TRANSIENT CONJUGATED FORCED CONVECTION HEAT TRANSFER IN THICK WALLED PIPES AND MINIPIPES WITH TIME PERIODICALLY CHANGING CONVECTIVE BOUNDARY CONDITION

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ABSTRACT

Transient conjugated heat transfer in thick walled pipes with thermally developing laminar flow is analyzed. Effects of axial fluid conduction as well as radial and axial wall conduction are involved. The problem is solved numerically in a two regional pipe, initially isothermal, for which the ambient temperature in the downstream region suddenly begins to change periodically in time. A parametric study is done to investigate the effects of five defining dimensionless parameters of the problem, which are namely; wall thickness ratio, wall-to-fluid thermal conductivity ratio, wall-to-fluid thermal diffusivity ratio, the Peclet number, the Biot number, and also the effect of the value of the angular frequency.

INTRODUCTION

Problems in which the boundary conditions change periodically with time may be faced in many engineering applications like solar heating systems, thermostatically controlled resistance heating systems, nuclear fuel rods, cooling of electronic devices and heat exchangers with variable mass flow rates. In conjugated problems, i.e., for thick walled pipes, the thermal boundary conditions along the solid-fluid interface are not known a priori, and the energy equations, both in the solid and in the fluid sides, should be solved under the conditions of continuity in temperature and/or heat flux. Conjugated problems were solved by using several approaches using analytical or numerical [1-3] and hybrid methods [4, 5].

When flow Peclet number is low, on the other hand, the axial conduction in the fluid may be comparable to convection and therefore such problems should be analyzed for two-regional pipes and heat transfer characteristics should be determined for both upstream and downstream regions. Both wall and fluid axial conduction cause some heat diffusing backward to the unheated upstream region and this results preheating the fluid prior to the heated downstream region. Conjugated heat transfer problems and fluid axial conduction effects are also the case in mini or microtubes, because of relatively thick walls, small diameters and therefore low Reynolds and Peclet numbers in flows. Literature surveys for conjugated pipe or channel flows with the effects of fluid axial conduction are given briefly in [6-10].

Problems were solved under periodic boundary conditions changing either with time [11, 12], spatially [13-18] and both spatially and with time [19]. There are also some investigations handled the problem with periodically changing inlet fluid temperature [20-22] and in mixed convection in inclined and vertical pipes [23, 24].

NOMENCLATURE

Bi	Biot number
c_p	specific heat at constant pressure
d	thickness of the pipe wall
Fo	Fourier number
Gz	Graetz number
h	convection heat transfer coefficient
k	thermal conductivity
Pe	Peclet number
q	heat flux
r	radial coordinate
t	time
T	temperature
T_o	initial temperature of the system
u	axial velocity
x	axial coordinate

Greek symbols

α	thermal diffusivity
ΔT	amplitude of the periodic temperature variation
ρ	density
ω	angular frequency
Ω	dimensionless angular frequency

Subscripts

f	fluid
i	inner wall
m	mean
o	outer wall
w	wall
wf	ratio of wall to fluid

Superscript

$'$	dimensionless quantity

PROBLEM FORMULATION

The problem schematic and the coordinate system are given in Fig. 1. The flow pipe is two-regional and infinite in length in both regions. At far upstream ($x = -\infty$) fluid enters the pipe with a uniform temperature T_o, which is also the initial temperature of the whole pipe. The flow is steady and laminar. The upstream region is externally insulated and long enough so that the flow is hydrodynamically developed prior to the heated downstream region. At time t=0, the downstream region of the pipe is suddenly subjected to a time periodically changing external fluid temperature, exchanging heat with the outer wall surface with a constant heat transfer coefficient of h_0. All thermo-physical properties of the wall and fluid are assumed to be constant and the viscous dissipation is neglected.

