TRANSITIONAL FLOW REGIME HEAT TRANSFER IN A HORIZONTAL ANNULAR PASSAGE ASSOCIATED WITH MIXED CONVECTION AND NON-UNIFORM WALL TEMPERATURE BOUNDARY CONDITION

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ABSTRACT

In this experimental study, hydrodynamic and thermally developing flow associated with mixed convection was examined in the transitional flow regime for a horizontal concentric annular passage. The test facility, which consisted of a counter-flow heat exchanger having an annular diameter ratio was 0.483 with an inner passage wall diameter of 15.90 mm, was operated at different degrees of longitudinal Wall Temperature Uniformity (WTU) for heated flow applications using a conventional annular inlet geometry type. The degree of WTU is described in this paper as a ratio of the wall temperature (value in Kelvin) at the outlet to the wall temperature (value in Kelvin) at the inlet of the inner surface of annular passage. Three different degrees of WTUs of 0.99, 0.975 and 0.965 were examined. Critical Reynolds numbers based on heat transfer were analyzed. It was found that the degree of WTU have an influence on critical Reynolds number and the heat transfer coefficient. A model which takes into account WTU is proposed to predict heat transfer in the transition regime.

INTRODUCTION

In literature most of the available studies on heat transfer and pressure drop deal with fully developed, pure forced convection associated with either Uniform Heat Flux (UHF) or Uniform Wall Temperature (UWT) boundary conditions. Such studies therefore leave out the effects of flow development, secondary flows and non-uniform wall temperature boundary conditions which are experienced in many heat exchange applications. Analytical and numerical solutions for heat transfer and fluid behaviour for fully developed flow with pure forced convection heat transfer in the laminar flow regime have been available for many years, whereas those for turbulent flow are normally calculated from empirical equations based on experimental data. On the other hand, heat transfer and pressure drop performances in the transitional flow regime is un-explored for many heat transfer applications including that of annular passages of tube-in-tube heat exchangers, one of the most common heat exchanger types. Consequently, heat exchanger design guidelines normally advise that designs should be done either for the laminar or for the turbulent flow regimes. However, design constraints and energy requirements have often lead to heat exchangers operating outside their design parameters. These parameters often involve the heat exchanger operating in the transitional flow regime [1].

At low Reynolds numbers heat transfer characteristics and fluid flow processes have been observed to be influenced by buoyancy and centrifugal forces. When both the secondary flow effects as well as the main pressure differential effects are significant in the heat exchanger passage, the heat transfer process is referred to as mixed convection process.

NOMENCLATURE

A_s	[m^2]	Surface area
c_p	[J/kg.K]	Specific heat at constant pressure
C	Constant	[-]
D	[m]	Diameter
Gr	[-]	Grashof number
h	[W/m^{2}K]	Convection heat transfer coefficient
k	[W/m.K]	Thermal conductivity
L_{hx}	[m]	Heat exchange length
$\dot{m}$	[kg/s]	Mass flow rate
Nu	[-]	Nusselt number
Pr	[-]	Prandtl number
Re	[-]	Reynolds number
$\dot{Q}$	[W]	Heat transfer rate
T	[°C]	Temperature
V	[m/s]	Average velocity

Special characters

ρ	[kg/m^3]	Density
μ	[kg/ms]	Dynamic viscosity

Subscripts

0	Inner wall of outer tube
1	Outer wall of inner tube
b	Bulk property
h	Hydraulic
i	Inner tube
iw	Inner wall
$LMTD$	Logarithmic mean temperature difference
o	Annular passage
oi	Annular passage inlet
oo	Annular passage outlet

Acronyms

WTU	Wall Temperature Uniformity ratio

In their experimental study of forced and free convective heat transfer in the thermal entry region of horizontal concentric annuli Mohammed *et al.* [2] found that the temperature on the inner surface was higher for low Reynolds numbers than that for high Reynolds numbers. This phenomenon was attributed to the dominance of free convection in the low Reynolds number range. Their investigation also revealed that the Nusselt number values for thermally developing flow were considerably greater than the corresponding values for fully developed mixed convection over a significant portion of the annulus. Lu and Wang [3, 4] investigated experimentally the characteristics of a developing flow with secondary flow in narrow annuli of hydraulic diameters 6.16 mm and 4.12 mm and pressure drop lengths of 1410 mm and 1500 mm, respectively. They also observed that secondary flows had significant influence on heat transfer in the low Reynolds number while none was observed in the high Reynolds number.

