CONSTRUCTAL DESIGN OF CAVITIES FOR INTENSIFIED COOLING PERFORMANCE ON HEAT GENERATING VOLUMES

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ABSTRACT

The study of heat conduction in micro systems is a topic of interest as heat generation is a common issue in electronics. This paper will study heat conduction, using finite element simulations of a cross-sectional copper surface with micro channels where the thermal conductivity of the coolant fluid is set to 0.591 W/(m·K), which corresponds to water at 293 K. The simulated models are made using COMSOL Multiphysics 5.2a. Heat Transfer module allows for the study of heat transfer in devices. The bottom and side surfaces of the heat sink are thermally isolated. The top surface is assigned a general inward heat flux of 10 MW/m^2. The channel structure is designed following the Allometric law. For a Y-design, 45˚ angles are used between one bifurcation level and the next, and 90˚ for the T-design. A steady state situation is defined, and a laminar flow at constant temperature is designated for the fluid in the channels. Looking at this as a fully developed region with constant surface temperature, the Nusselt number can be considered constant. The heat transfer coefficient assigned to the channels is obtained from calculations related to the channel dimensions and the previously mentioned boundary conditions. The respective Darcy friction factor, pressure drop and nominal pumping power is calculated for each of the designs. The resulting simulations show the diffusion dominated heat conduction of the heated area. The Y-design is shown to be the superior design for heat conduction.

INTRODUCTION

Heat fluxes in different materials and products are important issues to solve, hence the construction of electronic devices being smaller and closer packaged creates a need for micro cooling systems. Air-cooling systems have a limit of 100W/cm^2. By using cooling liquids and systems consisting of micro channels, it should be possible to deliver a heat dissipation rate closer to 1kW/cm^2, (10MW/m^2) with a junction-to-air temperature difference of 50˚C [1].

In nature, the tree shape is often found in different variations, such as veins in the body, river basins and roots and branches of trees. The branching networks consists of a main branch which bifurcates into thinner and shorter branches, and this is repeated n times. The structure is based on the constructal law which gives the most efficient transport of fluids and heat due to the hydrodynamics in such a system [2]. The constructional law first formulated by Adrian Bejan in 1996, states: "For a flow to persist in time (to survive), it must evolve in such a way that it provides easier and easier access to the currents that flow through it". According to this theory, if a flow system has sufficient freedom to change its configuration, pattern and geometry, then the system will progressively improve access routes for the currents that flow within that system over time [7].

Experiments are done using a tree structure with rectangular channels comparing this to parallel channels, arguing that this gives place to an increase of convective heat transfer and that it reduces the pressure drop [3].

Using branched network of micro channels embedded in a heat generating volume connected to a heat sink, shows that more complex structures increase the cooling performance, but also that there are limitations, which result in some designs being more optimal [4] [5].

Further experiments comparing Y-shaped and Ψ-shaped structures with various number of bifurcation levels, concludes that Ψ-shaped structures have better heat transfer characteristics such as lower thermal resistance and higher surface temperature uniformity. This is mainly caused due to a more uniform distribution of the channels [6].

This information is used as a base for considering how to design the structures desired for this project.

NOMENCLATURE

Nu_D	[-]	Nusselt number
h	[W/m^{2}K]	Heat transfer coefficient

D_h	[m]	Channel hydraulic diameter
L	[m]	Channel length
D_s	[m]	Distance between two parallel channels
B	[-]	Branch
k	[W/mK]	Thermal conductivity
T	[K]	Temperature
$\dot{q}'''$	[W/m^3]	Volumetric heat generation density
Δp	[Pa]	Pressure loss
f_D	[-]	Darcy friction factor
Re	[-]	Reynolds number
v	[m/s]	Velocity
Q_P	[W]	Nominal pumping power
A	[m^2]	Area

Special characters

φ	[-]	Phi Number
ζ	[-]	The Allometric Law
ρ	[kg/m^3]	Density
μ	[kg/m·s]	Viscosity

NUMERICAL METHOD

Governing Equations

Heat transfer can occur by different means, depending on the material and structure. Heat conduction is the transfer of energy between objects, or inside a material, through vibrating atoms or molecules passing energy to their neighbours, or migrating electrons. The energy moves from a warmer to a colder area [8]. Convection occurs when the heat is transported through movement of clusters of molecules from heated to colder areas, and through diffusion [9]. In this project the radiation is ignored.