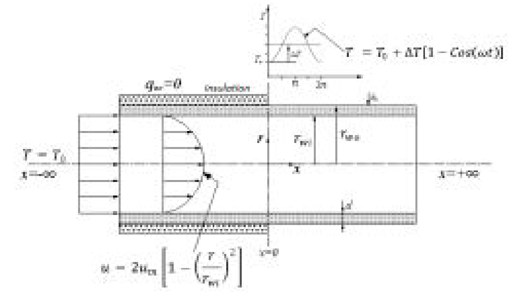

Figure 1 Schematics of the problem and the coordinate system

The problem described above can be formulated in non-dimensional form as follows: In the wall side the energy equation is

$$\frac{1}{\alpha_{wf}}\frac{\partial T'_w}{\partial t'} = \frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial T'_w}{\partial r'}\right) + \frac{1}{Pe^2}\frac{\partial^2 T'_w}{\partial x'^2} \tag{1a}$$

The initial and boundary conditions are;

$$\text{at } t' = 0, \quad T'_w = 0 \tag{1b}$$

$$\text{at } x' = -\infty, \quad T'_w = 0 \tag{1c}$$

$$\text{at } x' = \infty, \quad \frac{\partial T'_w}{\partial x'} = 0 \tag{1d}$$

$$\text{at } r' = 1 + d' \text{ for } x' < 0 \quad \frac{\partial T'_w}{\partial r'} = 0 \tag{1e}$$

$$\text{at } r' = 1 + d' \text{ for } x' \geq 0 \quad \frac{\partial T'_w}{\partial r'} + Bi\left\{T'_w - \left[1 - \mathrm{Cos}\left(\Omega t'\right)\right]\right\} = 0 \tag{1f}$$

$$\text{at } r' = 1, \quad T_f{}' = T_w{}' \text{ and } \frac{\partial T'_w}{\partial r'} = \frac{1}{k_{wf}}\frac{\partial T'_f}{\partial r'} \tag{1g, h}$$

In the fluid side the energy equation is

$$\frac{\partial T'_f}{\partial t'} + \left(1 - r'^2\right)\frac{\partial T'_f}{\partial x'} = \frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial T'_f}{\partial r'}\right) + \frac{1}{Pe^2}\frac{\partial^2 T'_f}{\partial x'^2} \tag{2a}$$

The initial and boundary conditions are;

$$\text{at } t' = 0, \quad T'_f = 0 \tag{2b}$$

$$\text{at } x' = -\infty, \quad T'_f = 0 \tag{2c}$$

$$\text{at } x' = \infty, \quad \frac{\partial T'_f}{\partial x'} = 0 \tag{2d}$$

$$\text{at } r' = 0, \quad \frac{\partial T'_f}{\partial r'} = 0 \tag{2e}$$

$$\text{at } r' = 1, \quad T'_f = T'_w \text{ and } \frac{\partial T'_f}{\partial r'} = k_{wf}\frac{\partial T'_w}{\partial r'} \tag{2f, g}$$

Non-dimensional parameters are defined as;

$$x' = \frac{x}{r_{wi}Pe} \equiv \frac{2}{Gz}, \quad r' = \frac{r}{r_{wi}}, \quad d' = \frac{d}{r_{wi}}, \quad T' = \frac{T - T_o}{\Delta T}$$

$$k_{wf} = \frac{k_w}{k_f}, \quad \alpha_{wf} = \frac{\alpha_w}{\alpha_f}, \quad t' = \frac{t\alpha_f}{r_{wi}^2} \equiv Fo$$

$$Pe = \frac{2r_{wi}u_m\rho_f c_{pf}}{k_f}, \quad Bi_0 = \frac{h_0 r_{wi}}{k_w}, \quad \Omega = \frac{r_{wi}^2\omega}{\alpha_f}$$

