COMPARISON BETWEEN METAL FOAM AND FINNED TUBE HEAT EXCHANGERS FOR HVAC APPLICATIONS

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ABSTRACT

Nowadays the most commonly used heat exchangers in residential air conditioning applications for heat exchange between air and refrigerant are of the round tube and fin type. Recent investigations have revealed that combining round tubes with open cell metal foam can lead to heat exchangers which outperform their finned counterparts. The thermal contact resistance is one of the issues which is very critical in order to compete with the finned heat exchangers. Using flat tubes allows for a larger base surface for the same flow area available to the medium in the tubes. Since a larger contact area between the foam and the tubes leads to a reduced thermal contact resistance, flat tubes and metal foam are expected to be a good combination. In this work a comparison is made between commonly used louvered fin and round tube heat exchangers and metal foam enhanced flat tube heat exchangers. The comparison is made using computational fluid dynamics (CFD) and a two-dimensional volume averaged model for the metal foam. The influence of the contact resistance, pore density, external porosity, tube spacing and tube width are investigated. The most important parameters and their optimal values are identified. It is revealed that the contact resistance and surface efficiency are absolutely critical in order for the metal foam heat exchangers to achieve good performance.

INTRODUCTION

Finned round tube heat exchangers are used in many different industrial and residential applications. Many different types of compact fin and round tube heat exchangers exist, such as the plain fin and the louvered fin. Enhanced fin types such as the louvered fin tend to have higher pressure drops per unit length, but compensate this with higher heat transfer coefficients. This tends to result in heat exchangers with shorter flow depths than unenhanced fin designs such as the plain fin [1]. A different concept is using open cell metal foams to replace the fins. Like fins, these structures have a high porosity and possibly a large surface-to-volume ratio. A significant difference with fins is that they do not have a preferential air flow direction. Several authors, such as Chumpia and Hooman [2], Boomsma et al. [3] and T'Joen et al. [4] have already investigated the potential of these metal foam heat exchangers to compete with tube and fin types. T'Joen et al. experimentally investigated the thermal-hydraulic performance of a single row of aluminium tubes covered with metal foam, shown in Figure 1. They found that the performance decreased as the thickness of the foam layer was increased. Even though the thermal

resistance decreased due to the larger surface area, the pressure increased to such an extent that the total performance deteriorated. They noted that the performance of the metal foam tubes increased as the spacing between the tubes decreased. They also investigated the effect of the pore size, indicated by the pores per inch (PPI) value, which showed that the foams with 20 PPI outperformed the 10 PPI foams. Furthermore, it was found that the best performing metal foam had the lowest porosity. Finally, the metal foam covered tubes were compared to the helically finned tube. This showed that the metal foam covered tubes could outperform the finned tube only if the thermal contact resistance was limited by brazing the foam to the tube.



Figure 1 Helically finned tube (top) and metal foam covered tube (bottom) [4]

In a later study, De Schampheleire et al. [5] investigated the performance of a heat exchanger which consists of blocks of aluminium foam into which tubes are inserted, instead of using individual tubes. This can be seen as a limiting case of the previous research, where the spacing between the foam covered tubes has become zero. This type of heat exchanger is shown in Figure 2.



Figure 2 Round tube and metal foam heat exchanger

They compared this heat exchanger with a louvered fin and tube type heat exchanger, consisting of copper tubes and aluminium fins. Both heat exchangers have the same flow depth and the same frontal surface area. The metal foam had a PPI value of 10, which resulted in a surface-to-volume ratio which was only 66% of that of the louvered fin heat exchanger. The airside heat transfer resistance of the metal foam heat exchanger was over twice as large as that of the louvered fin heat exchanger, for the same frontal velocity. The friction factor, which can be interpreted as a dimensionless representation of the pressure drop per unit length, was lower for the metal foam heat exchanger. This is explained by the relatively large pore sizes of the metal foam (10 PPI) and the very low fin pitch (1.4 mm) of the louvered fin.

