OPTIMAL PROCESS DURATIONS OF EXPANSION OF A HEATED GAS WITH LINEAR PHENOMENOLOGICAL TRANSFER LAW

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ABSTRACT

An optimal motion of a piston is examined. The piston is fitted inside one cylinder, which contains ideal gas. The gas contacts a heat bath and is heated with given rate $f(t)$ in finite time. The heat transfer between heat bath and gas obeys the linear phenomenological heat transfer law (LPHTL) $\dot{q} \propto \Delta(T^{-1})$. Optimal expansion problem of the heated gas is solved step by step. There exist an OMP for any given final time, an optimal process duration of expansion (OPDE) for maximum power output (MPO), and an OPDE for maximum work output (MWO) for fixed power output. Both the OPDE for MPO and the OPDE for MWO with fixed power output is obtained. The best performance of the system is obtained using finite time thermodynamics. Numerical calculations are performed. The results show that the leaking energy and the energy from the heat source change into work output of the expansion process. Both the MPO and MWO with fixed power increase with the increase of heat conductance K. The results obtained can provide some guidelines for real systems operating with LPHTL.

INTRODUCTION

In 1975, Curzon and Ahlborn [1] obtained efficiency of a Carnot engine at MPO. Since then, finite time thermodynamics has had great progresses [2-19]. Specially, References [20-25] applied finite time thermodynamics to study the optimal motion of a piston. The piston is fitted inside one cylinder, which contains ideal gas. The gas contacts a heat bath with temperature T_{ex}, and is heated with given rate $f(t)$ in finite time. The heat transfer between the bath and gas obeys Newtonian heat transfer law, $\dot{q} \propto \Delta T$. The configuration for MWO was obtained [20, 21], the MPO optimization was performed [23, 24]. The optimal configuration was applied in the performance optimizations for internal [25] and external [22] combustion engines. Those results were strongly related to the law of the heat transfer process. One should study the effects of heat transfer law [26-29]. Reference [30] investigated the MWO configuration for the same system with LPHTL $\dot{q} \propto \Delta(T^{-1})$ instead of Newtonian one. References [31, 32] carried out the similar work further using generalized radiative heat transfer law $\dot{q} \propto \Delta(T^{n})$ [31] and convective-radiation law [32]. Based on Ref. [31], References [33, 34] performed performance optimizations for internal [33] and external [34] combustion engines. This paper will make a further step based on Ref. [30], just as Refs. [23, 24] performed based on Refs. [20, 21]. The optimal expansion problem of the heated gas, which is inside a cylinder and with a piston and with LPHTL will be solved step by step. There will be an OMP for any given final time, an OPDE for MPO, and an OPDE for MWO for fixed power output. Both the OPDE for MPO and the OPDE for MWO with fixed power output will be obtained. The device performance will be determined. Numerical calculations will be performed.

NOMENCLATURE

C_V	[J/kg·K]	Constant volume specific heat
$\dot{E}$	[J/s]	Change rate of internal energy
f	[J/s]	Heated rate
K	[JK/s]	Heat conductance
P	[W]	Given power output
R	[J/(mol·K)]	Ideal gas constant
T	[K]	Gas Temperature
t	[s]	Time
V	[l]	Gas volume
$\dot{W}$	[J/s]	The power output of expansion

Special characters

τ_m	[s]	The optimal final time

Subscripts

ex	External
m	Final point
max	Maximum
0	Origin point

Abbreviations

LPHTL	Linear phenomenological heat transfer law
OMP	
OPDE	Optimal process duration of expansion
MPO	Maximum power output
MWO	Maximum work output

Conductive cooling of heat-generating volumes has been approached by other researchers as a volume- or area-to-point heat transfer problem [5]. Thermal tree theories have been developed to describe the distribution of low thermal resistant paths and heat transfer has been optimised for different thermal tree structures [6-8].