Interfacial heat flux values, q'_{wi}, can be calculated as follows;

$$q'_{wi} = -\left(\frac{\partial T'_f}{\partial r'}\right)_{r'=1} \tag{3}$$

SOLUTION METHODOLOGY

A numerical finite-difference approach is used to solve the system of Eqs. (1a-1h) and (2a-2g). The conductive terms are discretized by central-difference and the convective term by an exact scheme given in [25]. For the transient terms fully implicit formulation is used. In radial direction uniform grid lengths are used in the wall side, but the grids in the fluid side are contracted near the interface. The grids are bounded between the outer surface and the axis of the pipe, while the boundaries in the axial direction were found by the results of testing runs with coarse grids, so that the conditions at these boundaries are assured. Axial grids are also contracted in the vicinity of the beginning of the heating section. The first axial step size is taken 0.001 for both upstream and downstream regions and linearly stretched in both directions by taking the axial step size of a grid as 1.33 times higher than that of the previous grid. After successive grid refinements, the optimum number for the grid system is found as 60 X 24. Furthermore, to verify that the solutions are grid independent, results were assessed based on the generalized Richardson extrapolation and grid convergence index (GCI), suggested in [26].

In the solutions, the time step is taken $\pi/1800$ and constant as to be harmonious to periodic variation of the boundary condition. Successive computational refinements were also made in time mesh to test the numerical error associated with the temporal discretization. Temperature distributions are obtained by Gauss–Seidel iteration technique. In each time step iterations were made by the line-by-line method [27], by traversing from outer surface of the wall to the pipe axis in radial direction and by sweeping from upstream to downstream region. At the interface, the harmonic mean formulation [27] is used for the discretization of the boundary conditions and a consecutive procedure is used in the solutions. Previously calculated temperatures are used to transmit information from wall to fluid side and interfacial heat flux values from fluid to wall side in the iterations. At a time step solutions were obtained in 120-150 iterations. At the end of the transient state, system achieves to steady periodic state and, although the temperature at a grid point changes with time it will exactly be the same after a complete period. Solutions were obtained approximately in 10^4 total iterations. Some additional accuracy tests were also done by changing the convergence limit and by changing the traversing and sweeping directions for the solutions and no considerable difference in computed values was detected.

RESULTS AND DISCUSSIONS

The problem depends on five dimensionless parameters, namely the Peclet number, *Pe*, the Biot number, *Bi*, wall thickness ratio, d', wall-to-fluid thermal conductivity ratio, k_{wf}, wall-to-fluid thermal diffusivity ratio, α_{wf}, and also the dimensionless angular frequency, Ω. Solutions are made for different combinations of these parameters: Pe = 1, 5 and 20; Bi = 1, 10, 100 and 1000; d' = 0.02, 0.1 and 0.3; k_{wf} = 0.1, 1, 10, 100 and 1000; α_{wf} = 0.1, 1 and 10 and Ω = 1, 18, 60, 900, 3000 and 9000. These values are chosen as appropriate for engineering applications and from the range that the presumed effects of the problem, i.e., wall and fluid axial conduction are in a significant level. On the other hand, Ω values are chosen as multiples of time steps as to be compatible to the periodic variation of outside fluid temperature. Here, the results are only given by interfacial heat flux values.

In Fig.2, the variations of interfacial heat flux values with time are given at various axial positions for a run executed with average parameter values and for Ω=1. This and similar figures are given in two parts, separately for upstream and downstream regions for simplicity. Considerable amount of heat is seen diffused back to the upstream region due to the wall and fluid axial conduction, and this causes preheating of fluid before entering heated downstream region. Heat flux curves are changing periodically with time as of the change in external fluid temperature. The periodicity is seen for all axial positions including the beginning of the preheating length and also in the thermally developed region.