Micro-channels tend to create laminar flow due to the miniscule features, which gives a Reynolds Number (relation between the inertia forces and the viscous forces) often lower than 100 [10].

Using the idea of the constructal law, ramifications going into the heat generating body are designed so that the ramifications get shorter and thinner at higher levels. At conventional sized channels, the heat conduction from the duct walls can be ignored, hence the thickness of this layer is almost inappreciable compared to the hydraulic diameter. The opposite occurs in micro-channels, and with decreasing size of the hydraulic diameter, the heat conduction from the duct walls will have an increased effect on total heat transfer, and the system will move from a convective-dominant heat transfer to be increasingly more diffusion dominant [11].

In this project, a steady state situation is defined, and a laminar flow at constant temperature is designated for the fluid in the channels.

Considering the flow in each of the branch level as both hydraulically and thermally developed with constant surface temperature, the Nusselt number can be considered a constant. Equation (1) shows the relation between the Nusselt number (Nu_D) and the heat transfer coefficient (h). D_h is the hydraulic diameter of the flow path and k is the thermal conductivity of the fluid. For a developed flow with constant Nu, it is concluded that decreasing the diameter will increase the heat transfer coefficient, hence smaller channels should be more effective for heat transfer [12].

$$Nu_D = hD_h/k \qquad (1)$$

The hydraulic diameter D_h is by definition 4 times the cross-sectional area divided by the wetted perimeter, 2L. Hence, the area is the hydraulic diameter of the channel multiplied by 25% of the wetted perimeter.

$$D_h = 4LD_s/P = 2D_s \qquad (2)$$

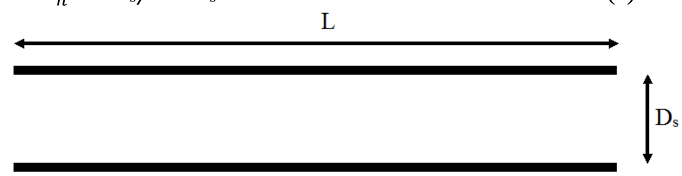

Figure 1 - Channel dimensions.

Considering a layer of fluid that moves, the heat flux could be described as Equation (3), where h is the heat transfer coefficient and ΔT is the temperature difference between the wall and the fluid [13].

$$q''\,(\text{convection}) = h\Delta T \qquad (3)$$

A layer of fluid that is not moving will have a diffusion-dominant heat flux. Equation (4), where k is the thermal conductivity, ΔT is the temperature difference [13].

$$q''\,(\text{conduction}) = k\Delta T/D_h \qquad (4)$$

The Nusselt number is the ratio between convection heat transfer and heat conduction transfer in the fluid as shown in Equation (5) [13].

$$Nu_D = \frac{q''\,(\text{convection})}{q''\,(\text{conduction})} = \frac{hD_h}{k} \qquad (5)$$

Solving Equation (5) gives the heat transfer coefficient shown in Equation (6).

$$h = Nu_{D_h}\frac{k}{D_h} \qquad (6)$$

The Allometric law and the Phi number are ratios between the length and hydraulic diameters of the different levels of branches in a structure based on constructal law. This is expressed in Equation (7) where φ is the Phi number, ζ is the Allometric law, L is the length of the branch, D_s is the distance between the two parallel walls in each level, i is the indicator of the branch level. φ has a value of approximately 1.618 and the Allometric law has a value of approximately $2^{1/3}$ [6]. In this project, the bifurcation angles are not taken into special consideration.

$$\varphi,\zeta = \frac{L_i}{L_{i+1}} = \frac{D_{s\,i}}{D_{s,i+1}} \qquad (7)$$

The pressure drop of the channels are of interest to consider the pumping power needed to maintain a steady flow. The pressure drop of each branch is studied to see how the dimensions of the channel effect the pumping power for a varying Reynold's number. The flow in channel (level i) is considered horizontal so hydrostatic and gravitational effects can be neglected.

$$\Delta p = f_{D_{h,i}} \cdot \frac{L_i}{D_{h,i}} \cdot \frac{\rho v^2}{2} \tag{8}$$

Pressure loss per unit depth (1μm) of the flow path Δp is due to viscous effects. The first term in right hand side of Equation (8) is the Darcy friction factor, and the last term is dynamic pressure. The Darcy friction factor is a function of the Reynolds number. For smooth pipe wall surface, the Darcy friction factor is,

$$f_{D_{h,i}} = 96/Re \tag{9}$$

Hence, the channel walls are considered to be parallel with unspecified depth.