They also estimated the thermal contact resistance and found that this could account for up to 58% of the total thermal resistance. As such, the thermal contact resistance was identified as one of the large issues for the thermal performance of metal foam heat exchangers, which is in agreement with the findings of the earlier research by T'Joen et al. [4].

Huisseune et al. [6] used the volume averaged metal foam model developed by De Jaeger [7] to predict the performance of the metal foam heat exchanger which was measured by De Schampheleire, in case different metal foams would be used and the dimensions would be changed. This predicted performance was compared to the measured performance of the louvered fin heat exchanger. The PPI was varied between 10 and 45, the volumetric porosity was decreased as the PPI increased, with a value of 95% for the 10 PPI foam and a value of 90% for the 45 PPI foam. The method of Cowell [1] was used to evaluate the performance of the different heat exchangers. This method evaluates the heat exchanger volume and pressure drop under constraint of having the same mass flow rate, heat transfer rate and inlet and outlet temperature. They predicted that the metal foam heat exchanger could outperform the louvered fin heat exchanger, provided that the global dimensions of the heat exchanger was changed. The metal foam heat exchanger required a larger frontal surface area and thus a lower frontal air velocity than the louvered fin heat exchanger. However, by reducing the flow depth, the volume could be reduced, while still achieving the same heat transfer rate at the same pressure drop as the louvered fin heat exchanger. This was only possible if a dense metal foam was used (with a PPI of 45) which was made out of copper. The authors concluded that compact metal foam heat exchangers have some potential, but that optimization of the metal foam parameters is crucial.

Another approach is to combine the metal foam with flat tubes instead of round tubes. This has the advantage that the perimeter of the tube is larger for a given flow area in the tube, which allows for a larger contact surface between the metal foam and the tubes. This could lead to a reduction of the thermal contact resistance, which was identified as a major problem. In this study a performance screening of a heat exchanger consisting of metal foam combined with flat tubes will be done. Using the Cowell method, the thermal performance will be compared with that of a typical louvered fin and round tube heat exchanger. It should be noted that this

only compares the thermohydraulic merits of these heat exchangers. In reality, other concerns are at least as important, such the production cost, reliability, fouling and noise. These aspects are however out of scope of the present study.

NOMENCLATURE

A	$[m^2]$	Heat transfer surface area
U	[W/m²K]	Overall heat transfer coefficient
h	$[W/m^2K]$	Heat transfer coefficient
ρ	[kg/m³]	Density
v	[m/s]	Frontal air velocity
c_p	[J/kgK]	Specific heat capacity
Pr	[-]	Prandtl number $Pr = \frac{c_p \mu}{k}$
T	[K]	Temperature
μ	[Pa s]	Dynamic viscosity
k	[W/mK]	Thermal conductivity
L	[m]	Length in the flow direction
Vol	[m³]	Heat exchanger volume
Q	[W]	Heat transfer rate
m	[kg/s]	Mass flow rate
NTU	[-]	Number of transfer units
PPI	[-]	Pores per linear inch
Special charac	rters	
Λ		Difference
ϵ	[-]	Heat exchanger effectiveness
	[m ² /m ³]	Surface-to-volume ratio
$_{\partial P}^{eta}$	[Pa/m]	Pressure drop per unit length
		T I
$\overline{\partial L}$		
Subscripts		
in		At the inlet of the computational domain
		dr une companie domain
Superscripts		
*		Relative

GEOMETRY AND NUMERICAL MODEL

The geometry of interest and the macroscopic geometrical parameters are shown in Figure 3. The numerical values of the parameters are given in Table 1. A 2D model is used, corresponding to a heat exchanger of infinite width. The upstream zone is extended over a distance which is equal to the maximum foam height which will be tested, equal to 30 mm. This allows taking the contraction of the flow upstream of the heat exchanger into account. A uniform temperature and velocity profile is imposed. The velocity varies over a range of 0.5 to 2.0 m/s, the inlet temperature is fixed to 45°C. At the foam outlet, the pressure is fixed to atmospheric pressure. This is an approximation of the reality, as this does not allow for static pressure recovery due to the expansion behind the heat exchanger.

