resulting in a given relative decrease in volume. See Figure 1.

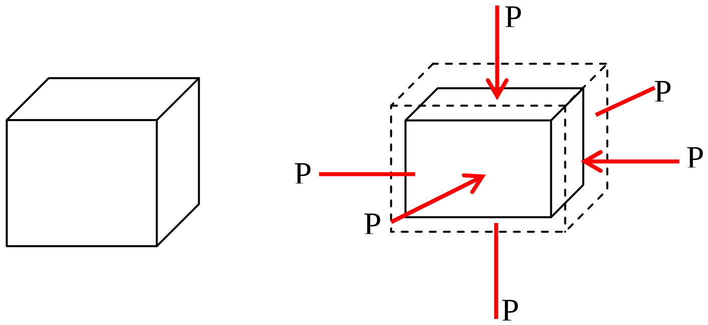

Figure 1 Isostatic pressure (P) compression of a cube

As an example, let us suppose that a spherical iron cannon ball with a bulk modulus of 160 GPa (gigapascal) is to be reduced in volume by 0.5 per cent. This would require a pressure increase of 0.005×160 GPa = 0.8 GPa. If the cannon ball were subjected to a pressure increase of only 100 MPa, it would decrease in volume by a factor of 100 MPa/160 GPa = 0.000625, or 0.0625 per cent. The bulk modulus *K* can be formally defined by Equation 3 below:

$$K = -V \frac{\partial p}{\partial V} \qquad (3)$$

Where: *p* is pressure, *V* is volume, and $\partial P/\partial V$ denotes the partial derivative of pressure with respect to volume.

The inverse of the bulk modulus indicates the *compressibility* of a substance. Strictly speaking, since bulk modulus is a thermodynamic quantity, it is necessary to specify how the temperature varies in order to specify a bulk modulus: constant-temperature (K_T), constant-enthalpy (adiabatic K_S) and other variations are possible. In practice, such distinctions are usually only relevant for gases. Bulk modulus values for some substances are given by way of example in Table 1. A very important piece of information for road pavement engineers is that the bulk modulus of water is 2 200 MPa. This could have great implications for saturated and partially saturated granular/particulate media (un-drained), which in essence are *multiphase* materials *(i.e. rock grains, water and air)* in pavements, especially under the impact (or impulse) loading of the FWD.

Table 1 Typical Bulk Moduli **for some known substances**

Substance	Bulk Modulus (MPa)	COMMENTS
Water	2 200	Value increases at higher pressures
Air	0.142	Adiabatic bulk modulus (K_S)
Air	0.101	Constant temperature bulk modulus (K_T)
Steel	1.6×10^5	Approximate
Solid Helium	50	Approximate
Glass	3.5×10^4 to 5.5×10^4	See- http://en.wikipedia.org/wiki/Glass (Wiki, 2008)

Saturated materials (or road layers) will therefore exhibit relatively large values of back-calculated "effective moduli" (Mavko *et al.,* 1998), which need to be understood when pavement performance results based on FWD testing are analysed or when these are compared with results obtained from creep speed or "static" devices such as the RSD (and/or the BB) where pore water pressure may decrease (i.e. drained conditions). For a gas (such as air within compacted road materials), the adiabatic bulk modulus K_S is approximately given by

$$K_S = \kappa p \tag{4}$$

Where: κ is the adiabatic index, (sometimes called γ); p is the pressure.

In a fluid (such as moisture in compacted road materials), the adiabatic bulk modulus K_S and the density ρ determine the speed of sound c (pressure waves), according to Equation 5 below:

$$c = \sqrt{\frac{K_S}{\rho}} \tag{5}$$

3.2 Young's Modulus (E)

In solid mechanics, Young's Modulus (E) is a measure of the stiffness of a given material. It is also known as the *Young modulus*, *Modulus of elasticity*, *Elastic modulus, and Tensile or Compression moduli*. (Bulk modulus (K) and Shear modulus (G, S or μ) are different types of elastic moduli, see later) For relatively small strains (i.e. < 100 microns) it is defined as the ratio of the rate of change of stress with strain (Mavko *et al.,*1998). This can be determined experimentally from the slope of a stress-strain curve created during tensile (or compressive) tests conducted on a sample of the material. Young's Modulus is named after Thomas Young, the 18th Century British scientist. The SI unit of modulus of elasticity, E is the *pascal*. Given the large values typical of many common materials, figures are usually quoted in megapascals or gigapascals. An alternative unit form, kN/mm², is sometimes used, which has the same numeric value as a gigapascal. Young's Modulus allows the behaviour of a material under load to be calculated. For instance, it can be used to predict how much a wire will extend under tension or to predict the load at which a thin column will buckle under compression. Some calculations also require the use of other properties of the material, such as shear modulus, density or Poisson's Ratio. Young's Modulus, E, can be calculated by dividing the tensile stress by the tensile strain:

$$E = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{\sigma}{\varepsilon} = \frac{F/A_0}{\Delta L/L_0} = \frac{FL_0}{A_0 \Delta L} \qquad (6)$$

Where:

- E is the Young's Modulus (modulus of elasticity) measured in pascal;
- F is the force applied to the object;
- A_0 is the original cross-sectional area through which the force is applied;
- ΔL is the amount by which the length of the object changes;
- L_0 is the original length of the object.

The above are also applicable to linear elastic materials in *compression*. The Young's Modulus of a material can be used to calculate the force exerted under a specific strain.

$$F = \frac{EA_0 \Delta L}{L_0} \qquad (7)$$

Where: F is the force exerted on the material (or layer) when compressed or stretched by ΔL. From this formulation *Hooke's Law* can be derived, which describes the stiffness of an ideal spring by Equation 8 below:

$$F = \left(\frac{EA_0}{L_0} \right) \Delta L = kx \qquad (8)$$

Where: $k = \frac{EA_0}{L_0}$ and $x = \Delta L$

Depending on the exact composition of the material, Young's Modulus can vary considerably. For example, the Young's Modulus for most metals can vary by 5 per cent or more, depending on the precise composition of the alloy and on any heat treatment applied during manufacture. Consequently, many of the values here (as well as those generally found and used in pavement engineering practice as given in Theyse *et al.,* 1996) are only *approximations* of reality. For the approximate Young's moduli (E) of various solids see http://en.wikipedia.org/wiki/Young%27s_modulus (Wiki, 2008).

3.3 Shear modulus (G, S or μ)

In materials science, shear modulus, G (sometimes S or μ) and sometimes referred to as the modulus of rigidity, is defined as the ratio of shear stress to the shear strain:

$$G = \frac{F/A}{\Delta x/h} = \frac{Fh}{\Delta x A} \qquad (9)$$

Where: F / A = shear stress; force F acts on area A; $\Delta x / h$ = shear strain; with initial length h and transverse displacement Δx.

The shear modulus is one of several parameters used for measuring the strength of materials. All of them arise in the generalized Hooke's Law. Young's Modulus describes the response of a material to linear strain (e.g. pulling on the ends of a wire), the bulk modulus describes the response of a material to uniform pressure and shear modulus relates to the response of a material to shearing strains. Shear modulus is usually measured in GPa (gigapascal). For typical values of shear modulus at room temperature (see http://en.wikipedia.org/wiki/Shear_modulus (Wiki, 2008)). Anisotropic materials, such as wood, paper and some road materials, exhibit different material responses to stress or strain when tested in different directions. Shear modulus is concerned with the deformation of a solid when it experiences a force parallel to one of its surfaces while its opposite face experiences an opposing force (such as friction). In the case of an object shaped like a rectangular prism, it will deform into a parallelepiped.

4. P-WAVE MODULUS (OR MASS MODULUS, *M OR E_{MASS}*)

In solids there are two kinds of seismic body waves, namely pressure waves and shear waves. The speed of shear waves is controlled by the shear modulus, G (or S or μ). Solids can also sustain transverse waves. In the case of solids an additional elastic modulus, for example the shear modulus, G (or μ), is needed to determine wave speeds. In linear elasticity, the P-wave modulus, M, is one of the elastic moduli available to describe linear isotropic homogeneous materials. M, the "Mass Modulus" (or E_{mass}) is generally measured under small strain impulse loading (i.e. PSPA) and/or under "seismic" conditions in road engineering, i.e. where impulse loads are applied over relatively short periods, typically less than 50 milli-seconds as is the case with the FWD. (However, more research is needed to quantify these effects for the FWD on southern African pavements).

