

ANALYTICAL THERMAL MODELING AND CALIBRATION METHOD FOR LITHIUM-ION BATTERIES

Fukui M., Kato K.,
Department of Electronic and
Computer Engineering,
Ritsumeikan University,
Kusatsu, Shiga, 525-8577 Japan
e-mail : mfukui@se.ritsumeai.ac.jp

Tsukiyama S.,
Dept. of Electrical, Electronic
and Communication Eng.,
Chuo University,
Tokyo, 112-8551 Japan

Shirakawa I.
Graduate School of
Applied Informatics
University of Hyogo
Kobe, Hyogo, 650-0047 Japan

ABSTRACT

The lithium-ion battery is an essential component to drive mobile systems. The battery behavior varies depending on its thermal conditions. Thus, to optimize the lifetime performance, it is important to estimate the inner temperatures of battery. This paper proposes a new analytical method to estimate the inner temperatures of battery with the use of the Joule heat as well as the Entropy heat. An evaluation is also attempted by means of a real battery sample.

INTRODUCTION

With the advent of high density lithium-ion batteries, high performance mobile systems have become realized in quite small packages. However, the lithium-ion battery characteristics, e.g., inner resistance, open circuit voltage, and state of charge, depend on their thermal conditions [1, 2]. Especially, they degrade rapidly when they operate in high temperature. Thus, it is of practical importance to manage accurately the inner temperature of lithium-ion battery.

In prior works on the thermal macro model for lithium-ion batteries, Vaidyanathan [3] analyzed basic heat behaviors of battery, Mousavi [4] examined a cooling system on the basis of the general theory of thermodynamics, and Huria [5] proposed a thermal model for high power lithium cells. However, they do not take account of differences between the surface and inner temperatures of lithium-ion battery.

The present paper newly proposes an accurate inner temperature estimation method for lithium-ion batteries through the analysis of thermodynamics. The transfer functions of natural convection and heat process are also considered, which can be practically applied to controlling battery temperatures. Furthermore, a calibration method of the heat resistance and heat capacity of battery is also proposed.

These heat parameters are essential for the inner temperature analysis. It is practically valuable to calibrate them without decomposing the battery. Furthermore, these heat parameters are calibrated in full range of state of charge. We propose a practical and efficient calibration technique using a pseudo battery cell by assembled batteries. Finally, experiments show that the inner temperature is obviously higher than the surface temperature in low SOC area. Especially, it is remarkable when the ambient

temperature is low. We have confirmed by experiments using the assembled battery that the estimation formula is highly accurate.

NOMENCLATURE

T_{in}	[K]	Inner temperature
Q_s	[W]	Entropy heat
Q_p	[W]	Joule heat
R_{in}	[K/W]	Inner thermal resistance
R_a	[K/W]	Thermal resistance located surface of the lithium-ion battery in air
C_{in}	[J/K]	Inner heat capacity of the lithium-ion battery
T_a	[K]	Ambient temperature
A	[m ²]	Surface area size
h	[W/m ² K]	Heat transfer coefficient
T_{surf}	[K]	Surface temperature
P_s	[W]	Heat generation
V_o	[V]	Output voltage
V_{oc}	[V]	Open circuit voltage

THERMAL MODELING

The work when electric charge of 1mol is slowly moved through a big resistance connected to the lithium-ion battery whose electromotive force is E [V] can be expressed by the following thermodynamic formula:

$$-nFE = \Delta G \quad (1)$$

In this equation, n is the electric charge quantity pertaining to the reaction, F is the Faraday constant, and ΔG is the Gibbs free energy [W]. Next, eqs. (2)-(3) are obtained by the Gibbs free energy formula when the temperature and pressure are constant:

$$\Delta G = \Delta H - T_{in} \Delta S \quad (2)$$

$$\Delta S = nF \frac{\partial E}{\partial T_{in}} \quad (3)$$

In these equations, T_{in} [K] is the inner temperature of lithium-ion battery, ΔH [J] is the amount of change of enthalpy, and ΔS [J/K] is the rate of change of entropy. The heat generation quantity Q_s [W] due to the entropy is equivalent to the heat energy $T_{in} \Delta S$ [J] in eq. (2). Electric charge quantity I/nF moves between the electrode gap per unit time. Therefore, eqs. (4)- (5) are derived. In eq. (5), electromotive force E is replaced by the OCV (open circuit voltage) V_{oc} .