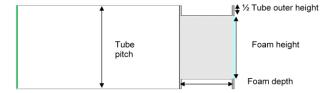


Figure 3 Macroscopic geometrical parameters

At the outer tube wall the temperature is fixed to 75 °C. For the material properties of the air, the Sutherland approximation was used for the molecular viscosity and the molecular thermal conductivity was based on the kinetic theory. The density was calculated using the incompressible ideal gas law and the specific heat capacity using a polynomial curve fit. The density, specific heat and thermal conductivity of the metal foam materials were considered constant (for aluminum: $\rho = 2710$ kg/m3, cp = 871 J/kgK and ks = 220 W/mK; for copper: ρ = 8960 kg/m3, cp = 380 J/kgK and ks = 390 W/mK).

The foam zone is described using the volume averaged equations of the macroscopic model that was also used by Huisseune et al. [3]. This model predicts the effect of the foam on the momentum and the energy of the volume averaged air flow. This effect depends on some properties of the metal foam, which are indicated in Table 2.

Table 1 Macroscopic geometrical parameters

1.5 mm - 4.5 mmTube outer height 10 mm - 45 mmFoam depth Foam height 5 mm - 30 mm

Table 2 Metal foam parameters

Foam material aluminium/copper PPI 10 - 100 External porosity 0.87 - 0.96Strut type Full/hollow

In this screening study, a full factorial design matrix is used to select the different heat exchanger designs to be evaluated. Using a full factorial matrix instead of fractional factorial designs or orthogonal arrays allows taking all possible interactions into account. The levels of the different parameters are given in Table 3. This results in a total of 6912 simulations, which is only feasible because the simple 2D VAT model calculates very quickly. The downside of using the VAT model is that the accuracy is lower than a fully resolved simulation of the microscopic flow around the heat exchanger, but that requires a computational effort which is orders of magnitude larger.

Table 3 Overview of the levels of the full factorial design

Parameter	Levels
Inlet velocity	4
Foam material	2
Foam depth	2
Tube outer height	2
Foam height	2
PPI	9
External porosity	6
Strut type	2

DATA REDUCTION

The different geometries identified by the full factorial sampling plan are simulated using CFD calculations with the

commercial software Fluent, incorporating the VAT model as used by Huisseune et al. [6]. From each simulation the heat transfer rate and the pressure drop over the heat exchanger is obtained. The heat transfer rate is calculated from the net enthalpy change of the air.

$$\dot{Q} = \dot{m}(h_{out} - h_{in})$$

 $\dot{Q} = \dot{m}(h_{out} - h_{in})$ Since the tube wall temperature is constant, the overall thermal conductance UA can then be calculated from the heat transfer rate using the effectiveness-NTU relation for flow with one fluid at constant temperature and by calculating the mass flow rate from the inlet density, velocity and the tube pitch. The fluid properties that are required are evaluated at the bulk temperature, equal to the average between inlet and outlet temperature.

$$\frac{\dot{Q}}{\dot{m}c_p \left(T_{innerwall}-T_{fluid,in}\right)} = 1 - \exp\left(-\frac{UA}{\dot{m}c_p}\right)$$
 This thermal overall thermal conductance relates to the

temperature difference between the metal foam and tube interface and the fluid and does not yet account for the thermal contact resistance. This needs to be taken into account by adding an additional thermal resistance in series. Denoting the overall heat transfer coefficient including the thermal contact resustance as U_{TCR} and the thermal contact resistance for a unit substrate area as $A_{contact}$, the thermal resistance network gives the following result.

$$\frac{1}{U_{TCR}A} = \frac{1}{UA} + \frac{TCR}{A_{contact}}$$

Note that this contact area consists of both the actual area where the metal foam connects to the tubes (which is difficult to determine) as well as the contact area between the air phase and the tube. Both sides of the equation can now be divided by the heat exchanger volume, resulting in an expression incorporating the surface to volume ratio β .