Even though thermal tree schemes present optimised heat transfer performance, it requires complex geometric layouts which at small dimensional scales can lead to high manufacturing costs. In passive power electronic modules, which typically have inductive, capacitive and transformative functions, restrictions imposed by the electromagnetic fields, dictates that only parallel-running internal embedded solid geometries can be considered. Such layouts, when placed in-line with magnetic field lines reduces the interference a cooling insert may have on magnetic and electric field distribution. Three-dimensional thermal path networks are thus not suitable for such applications

In a previous investigation [9], the thermal performance of a grid of discrete parallel-running rectangular solid inserts were studied and geometrically optimised in terms of fixed volume use. At the dimensional scale of interest in power electronics and electronics cooling, it was found that the geometric shape of embedded cooling inserts has a diminishing influence on thermal performance and that the fraction of volume occupied by the cooling system plays a much more dominant role [10]. With this in mind it may be appreciated that from an economic and manufacturing point of view, continuous cooling layers provides a more practical embedded conductive cooling configuration. This paper focuses on thermal characterisation of cooling layers and aims to provide some information on thermal cooling performance.

PHYSICAL MODEL

Considering the problem of MWO from a heated ideal gas, which is inside a cylinder, and the cylinder is with moveable piston. In the cylinder, there is 1*mol* ideal gas. The gas is heated uniformly by heating rate of $f(t)$ from a heat source. The heating rate is an arbitrary given function with time. The gas contacts an external heat bath, whose temperature is a constant, T_{ex}. The problem is to control the motion of the piston in order to obtain the MWO or MPO from this system with a fixed time interval, see Fig. 1. This model is the same as that adopted in Ref. [24], except the heat transfer between gas and heat bath. Reference [24] applied Newtonian law, while LPHTL $\dot{q} \propto \Delta(T^{-1})$ [30, 33-34] is applied herein, where $\dot{q}$ presents heat flux across cylinder wall with heat conductance K, T presents gas temperature, and T_{ex} presents bath temperature. K is taken to be a constant herein, for simplicity, as did in Refs. [20-25, 30-34]. Besides, the inertia of gas and piston is neglected, and the friction effect associated with piston movement is also neglected. In fact, the friction affects the optimization result [28]. However, for simplicity, it is neglected in this paper, as did in Refs. [20-25, 30-34].

The first law of thermodynamics gives

$$\begin{aligned}\dot{E}(t) &= f(t) - \dot{W}(t) - \dot{q}(t) \\ &= f(t) - \dot{W}(t) - K[T_{ex}{}^{-1} - T^{-1}(t)]\end{aligned} \tag{1}$$

where $\dot{E}$ is the change rate of internal energy, and $\dot{W}(t)$ is the instantaneous power output of expansion. The goal is to maximize the average power P over time t_m:

$$P(t_m) = W(t_m)/t_m \tag{2}$$

with given $f(t)$, initial volume (V_0), final volume (V_m), and initial internal energy (E_0) of the gas, where $W(t_m)$ is the total work output over time t_m.

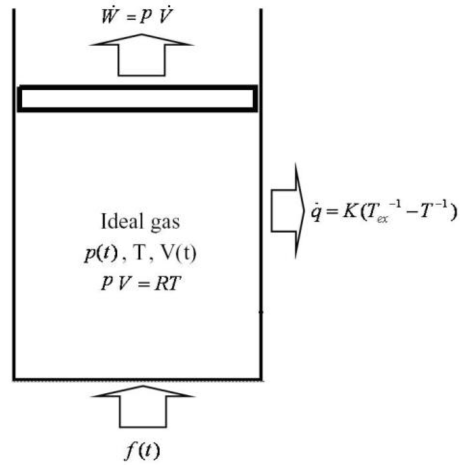

Figure 1 Schematic diagram of the model: a mole of ideal gas inside a cylinder is pumped by a given heating function $f(t)$ and is coupled to heat bath by the linear phenomenological heat transfer law

OPTIMAL MOTION OF THE PISTON FOR ANY GIVEN FINAL TIME

The problem is to optimize motion of the piston, which will be a function of time. This means that one must optimize the system operating time t_m subject to the constraint, Eq.(1). However, one can note that for any given final time t_m (also for the optimal final time τ_m) the maximum of $P(t_m)$ is provided by maximizing $W(t_m)$. The problem of MWO for given t_m was solved by Chen et al [30]. It is the must only task to determine the optimal final time τ_m.

