STUDY OF NATURAL CONVECTION IN THE PRESENCE OF MAGNETIC FIELD (CYLINDRIC ENCLOSURE)

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ABSTRACT

Buoyancy-driven magneto hydrodynamic flow in a liquid-metal filled cylindrical enclosure is investigated by numerical simulations, the finite volume method was applied to solve the mass, momentum and energy equations. The numerical result obtained shows the disappearance of the vortex break down under the effect of an axial temperature gradient between the bottom heated and the top cooled of cylinder. In the case of the natural convection with and without magnetic field for fluid with low Prandtl number, the instability appears in the form of regular oscillations for high values of the critical Grashof number, corresponding to the Hartmann numbers. These oscillations are produced by the multicellular mode of the flow. Diagrams of stability show the dependence of the critical Grashof number and critical frequency with the increase of the Hartmann number. we note that, the action of the magnetic field has a stabilizing effect on the flow.

NOMENCLATURE

Ar	[-]	aspect ratio
B	[T]	intensity of magnetic field
Br	[T]	radial magnetic field
Bz	[T]	axial magnetic field
Fr		dimensionless frequency pressure
g	[m^2/s]	gravitational acceleration
Gr		Grashof number
H	[m]	height of the cylinder
Nu		average Nusselt number
Cp		specific heat at constant pressure of liquid
FEMr		dimensionless Lorentz force in the r-direction
FEMz		dimensionless Lorentz force in z-direction
P		dimensionless pressure
Pr		Prandtl number
R	[m]	radius of the cylinder
r, z		gravitational acceleration
T		temperature
u, v		dimensionless radial and axial velocity
α		thermal diffusivity
β		thermal expansion coefficient
δ		orientation of the magnetic field
θ		dimensionless temperature
ρ		density of the fluid
σ		electric conductivity
τ		dimensionless time
υ		kinematic viscosity

Subscripts

c	cold
cr	critical value
h	hot
max	maximum value

INTRODUCTION

The natural convection heat transfer in cylindrical is an important research topic due to its wide application in engineering problems, such applications are found in energy conversion, storage and transmission systems, Examples of using annulus geometry include electrical cooling, solar energy collector, nuclear reactor design. A significant number of experimental and theoretical works have been carried out in the past decades in an attempt to understand heat transfer flow in a cavity. A comprehensive review of natural convection in cylindrical cavities has been documented in the literature. Among the very first investigations. For instance, in crystal growth processes from melts, it has been reported by [1] that larger transports rated are obtained by tilting the ampoule. Motivated by this, [2] have carried out a numerical and experimental investigation on threedimensional buoyancy-driven flows in a tilted cylinder (ampoule) with axial heating. [3] have found that the strength of the magnetic field is one of the important factors in determining the quality of the crystal. This is related to the fact that during a crystal growth process, some turbulence in the natural convection currents occurs. This can be suppressed by the application of a magnetic field. [4] studied numerically the effect of an external magnetic field with different magnitudes and orientations in a rectangular cavity. Stability diagrams for the dependence of the critical Grashof number on the Hartmann number were obtained by the authors.

They showed that a vertical magnetic field provides a strongest stabilizing effect, and also that the multiplicity of steady states is suppressed by the electromagnetic effect. [5] studied the effect of a magnetic field orientation on fluid flow and heat transfer during solidification from a melt in a cubic enclosure. They have shown a strong dependence between the interface shape and the intensity and orientation of magnetic field and the strongest stabilization of the flow field and heat transfer are shown when the magnetic field is oriented vertically ($\gamma = 90°$).

Recently, [6] presented the MHD mixed convection oscillatory flow over a vertical surface in a porous medium with chemical reaction and thermal radiation. They analysed the influence of a first-order homogeneous chemical reaction, heat source and Soret effects are analyzed. They conclude that, the velocity decreases with increasing the Prandtl number, and magnetic field parameter whereas reverse trend is seen with increasing the heat generation parameter, radiation parameter, porous parameter, Soret number, thermal and solutal Grashof numbers. The temperature decreases as the values of Prandtl number increase and reverse trend is seen by increasing the values of the thermal radiation parameter, heat source parameter. The concentration decreases as the values of the chemical reaction parameter and Schmidt number whereas concentration increases with increase the value of Soret number.

