Design and Implementation Issues for a Class of Distribution-Free Phase II EWMA Exceedance Control Charts

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ABSTRACT

Distribution-free (nonparametric) control charts can play an essential role in process monitoring when there is dearth of information about the underlying distribution. In this paper, we study various aspects related to an efficient design and execution of a class of nonparametric Phase II exponentially weighted moving average (denoted by NPEWMA) charts based on exceedance statistics. The choice of the Phase I (reference) sample order statistic used in the design of the control chart is investigated. We use the exact time-varying control limits and the median run-length as the metric in an in-depth performance study. Based on the performance of the chart, we outline implementation strategies and make recommendations for selecting this order statistic from a practical point of view and provide illustrations with a dataset. We conclude with a summary and some remarks.

Keywords: Average run-length (*ARL*); Exponentially Weighted Moving Average (EWMA), Median run-length (*MRL*), Nonparametric, Order Statistic, Precedence.

1. Introduction

Most processes in the real world do not follow a normal distribution and often their exact distributions are not known. In such situations, nonparametric (distribution-free) process monitoring is the best alternative as it comes with a key advantage of in-control (IC) robustness even when the underlying distribution is unknown. This is because the IC run-length distribution of an exactly distribution-free control chart remains the same for all continuous process distributions. The growing trend of research and practical utilities of nonparametric process control charts may be seen, for example, from Chakraborti et al. (2015) and Mukherjee and Marozzi (2016a). They noted a nearly 200% growth on research in nonparametric process control charts in the first half of the current decade. Interested readers may see Chakraborti and Graham (2007), Chakraborti et al. (2011), Graham et al. (2012, 2014), Mukherjee and Chakraborti (2012), Mukherjee et al. (2013), Balakrishnan et al. (2015) Mukherjee and Sen (2015), Li et al. (2016) and Mukherjee and Marozzi (2016b) among others for various aspects of nonparametric control charts. Some other recent works include Hawkins and Deng (2010) who considered a nonparametric control chart under a change-point set-up and Abbasi et al. (2013) who considered a nonparametric control chart for the progressive mean. For a comprehensive discussion on several nonparametric process control charts see the book by Qiu (2014).

The Shewhart-type charts are the most extensively implemented charts in practice over the last few decades because of its simplicity and efficiency in detecting abrupt and typically larger shifts in a process. Nevertheless, other types of charts, such as the exponentially weighted moving average (EWMA) charts are often more beneficial and appropriate in the process control environment in detecting smaller and persistent shifts in a process. Roberts (1959) first introduced the EWMA charts for subgroup averages and, following this, since then there has been an incredible amount of work on EWMA charts (see e.g. the overview by Ruggeri et al. (2007) and the citations therein). Some more recent references include Maravelakis and Castagliola (2009), Huwang et al. (2010), Su et al. (2011), Haq (2013), Lu et al. (2013), Abbas et al. (2013, 2014), Lu (2015), Liu et al. (2015) and Khaliq et al. (2016). Interested readers may also see Knoth (2015) for a nice discussion on the run-length quantiles of EWMA control charts for monitoring normal mean and/or variance. Traditional parametric EWMA charts based on subgroup averages usually assume that the underlying process distribution is exactly or closely normal. Such an assumption is often invalid in practice. Human et al. (2011) recently showed that the parametric EWMA chart can lack IC robustness for some non-normal distributions. The problem is aggravated when some of the true process parameters are unknown and are subsequently estimated from a reference sample. For a detailed account of non-robustness of traditional parametric EWMA charts under non-normality, readers may see Graham et al. (2012).

In the present work, we mainly focus on the design and execution issues concerning the nonparametric EWMA exceedance (denoted by NPEWMA-EX) chart proposed by Graham et al. (2012). While constructing their NPEWMA-EX chart, Graham et al. (2012) focused on the reference sample median as classically the median is robust and one of the most commonly used measures of location in practice. Most of the traditional works in the field of nonparametric hypothesis testing and control charts abundantly use sample median and Graham et al. (2012) is no exception. In the recent years, a key question of which order statistic (or percentile) from the reference sample should be chosen has surfaced. Graham et al. (2014) addressed this issue with reference to a class of Phase II exceedance cumulative sum (CUSUM) chart proposed earlier by Mukherjee et al. (2013). Mukherjee et al. (2013) also used the median of the reference sample order statistics and later Graham et al. (2014) found that more often the 25^{th} or the 75^{th} percentile is the better choice and, in fact, the median is more often is the poorest choice. These observations relate to the class of exceedance CUSUM charts (denoted CUSUM-EX) and to the best of our knowledge, the effects of the choice of different percentiles of the reference sample on the performance of the NPEWMA-EX chart have not been examined yet. To bridge this research gap, in this paper we investigate the performance of the NPEWMA-EX chart systematically, based on the 25th, 40th, 50th, 60th and 75th percentiles, respectively. Precisely, following the line of Graham et al. (2014), we search for the best performing order statistic (percentile), from the reference sample that will enhance the efficiency of the chart. Further, unlike Graham et al. (2012) we consider the exact time-varying control limits instead of asymptotic control limits (also referred to as steady-state control limits) and use the median run-length (*MRL*) as the performance metric. We discuss more on these issues in the subsequent sections.

The content of rest of the paper is presented in different Sections as follows: In the next section the NPEWMA-EX charts are introduced. In Section 3 we consider practical approach of implementation of the charts. In Section 4 the IC and out-of-control (OOC) control chart performance is studied with extensive simulations. Illustrative examples are given in Section 5. We conclude with a summary and some recommendations.

2. Statistical background: NPEWMA-EX chart

In this section, we clearly outline the notation and the statistical preliminaries used in the paper. We denote the reference sample (Phase I sample or training sample or retrospective/historical data) of size m by $X_1, X_2, \dots, X_m$ from an IC process with a cdf $F(x)$. Establishment of the reference sample is in itself a research issue but is beyond the scope of this paper. Here we assume that a reference sample is available a-priori. We further denote the j^{th} $(j = 1,2, \dots,)$ test sample (Phase II sample) of size n from a cdf $G(y)$ by $Y_{j1}, Y_{j2}, \dots, Y_{jn}$. In the present work, we assume that both F and G are unknown continuous cdfs and consider the model $G_Y(x) = F(x - \theta)$ where $\theta \epsilon (-\infty, \infty)$ is the location parameter. Clearly, the process is IC when $F = G$ or $\theta = 0$ and we are interested in detecting shifts in the location parameter θ.

Let the number of Y observations in the j^{th} test sample that exceeds $X_{(r)}$, the r^{th} ordered observation in the reference sample, be denoted by $U_{j,r}$. The statistic $U_{j,r}$ is popularly known as an exceedance statistic and the probability $p_r = P[Y > X_{(r)} | X_{(r)}]$ is referred to as the exceedance probability. It is worth mentioning that the number of Y observations in the j^{th} Phase II sample that precede $X_{(r)}$ is known as a precedence statistic, a term coined by Nelson (1963) and was used by Chakraborti et al. (2004) to study the Shewhart-type precedence charts. From the Result A.3 of the Appendix in Mukherjee et al. (2013), one can easily see that the joint distribution of the exceedance statistics is does not depend on the underlying cdf's when the process is IC. Hence, control charts based on exceedance statistics are distribution-free and therefore, the class of NPEWMA-EX charts is distribution-free.

Details of the construction of the NPEWMA-EX chart for the reference sample median are provided in Graham et al. (2012) and updating this for any order statistic from the reference sample is straight forward. Graham et al. (2012, 2014) and Mukherjee et al. (2013), among others, noted that conditionally on $X_{(r)}$, that is, given the value of the order statistic $X_{(r)} = x_{(r)}$, the variable $U_{j,r}$ follows a binomial(n, p_r) distribution. Consequently, one can construct a binomial-type EWMA chart using the $U_{j,r}$'s to monitor the process location using the charting statistic given by

$$Z_j = \lambda\, U_{j,r} + (1-\lambda) Z_{j-1} \quad \text{for} \quad j = 1,2,3, \ldots \tag{1}$$

where the starting value is generally taken as $Z_0 = E\left(U_{j,r}|X_{(r)}\right) = np_r$ and $0 < \lambda \leq 1$ is the smoothing constant. It is well-known that when $\lambda = 1$, the EWMA chart reduces to a Shewhart chart. Note that, np_r is random and can take any value in between 0 and n. Therefore, Graham et al. (2012) recommended switching to $Z_0 = EE\left(U_{j,r}|X_{(r)}\right) = nE(p_r) = n(1-a)$, where $a = r/(m+1)$, and did not explore other possible choices.

Now we need to derive the IC mean and IC standard deviation of Z_j to calculate the control limits and the centerline (CL) of the proposed chart. With an arbitrary starting value $Z_0 = \tau$, where τ lies between 0 and n, both inclusive, the unconditional IC mean and the unconditional IC standard deviation of Z_j are given by

$$E\left(Z_j\right) = n(1-a)\left(1-(1-\lambda)^j\right) + (1-\lambda)^j \tau \qquad \ldots \ldots \tag{2}$$

and

$$STDEV\left(Z_j\right) = \sqrt{\left(\frac{na(1-a)}{m+2}\right)\left\{n(1-(1-\lambda)^j)^2 + \frac{\lambda(m+1)}{2-\lambda}(1-(1-\lambda)^{2j})\right\}} \quad \ldots \tag{3}$$

respectively. This is true for any prefixed value of τ in the interval 0 and n. Detailed proofs are given in Appendix A. Hence, the NPEWMA-EX chart has a charting statistic, Z_j, as in Equation (1), with $Z_0 = \tau$, and the exact time-varying upper control limit (UCL), lower control limit (LCL) and CL of the chart are given by $CL = E\left(Z_j\right)$ and $UCL/LCL = E\left(Z_j\right) \pm L \times STDEV\left(Z_j\right)$ where the IC mean and the IC standard deviation are given in Equations (2) and (3), respectively. Irrespective of the value of τ, the corresponding unconditional asymptotic control limits and CL are given by

$$CL = n(1-a) \qquad \ldots \ldots \tag{4}$$

and

$$LCL/UCL = n(1-a) \pm L\sqrt{\left(\frac{na(1-a)}{m+2}\right)\left\{n + \frac{\lambda(m+1)}{2-\lambda}\right\}} \qquad \ldots . \tag{5}$$

respectively. Graham et al. (2012) primarily considered such asymptotic limits, that are obtained from Equations (2) and (3), by letting $j \to \infty$ so that the term $(1-\lambda)^j$ approaches zero and the terms $\left(1-(1-\lambda)^j\right)$ and $\left(1-(1-\lambda)^{2j}\right)$ approach one, respectively. The process is declared OOC if any Z_j plots on or outside either one of the control limits and a search for possible assignable causes is started. If not, the process is considered IC and we continue monitoring the process. It is also worth mentioning that the NPEWMA chart looks and operates alike the parametric EWMA chart (denoted EWMA-$\overline{X}$ hereafter) but, additionally, comes with the IC robustness property since distribution-free exceedance statistics are used in place of the averages.

Note that if steady-state limits are considered, $Z_0 = n(1-a)$ is the only choice. It can be empirically checked, using Monte-Carlo simulations, that other choices of τ are inadmissible when asymptotic control limits are used, since it will seriously impact the false alarm rate (*FAR*). For example, if $\tau = 0$ is used, along with asymptotic control limits, almost invariably a false alarm will be found right at the beginning and make the charting procedure unusable. There is no scope of adventure with other choices for Z_0 if asymptotic limits are used. Consequently, Graham et al. (2012) used $\tau = n(1-a)$ and omitted the effect of various choices of τ. Nevertheless, unlike the steady-state case, where $Z_0 = n(1-a)$ is the only possibility, the exact case is more accommodative regarding the choice of Z_0. The exact time-varying control limits depends on τ and empirical studies show that there is no performance difference if $\tau = 0$ is used instead of $\tau = n(1-a)$. Therefore, unlike Graham et al. (2012), we display the results $\tau = 0$ in this paper. Details are discussed in Appendix A. Computational results will be almost the same if $\tau = n(1-a)$ is used and, consequently, are omitted. Readers should not take it for granted that the choice $Z_0 = 0$ is superior to $Z_0 = n(1-a)$, or vice-versa. There is no clear winner when exact limits are considered.

In the current context, we propose using symmetrically placed upper and lower control limits. We typically make use of symmetrically placed control limits when the median of the Phase I observations is considered and the underlying population distribution is symmetric. In such cases, the distribution of $U_{j,r}$ is symmetric. For other order statistics or skewed distributions, such a design may be biased, but in the two-sided EWMA chart with an asymmetric statistic, such a design is often used for simplicity. The use of symmetrically placed control limits, for a two-sided chart for monitoring both decreasing and increasing shifts, with a plotting statistic having an asymmetrical distribution may lead to an *ARL*-biased chart, that is, some ARL_δ values are larger than the ARL_0 value. Recently, Knoth and Morais (2015) noted that: "problem of choosing the control limits of EWMA charts meant to monitor both increases and decreases in the process variance and based on asymmetrically distributed control statistics is not properly discussed in literature." They also pointed out that there are many instances in the literature where EWMA charts, for monitoring spread, have been developed that are *ARL*-biased (see, e.g. Wortham and Ringer (1971), Ng and Case (1989) and MacGregor and Harris (1993)) and that the problem of finding the (asymmetric) control limits of two-sided EWMA charts for monitoring spread has not been considered in the literature. They then go on to discuss the vanilla EWMA design and recommend that small values of λ be used, since this reduces the bias dramatically. We also follow the convention.

As noted earlier in the introduction, exact time-varying control limits are used in this paper. There are certain advantages to doing so. Steiner (1999) compared the run-length characteristics of the EWMA-$\overline{X}$ chart with the exact time-varying control limits to the run-length characteristics of the EWMA-$\overline{X}$ chart with asymptotic control limits. He used the average run-length (*ARL*) as a performance measure and showed that for an IC process, the IC *ARL* (denoted ARL_0) values of EWMA charts with time-varying control limits are nearly identical to those of EWMA charts with asymptotic control limits. However, if a shift in process location takes place soon after the monitoring starts, i.e. if the process goes out of control at an early stage, the OOC *ARL* (denoted ARL_δ) values may differ substantially depending on the value of the smoothing constant λ. Steiner (1999) concluded that, in general, exact time-varying control limits are useful when λ is small, say, less than 0.3. For an elaborate discussion on the differences between asymptotic and exact time-varying limits, we refer Knoth (2003, 2005). The choice of the two design parameters, λ and L, for the proposed charts is deliberated in more detail in Section 3.1.

3. Run-length distribution

Several authors have considered the *FAR* as the performance metric but this is no so well-accepted among the practitioners, particularly when parameters are unknown and are estimated. The performance of a control chart is popularly studied via its run-length distribution. The *ARL* and the standard deviation of the run-length (*SDRL*) distribution are commonly used as the performance indicators. Nevertheless, noting that the run-length distribution is significantly right-skewed, many researchers recommend examining a number of percentiles including the 5th, 25th, median, 75th and the 95th percentiles to better characterize the run-length distribution. Moreover, there are several shortcomings of the *ARL* as a performance measure as summarized in Graham et al. (2014). Therefore, in this paper we use the median run-length (*MRL*) to measure the chart performance. This is supported and motivated by the works of several authors including Gan (1994), Radson and Boyd (2005), Khoo et al. (2011), and Graham et al. (2014). To this end, we set the desired nominal MRL_0, say $MRL_0^* = 350$, meaning that there is at least a 50% chance that the first OOC signal will be witnessed at or before the 350th sample even though the process is actually IC. In other words, 50% of the IC run-lengths will be greater than or equal to 350 and 50% will be less. Graham et al. (2014) discussed the motivation behind choosing MRL_0 equal to 350. They showed that for a traditional Shewhart $\overline{X}$ chart when ARL_0 is set as 500; MRL_0 becomes close to 346. Naturally, in such charts, if one sets MRL_0=350, the actual ARL_0 will be marginally higher that 500, the current industry standard.

3.1 Implementation of the chart: Chart design parameters

Practical deployment of the NPEWMA-EX charts requires specifying the following parameters: (i) *m*: the size of the reference sample from the IC state, (ii) *n*: the size of each test sample (the rational subgroup size), (iii) *r*: the order of the reference sample order statistic, (iv) MRL_0^*: the desired MRL_0, (v) λ: the smoothing parameter and (vi) L: which determines the width of the control limits. It is up to the investigator to specify the parameters *m*, *n*, *r* and MRL_0^*. The choice of the design parameters (λ, L) of the chart consists of two steps: First, using a search algorithm to determine the (λ, L) combinations that produce the desired MRL_0 for a given m, n, r, λ and L. A detailed simulation algorithm is given in Appendix C of Graham et al. (2012) when using the median of the reference (Phase I) sample. SAS® v 9.3 was used to implement this simulation algorithm and the results were verified using R.3.2.2. These programs are easily adapted with minor modifications for the case when different order statistics from the reference (Phase I) sample is used.

To apply the chart in a practical situation, we first need to choose appropriate λ. For small shifts (which are approximately less than or equal to 0.5 standard deviations) a small value of λ is chosen, say λ = 0.01, 0.025 or 0.05. For moderate shifts (which are approximately between 0.5 and 1.5 standard deviation) a larger value of λ is chosen, say λ = 0.10. For large shifts (which are approximately more than 1.5 standard deviations) an even larger value of λ is chosen, say λ = 0.20 (see e.g. Montgomery (2009), page 423). Next we choose L, in combination with the chosen λ, so that a desired nominal MRL_0 is attained. In this paper, we investigate λ = 0.05, 0.10 and 0.20, respectively, following the guidelines set out by Steiner (1999), i.e. λ < 0.3. In fact, we also studied several higher values of λ and observe that the performance of the chart under higher values of λ is almost like that of a Shewhart chart and therefore we drop them from the subsequent discussions.

4. Performance comparisons

Several distributions, apart from the normal distributions, are considered for the performance study. This includes heavy-tailed symmetric, skewed non-normal and mixture of normal distributions. To be precise, we consider distributions in line with Graham et al. (2012): (a) the standard normal distribution, $N(0,1)$, (b) the exponential distribution with mean 1, $EXP(1)$, which is positively skewed, (c) the Double Exponential distribution $DE(0,1)$, also referred to as the Laplace distribution, with mean 0 and variance 2 which is symmetric but has heavier tails, (d) the Symmetric Mixture Normal distribution $[0.6N(\mu_1 = 0,\ \sigma_1 = 0.25) + 0.4N(\mu_2 = 0,\ \sigma_2 = 4)]$ denoted *SymmMixN*, (e) two Asymmetric Mixture Normal distributions with parameters $[0.6N(\mu_1 = 0.25,\ \sigma_1 = 0.25) + 0.4N(\mu_2 = 0,\ \sigma_2 = 4)]$ and $[0.6N(\mu_1 = -0.25,\ \sigma_1 = 0.25) + 0.4N(\mu_2 = 0,\ \sigma_2 = 4)]$, denoted *AsymmMixN1* and *AsymmMixN2* respectively, and (f) the Log-Logistic ($\alpha = 1$, $\beta = 2.5$) distribution. Graham et al. (2012) considered these mixture normal distributions which are fairly heavier tailed than the normal and with higher kurtosis. Note that all distributions included in the study have been standardized, that is, the mean is translated to 0 and the standard deviation is scaled to unity. As a consequence, the results are easily comparable across the distributions. Without loss of generality, we consider $Z_0 = 0$ throughout the numerical investigation. See the Appendix for more details.

Table 1 shows some (λ, L)-combinations for the NPEWMA-EX chart for a nominal MRL_0, $MRL_0^* =$ 350 for m = 100 and n = 5. The first row of each cell in Table 1 shows the attained MRL followed by the

Table 1. (λ, L)-Combinations for the NPEWMA-EX chart for nominal MRL_0= 350 for m = 100 and n = 5

		25th percentile		40th percentile		50th percentile		60th percentile		75th percentile	
Shift	λ	L	Attained values	L	Attained values	L	Attained values	L	Attained values	L	Attained values
Small	0.05	2.041	341 (1702) 3, 43, 1745, 8289	2.044	342 (1710) 4, 44, 1754, 8294	2.091	345 (1933) 1, 30, 1963, 10496	2.044	342 (1627) 4, 42, 1669, 8305	2.041	363 (1739) 3, 46, 1785, 8910
Moderate	0.10	2.347	344 (1054) 4, 80, 1134, 4139	2.380	351 (1031) 5, 81, 1112, 3921	2.384	352 (1036) 7, 88, 1124, 3847	2.380	355 (1053) 5, 84, 1137, 4028	2.347	347 (1055) 4, 78, 1133, 3931
Large	0.20	2.608	345 (961) 7, 92, 1053, 3451	2.653	356 (803) 15, 112, 915, 2540	2.676	353 (791) 17, 119, 910, 2581	2.653	345 (795) 13, 109, 904, 2581	2.608	349 (922) 9, 93, 1015, 3255

interquartile range (IQR) in parentheses, whereas the second row shows the values of the 5th, 25th, 75th and 95th percentiles (in this order). Note that Tables 1 to 8 are presented in this manner.

From Table 1 it is seen that the design parameter, L, is the same for the 25th and 75th percentiles and for the 40th and 60th percentiles, respectively. This is due to the fact that $STDEV(U_{j,r})$ is the same for the pair of percentiles $(M - \Delta_1, M + \Delta_1)$ where M denotes the median and Δ_1 is an integer between 1 and 49. Next, we study the OOC chart performance.

