THERMOSOLUTAL CONVECTION IN A DARCY POROUS MEDIUM WITH ANISOTROPIC PERMEABILITY AND THERMAL DIFFUSIVITY

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NOMENCLATURE

v_i	Velocity, where $i = 1,2,3$
p	Pressure
T	Temperature
C	Salt concentration
μ	Fluid viscosity
ρ_0	Fluid density
$\hat{k}$	The reaction rate
k_C	The molecular diffusivity of the solute through the fluid
g	The gravity
α_T	The thermal expansion coefficient
α_C	The solutal expansion coefficient
$\hat{\phi}$	The matrix porosity
M	The ratio of the heat capacity of the fluid to the heat capacity of the medium
$\mathbf{K}$	The permeability tensor
$\mathbf{k}_T$	The thermal diffusivity tensor
C_U	Salt concentration of the upper boundary
C_L	Salt concentration of the lower boundary
T_U	Temperature of the upper boundary
T_L	Temperature of the lower boundary
$\mathbf{k}$	$= (0,0,1)$

ABSTRACT

A problem of thermosolutal convection with reaction in an anisotropic porous medium of Darcy type is investigated. The Darcy model is employed as the momentum equation with the density being a linear function in temperature and salt concentration. Two cases are considered: heated below and salted above system and heated and salted below system. We allow the permeability and thermal diffusivity to be anisotropic tensors. Particularly, we restrict consideration to horizontal isotropy in mechanical and thermal properties of the porous medium. The energy method is used to study the non-linear stability aspect of the problem. The D^2 Chebyshev Tau method is implemented to solve the associated system of equations, with the corresponding boundary conditions, and to obtain the non-linear stability boundaries below which the solution of the system is globally stable. The effect of the reaction rate, the mechanical anisotropy parameter, and the thermal anisotropy parameter on the stability of the system is discussed and presented graphically. We find that the thermal anisotropy parameter has the opposite effect to that of the mechanical anisotropy parameter on the stability of the system.

INTRODUCTION

Convection in porous media, specifically double diffusive convection in porous media, has its wide implications in geological process and a variety of geotechnical applications (see Malashetty and Biradar [1]). Double diffusive convection in porous media is well investigated by Nield and Bejan [2], Ingham and Pop [3; 4], Vafai [5; 6], Nield [7], Rudraiah et al. [8], Wollkind and Frisch [9; 10], Bdzil and Frisch [11; 12], and Gutkowicz-Krusin and Ross [13]. Many recent studies in double and multi-component convection are investigated by Rionero [14; 15; 16]. Steinberg and Brand [17; 18] carried out the first study on reactive convection in porous media. They studied the linear stability analysis of a reactive binary mixture with a fast chemical reaction. More studies were carried out by Gatica et al. [19; 20], Viljoen et al. [21], and Malashetty and Gaikwad [22]. Pritchard and Richardson [23] explored a model similar to that of Steinberg and Brand [17; 18] in which they used the linear instability theory to study the onset of Darcy thermosolutal convection with reaction. Wang and Tan [24] extended the previous work of Pritchard and Richardson [23] in which Wang and Tan [24] considered the Darcy-Brinkman model to discuss how the onset of double-diffusive convection varies with the Darcy number, the Lewis number, and the reaction term by using the normal mode technique to carry out a linear instability analysis. Most studies have focused on studying convection in isotropic porous media, even though the porous materials in reality are anisotropic. A well documented review of research articles on convective flows in anisotropic porous media can be found in Storesletten [25]. Malashetty et al. [26; 27; 28] studied the onset of double diffusive convection in anisotropic porous media with different effects, like rotation, cross-diffusion effects, and soret effect. Recently, Malashetty and Biradar [1] studied the onset of double diffusive reaction convection in anisotropic porous layer of Darcy type. Srivastava and Bera [29] considered the onset of thermosolutal reaction convection in a couple-stress fluid saturated anisotropic porous medium. Gaikwad and Begum [30] investigated the onset of a rotating double-diffusive reaction convection in anisotropic Darcy type porous medium. The authors

in the six articles mentioned above used a linear theory and a weak non-linear theory to study the stability. The linear analysis is based on the normal mode technique, while the non-linear analysis is based on a truncated Fourier series representation.
We are studying non-linear stability using an energy stability technique which is used extensively by, for example, Straughan [31; 32; 33], Rionero et al. [34; 35], and Capone et al. [36]. Al-Sulaimi [37; 38] used the energy method to study the non-linear stability of Darcy and Brinkman thermosoultal convection with reaction, respectively, in isotropic porous medium. Malashetty and Biradar [1] studied the onset of the double-diffusive reaction convection in an anisotropic porous medium of Darcy type. They analysed the linear and weak non-linear stability of a reactive binary mixture in a horizontal porous layer with anisotropic permeability and thermal diffusivity. I use the energy method to study the non-linear stability aspect of the problem. The aim of the study is to obtain the non-linear stability boundaries below which the solution is globally stable. The effect of the reaction terms, the anisotropic permeability, and thermal diffusivity tensors on the onset of stability is analysed and compared with the relevant results obtained by Malashetty and Biradar [1].

