NATURAL CONVECTIVE HEAT TRANSFER FROM UPWARD FACING RECESSED AND PROTRUDING HEATED HORIZONTAL ISOTHERMAL CIRCULAR ELEMENTS WITH ISOTHERMAL VERTICAL SIDE SURFACES

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ABSTRACT

Heat transfer by natural convection from a horizontal upward facing circular isothermal heated element that is imbedded in a horizontal, adiabatic surrounding surface has been numerically investigated. The element is either recessed into or protruding from the surroundings surface by a small amount. The circular horizontal element surface is at a temperature higher than the surrounding fluid. A previous study of this type of situation assumed that the vertical side walls formed by the protrusion or recession of the element were adiabatic. However, the thermal conditions on these side walls could potentially have a significant effect on the heat transfer rate from the horizontal circular element surface and this has been investigated here by considering the case where the side walls are isothermal and at the same temperature as the horizontal circular heated element surface. The present study considers a range of conditions such that laminar, transitional, and turbulent flows can occur. The density changes with temperature have been treated using the Boussinesq approach. The standard *k*-epsilon turbulence model was used. Results have been obtained only for a Prandtl number of 0.74 which is essentially the value for air. A study of the effect of the dimensionless distance that the element is recessed or protrudes from the surrounding adiabatic surface on the variation of the Nusselt number with Rayleigh number has been undertaken. The variations for the case where the side wall is isothermal and the case where it is adiabatic have been compared in order to determine the effect of side wall heating.

INTRODUCTION

Natural convective heat transfer from a horizontal upward facing circular isothermal heated element that is imbedded in a large flat, horizontal, adiabatic surrounding surface has been numerically investigated. As illustrated in Fig. 1, this element is either recessed into or protruding from the surrounding adiabatic surface by a small amount. The temperature of the element, which is facing upward, is higher than that of the surrounding fluid. A previous study of this type of situation has assumed that the vertical side walls formed as a result of the protrusion or recession of the element were adiabatic [1]. However, the thermal conditions on these side walls could potentially have a significant effect on the heat transfer rate from the horizontal circular element surface and this has been investigated in the present study by considering the case where these side walls are isothermal and at the same temperature as the horizontal circular heated element surface. This study considers a range of conditions that allows for laminar, transitional, and turbulent flows to occur. The effect of the dimensionless distance that the element is recessed or is protruding from the surrounding adiabatic surface on the heat transfer rate has been examined here.

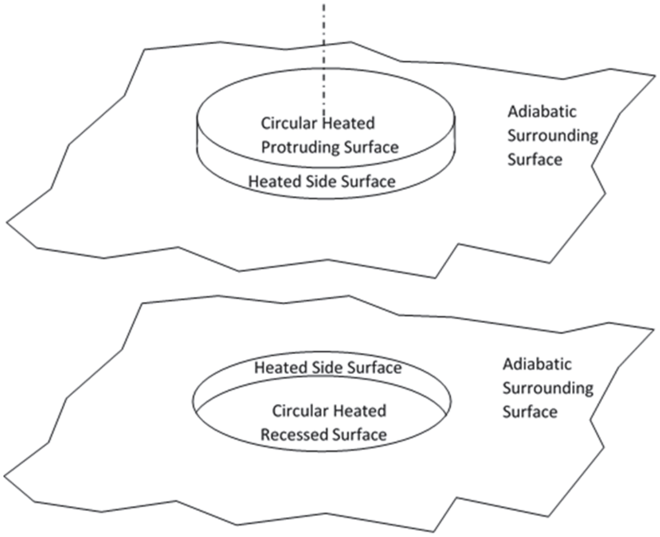

Figure 1 Flow situations considered

There have been many studies of natural convective heat transfer from heated horizontal surfaces, e.g., see [2-17]. These papers describe studies that are typical of the earlier work in this area most considering only conditions under which laminar flow exists. Most of these studies have also limited investigation to a heated element in the same plane as the surface of the adiabatic surrounding surface, i.e., have not considered the case of a heated element recessed or protruding from the surrounding surface.