4.1 Out-of-control chart performance comparisons

For the OOC chart performance comparison we ensure that the MRL_0 values of the competing charts are fixed at (or very close to) an acceptably high value, such as 350 in this case, and then compare their MRL_δ values, for specific values of the shift δ, and the chart with the smaller MRL_δ value is preferred. Graham et al. (2012) studied the effect of the reference sample size when using the median of the reference (Phase I) sample and concluded that the larger the reference sample size, the less the uncertainty and the better the performance of the chart, and that, generally, when the reference sample size is not less than 100, the proposed chart performs well. Accordingly, in this paper, we take the size of the IC Phase I reference sample to be 100, i.e. m = 100. Tables 2 to 8 show the OOC performance characteristics of the run-length distribution for various distributions and shifts $\delta = \gamma \frac{\sigma}{\sqrt{n}}$, where σ denotes the process standard deviation, γ = $\pm$0.25, $\pm$0.50, $\pm$0.75, $\pm$1.00, $\pm$1.50 and $\pm$2.00, represents the shift in the median, for $m = 100$ and $n = 5$. The Tables with Series a (2a to 8a) provide OOC performances for negative shifts and the Tables with Series b (2b to 8b) provide OOC performances for positive shifts, respectively, both under various distributions. We also compare the NPEWMA-EX chart to the nonparametric EWMA chart based on the Wilcoxon rank-sum statistic (denoted NPEWMA-Rank) chart proposed by Li et al. (2010).

Table 2a. Control chart performance comparison under the $N(0,1)$ distribution for m = 100 and n = 5 for negative shifts

Shift (γ)	NEWMA-EX chart					NPEWMA-Rank chart
	25th percentile	40th percentile	50th percentile	60th percentile	75th percentile	
	λ = 0.05					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 34.25
-0.25	122 (1339) 3, 35, 1374, 7885	117 (790) 3, 17, 807, 6060	137 (978) 1, 16, 994, 7896	165 (1013) 4, 4, 1017, 6674	225 (1221) 3, 34, 1255, 7364	139 (481) 21, 50, 531, 3367
-0.50	26 (136) 2, 7, 143, 2263	28 (126) 2, 8, 134, 2033	29 (139) 1, 7, 146, 2387	35 (162) 3, 11, 173, 2543	56 (278) 3, 15, 293, 3703	41 (68) 13, 24, 92, 695
-0.75	11 (27) 1, 5, 32, 278	11 (26) 2, 4, 30, 235	12 (29) 1, 4, 33, 249	15 (32) 3, 6, 38, 288	22 (55) 3, 9, 64, 616	22 (20) 10, 15, 35, 94
-1.00	6 (11) 1, 3, 14, 63	6 (10) 1, 3, 13, 47	7 (13) 1, 3, 16, 54	9 (14) 2, 4, 18, 63	12 (21) 3, 6, 27, 113	15 (10) 8, 11, 21, 39
-1.50	3 (4) 1, 2, 6, 13	4 (4) 1, 2, 6, 13	3 (5) 1, 2, 7, 14	4 (5) 2, 3, 8, 16	6 (6) 3, 5, 11, 25	9 (4) 6, 8, 12, 17
-2.00	2 (2) 1, 1, 3, 6	2 (2) 1, 2, 4, 7	2 (3) 1, 1, 4, 7	4 (2) 2, 2, 4, 8	5 (4) 3, 3, 7, 12	7 (2) 5, 6, 8, 11
Shift ($\boldsymbol{\gamma}$)	λ = 0.10					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 48.5
-0.25	132 (558) 2, 26, 584, 2999	166 (622) 3, 35, 657, 3023	180 (663) 4, 39, 702, 3019	219 (775) 5, 52, 827, 3376	306 (963) 7, 74, 1037, 3872	150 (490) 16, 49, 539, 2684
-0.50	34 (129) 2, 9, 138, 1241	41 (145) 2, 12, 157, 1269	49 (164) 3, 15, 179, 1340	64 (226) 4, 20, 246, 1730	109 (385) 6, 32, 417, 2428	40 (86) 10, 20, 106, 696
-0.75	14 (35) 1, 5, 40, 258	16 (37) 2, 6, 43, 273	19 (43) 2, 8, 51, 299	23 (53) 3, 10, 63, 367	40 (100) 6, 16, 116, 763	19 (23) 7, 12, 35, 117
-1.00	7 (14) 1, 3, 17, 64	9 (15) 1, 4, 19, 64	10 (17) 2, 5, 22, 76	13 (20) 2, 6, 26, 97	20 (35) 4, 10, 45, 188	12 (10) 6, 9, 19, 40
-1.50	3 (4) 1, 2, 6, 16	4 (5) 1, 2, 7, 16	5 (6) 2, 3, 9, 18	6 (7) 2, 3, 10, 22	10 (10) 4, 6, 16, 36	7 (4) 4, 6, 10, 15
-2.00	2 (2) 1, 1, 3, 7	3 (2) 1, 2, 4, 8	3 (3) 2, 2, 5, 9	4 (4) 2, 2, 6, 11	7 (6) 4, 4, 10, 17	5 (2) 4, 5, 7, 9
Shift ($\boldsymbol{\gamma}$)	λ = 0.20					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 69.5
-0.25	114 (393) 3, 31, 424, 2231	162 (488) 7, 49, 537, 2046	221 (572) 10, 64, 636, 2200	272 (659) 12, 82, 741, 2308	468 (1096) 19, 147, 1243, 3548	176 (478) 13, 55, 533, 2158
-0.50	38 (114) 2, 12, 126, 803	54 (149) 4, 18, 167, 922	75 (213) 6, 26, 239, 1192	109 (312) 8, 35, 347, 1515	250 (779) 16, 90, 869, 3090	49 (121) 8, 20, 141, 782
-0.75	17 (38) 1, 6, 44, 202	22 (49) 2, 9, 58, 265	29 (66) 4, 12, 78, 372	40 (91) 5, 16, 107, 533	109 (307) 11, 38, 345, 1638	20 (32) 6, 11, 43, 158
-1.00	9 (17) 1, 4, 21, 74	12 (19) 1, 6, 25, 83	15 (25) 3, 8, 33, 113	20 (34) 4, 10, 44, 155	46 (99) 8, 20, 119, 558	12 (13) 4, 7, 20, 52
-1.50	4 (6) 1, 2, 8, 18	5 (6) 1, 3, 9, 21	7 (7) 2, 4, 11, 26	9 (9) 3, 6, 15, 34	18 (20) 6, 11, 31, 81	6 (4) 3, 5, 9, 15
-2.00	3 (3) 1, 1, 4, 8	3 (3) 1, 2, 5, 10	4 (4) 2, 3, 7, 12	6 (4) 3, 4, 8, 15	11 (8) 6, 8, 16, 30	4 (3) 3, 3, 6, 8

Table 2b. Control chart performance comparison under the $N(0,1)$ distribution for m = 100 and n = 5 for positive shifts

Shift (γ)	NEWMA-EX chart					NPEWMA-Rank chart
	25th percentile	40th percentile	50th percentile	60th percentile	75th percentile	
	λ = 0.05					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 34.25
0.25	234 (1276) 3, 34, 1310, 8227	155 (983) 4, 23, 1006, 6054	139 (975) 1, 16, 991, 8595	129 (832) 3, 18, 850, 6247	118 (833) 2, 15, 848, 6616	130 (506) 20, 49, 555, 3836
0.50	60 (312) 3, 15, 327, 3759	37 (167) 3, 10, 177, 2619	31 (150) 1, 8, 158, 2532	28 (127) 2, 8, 135, 2160	27 (140) 2, 7, 147, 2547	41 (68) 13, 23, 91, 628
0.75	22 (57) 3, 9, 66, 635	15 (33) 3, 7, 40, 291	12 (30) 1, 4, 34, 225	11 (28) 1, 4, 32, 241	10 (25) 1, 4, 29, 262	22 (20) 10, 15, 35, 95
1.00	13 (23) 3, 6, 29, 112	9 (14) 2, 4, 18, 63	7 (13) 1, 3, 16, 55	7 (10) 1, 4, 14, 49	6 (11) 1, 3, 14, 56	15 (9) 8, 12, 21, 39
1.50	7 (9) 3, 3, 12, 25	4 (4) 2, 3, 7, 16	3 (5) 1, 2, 7, 14	4 (4) 1, 2, 6, 13	3 (4) 1, 2, 6, 13	9 (4) 6, 8, 12, 16
2.00	5 (5) 3, 3, 8, 13	4 (3) 2, 2, 5, 9	2 (3) 1, 1, 4, 8	2 (2) 1, 2, 4, 7	2 (2) 1, 1, 3, 7	7 (2) 5, 6, 8, 11
Shift ($\boldsymbol{\gamma}$)	λ = 0.10					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 48.5
0.25	312 (1007) 9, 78, 1085, 4070	229 (807) 5, 53, 860, 3295	188 (690) 5, 43, 733, 3172	150 (628) 2, 33, 661, 3011	137 (594) 2, 26, 620, 3204	154 (534) 15, 49, 583, 2688
0.50	121 (454) 8, 35, 489, 2540	64 (229) 3, 19, 248, 1728	52 (185) 3, 16, 201, 1548	41 (143) 2, 12, 155, 1346	35 (123) 1, 9, 132, 1255	40 (90) 10, 20, 110, 669
0.75	42 (109) 6, 17, 126, 826	24 (54) 2, 10, 64, 384	19 (43) 3, 8, 51, 298	16 (37) 2, 6, 43, 256	13 (33) 1, 5, 38, 267	19 (22) 7, 12, 34, 109
1.00	22 (37) 6, 11, 48, 203	13 (20) 2, 6, 26, 94	11 (17) 2, 5, 22, 76	9 (15) 1, 4, 19, 65	7 (14) 1, 3, 17, 70	12 (10) 6, 9, 19, 39
1.50	11 (10) 5, 7, 17, 38	6 (8) 2, 3, 11, 22	5 (6) 2, 3, 9, 18	4 (5) 1, 2, 7, 16	3 (4) 1, 2, 6, 15	7 (3) 4, 6, 9, 15
2.00	7 (4) 5, 6, 10, 18	4 (4) 2, 2, 6, 11	3 (3) 2, 2, 5, 9	2 (2) 1, 2, 4, 8	2 (3) 1, 1, 4, 7	5 (2) 4, 5, 7, 9
Shift ($\boldsymbol{\gamma}$)	λ = 0.20					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 69.5
0.25	500 (1193) 20, 158, 1351, 3920	272 (683) 13, 85, 768, 2388	230 (618) 11, 68, 686, 2344	169 (481) 6, 49, 530, 2091	123 (414) 4, 33, 447, 2116	186 (496) 13, 55, 551, 2211
0.50	337 (901) 18, 102, 1003, 3357	111 (323) 8, 35, 358, 1590	77 (226) 6, 26, 252, 1218	54 (152) 3, 18, 170, 957	38 (107) 2, 12, 119, 755	48 (116) 8, 20, 136, 672
0.75	130 (360) 12, 44, 404, 1912	40 (91) 6, 17, 108, 554	30 (64) 4,12, 76, 370	22 (49) 2, 9, 58, 266	16 (37) 1, 6, 43, 194	20 (31) 5, 11, 42, 159
1.00	54 (117) 9, 24, 141, 702	21 (36) 4, 10, 46, 170	16 (25) 3, 8, 33, 115	12 (19) 1, 6, 25, 84	9 (17) 1, 4, 21, 68	12 (12) 4, 8, 20, 51
1.50	19 (25) 6, 11, 36, 102	9 (9) 3, 6, 15, 35	7 (8) 2, 4, 12, 26	5 (6) 1, 3, 9, 20	4 (6) 1, 2, 8, 19	6 (4) 3, 5, 9, 16
2.00	11 (9) 6, 8, 17, 33	6 (4) 3, 4, 8, 15	4 (4) 2, 3, 7, 12	3 (3) 1, 2, 5, 10	3 (3) 1, 1, 4, 8	4 (3) 3, 3, 6, 8

Table 3a. Control chart performance comparison under the $EXP(1)$ distribution for m = 100 and n = 5 when target MRL_0 = 350 for negative shifts

Shift (γ)	NEWMA-EX chart					NPEWMA-Rank chart
	25th percentile	40th percentile	50th percentile	60th percentile	75th percentile	
	λ = 0.05					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 34.25
-0.25	23 (85) 2, 6, 91, 893	59 (320) 2, 12, 332, 4133	113 (786) 1, 15, 801, 7218	170 (964) 4, 26, 990, 6840	297 (1545) 3, 41, 1586, 8342	51 (91) 15, 28, 119, 622
-0.50	6 (11) 1, 3, 14, 39	13 (30) 2, 5, 35, 180	22 (73) 1, 6, 79, 657	39 (155) 3, 12, 167, 1951	119 (628) 3, 23, 651, 5229	20 (15) 9, 14, 29, 58
-0.75	4 (4) 1, 2, 6, 14	7 (10) 1, 4, 14, 36	10 (19) 1, 4, 23, 81	17 (36) 3, 7, 43, 224	44 (162) 3, 14, 176, 1983	13 (7) 7, 10, 17, 26
-1.00	3 (3) 1, 2, 5, 9	4 (5) 1, 3, 8, 17	6 (10) 1, 3, 13, 32	10 (16) 2, 5, 21, 63	24 (54) 3, 10, 64, 382	9 (4) 6, 8, 12, 17
-1.50	2 (2) 1, 1, 3, 5	3 (2) 1, 2, 4, 8	4 (5) 1, 2, 7, 13	6 (6) 2, 4, 10, 20	11 (17) 3, 6, 23, 62	7 (2) 5, 6, 8, 11
-2.00	1 (1) 1, 1, 2, 3	2 (1) 1, 2, 3, 6	3 (3) 1, 1, 4, 8	4 (3) 2, 3, 6, 12	8 (8) 3, 5, 13, 27	6 (2) 4, 5, 7, 8
Shift ($\boldsymbol{\gamma}$)	λ = 0.10					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 48.5
-0.25	30 (86) 2, 9, 95, 56	85 (315) 2, 21, 336, 2027	151 (566) 5, 37, 603, 2807	240 (818) 5, 54, 872, 3474	353 (1055) 7, 88, 1143, 4037	49 (94) 10, 23, 117, 500
-0.50	8 (14) 1, 3, 17, 47	19 (41) 2, 7, 48, 197	35 (86) 3, 13, 99, 571	68 (212) 4, 21, 233, 1606	204 (686) 8, 54, 740, 3232	16 (15) 6, 11, 26, 56
-0.75	4 (6) 1, 2, 8, 17	9 (14) 1, 4, 18, 47	16 (26) 2, 7, 33, 109	28 (57) 3, 12, 69, 296	85 (257) 7, 28, 285, 1770	10 (7) 5, 7, 14, 24
-1.00	3 (3) 1, 2, 5, 10	6 (7) 1, 3, 10, 22	9 (12) 2, 5, 17, 41	16 (24) 2, 8, 32, 90	43 (96) 6, 18, 114, 576	7 (4) 4, 6, 10, 15
-1.50	2 (2) 1, 1, 3, 5	3 (4) 1, 2, 6, 10	5 (5) 2, 3, 8, 16	8 (8) 2, 5, 13, 28	19 (26) 4, 10, 36, 97	5 (2) 3, 4, 6, 9
-2.00	1 (1) 1, 1, 2, 4	2 (2) 1, 2, 4, 6	4 (3) 2, 2, 5, 10	6 (5) 2, 3, 8, 15	12 (13) 4, 7, 20, 41	4 (1) 3, 4, 5, 7
Shift ($\boldsymbol{\gamma}$)	λ = 0.20					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 69.5
-0.25	35 (79) 2, 12, 91, 373	105 (293) 5, 32, 325, 1384	192 (524) 10, 59, 583, 2089	283 (668) 13, 90, 758, 2334	494 (1109) 17, 157, 1266, 3643	53 (104) 8, 22, 126, 439
-0.50	10 (16) 1, 5, 21, 53	28 (55) 3, 11, 66, 231	56 (129) 5, 21, 150, 663	115 (310) 9, 39, 349, 1359	424 (1036) 20, 139, 1175, 3457	15 (18) 5, 9, 27, 65
-0.75	5 (7) 1, 3, 10, 20	12 (18) 1, 6, 24, 63	23 (40) 4, 11, 51, 160	47 (93) 6, 20, 113, 440	251 (655) 16, 80, 735, 2638	9 (7) 3, 6, 13, 25
-1.00	4 (4) 1, 2, 6, 11	8 (9) 1, 4, 13, 30	14 (18) 3, 7, 25, 64	25 (39) 5, 12, 51, 153	123 (299) 12, 45, 344, 1456	6 (5) 3, 4, 9, 14
-1.50	2 (3) 1, 1, 4, 6	4 (4) 1, 3, 7, 12	7 (7) 2, 4, 11, 21	12 (13) 4, 7, 20, 45	43 (74) 9, 20, 94, 295	4 (2) 2, 3, 5, 8
-2.00	1 (1) 1, 1, 2, 4	3 (3) 1, 2, 5, 8	5 (4) 2, 3, 7, 12	8 (7) 3, 5, 12, 22	22 (28) 7, 13, 41, 97	3 (1) 2, 3, 4, 6

Table 3b. Control chart performance comparison under the *EXP*(1) distribution for m = 100 and n = 5 when target MRL_0 = 350 for positive shifts

Shift (γ)	NEWMA-EX chart					NPEWMA-Rank chart
	25[th] percentile	40[th] percentile	50[th] percentile	60[th] percentile	75[th] percentile	
	λ = 0.05					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 34.25
0.25	23 (96) 3, 8, 104, 2391	49 (322) 3, 11, 333, 4271	68 (534) 1, 11, 545, 6466	98 (686) 2, 15, 701, 5805	134 (985) 2, 18, 1003, 6998	62 (183) 15, 29, 212, 3427
0.50	5 (5) 3, 3, 8, 32	9 (19) 2, 4, 23, 233	12 (40) 1, 5, 45, 904	20 (90) 2, 6, 96, 1892	40 (288) 2, 8, 296, 4190	21 (19) 10, 15, 34, 15
0.75	3 (0) 3, 3, 3, 5	4 (4) 2, 3, 7, 23	5 (10) 1, 2, 12, 61	7 (15) 1, 4, 19, 160	14 (52) 1, 5, 57, 1262	13 (8) 8, 10, 18, 32
1.00	3 (0) 3, 3, 3, 3	2 (2) 2, 2, 4, 7	3 (4) 1, 1, 5, 15	4 (7) 1, 2, 9, 32	7 (17) 1, 3, 20, 206	10 (4) 7, 8, 12, 18
1.50	3 (0) 3, 3, 3, 3	2 (0) 2, 2, 2, 2	1 (0) 1, 1, 1, 3	2 (2) 1, 1, 3, 7	3 (4) 1, 2,6, 21	7 (2) 5, 6, 8, 10
2.00	3 (0) 3, 3, 3, 3	2 (0) 2, 2, 2, 2	1 (0) 1, 1, 1, 2	1 (0) 1, 1, 1, 1	2 (2) 1, 1, 1, 2	6 (1) 5, 5, 6, 7
Shift ($\boldsymbol{\gamma}$)	λ = 0.10					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 48.5
0.25	43 (170) 6, 15, 185, 1546	85 (370) 3, 21, 391, 2315	114 (491) 4, 26, 518, 2704	134 (576) 3, 27, 603, 2941	156 (658) 2, 30, 688, 3193	69 (245) 12, 27, 272, 3121
0.50	7 (8) 5, 5, 13, 53	14 (31) 2, 6, 37, 297	21 (60) 2, 8, 68, 718	29 (111) 2, 9, 120, 1206	48 (232) 2, 11, 243, 1974	19 (23) 7, 12, 35, 183
0.75	5 (0) 5, 5, 5, 7	5 (7) 2, 3, 10, 32	8 (13) 2, 4, 17, 85	10 (23) 1, 4, 27, 213	18 (62) 1, 5, 67, 763	11 (8) 6, 8, 16, 35
1.00	5 (0) 5, 5, 5, 5	2 (2) 2, 2, 4, 9	4 (5) 2, 2, 7, 20	5 (9) 1, 2, 11, 42	9 (24) 1, 3, 27, 200	8 (4) 5, 6, 10, 16
1.50	5 (0) 5, 5, 5, 5	2 (0) 2, 2, 2, 4	2 (0) 2, 2, 2, 2	2 (2) 1, 1, 3, 8	3 (5) 1, 2, 7, 26	5 (1) 4, 5, 6, 8
2.00	5 (0) 5, 5, 5, 5	2 (0) 2, 2, 2, 2	2 (0) 2, 2, 2, 2	1 (0) 1, 1, 1, 3	2 (2) 1, 1, 3, 8	4 (1) 3, 4, 5, 6
Shift ($\boldsymbol{\gamma}$)	λ = 0.20					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 69.5
0.25	135 (489) 10, 37, 526, 2668	141 (450) 8, 40, 490, 1911	155 (474) 8, 44, 518, 1994	148 (456) 5, 40, 496, 1975	145 (501) 4, 36, 537, 2398	113 (388) 11, 37, 425, 3134
0.50	10 (17) 6, 6, 23, 156	23 (55) 4, 10, 65, 442	32 (97) 4, 12, 109, 718	40 (126) 3, 13, 139, 869	52 (183) 2, 15, 198, 1345	22 (46) 6, 11, 57, 347
0.75	6 (0) 6, 6, 6, 11	7 (10) 3, 4, 14, 58	11 (19) 3, 5, 24, 128	15 (31) 1, 6, 37, 218	22 (67) 1, 8, 75, 531	10 (10) 4, 7, 17, 52
1.00	6 (0) 6, 6, 6, 6	3 (2) 3, 3, 5, 13	5 (6) 2, 3, 9, 29	7 (11) 1, 4, 15, 59	11 (27) 1, 4, 31, 172	7 (4) 4, 5, 9, 20
1.50	6 (0) 6, 6, 6, 6	3 (0) 3, 3, 3, 3	2 (1) 2, 2, 3, 5	3 (3) 1, 1, 4, 10	4 (7) 1, 2, 9, 30	4 (1) 3, 4, 5, 7
2.00	6 (0) 6, 6, 6, 6	3 (0) 3, 3, 3, 3	2 (0) 2, 2, 2, 2	1 (0) 1, 1, 1, 3	2 (3) 1, 1, 4, 9	3 (1) 3, 3, 4, 5

*** The run-length characteristics become unreasonably large, these values are omitted as they are not considered useful in practice.