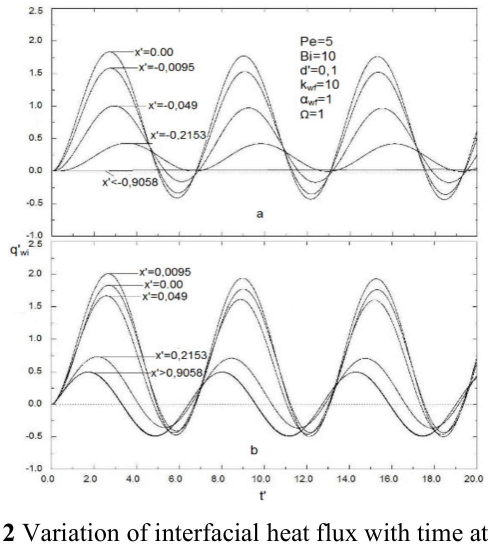

Figure 2 Variation of interfacial heat flux with time at various axial positions (Ω=1) (a-upstream, b-downstream region)

The period is large for small Ω values and as can be seen from the figure, the system reaches to steady periodic state at the beginning of the third period (t'=11.75) and, then after at an axial position the change in heat flux values with time is the same for all periods.

Due to the fast backward axial conduction in the wall side, the interface temperatures increase in the upstream region and generally positive heat flux values are seen in this region. The average values of interfacial heat flux and also the amplitudes of variation are increasing in the flow direction. This trend continues in the downstream region until at a certain axial position. Then the convection increases its effect and the values and amplitudes become decreasing. In the temperature rise term of a period, since the increase in temperatures becomes earlier in the wall than in the fluid, the temperatures are higher in the wall side and heat flux values are positive. The opposite is true for the temperature drop term and since the fluid bulk temperatures are higher than the interfacial temperatures, heat flux values are negative. This becomes clearer as approaching to the fully developed region. In the fully developed region and at steady periodic state, the average of the heat flux values is zero, the amplitude of variation is 0.5 and the net heat transfer is zero as expected. Phase lag is seen in the curves at different axial positions, which is more evident in the far upstream and as approaching to the fully developed region.

For the development of the problem, the most important parameter is probably the angular frequency. For the runs with Ω values less than 1, the results are obtained completely similar to the results obtained for Ω=1 but only the period is increasing. Again, for Ω values up to 18 the results are very similar, but the shape of the figures change after this value. A typical result is given in Fig.3 for Ω=900, again with the same average values of the other parameters.

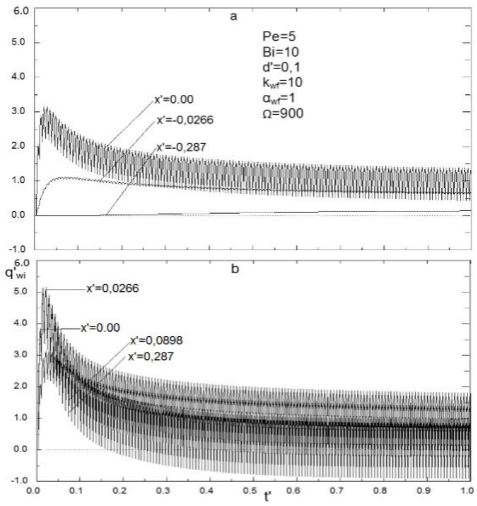

Figure 3 Variation of interfacial heat flux with time at various axial positions (Ω=900) (a-upstream, b-downstream region)

For high Ω values, at early times of transient, at all axial positions the heat flux values first increase to a maximum and then decrease. With increasing Ω, these maximums also increase but with further increase in Ω a decrease is seen. This is also the case for the amplitude of variation in heat flux values. For too high Ω values (>9000), the period of variation in the external fluid temperature is too small and the inertia of the system probably prevents to respond to the change. The amplitude of variation is also vanishing for very high Ω values and the results are becoming similar to the work in [28], for which the external fluid temperature is suddenly increased by a value of ΔT in the downstream region as a boundary condition. The number of periods is increasing as Ω increases in the transient state. On the other hand, for all Ω values the thermal development is achieved at around x'=1 and steady periodic state is reached at around t'=11.75.

