ANALYSIS OF PIN FINS WITH RADIATION

Vincent T.G., Schetz J.A.* and Lowe K.T.
*Author for correspondence
Departments of Aerospace & Ocean and Mechanical Engineering,
Virginia Tech,
Blacksburg, VA 24061,
USA,
E-mail: ptiger@vt.edu

ABSTRACT

The design of fins for heat transfer enhancement remains a topic of great interest in a number of engineering areas and applications, despite a broad and deep prior literature on the subject. Rapid prediction of the effects of convection, conduction and radiation is still an area of concern. For hot-flow conditions, the fin is normally mounted in a cooled surface, leading to substantial axial conduction. Also, radiation plays a very important role in hot flow conditions. One can apply detailed computational methods for simultaneous convection, conduction and radiation heat transfer, but such approaches are not suitable for rapid, routine design studies. So, there is still a place for approximate analytic methods, and that is the subject of this paper. We have extended the traditional pin fin analysis to include a more realistic radiation treatment and also considered variable thermal conductivity, variable heat transfer coefficients over the tip and sides of the fin with variable area distribution, variable internal heat generation and then produced a *MATLAB* solution procedure for routine use by designers and analysts.

INTRODUCTION

The use of fins to enhance heat transfer is ubiquitous in industrial and consumer applications and even in nature,[1], [2]. So, the design and analysis of fins for heat transfer enhancement remains a topic of great interest, despite a broad and deep prior literature on the subject (e.g. [3], [4]). Prediction of the effects of convection, conduction and radiation remains an area of concern. For hot-flow conditions, the fin is normally mounted in a cooled surface, leading to substantial axial conduction. Also, radiation plays a very important role in hot conditions. One can apply detailed computational methods for simultaneous convection, conduction and radiation heat transfer. We have used *ANSYS Fluent* [5] for related studies, but such approaches are not suitable for rapid, routine design studies. So, there is still a place for approximate analytic methods, and that is the topic of this paper.

Some useful, early analytical methods treated a so-called "pin fin" (a straight or tapered rod projecting from a wall) with combined convection and conduction. Reference [3] provides a very thorough review of the literature. While useful, these treatments were quite restricted in a number of ways. Some, but not all, of the restrictions are covered in the so-called: Murray-Gardner-Kern Assumptions [3]: (1) The heat flow and temperature distribution are steady in time, (2) The fin material is homogeneous and isotropic, (3) There are no heat sources in the fin, (4) The heat flow to or from the fin surface is proportional to the temperature difference between the surface and the surrounding fluid, (5) The thermal conductivity of the fin is constant, (6) The heat transfer coefficient is the same over the fin surface, (7) The temperature of the surrounding fluid is uniform, (8) The temperature of the base of the fin is uniform, (9) Temperature gradients normal to the surface may be neglected (1D assumption), (10) The heat transferred through the tip of the fin is negligible compared to that passing through the sides, (11) The joint between the fin and the prime surface offers no bond or contact resistance. One can add an additional common assumption: (12) Radiation is neglected, or radiation is treated without convection.

NOMENCLATURE

$A(x)$	[m^2]	area of fin
$d(x)$	[m]	diameter of fin
G	[1/m^{2}K^4]	$=2/5(\sigma\varepsilon P/kA)$
$h(x)$	[W/m^{2}K]	convection heat transfer coefficient on fin side
h_{tip}	[W/m^{2}K]	convection heat transfer coefficient on fin tip
i	[-]	node index
$k(T)$	[W/mK]	thermal conductivity
k_0	[W/mK]	reference thermal conductivity
k_1	[W/mK^2]	slope of linear thermal conductivity
L	[m]	length of the fin
m	[1/m]	$=(hP/kA)^{1/2}$
n	[-]	number of nodes
$P(x)$	[m]	perimeter of fin
$\dot{q}(x)$	[W/m^3]	internal heat generation
Q_b	[W/m^2]	base heat transfer
$T(x)$	[K]	temperature
T_0	[K]	reference temperature
T_f	[K]	fluid temperature
T_b	[K]	base temperature
T_{surr}	[K]	surroundings temperature
x	[m]	base to tip coordinate
Δx	[m]	discretized element length
α	[-]	absorptivity
σ	[WK^4/m^2]	Stefan-Boltzmann constant
ε	[-]	emissivity

Analyses of the pin fin for heat transfer enhancement have also been used to study total temperature probes where the fluid flow is directed towards the tip of the fin. That was the topic of some recent work of ours [6].