Therefore, the objective of the present contribution is to study the effect of magnetic field orientation on the oscillatory natural convection. The study was carried out in oscillatory state, for different orientations (δ = 0 at 90°, Ha ≤ 50). The critical Grashof number Gr_{cr} and corresponding critical frequency Fr_{cr} and Hartmann numbers/different orientations are determined in each case and discussed.

GEOMETRY AND MATHEMATICAL MODEL

The geometry considered is a cylindrical enclosure (Fig.1) of radius R and height H, thus with an aspect ratio Ar = H/R = 4. The enclosure filled completely with a molten metal, have a Prandtl number equal Pr = 0.015. The horizontal walls of the enclosure are maintained at different temperatures, the bottom wall is maintained at the hot temperature Th while the top wall maintained at the cold temperature Tc (Th >Tc). The side wall is supposed adiabatic. The flow is subjected to the action of an external uniform magnetic field with different orientations. Electrically, the walls of the cylindrical enclosure are insulated. The induced magnetic field is negligible because the magnetic Reynolds number Rem is much smaller than unity [7]. By neglecting the dissipation and Joule heating, and using R, υ/R, R^2/υ, ρ $(\upsilon R)^2$ and (Th-Tc) as typical scales for lengths, velocities, time, pressure and temperature, respectively. The dimensionless governing equations for the conservation of mass, momentum, and energy with the Boussinesq approximation, together with appropriate initial and boundary conditions in the cylindrical coordinates system (r, z), are written in dimensionless form, as follows:

Continuity equation

$$\frac{1}{R}\frac{\partial}{\partial R}(R\,U)+\frac{\partial V}{\partial Z}=0 \qquad (1)$$

R-Momentum equation

$$\frac{\partial U}{\partial \tau}+\frac{1}{R}\frac{\partial}{\partial R}\left(RU^2\right)+\frac{\partial}{\partial Z}(VU)-\frac{W^2}{R}=-\frac{\partial P}{\partial R}+\frac{1}{\mathrm{Re}}\left(\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial U}{\partial R}\right)+\frac{\partial^2 U}{\partial Z^2}-\frac{U}{R^2}\right)+F_{EMr} \qquad (2)$$

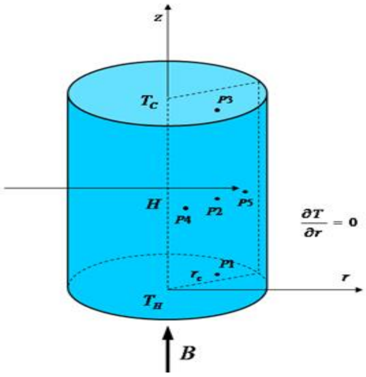
Z-Momentum equation

$$\frac{\partial V}{\partial \tau}+\frac{1}{R}\frac{\partial}{\partial R}(RUV)+\frac{\partial}{\partial Z}\left(V^2\right)=-\frac{\partial P}{\partial Z}+\frac{1}{\mathrm{Re}}\left(\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial V}{\partial R}\right)+\frac{\partial^2 V}{\partial Z^2}\right)+\frac{Gr}{\mathrm{Re}^2}\theta+F_{EMz} \qquad (3)$$

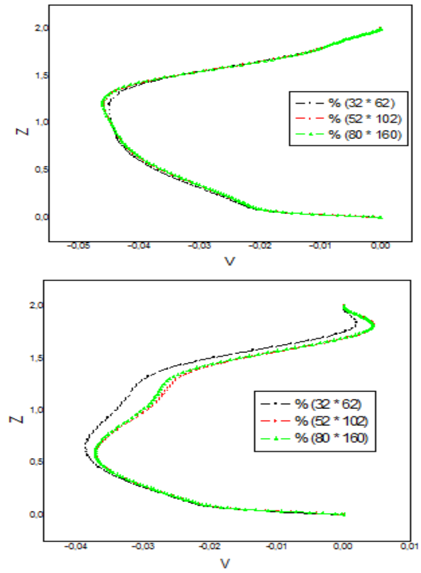
Energy equation

$$\frac{\partial \theta}{\partial \tau}+\frac{1}{R}\frac{\partial}{\partial R}(RU\theta)+\frac{\partial}{\partial Z}(V\theta)=\frac{1}{\mathrm{Re.Pr}}\left(\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \theta}{\partial R}\right)+\frac{\partial^2 \theta}{\partial Z^2}\right) \qquad (4)$$