Table 4a. Control chart performance comparison under the $DE(0,1)$ distribution for m = 100 and n = 5 when target MRL_0 = 350 for negative shifts

Shift (γ)	NEWMA-EX chart					NPEWMA-Rank chart
	25th percentile	40th percentile	50th percentile	60th percentile	75th percentile	
	λ = 0.05					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 34.25
-0.25	94 (686) 2, 14, 700, 6374	88 (299) 4, 26, 325, 1490	51 (266) 1, 10, 276, 4033	95 (523) 4, 18, 541, 4914	215 (1152) 3, 34, 1186, 7867	92 (317) 18, 39, 356, 2806
-0.50	17 (80) 2, 5, 85, 1850	19 (43) 1, 8, 51, 295	11 (24) 1, 4, 28, 132	20 (45) 3, 8, 53, 315	53 (210) 3, 15, 225, 2662	29 (34) 11, 19, 53, 228
-0.75	6 (15) 1, 3, 18, 171	8 (13) 1, 4, 17, 54	6 (9) 1, 3, 12, 30	9 (14) 2, 4, 18, 51	21 (48) 3, 9, 57, 298	17 (12) 8, 12, 24, 50
-1.00	4 (6) 1, 2, 8, 30	5 (6) 1, 3, 9, 19	4 (5) 1, 2, 7, 15	6 (7) 2, 4, 11, 23	13 (19) 3, 6, 25, 73	12 (6) 7, 9, 15, 25
-1.50	2 (2) 1, 1, 3, 7	3 (3) 1, 2, 5, 8	3 (3) 1, 1, 4, 7	4 (3) 2, 3, 6, 10	7 (7) 3, 5, 12, 24	8 (3) 5, 7, 10, 13
-2.00	1 (1) 1, 1, 2, 4	2 (2) 1, 1, 3, 5	2 (2) 1, 1, 3, 5	3 (2) 2, 2, 4, 7	5 (5) 3, 3, 8, 14	6 (2) 5, 5, 7, 9
Shift (γ)	λ = 0.10					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 48.5
-0.25	106 (464) 2, 20, 484, 2807	72 (329) 2, 18, 347, 2287	76 (264) 4, 21, 285, 1801	137 (518) 5, 36, 554, 2738	306 (973) 7, 72, 1045, 3872	112 (351) 14, 39, 390, 2251
-0.50	21 (81) 1, 6, 87, 1054	14 (33) 2, 6, 39, 321	18 (35) 2, 8, 43, 181	30 (68) 3, 12, 80, 383	98 (302) 7, 30, 332, 1956	27 (43) 8, 15, 58, 294
-0.75	8 (20) 1, 3, 23, 154	6 (10) 1, 3, 13, 37	9 (12) 2, 4, 16, 39	14 (20) 2, 7, 27, 76	37 (79) 6, 16, 95, 449	14 (12) 6, 10, 22, 50
-1.00	4 (7) 1, 2, 9, 38	4 (5) 1, 2, 7, 15	6 (6) 2, 3, 9, 18	9 (10) 2, 5, 15, 32	21 (30) 4, 11, 41, 122	9 (6) 5, 7, 13, 24
-1.50	2 (2) 1, 1, 3, 8	2 (2) 1, 2, 4, 7	4 (3) 2, 2, 5, 9	5 (5) 2, 3, 8, 14	11 (10) 4, 7, 17, 35	6 (3) 4, 5, 8, 11
-2.00	1 (1) 1, 1, 2, 3	2 (1) 1, 1, 2, 4	2 (2) 2, 2, 4, 6	3 (3) 2, 2, 5, 9	8 (5) 4, 6, 11, 19	5 (2) 3, 4, 6, 7
Shift (γ)	λ = 0.20					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 69.5
-0.25	100 (355) 3, 25, 380, 2107	90 (301) 4, 26, 327, 1565	115 (300) 8, 38, 338, 1562	214 (559) 12, 69, 628, 2064	491 (1116) 21, 155, 1271, 3586	129 (376) 12, 42, 418, 1907
-0.50	27 (82) 1, 9, 91, 693	19 (42) 2, 8, 50, 296	27 (53) 4, 12, 65, 247	55 (120) 7, 22, 142, 538	278 (743) 17, 85, 828, 2916	30 (58) 6, 14, 72, 410
-0.75	10 (23) 1, 4, 27, 147	8 (13) 1, 4, 17, 55	12 (16) 3, 7, 23, 58	22 (36) 5, 11, 47, 130	104 (237) 12, 40, 277, 1171	13 (17) 5, 8, 25, 76
-1.00	5 (9) 1, 2, 11, 42	5 (6) 1, 3, 9, 20	8 (8) 3, 5, 13, 26	13 (16) 4, 7, 23, 51	49 (89) 9, 23, 112, 374	8 (7) 4, 6, 13, 27
-1.50	2 (3) 1, 1, 4, 9	3 (3) 1, 2, 5, 8	5 (4) 2, 3, 7, 11	7 (6) 3, 5, 11, 19	20 (23) 6, 12, 35, 82	5 (2) 3, 4, 6, 10
-2.00	1 (1) 1, 1, 2, 4	2 (2) 1, 1, 3, 5	4 (2) 2, 3, 5, 7	5 (3) 3, 4, 7, 12	12 (11) 6, 8, 19, 36	4 (2) 2, 3, 5, 6

Table 4b. Control chart performance comparison under the $DE(0,1)$ distribution for m = 100 and n = 5 when target MRL_0 = 350 for positive shifts

Shift (γ)	NEWMA-EX chart					NPEWMA-Rank chart
	25th percentile	40th percentile	50th percentile	60th percentile	75th percentile	
	λ = 0.05					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 34.25
0.25	233 (1351) 3, 36, 1387, 8103	96 (549) 4, 18, 567, 5159	53 (281) 1, 10, 291, 4045	46 (311) 2, 10, 321, 4316	85 (643) 2, 12, 655, 5990	93 (334) 18, 39, 373, 3213
0.50	54 (225) 3, 15, 240, 2679	20 (45) 3, 8, 53, 334	11 (25) 1, 4, 29, 151	10 (22) 2, 4, 26, 281	16 (69) 1, 5, 74, 1770	29 (35) 11, 19, 54, 230
0.75	23 (53) 3, 9, 62, 364	9 (14) 2, 5, 19, 54	6 (9) 1, 3, 12, 30	5 (7) 1, 3, 10, 26	7 (14) 1, 3, 17, 133	17 (12) 9, 12, 24, 49
1.00	13 (21) 3, 7, 28, 88	6 (7) 2, 4, 11, 25	4 (5) 1, 2, 7, 15	3 (5) 1, 2, 7, 27	4 (4) 1, 2, 6, 12	12 (6) 7, 10, 16, 25
1.50	8 (8) 3, 5, 13, 25	4 (3) 2, 3, 6, 11	3 (3) 1, 1, 4, 7	2 (2) 1, 1, 3, 6	2 (2) 1, 1, 3, 7	8 (3) 5, 7, 10, 13
2.00	5 (5) 3, 3, 8, 15	3 (2) 2, 2, 4, 7	2 (2) 1, 1, 3, 5	2 (1) 1, 1, 2, 4	1 (1) 1, 1, 2, 3	6 (2) 5, 5, 7, 9
Shift (γ)	λ = 0.10					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 48.5
0.25	332 (1028) 9, 84, 1112, 3898	148 (513) 4, 37, 550, 2735	83 (280) 4, 24, 304, 1830	67 (308) 2, 16, 324, 2164	96 (467) 2, 19, 486, 2900	103 (354) 14, 38, 392, 2356
0.50	111 (377) 8, 35, 412, 2329	34 (77) 3, 13, 90, 461	19 (35) 3, 8, 43, 187	14 (31) 1, 5, 36, 265	22 (78) 1, 6, 84, 885	26 (41) 8, 15, 56, 303
0.75	44 (92) 6, 18, 110, 542	15 (22) 2, 7, 29, 81	9 (13) 2, 4, 17, 41	6 (9) 1, 3, 12, 37	8 (18) 1, 3, 21, 146	14 (13) 6, 10, 23, 54
1.00	23 (35) 6, 12, 47, 152	9 (11) 2, 5, 16, 34	6 (7) 2, 3, 10, 20	4 (5) 1, 2, 7, 15	4 (7) 1, 2, 9, 33	10 (6) 5, 7, 13, 23
1.50	12 (11) 5, 8, 19, 38	5 (5) 2, 3, 8, 14	4 (3) 2, 2, 5, 9	2 (3) 1, 1, 4, 7	2 (2) 1, 1, 3, 7	6 (3) 4, 5, 8, 11
2.00	8 (6) 5, 6, 12, 21	3 (4) 2, 2, 6, 9	3 (2) 2, 2, 4, 6	2 (1) 1, 1, 2, 4	1 (1) 1, 1, 2, 4	5 (2) 3, 4, 6, 7
Shift (γ)	λ = 0.20					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 69.5
0.25	564 (1269) 23, 181, 1450, 4153	219 (597) 12, 67, 644, 2030	124 (332) 8, 40, 372, 1570	87 (294) 4, 25, 319, 1518	92 (330) 3, 24, 354, 1944	127 (366) 10, 41, 407, 1927
0.50	346 (894) 18, 108, 1002, 3288	60 (129) 6, 23, 152, 588	29 (55) 4, 13, 68, 282	19 (41) 1, 8, 49, 297	26 (79) 1, 9, 88, 599	30 (61) 7, 14, 75, 369
0.75	127 (303) 12, 46, 349, 124	24 (38) 5, 12, 50, 143	13 (18) 3, 7, 25, 63	9 (13) 1, 4, 17, 53	10 (21) 1, 4, 25, 147	13 (16) 5, 8, 24, 70
1.00	61 (116) 10, 26, 142, 506	13 (16) 4, 8, 24, 55	8 (8) 3, 5, 13, 27	5 (6) 1, 3, 9, 21	5 (8) 1, 2, 10, 38	8 (7) 4, 6, 13, 29
1.50	22 (28) 6, 13, 41, 98	7 (6) 3, 5, 11, 21	5 (4) 2, 3, 7, 12	3 (4) 1, 1, 5, 8	2 (3) 1, 1, 4, 8	5 (2) 3, 4, 6, 10
2.00	14 (12) 6, 9, 21, 41	5 (3) 3, 4, 7, 12	4 (2) 2, 3, 5, 8	2 (2) 1, 1, 3, 5	1 (1) 1, 1, 2, 4	4 (2) 2, 3, 5, 6

Table 5a. Control chart performance comparison under the *SymmMixN*[0.6*N*($\mu_1 = 0$, $\sigma_1 = 0.25$) + 0.4*N*($\mu_2 = 0$, $\sigma_2 = 4$)] distribution for m = 100 and n = 5 when target MRL_0 = 350 for negative shifts

Shift (γ)	NEWMA-EX chart					NPEWMA-Rank chart
	25th percentile	40th percentile	50th percentile	60th percentile	75th percentile	
	λ = 0.05					
	For control limits see Table 1					*LCL*/*UCL* = 265 $\pm$ 34.25
-0.25	4 (7) 1, 2, 9, 109	4 (4) 1, 2, 6, 12	4 (5) 1, 2, 7, 15	7 (8) 2, 4, 12, 24	74 (136) 6, 28, 164, 452	16 (10) 8, 12, 22, 41
-0.50	1 (1) 1, 1, 2, 4	2 (2) 1, 1, 3, 4	3 (3) 1, 1, 4, 6	4 (3) 2, 4, 7, 11	34 (51) 5, 16, 67, 143	9 (4) 6, 7, 11, 15
-0.75	1 (0) 1, 1, 1, 2	2 (1) 1, 1, 2, 4	2 (2) 1, 1, 3, 5	4 (3) 2, 3, 6 10	23 (32) 3, 11, 43, 89	7 (3) 5, 6, 9, 12
-1.00	1 (0) 1, 1, 1, 2	1 (2) 1, 1, 2, 3	2 (2) 1, 1, 3, 5	4 (3) 2, 3, 6, 9	17 (21) 3, 9, 30, 60	7 (2) 5, 6, 8, 10
-1.50	1 (0) 1, 1, 1, 2	1 (2) 1, 1, 2, 3	2 (2) 1, 1, 3, 4	4 (1) 2, 3, 4, 7	11 (12) 3, 6, 18, 35	6 (2) 4, 5, 7, 8
-2.00	1 (0) 1, 1, 1, 2	1 (2) 1, 1, 2, 3	2 (2) 1, 1, 3, 5	3 (2) 2, 2, 4, 6	8 (8) 3, 5, 13, 22	5 (1) 4, 5, 6, 7
Shift (γ)	λ = 0.10					
	For control limits see Table 1					*LCL*/*UCL* = 265 $\pm$ 48.5
-0.25	4 (9) 1, 2, 11, 132	4 (5) 1, 2, 7, 16	6 (6) 2, 4, 10, 19	11 (12) 2, 6, 18, 36	161 (284) 14, 67, 351, 900	13 (11) 6, 9, 20, 46
-0.50	1 (1) 1, 1, 2, 4	2 (2) 1, 1, 3, 5	4 (3) 2, 2, 5, 7	7 (6) 2, 4, 10, 15	69 (103) 9, 32, 135, 293	7 (3) 4, 6, 9, 13
-0.75	1 (0) 1, 1, 1, 2	2 (1) 1, 1, 2, 4	3 (2) 2, 2, 4, 6	6 (4) 2, 4, 8, 13	43 (59) 7, 22, 81, 170	6 (2) 4, 5, 7, 10
-1.00	1 (0) 1, 1, 1, 2	2 (1) 1, 1, 2, 3	3 (2) 2, 2, 4, 6	5 (4) 2, 3, 7, 12	30 (38) 6, 16, 54, 106	5 (2) 4, 4, 6, 8
-1.50	1 (0) 1, 1, 1, 2	2 (1) 1, 1, 2, 3	2 (2) 2, 2, 4, 5	4 (3) 2, 3, 6, 10	18 (19) 4, 10, 29, 55	5 (1) 3, 4, 5, 7
-2.00	1 (0) 1, 1, 1, 2	2 (1) 1, 1, 2, 3	1 (2) 2, 2, 3, 4	4 (3) 2, 2, 5, 8	12 (11) 4, 8, 19, 33	4 (1) 3, 4, 5, 6
Shift (γ)	λ = 0.20					
	For control limits see Table 1					*LCL*/*UCL* = 265 $\pm$ 69.5
-0.25	6 (10) 1, 3, 13, 132	5 (6) 1, 3, 9, 20	8 (8) 3, 5, 13, 28	17 (20) 5, 10, 30, 64	734 (1291) 56, 281, 1572, 4010	12 (14) 4, 8, 22, 58
-0.50	1 (2) 1, 1, 3, 5	3 (3) 1, 1, 4, 6	5 (3) 2, 3, 6, 10	9 (8) 4, 6, 14, 25	282 (439) 25, 118, 557, 1235	6 (4) 3, 4, 8, 13
-0.75	1 (0) 1, 1, 1, 3	3 (2) 1, 1, 3, 5	4 (3) 2, 3, 6, 8	8 (6) 4, 6, 12, 20	158 (240) 19, 69, 309, 666	5 (2) 3, 4, 6, 9
-1.00	1 (0) 1, 1, 1, 2	2 (2) 1, 1, 3, 4	4 (2) 2, 3, 5, 7	7 (5) 3, 5, 10, 17	92 (138) 14, 42, 180, 386	4 (2) 3, 3, 5, 8
-1.50	1 (0) 1, 1, 1, 2	2 (2) 1, 1, 3, 4	3 (1) 2, 3, 4, 6	6 (4) 3, 4, 8, 13	43 (57) 9, 22, 79, 167	4 (1) 2, 3, 4, 6
-2.00	1 (0) 1, 1, 1, 2	1 (2) 1, 1, 3, 4	3 (2) 2, 2, 4, 6	5 (3) 3, 4, 7, 11	24 (27) 7, 14, 41, 81	3 (1) 2, 3, 4, 5

Table 5b. Control chart performance comparison under the *SymmMixN*[0.6$N(\mu_1 = 0, \sigma_1 = 0.25)$ + 0.4$N(\mu_2 = 0, \sigma_2 = 4)$] distribution for m = 100 and n = 5 when target MRL_0 = 350 for positive shifts

Shift (γ)	NEWMA-EX chart					NPEWMA-Rank chart
	25th percentile	40th percentile	50th percentile	60th percentile	75th percentile	
	λ = 0.05					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 34.25
0.25	116 (226) 7, 40, 266, 778	8 (9) 2, 4, 13, 26	5 (5) 1, 2, 7, 15	4 (3) 1, 2, 5, 12	4 (6) 1, 2, 8, 47	16 (10) 8, 12, 22, 41
0.50	46 (77) 5, 19, 96, 216	5 (3) 2, 4, 7, 12	3 (3) 1, 1, 4, 6	2 (2) 1, 1, 3, 4	1 (1) 1, 1, 2, 3	9 (4) 6, 7, 11, 15
0.75	29 (43) 3, 14, 57, 122	4 (3) 2, 4, 7, 11	3 (2) 1, 1, 3, 6	2 (1) 1, 1, 2, 4	1 (0) 1, 1, 1, 2	7 (3) 5, 6, 9, 12
1.00	21 (29) 3, 10, 39, 78	4 (3) 2, 3, 6, 10	2 (2) 1, 1, 3, 5	2 (1) 1, 1, 2, 3	1 (0) 1, 1, 1, 2	7 (2) 5, 6, 8, 10
1.50	13 (15) 3, 7, 22, 42	4 (2) 2, 3, 5, 8	2 (2) 1, 1, 3, 5	2 (1) 1, 1, 2, 3	1 (0) 1, 1, 1, 2	6 (2) 4, 5, 7, 9
2.00	9 (9) 3, 5, 14, 26	4 (2) 2, 2, 4, 7	2 (2) 1, 1, 3, 4	2 (1) 1, 1, 2, 3	1 (0) 1, 1, 1, 2	5 (1) 4, 5, 6, 7
Shift (γ)	λ = 0.10					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 48.5
0.25	254 (465) 19, 102, 567, 1466	12 (12) 2, 7, 19, 39	6 (6) 2, 4, 10, 20	4 (5) 1, 2, 7, 15	4 (8) 1, 2, 10, 78	13 (11) 6, 9, 20, 45
0.50	102 (153) 12, 45, 198, 444	7 (6) 2, 4, 10, 17	4 (2) 2, 3, 5, 8	2 (2) 1, 1, 3, 5	1 (1) 1, 1, 2, 4	7 (3) 4, 6, 9, 13
0.75	61 (88) 9, 29, 117, 242	6 (5) 2, 4, 9, 15	3 (2) 2, 2, 4, 7	2 (1) 1, 1, 2, 4	1 (0) 1, 1, 1, 2	6 (2) 4, 5, 7, 10
1.00	39 (52) 8, 20, 72, 148	6 (5) 2, 3, 8, 13	3 (2) 2, 2, 4, 6	2 (1) 1, 1, 2, 3	1 (0) 1, 1, 1, 2	5 (2) 4, 4, 6, 8
1.50	21 (24) 6, 12, 36, 69	4 (4) 2, 3, 7, 10	3 (2) 2, 2, 4, 6	2 (1) 1, 1, 2, 3	1 (0) 1, 1, 1, 2	5 (1) 3, 4, 5, 7
2.00	14 (13) 5, 9, 22, 41	4 (3) 2, 2, 5, 8	3 (1) 2, 2, 3, 5	2 (1) 1, 1, 2, 3	1 (0) 1, 1, 1, 2	4 (1) 3, 4, 5, 6
Shift (γ)	λ = 0.20					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 69.5
0.25	1297 (2252) 95, 522, 2774, 7188	18 (23) 5, 10, 33, 95	9 (9) 3, 5, 14, 29	5 (6) 1, 3, 9, 19	5 (9) 1, 3, 12, 69	12 (14) 4, 8, 22, 58
0.50	489 (792) 40, 200, 992, 2227	10 (8) 4, 7, 15, 28	5 (4) 2, 3, 7, 10	3 (3) 1, 1, 4, 6	1 (2) 1, 1, 3, 5	6 (4) 3, 4, 8, 13
0.75	254 (396) 26, 109, 505, 1136	9 (7) 4, 6, 13, 22	4 (3) 2, 3, 6, 9	3 (2) 1, 1, 3, 5	1 (0) 1, 1, 1, 3	5 (2) 3, 4, 6, 9
1.00	144 (219) 18, 64, 283, 626	8 (6) 3, 5, 11, 19	4 (2) 2, 3, 5, 8	2 (2) 1, 1, 3, 4	1 (0) 1, 1, 1, 2	4 (2) 3, 3, 5, 8
1.50	58 (80) 11, 29, 109, 237	6 (4) 3, 5, 9, 14	4 (2) 2, 3, 5, 7	2 (2) 1, 1, 3, 4	1 (0) 1, 1, 1, 2	4 (1) 2, 3, 4, 6
2.00	30 (37) 8, 17, 54, 111	5 (3) 3, 4, 7, 11	3 (2) 2, 2, 4, 6	2 (2) 1, 1, 3, 4	1 (0) 1, 1, 1, 2	3 (1) 2, 3, 4, 5