The effects of the parameter values are presented in this section, for Ω=1 except α_{wf}. In Fig. 4, the variations of interfacial heat fluxes with time at various axial positions are given for three different wall thickness ratios. In thick walled pipes, at far upstream heat flux values are usually positive and higher due to the increased backward wall axial conduction. Whereas, as approaching to the heated region, fast radial conduction in thin walls results higher interfacial temperatures and higher heat flux values. The same is valid at the beginning of the heated downstream region. Further downstream, by the increase in convection effect the average values and the amplitude of variation decrease. At fully developed region all the curves match for all thickness ratios. The length of thermal development and the time to reach to periodic steady state do not change with the thickness ratio.

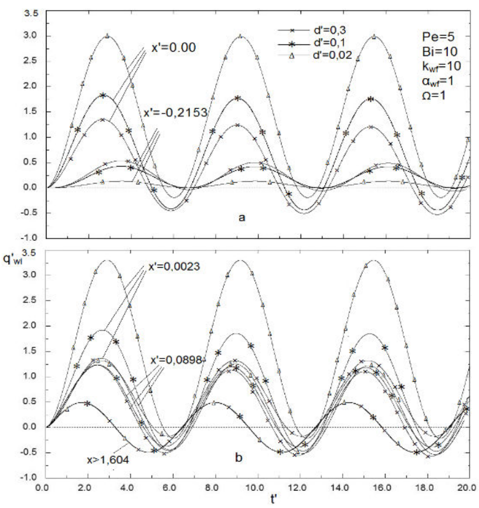

Figure 4 Effect of wall thickness ratio on interfacial heat flux (Ω=1) (a-upstream, b-downstream region)

Fig. 5 gives the effect of wall-to-fluid thermal conductivity ratio. For high k_{wf} values, due to increased wall conduction higher heat flux values are seen, especially in the upstream region. The curves are matching in the fully developed region except for k_{wf} =0.1. Besides, the phase lag is more evident for k_{wf} =0.1 in this region. The length for thermal development is increased considerably for small values of $k_{wf,}$ but the required time to rich to steady periodic state is not changed with this parameter. The effect of k_{wf} can be said disappearing for k_{wf}>100.

The effect of Peclet number on interfacial heat flux is shown in Fig. 6. Since the fluid axial conduction is high for small Pe number flows, fluid temperatures are higher in the upstream region and therefore, interfacial heat flux values are low. Due to the same reason, the negative heat flux values, arising in the decrease term of the temperature in a period, are also higher in the upstream region. The heat conveyed by the flow from upstream causes the high fluid temperatures continue

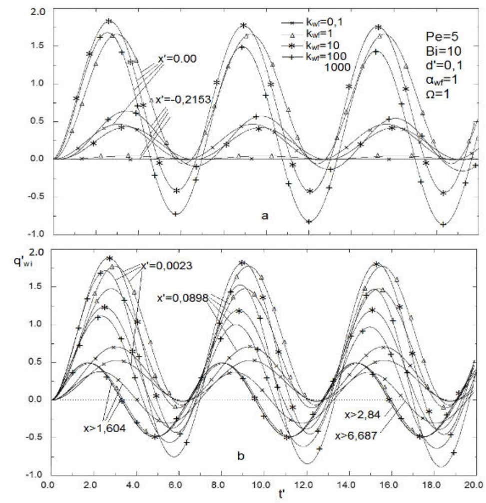

Figure 5 Effect of wall-to-fluid thermal conductivity ratio on interfacial heat flux (Ω=1) (a-upstream, b-downstream region)

also in the downstream region, and lower heat flux values are seen for small Pe numbers. All the curves are also matching in the fully developed region. The thermal development length is increased almost threefold for Pe =1, but the time to reach to periodic steady state does not change with Pe number.