It is very difficult to treat radiation from/to the fin within a purely analytical method. The only available solutions in the literature are limited to very restricted cases. Here, we have extended the pin fin analysis to include a more realistic radiation treatment and also considered variable thermal conductivity, variable heat transfer coefficients over the tip and sides of the fin with a varying cross-sectional fin area and then produced a *MATLAB* solution procedure for routine use by designers and analysts.

DISCUSSION OF PRIOR ANALYSES

Assuming 1D conduction along a fin and neglecting radiation, the analysis is simple, especially when one makes additional assumptions [3].

The nonlinear ODE equation to be treated with radiation included can be written [7]

$$\frac{d}{dx}\left[k(T)A(x)\frac{dT}{dx}\right] = h(x)P(x)\left[T(x) - T_f\right] + \sigma P(x)\left[\varepsilon T^4(x) - \alpha T_{surr}^4\right] + A(x)\dot{q}(x) \quad (1)$$

The term on the left-hand-side represents axial conduction, the first term on the right-hand-side represents convection, the second term on the right-hand-side represents radiation, and the third term on the right-hand-side allows for internal heat generation.

Assuming, for the moment, a constant cross-section area (and hence perimeter), h and k constant, no internal heat generation and $\varepsilon=\alpha$ (grey bodies), the governing equation is [7].

$$\frac{d^2T}{dx^2} = \frac{hP}{kA}\left[T(x) - T_f\right] + \frac{\sigma\varepsilon P}{kA}\left[T^4(x) - T_{surr}^4\right] \quad (1a)$$

The simplest cases neglect radiation, and the equation above reduces to a linear ODE with a well-known solution [3], but, the assumption of constant h is unrealistic. It is an easy matter to extend the solution to have $h_{tip} \neq h$ by modifying the boundary condition at $x=L$. The results are [6]:

$$\frac{T(L) - T_f}{T_b - T_f} = \frac{1}{\cosh(mL) + \frac{h_{tip}}{mk}\sinh(mL)}$$

$$Q_b = \sqrt{hPkA}\left(T_b - T_f\right)\frac{\sinh(mL) + \left(\frac{h_{tip}}{mk}\right)\cosh(mL)}{\cosh(mL) + \left(\frac{h_{tip}}{mL}\right)\sinh(mL)} \quad (2)$$

where Q_b is the heat transfer at the base.

Consider next prior treatments of the pin fin problem including radiation using eqn. (1a). The BC at $x=L$ is now

$$h_{tip}\left[T(L) - T_f\right] + \sigma\varepsilon\left[T(L)^4 - T_{surr}^4\right] = -k\left[\frac{dT}{dx}\right]_{x=L} \quad (3)$$

Since eqn. (1a) is a non-linear ODE, analytical solutions are only obtained for limited cases. If $T_{surr}=T_f$, multiply eqn (1a) by dT/dx and integrate once to get [7]:

$$\frac{1}{2}\left(\frac{dT}{dx}\right)^2 = \frac{hP}{kA}\left[\frac{1}{2}T^2 - TT_f\right] + \frac{\sigma\varepsilon P}{kA}\left[\frac{1}{5}T^5 - TT_f^4\right] + C \quad (4)$$

For the very restrictive case where $T_f=0$ and a very long fin so that both $T(L)$ and $(dT/dx)_{x=L} \rightarrow 0$, $C=0$

$$\frac{dT}{dx} = \pm\sqrt{\frac{2}{5}\frac{\sigma\varepsilon P}{kA}T^5 + \frac{hP}{kA}T^2} \quad (5)$$