Dawood *et al.* [5] reviewed previous studies on forced, natural and mixed-convection heat transfer and fluid flow in annulus. They reviewed a total of 28 numerical and 13 experimental studies of various parameters. Out of the list 9 studies had Uniform Wall Temperature (UWT) boundary conditions, 20 had Uniform Heat Flux (UHF) boundary conditions and 6 were not specified. Although it was not clearly elaborated on in their papers, a study by Lu and Wang [3, 4] which is very close to the present study had non-uniform wall temperature boundary condition.

Comparatively, due to the wide range of application of circular tubes, a large number of heat transfer correlations for tubes exist. Most of these correlations are available for turbulent flow, followed in number by the correlations for laminar flow, and least in number for the transition flow regime. Some correlations also exist specifically for annular passages, most of which for turbulent flow as listed in [6, 7], but not transitional flow regimes where a non-uniform longitudinal wall temperature profiles are present.

Therefore, based on the foregoing literature review a knowledge gap exists for heat transfer and pressure drop characteristics in the transition regime of annular passage and when the thermal boundary wall conditions are not uniform. This paper only deals with heat transfer.

EXPERIMENTAL SETUP

Figure 1 shows the schematic layout of the experimental facility containing two closed water loops (hot water and cold water) and a test section portion consisting of a tube-in-tube heat exchanger. By switching the connectors at the test section inlets and outlets between the hot and cold loops, either cooled or heated annular cases could be investigated depending on the test requirements. In this paper all results are for cold water in the annulus and hot water in the inner tube.

The hot water loop was equipped with a 1000 litre reservoir (item R1) fitted with a 36 kW electrical resistance heater. The hot water was circulated by a positive displacement pump, CB410 (item P1) with a delivery range of 0.3 – 1.6 kg/s. Since flow rates were often required that were much less than what the pump was rated for, a bypass line with a hand-operated valve (item RV1) was utilized to assist control the flow rate through the test section. An accumulator (item A1) was installed next to the pump to arrest pulsations. Flow rates were measured using a Coriolis flow meter with an effective range of 0 – 1.833 kg/s (item M1). To avoid loose particles settling in the test section, and thereby compromising results, a filter (item F1) was installed in the loop.

The cold water loop was similar to the hot water loop and also had a bypass line. The cold water loop, however, had a 45 KW chiller unit connected to a 5000 litre reservoir (item R2). A SP4 pump (item P2) with a delivery range of 0.032 – 0.775 kg/s was utilized to circulate the water. A Coriolis flow meter (item M2) with a range of 0 – 0.604 kg/s was used to obtain the mass flow rate. The flow meters had measurement uncertainty of ±0.1%. Other components such as an accumulator (item A2) and a filter (item F2) were also fitted in this loop.

In addition, both loops were fitted with relevant pressure relief valves, pressure gauges, non-return valves (items NV1 and NV2) and appropriate pipes and pipe fittings.

The test section, represented in Figure 2, was a counter flow double tube heat exchanger made from hard drawn copper. The inner tube had inner and outer diameters of 14.49 mm and 15.90 mm (D_1) respectively while for the outer tube these diameters were 32.9 mm (D_0) and 35 mm, respectively. The tube diameters and lengths resulted in an annular diameter ratio (D_1/D_0) of 0.483 and a heat transfer length of 5.08 m (L_{hx}).