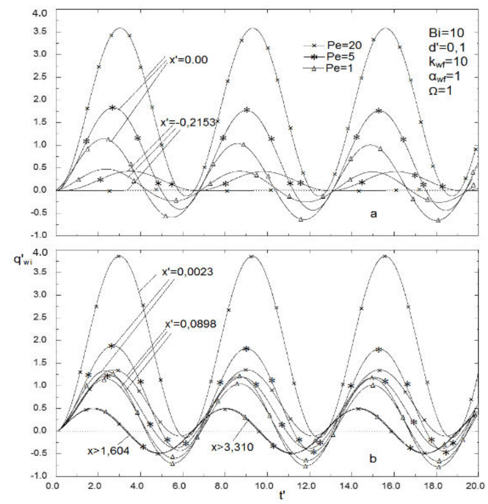

Figure 6 Effect of Peclet number on interfacial heat flux (Ω=1) (a-upstream, b-downstream region)

In Fig.7, the interfacial heat flux values are given as parameterized for four different Biot numbers. High Bi numbers means faster heat transfer in the outer surface and higher interfacial temperatures, and therefore the heat flux values are high. The curves for all Bi match at the fully developed region. The length required for thermal development and the time required to achieve steady periodic state are not affected by the *Bi* number. The effect of this parameter also disappears for values greater than 100.

Wall-to-fluid thermal conductivity ratio, α_{wf}, is probably the most interacting parameter with the angular frequency, Ω. α_{wf} is affecting the results of the problem in a different manner for small and large Ω values. For this reason, the effects of α_{wf} on interfacial heat flux are given for a small value, Ω=1 in Fig. 8, and additionally for a medium value, Ω=60 in Fig. 9. As discussed before, for very large Ω its effect is disappearing.

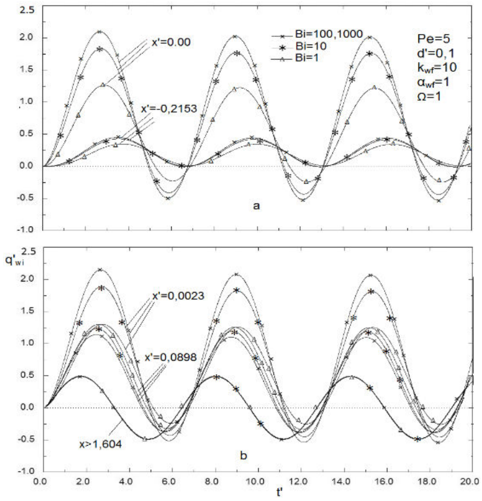

Figure 7 Effect of Biot number on interfacial heat flux (Ω=1) (a-upstream, b-downstream region)

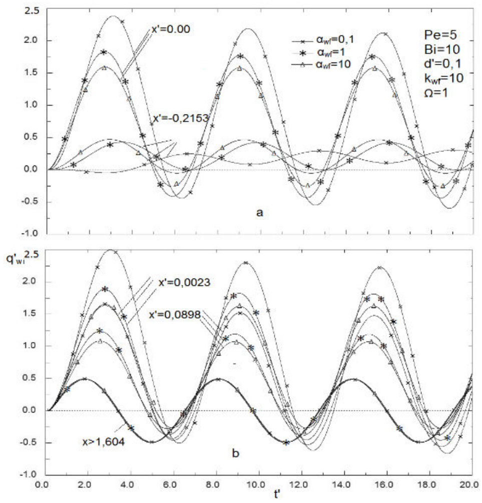

Figure 8 Effect of wall-to-fluid thermal diffusivity ratio on interfacial heat flux (Ω=1) (a-upstream, b-downstream region)