Velocity is be found by rearranging the definition of Reynolds number where μ and ρ is the viscosity and density of the fluid respectively.

$$v = \frac{\mu Re}{\rho D_{h,i}} \tag{10}$$

Combing Equation (8), Equation (9) and Equation (10) gives the pressure loss in the channels.

$$\Delta p = \frac{96}{Re} \cdot \frac{L_i}{D_{h,i}} \cdot \frac{\rho \mu^2 Re^2}{2\rho^2 D_{h,i}^2} = \frac{48 Re L_i \mu^2}{\rho D_{h,i}^3} \tag{11}$$

For the ith level of the constructal channel network, the pumping power $Q_{P,i}$ to compensate for the pressure loss is given as the sum of pressure loss multiplied by velocity and cross sectional area, according to Poiseuille's law. The area is by definition the hydraulic diameter of the channel multiplied by 25% of the wetted perimeter.

$$Q_{P,i} = \Delta p v A = \frac{48 Re L_i \mu^2}{\rho D_{h,i}^3} \frac{\mu Re}{\rho D_{h,i}} \frac{D_{h,i} L_i}{2} = \frac{24 Re^2 L_i^2 \mu^3}{\rho^2 D_{h,i}^3} \tag{12}$$

Simulation Method

The simulated models are made using COMSOL Multiphysics 5.2a [14]. Heat Transfer module allows for the study of heat transfer in devices. A stationary study of different heat sink designs is performed to evaluate the efficiency of heat conduction in micro channels.

The geometry, representing the cross-sectional area of a heat sink is built of using copper as the material for the heat sink. Copper was chosen due to its high thermal conductivity compared to water, used as the coolant fluid [15].

The channel structure is designed following the Allometric law shown in Equation (7). The values listed in TABLE I are used to construct the COMSOL design of the two selected structures. For the Y-design, 45˚ angles are used between one bifurcation level and the next, and 90˚ for the T-design.

Table 1
Design Parameters

Branch	Length (μm)	Hydraulic Diameter (μm)	Heat transfer coefficient (W/(m ²·K))
B1	100.00	10.00	486688.50
B2	79.37	7.94	612957.81
B3	62.99	6.30	772521.43
B4	50.00	5.00	973377.00

The channel walls are assigned the boundary condition convective heat flux, where temperature of the coolant fluid and the heat transfer coefficient are assigned. The heat transfer coefficient is calculated from Equation (6) where the thermal conductivity is set to 0.591 W/(m·K) to represent water at 293 K, which is set as the temperature of the coolant fluid [16].

The Nu_D number applied is 8.235, hence the channels are considered to be of unspecified depth with a uniform heat flux and a fully developed laminar flow of the coolant fluid. The system is considered as a steady state situation [17].

The bottom and side surfaces of the heat sink are thermally isolated. The top surface is assigned a general inward heat flux of 10 MW/m².

The models are meshed using a free triangular mesh with a preset element size of fine which gives a minimum element size of 0.12 μm.

Matlab 2016b is used to plot the pressure drop and nominal pumping power for increasing Reynold's number from 100 to 1000 for each of the branches. The dimensions listed in TABLE I are used for the calculations.

RESULTS AND DISCUSSION

As Equation (6) indicates, the heat transfer coefficient is inverse proportional to the hydraulic diameter of the channel. This is clearly visible in Figure 2 where a steady state heat conduction simulation is performed. Heat is applied to the top surface of a 10 by 5 mm copper block with three channels of equal length of 600 μm and variable hydraulic diameter.

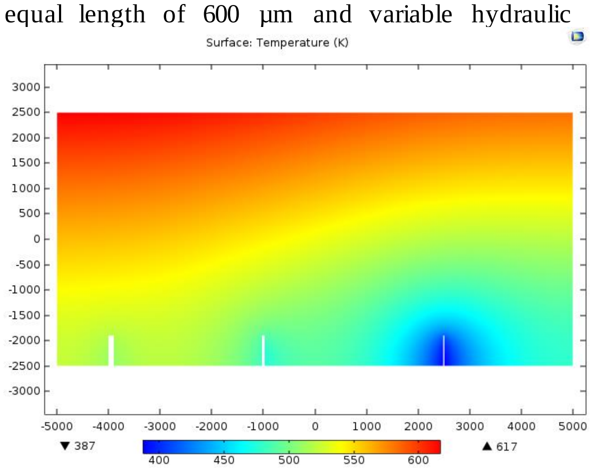

Figure 2 - Three channels of length 600 μm and varying hydraulic diameter. From left: 100 μm, 50 μm, 10 μm.