Table 6a. Control chart performance comparison under the *AsymmMixN1*[0.6*N*($\mu_1 = 0.25, \sigma_1 = 0.25$) + 0.4*N*($\mu_2 = 0, \sigma_2 = 4$)] distribution for m = 100 and n = 5 when target MRL_0 = 350 for negative shifts

Shift (γ)	NEWMA-EX chart					NPEWMA-Rank chart
	25th percentile	**40th percentile**	**50th percentile**	**60th percentile**	**75th percentile**	
	λ = 0.05					
	For control limits see Table 1					*LCL*/*UCL* = 265 $\pm$ 34.25
-0.25	5 (9) 1, 2, 11, 391	4 (4) 1, 2, 6, 13	4 (5) 1, 2, 7, 14	7 (7) 2, 4, 11, 22	51 (84) 5, 21, 105, 282	16 (10) 8, 12, 22, 42
-0.50	2 (1) 1, 1, 2, 5	2 (2) 1, 1, 3, 4	3 (2) 1, 1, 3, 6	4 (3) 2, 3, 6, 10	25 (35) 3, 12, 47, 96	9 (4) 6, 7, 11, 15
-0.75	1 (0) 1, 1, 1, 2	2 (1) 1, 1, 2, 3	2 (2) 1, 1, 3, 5	4 (3) 2, 3, 6, 9	18 (22) 3, 10, 32, 63	7 (3) 5, 6, 9, 11
-1.00	1 (0) 1, 1, 1, 2	2 (1) 1, 1, 2, 3	2 (2) 1, 1, 3, 4	4 (2) 2, 3, 5, 8	14 (16) 3, 8, 24, 46	7 (2) 5, 6, 8, 10
-1.50	1 (0) 1, 1, 1, 2	2 (1) 1, 1, 2, 3	2 (2) 1, 1, 3, 4	4 (1) 2, 3, 4, 7	10 (10) 3, 6, 16, 28	6 (2) 4, 5, 7, 8
-2.00	1 (0) 1, 1, 1, 2	1 (1) 1, 1, 2, 3	1 (2) 1, 1, 3, 3	3 (2) 2, 2, 4, 6	7 (6) 3, 5, 11, 19	5 (1) 4, 5, 6, 7
Shift (γ)	λ = 0.10					
	For control limits see Table 1					*LCL*/*UCL* = 265 $\pm$ 48.5
-0.25	5 (12) 1, 2, 14, 450	4 (5) 1, 2, 7, 15	5 (6) 2, 3, 9, 18	10 (10) 2, 6, 16, 31	104 (174) 11, 44, 218, 550	13 (11) 6, 9, 20, 43
-0.50	1 (1) 1, 1, 2, 6	2 (2) 1, 1, 3, 4	3 (2) 2, 2, 4, 7	6 (5) 2, 4, 9, 14	47 (66) 8, 23, 89, 186	7 (3) 4, 6, 9, 13
-0.75	1 (0) 1, 1, 1, 2	2 (1) 1, 1, 2, 3	3 (2) 2, 2, 4, 6	5 (5) 2, 3, 8, 12	32 (40) 6, 17, 57, 116	6 (2) 4, 5, 7, 10
-1.00	1 (0) 1, 1, 1, 2	2 (1) 1, 1, 2, 3	2 (2) 2, 2, 4, 6	5 (4) 2, 3, 7, 11	24 (28) 6, 13, 41, 80	5 (2) 4, 4, 6, 8
-1.50	1 (0) 1, 1, 1, 2	2 (1) 1, 1, 2, 3	2 (1) 2, 2, 3, 5	4 (3) 2, 3, 6, 9	16 (14) 4, 10, 24, 44	5 (1) 3, 4, 5, 7
-2.00	1 (0) 1, 1, 1, 2	1 (1) 1, 1, 2, 3	2 (1) 2, 2, 3, 4	3 (3) 2, 2, 5, 7	11 (10) 4, 7, 17, 28	4 (1) 3, 4, 5, 6
Shift (γ)	λ = 0.20					
	For control limits see Table 1					*LCL*/*UCL* = 265 $\pm$ 69.5
-0.25	6 (14) 1, 3, 17, 480	5 (6) 1, 3, 9, 19	8 (8) 3, 5, 13, 26	15 (16) 4, 9, 25, 55	442 (733) 37, 183, 916, 2273	12 (14) 4, 8, 22, 59
-0.50	2 (2) 1, 1, 3, 7	3 (3) 1, 1, 4, 5	4 (3) 2, 3, 6, 9	8 (7) 4, 6, 13, 21	174 (268) 20, 75, 343, 765	6 (4) 3, 4, 8, 13
-0.75	1 (0) 1, 1, 1, 3	2 (2) 1, 1, 3, 4	4 (2) 2, 3, 5, 8	7 (6) 3, 5, 11, 18	103 (150) 14, 48, 198, 433	5 (2) 3, 4, 6, 9
-1.00	1 (0) 1, 1, 1, 2	2 (2) 1, 1, 3, 4	4 (2) 2, 3, 5, 7	7 (4) 3, 5, 9, 15	68 (94) 12, 33, 127, 269	4 (2) 3, 3, 5, 8
-1.50	1 (0) 1, 1, 1, 2	2 (2) 1, 1, 3, 4	3 (1) 2, 3, 4, 6	6 (4) 3, 4, 8, 12	33 (41) 8, 18, 59, 119	4 (1) 2, 3, 4, 6
-2.00	1 (0) 1, 1, 1, 2	1 (2) 1, 1, 3, 3	3 (2) 2, 2, 4, 5	5 (2) 3, 4, 6, 10	21 (20) 7, 13, 33, 65	3 (1) 2, 3, 4, 5

Table 6b. Control chart performance comparison under the *AsymmMixN1*[0.6$N(\mu_1 = 0.25, \sigma_1 = 0.25)$ + 0.4$N(\mu_2 = 0, \sigma_2 = 4)$] distribution for m = 100 and n = 5 when target MRL_0 = 350 for positive shifts

Shift (γ)	NEWMA-EX chart					NPEWMA-Rank chart
	25th percentile	**40th percentile**	**50th percentile**	**60th percentile**	**75th percentile**	
	λ = 0.05					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 34.25
0.25	213 (454) 8, 69, 523, 1581	8 (10) 2, 4, 14, 29	5 (5) 1, 3, 8, 15	4 (4) 1, 2, 6, 12	3 (5) 1, 2, 7, 30	16 (10) 8, 12, 22, 42
0.50	76 (133) 5, 29, 162, 360	5 (4) 2, 4, 8, 14	3 (3) 1, 1, 4, 7	2 (2) 1, 1, 3, 4	1 (1) 1, 1, 2, 3	9 (4) 6, 7, 11, 15
0.75	41 (65) 5, 18, 83, 186	5 (3) 2, 4, 7, 12	3 (2) 1, 1, 3, 6	2 (1) 1, 1, 2, 4	1 (0) 1, 1, 1, 2	7 (3) 5, 6, 9, 12
1.00	27 (39) 3, 13, 52, 110	4 (4) 2, 3, 7, 10	2 (2) 1, 1, 3, 6	2(1) 1, 1, 2, 3	1 (0) 1, 1, 1, 2	7 (2) 5, 6, 8, 10
1.50	15 (18) 3, 8, 26, 52	4 (2) 2, 3, 5, 8	2 (2) 1, 1, 3, 5	2(1) 1, 1, 2, 3	1 (0) 1, 1, 1, 2	6 (2) 4, 5, 7, 9
2.00	10 (10) 3, 6, 16, 31	4 (1) 2, 3, 4, 7	2 (2) 1, 1, 3, 4	2(1) 1, 1, 2, 3	1 (0) 1, 1, 1, 2	5 (1) 4, 5, 6, 7
Shift (γ)	λ = 0.10					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 48.5
0.25	457 (873) 30, 174, 1047, 2857	13 (15) 2, 7, 22, 45	6 (6) 2, 4, 10, 21	4 (5) 1, 2, 7, 15	4 (6) 1, 2, 8, 38	13 (11) 6, 9, 20, 43
0.50	169 (274) 16, 69, 344, 784	8 (7) 2, 5, 12, 19	4 (2) 2, 3, 5, 8	2 (2) 1, 1, 3, 5	1 (1) 1, 1, 2, 4	7 (3) 4, 6, 9, 13
0.75	88 (136) 11, 40, 176, 389	7 (6) 2, 4, 10, 16	4 (3) 2, 2, 5, 7	2 (1) 1, 1, 2, 4	1 (0) 1, 1, 1, 2	6 (2) 4, 5, 7, 10
1.00	53 (75) 9, 26, 101, 215	6 (6) 2, 3, 9, 14	3 (2) 2, 2, 4, 7	2 (1) 1, 1, 2, 4	1 (0) 1, 1, 1, 2	5 (2) 4, 4, 6, 8
1.50	26 (31) 6, 15, 46, 91	5 (4) 2, 3, 7, 11	3 (2) 2, 2, 4, 6	2 (1) 1, 1, 2, 3	1 (0) 1, 1, 1, 2	5 (1) 3, 4, 5, 7
2.00	17 (16) 6, 10, 26, 49	4 (4) 2, 2, 6, 9	3 (1) 2, 2, 3, 5	2 (1) 1, 1, 2, 3	1 (0) 1, 1, 1, 2	4 (1) 3, 4, 5, 6
Shift (γ)	λ = 0.20					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 69.5
0.25	2291 (4167) 153, 902, 5069, 13072	21 (28) 5, 12, 40, 87	9 (9) 3, 6, 15, 31	6 (6) 1, 3, 9, 20	5 (7) 1, 3, 10, 45	12 (13) 4, 8, 21, 57
0.50	855 (1426) 64, 346, 1773, 4084	12 (11) 4, 7, 18, 33	5 (3) 2, 4, 7, 11	3 (3) 1, 1, 4, 6	1 (2) 1, 1, 3, 4	6 (4) 3, 4, 8, 13
0.75	408 (670) 36, 170, 840, 1880	10 (9) 4, 16, 15, 25	5 (3) 2, 3, 6, 10	3 (2) 1, 1, 3, 5	1 (1) 1, 1, 2, 3	5 (2) 3, 4, 6, 9
1.00	216 (334) 23, 93, 427, 986	9 (7) 4, 6, 13, 22	4 (3) 2, 3, 6, 9	2 (2) 1, 1, 3, 5	1 (0) 1, 1, 1, 2	4 (2) 3, 3, 5, 8
1.50	81 (119) 13, 38, 157, 334	7 (5) 3, 5, 10, 16	4 (2) 2, 3, 5, 7	2 (2) 1, 1, 3, 4	1 (0) 1, 1, 1, 2	4 (1) 2, 3, 4, 6
2.00	38 (50) 9, 21, 71, 147	6 (4) 3, 4, 8, 13	3 (1) 2, 3, 4, 6	2 (2) 1, 1, 3, 4	1 (0) 1, 1, 1, 2	3 (1) 2, 3, 4, 5

Table 7a. Control chart performance comparison under the *AsymmMixN2*[0.6*N*($\mu_1 = -0.25$, $\sigma_1 = 0.25$) + 0.4*N*($\mu_2 = 0$, $\sigma_2 = 4$)] distribution for m = 100 and n = 5 when target MRL_0 = 350 for negative shifts

Shift (γ)	NEWMA-EX chart					NPEWMA-Rank chart
	25th percentile	**40th percentile**	**50th percentile**	**60th percentile**	**75th percentile**	
	λ = 0.05					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 34.25
-0.25	4 (6) 1, 2, 8, 50	4 (4) 1, 2, 6, 12	4 (5) 1, 3, 8, 15	8 (9) 3, 4, 13, 27	123 (253) 6, 43, 296, 867	16 (10) 8, 12, 22, 41
-0.50	1 (1) 1, 1, 2, 4	2 (2) 1, 1, 3, 4	3 (3) 1, 1, 4, 7	5 (4) 2, 4, 8, 12	49 (79) 5, 20, 99, 219	9 (4) 6, 7, 11, 15
-0.75	1 (1) 1, 1, 2, 2	2 (1) 1, 1, 2, 4	3 (2) 1, 1, 3, 6	4 (3) 2, 4, 7, 11	30 (44) 5, 14, 58, 128	8 (3) 5, 6, 9, 12
-1.00	1 (0) 1, 1, 1, 2	2 (1) 1, 1, 2, 4	2 (2) 1, 1, 3, 5	4 (3) 2, 3, 6, 10	22 (29) 3, 11, 40, 82	7 (2) 5, 6, 8, 10
-1.50	1 (0) 1, 1, 1, 2	2 (1) 1, 1, 2, 3	2 (2) 1, 1, 3, 4	4 (2) 2, 3, 5, 8	13 (15) 3, 7, 22, 42	6 (2) 4, 5, 7, 8
-2.00	1 (0) 1, 1, 1, 2	2 (1) 1, 1, 2, 3	2 (2) 1, 1, 3, 4	4 (1) 2, 3, 4, 7	9 (9) 3, 6, 15, 26	5 (1) 4, 5, 6, 7
Shift (γ)	λ = 0.10					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 48.5
-0.25	4 (7) 1, 2, 9, 63	4 (5) 1, 2, 7, 15	6 (6) 2, 4, 10, 20	12 (13) 2, 7, 20, 40	273 (495) 22, 106, 601, 1631	13 (11) 6, 9, 20, 45
-0.50	1 (1) 1, 1, 2, 4	2 (1) 1, 2, 3, 5	4 (3) 2, 2, 5, 8	7 (5) 2, 5, 10, 17	108 (165) 12, 47, 212, 465	7 (3) 4, 6, 9, 13
-0.75	1 (1) 1, 1, 2, 2	2 (1) 1, 1, 2, 4	3 (2) 2, 2, 4, 7	7 (5) 2, 4, 9, 14	61 (91) 9, 28, 119, 250	6 (2) 4, 5, 7, 10
-1.00	1 (0) 1, 1, 1, 2	2 (1) 1, 1, 2, 4	3 (2) 2, 2, 4, 6	6 (5) 2, 3, 8, 13	39 (53) 7, 21, 74, 154	5 (2) 4, 4, 6, 8
-1.50	1 (0) 1, 1, 1, 2	2 (1) 1, 1, 2, 3	2 (2) 2, 2, 4, 5	5 (4) 2, 3, 7, 10	21 (24) 6, 12, 36, 71	5 (1) 3, 4, 5, 7
-2.00	1 (0) 1, 1, 1, 2	2 (1) 1, 1, 2, 3	2 (1) 2, 2, 3, 5	4 (3) 2, 3, 6, 8	14 (13) 4, 9, 22, 40	4 (1) 3, 4, 5, 6
Shift (γ)	λ = 0.20					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 69.5
-0.25	5 (9) 1, 2, 11, 65	6 (6) 1, 3, 9, 20	9 (9) 3, 5, 14, 29	19 (22) 5, 11, 33, 77	1228 (2097) 91, 486, 2583, 6737	12 (14) 4, 8, 22, 60
-0.50	1 (1) 1, 1, 2, 4	3 (3) 1, 1, 4, 6	5 (3) 2, 4, 7, 10	10 (9) 4, 7, 16, 28	454 (748) 40, 192, 940, 2129	6 (4) 3, 4, 8, 13
-0.75	1 (1) 1, 1, 2, 3	3 (2) 1, 1, 3, 5	4 (3) 2, 3, 6, 9	9 (7) 4, 6, 13, 23	245 (388) 25, 107, 495, 1062	5 (2) 3, 4, 6, 9
-1.00	1 (0) 1, 1, 1, 2	3 (2) 1, 1, 3, 4	4 (2) 2, 3, 5, 8	8 (6) 4, 5, 11, 19	141 (216) 17, 60, 276, 613	4 (2) 3, 3, 5, 8
-1.50	1 (0) 1, 1, 1, 2	2 (2) 1, 1, 3, 4	4 (2) 2, 3, 5, 7	6 (4) 3, 5, 9, 15	57 (79) 11, 28, 107, 224	4 (1) 2, 3, 4, 6
-2.00	1 (0) 1, 1, 1, 2	2 (2) 1, 1, 3, 4	3 (1) 2, 3, 4. 6	5 (3) 3, 4, 7, 12	30 (35) 8, 17, 52, 103	3 (1) 2, 3, 4, 5

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Table 7b. Control chart performance comparison under the $AsymmMixN2[0.6N(\mu_1 = -0.25, \sigma_1 = 0.25) + 0.4N(\mu_2 = 0, \sigma_2 = 4)]$ distribution for m = 100 and n = 5 when target MRL_0 = 350 for positive shifts

Shift (γ)	NEWMA-EX chart					NPEWMA-Rank chart
	25th percentile	**40th percentile**	**50th percentile**	**60th percentile**	**75th percentile**	
	λ = 0.05					
	For control limits see Table 1					*LCL*/*UCL* = 265 $\pm$ 34.25
0.25	70 (132) 5, 26, 158, 450	7 (8) 2, 4, 12, 23	4 (5) 1, 2, 7, 15	4 (4) 1, 2, 6, 12	4 (7) 1, 2, 9, 124	16 (10) 8, 12, 22, 44
0.50	33 (48) 4, 15, 63, 136	4 (3) 2, 4, 7, 11	3 (2) 1, 1, 3, 6	2 (2) 1, 1, 3, 4	1 (1) 1, 1, 2, 4	9 (4) 6, 7, 11, 15
0.75	22 (30) 3, 11, 41, 84	4 (3) 2, 3, 6, 10	2 (2) 1, 1, 3, 5	2 (1) 1, 1, 2, 3	1 (0) 1, 1, 1, 2	8 (3) 5, 6, 9, 11
1.00	17 (21) 3, 9, 30, 60	4 (2) 2, 3, 5, 9	2 (2) 1, 1, 3, 5	2 (1) 1, 1, 2, 3	1 (0) 1, 1, 1, 2	7 (2) 5, 6, 8, 10
1.50	11 (12) 3, 6, 18, 33	4 (1) 2, 3, 4, 7	2 (2) 1, 1, 3, 4	2 (1) 1, 1, 2, 3	1 (0) 1, 1, 1, 2	6 (2) 4, 5, 7, 8
2.00	8 (8) 3, 5, 13, 22	3 (2) 2, 2, 4, 6	1 (1) 1, 1, 2, 3	1 (1) 1, 1, 2, 3	1 (0) 1, 1, 1, 2	5 (1) 4, 5, 6, 7
Shift (γ)	λ = 0.10					
	For control limits see Table 1					*LCL*/*UCL* = 265 $\pm$ 48.5
0.25	158 (272) 16, 66, 338, 889	11 (12) 2, 6, 18, 35	6 (6) 2, 4, 10, 19	4 (5) 1, 2, 7, 15	5 (9) 1, 2, 11, 137	13 (11) 6, 9, 20, 46
0.50	66 (95) 10, 32, 127, 273	7 (5) 2, 4, 9, 15	4 (3) 2, 2, 5, 7	2 (2) 1, 1, 3, 4	1 (1) 1, 1, 2, 5	7 (3) 4, 6, 9, 13
0.75	42 (56) 8, 22, 78, 165	6 (5) 2, 3, 8, 13	3 (2) 2, 2, 4, 6	2 (1) 1, 1, 2, 3	1 (0) 1, 1, 1, 2	6 (2) 4, 5, 7, 9
1.00	30 (36) 7, 17, 53, 106	5 (5) 2, 3, 8, 11	3 (2) 2, 2, 4, 6	2 (1) 1, 1, 2, 3	1 (0) 1, 1, 1, 2	5 (2) 4, 4, 6, 9
1.50	18 (18) 6, 11, 29, 55	4 (3) 2, 3, 6, 9	3 (2) 2, 2, 4, 5	2 (1) 1, 1, 2, 3	1 (0) 1, 1, 1, 2	5 (1) 3, 4, 5, 7
2.00	13 (11) 5, 8, 19, 33	3 (3) 2, 2, 5, 8	2 (1) 2, 2, 3, 4	1 (1) 1, 1, 2, 3	1 (0) 1, 1, 1, 2	4 (1) 3, 4, 5, 6
Shift (γ)	λ = 0.20					
	For control limits see Table 1					*LCL*/*UCL* = 265 $\pm$ 69.5
0.25	760 (1281) 58, 306, 1587, 4020	16 (20) 5, 9, 29, 64	8 (8) 3, 5, 13, 27	5 (6) 1, 3, 9, 20	6 (11) 1, 3, 14, 185	12 (14) 4, 7, 21, 56
0.50	291 (462) 28, 124, 586, 1322	9 (8) 4, 6, 14, 24	4 (3) 2, 3, 6, 10	3 (3) 1, 1, 4, 6	1 (2) 1, 1, 3, 5	6 (4) 3, 4, 8, 13
0.75	164 (252) 19, 73, 325, 710	8 (6) 3, 5, 11, 19	4 (3) 2, 3,6, 8	2 (2) 1, 1, 3, 4	1 (0) 1, 1, 1, 3	5 (2) 3, 4, 6, 9
1.00	98 (145) 15, 46, 191, 415	7 (5) 3, 5, 10, 17	4 (2) 2, 3, 5, 8	2 (2) 1, 1, 3, 4	1 (0) 1, 1, 1, 2	4 (2) 3 , 3, 5, 7
1.50	44 (58) 10, 23, 81, 165	6 (4) 3, 4, 8, 13	3 (1) 2, 3, 4, 6	2 (2) 1, 1, 3, 4	1 (0) 1, 1, 1, 2	3 (1) 2, 3, 4, 6
2.00	26 (28) 8, 15, 43, 85	5 (3) 3, 4, 7, 10	3 (2) 2, 2, 4, 6	1 (2) 1, 1, 3, 3	1 (0) 1, 1, 1, 2	3 (1) 2, 3, 4, 5