One selects the appropriate sign for the expected behavior of dT/dx. For example, if $T(x)$ is decreasing along the fin, the minus sign is appropriate. Using $T(0)=T_b$ one can integrate to obtain:

$$\int_0^x dx = -\int_{T_b}^T \frac{dT}{T\left[\frac{2}{5}\left(\frac{\sigma\epsilon P}{kA}\right)T^3 + \frac{hP}{kA}\right]} \quad (6)$$

Further integration yields the closed form solution [7]:

$$x = \frac{1}{3m}\left[ln\frac{\sqrt{GT_b^3 + m^2} - m}{\sqrt{GT_b^3 + m^2 + m}} - ln\frac{\sqrt{GT^3 + m^2} - m}{\sqrt{GT^3 + m^2 + m}}\right] \quad (7)$$

where $G=2/5(\sigma\varepsilon P/kA)$. Even though this solution is only obtained following very limiting assumptions, it is still useful by displaying the key lumped parameters, G and m.

One can find interesting solutions for other restricted cases in the literature, all for $h_{tip}=h$ or an insulated tip, $(dT/dx)_{x=L} \rightarrow 0$. See for example Refs. [8] and [9]. Generally, numerical evaluation of complex integrals is required even for such restricted cases.

Cases with variable cross-section area, $A(x)$, and variable heat transfer coefficient, $h(x)$, (except for the simplest case of $h_{tip} \neq h$) are much more complicated. See Ref. [4] for a thorough exposition of available results. Reference [10] considered variable $k(T)$ and simplified $h(x)$ variations. There is much less work in the literature for cases with varying $A(x)$ and $h(x)$ in the presence of radiation.

Based on the discussions above, we conclude that there is no suitable analytic solution for general application to the pin fin case. That conclusion lead to the work described in the following sections.

NUMERICAL METHODS

It is our goal to implement a numerical solution that generalizes the thermal behaviour of a 1-D conducting pin fin with radiation. This means allowing for variable heat transfer coefficient and area along the fin's length, $h(x)$ and $A(x)$, as well as variable internal heat generation $\dot{q}(x)$, respectively. For cases with large thermal gradients along the fin, model accuracy can be further improved by replacing the constant solid conductivity assumption with a local temperature dependent conductivity, *k(T)*. Thus, the governing differential equation takes the form:

$$\frac{d}{dx}\left[k(T)A(x)\frac{dT}{dx}\right] = h(x)P(x)\left(T(x) - T_f\right) + \varepsilon\sigma P(x)(T(x)^4 - T_{surr}^4) + A(x)\dot{q}(x) \quad (7)$$

The pin fin problem further requires two boundary conditions. At $x = 0, T(0) = T_b$ and at x = L:

$$\left.\frac{dT}{dx}\right|_{x=L} = \frac{-h_{tip}}{k\left(T(L)\right)}\left(T(L) - T_f\right) - \frac{\varepsilon\sigma}{k\left(T(L)\right)}(T(L)^4 - T_{surr}^4) \quad (8)$$

This is a "two-point boundary value problem" since the two required boundary conditions are applied at two values of the independent variable, *x*. There are three main ways to treat such problems [11]: *Shooting, Finite Differences and Projections.* In the *Shooting* method, one estimates the value of the slope at *x* = 0, and then iterates the solutions until the desired boundary condition at the other point is matched. In the *Finite Differences* approach, a mesh is introduced over the domain, and derivatives in the ODE are replaced by finite differences. This leads to an algebraic system that may be solved to produce a discrete approximation to the problem. In the *Projections* method, the solution is approximated by simpler functions, *e.g.* polynomials, and the differential equation and boundary conditions are satisfied approximately. Collocation or finite element methods can furnish these approximations. We have selected an implicit *Finite Differences* approach.