The results of Refs. [30, 31] showed that the optimal configuration for this problem consists of three steps: the first is an initial instantaneous adiabatic process, the second is an intermediate Euler-Lagrange (E-L) arc, and the third is a final instantaneous adiabatic process.

Step (1) is the initial adiabatic process from $V(0)$ to $V'(0)$ at $t = 0$. Its equation is

$$E'(0) = E(0)[V'(0)/V(0)]^{-R/C_V} \tag{3}$$

where $E^{'}(0)$ is the initial internal energy of the optimal process, but not the initial internal energy ($E(0)$) of the whole optimal expansion process.

Step (2) is the E-L arc and proceeds from the initial $V^{'}(0)$ and $E^{'}(0)$ at $t=0$ until time $t=t_m$:

$$E(t)=\frac{2KC_V E^{'}(0)}{2KC_V - E^{'}(0)[F(t)-F(0)]} \quad (4)$$

$$V(t)=V^{'}(0)\{1-\frac{E^{'}(0)}{2KC_V}[F(t)-F(0)]\}^{C_V/R} \exp(\frac{[2KC_V+E^{'}(0)F(0)]^2}{4KRE^{'2}(0)}t-\frac{1}{4KR}\int_0^t F^2(t)dt) \quad (5)$$

and $E^{'}(0)$ and $V^{'}(0)$ satisfy following equations:

$$[2KC_V+E^{'}(0)F(0)]\exp(\frac{[2KC_V+E^{'}(0)F(0)]^2}{4KC_V[E^{'}(0)]^2}t_m - \frac{1}{4KC_V}\int_0^{t_m}F^2(t)dt)=\frac{E^{'}(0)KC_V}{E(0)}(\frac{V_m}{V(0)})^{R/C_V} \quad (6)$$

$$\frac{[2KC_V+(\frac{V^{'}(0)}{V(0)})^{-R/C_V}E(0)F(0)]}{(\frac{V^{'}(0)}{V(0)})^{-R/C_V}E(0)} \exp\left(\frac{[2KC_V+(\frac{V^{'}(0)}{V(0)})^{-R/C_V}E(0)F(0)]^2}{4KC_V E^2(0)(\frac{V^{'}(0)}{V(0)})^{-2R/C_V}}t_m\right) =\frac{KC_V}{E(0)}(\frac{V_m}{V(0)})^{R/C_V}\exp\left(\frac{1}{4KC_V}\int_0^{t_m}F(t)dt\right) \quad (7)$$

where $F(t)=f(t)-KT_{ex}^{-1}$.

Step (3) is the final adiabatic process from $V(t_m)$ to V_m at $t=t_m$

$$E_m=E(t_m)[V_m/V(t_m)]^{-R/C_V} \quad (8)$$

where $V(t_m)$ and $E(t_m)$ can be obtained from Eqs. (6) and (7) at time t_m.

OPDE FOR MPO

The work performed by the gas in this motion is given by

$$W(t_m)=\frac{1}{2}\int_0^{t_m}F(t)dt+E(0)-E_m+[\frac{KC_V}{(\frac{V^{'}(0)}{V(0)})^{-R/C_V}E(0)}+\frac{F(0)}{2}] \quad (9)$$

This is the expression for the maximum work for any t_m. One needs only find the extremum with respect to t_m of

$$P(t_m)=\frac{W(t_m)}{t_m}=\frac{1}{2t_m}\int_0^{t_m}F(t)dt+\frac{E(0)-E_m}{t_m}+\frac{1}{t_m}[\frac{KC_V}{(\frac{V^{'}(0)}{V(0)})^{-R/C_V}E(0)}+\frac{F(0)}{2}] \quad (10)$$

where

$$E_m=(\frac{V_m}{V(0)})^{-R/C_V}E(0)\exp\left(\frac{[2KC_V+(\frac{V^{'}(0)}{V(0)})^{-R/C_V}E(0)F(0)]^2}{4KC_V E^2(0)(\frac{V^{'}(0)}{V(0)})^{-2R/C_V}}t_m-\frac{1}{4KC_V}\int_0^{t_m}F^2(t)dt\right) \quad (11)$$