The area on the top left corner is the warmest at approximately 617 K. The lowest temperature of approximately 387 K is found closest to the walls of the 10µm channel, and it is evident that this channel affects the totality of the block, showing a much more efficient cooling than the thicker channels. Hence, 10 µm is chosen as the hydraulic diameter for first branch-level of the following simulations.

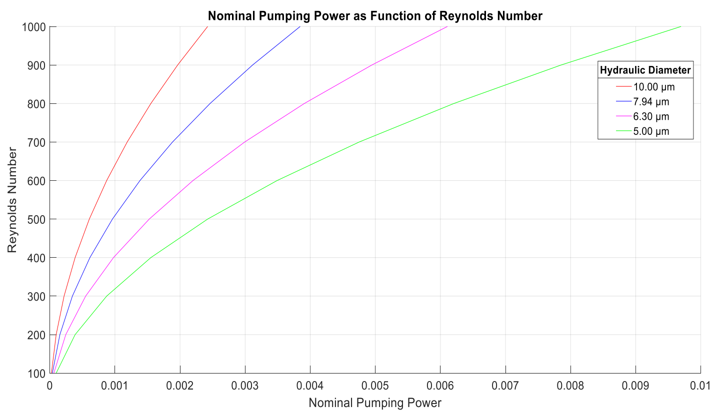

Figure 3 – Nominal pumping power as function of Re

The total nominal pumping power (for all levels of branches) increases quadratically with *Re* and while decreasing linearly with the increasing cubed hydraulic diameter of the channel.

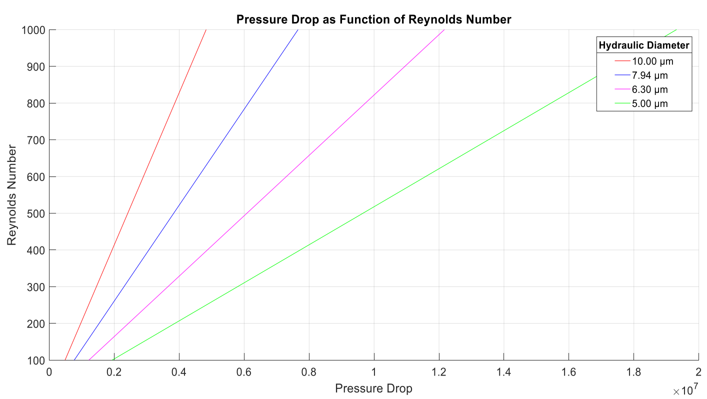

Figure 4 – Pressure drop as function of *Re*

Two different heat-sink designs are studied to see the efficiency of channel geometry. Heat is applied to the top surface of the blocks of copper with height and width, 300 µm and 400 µm, respectively. One "T-shaped channel" structure and one "Y-shaped channel" structure is designed to cool the copper blocks, as seen in Figure 5 and Figure 6 respectively. Both models are designed using the same length dimensions, listed in TABLE I. The "T-shaped" design has 90° angle ramifications, whereas the "Y-shaped" design has 45° angle ramifications.

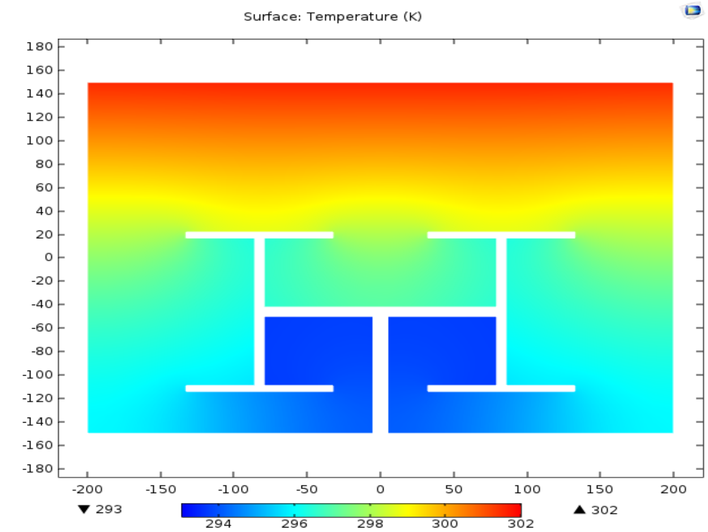

Figure 5 - "T-shape" design with four levels of branches. Colors indicate the thermal gradient distribution in the block. The heat from the top is drawn towards the fourth- and second-level branch. This design gives an average temperature of 297.75 K.