This work presented in this paper is part of a more comprehensive overall study of natural convective heat transfer from horizontal and near horizontal heated elements for conditions in which laminar, transitional, and turbulent flow occur. Typical of the studies that are part of this more comprehensive investigation are those described in [18-23].

NOMENCLATURE

A_{bot}	[m^2]	area of bottom circular surface with a recessed element
A_{side}	[m^2]	area of vertical side surface
A_{top}	[m^2]	area of top circular surface with a protruding element
A_{total}	[m^2]	area of total heated surface of element
d	[m]	diameter of circular element
g	[m/s^2]	gravitational acceleration
h	[m]	recess depth or protrusion height
H	[-]	dimensionless recess depth or protrusion height, *h/d*
k	[W/mK]	thermal conductivity
Nu_{bot}	[-]	mean Nusselt number for bottom circular surface with a recessed element
Nu_{side}	[-]	mean Nusselt number for vertical side surface
Nu_{top}	[-]	mean Nusselt number for top circular surface with a protruding element
Nu_{total}	[-]	mean Nusselt number for total heated surface of element
Pr	[-]	Prandtl number
Q'_{bot}	[W]	heat transfer rate from bottom circular surface with a recessed element
Q'_{side}	[W]	heat transfer rate from vertical side surface
Q'_{top}	[W]	heat transfer rate from top circular surface with a protruding element
Q'_{total}	[W]	heat transfer rate from total heated surface area of element
Ra	[-]	Rayleigh number
T_f	[K]	undisturbed fluid temperature
T_w	[K]	element surface temperature

Greek Symbols

α	[m^2/s]	thermal diffusivity
β	[1/K]	bulk coefficient of thermal expansion
ν	[m^2/s]	kinematic viscosity

SOLUTION PROCEDURE

In obtaining the results reported here it has been assumed that the flow is steady and axisymmetric about the vertical center-line through the heated element. Except for the density change with temperature which gives rise to the buoyancy forces, the fluid properties are assumed constant and the Boussinesq approximation was utilized. Consideration has been limited to the case of upward facing elements with no consideration being given to radiant heat transfer effects. This study used the standard *k*-epsilon turbulence model with standard wall functions and with account being taken of buoyancy force effects. This turbulence model, which is applied under all conditions, is used to predict when transition occurs. It has been determined by many previous studies, e.g., [24 to 30] that this method provides relatively good predictions of turbulence development in flows similar to those considered here. The solution to the governing equations subject to the boundary conditions have been obtained using the commercial CFD solver ANSYS FLUENT©.

The results of grid independence and convergence-criteria independence testing indicated that the heat transfer results are to within approximately one per cent grid- and convergence criteria independent with the meshes employed here.

RESULTS

The solution has the following governing parameters:

- *Ra*, the Rayleigh number based on the diameter, *d*, of the circular heated element and on the difference between the surface temperature of this heated element and the side wall, T_w, and the temperature of the undisturbed fluid well away from the system, T_f, i.e.:

$$Ra = \frac{\beta g d^3 (T_w - T_f)}{\nu\alpha} \tag{1}$$

- the dimensionless recession or protrusion distance of the heated element, i.e.:

$$H = \frac{h}{d} \tag{2}$$

- *Pr*, the Prandtl number.

Results have only been obtained for a Prandtl number of 0.74, i.e., effectively the value for air. Consideration has been given to Rayleigh numbers of between approximately 10^5 and 10^{16} and to recess and protrusion height-to-diameter ratios of between 0 and 0.25.