The P-wave modulus is defined as the ratio of axial stress to axial strain in a uniaxial strain state:

$$\sigma_{zz} = M\varepsilon_{zz} \qquad (10)$$

With strains: $\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{xy} = \varepsilon_{xz} = \varepsilon_{yz} = 0$

This is equivalent to stating that: $M = \rho V_P^2$, where V_P is the velocity of a P-wave and ρ the material density. The elastic moduli P-wave modulus, M, is defined so that M = K + 4μ / 3 and M can then be determined by Equation 11, with a known speed V_p

$$V_P^2 = \frac{M}{\rho} \qquad (11)$$

It should however also be noted that E_{mass} (or M or Constrained moduli) are obtained from the speed of body waves that are sufficiently far from the source that there is no or negligible deformation (i.e. small strain) perpendicular to the propagation direction of the wave inside a solid medium. This is in fact the modulus determined by the PSPA apparatus but probably not in the case of FWD. For improved FWD back-calculation a dynamic method is proposed (i.e. inertia and damping included) by various authors (Magnuson, 1988, Kikuta *et al.,* 2004, Lourens, 1992, 1995).

5. POISSON'S RATIO, ν

When a sample of material is stretched in one direction, it tends to get thinner in the other two directions and, when compressed, it tends to get thicker in the other two directions. Poisson's Ratio (ν), named after Simeon Poisson, is a measure of this tendency. Poisson's Ratio is the ratio of the relative contraction strain or transverse strain (normal to the applied load to the relative extension strain, or axial strain (in the direction of the applied load). In the case of a perfectly incompressible material (like rubber) that is deformed elastically at small strains, its Poisson's Ratio would be exactly 0.5. Most practical engineering materials have ν - values of between 0.0 and 0.5. The ν - value of cork is close to 0.0, that of most steels around 0.3 and that of rubber is almost 0.5. See

Figure 2. *Note: Auxetic materials, such as polymer foams, have negative Poisson's Ratios. If these materials are stretched in one direction, they become thicker in perpendicular directions*.

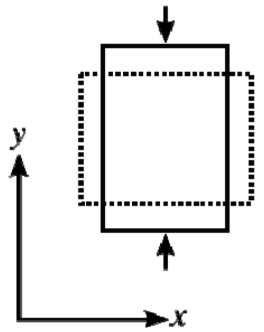

Figure 2 Rectangular *incompressible* specimen subject to compression, with Poisson's ratio of approximately 0.5 (i.e. rubber)

If it is assumed that the material is *compressed* along the axial direction then:

$$\nu = -\frac{\varepsilon_{\text{trans}}}{\varepsilon_{\text{axial}}} \qquad (12)$$

Where: ν is the resulting Poisson's ratio; $\varepsilon_{\text{trans}}$ is transverse strain and $\varepsilon_{\text{axial}}$ is axial strain.

At first glance, a Poisson's Ratio greater than 0.5 does not make sense because, at a specific strain, the material would have a zero volume and any further strain would give the material a "negative volume". Unusual Poisson's Ratios are usually found in materials with a complex architecture. For Poisson's Ratio values for different materials – see http://en.wikipedia.org/wiki/Poisson%27s_ratio (Wiki, 2008). Tensile deformation is usually considered *positive* and compressive deformation is considered *negative*. This definition of Poisson's Ratio contains a minus sign so that normal materials have a positive ratio. Poisson's ratio is usually represented as a lower case Greek nu, ν. The cross sections of virtually all common materials become narrower when these materials are stretched. The reason for this, in the continuum view, is that most materials are more resistant to a change in volume as determined by the bulk modulus K than to a change in shape, as determined by the shear modulus G. Poisson's ratio is directly proportional to the material properties of Bulk modulus (K), Shear modulus (G), and Young's Modulus (or strain modulus, E). These moduli all reflect some aspect of the stiffness of the material and are themselves a derivation of stress-to-strain ratios. The following equations show how these properties are all related: (See also Table 2):

$$\nu = (3K - 2G)/(6K + 2G) \qquad (13)$$

$$E = 2G(1+\nu) \qquad (14)$$

The theory of isotropic elasticity makes provision for Poisson's Ratios in the range from -1 to +0.5. Physically it means that the material needs to be stable and that the stiffnesses must be positive: the bulk and shear stiffnesses are interrelated by formulae which incorporate Poisson's Ratio. As the elastic properties of homogeneous isotropic linear elastic materials are uniquely determined by any two of the above moduli (M, E), any of the other elastic moduli can be calculated. See the interrelationships of M and E in Table 2.