Table 8a. Control chart performance comparison under *Log-Logistic*($\alpha = 1$, $\beta = 2.5$) distribution for m = 100 and n = 5 when target MRL_0 = 350 for negative shifts

Shift (γ)	NEWMA-EX chart					NPEWMA-Rank chart
	25th percentile	40th percentile	50th percentile	60th percentile	75th percentile	
	λ = 0.05					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 34.25
-0.25	9 (18) 1, 4, 22, 97	14 (34) 2, 5, 39, 224	21 (68) 1, 6, 74, 710	38 (142) 3, 11, 153, 1844	144 (784) 3, 27, 811, 6404	26 (27) 11, 17, 44, 123
-0.50	3 (3) 1, 2, 5, 11	4 (5) 1, 3, 8, 18	6 (9) 1, 3, 12, 31	10 (16) 2, 5, 21, 60	28 (69) 3, 11, 80, 468	11 (6) 6, 9, 15, 22
-0.75	2 (2) 1, 1, 3, 5	3 (2) 1, 2, 4, 9	4 (4) 1, 2, 6, 13	6 (6) 2, 4, 10, 20	14 (21) 3, 6, 27, 80	8 (3) 5, 6, 9, 12
-1.00	2 (1) 1, 1, 2, 4	2 (1) 1, 2, 3, 6	3 (3) 1, 1, 4, 8	4 (4) 2, 3, 7, 12	10 (10) 3, 6, 16, 35	6 (2) 4, 5, 7, 9
-1.50	1 (1) 1, 1, 2, 2	2 (1) 1, 1, 2, 4	2 (2) 1, 1, 3, 5	3 (2) 2, 2, 4, 7	6 (7) 3, 3, 10, 17	5 (1) 4, 4, 5, 6
-2.00	1 (0) 1, 1, 1, 2	1 (1) 1, 1, 2, 3	1 (1) 1, 1, 2, 3	3 (2) 2, 2, 4, 5	5 (4) 3, 3, 7, 12	4 (1) 3, 4, 5, 5
Shift (γ)	λ = 0.10					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 48.5
-0.25	11 (23) 1, 4, 27, 110	21 (47) 2, 8, 55, 272	35 (91) 2, 12, 103, 614	67 (207) 4, 21, 228, 1419	216 (721) 8, 61, 782, 3259	23 (29) 8, 14, 43, 136
-0.50	3 (4) 1, 2, 6, 13	6 (8) 1, 3, 11, 24	9 (12) 2, 5, 17, 41	15 (23) 2, 8, 31, 89	51 (116) 6, 20, 136, 655	9 (5) 5, 7, 12, 20
-0.75	2 (2) 1, 1, 3, 6	3 (4) 1, 2, 6, 10	5 (5) 2, 3, 8, 16	8 (9) 2, 5, 14, 28	23 (33) 4, 12, 45, 124	6 (2) 4, 5, 7, 10
-1.00	2 (1) 1, 1, 2, 4	2 (2) 1, 2, 4, 7	4 (3) 2, 2, 5, 10	6 (6) 2, 3, 9, 16	14 (16) 4, 9, 25, 54	5 (2) 3, 4, 6, 7
-1.50	1 (1) 1, 1, 2, 2	2 (1) 1, 1, 2, 4	2 (2) 2, 2, 4, 6	4 (4) 2, 2, 6, 9	9 (7) 4, 6, 13, 23	4 (1) 3, 3, 4, 5
-2.00	1 (0) 1, 1, 1, 2	1 (1) 1, 1, 2, 3	2 (1) 2, 2, 3, 4	3 (2) 2, 2, 4, 7	7 (6) 4, 4, 10, 16	3 (1) 3, 3, 4, 4
Shift (γ)	λ = 0.20					
	For control limits see Table 1					LCL/UCL = 265 $\pm$ 69.5
-0.25	13 (25) 1, 6, 31, 110	29 (61) 3, 12, 73, 285	54 (124) 5, 20, 144, 681	114 (297) 8, 40, 337, 1421	481 (1099) 22, 156, 1255, 3543	24 (38) 6, 12, 50, 166
-0.50	4 (5) 1, 2, 7, 14	8 (10) 1, 4, 14, 32	13 (18) 3, 7, 25, 62	25 (41) 5, 12, 53, 155	154 (367) 14, 57, 424, 1648	7 (6) 3, 5, 11, 21
-0.75	2 (3) 1, 1, 4, 7	4 (4) 1, 3, 7, 13	7 (7) 3, 4, 11, 22	12 (14) 4, 7, 21, 45	56 (101) 9, 25, 126, 395	5 (2) 3, 4, 6, 10
-1.00	2 (2) 1, 1, 3, 5	3 (3) 1, 2, 5, 8	5 (4) 2, 3, 7, 13	8 (7) 3, 5, 12, 23	30 (42) 8, 16, 58, 143	4 (2) 2, 3, 5, 7
-1.50	1 (1) 1, 1, 2, 3	2 (2) 1, 1, 3, 5	3 (2) 2, 3, 5, 7	5 (3) 3, 4, 7, 12	15 (14) 6, 10, 24, 47	3 (1) 2, 2, 3, 4
-2.00	1 (0) 1, 1, 1, 2	1 (2) 1, 1, 3, 4	3 (2) 2, 2, 4, 5	4 (3) 3, 3, 6, 8	11 (8) 6, 8, 16, 27	2 (1) 2, 2, 3, 4

Table 8b. Control chart performance comparison under *Log-Logistic*($\alpha = 1, \beta = 2.5$) distribution for m = 100 and n = 5 when target MRL_0 = 350 for positive shifts

Shift (γ)	NEWMA-EX chart 25th percentile	NEWMA-EX chart 40th percentile	NEWMA-EX chart 50th percentile	NEWMA-EX chart 60th percentile	NEWMA-EX chart 75th percentile	NPEWMA-Rank chart
	λ = 0.05					
	For control limits see Table 1					LCL/UCL = 265 ± 34.25
0.25	12 (23) 3, 6, 29, 187	13 (30) 2, 5, 35, 327	14 (46) 1, 5, 51, 856	19 (81) 2, 6, 87, 1673	37 (270) 2, 8, 278, 4180	28 (36) 11, 18, 54, 273
0.50	3 (3) 3, 3, 6, 11	4 (3) 2, 3, 6, 13	3 (6) 1, 1, 7, 18	4 (7) 1, 2, 9, 32	7 (20) 1, 3, 23, 304	12 (6) 7, 9, 15, 23
0.75	3 (0) 3, 3, 3, 5	2 (1) 4, 3, 2, 2	1 (2) 1, 1, 3, 5	2 (3) 1, 1, 4, 7	3 (4) 1, 2, 6, 24	8 (2) 6, 7, 9, 12
1.00	3 (0) 3, 3, 3, 3	2 (0) 2, 2, 2, 3	1 (0) 1, 1, 1, 3	1 (1) 1, 1, 2, 4	2 (2) 1, 1, 3, 8	6 (2) 5, 5, 7, 8
1.50	3 (0) 3, 3, 3, 3	2 (0) 2, 2, 2, 2	1 (0) 1, 1, 1, 1	1 (0) 1, 1, 1, 1	1 (0) 1, 1, 1, 2	5 (1) 4, 4, 5, 6
2.00	3 (0) 3, 3, 3, 3	2 (0) 2, 2, 2, 2	1 (0) 1, 1, 1, 1	1 (0) 1, 1, 1, 1	1 (0) 1, 1, 1, 1	4 (0) 4, 4, 4, 5
Shift (γ)	λ = 0.10					
	For control limits see Table 1					LCL/UCL = 265 ± 48.5
0.25	21 (38) 5, 11, 49, 308	20 (48) 2, 8, 56, 433	23 (70) 3, 9, 79, 763	28 (102) 2, 9, 111, 1086	50 (239) 2, 11, 250, 2028	26 (51) 8, 14, 65, 505
0.50	6 (4) 5, 5, 9, 15	4 (5) 2, 3, 8, 17	5 (6) 2, 3, 9, 24	5 (10) 1, 2, 12, 42	9 (25) 1, 3, 28, 276	9 (6) 5, 7, 13, 22
0.75	5 (0) 5, 5, 5, 6	2 (1) 2, 2, 3, 6	3 (2) 2, 2, 4, 7	2 (3) 1, 1, 4, 9	3 (5) 1, 2, 7, 30	6 (2) 4, 5, 7, 10
1.00	5 (0) 5, 5, 5, 5	2 (0) 2, 2, 2, 3	2 (0) 2, 2, 2, 3	1 (1) 1, 1, 2, 4	2 (2) 1, 1, 3, 9	5 (1) 4, 4, 5, 6
1.50	5 (0) 5, 5, 5, 5	2 (0) 2, 2, 2, 2	2 (0) 2, 2, 2, 2	1 (0) 1, 1, 1, 1	1 (0) 1, 1, 1, 2	4 (1) 3, 3, 4, 4
2.00	5 (0) 5, 5, 5, 5	2 (0) 2, 2, 2, 2	2 (0) 2, 2, 2, 2	1 (0) 1, 1, 1, 1	1 (0) 1, 1, 1, 1	3 (0) 3, 3, 3, 4
Shift (γ)	λ = 0.20					
	For control limits see Table 1					LCL/UCL = 265 ± 69.5
0.25	50 (132) 8, 21, 153, 887	35 (86) 5, 14, 100, 583	37 (105) 4, 14, 119, 688	39 (122) 3, 13, 135, 797	54 (194) 2, 15, 209, 1443	34 (79) 7, 15, 94, 646
0.50	9 (7) 6, 6, 13, 27	7 (6) 3, 5, 11, 26	7 (8) 2, 4, 12, 35	7 (11) 1, 4, 15, 57	11 (26) 1, 4, 30, 209	9 (7) 4, 6, 13, 39
0.75	6 (0) 6, 6, 6, 9	4 (2) 3, 3, 5, 7	3 (3) 2, 2, 5, 8	3 (4) 1, 1, 5, 11	4 (7) 1, 2, 9, 33	5 (2) 3, 4, 6, 10
1.00	6 (0) 6, 6, 6, 6	3 (0) 3, 3, 3, 4	2 (1) 2, 2, 3, 4	1 (2) 1, 1, 3, 5	2 (3) 1, 1, 4, 10	4 (1) 3, 3, 4, 6
1.50	6 (0) 6, 6, 6, 6	3 (0) 3, 3, 3, 3	2 (0) 2, 2, 2, 2	1 (0) 1, 1, 1, 1	1 (0) 1, 1, 1, 3	3 (0) 2, 3, 3, 3,
2.00	6 (0) 6, 6, 6, 6	3 (0) 3, 3, 3, 3	2 (0) 2, 2, 2, 2	1 (0) 1, 1, 1, 1	1 (0) 1, 1, 1, 1	2 (1) 2, 2, 3, 3

From Table 2 it can be seen that when the underlying process distribution is $N(0,1)$, the NPEWMA-EX chart based on the 25th percentile is overall good for detecting negative shifts while the NPEWMA-EX chart based on the 75th percentile performs the best irrespective of the size of the positive shift and choice of the smoothing constant λ. When, $\lambda = 0.05$, the NPEWMA-EX chart based on the 40th percentile is also a very competitive choice. These are illustrated in Figures 1a,b,c for $\lambda = 0.05$, 0.10 and 0.20, respectively. For brevity, we only consider some positive shifts.

From Table 3a, we see that the NPEWMA-EX chart based on the 25th percentile always performs best in detecting negative shifts when the underlying process distribution is $EXP(1)$. However, from Table 3b, it can be seen that the decision is not so straightforward in case of positive shifts. For γ = 0.25 the NPEWMA-EX chart based on the 25th percentile performs best for λ = 0.05 and 0.10, however, the NPEWMA-Rank chart performs best for λ = 0.20. For γ = 0.50 and 0.75 the NPEWMA-EX chart based on the 25th percentile performs best for all λ. For $\gamma = 1.00$ the NPEWMA-EX chart based on the 40th percentile performs best and for γ = 1.50 the NPEWMA-EX chart based on the median performs best for all λ. For the largest shift under consideration, i.e. $\gamma = 2.00$, the NPEWMA-EX chart based on the 60th percentile performs best for all λ. These are briefly illustrated in Figures 2a,b,c for $\lambda = 0.05, 0.10$ and 0.20, respectively, for positive shifts.

From Table 4a it can be seen that, when the underlying process distribution is $DE(0,1)$ and when the smoothing constant $\lambda = 0.05$, the NPEWMA-EX chart based on the 50th percentile is good in detecting negative shifts. Further, for $\lambda = 0.1$ and 0.2, the NPEWMA-EX chart based on the 40th percentile performs the best for smaller negative shifts ($-1.00 \leq \gamma < 0$) and the NPEWMA-EX chart based on the 25th percentile performs the best for larger negative shifts ($\gamma =$ -1.50 and -2.00). From Table 4b it can be seen that for positive shifts the choice of the order statistic from the reference sample stays the same regardless of the value of the smoothing constant λ. In summary, for all λ, the NPEWMA-EX chart based on the 60th percentile performs best for smaller shifts ($\gamma \leq 1.00$), the NPEWMA-EX chart based on the 75th percentile performs best for larger shifts ($\gamma =$ 1.50 and 2.00). Since the run-length characteristics seem to converge as the size of this shift increases, the recommendation would be to use the NPEWMA-EX chart based on the 75th percentile when large shifts are of concern. These are illustrated in Figures 3a,b,c for $\lambda = 0.05, 0.10$ and 0.20, respectively, with some positive shifts, where it can also clearly be seen that the NPEWMA-EX chart based on the 25th percentile is performing the worst.

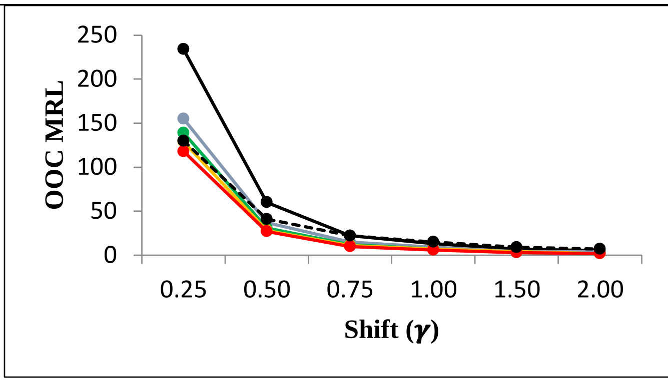


Figure 1a. *MRL* performance comparison of the NPEWMA-EX chart based on various percentiles of the reference sample and the NPEWMA-Rank chart under the $N(0,1)$ distribution with m = 100, n = 5 and λ = 0.05

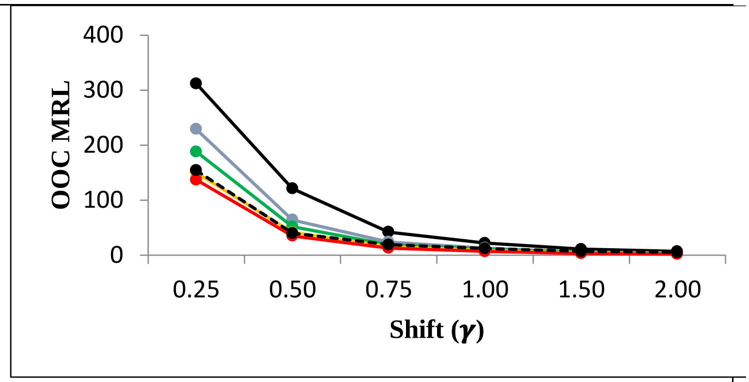


Figure 1b. *MRL* performance comparison of the NPEWMA-EX chart based on various percentiles of the reference sample and the NPEWMA-Rank chart under the $N(0,1)$ distribution with m = 100, n = 5 and λ = 0.10

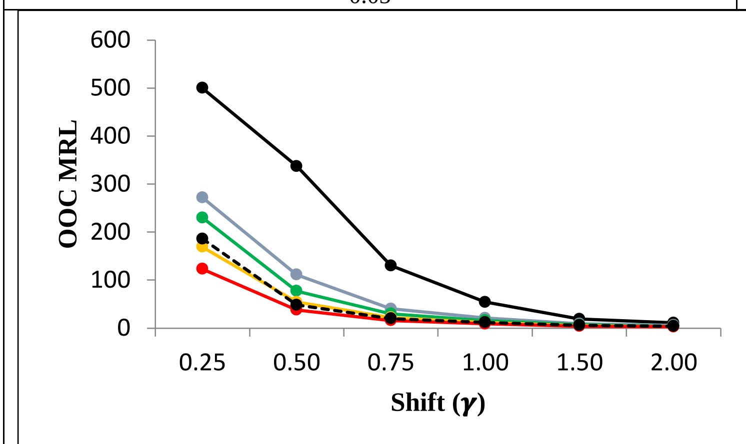


Figure 1c. *MRL* performance comparison of the NPEWMA-EX chart based on various percentiles of the reference sample and the NPEWMA-Rank chart under the $N(0,1)$ distribution with m = 100, n = 5 and λ = 0.20

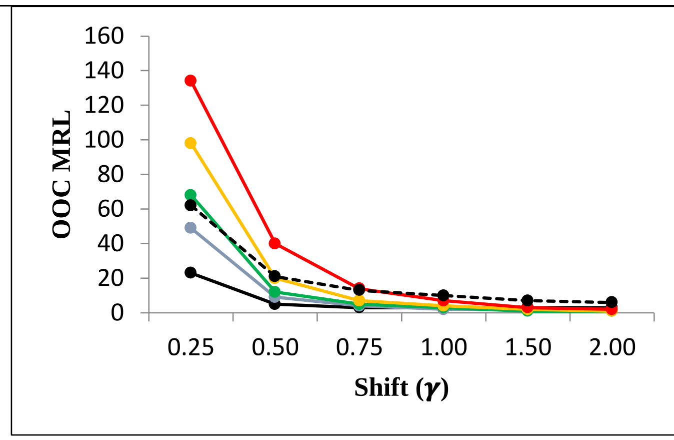


Figure 2a. *MRL* performance comparison of the NPEWMA-EX chart based on various percentiles of the reference sample and the NPEWMA-Rank chart under the *EXP*(1) distribution with m = 100, n = 5 and λ = 0.05

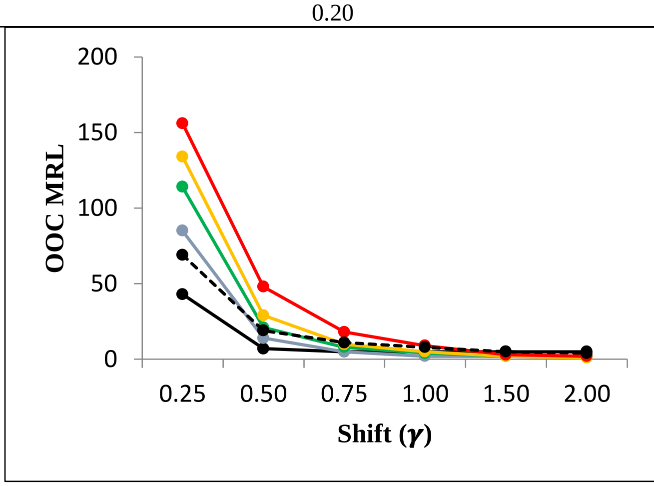


Figure 2b. *MRL* performance comparison of the NPEWMA-EX chart based on various percentiles of the reference sample and the NPEWMA-Rank chart under the *EXP*(1) distribution with m = 100, n = 5 and λ = 0.10

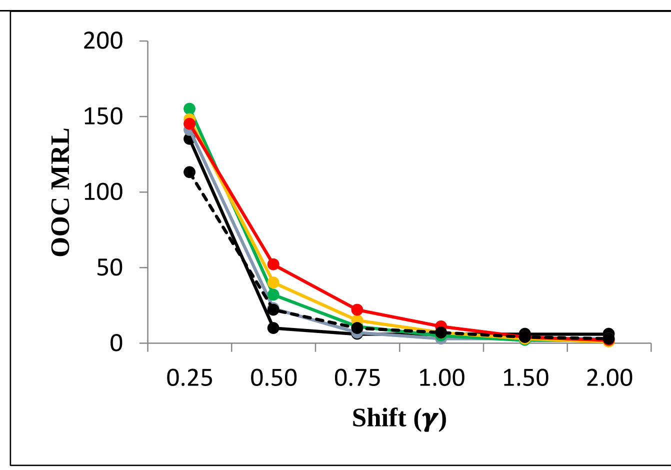


Figure 2c. *MRL* performance comparison of the NPEWMA-EX chart based on various percentiles of the reference sample and the NPEWMA-Rank chart under the *EXP*(1) distribution with m = 100, n = 5 and λ = 0.20