$$Q_S = T_{in} \Delta S \frac{I}{nF} \quad (4)$$

$$Q_S = T_{in} I \frac{\partial V_{OC}}{\partial T_{in}} \quad (5)$$

The electric current quantity I [A] is defined as a positive value during charge and a negative value during discharge. The OCV and the output voltage V_O are different because of the inner resistance of lithium-ion battery. The difference generates the energy loss, and the Joule heat Q_P [W]:

$$Q_P = I(V_O - V_{OC}) \quad (6)$$

The formula to estimate the inner temperature from the ambient temperature is discussed here. The formula (7) is obtained from the Fourier's law of heat conduction and the Newton's law of cooling:

$$Q = C_{in} \frac{\partial T_{in}(t)}{\partial t} = -\frac{T_{in}(t) - T_a}{R_{in} + R_a} \quad (7)$$

In this equation, Q is the heat flow [W], C_{in} is the inner heat capacity of the lithium-ion battery [J/K], R_{in} is the inner thermal resistance of the lithium-ion battery [K/W], R_a is the thermal resistance located surface of the lithium-ion battery in air [K/W], T_a is the ambient temperature [K], A is the surface area size of the heat transfer [m²], and h is the heat transfer coefficient [W/m²K]. The thermal resistance R_a is expressed by $1/Ah$. Fig.1 illustrates temperatures at each place, T_{in} is inner temperature, T_{surf} is surface temperature, and T_a is ambient temperature.

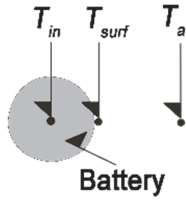


Figure 1 Three external boundary condition types

Next, eq. (8) is obtained by adding the total quantity heat generation P_S (W) of the lithium-ion battery:

$$C_{in} \frac{\partial T_{in}(t)}{\partial t} = -\frac{T_{in}(t) - T_a}{R_{in} + R_a} + P_S \quad (8)$$

Here, P_S is the sum of the joule heat and the entropy heat. The heating process is represented by the step response in time domain. Therefore, eq. (8) is multiplied by the step function $u(t)$. Here, we can transform from eq. (8) to eq. (10) by using eq. (9) which we defined. Finally, eq. (13) is obtained by transforming from eq. (10):

$$\theta(t) = T_{in}(t) - T_a \quad (9)$$

$$\left\{ 1 + (R_{in} + R_a) C_{in} \frac{\partial}{\partial t} \right\} \theta(t) = P_S (R_{in} + R_a) u(t) \quad (10)$$

$$\frac{\theta(s)}{P_S(s)} = \frac{R_{in} + R_a}{s \{ 1 + (R_{in} + R_a) C_{in} s \}} \quad (11)$$

$$\frac{\theta(s)}{P_S(s)} = \frac{R_{in} + R_a}{s} - \frac{R_{in} + R_a}{\frac{1}{(R_{in} + R_a) C_{in}} + s} \quad (12)$$

$$\theta(t) = \left[1 - \exp \left\{ -\frac{t}{(R_{in} + R_a) C_{in}} \right\} \right] P_S (R_{in} + R_a) \quad (13)$$