Since transitional flow range results could be affected by the inlet geometry, special care was taken with the inlet of the annular passage to ensure repeatable results. The annular inlet geometry was that of a 90° T-section fitting, similar to that found in most practical applications.

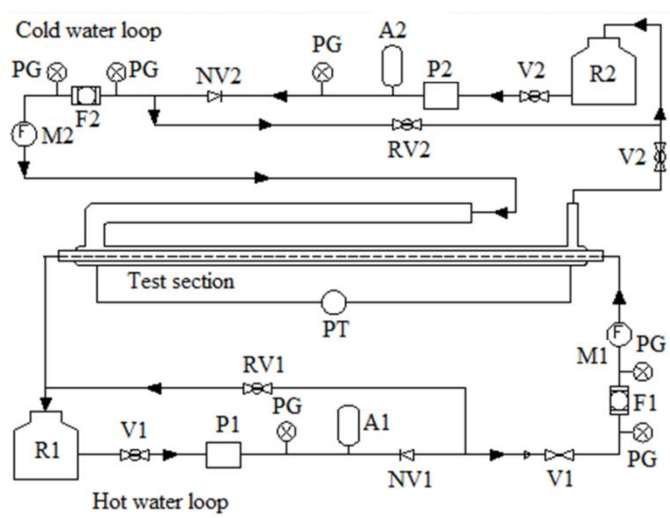

Figure 1 Schematic diagram of experimental facility.

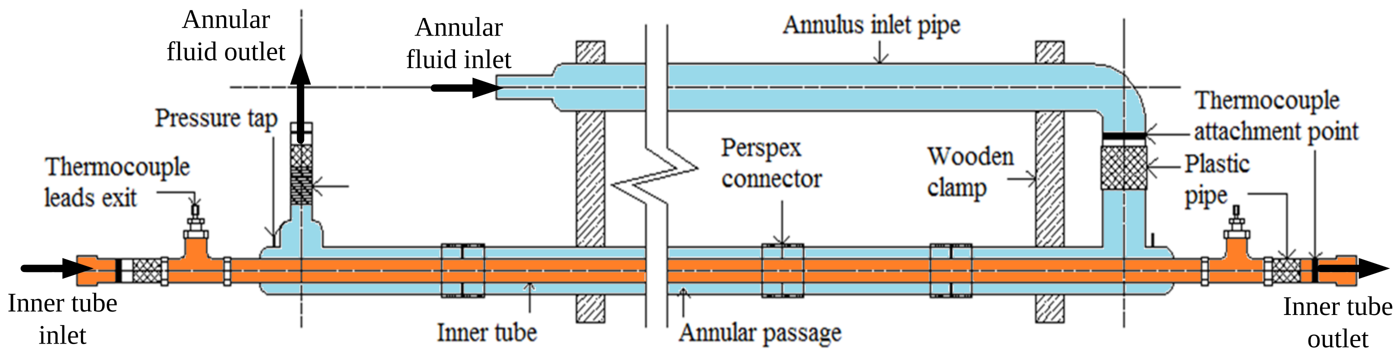

Figure 2 Tube-in-tube heat exchanger test section.

The T-section was preceded by an adiabatic inlet length which ran parallel to the main test section length and which was connected to the heat exchanger by an elbow with a mid-pipe radius of 25 mm. The inlet length was 4.2 m with an inner diameter of 32.9 mm and was secured to the outer tube of the heat exchanger by three wooden clamps.

To ensure concentricity of the annular passage, the inner tube was supported by hypodermic needles (0.8 mm in diameter) at eight equally spaced axial positions. Each support position had four needles spaced 90° apart circumferentially, and was held in place on the outer annular wall in thick-walled acrylic glass connectors. The outer tube was therefore an assembly of nine copper sections linked to each other via the carefully manufactured acrylic glass connectors, such that the outer wall of the annulus was smooth and straight.