Table 2. Conversion formulas between elastic properties of homogeneous isotropic linear elastic materials (modified from Mavko et al., 1998). [The symbols are defined below.]

	(λ,μ)	(E,μ)	(K,λ)	(K,μ)	(λ,ν)	(μ,ν)	(E,ν)	(K,ν)	(K,E)
$K=$	$\lambda+\frac{2\mu}{3}$	$\frac{E\mu}{3(3\mu-E)}$	--	--	$\lambda\frac{1+\nu}{3\nu}$	$\frac{2\mu(1+\nu)}{3(1-2\nu)}$	$\frac{E}{3(1-2\nu)}$	--	--
$E=$	$\mu\frac{3\lambda+2\mu}{\lambda+\mu}$		$9K\frac{K-\lambda}{3K-\lambda}$	$\frac{9K\mu}{3K+\mu}$	$\frac{\lambda(1+\nu)(1-2\nu)}{\nu}$	$2\mu(1+\nu)$		$3K(1-$	
$\lambda=$		$\mu\frac{E-2\mu}{3\mu-E}$		$K-\frac{2\mu}{3}$		$\frac{2\mu\nu}{1-2\nu}$	$\frac{E\nu}{(1+\nu)(1-2\nu)}$	$\frac{3K}{1+\nu}$	$\frac{3K(3K-E)}{9K-E}$
$\mu=$			$3\frac{K-\lambda}{2}$		$\lambda\frac{1-2\nu}{2\nu}$		$\frac{E}{2+2\nu}$	$3K\frac{1-}{2-}$	$\frac{3KE}{9K-E}$
$\nu=$	$\frac{\lambda}{2(\lambda+2\mu)}$	$\frac{E}{2\mu}-1$	$\frac{\lambda}{3K-\lambda}$	$\frac{3K-2\mu}{2(3K+\mu)}$					$\frac{3K-E}{6K}$
$M=$	$\lambda+2\mu$	$\mu\frac{4\mu-E}{3\mu-E}$	$3K-2\lambda$	$K+\frac{4\mu}{3}$	$\lambda\frac{1-\nu}{\nu}$	$\mu\frac{2-2\nu}{1-2\nu}$	$E\frac{(1-\nu)}{(1+\nu)(1-2\nu)}$	$3K\frac{1-}{1+}$	$3K\frac{3K+E}{9K-E}$

Definition of Symbols in Table 2:

$K=$ Bulk Modulus; $E=$ Young's Modulus (or Static Modulus); $\lambda=$ Lame Modulus; $\mu=$ Shear Modulus ; $\nu=$ Poisson's Ratio; $M=$ Mass Modulus, P-Wave Modulus, Constrained Modulus, or Seismic Modulus.

6. SEISMIC LOADING AND THE RELATIONSHIP BETWEEN YOUNG'S MODULUS (E) AND MASS MODULUS (M)

6.1 Seismic Loading

Seismic loading normally refers to relatively small engineering strains (λ) on the body being tested (Roesset *et al.,* 1990). Saeed and Hall (2002) showed the changes in the complex moduli of asphalt, especially for asphaltic base layers, depending on loading frequency.

