ENTROPY GENERATION ANALYSIS OF A GAS COOLER FOR TRANSCRITICAL CO_2 SYSTEMS

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ABSTRACT

The performance of a transcritical CO_2 based refrigeration system is highly sensitive to the gas cooler pressure and exit temperature. This is due to the peculiar s-shape of the isotherm in the supercritical region. Hence proper design of the gas cooler is crucial to extract optimum performance from the CO_2 based systems. The gas cooler considered for the current application is an air cooled, counter cross-flow spiral fin-and-tube heat exchanger. A discretized node-by-node approach is adopted here to model the gas cooler. The governing equations for each node are formulated based on the NTU-ε method. Using the developed model, a study has been conducted to optimize the geometrical dimensions of the heat exchanger based on the minimization of the total entropy generation due to fluid friction and heat transfer. The total entropy generation is indicated in terms of a non-dimensional entropy generation number. The variations of this non-dimensional number with changes in face velocity, fin pitch, and total number of tubes in each row are investigated in the present study. It is observed that entropy generation number decreases with the increase in fin pitch for higher flow rate. However, at a lower flow rate, entropy generation number lines for different fin pitches overlap. On the other hand, entropy generation number decreases with increase in the total number of tubes. Finally, from the results, the mentioned parameters are optimized to achieve lower entropy generation number with lower fan power consumption.

INTRODUCTION

With the growing interest in CO_2 as a refrigerant, research efforts on CO_2 based refrigeration systems are increasing exponentially over the past few years. The primary goal of these efforts is to make CO_2 based systems competitive in today's market. It is well known that as the critical temperature of CO_2 is low (31.1°C), for hot climatic conditions, the heat rejection process has to be carried out under supercritical conditions for CO_2 based air conditioning systems, while the heat extraction is subcritical. Lorentzen and Pettersen [1] first demonstrated a transcritical CO_2 based mobile air conditioner where heat rejection process is carried out at supercritical pressure. In the supercritical zone, pressure becomes independent of temperature. Furthermore, the isotherm characteristic is unique in the supercritical zone. Therefore, performance of the system becomes very sensitive to the exit pressure and temperature of gas cooler. Hence, prediction of the optimum dimensions for design of the gas cooler is very critical. The common procedure to predict optimum configurations for heat exchanger is formulated based on the second law of thermodynamics. For a shell-and-tube heat exchanger, Bejan [2] first showed that the total irreversibility due to heat transfer and fluid friction can be expressed in the form of a non-dimensional number which he termed as entropy generation number. Based on the minimization of this non-dimensional number, he proposed a heat exchanger design method. This new technique of heat exchanger design has become popular in the recent times as it relates the total irreversibility with the dimensions of heat exchanger. Saechan et al. [3] predicted optimal configuration for a fin-and-tube condenser used in an R134a system based on the minimization of entropy generation number. Similarly, number of researches [4-7] has been carried out to design heat exchanges by applying this procedure.

This study presents an entropy generation analysis of a gas cooler for a transcritical CO_2 system. Here the total number of tubes in each row, and fin pitch are changed and their effect on total entropy generation rate is analyzed for different face velocity. From the results, these two geometrical parameters are optimized to achieve the lower entropy generation number with lower fan power consumption.

NOMENCLATURE

A_s	[m^2]	Unfinned base surface area
A_f	[m^2]	Fin surface area
A_o	[m^2]	Total surface area
A_{min}	[m^2]	Minimum free flow area
a_{node}	[m^2]	External surface area of elementary node
a_i	[m^2]	Inner surface area of elementary node
C	[W/K]	Heat capacity rate
C_p	[J/kg.K]	Specific heat
$d\dot{Q}$	[W]	Heat transfer rate for elementary node
dL	[m]	Length of elementary node
d_i	[m]	Inner diameter of tube
f	[-]	Fanning friction factor
G	[kg/m^2.sec]	Mass flux

h	[J/kg]	Specific enthalpy
$\dot{m}$	[kg/sec]	Mass flow rate
N_s	[-]	Entropy generation number
P	[Pa]	Pressure
$\dot{S}_{gen}$	[J/K]	Entropy generation rate
T	[K]	Temperature
U	[W/m^2.K]	Universal heat transfer coefficient

Special characters

α	[W/m^2.K]	Heat transfer coefficient
ε	[-]	Effectiveness of heat exchanger
ρ	[kg/m^3]	Density