where surface to volume ratio
$$\beta$$
.
$$U_{TCR}\beta = \frac{1}{\frac{1}{U\beta} + TCR \frac{V}{A_{contact}}}$$

Reducing the foam height does not change the contact area between the tube and the metal foam, but it does reduce the overall heat exchanger volume. Keeping this in mind, this expression shows that for a given overall heat transfer coefficient, surface to volume ratio and thermal contact resistance for a unit area, it is beneficial to reduce the foam height from a thermal contact resistance point of view. This can be understood physically, as this means that for a given foam volume, there is a larger contact area over which the metal foam can make physical contact with the tubes. As a result, the thermal contact resistance in K/W is decreased. It can be expected that if the thermal contact resistance is large, that this will favour designs with smaller foam heights.

RESULTS

In a first step, the thermal contact resistance per unit area is set to zero, and the overall heat transfer coefficient per unit volume $U\beta$ is represented as a function of the pressure drop for all designs evaluated in the sampling plan. In a second step, for each frontal air velocity the dominated designs are removed. A

design is said to be dominated if there is some other design in the design space, which achieves the same or better heat transfer coefficient $U\beta$, while having a lower pressure drop. For each velocity, this results in a set of Pareto optimal designs. The result is shown in Figure 4. It is clear that as the velocity increases, the pressure drop to achieve a certain heat transfer coefficient is increased. The maximum heat transfer coefficient which can be reached also increases with the air velocity.

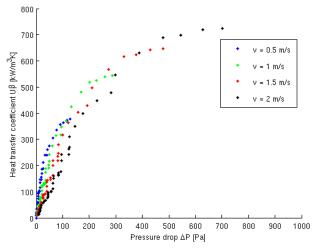


Figure 4 Pareto optimal designs without TCR

For a brazed connection between the metal foam and the flat tube, the thermal contact resistance (TCR) is 7e-4 m²K/W, as determined by De Jaeger et al. [8]. The same analysis is now repeated but incorporating this value for the thermal contact resistance into the overall heat transfer coefficient per unit heat exchanger volume. The Pareto analysis was repeated starting from the full sampling plan results incorporating the thermal resistance. This results in Figure 5.

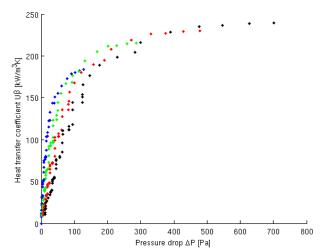


Figure 5 Pareto optimal designs with TCR

It is clear that the performance is strongly compromised by taking the contact resistance into account. The maximum heat transfer coefficient is reduced from more than 700 to less than 250 kW/m³K. There is also a clear flattening of the curve,

which can be understood from substituting an infinitely large β value into the equation of the overall heat transfer coefficient with thermal contact resistance. This leads to the following equation:

$$U_{TCR}\beta_{max} = \frac{1}{TCR} \frac{A_{contact}}{V}$$

 $U_{TCR}\beta_{max}=\frac{1}{TCR}\frac{A_{contact}}{V}$ The volume of the heat exchanger can be written as half of the contact area, multiplied by the sum of the outer tube height and the foam height. The largest value is obtained for the smallest value of the tube and foam heights, equal to 1.5 mm and 5mm, respectively. This results in a maximum value of 440 kW/m³K for the design space under consideration, just due to the thermal contact resistance. Since the best cases under consideration have a total thermal conductance in the order of 240 kW/m³K, the thermal contact resistance in these cases accounts for nearly half of the total resistance. This clearly shows that the thermal contact resistance is absolutely critical to achieve good performance when going to high values of the overall heat transfer coefficient per unit volume.

The metal foam heat exchangers should now be compared to a louvered fin and round tube heat exchanger. For the louvered fin, the Wang correlation [9] is used to predict the performance. The geometrical parameters of the louvered fin are given in Table 4.