BASIC EQUATIONS

We consider an anisotropic porous layer of the Darcy model for the momentum equation with the density ρ being a linear function in temperature T and salt concentration C. In addition, we need the continuity equation, the advection-diffusion equation for the transport of heat, and advection-diffusion equation for the transport of solute with reaction. The governing system of equations is

$$\begin{aligned}
&\mu v_i = -K_{ij}p_{,j} - K_{ij}k_j g\rho_0[1-\alpha_T(T-T_0)+\alpha_C(C-C_0)],\\
&v_{i,i} = 0,\\
&\frac{1}{M}T_{,t} + v_i T_{,i} = \nabla.(k_{Tij}.\nabla T),\\
&\hat{\phi}C_{,t} + v_i C_{,i} = \hat{\phi}k_C\Delta C + \hat{k}[f_1(T-T_0)+f_0-C],
\end{aligned} \tag{1}$$

where $\mathbf{K} = K_x\mathbf{ii} + K_y\mathbf{jj} + K_z\mathbf{kk}$ is the permeability tensor and $\mathbf{k}_T = k_{Tx}\mathbf{ii} + k_{Ty}\mathbf{jj} + k_{Tz}\mathbf{kk}$ is the thermal diffusivity tensor. We restrict consideration to horizontal isotropy in permeability and thermal diffusivity, so that $K_x = K_y$ and $k_{Tx} = k_{Ty}$. The system is taken in the domain $\mathbb{R}^2 \times (0,d) \times \{t > 0\}$, and the corresponding boundary conditions are

$$\begin{aligned}
&v_i n_i = 0 \; on \; z = 0, d,\\
&T = T_L \; on \; z = 0, \; and \; T = T_U \; on \; z = d,\\
&C = C_L \; on \; z = 0, \; and \; C = C_U \; on \; z = d.
\end{aligned} \tag{2}$$

Where $T_L > T_U$ since the systems are heated below, while $C_U > C_L$ for the salted above system, and $C_L > C_U$ for the salted below system. The steady state whose stability is under investigation is

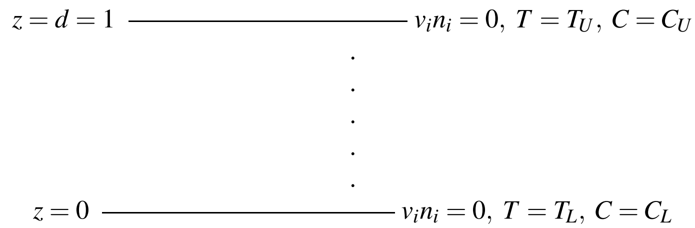


Figure 1. The test domain with the boundary conditions.

$$\begin{aligned}
\bar{v}_i =& 0,\\
\bar{T}(z) =& -\beta_T z + T_L,\\
\bar{C}(z) =& -\beta_C z + C_L,\\
\bar{p}(z) =& -\frac{1}{2}g\rho_0(\alpha_T\beta_T - \alpha_C\beta_C)z^2\\
& - g\rho_0[1-\alpha_T(T_L-T_0)+\alpha_C(C_L-C_0)]z + p_0,
\end{aligned} \tag{3}$$

where $\beta_T = (T_L - T_U)/d$, $\beta_C = (C_L - C_U)/d$, and p_0 is the pressure at $z = 0$. To study the stability, we introduce perturbations (u_i, π, θ, ϕ) to the steady solutions (3) in such a way that

$$v_i = \bar{v}_i + u_i \; , \; p = \bar{p} + \pi \; , \; T = \bar{T} + \theta \; , \; C = \bar{C} + \phi. \tag{4}$$

We substitute (4) in (1) and derive equations for (u_i, π, θ, ϕ). We introduce an inverse permeability tensor M_{ij} which satisfies $M_{ij}K_{jk} = \delta_{ik}$, where $M_{ij} = diag\{\kappa, \kappa, \kappa_3\}; \kappa \neq \kappa_3$. The perturbed equations are non-dimensionalized by defining the non-dimensional variables

$$\pi = P\pi^*, \; u_i = Uu_i^*, \; \theta = T^\sharp\theta^*, \; \phi = C^\sharp\phi^*, \; x_i = dx_i^*, \; t = \tau t^*. \tag{5}$$