Applying the condition $\partial P(t_m)/\partial t_m=0$ yields the equation that the OPDE (τ_m) should satisfy

$$\begin{aligned}&\frac{1}{2}F(\tau_m)+(\frac{V_m}{V(0)})^{-R/C_V}\frac{E(0)}{\tau_m}\exp\left(\frac{[2KC_V+(\frac{V^{'}(0)}{V(0)})^{-R/C_V}E(0)F(0)]^2}{4KC_V E^2(0)(\frac{V^{'}(0)}{V(0)})^{-2R/C_V}}\tau_m-\frac{1}{4KC_V}\int_0^{\tau_m}F^2(t)dt\right)\\&=\frac{E(0)}{\tau_m}+(\frac{V_m}{V(0)})^{-R/C_V}E(0)\exp\left(\frac{[2KC_V+(\frac{V^{'}(0)}{V(0)})^{-R/C_V}E(0)F(0)]^2}{4KC_V E^2(0)(\frac{V^{'}(0)}{V(0)})^{-2R/C_V}}\tau_m-\frac{1}{4KC_V}\int_0^{\tau_m}F^2(t)dt\right)\\&\times\left(\frac{[2KC_V+(\frac{V^{'}(0)}{V(0)})^{-R/C_V}E(0)F(0)]^2}{4KC_V E^2(0)(\frac{V^{'}(0)}{V(0)})^{-2R/C_V}}\tau_m-\frac{1}{4KC_V}\int_0^{\tau_m}F^2(t)dt\right)\\&+\frac{1}{2\tau_m}\int_0^{\tau_m}F(t)dt\end{aligned} \quad (12)$$

The system of Eqs. (7) and (12) determines the OPDE τ_m, and the MPO ($P_{\max}$) is determined by Eq. (10).

OPDE FOR MWO FOR ANY GIVEN POWER OUTPUT

To find the MWO ($W_{\max}$) for any given power output P ($P \le P(\tau_m)=P_{\max}$), one should solve Eqs. (7) and (10) simultaneously, obtain some optimal roots, and find the largest one, $\tau^{\max}$, of the system. This largest root is just the OPDE for MWO for any given power output. Therefore, the corresponding MWO is

$$W_{\max}=P\tau^{\max} \quad (13)$$

If the power output (P) is the maxima ($P_{\max}$) obtained in section 2.3, the OPDE for MWO for any given power output is the OPDE for MPO, and the obtained work output is the double-MWO.

NUMERICAL EXAMPLES AND DISCUSSIONS

Numerical examples for the OPDE of a heated gas are performed. In the numerical calculations, it is set that $V_m = 8l$, $E(0) = 3780J$, $T_{ex} = 300K$, $C_V = 3R/2$ and $f(t) = Ate^{-t/B}$, where $A = 4200J/s^2$ and $B = 1\sec$. In order to analyze the effect of heat transfer law on the optimal expansion process, heat conductance K is set as a variable parameter.

Figure 2 shows OPDE τ_m versus heat conductance K. Figure 3 shows the corresponding MPO $P_{\max}$ versus heat conductance K.