Inlet and outlet fluid temperatures for the inner tube were measured at adiabatic measuring stations which each consisted of a short copper length equipped with four thermocouples connected 90° apart. A combined measurement uncertainty of these four thermocouples was $\pm$0.053°C when using the arithmetic average of the individual measurements. Thermally, these measuring stations were insulated from the heat exchanger by means of rubber hoses.

The inlet and outlet temperatures of the annular passage fluid were measured in a similar manner as for the inner tube, except that each measuring station was equipped with eight thermocouples so that the measurement uncertainty was reduced to $\pm$0.038°C based on the arithmetic average. A mixing section was placed before the outlet temperature measuring station to disrupt any thermal boundary layers against the wall and to ensure that the correct water temperature was captured.

In order to measure local wall temperatures, nine equally spaced measuring stations were manufactured along the length of the inner tube. At each station two T-type thermocouples, with combined measurement uncertainty of about $\pm$0.075°C (based on the arithmetic average), were embedded into the wall. In order to keep the annular passage clear from unnecessary obstructions, the thermocouple leads were passed through the inside of the inner tube. Each thermocouple junction was soldered in a groove (10 mm long with a depth of 0.46 mm) that was machined into the tube wall. The grooves were filled with solder at the measuring tips and with epoxy in the remainder of the grooves (facing downstream) such that the outer surface of the tube remained smooth. The inlet and outlet ends for the inner tube provided exit ports for thermocouple leads out of the inner tube.

To measure the local outer wall temperatures along the annular passage sets of two thermocouples each were attached on the outer surface of outer tube wall at intervals exactly midway between the inner tube measuring stations. The entire set-up was thermally well insulated from the laboratory.

Temperature and water flow rate readings were captured using National Instruments data acquisition system using Lab-view software.

EXPERIMENTAL PROCEDURE

All thermocouples were calibrated *in situ* by using two PT100 resistance temperature detectors with an accuracy of 0.1 °C each. Calibration curves were created with which measured data were conditioned during the data-processing stages.

Three types of experimental tests related to the longitudinal inner wall temperature profiles were conducted. These were identified by the degree of the Wall Temperature Uniformity (WTU) which was defined as the ratio of the average wall temperature at the outlet to that at the inlet of inner wall of annular passage. Since the inner wall temperatures at the inlet and outlet were not directly measured, line-fit extrapolations from the measured wall temperature profiles where used.

In all cases, hot water with an inlet temperature range of 49.01 °C – 50.98 °C (average of 50.13 °C) and cold water with an inlet temperature range of 18.01 °C - 20.76 °C (average of 19.23 °C) was used.

Since the annular passage was the focus of this investigation, its flow rate was the independent property while the inner tube flow rate was depended on the annular flow rate in order to create a particular WTU condition. The annular Reynolds number based on the hydraulic diameter ranged from slightly below 200 to approximately 9300 in order to

Table 1 Averaged degrees of WTU and the corresponding water temperature difference between the inlet and out of the inner tube.

Averaged degree of WTU	Approximate inner tube water temperature difference [°C]
0.99	2
0.975	6
0.965	10

ensure that the transitional flow regime was adequately covered. Specific WTUs were achieved by running the hot water in the inner tube at a rate relative to the flow of the cold water in the annulus that would give a constant temperature difference between inlet and outlet of the inner tube wall.

Experiments with averaged degrees of WTU of 0.99, 0.975 and 0.965 were conducted. The corresponding temperature differences to these degrees of WTU are shown in Table 1. Going by the definition of WTU to achieve Uniform Wall Temperature (UWT) the inlet to outlet ratio of the inner wall of the annular passage should be equal to 1. Since it was practically impossible to achieve this with the test facility (because an infinite inner flow rate would be needed) the WTU of 0.99, being closest to 1, is considered in this paper as an approximate UWT.