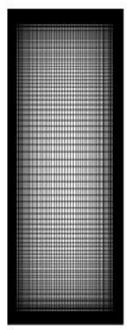
In Eqs. (2) and (3), F_{EMr} and F_{EMz} represent the Lorentz forces components in the r and z directions respectively [8], u and v are the dimensionless velocity components in the radial and axial directions, P is the dimensionless pressure and θ is the dimensionless temperature. So, the resulting dimensionless numbers are:



(a) Physical problem



(b) Effect of grid size on the velocity axial variation V with a Z *for R=0.5 and R=0.6*



(c) Computational domain (52 × 102)

Fig. 1. Geometry and computational domain with boundary conditions.

Grashof number $Gr=\frac{g\,B(T_H-T_C).\,r_C^{\;3}}{\nu^2}$ Prandtl number $\Pr=\frac{\nu}{\alpha}$ and Hartmann number $Ha=B\,r_c\sqrt{\sigma/\rho\upsilon}$, which indicate the ratio of the electromagnetic forces to the viscosity forces. The quantities g, β, ρ, ν and σ are the gravity acceleration, the thermal expansion coefficient, the density, the kinematic viscosity and the electric conductivity of the fluid, respectively.

The above equations are solved subject to the following initial and boundary conditions:
The initial conditions,
at $\tau = 0$, $u = v = \theta = 0$ (5a)
The boundary conditions of the dimensionless quantities (u, v and θ)
for $\tau > 0$ are:
- Along the bottom wall
($z = 0, 0 \le r \le 1$); $u = v = 0, \theta = 1$ (5b)
- Along the top wall
($z = Ar; 0 \le r \le 1$); $u = v = 0, \theta = 0$ (5c)
- Along the sidewall
($0 \le z \le Ar; r = 1$); $u = v = 0, \dfrac{\partial \theta}{\partial R} = 0$ (5d)
- Along the symmetry axe ($0 \le z \le Ar; r = 0$);

NUMERICAL METHOD

The governing equations (Eqs. (1)-(4)) with the associated boundary and initial conditions (Eqs. 5a-d) are solved using a finite volume method. Scalar quantities are stored at the center of control volume, whereas the vectorial quantities are stored on the faces of each volume. For the discretization of spatial terms, a second-order central difference scheme is used for the diffusion and convection terms of the mathematical model, and the SIMPLER Algorithm [9] is used to determine the pressure from continuity equation. The obtained algebraic equations are solved by the line-by-line tri-diagonal matrix algorithm (TDMA). The convergence is declared when the maximum relative change between two consecutive iteration levels fell below than 10^{-5}. The increments Δr and Δz of the grid are not regular, they are chosen according to geometric progressions of ratio equal to 1.05 [10], which permitted grid refinement near the walls, in the Hartmann layer where large velocity and temperature gradients exist, thus requiring a larger number of nodes in order to resolve the specific characteristics of the magnetohydrodynamic flow, also in order to reduce numerical errors.

The grid independency tests are presented in Fig 1 b which presented the variation the velocity with Z for different grid sizes. We can notice that the relative error between all grids is very low. We also see that the low relative error occurs between the two meshes 52 × 102 and80 × 160. So, the grid used has 52 × 102 nodes.

To allocate more confidence in our numerical results, we have established some comparisons with other experimental and numerical investigations available in the literature. Firstly, Fig. 2 A good agreement between the obtained and reported results was observed. and Secondly, Fig. 3 It is clear that the computed values can be seen to be in excellent agreement with the measurements. These comparisons validate our computer code by assigning the desired confidence to use.