The local first and second spatial derivatives of temperature and geometry are approximated using a second-order accurate, 4-point centered-differencing scheme, derived to allow for non-uniform element lengths. Non-uniform gridding can help improve accuracy with a reduction in computational cost by allowing for finer grids near regions of large spatial gradients (thermal, geometric, or film coefficient) without the need to refine regions of small gradients where coarser grids are adequate. Since we wish to solve for the temperature at each node along the fin, the implicit *Finite Difference* method requires n equations for n node temperatures. At nodes $i = 1$ and $i = n$, the two boundary conditions described above are applied $(x(i = 1) = 0$ and $x(i = n) = L)$. For each of the nodes $i = 2:(n - 1)$, the discrete form of eqn (7) is used. Due to the non-linearity of the governing equation with radiation, an iterative approach is required to solve the system of equations. MATLAB's *fsolve* function is built to solve such a system of non-linear equations. To use *fsolve*, one must provide an initial guess for the temperature solution along the length of the fin. While a good initial guess is not critical for convergence, it can reduce computational cost. A simple, yet effective initial guess for *fsolve* is a linear temperature profile along the length, where the base is equal to the known base temperature and the tip is equal to the surrounding fluid temperature.

Following evaluation of the temperature solution, we can compute fin performance. It is common to represent the performance of a fin using the fin efficiency, η_f, which is defined as the ratio of actual fin heat transfer rate, Q_b, to the idealized maximum amount of heat transfer through the base, Q_{max}, assuming the fin is entirely at its specified base temperature [4]:

$$\eta_f = \frac{Q_b}{Q_{\max}} \quad (9)$$

The fin heat transfer rate can be determined by calculating the conduction heat transfer at the fin base using a numerically estimated temperature gradient from the temperature solution. To achieve a reasonable level of accuracy, a second-order accurate, 4-point forward-differencing scheme was used for the base temperature gradient. It is critical that we consider how the definition of fin efficency handles the addition of radiation heat transfer. If radiation affects the fin heat transfer rate (i.e. $\varepsilon \neq 0$), it is logical to adjust the denominator of fin efficiency to include idealized radiation to the surroundings. Further generalizing fin efficiency to account for variable film coefficient and geometry, we get:

$$\eta_f = \frac{Q_b}{\left(T_b - T_f\right)\sum_{i=1}^{n} h_i P_i \Delta x_i + (T_b^4 - T_{surr}^4)\sum_{i=1}^{n} \sigma\varepsilon P_i \Delta x_i} \quad (10)$$

If radiation is negected, eqn (9) simplifies to the standard definition of fin efficiency for a conducting fin with convection.

Due to truncation errors arising from necessarily neglecting high-order terms in the estimation of local derivatives, it is necessary to check the final temperature solution for grid independence. For solutions presented in this paper, a 1000 node grid with a refinement bias towards the base (finest elements near fin base) produced converged results, although 100 elements also produces adequate results with much reduced computational cost.

Constant cross-section area fins with constant thermal conductivity, k, and heat transfer coefficient, h.

There are a few limiting examples where exact solutions are available that can be used to validate our numerical method. Let us consider pin fins with constant cross-section area, *A*, and heat transfer coefficient, *h*, (except for the simplest case of $h_{tip} \neq h$). The first has the solution in eqn. (2) where radiation is neglected. The second is the very restrictive radiation situation with the solution given here as eqn. (7). We selected a test case for a fin of a metal like stainless steel: *d* = 1.0 mm, *L* = 10 mm, *k* = 16 W/mK, ε =0 and 1.0, T_b= 300K, T_f= 1000K, T_{surr} = 300K, *h* = 1000 W/m^{2}K and h_{tip}/h = 1.0.

Using eqn. (2) with the above test case conditions, one finds that Q_b = -4.398W for this simulated fin with a cooled base in a high convection environment. The negative sign here indicates this is a heating fin since the surrounding fluid is warmer than the fin base. The result from the current numerical solution with ε=0 using 1000 nodes yields a fin heat transfer that matches the analytical value of -4.398W, as it should. The corresponding solution including radiation with ε =1.0 yields a base heat transfer of -4.310W. Radiation reduces the magnitude of heat transfer through the base in this example.