In Fig. 8, it can be seen that for high α_{wf} values, due to fast backward diffusion in the wall side interfacial heat flux values are high in the far upstream. However, towards downstream, the warmed fluid coming from far upstream increases bulk temperatures and heat flux values decrease. The curves are also matching in the fully developed region. For different α_{wf} values noticeable phase lags are seen in the curves, especially in the far upstream. The effect of α_{wf} is sensed more for high Ω values, which means shorter periods of temperature variation, as is shown in Fig. 9. The increase in the thermal inertia of the system for small α_{wf}, results a decrease in both average values and the amplitude of variation of heat flux. On the other hand, the effect of α_{wf} is also seen in the fully developed region as well as in the thermal entrance region. In the fully developed region, the amplitude of variation considerably decreases for small α_{wf}, though Fig. 9 gives the results for only the initial times of transient. The length of thermal entrance region and the time required to reach to steady periodic state are not changed with the value of α_{wf}. For α_{wf}>10, the effect of this parameter is negligible.

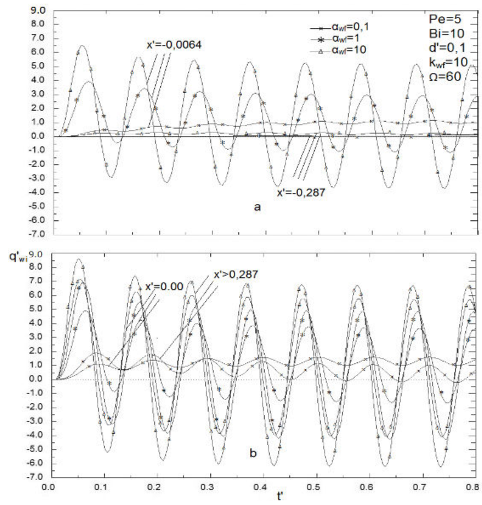

Figure 9 Effect of wall-to-fluid thermal diffusivity ratio on interfacial heat flux (Ω=60) (a-upstream, b-downstream region)

CONCLUSIONS

In thick walled pipes, laminar flow, thermal entrance region, transient heat transfer problem is investigated involving two dimensional wall and axial fluid conduction. An initially isothermal, two-regional pipe is considered, where upstream region is externally insulated and the downstream region is suddenly subjected to a time periodically changing ambient temperature. The problem is solved numerically by a finite-difference method and the effects of five defining parameters, *Pe*, *Bi*, *d'*, k_{wf} and α_{wf}, and also the effect of non-dimensional angular frequency, Ω, are analyzed. The results obtained can be outlined as follows:

Due to axial conduction both in the wall and in the fluid sides, a considerable amount of heat is diffused backward to the upstream region which causes preheating of the fluid prior to the heated downstream region. The length of this preheating region increases with increasing *d'* and k_{wf} and with decreasing *Pe*.

Heat transfer characteristics at all axial positions change in harmony with the change in temperature of the outside fluid temperature in the downstream region. At steady periodic state, the change in heat transfer characteristics with time is the same for all periods. At the initial sections of the downstream region, due to fast radial conduction in the wall side, interfacial heat flux values increase first, reach to a maximum and then further downstream convection increases the fluid temperature and heat flux values begin to decrease. In the fully developed region and at steady periodic state, for all parameter and for all angular frequency values, the average value of the interfacial heat flux and therefore the net heat transfer from wall to fluid is zero. Phase lag is seen in heat flux curves at different axial positions, which is more evident in far upstream and as approaching to the fully developed region.

For large Ω, at early times of transient heat flux values increase first to a maximum and then decrease. With increasing Ω, these maximums and the amplitude of variation increase first and then decrease. The total number of periods in the transient regime is increasing with increasing Ω. For too large Ω, the amplitude of variation in heat transfer characteristics disappears.

The results are highly affected by the parameter values, but except some very extreme values, the length of thermal development and the time to reach to steady periodic state do not change. It can be said that, for k_{wf}>100, d'<0.02, Pe>20, α_{wf}>10, Bi>100 and Ω>9000, the effects of these parameters are disappearing. The effects of the parameter values are sensed more at the initial transient and less as time approaches to steady periodic state.

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