Here, the ambient temperature and the initial inner temperature are assumed to be equal. We obtain the formula to estimate the lithium-ion battery inner temperature by expanding $\theta(t)$:

$$T_{in}(t) = \frac{\left[1 - \exp \left\{ -\frac{t}{(R_{in} + R_a) C_{in}} \right\} \right] \{ (V_O - V_{OC}) I (R_{in} + R_a) \} + T_{in}(0)}{1 - \left[1 - \exp \left\{ -\frac{t}{(R_{in} + R_a) C_{in}} \right\} \right] \{ I (R_{in} + R_a) \} \left(\frac{\partial V_{OC}}{\partial T_{in}} \right)} \quad (14)$$

The inner heat capacity C_{in} which is included in eq. (14) is obtained by observing a heating process of the lithium-ion battery. It is the heat generation quantity to raise the object temperature by 1 degree. SOC (state of charge) is defined by the remained charge ratio in the battery. Full charge state is represented by SOC = 1, and empty state is represented by SOC = 0.

This paper examines two lithium-ion batteries. One is lithium-ion battery A whose shape is square, the material of cathode is lithium cobalt oxide, the nominal voltage is 3.7V, and nominal capacity is 600mAh. The other is lithium-ion battery B whose shape is cylindrical, the material of cathode is lithium nickel and manganese oxide, the nominal voltage is 3.6 [V], and nominal capacity is 2250 [mAh].

The thermal circuit which includes variables of thermal resistances and heat capacities of lithium-ion battery B is shown Fig.2. The thermal resistance R_a [K/W] is located surface of the lithium-ion battery in air. It is expressed by formula eq. (15), here, A [m²] is the surface area of heat transfer of the Lithium-ion, h [W/m²K] is the heat transfer coefficient.

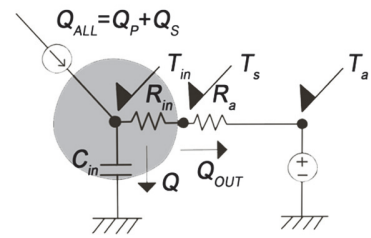


Figure 2 Three external boundary condition types

$$R_a = \frac{1}{Ah} \quad (15)$$

The heat transfer coefficient of between surface of the lithium-ion battery and air is calculated by using general formula of thermo-dynamics:

$$Gr = \frac{d^3 g \beta (T_{surf} - T_a)}{\nu^2} \quad (16)$$

$$Nu = 0.53 (Gr \cdot Pr)^{1/4} \quad (17)$$

$$h = \frac{Nu \cdot \lambda}{d} \quad (18)$$

Here, g is gravitational acceleration [m/s^2], β is the thermal expansion coefficient [$1/K$], d is the representative length [m], ν is the kinematic viscosity [m^2/s], λ is the thermal conductivity [W/mK], Nu is Nusselt number, Gr is Grashof number, and Pr is Prandtl number. Nu , Gr , and Pr are dimensionless quantity which used thermodynamics. The kinematic viscosity, the thermal conductivity, and Prandtl number, as the precondition, the fluid which touches the battery is air. This equation is in the case for a cylindrical lithium-ion battery [6]. (16, 17) are changed in square shape case.

The thermal resistance R_{in} [K/W] is the inner thermal resistance of the lithium-ion battery between current collector and surface of the lithium-ion battery. To measure R_{in} , we assembled lithium-ion battery Fig.3.

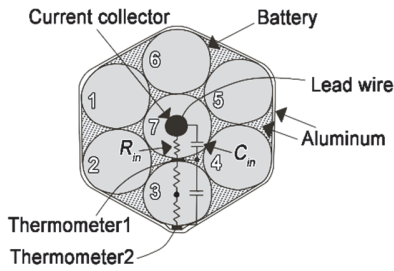


Figure 3 A method of R_{in} measurement of the lithium-ion battery B

We connect a lead wire to the center lithium-ion battery. Next, equivalent lithium-ion batteries are placed as the center lithium-ion battery is surrounded, and then aluminum foils are implanted in gap. Furthermore, aluminum foils are put in as we surround the ambient lithium-ion battery. By creating the thermal network, using only the center lithium-ion battery, eq. (19) is obtained:

$$R_{in} = \frac{T_{surf}(t) - T_{in}(t)}{C_{in}(\partial T_{in}(t)/\partial t) - P_S} \quad (19)$$