The second restricted case for which an exact solution exists (eqn. (7)) allows radiation, but requires assuming $T_f = T_{surr} = 0$ and a very long fin so that both $T(L)$ and $(dT/dx)_{x=L} \rightarrow 0$. It is obvious that these are severe restrictions, but we can still use the predictions to compare with our numerical solution as a validation case. Radiation can be enhanced by choosing T_b= 1500K and reducing convection with h = 50 W/m^{2}K and an adiabatic tip (h_{tip}/h = 0). The temperature decay is also likely slower, so L was increased to 100 mm to better simulate the infinite length required for the analytic solution. The analytical solution in eq. (7) for this case gives x = 8.37 mm for $T(x)$ = 500K, and the numerical result agrees very well with that. Our numerical calculation further shows a base heat transfer Q_b = 3.354W. Neglecting radiation would lead to an error of more than 100K at that location.

Constant cross-section area fins with variable heat transfer coefficients over the surface

There has long been considerable interest in fin analyses that permit the heat transfer coefficient to vary along the length of the fin, $h(x)$, because that is the situation in most practical applications. See for example Huang and Chen [12] and the extensive review of older work in Kraus [3].

Unfortunately, there is no simple, analytic solution with variable heat transfer coefficient even without radiation available that can be used to validate the current numerical procedure. We can, however, show predictions for a few cases and then compare the trends observed with those found in the literature with more elaborate analyses. The cases selected for these comparisons had: k = 16 W/mK, h_{ave} = 500 W/m^{2}K, h_{tip} = 0 (for better comparison with elaborate analysis from the literature, Huang and Chen [12]), L = 10 mm, d = 1.0 mm, T_f = 1000K, T_b = 300K, and T_{surr} = 300K.

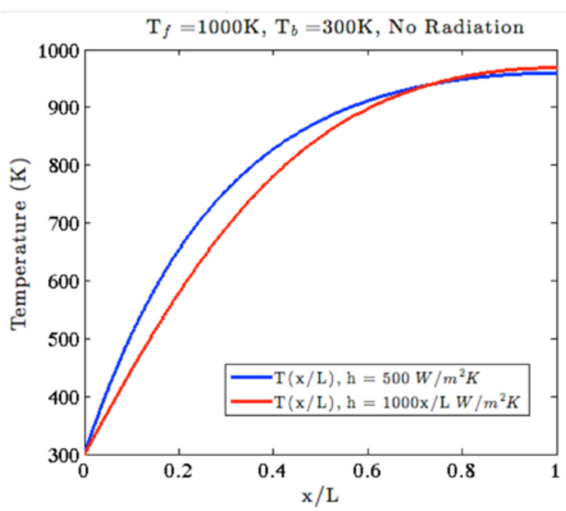

Fig. 1. Numerical solution results for Validation Case no. 3: d = 1.0 mm, L = 10 mm, k = 16 W/mK, ε = 0, T_b = 300K, T_f = 1000K, h_{ave} = 500 W/m^{2}K and h_{tip}/h = 0.0. h=*constant* (top/blue curve) and linear $h(x)$ (bottom/red curve), both with h_{ave} = 500 W/m^{2}K.

In Fig. 1, we show the current results for this case excluding radiation, but with both h=*constant*=h_{ave}=500 W/m^{2}K and also a linear $h(x)$ starting from 0 at the fin base $x/L=0$ and having the same h_{ave} = 500 W/m^{2}K. A low value of the film coefficient near the base and a high value near the tip can be expected in practical situations. One can easily see that a varying $h(x)$ has a large effect on the temperature distribution along the fin. Here, we found η_f = 0.2849 for the constant h assumption and 0.1706 for the linear $h(x)$. This behavior of a constant h assumption over-predicting η_f is in agreement with the literature [12] for other assumed $h(x)$ cases. We take this as at least partial validation of the current numerical method.

Variable cross-section area fins with constant heat transfer coefficients over the surface

In many heat transfer applications, it is desirable to design a pin fin with a variable cross sectional area along its length, $A(x)$. This may be motivated by several factors including the fin's structural capabilities, volumetric constraints, or various heat transfer requirements.

For the case of constant heat transfer coefficient without radiation, several analytical solutions exist in the literature [4]. For validation of variable area incorporation into the current numerical method, we computed the temperature solution for a conical fin with a base diameter d = 1.0 mm, L = 10 mm, k = 16 W/mK, ε = 0, T_b = 300K, T_f = 1000K, h = 1000 W/m^{2}K and h_{tip}/h = 0.0. The analytical solution for the fin efficiency of a conical fin is:

$$\eta_f = \frac{2}{mL}\frac{I_2(2mL)}{I_1(2mL)} \tag{11}$$

Evaluation of the analytic fin efficiency (eqn (11)) for the input parameters described above yields $\eta_f = 0.3417$. Our numerical calculations show excellent agreement, thus validating the numerical approach for variable area fins.