Effect of Magnetic Field Orientation on the Oscillatory Natural Convection (Ha ≠ 0)

In this section, we are interested in the oscillatory solution of the flow convection with different orientations of magnetic field ($\delta = 0°, 30°, 45°$ and $90°$), the same results are obtained in the range $0 \le \delta \le 90°$.

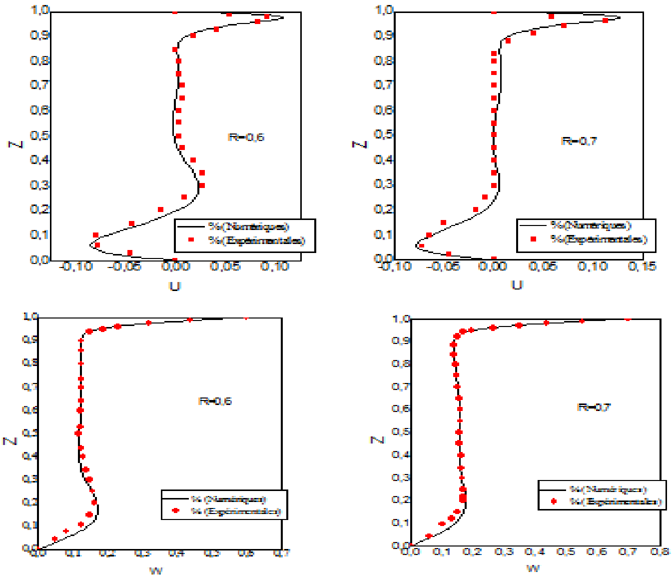


Fig. 2. Comparison between our results and previous numerical/experimental studies. Distribution the radial and azimutal velocity for R=0.6 and R=0.7 with Re=1800 and H/r_c=1

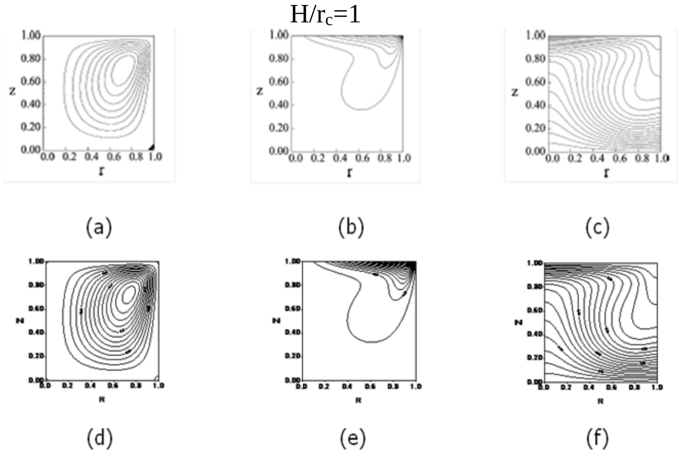


Fig. 3. Comparison between our results and previous numerical/experimental studies.

In general, the magnetic field suppresses the fluid motion and reduces the heat transfer rate. As the Hartmann number increases, the temperature gradients become less abrupt and the convection effect become less intense, resulting in smaller velocities. Thus, the increase of the magnetic field favours the conduction heat transfer. Our numerical simulations are presented for various values of the Hartmann number.

To see the effect of magnetic field on the oscillatory flow regime, we applied the magnetic field in radial direction ($\delta = 0°$ parallel to the cylinder axis). The Fig. 7, present the time dependent of the axial velocity component in one period, for Grashof number $Gr = 2.1\times10^{6}$ and for various Hartmann numbers (Ha = 10, 20, 30, 40 and 50). It is clearly that the increase of Hartmann number (intensity of magnetic field), stabilize the oscillatory flow and reduce the magnitude of velocity [11]. The flow regime is oscillatory for Ha = 10, and stabilized to steady state flow when Ha > 10. This is translates the ability of the magnetic field on the stability of convective flows, this reduction due to radial Lorentz force which slows the velocity of particles.