From the temperature of the thermometers, T_{in} in Fig. 3 is derived. The SOC of ambient lithium-ion batteries from number 1 to 6 are changed from 0.7 to 0.5, and to 0.3. In each case, we carry out full discharge from SOC = 1 to SOC = 0 at the center lithium-ion battery of number 7, and then we decide the value of R_{in} of the SOC when the SOC of the center cell and ambient cells corresponded. R_{in} is measured from the state of full charge by 1 [C] discharging in various ambient temperatures. Here, 1 [C] is the electric current which discharges all the full charge battery all in 1 hour. Fig.4 is the result of the calibration. We can see the thermal resistance R_{in} of the inner lithium-ion battery depends on SOC and the ambient temperature. In addition, the R_{in} tends to

decrease in all ambient temperatures when SOC is decreasing gradually.

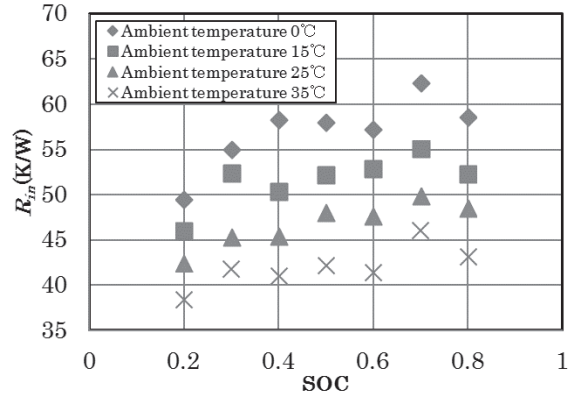


Figure 4 Three external values of R_{in} .

Next, the current collector is covered by aluminum in case of the square shape lithium-ion battery A, R_{in} is obtained by eq. (20):

$$R_{in} = \frac{d}{k_A A} \quad (20)$$

Here, k_A is the heat transfer coefficient of aluminum. We estimate the inner temperature of the lithium-ion battery by using eq. (14).

Then, the heat capacity is represented:

$$C_{in} = \frac{\Delta t * (P_S(t_1 + \Delta t) - P_S(t_1))}{(T_{surf}(t_1 + \Delta t) - T_{surf}(t_1))} \quad (21)$$

The Δt is a small time (s). Based on the definition of the heat capacity, a denominator is the difference of temperature between t_1 and $t_1 + \Delta t$ while a numerator amounts to the heating value between t_1 and $t_1 + \Delta t$.

Fig.5 plots the measured heat capacities. In this calibration, we can assume that the C_{in} calibrated by the surface temperature is equal to that of the inner temperature since the material is same. Dependency between the heat capacity and the ambient temperature is observed.

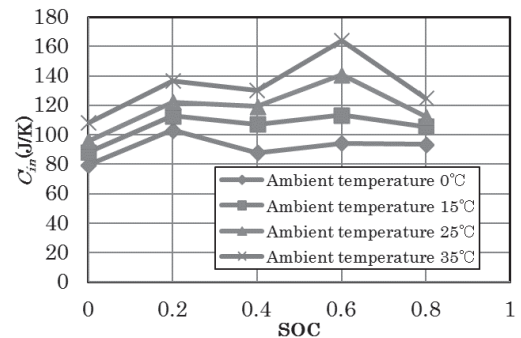


Figure 5 Calibration results of the inner heat capacity of the lithium-ion battery B.

MEASUREMENT OF THE HEAT COMPONENTS

To measure the Joule heat, the lithium-ion battery is fully charged. Then, it is discharged by 1[C]. In conditions of ambient temperatures, 0 [°C], 15 [°C], 25 [°C], and 35 [°C], we measure the OCV and output voltage for various SOC, and calculate the Joule heat by eq. (6).