Data was logged at steady state conditions which were deemed to have been reached when the change in temperature of the water at the outlet of annular passage fluctuated by 0.1 °C or less over a period of 1 minute. Up to 120 data points were collected for each data log which was averaged to produce one experimental data point.

PROCESSING OF RESULTS

The convection heat transfer rate at a specified temperature difference between a surface and a fluid is determined by Newton's law of cooling:

$$\dot{Q} = hA_s \Delta T_{LMTD} \tag{1}$$

where h is the convection heat transfer coefficient, A_s is the surface area of the inner wall of the annulus where convection heat transfer took place, and ΔT_{LMTD} is the logarithmic mean temperature difference.

Our interest is to get the convection heat transfer coefficient; therefore, the rest of the parameters including the heat transfer rate in equation (1) should be known or analysed first. The heat transfer rate between the water in annular passage and the inner wall can be found by considering the temperature change in the annular fluid:

$$\dot{Q}_o = \dot{m}_o c_p \left(T_{oo} - T_{oi}\right) \tag{2}$$

Here the annular mass flow rate ($\dot{m}_o$) was obtained from the reading of the relevant flow meter, cp was evaluated using the method of Popiel and Wojtkowiak [8] at the average bulk fluid temperature, and the inlet and outlet temperatures (T_{oi} and T_{oo}) were obtained by taking the arithmetic average of the thermocouples installed at the inlet and outlet measuring stations.

The surface area of the outside wall of the inner tube in equation (1) was calculated as:

$$A_s = \pi L_{hx} D_1 \tag{3}$$

The logarithmic mean temperature difference for the annular passage can be calculated as:

$$T_{LMTD} = \frac{(\bar{T}_{iw} - T_{oi}) - (\bar{T}_{iw} - T_{oo})}{\ln[(\bar{T}_{iw} - T_{oi})/(\bar{T}_{iw} - T_{oo})]} \tag{4}$$

The mean dimensionless Nusselt number for annular passage was based on the hydraulic diameter and calculated as:

$$Nu = hD_h / k \tag{5}$$

Where the hydraulic diameter D_h of the annulus was calculated as:

$$D_h = D_0 - D_1 \tag{6}$$

Here D_0 and D_1 represent the outer and inner annular wall diameters respectively. The Reynolds number for flow in annular passage was calculated as:

$$\mathrm{Re}_o = \frac{\dot{m}_o D_h}{\mu_o A_o} \tag{7}$$

All other water properties were also calculated with the method of Popiel and Wojtkowiak [8] at the average bulk fluid temperature. At low Reynolds numbers the annular bulk fluid temperature along the longitudinal direction was not linear. Therefore, to increase the accuracy of the calculated fluid properties, the effective bulk fluid temperature was obtained from the local temperature measurements on the adiabatic outer tube wall by using the trapezoidal rule. For this purpose it was assumed that the bulk fluid temperature profile and outer wall temperature profiles were approximately the same.

VALIDATION OF TEST PROCEDURE

Although this investigation focused mainly on the transition flow regime, the validation was done in the turbulent regime because no relevant correlations were found for transitional flow regime cases. The present experimental sets were validated against the correlation by Gnielinski [9] as shown in Figure 3 for an arbitrary wall temperature uniformity case. It can be observed that the calculated Nusselt numbers in this study compared well with Gnielinski correlation. The Gnielinski correlation was found to under-predict experimental results by 0.82% for a Reynolds number range of $4682 \le \mathrm{Re} \le 8659$. Uncertainty bars are also included in Figure 3 based on an uncertainty propagation analysis. A maximum uncertainty of 26.80% is present at the lowest Reynolds number of 198.06 while in the transitional flow