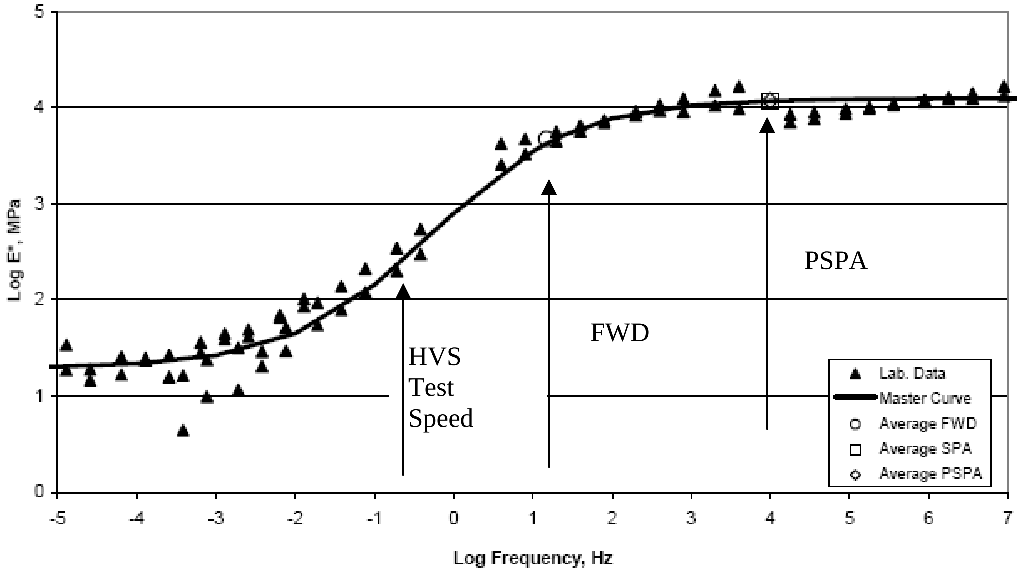

Figure 3 Dependence of complex moduli of asphalt (25 degrees C) on loading frequency, after Saeed and Hall (2002) (modified in this paper)

Figure 3 shows an increase in complex moduli with loading frequency (asphalt in this case) depending on loading frequency. Similar findings were reported by Roesset *et al.,* (1990) and Nazarian *et al.,* (2002, 2005). Also shown on the figure are the relative loading frequencies of the Heavy Vehicle Simulator (HVS), Falling Weight Deflectometer (FWD) and the Portable Seismic Pavement Analyser (PSPA). It is clear that, for asphalt materials at least, care should be taken when measuring field moduli with different devices. However, more dedicated research is needed to evaluate the effect of load frequency on typical "thin" pavement structures in southern Africa. The effect of inertia and damping also need further quantification for these pavement structures, as it is well known that these factors (i.e. inertia and damping) are completely ignored with back-analyses of routine testing e.g. with the FWD, as was demonstrated adequately by Lourens (1992, 1995).

6.2 Mass Modulus vs Static Modulus

There is a very important difference (which may not be so well known by pavement engineers) between moduli measured in static (or creep) loading conditions (i.e. low frequency testing) and those measured at relatively high loading frequencies, but under small engineering strain conditions. The former modulus is known as "E_{static}" (or E) and the latter "E_{mass}" (or M, or also known as Constrained Modulus). In the case of linear elastic isotropic homogenous materials these two moduli are related via the Poisson's Ratio, ν. This relationship is shown below in Equation 15 as well as in Table 2.

$$M = E_{mass} = E_{static}\left[\frac{(1-\upsilon)}{(1+\upsilon)(1-2\upsilon)}\right] \qquad (15)$$

Typical values of Poisson's Ratio for road materials range between 0.25 and 0.45. A graphical representation of Equation 14 is illustrated in Figure 3. Figure 3 shows that the increase in the multiplying factor *(i.e. Poisson's factor)* E_{static} for above is exponential and increases drastically above Poisson's Ratios of about 0.4. Typical Poisson's ratios used in South Africa for granular and asphaltic materials are 0.35 and 0.44 respectively. Thus it can be seen in Figure 4 that the ratios between *M* (E_{mass}) and E_{static} (i.e. E_{mass} / E_{static}) are

1.6 and 3.24 for granular and asphalt materials respectively, (assuming of course that these are homogeneous, isotropic linear elastic materials).