In the eq. (5) of the entropy heat, I is set to 1[C], and T_{in} is the inner temperature, which is approximated by the surface temperature. Then, $\partial V_{OC}/\partial T_{in}$ is calibrated in the following manner. For each SOC, the ambient temperature is changed from 35 [°C] to 0 [°C], then to 15[°C], 25[°C], and 35[°C], sequentially. OCV is measured at each temperature. The $\partial V_{OC}/\partial T_{in}$ from 35 [°C] to 0 [°C] is calibrated. Then, $\partial V_{OC}/\partial T_{in}$ from 0 [°C] to 15 [°C], that from 15 [°C] to 25 [°C], and that from 25 [°C] to 35 [°C] are calibrated in the same manner. Fig.6 depicts the calibrated values of $\partial V_{OC}/\partial T_{in}$ in different condition of temperature and SOC. From this result, we can observe that the dependency of the $\partial V_{OC}/\partial T_{in}$ to temperature is small. Therefore, we can treat $\partial V_{OC}/\partial T_{in}$ as a value which depends only on SOC.

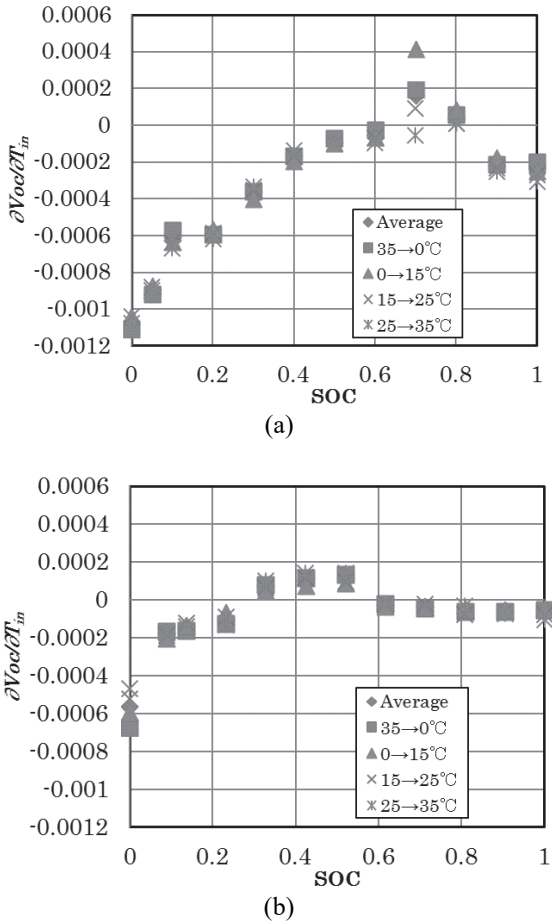


Figure 6 $\partial V_{OC}/\partial T_{in}$ in different SOC and temperature of (a) battery A and (b) battery B.

ESTIMATION RESULTS

Fig.7 shows the results of inner temperature estimation by means of eqs. (14)-(21). Dotted lines are measured values of surface temperature, and solid lines are estimated values of inner temperature. These values are measured, from the state of full charge, by discharging by 1[C], in various ambient temperatures. The inner temperature is obviously higher than surface temperature, in low SOC area. It is remarkable when the ambient temperature is 0 [°C]. To understand these phenomena, Fig.8 shows values of Entropy heat Q_S , Joule heat Q_P , and total heat P_S , in 0 [°C] and 35 [°C]. Not big difference by the difference of the ambient temperatures is observed for the Entropy heats Q_S .