Choose the time, velocity, pressure, temperature, and salt scales as $\tau = d/MU$, $U = k_{Tz}/d$, $P = dU\mu$, $T^{\sharp^2} = \beta_T\mu k_{Tz}/g\rho_0\alpha_T$, $C^{\sharp^2} = \beta_C\mu k_{Tz}/g\rho_0\alpha_C$ and define the temperature and salt Rayleigh numbers by $R = \sqrt{g\rho_0\alpha_T\beta_T d^2/\mu k_{Tz}}$, $R_s = \sqrt{g\rho_0\alpha_C|\beta_C|d^2 Le/\mu k_{Tz}\hat{\phi}}$ *when* $C_L < C_U$, *or* $R_s = \sqrt{g\rho_0\alpha_C\beta_C d^2 Le/\mu k_{Tz}\hat{\phi}}$ *when* $C_L > C_U$, where $Le = k_{Tz}/k_C$ is the Lewis number. The non-linear, non-dimensional system of equations is

$$\begin{aligned}
M_{ij}u_j &= -\pi_{,i} + k_i R\theta - k_i R_s\phi \; ,\\
u_{i,i} &= 0 \; ,\\
\theta_{,t} + u_i\theta_{,i} &= Rw + \alpha\Delta^*\theta + D^2\theta \; ,\\
\varepsilon\phi_{,t} + \frac{Le}{\hat{\phi}}u_i\phi_{,i} &= \mp R_s w + \Delta\phi + h\theta - \eta\phi \; ,
\end{aligned} \tag{6}$$

where $\alpha = k_{Tx}/k_{Tz}$, $\varepsilon = MLe$, $D = d/dz$, Δ^* is the horizontal Laplacian and h and η are the reaction coefficients

$$h = \frac{\hat{k} f_1 T^\sharp d^2 Le}{k_{Tz} C^\sharp \hat{\phi}} \quad and \quad \eta = \frac{\hat{k} d^2 Le}{k_{Tz} \hat{\phi}}.$$

Moreover, $-R_s$ for the salted above system and $+R_s$ for the salted below system. The corresponding boundary conditions are

$$u_i n_i = 0\ ,\ \theta = 0\ ,\ \phi = 0\ at\ z = 0,1\ , \tag{7}$$

with $\{u_i, \theta, \phi\}$ satisfying a plane tiling periodicity in (x,y) direction.

LINEAR INSTABILITY THEORY

In order to study the linear instability, we drop the non-linear terms in system (6) and retain the third component of the double curl of equation $(6)_1$. Assuming a normal mode representation, one finds

$$\begin{aligned} &(D^2 - \frac{a^2\kappa_3}{\kappa})W + \frac{a^2}{\kappa} R\Theta - \frac{a^2}{\kappa} R_s \Phi = 0\ , \\ &\sigma\Theta = RW + (D^2 - a^2\alpha)\Theta\ , \\ &\varepsilon\sigma\Phi = \mp R_s W + (D^2 - a^2 - \eta)\Phi + h\Theta\ . \end{aligned} \tag{8}$$

Equations (8) are to be solved subject to the boundary conditions

$$W = \Theta = \Phi = 0\ on\ z = 0,1\ , \tag{9}$$

using D^2 Chebyshev-Tau method, see Dongarra et.[39]. The analysis is presented in the last section.

NON-LINEAR ENERGY STABILITY THEORY

Returning to the non-linear, non-dimensional perturbed system of equations (6) and the corresponding boundary conditions (7). Multiply equation $(6)_1$ by u_i, equation $(6)_3$ by θ, and equation $(6)_4$ by ϕ and integrate over the domain. In this way we may derive the energy identity in the form

$$\frac{dE}{dt} = I - D\ , \tag{10}$$

where

$$\begin{aligned} &E(t) = \frac{1}{2}\|\theta\|^2 + \frac{\varepsilon\lambda}{2}\|\phi\|^2\ , \\ &I = 2R(\theta, w) + \lambda h(\theta,\phi) - (1 \pm \lambda) R_s(\phi, w)\ , \\ &D = (M_{ij}u_j, u_i) + \alpha\|\nabla\theta\|^2 + (1-\alpha)\|\theta_{,z}\|^2 + \lambda\|\nabla\phi\|^2 + \lambda\eta\|\phi\|^2, \end{aligned} \tag{11}$$

and $\lambda > 0$ is a coupling parameter to be selected optimally.