Subscripts

a	Air
ex	Exit
in	Inlet
min	Minimum
max	Maximum
ref	Refrigerant

Table 1: *Surface geometrical characteristics for fin-and-tube heat exchanger*

Unfinned base surface area [8]

$$A_s = n_t N_{row}\left(\pi d_o L - \left(\sqrt{f_p^2 + (\pi d_o)^2}\right) \times f_t\left(\frac{L}{f_p}\right)\right) \quad (1)$$

Fin surface area [8]

$$A_f = n_t N_{row}\left(\frac{L}{f_p}\right) \times \left(\frac{1}{2}\pi\left(d_f^2 - d_o^2\right) + \pi d_f f_t\right) \quad (2)$$

Total surface area

$$A_o = A_s + A_f \quad (3)$$

Minimum free flow area for staggered arrangement of tubes [9]

$$A_{\min} = \left[(n_t - 1)c' + (P_t - d_o) - (d_f - d_o) f_t\left(\frac{1}{f_p}\right)\right] \times L \quad (4a)$$

$$\text{where, } c' = \begin{cases} 2a' & if \quad 2a' < 2b' \\ 2b' & if \quad 2b' < 2a' \end{cases} \quad (4b)$$

$$2a' = (P_t - d_o) - (d_f - d_o) f_t\left(\frac{1}{f_p}\right) \quad (4c)$$

$$b' = \left(\left(\left(\frac{P_t}{2}\right)^2 + P_l^2\right)^{1/2} - d_o\right) - (d_f - d_o) f_t\left(\frac{1}{f_p}\right) \quad (4d)$$

MATHEMATICAL MODEL

Heat exchanger with spiral fins has been considered for the gas cooler. For this heat exchanger, conduction between adjacent tubes through fins is minimized as there is no direct contact between fins of adjacent tubes. Also it is easier to fabricate this heat exchanger with spiral fin and tube configuration.

Modelling Procedure

To design the heat exchangers, a discretized approach has been adopted. The procedures adopted to calculate the surface geometrical characteristics are shown in table 1. The procedures and assumptions considered while modelling the heat exchangers are:

(a) The whole heat exchanger is divided into three dimensional array of nodes.

(b) In the three dimensional configuration, nodes are arranged by using i, j and k co-ordinates. ‘k’ represents the number of rows of tube; ‘j’ represents number of tubes in each row; while ‘i’ represents number of nodes in each tube.

(c) Each node is considered as an independent heat exchanger and the transfer of heat among the nodes is neglected. Figure 1 shows isometric view of an elementary node of gas cooler.

(d) Conduction of heat along the length of tube and fin is neglected.

(e) All the air leaving each node is assumed to enter immediately the node which is in the downstream.

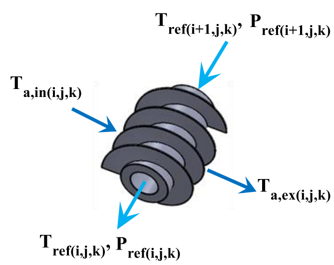

Figure 1 Isometric view of an elementary node of the gas cooler

Governing Equations

As counter cross flow arrangement is considered for the gas cooler, calculation is carried out by an iterative procedure. Calculation is first started from refrigerant exit based on some guessed value of pressure and temperature at the exit. Marching is then started node by node from refrigerant exit to refrigerant inlet. At the end of this march, pressure and temperature are obtained at refrigerant inlet. These pressure and temperature at refrigerant inlet are then matched with actual condition by iterative procedure.

An elementary node of the gas cooler is shown in figure 1. The NTU-ε method [10] is used to calculate the heat transfer rate for each elementary node. The governing equations are expressed in the following form:

Air side energy balance equation is:

$$d\dot{Q}(i,j,k) = \varepsilon \times C_{\min} \times \left(T_{ref}(i,j,k) - T_{a,in}(i,j,k)\right) = \dot{m}_a(i,j,k) \times Cp_a \times \left(T_{a,ex}(i,j,k) - T_{a,in}(i,j,k)\right) \quad (5)$$