In order to explain the nature of the flow oscillatory, we connect the temporal evolution of the dimensionless axial velocity v at point S8 during one period with evolution of the flow structure (streamline and temperature field) at various dimensionless times: τa, τb, τc, τd, τe and τf, for $Gr_{cr} = 0.8\times10^6$, $\delta = 0°$ and Ha = 30 (Fig. 4). The flow field presents two cells. These cells dilate and narrow during the time (τa, τb, τc, τd, τe and τf). At time τa, the structure characterized by a small cell located in bottom of liquid and separated by another secondary recirculation cell (dashed lines) in top of enclosure with a negative mass flow. At times τb, τc, τd and τe the size of secondary recirculation cell change and narrowing gradually in the axial direction, after (at time τf) this cell dilate gradually.

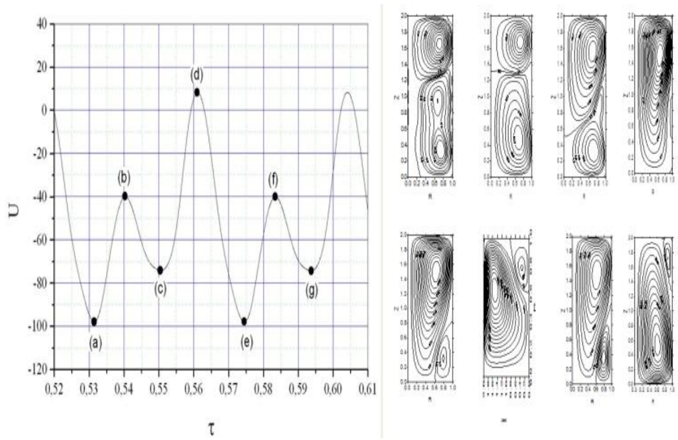


Fig. 4. Time evolution of the dimensionless axial velocity U in oscillatory flow at point S8, with streamlines at various dimensionless time: τa, τb, τc, τd, τe and τf, for $Gr_{cr} = 9\times10^6$ The magnetic field is applied in the radial direction ($\delta = 0°$ and Ha = 30).

We note that, the streamlines structure at the time τa is identical at the time τf, which means that the oscillatory flow is periodic. The temperature field is very significant in this case where the magnetic field is applied in radial direction ($\delta = 0°$), and shows the existence and the predominance of the convective mode compared to the diffusive mode (deformation of the isotherms). Figures 5a-b In order to obtain the energy spectrum of oscillations, we have used the fast Fourier transform (FFT) of a number Nech of samples of the time variations of various dimensionless parameters. This transform, once multiplied by the half of its conjugate quantity, gives the power spectrum density (PSD) E(F) as a function of the oscillation frequencies (Fig. 5), defined by: $F = k / (Nech\times\Delta\tau)$, where $\Delta\tau$ is the dimensionless time step and k = 1, 2 and Nech / 2. Energy has been normalized by N^2ech. The dimensionless predominant frequencies are considered as those playing the main role in the flow oscillation there can exist several others frequencies which are multiples of the dominant one [12] and Figure 6, show the magnetohydrodynamic stability diagram (Fr_{cr} –Ha). We can see that the strong dependence between the onset of oscillatory flow (the critical Grashof number and corresponding frequency) and the orientation of magnetic field, where the strongest damping of the flow is obtained when the magnetic field is applied along the radial direction ($\delta = 0°$). They found a better stabilization of the flow where the magnetic field applied in the radial direction.