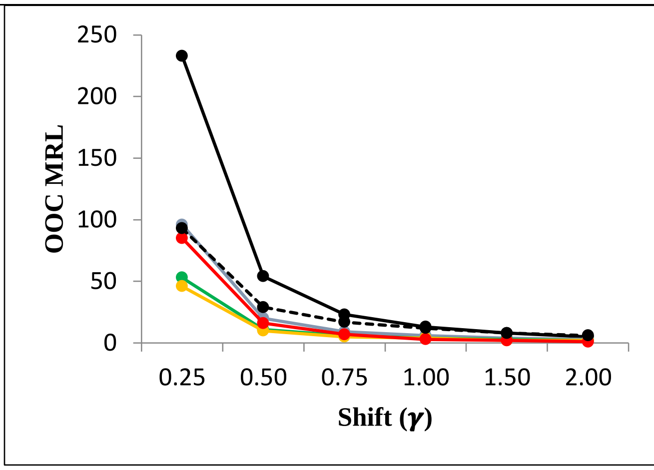


Figure 3a. *MRL* performance comparison of the NPEWMA-EX chart based on various percentiles of the reference sample and the NPEWMA-Rank chart under the *DE*(0,1) distribution with m = 100, n = 5 and λ = 0.05

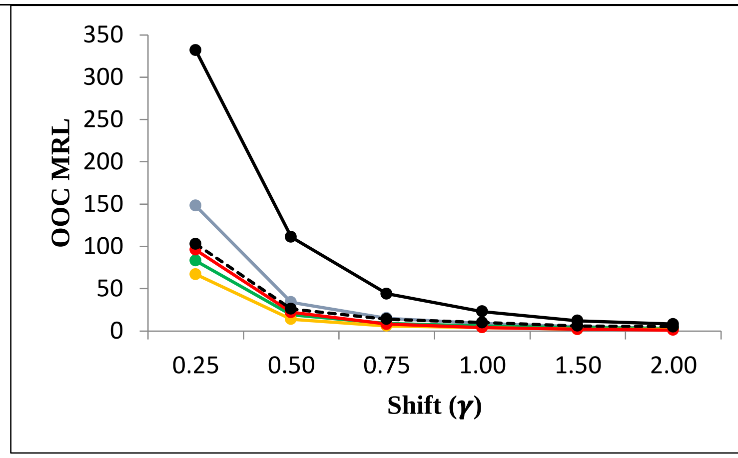


Figure 3b. *MRL* performance comparison of the NPEWMA-EX chart based on various percentiles of the reference sample and the NPEWMA-Rank chart under the *DE*(0,1) distribution with m = 100, n = 5 and λ = 0.10

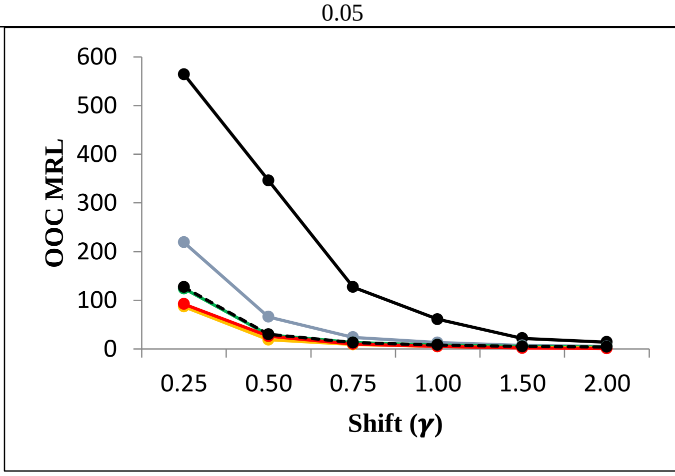


Figure 3c. *MRL* performance comparison of the NPEWMA-EX chart based on various percentiles of the reference sample and the NPEWMA-Rank chart under the *DE*(0,1) distribution with m = 100, n = 5 and λ = 0.20

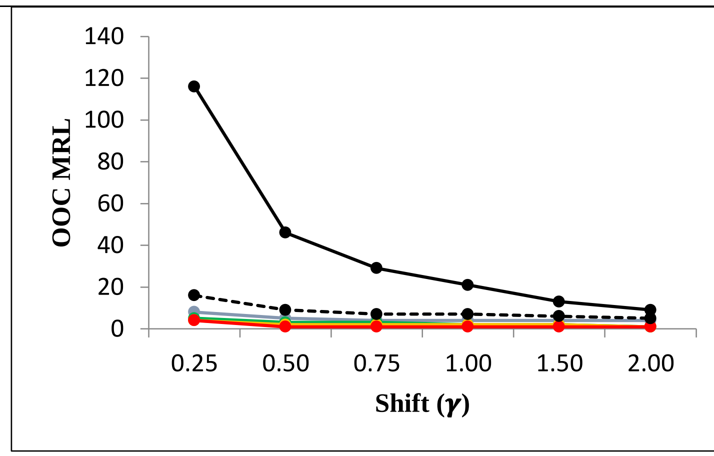


Figure 4a. *MRL* performance comparison of the NPEWMA-EX chart based on various percentiles of the reference sample and the NPEWMA-Rank chart under the *SymmMixN* distribution with m = 100, n = 5 and λ = 0.05

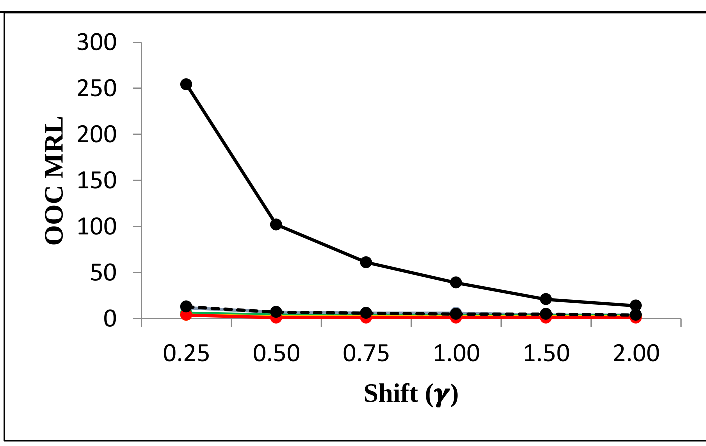


Figure 4b. *MRL* performance comparison of the NPEWMA-EX chart based on various percentiles of the reference sample and the NPEWMA-Rank chart under the *SymmMixN* distribution with m = 100, n = 5 and λ = 0.10

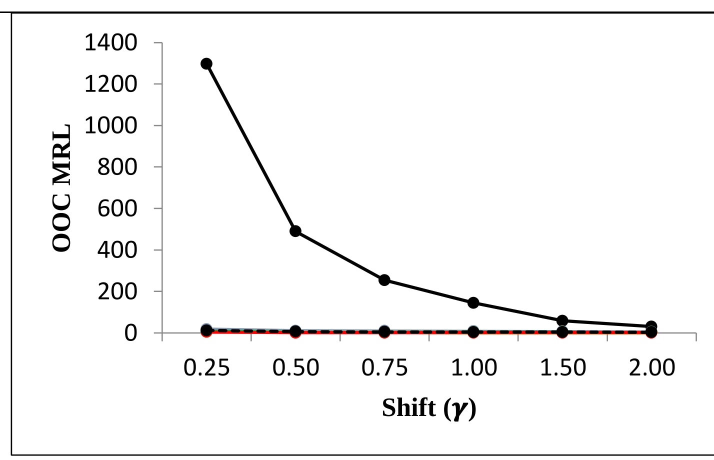


Figure 4c. *MRL* performance comparison of the NPEWMA-EX chart based on various percentiles of the reference sample and the NPEWMA-Rank chart under the *SymmMixN* distribution with m = 100, n = 5 and λ = 0.20

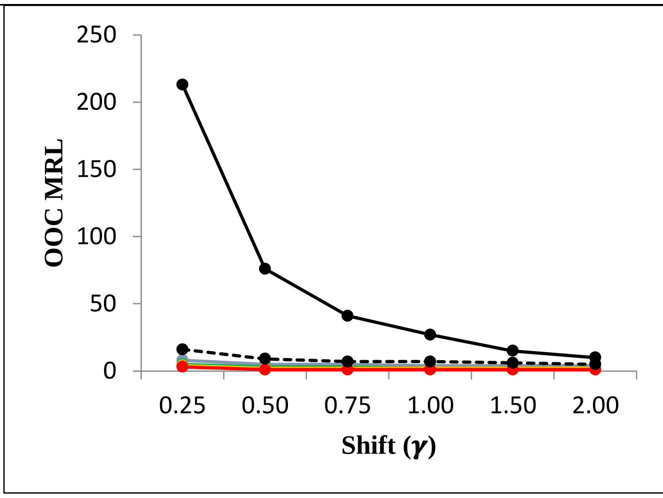


Figure 5a. *MRL* performance comparison of the NPEWMA-EX chart based on various percentiles of the reference sample and the NPEWMA-Rank chart under the *AsymmMixN1* distribution with m = 100, n = 5 and λ = 0.05

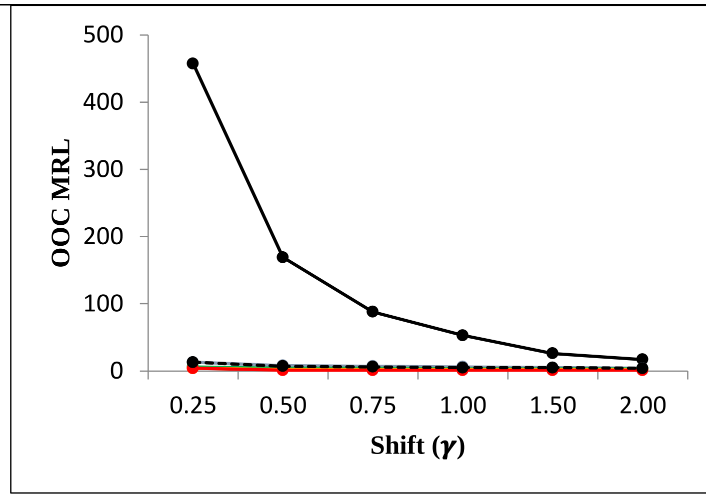


Figure 5b. *MRL* performance comparison of the NPEWMA-EX chart based on various percentiles of the reference sample and the NPEWMA-Rank chart under the *AsymmMixN1* distribution with m = 100, n = 5 and λ = 0.10

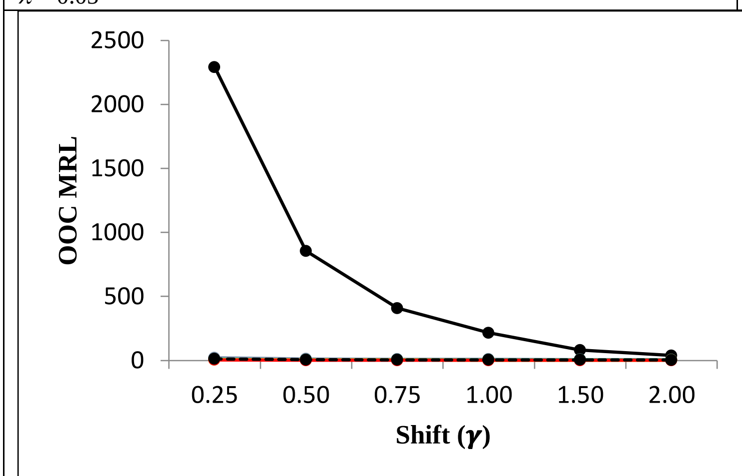


Figure 5c. *MRL* performance comparison of the NPEWMA-EX chart based on various percentiles of the reference sample and the NPEWMA-Rank chart under the *AsymmMixN1* distribution with m = 100, n = 5 and λ = 0.20

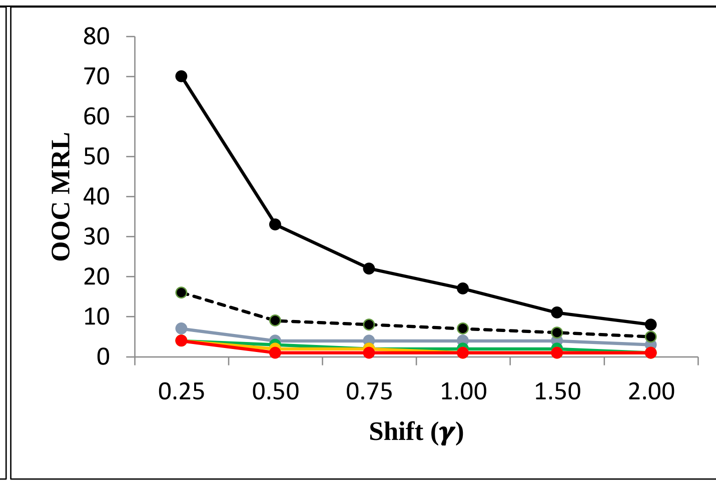


Figure 6a. *MRL* performance comparison of the NPEWMA-EX chart based on various percentiles of the reference sample and the NPEWMA-Rank chart under the *AsymmMixN2* distribution with m = 100, n = 5 and λ = 0.05

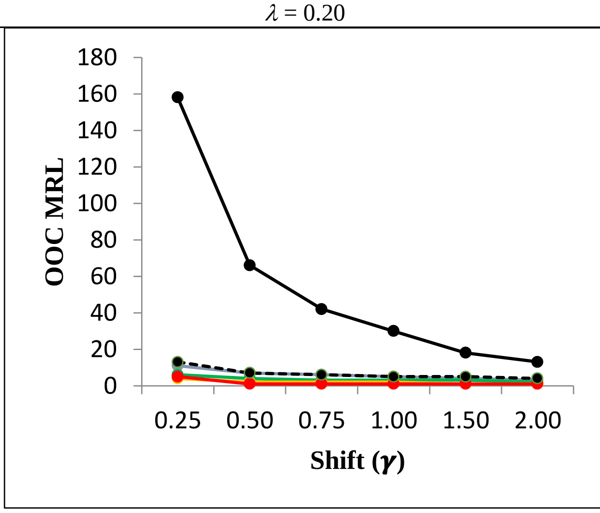


Figure 6b. *MRL* performance comparison of the NPEWMA-EX chart based on various percentiles of the reference sample and the NPEWMA-Rank chart under the *AsymmMixN2* distribution with m = 100, n = 5 and λ = 0.10

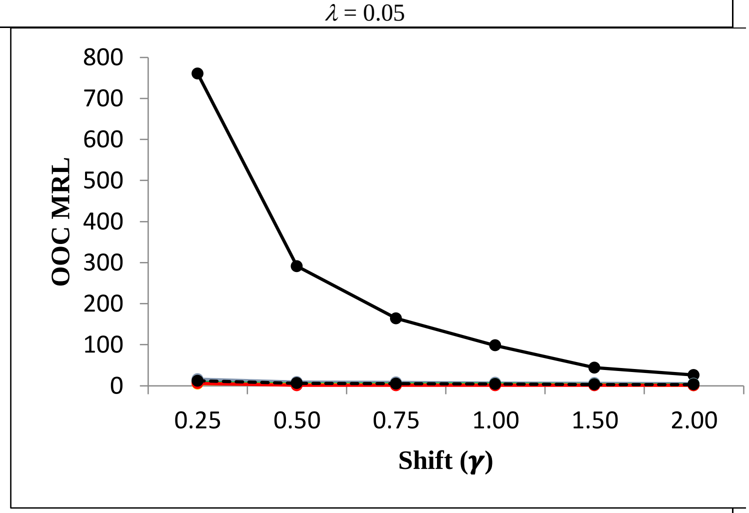


Figure 6c. *MRL* performance comparison of the NPEWMA-EX chart based on various percentiles of the reference sample and the NPEWMA-Rank chart under the *AsymmMixN2* distribution with m = 100, n = 5 and λ = 0.20

From Tables 5a and 5b we see that when the underlying process distribution is *SymmMixN*, the choice of the order statistic from the reference sample stays the same regardless of the value of the smoothing constant λ. Thus, for all λ, the NPEWMA-EX chart based on the 25th percentile performs the best for negative shifts and the NPEWMA-EX chart based on the 40th percentile is also competitive for γ = -0.25. Further, the NPEWMA-EX chart based on the 60th percentile performs best for γ = 0.25, whereas the NPEWMA-EX chart based on the 75th percentile performs best for all other shifts under consideration. Again we find that for the largest shift under consideration, i.e. γ = 2.00, the NPEWMA-EX chart based on the 75th percentile performs best. Since the run-length characteristics seem to converge as the size of this shift increases, the recommendation would be to use the NPEWMA-EX chart based on the 75th percentile when large shifts are of concern. This is illustrated in Figures 4a,b,c for $\lambda = 0.05, 0.10$ and 0.20, respectively, for some positive shifts, where it can also clearly be seen that the NPEWMA-EX chart based on the 25th percentile is performing the worst.

From Tables 6a and 6b we see that when the underlying process distribution is *AsymmMixN1* the NPEWMA-EX chart based on the 75th percentile performs the best for all possible positive shifts under consideration while for negative shifts, in general, the NPEWMA-EX chart based on the 25th percentile performs the best. The only minor exception is when γ = -0.25. In this case, for any λ, the NPEWMA-EX chart based on the 40th percentile is marginally better. Parts of the results are illustrated in Figures 5a,b,c for $\lambda = 0.05, 0.10$ and 0.20, respectively, where it can also clearly be seen that the NPEWMA-EX chart based on the 25th percentile is performing the worst.

From Tables 7a and 7b it can be seen that, when the underlying process distribution is *AsymmMixN2*, the NPEWMA-EX chart based on the 40th percentile performs the best for $\gamma = -0.25$ and that based on the 60th percentile performs the best for $\gamma = 0.25$, whereas the NPEWMA-EX chart based on the 25th percentile performs the best for other negative shifts and that based on the 75th percentile performs the best for all other positive shifts under consideration. Some of these phenomena are illustrated in Figures 6a,b,c for $\lambda = 0.05, 0.10$ and 0.20, respectively, where it can also clearly be seen that the NPEWMA-EX chart based on the 25th percentile is performing the worst.

From Tables 8a and 8b, we observe that when the underlying process distribution is *Log-Logistic*, for all negative shifts and small positive shifts, that is, when $\gamma = 0.25$ and 0.50, the NPEWMA-EX charts based on lower order percentiles perform the best, specifically, for

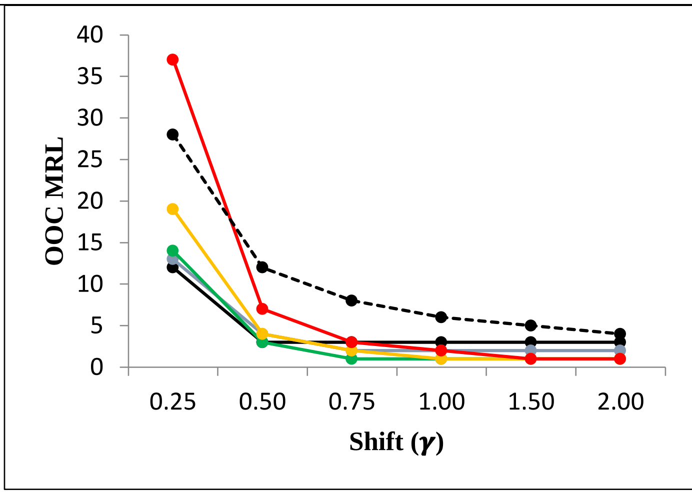


Figure 7a. *MRL* performance comparison of the NPEWMA-EX chart based on various percentiles of the reference sample and the NPEWMA-Rank chart under the *Log-Logistic* distribution with m = 100, n = 5 and λ = 0.05

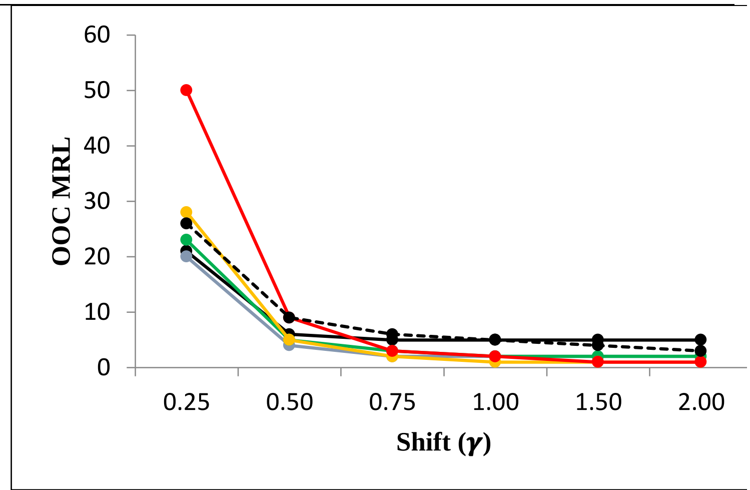


Figure 7b. *MRL* performance comparison of the NPEWMA-EX chart based on various percentiles of the reference sample and the NPEWMA-Rank chart under the *Log-Logistic* distribution with m = 100, n = 5 and λ = 0.10

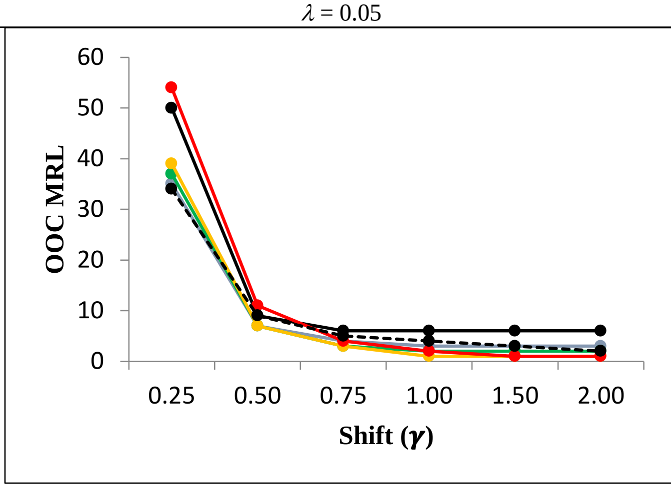
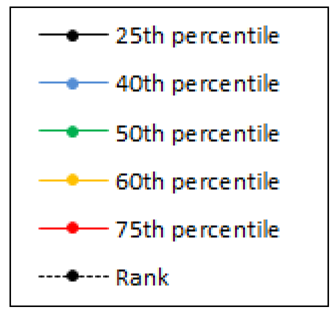