EXAMPLES

Constant cross-section area fins with constant heat transfer coefficient

A designer is always interested in the effects of fin length, and we can investigate that with the tools developed here. Consider a range of cylindrical fins with length to diameter ratio, L/d, ranging from 1.0 to 10.0 under the conditions presented in Fig. 2, with diameter fixed at d = 1mm. To emphasize the impact of radiation, we used a side film coefficient of h = 500 W/m^{2}K. Results for the fin base heat transfer for each fin length is given in Fig. 1. The influence of conduction to the cooled base reduces the fin temperature as the fin becomes shorter which increases the local heat transfer rate into the sides of the fin, but the smaller overall surface area drives total fin heat transfer down. Also, it can be observed that there is a point at which further increasing the fin length no longer improves heat transfer rate. Lastly, it can be seen that the effects of radiation are diminished as the fin becomes shorter.

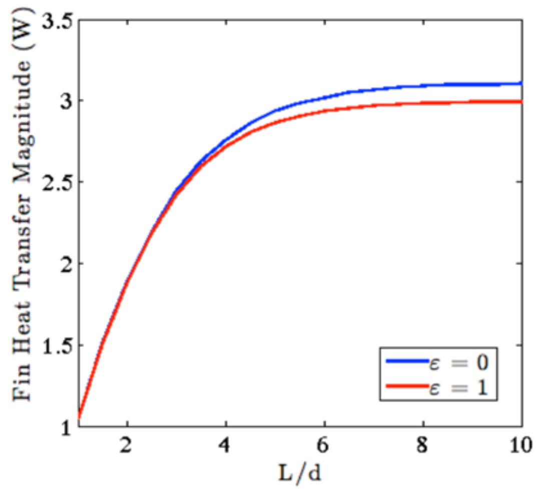

Fig. 2. Numerical solution results for: d = 1.0 mm, k = 16 W/mK, ε = 0 (top/blue curve) and ε = 1.0 (bottom/red curve), T_b = 300K, T_f =1000K, T_{surr} = 300K, h = 500 W/m^{2}K and h_{tip}/h = 0.0.

Variable cross-section area fins with constant heat transfer coefficient over the surface

Another interesting way to alter pin fin performance is to alter the fin shape. Changing the fin diameter as a function of the fin length allows the designer to control the behavior of the fin temperature profile. To investigate this effect, we can compare the temperature solution for three fins all with a base diameter d = *1mm* and L = *10mm* with differing profiles: cylindrical, conical, and convex parabolic. Setting T_b= 300K, T_f = 1000K, T_{surr} = 300K, h = 100 W/m^{2}K and h_{tip}/h = 0.0, we can compare the fins thermal response as a function of position as shown in Fig. 3.

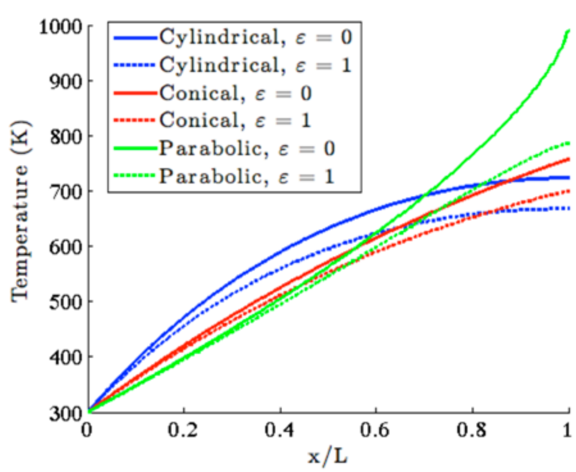

Fig. 3. Numerical solution results for varying fin profiles with: d_{base} = 1.0 mm, L = 10 mm, k = 16 W/mK, ε = 0 (solid curves) and ε = 1.0 (dotted curves), T_b = 300K, T_f =1000K, T_{surr} = 300K, h = 100 W/m^{2}K and h_{tip}/h = 0.0.