Figure 5 shows the temperature distribution of the block in Kelvin. The colors indicate the thermal gradient moving from warm at the top surface to colder towards the bottom. The areas around to the first branch and bottom surface is effectively cooled. From the fourth branch, it is visible that the heat is drawn into channels, seen by the dip in the yellow gradient. An integral operation over the surface gives the average surface temperature to be 297.75 K.

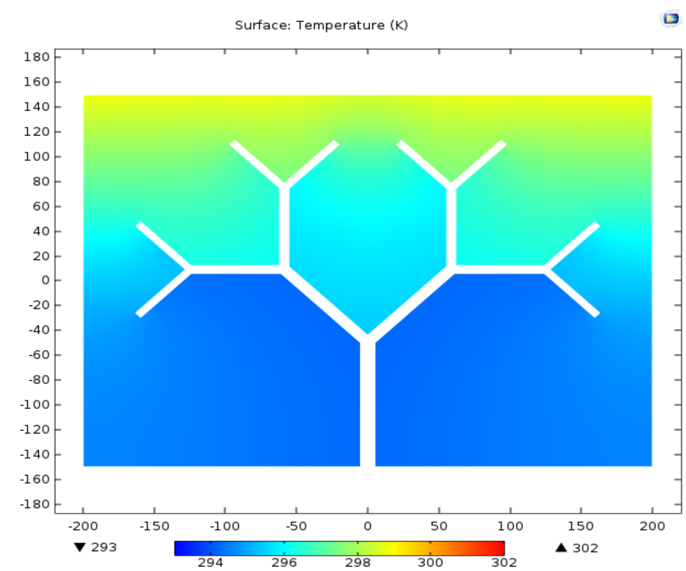

Figure 6 - "Y-shape" design with four levels of branches. Colors indicate the thermal gradient distribution in the block. The heat is drawn towards the openings of the "Y-shape". This design gives an average temperature of 295.74 K.

Figure 6 shows the temperature distribution of the block in Kelvin. The colors indicate the thermal gradient moving from warm at the top surface to colder towards the bottom. The areas around the first, second and third branch are effectively cooled. The heat from the top surface is draw towards the opening of the "Y-shape". The average surface temperature using this channel design is 295.74 K.

By comparing Figure 5 and Figure 6, it is clear that the "Y-shape" surpasses the "T-shape" in terms of cooling the block both more efficiently and evenly. The design dimensions of the block and channels, as well as the temperature range, is the same for both figures. The average temperature of Figure 6 is lower than that of Figure 5, indicating a better cooling performance.

One of the reasons the Y-shaped design is more efficient than the T-shaped design is the more conveniently distributed channels over the cross-sectional area of the heat sink. The "T-shaped channel" design is proven less effective due to a more compact area covered by cooling channels. By opening the ramifications at 45° angles is more effective than 90° angles. It is also a possibility that the bifurcation angle may have some effect on the heat transfer inside the channels, but this was not considered in this project.

CONCLUSION

For an inward heat flux of 10 MW/m^2, the "Y-shape" design conducts the heat more efficiently. This design gives a lower average surface temperature, which is clearly beneficial. It can therefore be concluded to that the "Y-shape" design is the superior design. It is also clear that the distance between the parallel channels is an important factor to create an effective design, where the thinner channels removes heat at a higher rate than thicker channels, at least in the order of dimensions used in this project.

The pressure drop in the thinner channels are higher than that of the thicker channels, thus a higher pumping power is required to maintain steady flow.

The simulations presented in this paper will be followed up by experimental research. The future work will be to test the channel systems in a realistic case where the flows in the channel are taken into account. This can be done by fabricating the designed models and building a pumping circuit for the channels.

ACKNOWLEDGMENTS

The present work is originated from the final report for the course MN-HMT5900 at HSN. Mette Varegg has made a great contribution to the original report. The financial support from Chongqing Research Program of Basic Research and Frontier Technology (No. cstc2015jcyjBX0004), Science and Technology Research Program of Chongqing Education Commission (No. KJ1600602 & No. KJ15006XX), Forskningsradet Nærings-Ph.D. (Project No.: 251129), Regionale Forskningsfond Oslofjordfondet (Project No.: 258902) and NANO2021 (Project No.: 263783) is hereby acknowledged.

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