Figure 7c. *MRL* performance comparison of the NPEWMA-EX chart based on various percentiles of the reference sample and the NPEWMA-Rank chart under the *Log-Logistic* distribution with m = 100, n = 5 and λ = 0.20



Legend

Table 9. Summary of the efficacy of different reference sample percentiles for the NPEWMA-EX chart and the NPEWMA-Rank chart

<table>
<tr><td>$\gamma = -2.00$</td><td>$\gamma = -1.50$</td><td>$\gamma = -1.00$</td><td>$\gamma = -0.75$</td><td>$\gamma = -0.50$</td><td>$\gamma = -0.25$</td><td></td><td>γ = 0.25</td><td>γ = 0.50</td><td>γ = 0.75</td><td>γ = 1.00</td><td>γ = 1.50</td><td>γ = 2.00</td></tr>
<tr><td colspan="13">Symmetric distributions</td></tr>
<tr><td colspan="6">For all λ the EX(25) chart performs best, for λ = 0.05: EX(40) is also good</td><td>N(0,1)</td><td colspan="6">For all λ the EX(75) chart performs best</td></tr>
<tr><td>For all λ the EX(25) chart performs best</td><td>λ = 0.05: EX(25)
λ = 0.10: EX(25) and EX(40) and λ = 0.20: EX(25)</td><td colspan="4">λ = 0.05: EX(50)
λ = 0.10 and 0.20: EX(40)</td><td>DE(0,1)</td><td colspan="4">For all λ the EX(60) chart performs best</td><td colspan="2">For all λ the EX(75) chart performs best</td></tr>
<tr><td colspan="5">For all λ the EX(25) chart performs best</td><td>For all λ the EX(25 and EX(40) chart performs best</td><td>SymmMixN</td><td>For all λ the EX(60) chart performs best</td><td colspan="5">For all λ the EX(75) chart performs best</td></tr>
<tr><td colspan="13">Asymmetric distributions</td></tr>
<tr><td colspan="6">For all λ the EX(25) chart performs best</td><td>EXP(1)</td><td>λ = 0.05: EX(25)
λ = 0.10: EX(25)
λ = 0.20: Rank</td><td colspan="2">For all λ the EX(25) chart performs best</td><td>For all λ the EX(40) chart performs best</td><td>For all λ the EX(50) chart performs best</td><td>For all λ the EX(60) chart performs best</td></tr>
<tr><td colspan="5">For all λ the EX(25) chart performs best</td><td>EX(40) performs best</td><td>AsymmMixN1 and AsymmMixN2</td><td>N1: EX(75) performs best
N2: EX(60) performs best</td><td colspan="5">For all λ the EX(75) chart performs best</td></tr>
<tr><td colspan="6">For all λ the EX(25) chart performs best</td><td>Log-Logistic</td><td>λ = 0.05: EX(25)

λ = 0.10
λ = 0.20: EX(40)</td><td colspan="2">λ = 0.05
λ = 0.20: EX(50)

λ = 0.10: EX(40)</td><td>λ = 0.05: EX(50)

λ = 0.10
λ = 0.20: EX(60)</td><td colspan="2">λ = 0.05: EX(50,60,75)

λ = 0.10
λ = 0.20: EX(60,75)</td></tr>
</table>

$\lambda = 0.05$ the NPEWMA-EX chart based on the 1st quartile performs best, whereas for $\lambda = 0.10$ and 0.20 the NPEWMA-EX chart based on the 40th percentile performs best. As the magnitude of the shift increases, we find that the NPEWMA-EX charts based on higher order percentiles perform best. For example, for $\gamma = 1.50$ and 2.00 the NPEWMA-EX charts based on the 50th, 60th and 75th percentiles performs best for $\lambda = 0.05$, whereas the latter two charts performs best for $\lambda = 0.10$ and 0.20, respectively. The situation under positive shifts are illustrated in Figures 7a,b,c for $\lambda = 0.05, 0.10$ and 0.20, respectively.

The observations from Tables 2 to 8 are summarized in Table 9 along with some recommendations. Note that for conciseness, a shorthand notation is used to describe the charts. For example, the NPEWMA-EX chart based on the 50th percentile is denoted by EX(50), and if two charts perform similarly, for example, if the NEWMA-EX chart based on the 50th and 60th percentiles perform similarly, the notation EX(50,60) is used. Finally, in almost all cases, we see that the NPEWMA-EX chart performs better than the NPEWMA-Rank chart when the chart design parameters are appropriately chosen.

5. Examples

Example 1

First we illustrate the NEWMA-EX chart using a well-known dataset from Montgomery (2001; Tables 5.1 and 5.2). This data contains the inside diameters of piston rings produced by a forging process. More specifically, Table 5.1 contains twenty-five Phase I samples, each of five observations, that were collected when the process was believed to be IC, i.e. m = 125. An analysis in Montgomery (2001) showed that these data are from an IC process and thus can be considered to be Phase I reference data. Note also that for these data, a goodness of fit test for normality is not rejected. This does not guarantee that the normality assumption for a parametric EWMA chart is valid but often the practical implication is as such. We instead apply and contrast the proposed nonparametric exceedance charts based on the 25th, 40th, 50th (median), 60th and the 75th percentile, respectively, of the reference sample. The values of the respective reference sample percentiles are as follows: 25th percentile = 73.995, 40th percentile = 73.998, median = 74.001, 60th percentile = 74.004 and 75th percentile = 74.008. All of the measurements are in mm. The NPEWMA-Rank chart is also considered.

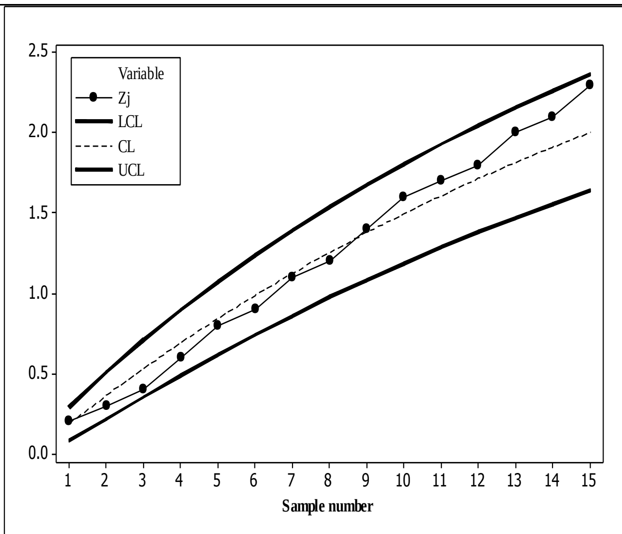


Figure 8a. The NPEWMA-EX chart based on the 25th percentile for the Montgomery (2001) piston-ring data

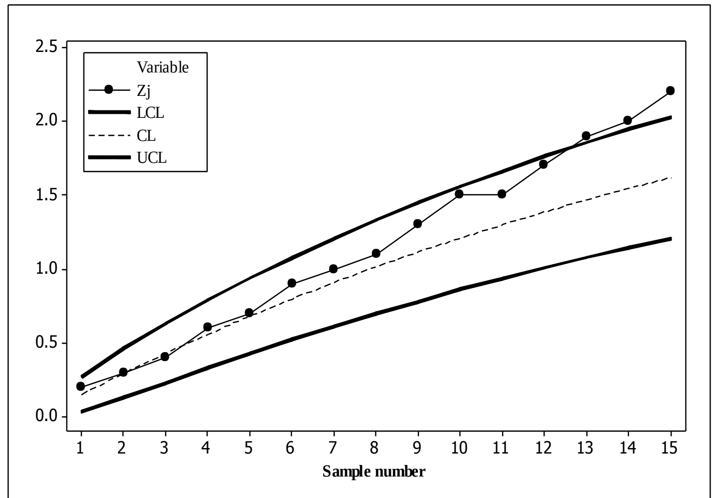


Figure 8b. The NPEWMA-EX chart based on the 40th percentile for the Montgomery (2001) piston-ring data

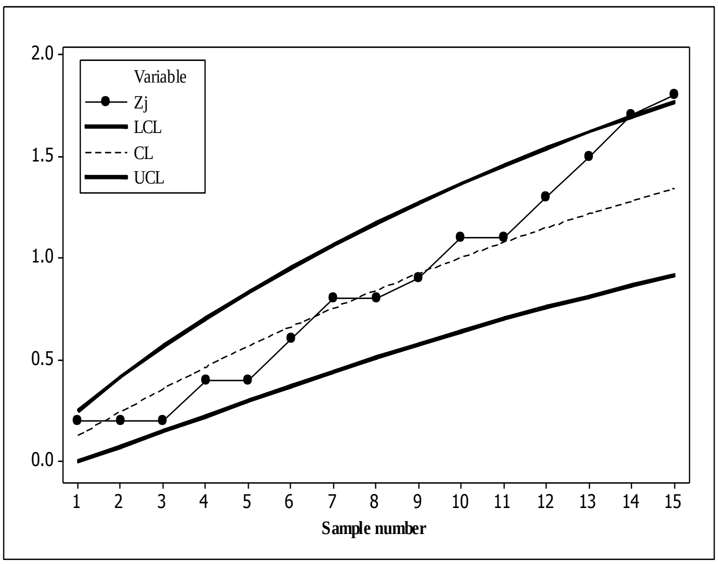


Figure 8c. The NPEWMA-EX chart based on the median for the Montgomery (2001) piston-ring data

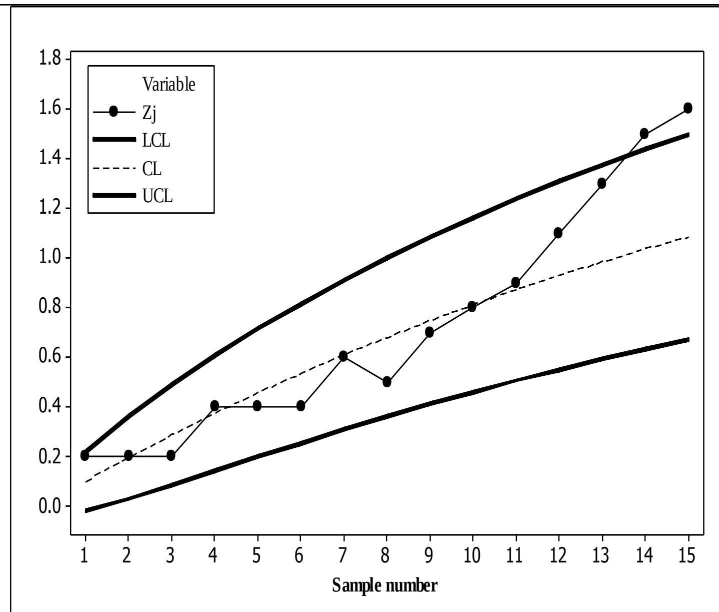


Figure 8d. The NPEWMA-EX chart based on the 60th percentile for the Montgomery (2001) piston-ring data

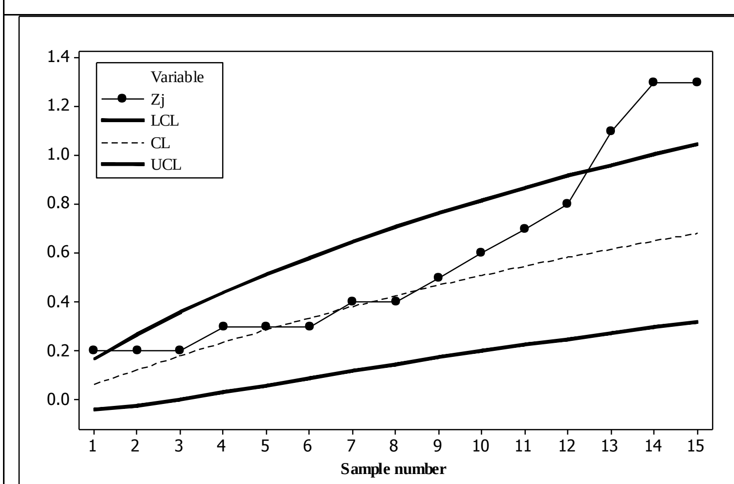


Figure 8e. The NPEWMA-EX chart based on the 75th percentile for the Montgomery (2001) piston-ring data

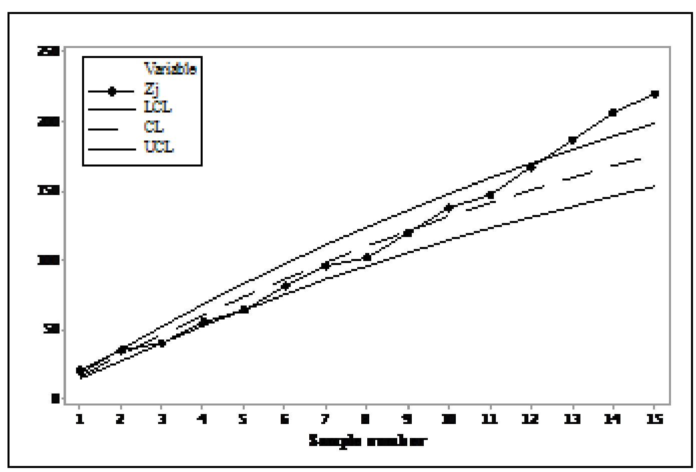


Figure 8f. The NPEWMA-Rank chart for the Montgomery (2001) piston-ring data

In order to calculate the Phase II exceedance control charts, we use the data in Table 5.2 of Montgomery (2001) that contains fifteen Phase II samples each of five observations (n = 5). The smoothing constant is taken to be λ = 0.05 and L is found such that MRL_0 = 350.

Table 9 suggests that the NPEWMA-EX chart based on 75th percentile performs best. However, if we investigate Table 2 in detail, we find that the performance of the NPEWMA-Rank chart is not too far from that of the NPEWMA-EX chart based on 75th percentile. In this example, both these charts perform similarly and the best by signaling on sample number 1. This is shown in Figures 8e and 8f for the NPEWMA-EX chart based on 75th percentile and the NPEWMA-Rank charts, respectively. From Figures 8b, 8c and 8d it can be seen that the NPEWMA-EX chart based on the 40th, 50th and 60th percentiles signal on samples number 13, 15 and 14, respectively, whereas the NPEWMA-EX chart based on the 25th percentile performs worst, since it doesn't signal at all (see Figure 8a).

For our first example, the data did not reject a goodness of fit test for normality. Nonparametric charts are useful for all continuous distributions and heavier tailed distributions are of particular interest in practice as they can give rise to more outliers which do not necessarily indicate an OOC process. So we illustrate the NPEWMA-EX chart when the data follow a $DE(0,1)$ distribution which is heavier tailed than the normal, but also symmetric .

Example 2

In practice the underlying process distribution is often unknown (or may not be normal) and this is where the nonparametric charts are particularly useful. To illustrate this the application of the NPEWMA-EX chart is shown when the data is non-normal, specifically, in this example it follows a $DE(0,1)$ distribution which is known to have a median of zero and a standard deviation equal to $\sqrt{2}$. An IC reference sample of size 100 (m = 100) was generated from this distribution and each data point was scaled so that the transformed observations have a standard deviation of 1. For the reference data we find the median equal to -0.052. Next the Phase II samples, each of size 5 (n = 5), were independently and sequentially generated by transforming the observations from a $DE(0,1)$ distribution so that the resulting observations have a median of $\gamma/\sqrt{n}$ $(= 0.112$ for γ = 0.25 and n = 5) and a standard deviation of 1. Consequently, the Phase II samples can be thought of as having been drawn from a process that is OOC in the median. The smoothing constant is taken to be λ = 0.05 and L is found such that MRL_0 = 350.

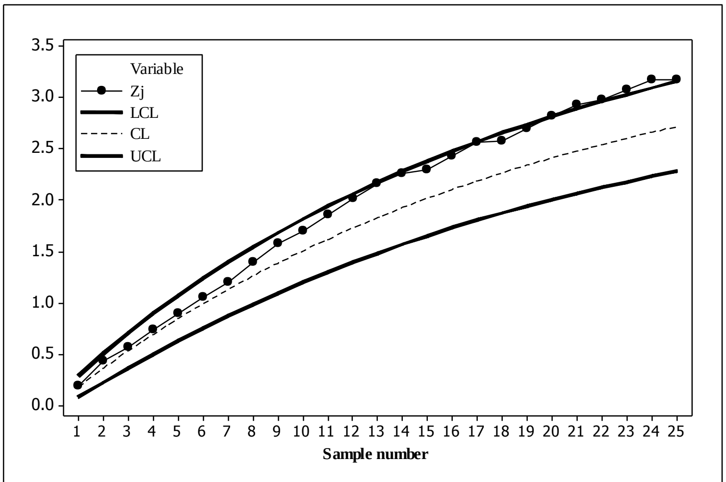


Figure 9a. The NPEWMA-EX chart based on the 25th percentile for the simulated

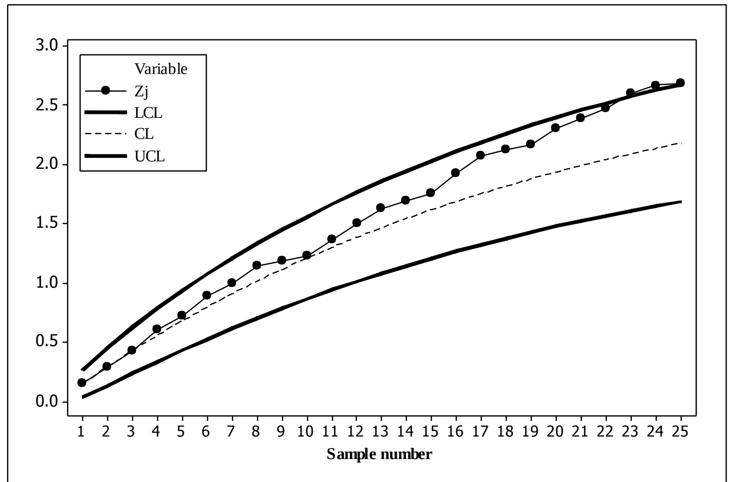


Figure 9b. The NPEWMA-EX chart based on the 40th percentile for the simulated data

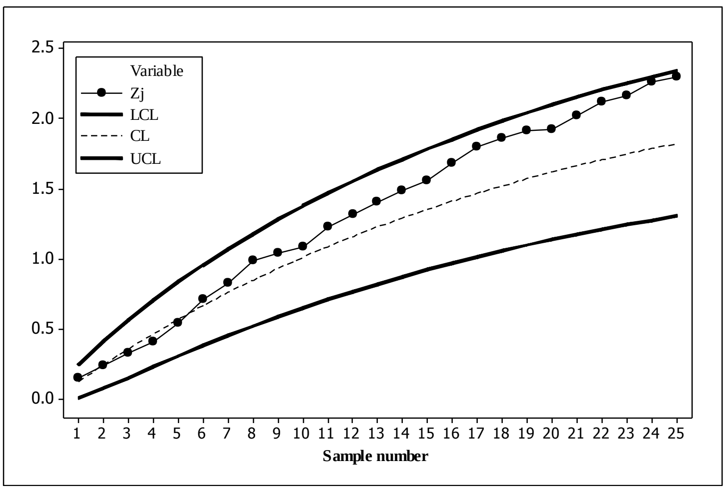


Figure 9c. The NPEWMA-EX chart based on the median for the simulated data

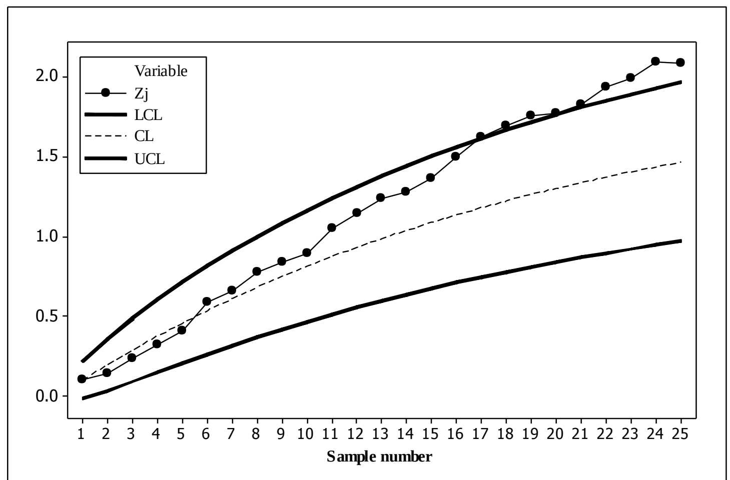


Figure 9d. The NPEWMA-EX chart based on the 60th percentile for the simulated data

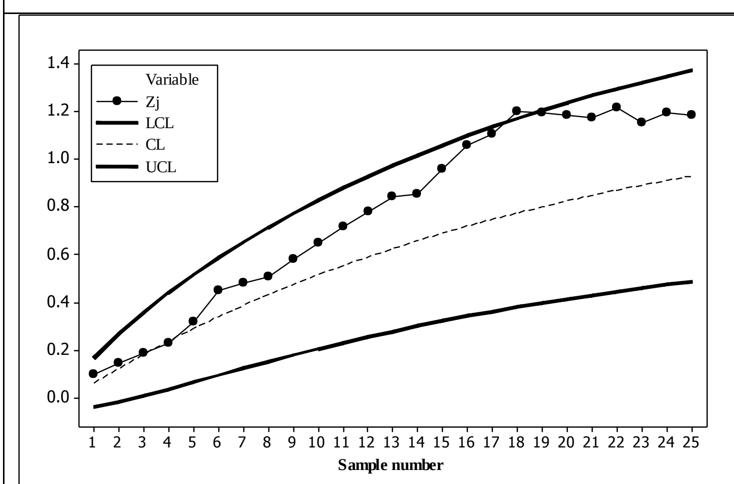


Figure 9e. The NPEWMA-EX chart based on the 75th percentile for the simulated data

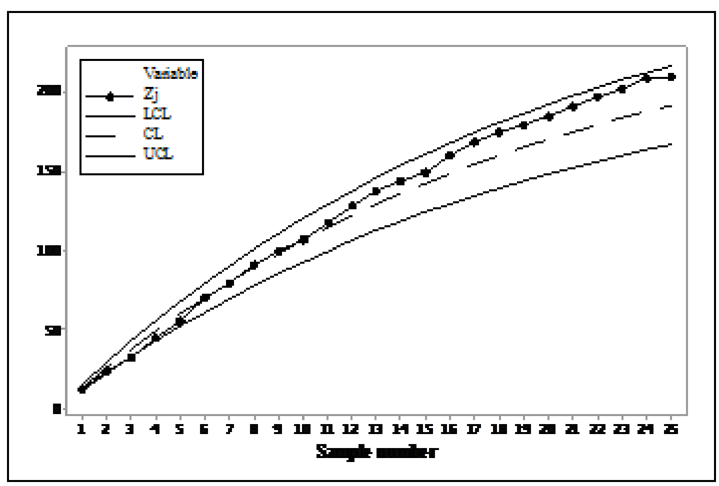


Figure 9f. The NPEWMA-Rank chart for the simulated data

From Figure 9d we can see that the NPEWMA-EX chart based on the 60^{th} percentile is performing best, since it signals the earliest at sample number 17. Performing second best is the NPEWMA-EX chart based on the 75^{th} percentile, signaling on sample number 18. This is consistent with the conclusions drawn in Table 9. The NPEWMA-EX charts based on the 25^{th} and 40^{th} percentiles signal on sample numbers 21 and 23, respectively, and the NPEWMA-EX chart based on the median and the NPEWMA-Rank chart perform the worst, since they don't signal at all.