Effectiveness is calculated from:

$$\varepsilon = 1 - \exp\left(-\gamma \frac{C_{\max}}{C_{\min}}\right) \qquad \gamma = 1 - \exp\left(-\frac{UA}{C_{\max}}\right) \quad (6)$$

for C_{max}=C_{ref}

$$\varepsilon = \frac{C_{\max}}{C_{\min}}\left(1 - \exp\left(-\gamma \frac{C_{\min}}{C_{\max}}\right)\right) \qquad \gamma = 1 - \exp\left(-\frac{UA}{C_{\min}}\right) \quad (7)$$

for C_{min}=C_{ref}

where, $$\frac{1}{UA} = \frac{1}{\alpha_a(i,j,k) \times \eta_{fin,overall} \times a_{node}} + \frac{1}{\alpha_{ref}(i,j,k) \times a_i} \quad (8)$$

Refrigerant side conservations equations are:

$$d\dot{Q}(i,j,k) = \dot{m}_{ref}(i,j,k) \times \left(h_{ref}(i+1,j,k) - h_{ref}(i,j,k)\right) \quad (9)$$

$$P_{ref}(i+1,j,k) - P_{ref}(i,j,k) = \Delta P_{fric} = f \frac{G_{ref}^2 \times dL}{2 \times \rho_{ref} \times d_i} \quad (10)$$

The heat transfer coefficient for supercritical in-tube cooling of carbon dioxide is calculated using the correlation proposed by Pitla et al. [11]. Air side heat transfer coefficient and pressure drop are calculated using the correlation proposed by Pongsoi et al. [12]. Fin efficiency is calculated using the method proposed by Gardner [12].

Table 2: *Input parameters supplied during simulation*

Parameter	Value
Discharge pressure (bar)	100
Discharge temperature (°C)	90
Mass flow rate of refrigerant (gm/sec)	22.4
Ambient temperature (°C)	35
Face velocity (m/sec)	1-8

Table 3: *Geometrical dimensions of the gas cooler*

Tube		Fin	
Parameter	Value	Parameter	Value
Material	Copper	Material	Copper
Inner diameter	5.5 mm	Outer diameter	26 mm
Outer diameter	9.5 mm	Fin thickness	0.19 mm
Longitudinal tube pitch (P_l)	30 mm	Fin pitch (f_p)	2-9 mm
Transverse tube pitch (P_t)	27 mm		
Finning length of tube	480 mm		
Total number of tubes per row	12-22		
Number of tube rows	3		

Based on the 2nd law of thermodynamics, the total entropy generation rate is calculated from [3]:

$$\dot{S}_{gen} = \dot{S}_{gen,a} + \dot{S}_{gen,ref} \quad (11)$$

where, $$\dot{S}_{gen,a} = \sum \left(\dot{S}_{gen,\Delta T} + \dot{S}_{gen,\Delta P}\right)_{i,j,k} \quad (12)$$

$$\dot{S}_{gen,ref} = \sum \left(\dot{S}_{gen,\Delta T} + \dot{S}_{gen,\Delta P}\right)_{i,j,k} \quad (13)$$

The non-dimensional entropy generation number is then calculated from:

$$N_S = \frac{\dot{S}_{gen}}{\left(\dot{m}C_p\right)_{\min}} = \frac{\dot{S}_{gen}}{\left(\dot{m}C_p\right)_a} \quad (14)$$

RESULTS AND DISCUSSION

Based on the descriptions presented in the earlier section, the steady-state model for the gas cooler is developed on the MATLAB platform. To evaluate the thermodynamic properties of CO_2, REFPROP 9 is integrated with the MATLAB code. Input parameters considered during simulation are mentioned in table 2, while the geometrical dimensions of the gas cooler are presented in table 3. Using the developed model, simulations are carried out to investigate the variations in entropy generation number with changes in the total number of tubes, fin pitch, and face velocity. The results obtained are presented in this section.

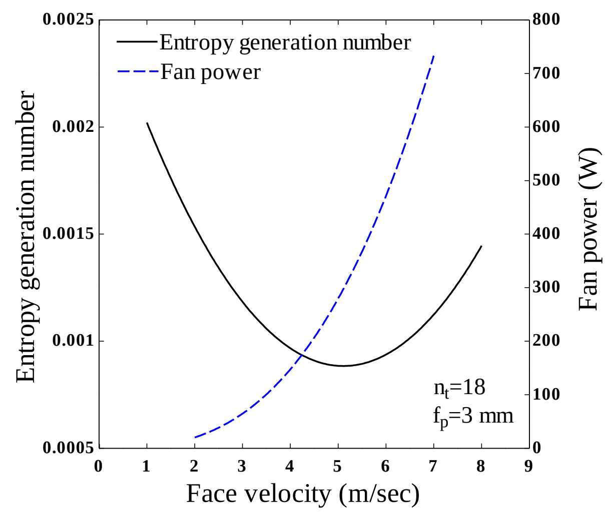

Figure 2 Variation in entropy generation number and fan power consumption with changes in face velocity