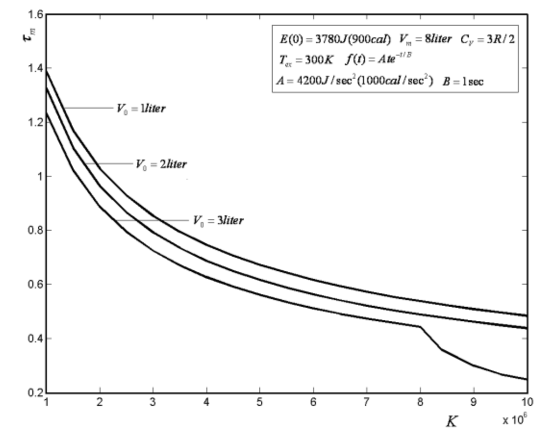

Figure 2 Optimal process duration of expansion τ_m vs. heat conductance K

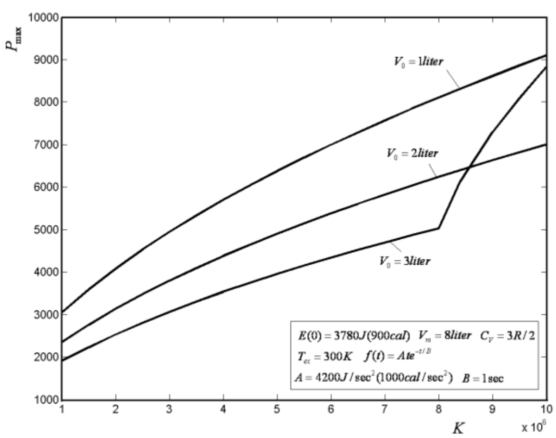

Figure 3 Maximum power output $P_{\max}$ vs. heat conductance K

Figure 4 shows OPDE $\tau^{\max}$ versus heat conductance K with fixed power output $P = 5000W$. Figure 5 shows the corresponding MWO $W_{\max}$ versus heat conductance K with fixed power output $P = 5000W$.

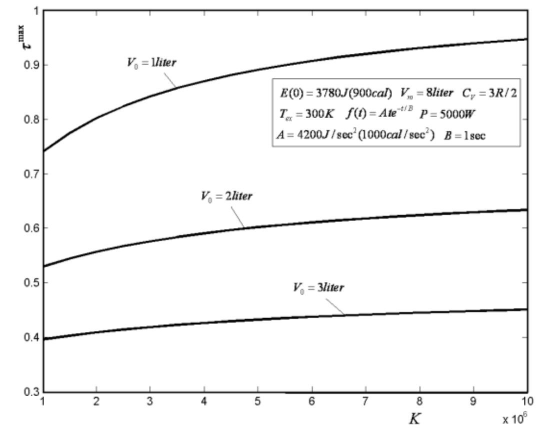

Figure 4 Optimal process duration of expansion $\tau^{\max}$ vs. heat conductance K with fixed power output $P = 5000W$

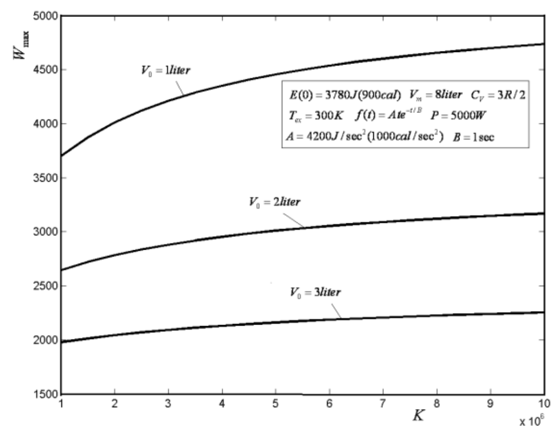

Figure 5 Maximum work output $W_{\max}$ vs. heat conductance K with fixed power output $P = 5000W$

From the figures one can see clearly that the potential of each parameter versus initial volume $V(0)$ of the gas and heat conductance K. With the increase of initial volume $V(0)$ of the gas, because of decrease of volume for expansion, the OPDEs τ_m and $\tau^{\max}$, and both the MPO and the MWO with fixed power decreases. Because the whole *E-L* arc begins at the temperature below that of heat bath ($T_{ex} = 300K$), the initial adiabatic expansion, step 1, cools the gas. The result is leading the energy leaks from the bath into the gas [20, 21]. Both this leaking energy and the energy from the heat source change into work output of the expansion process. Both the MPO and MWO with fixed power increase with the increase of heat conductance K.

CONCLUSION

The OPDE of a heated working gas inside a cylinder with a moveable piston and LPHTL is investigated. The OPDE for MPO and the OPDE for MWO for any fixed power output are determined. The results show that the optimal expansion problem can be solved step by step, and there exist an OMP for any given final time, an OPDE for MPO, and an OPDE for MWO for any fixed power output. The best performance is obtained using finite time thermodynamics. Numerical calculations are performed. The results show that the leaking energy and the energy from the heat source change into work output of the expansion process. Both the MPO and MWO with fixed power increase with the increase of heat conductance K. The results obtained can provide some guidelines for real systems operating with LPHTL.

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