The mean heat transfer rate from the lower horizontal circular surface when the element is recessed, Q'_{bot}, or from the upper horizontal circular surface when the element is protruding, Q'_{top}, from the heated side walls, Q'_{side}, and from the entire heated surface, Q'_{total} has been expressed in terms of a mean Nusselt numbers based on the diameter of the circular element, *d*, and on the difference between the heated element surface temperature and the temperature of the undisturbed fluid existing well away from the system, i.e.:

$$Nu_{bot} = \frac{Q'_{bot} d}{k A_{bot} (T_w - T_f)} \quad , \quad Nu_{top} = \frac{Q'_{top} d}{k A_{top} (T_w - T_f)} \quad ,$$
$$Nu_{side} = \frac{Q'_{side} d}{k A_{side} (T_w - T_f)} \quad , \quad Nu_{total} = \frac{Q'_{total} d}{k A_{total} (T_w - T_f)} \tag{3}$$

where $A_{\mathrm{bot}} = A_{\mathrm{top}} = \pi d^2/4$, $A_{\mathrm{side}} = \pi d h$, and $A_{\mathrm{total}} = \pi d^2/4 + \pi d h$. Since the value of *Pr* considered is fixed, the *Nu* values are a function of *Ra*, and of the dimensionless recession or protrusion distance, *H*.

Attention will first be given to the situation where the heated element is recessed into the adiabatic surrounding surface. Figures 2 to 4 show typical variations of the mean Nusselt numbers for the circular bottom surface, for the vertical side surface, and for the entire heated surface with Rayleigh number for various values of the dimensionless distance to which the element is recessed, *H*. Figures. 5 and 6, which show variations of the mean Nusselt number for the bottom surface with the dimensionless recess depth for various Rayleigh number values, provides a further illustration of the effect of the dimensionless recess depth. The changes in the form of the variation of Nusselt number with dimensionless recess depth that arise as a result of changes in the Rayleigh number are a consequence of the flow pattern changes over the heated element with Rayleigh number.

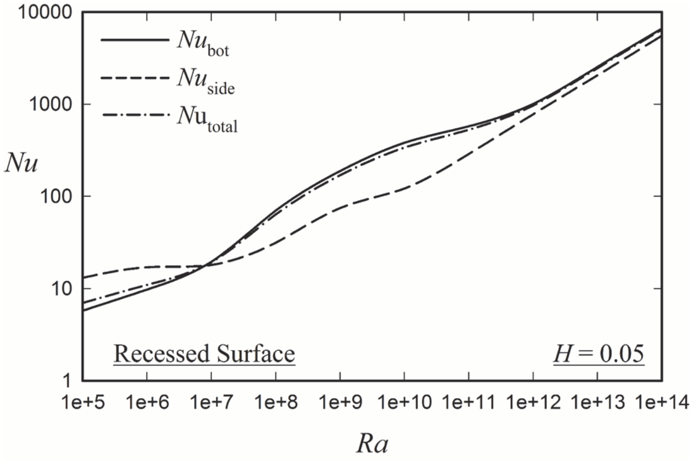

Figure 2 Variations of Nusselt numbers with Rayleigh number for a recessed element for a dimensionless recess depth of 0.05

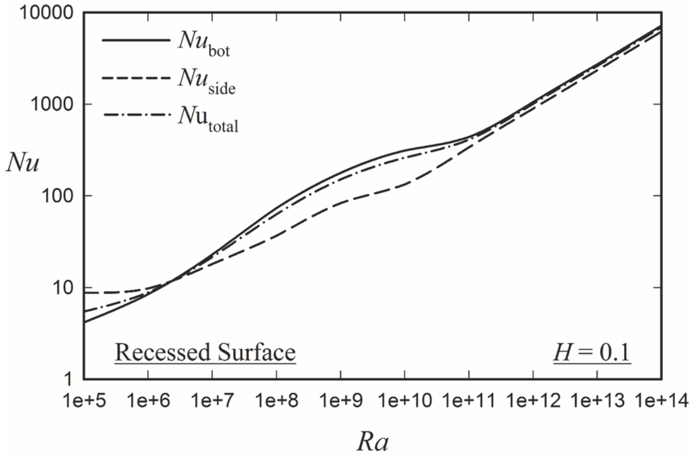

Figure 3 Variations of Nusselt numbers with Rayleigh number for a recessed element for a dimensionless recess depth of 0.1