Effect of Face Velocity

As the face velocity increases, the entropy generation number first decreases, reaches a minimum and then increases (figure 2). Entropy generation rate due to heat transfer as well as fluid friction increases with the increase in the face velocity. At the lower face velocity, the rate of increase in entropy generation rate is lower than the rate of increase in air flow rate. As a result, entropy generation number first decreases.

However, at higher face velocity, the rate of increase in entropy generation rate becomes higher than the rate of increase in air flow rate due to the rapid increase in the air side pressure drop. This results in higher entropy generation number at higher flow rate. Also fan power consumption increases exponentially with the increase in face velocity (figure 2). Hence, the face velocity in the range of 3-4 m/sec is found suitable for the current application since it gives the lower entropy generation number with lower fan power consumption.

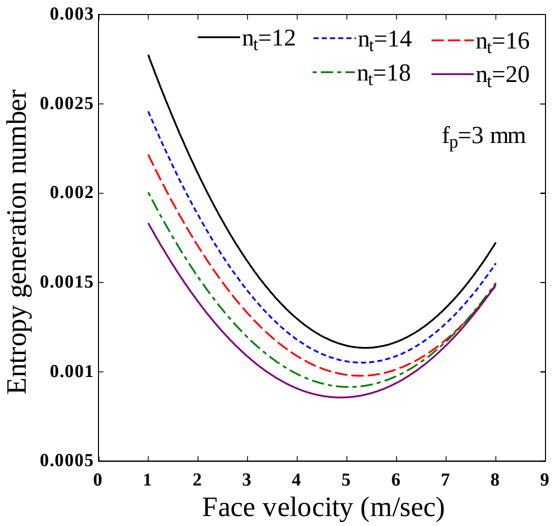

Figure 3 Variation in entropy generation number with changes in total number of tubes in each row

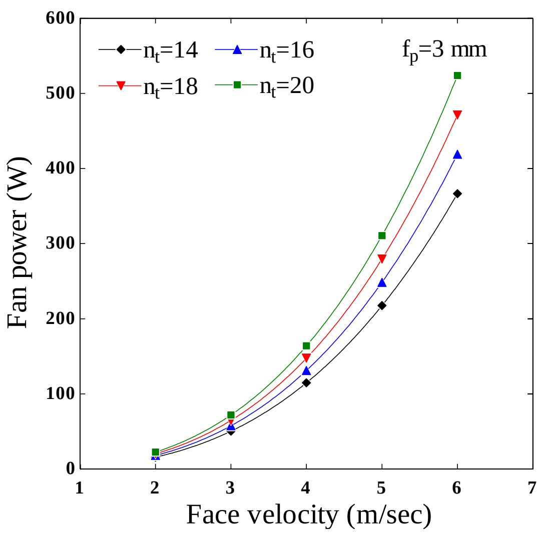

Figure 4 Variation in fan power consumption with changes in total number of tubes

Effect of Total Number of Tubes per Row

An increase in the total number of tubes in each row increases the surface area as well as face area leading to higher heat rejection rate as well as pressure drop. As a result, entropy generation rate increases. However air flow rate also increases with the increase in the total number of tubes in each row. Since the rate of increase in air flow rate is higher than the rate of increase in entropy generation rate, entropy generation number decreases with the increase in total number of tubes in each row (figure 3). On the other hand, an increase in the total number of tubes results in higher air flow rate that leads to higher pressure drop and higher fan power consumption. Figure 4 shows the fan power consumption with changes in total number of tubes in each row. Hence it can be concluded that too low an entropy generation number results in very high fan power consumption. Therefore, compromise has to be made while deciding the total number of tubes in each row. The total number of tubes in each row should be within 18-20 for the current application to achieve lower entropy generation number with lower fan power consumption.