6. Concluding remarks

Nonparametric EWMA (denoted NEWMA) charts may be an attractive substitute in practice as they combine the inherent advantages of nonparametric charts with the better small shift detection capability of EWMA-type charts. We examine a class of NPEWMA charts based on the exceedance statistic by investigating which order statistic (percentile), from the reference sample, should be used for good overall performance. We conclude that the NPEWMA-EX chart, based on higher order percentiles, such as the 60^{th} or the 75^{th} percentiles of the reference sample, are good overall charts for detecting a larger location shift. Other reference sample percentiles, such as the 25^{th} or the 40^{th}, can also be used when a smaller shift in location is expected. We also compare the NPEWMA-EX charts to the nonparametric EWMA chart based on the Wilcoxon rank-sum statistic (denoted NPEWMA-Rank) chart and, in almost all cases, we see that the NPEWMA-EX chart performs better than the NPEWMA-Rank chart if design parameters are appropriately chosen. Overall, it is seen that the exceedance EWMA chart based on higher percentiles performs better than its competitors in many cases for a number of distributions. More specifically, for moderate to large shifts there is little doubt that the end-user should use the exceedance chart based on the 75^{th} percentile which signals quickly for all reference values under consideration. This is an interesting result in the literature on nonparametric exceedance/precedence tests and control charts. Note that our metric of comparison is the *MRL*, which we endorse over the *ARL*.

In this context, it is worth mentioning that designing a mixed CUSUM-EWMA type chart, in the line of Zaman et al. (2014), based on the exceedance statistic, will be an interesting future research problem. Also, it is worth exploring how to to use auxiliary information in the EWMA-EX chart using ideas of Abbas et al. (2014). From a statistical point of view, one may also like to identify the relationship between the order of the reference sample order statistic and the underlying distribution.

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Appendix A: Various results, statistical derivations and discussions in the light of Graham et al. (2012)

Note that, under IC, given (conditionally on) $X_{(r)}$, the $U_{j,r}$'s are independently binomially distributed with parameters (n, p_r) for any $j = 1,2, \dots$.

Since $U_{j,r}$ is the number of Y-observations in the j^{th} Phase II sample that exceeds $X_{(r)}$, given $X_{(r)}$, the random variable $U_{j,r}$ follows a binomial distribution with parameters (n, p_r) under IC where $p_r = P[Y > X_{(r)} | X_{(r)}] = 1 - F(X_{(r)})$. Interested readers may also see Mukherjee et al. (2013).

Therefore,

$$E(U_{j,r}|X_{(r)}) = np_r \quad and \ Var\left(U_{j,r}|X_{(r)}\right) = np_r(1 - p_r) \quad \forall\, j = 1,2, \dots \qquad (A.1)$$

When there is no shift in the process $F(X_{(r)})$ has same distribution as the r^{th} order statistic from a random sample of size m from a Uniform distribution over the interval [0,1]. That is, $F(X_{(r)})$ follows a Beta distribution with parameters r and $m + 1 - r$ irrespective of choice of F. This result actually ensures distribution-free characteristics of the charting scheme. This can be used to obtain various moments of p_r using the properties of the Beta distribution with parameters r and $m - r + 1$. Therefore, we have:

$$E\left(F(X_{(r)})\right) = \frac{r}{r + m + 1 - r} = \frac{r}{m + 1} \text{ and } Var\left(F\left(X_{(r)}\right)\right) = \frac{r(m + 1 - r)}{(m + 1)^2(m + 2)}.$$

$$E\left(F(X_{(r)})^2\right) = \frac{r(r + 1)}{(m + 1)(m + 2)}.$$

Properties of p_r

Noting that $p_r = P[Y > X_{(r)} | X_{(r)}] = 1 - F(X_{(r)})$, we have,

I. $E(p_r) = 1 - E\left(F\left(X_{(r)}\right)\right) = 1 - \frac{r}{m+1} = \frac{m+1-r}{m+1} = 1 - a$ (say),

II. $E(p_r^2) = 1 - 2E\left(F\left(X_{(r)}\right)\right) + E\left(F\left(X_{(r)}\right)^2\right)$

$$= 1 - 2\frac{r}{m + 1} + \frac{r(r + 1)}{(m + 1)(m + 2)} = \frac{m^2 + 3m + 2 - 2rm - 3r + r^2}{(m + 1)(m + 2)}.$$

III. $E(p_r) - E(p_r^2) = \frac{r(m+1-r)}{(m+1)(m+2)}$

The conditional IC mean of the plotting statistic given $X_{(r)}$ and the choice of Z_0

Using recursive substitution, we have

$$E\left(Z_j|X_{(r)}\right) = E\left(\lambda U_{j,r} + (1 - \lambda)Z_{j-1}|X_{(r)}\right)$$

$$= E\left(\lambda \sum_{k=0}^{j-1} (1 - \lambda)^k\, U_{j-k,r} + (1 - \lambda)^j Z_0|X_{(r)}\right)$$

$$= \lambda \sum_{k=0}^{j-1} (1-\lambda)^k E\big(U_{j-k,r}|X_{(r)}\big) + (1-\lambda)^j E\big(Z_0|X_{(r)}\big).$$

Using the sum of a geometric series (in general $\sum_{k=0}^{j-1} r^k = \frac{1-r^j}{1-r}$) and since $E\big(U_{j-k,r}|X_{(r)}\big) = np_r\ \forall j,k,$ we obtain

$$E\big(Z_j|X_{(r)}\big) = np_r\big(1-(1-\lambda)^j\big) + (1-\lambda)^j E\big(Z_0|X_{(r)}\big). \qquad (A.2)$$

Graham et al. (2012) framed the NPEWMA-EX keeping in parity with the EWMA chart for binomial proportion. Consequently, they started with a natural choice: $Z_0 = E\big(U_{j-k,r}|X_{(r)}\big) = np_r$. Interestingly, in such a case, we have, from (A.2)

$$E\big(Z_j|X_{(r)}\big) = np_r\big(1-(1-\lambda)^j\big) + (1-\lambda)^j E\big(Z_0|X_{(r)}\big)$$
$$= np_r\big(1-(1-\lambda)^j\big) + (1-\lambda)^j\, n\, p_r = n\, p_r,$$

since, given $X_{(r)}$, the $Z_0 = np_r$ behaves as a constant. This result can be seen from Appendix A of Graham et al. (2012). In this case, $E\big(Z_j\big) = EE\big(Z_j|X_{(r)}\big) = nE(p_r) = n(1-a)$. Readers may note that, apparently, there was a typo in $E\big(Z_j\big)$ in Equation (2) of Graham et al. (2012). If $Z_0 = np_r$, then $E\big(Z_j\big) = EE\big(Z_j|X_{(r)}\big) = nE(p_r) = n(1-a)$ and not $np_r\big(1-(1-\lambda)^j\big)$. The derivation in the Appendix A of Graham et al. (2012) is however, correct.

Nevertheless, one may note that in reality, the choice $Z_0 = E\big(U_{j-k,r}|X_{(r)}\big) = np_r$ is not admissible. Unconditionally p_r is a random variable and np_r can take any value between 0 and n. Therefore, before introducing the NPEWMA-EX chart Graham et al. (2012) actually switched to a more realistic choice, namely, $Z_0 = EE\big(U_{j-k,r}|X_{(r)}\big) = nE\big(p_r\big) = n\,\frac{m+1-r}{m+1} = n(1-a)$. This can be seen from the statement immediately after Equation (2); though the reason was not explicitly mentioned. Under such a choice,

$$E\big(Z_j|X_{(r)}\big) = np_r\big(1-(1-\lambda)^j\big) + (1-\lambda)^j E\big(Z_0|X_{(r)}\big)$$
$$= np_r\big(1-(1-\lambda)^j\big) + (1-\lambda)^j\, n\,(1-a).$$

In this case,

$$E\big(Z_j\big) = EE\big(Z_j|X_{(r)}\big) = nE(p_r)\big(1-(1-\lambda)^j\big) + (1-\lambda)^j\, n\, \frac{m+1-r}{m+1}$$
$$= n\,\frac{m+1-r}{m+1}\big(1-(1-\lambda)^j\big) + (1-\lambda)^j\, n\,\frac{m+1-r}{m+1} = n\,\frac{m+1-r}{m+1} = n(1-a).$$

Note that, unconditionally np_r is random and can take any value in between 0 and n. Therefore, one may suggest an arbitrary starting value $Z_0 = \tau$, belonging between 0 and n, both inclusive. Then, we have

$$E(Z_j|X_{(r)}) = np_r(1-(1-\lambda)^j) + (1-\lambda)^j\tau. \qquad (A.3)$$

Graham et al. (2012) did not mention other possible choices. Here, we discuss the results in the light of two different starting values for better clarity.

Note that, p_r is a random variable and hence, the expected starting value could be $Z_0 = E(np_r) = n\,\frac{m+1-r}{m+1} = n(1-a)$. We may consider this as one of the possible choices as in Graham et al. (2012). Another possible choice is $Z_0 = 0$. This choice may appear unconventional but it works very nicely and performs equivalently to the choice $Z_0 = n(1-a)$ if exact time-varying limits are considered. We shall discuss this in details later.

From (A.3) we have,

$$E(Z_j|X_{(r)}) = \begin{cases} np_r(1-(1-\lambda)^j) + (1-\lambda)^j\, n(1-a) & \text{if} \quad Z_0 = \tau = n(1-a) \\ np_r(1-(1-\lambda)^j) & \text{if} \quad Z_0 = \tau = 0 \end{cases}. \qquad (\text{A}.4)$$

The unconditional IC mean of the plotting statistic given $X_{(r)}$.

With an arbitrary starting value $Z_0 = \tau$, where τ lies between 0 and n, both inclusive, the unconditional IC mean of Z_j can be obtained from (A.3) as

$$\begin{aligned} E(Z_j) &= EE(Z_j|X_{(r)}) = nE(p_r)(1-(1-\lambda)^j) + (1-\lambda)^j\,\tau \\ &= n(1-a)(1-(1-\lambda)^j) + (1-\lambda)^j\tau. \end{aligned} \qquad (A.5)$$

Next we discuss two exact cases. From (A.5) we have,

$$E(Z_j) = \begin{cases} n(1-a)(1-(1-\lambda)^j) + (1-\lambda)^j\, n(1-a) = n(1-a) & \text{if} \quad Z_0 = \tau = n(1-a) \\ n(1-a)(1-(1-\lambda)^j) & \text{if} \quad Z_0 = \tau = 0 \end{cases}. \ (\text{A}.6)$$

Clearly, whatever τ, we have, under steady-state (where j tends to ∞),

$$E(Z_j) = n\frac{m+1-r}{m+1} = n(1-a).$$

Graham et al. (2012) considered this as a steady-state mean with $Z_0 = \tau = n(1-a)$. Therefore, apart from a small typo in expression of $E(Z_j)$ in Equation (2) of Graham et al. (2012), Equation (3) and successive parts of their article are accurate.

The conditional IC variance and IC standard deviation of the plotting statistic

Further, from recursive substitution, we see,

$$Var(Z_j|X_{(r)}) = Var\left(\lambda \sum_{k=0}^{j-1} (1-\lambda)^k U_{j-k,r} + (1-\lambda)^j Z_0 | X_{(r)}\right)$$
$$= Var\left(\lambda \sum_{k=0}^{j-1} (1-\lambda)^k U_{j-k,r} | X_{(r)}\right), \qquad (A.7)$$

irrespective of the choice of Z_0, as given $X_{(r)}$, Z_0 behaves as a constant. This is true, even if Z_0 is not prefixed but $Z_0 = np_r$. Equation (A.7) holds when $Z_0 = np_r$, as given $X_{(r)}$, p_r is a constant.

Suppose, from Equation (A.7) we have

$$VAR(Z_j|X_{(r)}) = VAR\left(\lambda \sum_{k=0}^{j-1} (1-\lambda)^k U_{j-k,r} | X_{(r)}\right)$$
$$= \lambda^2 \sum_{k=0}^{j-1} (1-\lambda)^{2k} VAR(U_{j-k,r}|X_{(r)}) = np_r(1-p_r)\lambda^2 \sum_{k=0}^{j-1} (1-\lambda)^{2k}$$
$$= np_r(1-p_r)\lambda^2 \left(\frac{1-(1-\lambda)^{2j}}{1-(1-\lambda)^2}\right)$$
$$= np_r(1-p_r)\lambda \left(\frac{1-(1-\lambda)^{2j}}{2-\lambda}\right). \qquad (A.8)$$

Since, $VAR(U_{j-k,r}|X_{(r)}) = np_r(1-p_r)$ and, given $X_{(r)}$, the $U_{j,r}'s$ are independent.

Therefore, we always have $STDEV(Z_j|X_{(r)}) = \sqrt{\frac{\lambda}{2-\lambda}(1-(1-\lambda)^{2j}) np_r(1-p_r)}$.

The unconditional IC variance and IC standard deviation of the plotting statistic

When $Z_0 = np_r$, using previous results of conditional mean and variance we find

$$Var(Z_j) = Var[E(Z_j|X_{(r)})] + E[Var(Z_j|X_{(r)})]$$
$$= Var[np_r] + E\left[\frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2j}\right) np_r(1-p_r)\right]$$
$$= n^2 Var(p_r) + \frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2j}\right) n\left(E(p_r) - E(p_r^2)\right)$$
$$= n^2 \frac{r(m-r+1)}{(m+1)^2(m+2)} + \frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2j}\right) n \frac{r(m-r+1)}{(m+1)(m+2)}$$

This expression was actually not derived in Graham et al. (2012). They derived $Var(Z_j)$ only under the admissible choices of Z_0, such as $n(1-a)$; though not explicitly mentioned in the

Appendix. Therefore, we show that the exact expression of $Var(Z_j)$ is the same irrespective of the choice of Z_0, as $n(1-a)$ or 0.

Suppose $Z_0 = \tau$. Using the results under Equations (A.2) and (A.8) we find

$$
\begin{aligned}
&Var(Z_j) = Var[E(Z_j|X_{(r)})] + E[Var(Z_j|X_{(r)})] \\
&= Var[np_r(1-(1-\lambda)^j) + (1-\lambda)^j\,\tau] + E\left[\frac{\lambda}{2-\lambda}(1-(1-\lambda)^{2j})np_r(1-p_r)\right] \\
&= n^2(1-(1-\lambda)^j)^2 Var(p_r) + \frac{\lambda}{2-\lambda}(1-(1-\lambda)^{2j})n(E(p_r)-E(p_r^2)) \\
&= n^2\frac{r(m-r+1)}{(m+1)^2(m+2)}(1-(1-\lambda)^j)^2 + \frac{\lambda}{2-\lambda}(1-(1-\lambda)^{2j})n\,\frac{r(m-r+1)}{(m+1)(m+2)}
\end{aligned}
$$

since $Var\left((1-\lambda)^j\, n\,\frac{m+1-r}{m+1}\right) = 0$. This is true whatever value of prefixed τ is used; including 0 and $n(1-a)$. Therefore, under both the admissible choices,

$$STDEV(Z_j) = \sqrt{n^2\frac{r(m-r+1)}{(m+1)^2(m+2)}(1-(1-\lambda)^j)^2 + \frac{\lambda}{2-\lambda}(1-(1-\lambda)^{2j})n\,\frac{r(m-r+1)}{(m+1)(m+2)}}.$$

Consequently, the unconditional steady-state (asymptotic) control limits and the CL are given by

$$UCL/LCL = n\left(\frac{m-r+1}{m+1}\right) \pm L\sqrt{n^2\frac{r(m-r+1)}{(m+1)^2(m+2)} + \frac{\lambda}{2-\lambda}n\,\frac{r(m-r+1)}{(m+1)(m+2)}}$$

and

$$CL = n\left(\frac{m-r+1}{m+1}\right)$$

respectively. Graham et al. (2012) used these asymptotic limits with starting value $Z_0 = n(1-a)$. Their derivations and results are correct expect the typo mentioned above. In this context, it is worth noting that even in the parametric situation, there is a dearth of literature that uses the exact variance expression of the plotting statistic taking account of conditioning on the Phase I sample. The potential impact of the use of exact variance, in control chart design, in the parametric case for unknown in-control parameters, will also be worth exploring, which is out of scope of the present work.

Choices of Z_0

In summary, we observe that for any pre-fixed Z_0 between 0 and n, both the expressions of the exact and the steady-state $Var(Z_j)$ will remain invariant. Similarly, the expression for the steady-state $E(Z_j)$ will remain invariant. This is, however, not true for the exact $E(Z_j)$. As a consequence, for any pre-fixed Z_0 between 0 and n, the asymptotic control limits, introduced in Graham et al. (2012), will remain valid. Nevertheless, steady-state chart performance is seriously affected by the choice of Z_0. For example, if $Z_0 = 0$ is considered with asymptotic limits, there could be a large number of early false alarms, especially when λ is small. For example, when $\lambda = 0.05$, taking $Z_0 = 0$ will almost everywhere give a signal at the beginning. Thus, we recommend using $Z_0 = n(1 - a)$ as in Graham et al. (2012) instead of other choices if asymptotic control limits are used.

The situation is, however, slightly different if we consider exact time-varying control limits. For example, with $Z_0 = n(1 - a)$, UCL/LCL are given by

$$n\left(\frac{m - r + 1}{m + 1}\right) \pm L\sqrt{n^2\frac{r(m - r + 1)}{(m + 1)^2(m + 2)}(1 - (1 - \lambda)^j)^2 + \frac{\lambda}{2 - \lambda}(1 - (1 - \lambda)^{2j})n\,\frac{r(m - r + 1)}{(m + 1)(m + 2)}}$$

and $CL = n\left(\frac{m - r + 1}{m + 1}\right)$. On the other hand, with $Z_0 = 0$, UCL/LCL are given by

$$n\left(\frac{m - r + 1}{m + 1}\right)(1 - (1 - \lambda)^j) \pm L\sqrt{n^2\frac{r(m - r + 1)}{(m + 1)^2(m + 2)}(1 - (1 - \lambda)^j)^2 + \frac{\lambda}{2 - \lambda}(1 - (1 - \lambda)^{2j})n\,\frac{r(m - r + 1)}{(m + 1)(m + 2)}}$$

and $CL = n\left(\frac{m - r + 1}{m + 1}\right)(1 - (1 - \lambda)^j)$. Clearly, the exact time-varying control limits are different according to the choice of Z_0. Here control limits are statistically adjusted according to the starting value and therefore, the impact of choosing an arbitrary starting value is expected to have a minimal impact.

A simulation study was carried out and it was observed that the same charting constant L, as obtained with $Z_0 = 0$ with target nominal MRL_0 for a given m, n and λ, returns almost the same MRL_0 if in the similar set-up, $Z_0 = n(1 - a)$ is used instead. The OOC performance of the charts are also similar except minor sampling fluctuations if $Z_0 = n(1 - a)$ is used instead of $Z_0 = 0$. There will be practically no variation in the chart performance $Z_0 = 0$ is chosen instead of $Z_0 = n(1 - a)$. We omit further details for brevity.