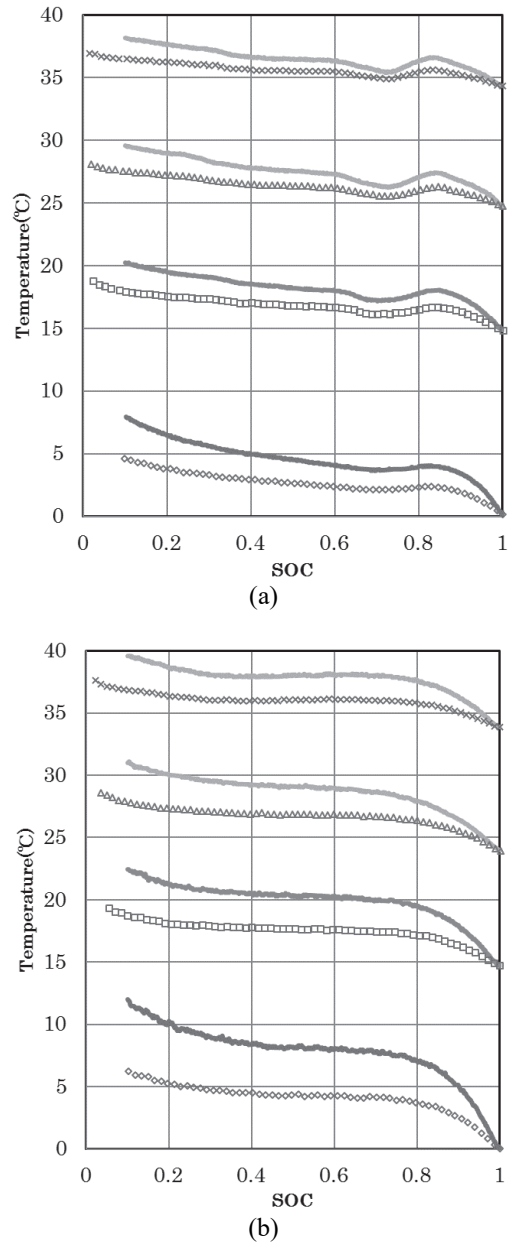


Figure 7 Estimated values of inner temperature of (a) battery A, and (b) battery B.