Table 4 Louvered fin parameters

Outer tube diameter	6.75 mm
Transversal tube pitch	17.6 mm
Longitudinal tube pitch	13.6 mm
Fin pitch	1.71 mm
Fin thickness	0.12 mm
Fin material	aluminium
Number of tube rows	2
Louver angle	35°

The Schmidt correlation as described by Shah and Sekulic [10] is used to predict the fin efficiency, the fluid properties are evaluated at the bulk temperature resulting from the same inlet temperature of 45° and the calculated outlet temperature. The incompressible ideal gas model is used to model the density and the Sutherland approximation [11] was used to determine the viscosity. A constant Prandtl number was used to determine corresponding thermal conductivity, approximation of the real behaviour.

In order to compare the metal foam heat exchangers to the louvered fin and tube counterparts, it is more useful to present the heat transfer coefficient in function of the pressure drop per unit length in the flow direction. This is shown in Figure 6. It is immediately clear that the louvered fin has a lower pressure drop per unit length than the metal foam heat exchangers, for the same frontal air velocity. Over the velocity range from 0.5 to 2 m/s, the louvered fin pressure drop is lower than the metal foam pressure drop at 0.5 m/s, even when the velocity for the louvered fins is as high as 2 m/s. On the other hand, the metal foam heat exchangers manage to reach higher values for the heat transfer coefficient per unit volume. There are also metal foam designs which do not achieve such a high heat transfer coefficient. Note that at this stage only Pareto optimal metal foam heat exchangers are considered.

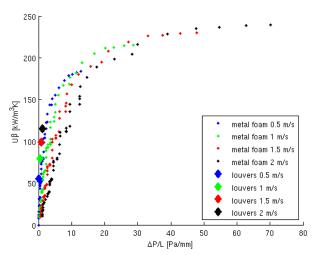


Figure 6 Overall heat transfer coefficient per unit volume as f function of pressure drop per unit length

In order to assess the relative merit of both heat exchanger concepts, it is helpful to do a performance comparison in the same vein as the Cowell method. This means that heat exchangers are designed using the different fin concepts, which achieve the same heat transfer rate, when operated using fluid with the same mass flow rate and the same inlet temperature. The fluid flowing in the tubes of the heat exchanger is assumed to be at a constant temperature. Furthermore, it is assumed that the friction factor is independent of the length of the heat exchanger in the flow direction.

Under these conditions, the heat transfer rate is the same if the overall heat transfer coefficient U is the same. This follows from the effectiveness-NTU relation. The maximum heat transfer rate is the same because the temperatures and mass flow rates are the same, so the heat transfer is the same if the effectiveness is the same. The effectiveness is only a function of the number of transfer units, which is itself a function of the overall heat transfer coefficient and the mass flow rate. Since the mass flow rates are the same, that means that the NTU is the same when the overall heat transfer coefficient is the same.

This allows stating that the volume of the heat exchanger is inversely proportional to the heat transfer coefficient per unit volume. The relative volume is then defined as one divided by this heat transfer coefficient. This is equal to the volume of the heat exchanger, divided by the volume of a heat exchanger with a heat transfer coefficient per unit volume of unity under the same mass flow rate, heat transfer and temperature constraints.

$$Vol^* = \frac{1}{U\beta}$$

Since the mass flow rate is constant and the thermodynamic state at the inlet is the same, this means that the frontal area of the heat exchanger is inversely proportional to the frontal air velocity. The volume of the heat exchanger can also be written as the product of the frontal area and the length in the flow

direction L. This allows expressing the relative length of the heat exchanger as the relative volume multiplied by the frontal air velocity. For two heat exchangers with the same volume and the same mass flow rate, a heat exchanger with a larger frontal air velocity requires a smaller frontal surface area and therefore a longer flow depth.

$$L^* = v.Vol^*$$

Finally the pressure drop of the heat exchanger can be expressed as the pressure drop per unit length, multiplied by the length. This is possible because of the assumption that was made that the friction factor and therefore the pressure drop per unit length do not depend on the length of the heat exchanger. This results in an expression for the relative pressure drop.