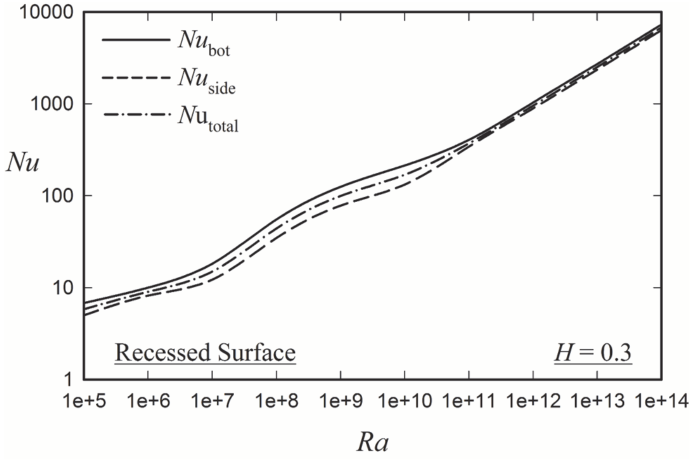

Figure 4 Variations of Nusselt numbers with Rayleigh number for a recessed element for a dimensionless recess depth of 0.3

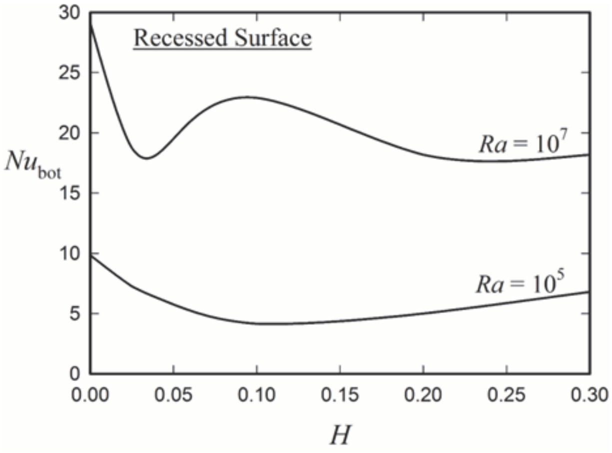

Figure 5 Variation of mean Nusselt number for the bottom heated surface of a recessed element with dimensionless recess depth for Rayleigh number values of 10^5 and 10^7

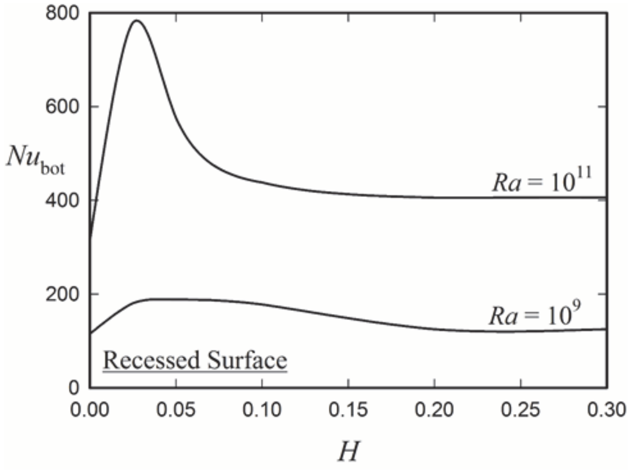

Figure 6 Variation of mean Nusselt number for the bottom heated surface of a recessed element with dimensionless recess depth for Rayleigh number values of 10^9 and 10^{11}

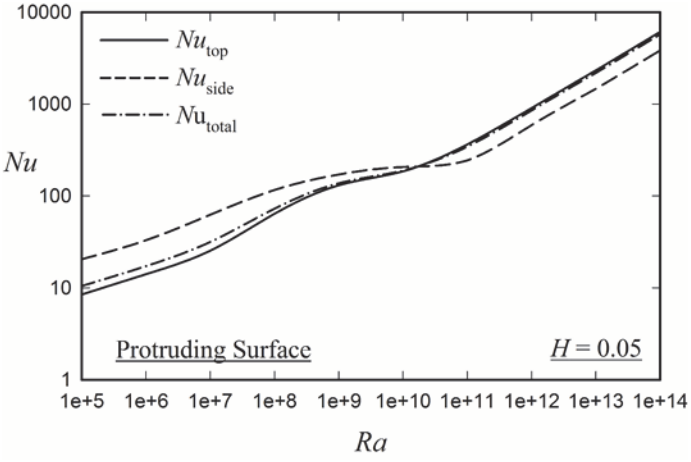

Figure 7 Variations of Nusselt numbers with Rayleigh number for a protruding element for a dimensionless protrusion height of 0.05