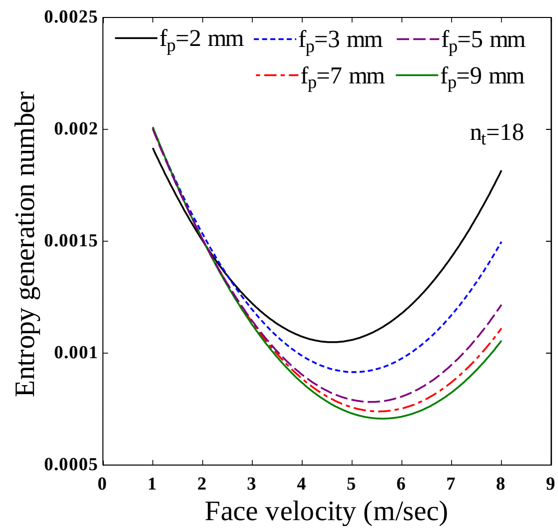

Figure 5 Variation in entropy generation number with changes in fin pitch

Effect of Fin Pitch

A decrease in the fin pitch results in higher external surface area and hence better heat exchange rate. As the external surface area increases, entropy generation rate due to heat transfer decreases marginally; while entropy generation rate due to pressure drop increases. Higher the face velocity, higher will be the entropy generation rate due to pressure drop. As a result, entropy generation number increases for lower fin pitches at higher face velocity (figure 5). However, at lower face velocity the rate of increase in entropy generation rate due to pressure drop becomes equal to the rate of decrease in entropy generation rate due to heat transfer. As a result, entropy generation number for different fin pitches overlaps at low velocity (figure 5). On the other hand, a decrease in the fin pitch results in higher pressure drop and hence higher fan power consumption. Figure 6 shows the fan power consumption for different fin pitches. From Figs. 5 and 6, it is observed that larger the fin pitch lower will be the entropy generation number as well as fan power consumption. But the heat exchange rate is poor for the larger fin pitch. Therefore, a compromise has to be made while selecting the fin pitch to achieve the higher heat exchange rate and the lower entropy generation number. For the current application, fin pitch of 3 mm is selected to achieve lower entropy generation number with better heat exchange rate.

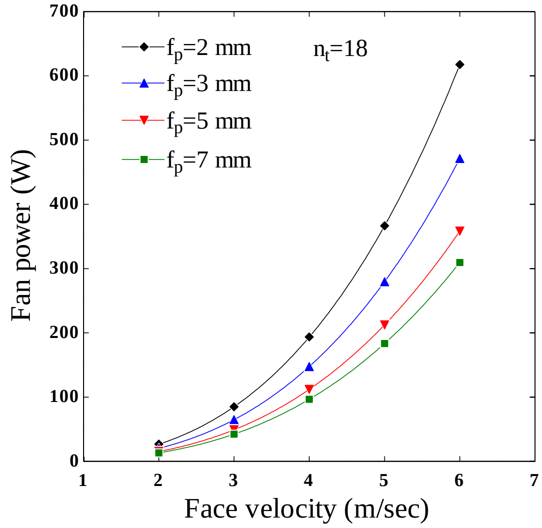

Figure 6 Variation in fan power consumption with changes in fin pitch

CONCLUSIONS

This study presents an entropy generation analysis of a gas cooler for transcritical CO_2 system. To investigate the variations in entropy generation number with changes in the total number of tubes, fin pitch, and face velocity; simulations are carried out using the developed steady-state model for the gas cooler. The results obtained from the numerical simulations are summarized here:

(a) As the face velocity increases, the entropy generation number first decreases, reaches a minimum and then increases. On the other hand, fan power consumption increases rapidly with the increase in face velocity. To achieve the lower entropy generation number with lower fan power consumption, the face velocity should be maintained within 3-4 m/sec for the current application.

(b) An increase in the total number of tubes in each row results in lower entropy generation number but higher fan power consumption. For the present application, the total number of tubes in each row should be within 18-20 to achieve lower entropy generation number with lower fan power consumption.

(c) As the fin pitch increases, both the entropy generation number as well as the fan power consumption decreases. However, the larger fin pitch results in poor heat exchange rate. Therefore to achieve better heat exchange rate with lower entropy generation number, fin pitch selected for the current application is 3 mm.

(d) For the predicted dimensions of the heat exchanger based on the second law analysis, the heat exchange rate varies from 4.27-4.36 kW while the fan power consumption varies from 65-164 W.

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