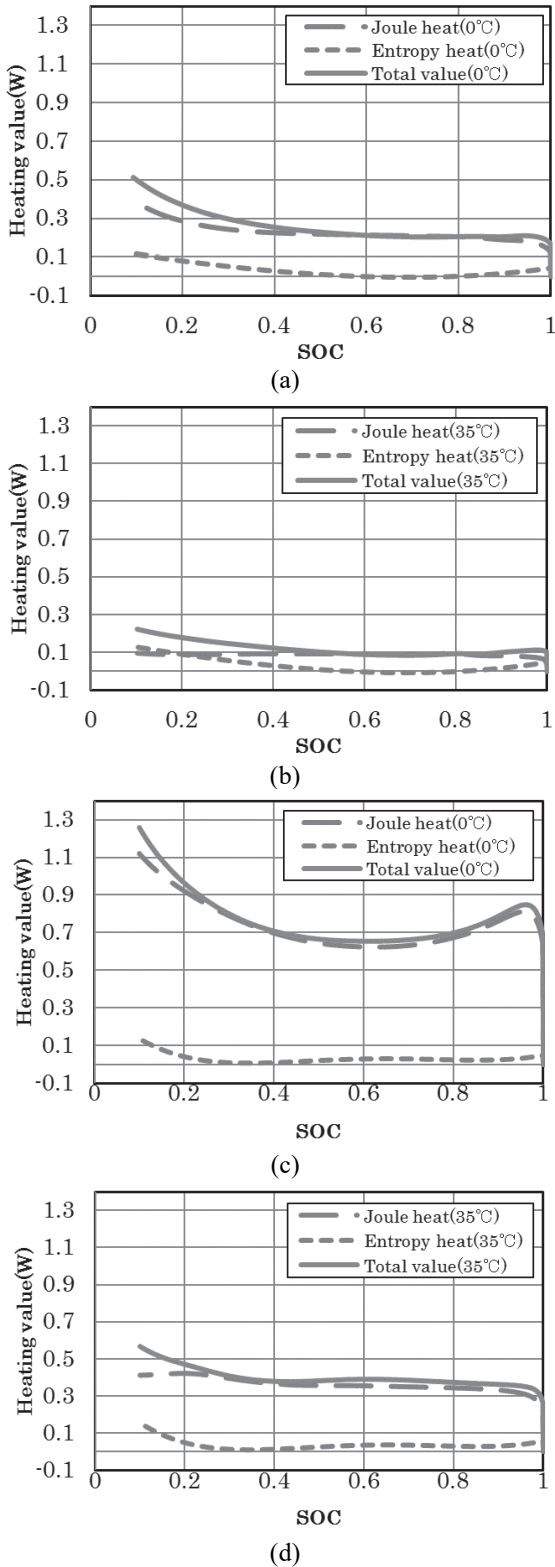


Figure 8 Joule and Entropy heat during discharge, (a) battery A ($T_a = 0^\circ\text{C}$), (b) battery A ($T_a = 35^\circ\text{C}$), (c) battery B ($T_a = 0^\circ\text{C}$), and (d) battery B ($T_a = 35^\circ\text{C}$).

Positive and negative values of the Entropy heat reverse because current becomes the value of positive when the ambient temperature of the lithium-ion battery A is 35°C . Therefore, we are considered that it will occur endothermic reactions when the end of charge nearly. The heat generation increases where the SOC is relatively small in all cases in Fig.8. Joule heat decreases where the ambient temperature is high. It is because the inner resistance decreases and the difference between V_O and V_{OC} also decrease. Fig.9 shows measured values of $V_O - V_{OC}$.

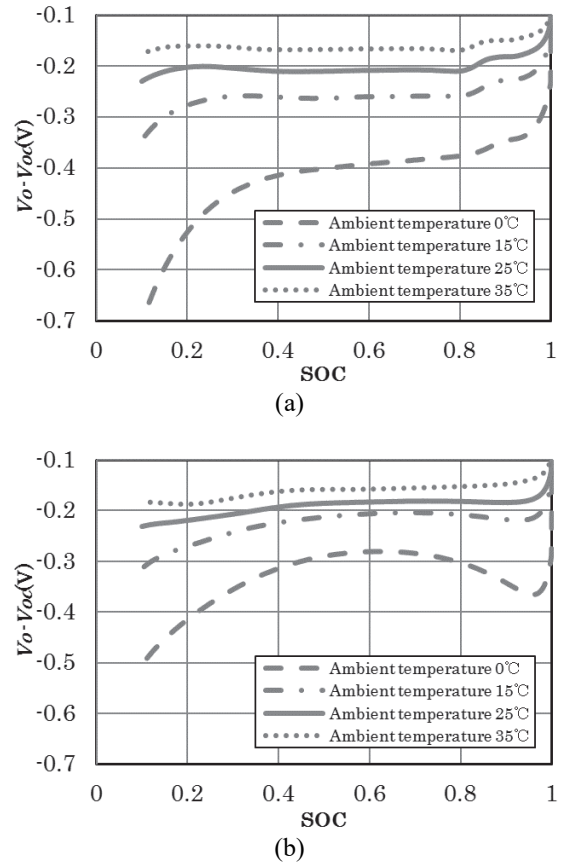


Figure 9 $V_O - V_{OC}$ when discharge of 1C, (a) battery A, (b) battery B.

VERIFICATION BY REAL BATTERIES

We examined a verification for the inner temperature estimation. Fig.10 shows an assembled battery composed of a series of four cells. There are spaces to measure the inner temperature. Around the surface of the assembled battery, it is covered with an aluminum foil. The upper and bottom sides of the assembled battery are covered with a tape so that the inside air does not escape. The assembled battery is treated as a large battery.

We measure the surface temperature and the inner temperature of the assembled battery for 1 [C] discharging, and compare it with the value of the inner temperature estimation.