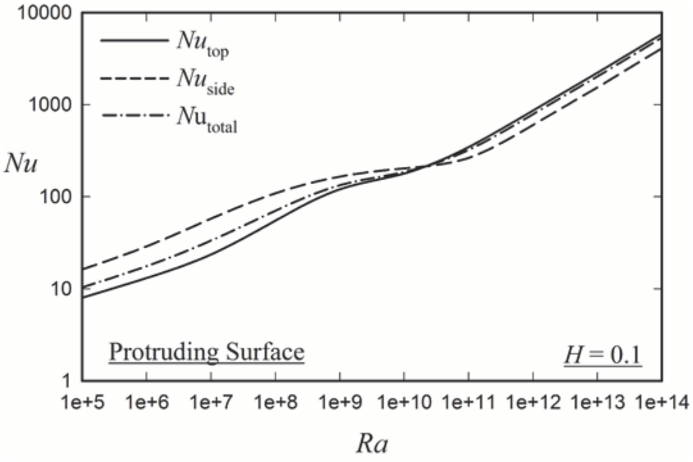

Figure 8 Variations of Nusselt numbers with Rayleigh number for a protruding element for a dimensionless protrusion height of 0.1

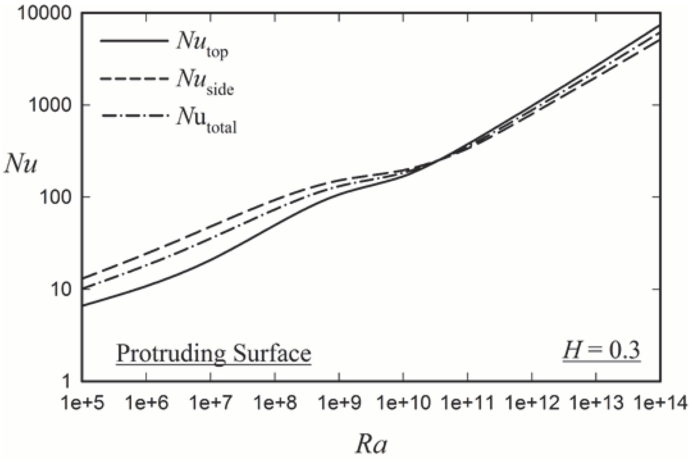

Figure 9 Variations of Nusselt numbers with Rayleigh number for a protruding element for a dimensionless protrusion height of 0.3

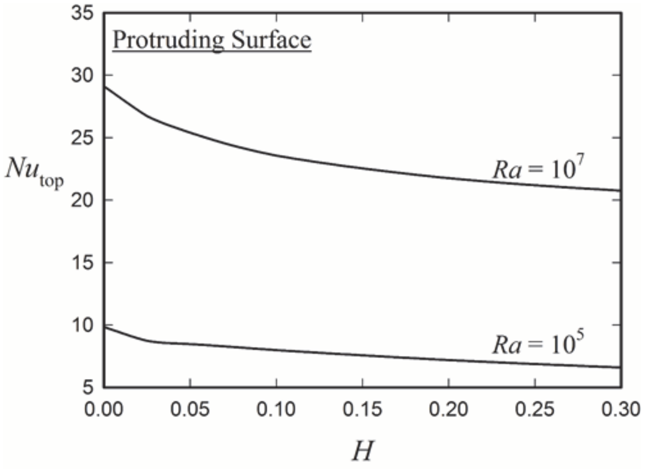

Figure 10 Variation of mean Nusselt number for the top heated surface of a protruding element with dimensionless protrusion height for Rayleigh number values of 10^5 and 10^7