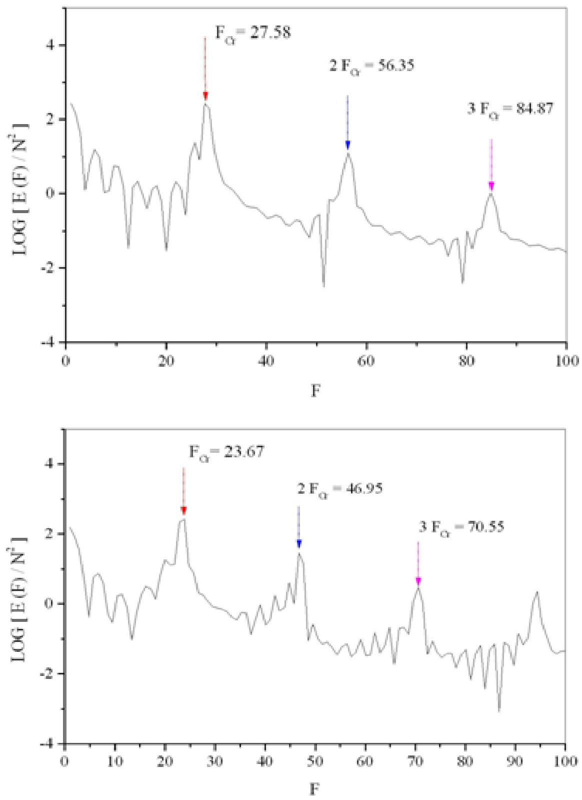


Fig. 5. Power spectrum of the dimensionless radial velocity component u, for $Gr_{cr} = 0.8\times10^6$ and $Gr_{cr} = 1.1\times10^6$ respectivoly for Ha = 0. $Fr_{cr} = 27.58$, represent the dimensionless critical frequency.

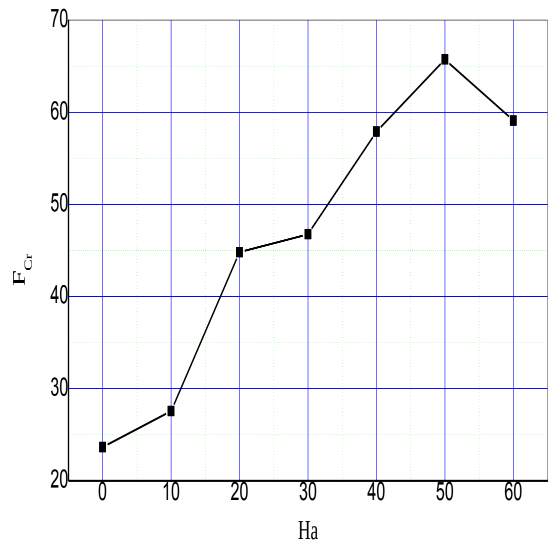


Fig.6. Magnetohydrodynamic stability diagram for different Hartmann number

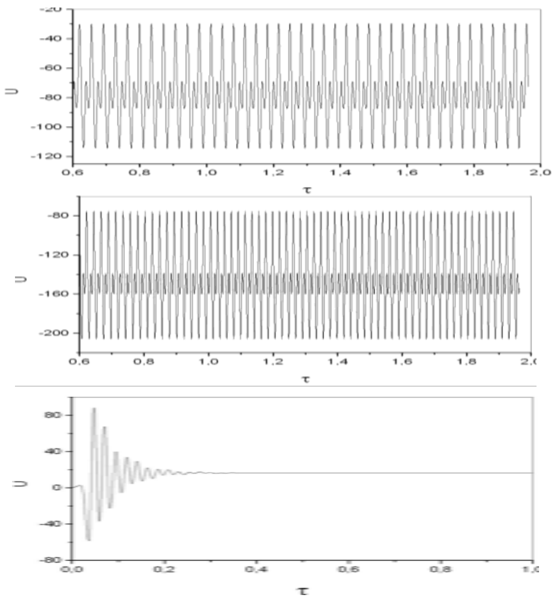


Fig. 7. Effect of magnetic field on the time dependent U-velocity component at monitoring point S7, for Gr = 2.1×10^6 and Ha=0, 10 and 20

CONCLUSION

The effect of magnetic field orientation on the oscillatory natural convection has been numerically studied. The obtained results have been compared with the available data (experimental/ numerical) from the literature and good agreement has been found.

The main results are as follows:

Circulation and convection become stronger with increasing Grashof numbers but they are significantly suppressed by the presence of a strong magnetic field.

A radial magnetic field, provides a strong stabilization of the flow field, where the high value of critical Grashof number is obtained for this case.

The concordance of increasing the magnetic field intensity with respect to the critical frequency Fcr, except for case Ha=60.

The obtained results in this study may allow researchers and industrialists to know the oscillatory modes of conducting fluid enclose with and without magnetic field, and help them for the stabilization according to the available possibilities, in order to improve the quality of the semiconductors obtained during the crystal growth.

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