The experimental results of the surface temperature, the inner temperature, and also the inner temperature estimation of the assembled battery are shown in Fig.11. The error between the estimation value and the measurement value was 0.34 [°C]. The 3[C] discharge was also examined, and the error was 1.3 [°C]. These errors are small enough, thus we could confirm the validity of the inner temperature estimation. According to [7], for a small lithium-ion battery, it is said that the distribution of inner temperature is ignorable because the distribution of inner temperature is 1.6 [°C] for 3[C] discharge. However, the experiment of Fig.11 shows the temperature distribution for 1[C] discharge is 2.7 [°C] or more. Accordingly, it is not ignorable and the proposed inner temperature estimation is effective for appropriate battery temperature control.

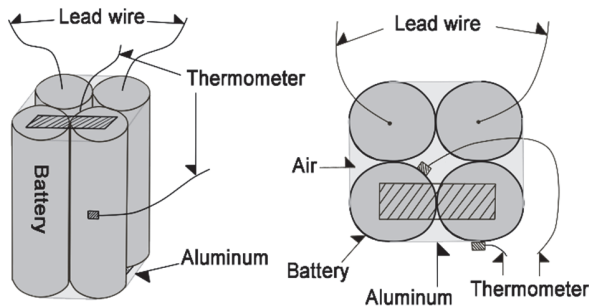


Figure 10 Assembled battery for verification.

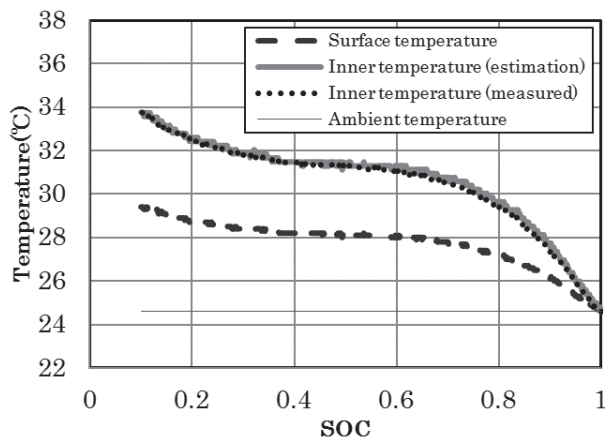


Figure 11 Measured values of inner and surface temperatures and estimated values of inner temperature. ($T_a=25^{\circ}\text{C}$)

CONCLUSION

We have proposed an accurate inner temperature estimation model for lithium-ion battery by analytical analysis of thermodynamics. Transfer function of the natural convection and the heat process are considered, and it can be used for practical control of lithium-ion battery temperature. It is shown that the inner temperature is obviously higher than surface temperature, in low SOC area. It is remarkable when the ambient temperature is low. We confirmed that the estimation formula is high accurate by experiment using the assembled battery.

REFERENCES

- [1] S.X. Chen, H.B. Gooi, N.Xia, and M.Q. Wang, "Modelling of lithium-ion battery for online energy management systems," *IET Electrical Systems in Transportation*, vol. 2, pp. 202-210, Dec. 2012.
- [2] Y. Chen, L. Song, and J.W.Evans, "Modeling studies on battery thermal behaviour, thermal runaway, thermal management, and energy efficiency," in *Proc. IECEC*, vol. 2, pp. 1465-1470, Aug. 1996.
- [3] H. Vaidyanathan and G. Rao, "Electrical and thermal characteristics of lithium-ion cells," in *Proc. IBCAA*, pp. 79-84, 1999.
- [4] M. Mousavi, S. Hoque, S. Rahnamayan, I. Dincer, and G.F. Naterer, "Optimal design of an air-cooling system for a lithium-ion battery pack in electric vehicles with a genetic algorithm," in *Proc. CEC*, pp. 1848-1855, Jun. 2011.
- [5] T. Huria, M.Ceraolo, J. Gazzarri, and R. Jackey, "High fidelity electrical model with thermal dependence for characterization and simulation of high power lithium battery cells," in *Proc. IIEVC*, pp. 1-8, Mar. 2012.