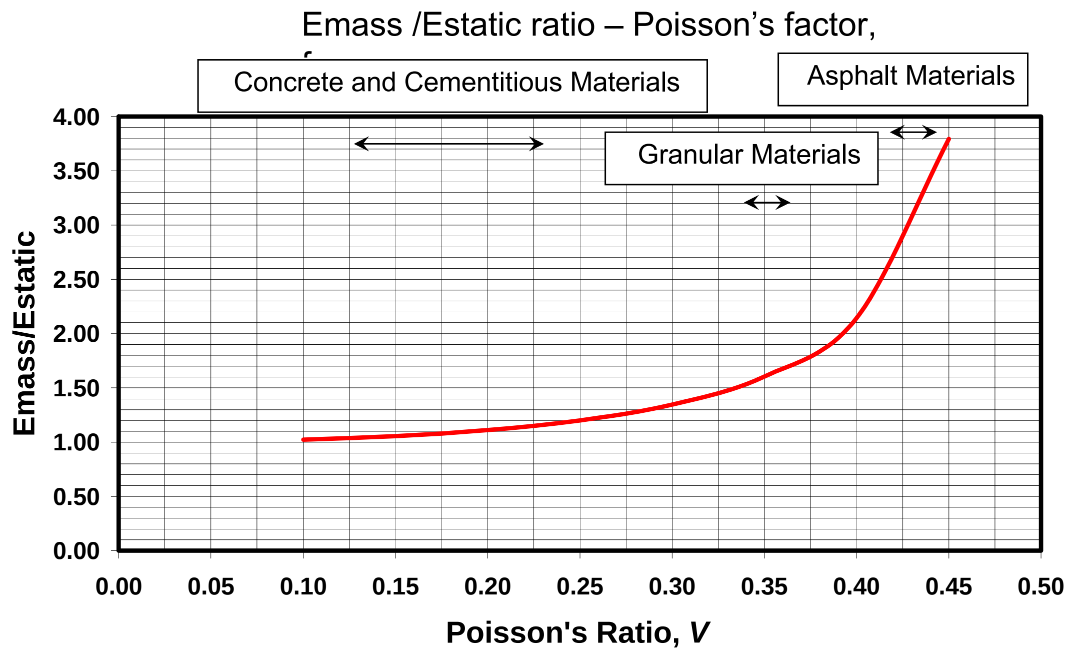

Figure 4 Ratio of E_{mass} (or M) vs E_{static} (or E) for different Poisson's Ratios with ranges generally applicable to road materials

The importance of this is that this factor (referred to here as "Poisson's factor") should be clear from the fundamentals and, ideally, should be introduced into practical engineering as soon as possible. Field measurements are, however, needed to evaluate this concept before its practical application in, for example, the conversion of moduli values from M to E and *vice versa* during the back-calculation of these constants and its application in pavement rehabilitation engineering. This obviously relates to the type of testing on the pavement being carried out (i.e. small-strain seismic, impulse or creep-speed loading, together with the position of receivers relative to the loading (impact) source.

7. DISCUSSION

From the fundamental descriptions of material properties such as moduli and Poisson's Ratios for homogeneous isotropic linear elastic materials it is clear that pavement engineers should be aware of the *different effective moduli* that are available and that can be used in practice. Some of these basic differences might account for the current confusion arising from the use of moduli obtained from PSPA (and possibly the FWD) impact measurements by comparison with those developed over the years in South Africa under creep loading of the Heavy Vehicle Simulator (HVS) and RSD testing, as well as from Multi-Depth Deflectometer (MDD) tests, in which the "creep" or "static" loading mode was used for load application during measurements of elastic road deflections. Further, the importance of obtaining the "correct" or appropriate Poisson's Ratios cannot be over-estimated. However, more research is needed to quantify these effects, especially those associated with the FWD, in order to provide more appropriate practical guidelines for design and maintenance of road pavements in southern Africa.

8. CONCLUSIONS AND RECOMMENDATIONS

8.1 Conclusion

A basic understanding of the fundamental definitions of the elastic parameters for homogeneous isotropic linear elastic materials in road pavement design and analysis is urgently needed for modern mechanistic analysis of road pavement by engineers and technicians.

8.2 Recommendation

The information in this paper should be made available to students and practitioners at as early a stage as possible in road pavement design courses. It is also recommended that, in particular, the Poisson's factor between Mass Modulus (*M or* E_{mass}) and Young's Modulus (*E or* E_{static}) be evaluated from practical experience and from measurements on road materials/layers as a matter of urgency.

9. ACKNOWLEDGEMENT

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