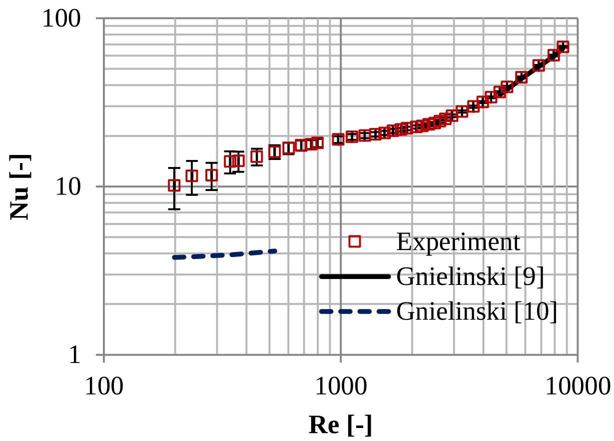

Figure 3 Comparison of the experimental heat transfer coefficient against some correlations predictions.

regime uncertainties are between 1.66% and 7.79%. The uncertainties for averaged Nusselt numbers in the transition regime for WTUs of 0.99, 0.975 and 0.965 were1.67% – 7.85%, 1.67% - 7.81% and 1.66% – 7.79%, respectively.

RESULTS AND DISCUSSIONS

As mentioned earlier, a non-uniform wall temperature boundary on the inner annular surface was considered for varying annular mass flow rates that covered all flow regimes from laminar to turbulent.

As other researchers have already shown, the heat transfer coefficients for mixed convection are higher than those for forced or natural convection only. The laminar flow correlation for a fully developed forced flow is incorporated in Figure 3. The correlation was developed by Gnielinski [10] for tubes with UWT boundary condition but it was applied here using a relevant hydraulic diameter for annular passage.

Considering the transitional flow regime region, the experimental results showed that the heat transfer coefficient depends on the degree of wall temperature uniformity. Figure 4 presents averaged heat transfer results in terms of the Nusselt number plotted against the Reynolds number on a logarithmic scale. In the transition flow regime range (approximately $470 \le \mathrm{Re} \le 3300$) a definite increase in the heat transfer coefficient is observed with an increase in the degree of wall temperature uniformity on the inner wall. This is so because at higher temperature difference buoyancy driven flow is stronger than at lower temperature difference. Therefore, improved fluid mixing is obtained. In the transition regime (at Re = 1600), as indicated in Figure 4, on average the Nusselt number for wall temperature uniformity cases of 0.975 and 0.965 were 6.58% and 11.33% lower than the 0.99 wall temperature uniformity case.

Correlations

The averaged heat transfer coefficients in the transitional flow regime, for the heat exchanger under consideration here,

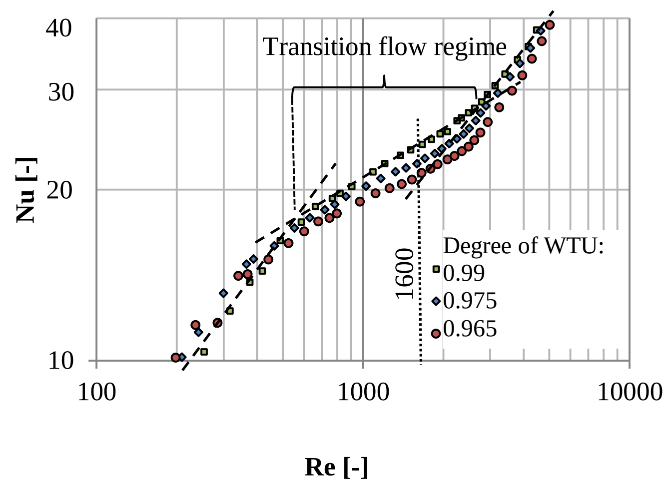

Figure 4 Heat transfer characteristics in terms of the Nusselt number for varying WTU.