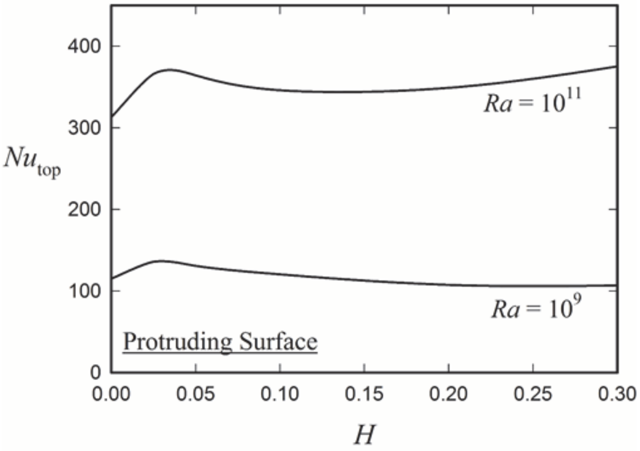

Figure 11 Variation of mean Nusselt number for the top heated surface of a protruding element with dimensionless protrusion height for Rayleigh number values of 10^9 and 10^{11}

The case of the heated element protruding from the surrounding adiabatic surface will next be considered. Figures 7 to 9 show typical variations of the mean Nusselt numbers for the circular top surface, the vertical side surface, and the entire heated surface with Rayleigh number for various values of the dimensionless distance to which the element protrudes, *H*. Figures 10 and 11 show variations of the mean Nusselt number for the top surface with the dimensionless protrusion height for various Rayleigh number values which further illustrates the effect of the dimensionless protrusion height.

Comparison of the results shown in Figs. 5 and 6 for a recessed element with those shown in Figs. 10 and 11 for a protruding element illustrate the very significant differences between the forms of the variations of Nusselt number with dimensionless recess depth for a recessed element compared to the forms of Nusselt number variations with dimensionless protrusion height for a protruding element. These differences arise from the flow patterns differences that occur in the two situations.

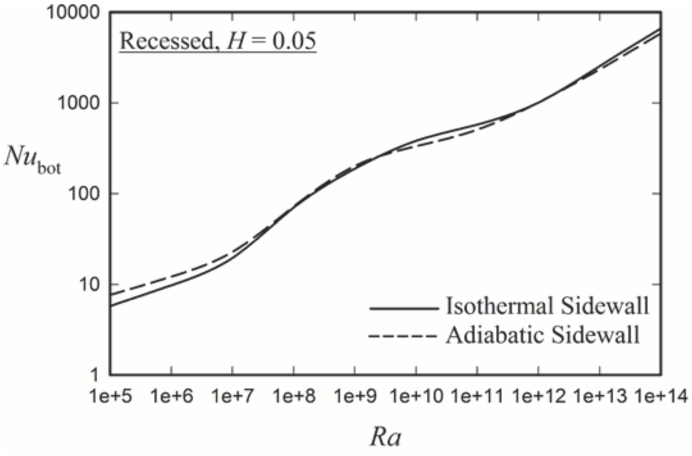

Figure 12 Variations of mean Nusselt number for the bottom heated surface of a recessed element with Rayleigh number for a dimensionless recess depth of 0.05 for the cases where the side wall is heated and where it is adiabatic

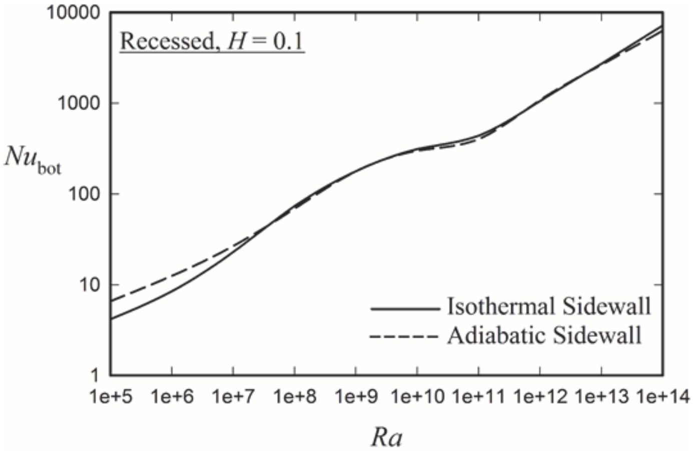

Figure 13 Variations of mean Nusselt number for the bottom heated surface of a recessed element with Rayleigh number for a dimensionless recess depth of 0.1 for the cases where the side wall is heated and where it is adiabatic