It is apparent that as the tip becomes more slender due to the profile shape definition, the tip temperature reaches closer to the surrounding fluid temperature (T(L) = 723K, 757K and 990K for the cylindrical, conical and parabolic fins respectively without radiation). By accounting for radiation to the cooler surroundings, the fin temperatures can be seen to drop as expected. Further investigation of the parabolic fin solution shows only a 2% discrepancy in the total fin heat transfer through the base when neglecting radiation (-0.585W w/ radiation and -0.598W w/o), which in most cases is admittedly insignificant for low-order model predictions. However, more notable is a 203K reduction in temperature at the tip of the parabolic fin when compared with the solution without radiation. This tip temperature discrepancy would be easily overlooked if only heat transfer rates were investigated. Thus, a quick, yet accurate calculation the tip temperature can be valuable if fluid temperatures are near the melting point of the fin material.

Variable cross-section area fins with variable heat transfer coefficient over the surface and temperature dependent thermal conductivity

An exhaustive literature review of 1-D pin fin solutions revealed very little consideration with regard to varying thermal conductivity based on local temperature. A constant thermal conductivity assumption can prove reasonably accurate if the temperature gradients along the fin are small. However, in cases where the fin base is significantly cooler than the surrounding fluid, large thermal gradients can exist along the fin's length. In extreme cases, temperatures gradients can exist on the order of 1000K per inch [14]. It is for these specific cases that relaxing the constant thermal conductivity assumption is beneficial.

Consider now a conical fin made with a material representative of iron. At 300K, iron has $k \approx$ 80 W/mK and at 1000K, $k \approx$ 32.5 W/mK [13]. As a first major step towards temperature dependent thermal properties, let us assume that k varies in a locally linear manner with temperature as in eqn (12):

$$k(T) = k_0 + k_1(T - T_0) \tag{12}$$

For iron, we may approximate $k(T)$ as linear using k_0 = 80 W/mK at a reference temperature T_0 = 300K and slope k_1= -0.0679 W/mK^2. To utilize the full generality of our numerical approach, we may apply a non-uniform film coefficient profile along the side of the fin, which varies linearly from $h(x=0)$ = 0 W/m^{2}K to $h(x=L)$ = 500 W/m^{2}K, as well as compare solutions with and without radiation. Taking base diameter d = 1.0 mm, L = 10 mm, T_b = 300K, T_f = 1000K, T_{surr} = 1000K, we get the results shown in Fig. 4. Also shown is the solution if k is assumed constant at the average of the base and fluid temperatures. Note that in our previous examples, the surroundings were cooler than the fluid temperature resulting in radiation away from the fin. This example has surroundings that are equal to the fluid temperature resulting in radiation into the fin.

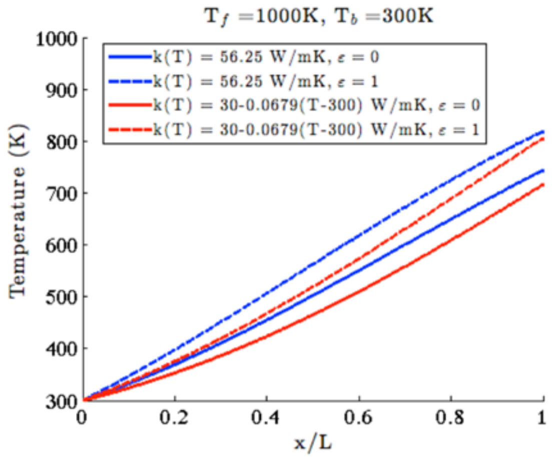

Fig. 4. Numerical solution results conical fin with: d_{base} = 1.0 mm, L = 10 mm, k = 56.25 W/mK (blue curves) and k(T) = 30-0.0679(T-300) W/mK (red curves), ε = 0 (solid curves) and ε = 1.0 (dotted curves), T_b = 300K,T_f=1000K, T_{surr} = 1000K, $h(x/L)$ = 500x/L W/m^{2}K and h_{tip}/h = 0.0.