$$\Delta P^* = \frac{\partial P}{\partial I} L^* = v. \frac{\partial P}{\partial I}. Vol^*$$

 $\Delta P^* = \frac{\partial P}{\partial L} L^* = v. \frac{\partial P}{\partial L}. Vol^*$ Now the performance of two entirely different heat exchangers can easily be compared. For example, the ratio of the volumes is simply the ratio of the relative volumes. Note that these quantities are the same as the ones used in the performance analysis by Cowell. The main difference is that the data was not reduced to friction factors and Colburn j factors. In the authors' opinion, this allows a more intuitive understanding of the reasons for the differing performance of different heat exchangers.

For each of the different Pareto optimal metal foam heat exchangers as well as the louvered fin heat exchangers, the relative volume is shown as a function of the relative pressure drop in Figure 7.

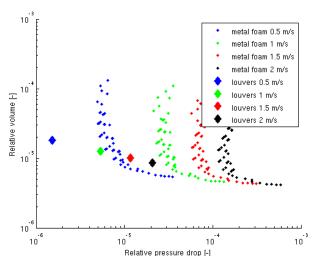


Figure 7 Relative volume as function of relative pressure drop

This figure reveals that metal foam heat exchangers need to operate at much lower velocities than the louvered fin heat exchanger. For example, in order to obtain a similar pressure drop as a louvered fin heat exchanger working with a frontal air velocity of 2 m/s, the metal foam heat exchanger needs to work with a frontal air velocity of 0.5 m/s, which is a factor of 4 lower. This also means that the frontal surface area of the metal foam heat exchanger is 4 times as large. However, the metal foam heat exchanger does manage to have a lower volume that the louvered fin heat exchanger. In other words, the flow length of the metal foam heat exchanger is more than 4 times shorter

than that of the louvered fin heat exchanger. This is in agreement with the observations of Cowell, that enhanced fin designs tend to result in heat exchangers which operate at a lower velocity, with a larger frontal surface area but a lower flow depth. A louvered fin has a much larger frontal surface area than a plain fin under the mentioned constraints. The metal foam heat exchanger is an even more enhanced structure than the louvered fin, in that it has both a larger pressure drop per unit length and a larger heat transfer coefficient at the same velocity as the louvered fin.

This poses some concerns, as louvered fin heat exchangers already tend to have rather short flow lengths. The louvered fin data was predicted for a heat exchanger with a flow length of two tube rows with a longitudinal pitch of 13.6 mm, resulting in a flow depth of just 2.7 cm. Since the metal foam heat exchanger will be around four times shorter in flow length, this would result in a length of 7 mm for this application, assuming that the same pressure drop has to be met. The assumptions of the VAT model will no longer be valid, since there will be only very few metal foam cells in the flow length.

CONCLUSION

Heat exchangers consisting of flat tubes and aluminium metal foam instead of fins can achieve comparable performance as the commonly used louvered fin and round tube heat exchanger designs. The Pareto optimal metal foam heat exchanger designs have a larger pressure drop per unit length and a larger heat transfer coefficient per unit volume for the same frontal air velocity as the louvered fin. This results in heat exchangers with larger frontal surface areas and lower frontal air velocities as the louvered fin, when exchanging the same amount of heat for the same mass flow rate, temperatures and pressure drop. The frontal air velocity is around four times lower than for the louvered fin, the flow length is more than four times shorter. This poses concerns regarding the applicability of simplified models such as VAT.

If the pressure drop is not a concern, metal foam heat exchangers can reach higher heat transfer coefficients per unit volume than the louvered fin. For the cases in this study, an improvement by a factor two is predicted, at the cost of an approximately hundredfold increase in the pressure drop.

The performance of the metal foam heat exchanger is strongly impacted by the thermal contact resistance. Achieving good thermal contact between the metal foam and the tube walls is absolutely crucial to get good performance. This is especially important when high volumetric heat transfer coefficients are desired.

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