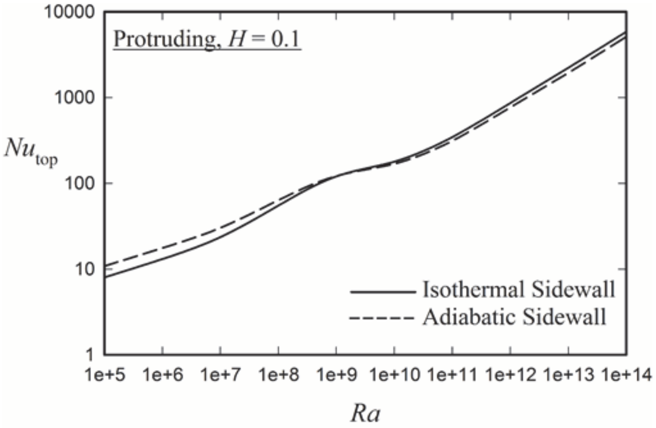

Figure 14 Variations of mean Nusselt number for the top heated surface of a protruding element with Rayleigh number for a dimensionless protrusion height of 0.05 for the cases where the side wall is heated and where it is adiabatic

The results given in Figs. 12 to 15 show that in both the case of a recessed element and the case of a protruding element the largest differences between the heat transfer rates for the heated sidewall case and those for the adiabatic side wall case occur at the lower Rayleigh number values considered. Under these conditions the mean Nusselt number for the heated sidewall case at a particular Rayleigh is less than that for the adiabatic sidewall case at the same Rayleigh number. At the larger Rayleigh number values considered the mean Nusselt number for the heated sidewall case at a particular Rayleigh number is greater than that for the adiabatic sidewall case at the same Rayleigh number, the difference however being relatively small and somewhat larger in the case of a protruding element than in the case of a recessed element.

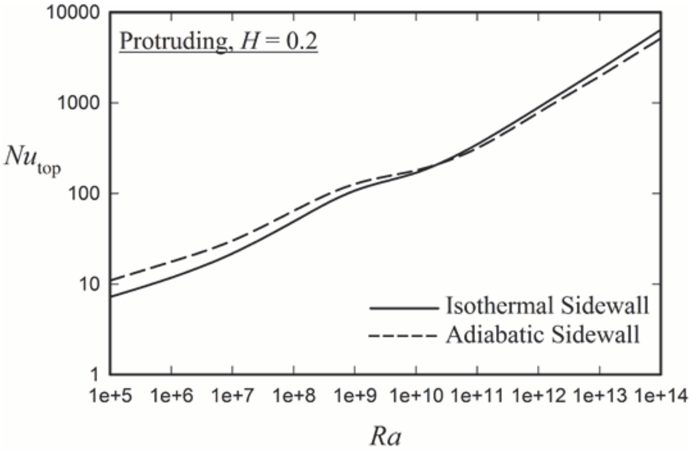

Figure 15 Variations of mean Nusselt number for the top heated surface of a protruding element with Rayleigh number for a dimensionless protrusion height of 0.2 for the cases where the side wall is heated and where it is adiabatic

CONCLUSIONS

The results obtained in the present study show that the thermal boundary conditions on the sides of a recessed or a protruding element, i.e., either adiabatic or isothermal, has a relatively small effect on the mean heat transfer rate from the bottom circular surface in the case of recessed element or on the mean heat transfer rate from the top circular surface in the case of protruding element. The biggest effect of the thermal boundary conditions on the sides of a recessed or a protruding element was found to occur at the lower Rayleigh numbers considered.

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