were correlated in terms of the relevant parameters with an empirical equation. Firstly, the results of the degree of WTU of 0.99 were correlated using a heat transfer equation for mixed convection of the following form [2]:

$$\mathrm{Nu} = C(\mathrm{Gr.Pr/Re})^{n} \tag{8}$$

The correlation is represented in Figure 5 for 1000 ≤ Re ≤ 2300 and $4.08\times10^{5} \le Gr \le 4.7\times10^{5}$ by a power line of the following equation:

$$\mathrm{Nu} = 175.38(\mathrm{Gr.Pr/Re})^{-0.276} \tag{9}$$

Secondly, the correlation for averaged heat transfer coefficients for different degrees of WTU was formulated for the same range of the Reynolds and Grashof numbers. The correlation is of the same form as equation (9) but the constant value C and exponent *n* are represented by equations (10) and (11):

$$C = 214.42 WTU_{ha}^{20} \tag{10}$$

$$n = -1.992\ln(WTU_{ha}) - 0.2961 \tag{11}$$

Equation (9) together with equations (11) and (12) are applicable to transition mixed convection and developing flow and can be used for WTU range 0.965 – 0.99. The equation represents the experimental data to ±1.84% for WTU = 0.99, ±1.87% for WTU = 0.975 and ±1.32% for WTU = 0.965. Figure 6 shows the comparison between the experimental Nusselt numbers and the proposed correlations for all the three degrees of WTU. It can be seen that good agreement between predictions and the experimentally obtained values are present for the entire Nusselt number range under consideration.

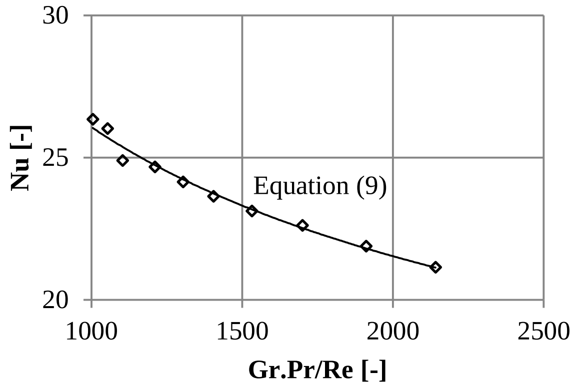

Figure 5 Correlation of the averaged heat transfer results for mixed convection in a horizontal concentric annulus for an approximate UWT boundary condition.

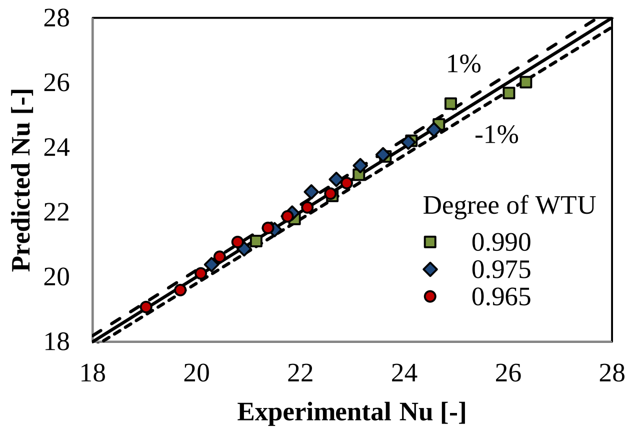

Figure 6 Comparison between experimental results and the proposed correlations in the transition regime of a horizontal concentric annulus.

CONCULUSIONS

An experimental investigation was conducted on heat transfer characteristics in a horizontal annular passage of annular ratio diameter of 0.483. The boundary condition on the inner wall of the annulus was non-uniform which was expressed in terms of the degree of wall temperature uniformity. Three types of WTU of 0.99, 0.975 and 0.965 were applied. The heat transfer coefficients were found to be directly proportional to the degree of WTU and the differences between them was quite significant. This phenomenon was attributed to variation in temperature difference amongst the three tests. The averaged heat transfer results for WTU of 0.99 (which was very close to uniform wall temperature boundary condition) only and for all the WTUs were correlated in terms of the relevant dimensionless parameters with an empirical correlation. The correlations represented the data with only few exceptions within ±1%.

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