Due to the large variability in profile temperatures, using conductivity with linear temperature dependence give a much better answer with minimal increase in computational cost. A fin heat transfer rate comparison between results including radiation shows a 25% reduction in magnitude when using *k(T)* compared to *k* = constant (-2.079W and -2.804W respectively).

CONCLUSION

In this work, we have greatly extended the classical pin fin analysis to now include a more general radiation treatment with variable convection heat transfer coefficient, *h(x)*, area change, *A(x)* variable thermal conductivity, *k(T)*, variable internal heat generation, $\dot{q}(x)$, and then produced a user-friendly *MATLAB* solution procedure for routine use by designers and analysts. The method was validated by comparing results with available analytic solutions that are applicable for restricted cases.

To illustrate the utility of the new analysis tool presented here, a few example cases were selected to study the influences of some key parameters. In particular, radiation on or off, fin length, fin shape, material choices, constant *h* versus variable *h(x)*, and constant *k* versus variable *k(T)* were considered. The tool can be used to thoroughly study these items and any other combinations of parameters that describe the general problem. Of course, in order to use this new tool for specific applications, one needs simple methods to find values of h_{tip} and *h* and/or *h(x)*.

The *MATLAB* code will be posted at: http://www.aoe.vt.edu/research/onlinesoft.html/.

REFERENCES

[1]. Farlow J O, Thompson C V and Rosner D E (1976) ``Plates of the Dinosaur Stegosaurus: Forced Convection Heat Loss Fins?", Science, vol. 192, no. 4244, pp. 1123-1125.
[2]. Hill R W and Veghte J H (1976) "Jackrabbit Ears: Surface Temperatures and Vascular Responses," Science, Volume 194, Issue 4263, pp. 436-438.
[3]. Kraus A D (1988) "Sixty-five years of extended surface technology (1922–1987)," Applied Mechanics Reviews, vol. 41, Issue 9, pp. 621–364.
[4]. Kraus D A, Aziz D and Welty J R (2001) *Extended Surface Heat Transfer*, John Wiley & Sons.
[5]. Reardon J.P., Schetz J.A. and Lowe K.T. (2017) "Computational Modelling of Total-Temperature Probes" J. Thermophysics and Heat Transfer.
[6]. Schetz JA, Vincent T and Lowe K T (2016) "Analysis of Base-cooled Total temperature Probes with Radiation," ASME Paper IMECE2016-65130.
[7]. Siegel R and Howell J R (1992) *Thermal Radiation Heat Transfer*, Hemisphere Publishing Corp.
[8]. Shouman A R (1967) "An Exact General Solution for the Temperature Distribution and Composite Radiation Convection Heat Exchange along a Constant Cross-sectional Area Fin," Quart. Appl. Math., vol. 25, pp. 458-462.
[9]. Asadi M and Khoshkho R H (2013) "Temperature Distribution along a Constant Cross Sectional Area Fin," Int. Journal of Mechanics and Applications, vol. 3, no. 5, pp. 131-137.
[10]. Sadri S, Raveshi MR and Amiri S (2012) "Efficiency analysis of straight fin with variable heat transfer coefficient and thermal conductivity," J. Mech. Sci. Tech, 26 (4), pp. 1283-1290.
[11]. Ascher U M, Mattheij R and Russel R (1995) *Numerical Solution of Boundary Value Problems for Ordinary Differential Equations* (2nd Edition), SIAM.
[12]. Huang M J and Chen C K (1985) "Conjugate mixed convection and conduction heat transfer along a vertical circular pin," Int J Heat Mass Transfer, vol. 28, no. 3, 523.
[13]. Powell, R. W., C. Y. Ho, and P. E. Liley. *Thermal Conductivity of Selected Materials*. Tech. West Lafayette: Thermophysical Properties Research Center, 1966. Print.
[14] Englerth, Steven T. "An Experimental Conduction Error Calibration Procedure for Cooled Total Temperature Probes." Thesis. Virginia Polytechnic Institute and State University, 2015. Print.