Then, provided that $R_E > 1$

$$\frac{dE}{dt} \le -D(1 - \frac{1}{R_E}) \tag{12}$$

is an energy inequality which follows from the energy identity (10). Where

$$\frac{1}{R_E} = \max_H \frac{I}{D}\ , \tag{13}$$

and $H = \{u_i, \theta, \phi\ |u_i \in L^2(V),\ \theta, \phi \in H^1(V),\ u_{i,i} = 0\}$ and u_i, θ, ϕ are periodic in x, y. We can show

$$(M_{ij}u_j, u_i) \ge \kappa_0\|\mathbf{u}\|^2\ ;\ \kappa_0 = \min\{\kappa, \kappa_3\}\ ,$$

and then

$$D \ge 2k\pi^2 E(t)\ ,$$

where $k = \min\{\frac{1}{\alpha MLe}, 1\}$. Then from (12) we may derive the inequality $dE/dt \le -2a_1 k\pi^2 E(t)$, where the coefficient a_1 is defined by $a_1 = (R_E - 1)/R_E$.

Upon integration we obtain

$$E(t) \le E(0) e^{-2a_1 k\pi^2 t},$$

which shows that $E(t) \to 0$ as $t \to \infty$. Therefore, $\|\theta(t)\| \to 0$ and $\|\phi(t)\| \to 0$ as $t \to \infty$ according to $(11)_1$.

To show the decay of $\|\mathbf{u}\|$, we have to employ the Arithmetic-Geometric Mean inequality in the balance equation obtained by integrating $(6)_1 \times u_i$ and using the fact $\|w\|^2 \le \|\mathbf{u}\|^2$ to obtain

$$\left(\kappa_0 - \frac{R\alpha_1 + R_s\beta_1}{2}\right)\|\mathbf{u}\|^2 \le \frac{R}{2\alpha_1}\|\theta\|^2 + \frac{R_s}{2\beta_1}\|\phi\|^2, \tag{14}$$

which shows the decay of $\|\mathbf{u}\|^2$ under the condition

$$\kappa_0 - \frac{R\alpha_1 + R_s\beta_1}{2} > 0.$$

Regarding the maximum equation (13), the nonlinear stability threshold is given by the variational problem

$$\frac{1}{R_E} = \max_H \frac{2R(\theta, w) - (1 \pm \lambda) R_s(\phi, w) + h\lambda(\theta,\phi)}{(M_{ij}u_j, u_i) + \alpha\|\nabla\theta\|^2 + (1-\alpha)\|\theta_{,z}\|^2 + \lambda\|\nabla\phi\|^2 + \eta\lambda\|\phi\|^2}. \tag{15}$$

We have to determine the Euler-Lagrange equations and maximize in the coupling parameter λ. By standard calculation, the Euler-Lagrange equations which arise from the variational problem (13) may be reduced to the normal mode form

$$\begin{aligned} &\left(D^2 - \frac{a^2\kappa_3}{\kappa}\right)W + \frac{a^2}{\kappa}RR_E\Theta - \left(\frac{1\pm\lambda}{2}\right)\frac{a^2}{\kappa}R_sR_E\Phi = 0\,, \\ &RR_EW + \left(D^2 - a^2\alpha\right)\Theta + \frac{\lambda h}{2}R_E\Phi = 0\,, \qquad (16) \\ &-R_sR_E\left(\frac{1\pm\lambda}{2\lambda}\right)W + \frac{h}{2}R_E\Theta + \left(D^2 - a^2 - \eta\right)\Phi = 0\,, \end{aligned}$$

and the corresponding boundary conditions are

$$W = \Theta = \Phi = 0 \;\; at\; z = 0, 1\,. \qquad (17)$$

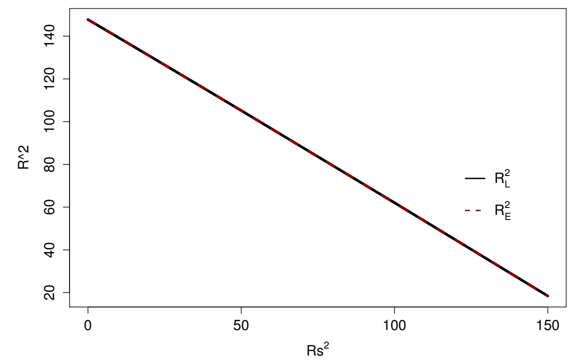
We solved the system (16)-(17) numerically using the D^2 Chebyshev Tau method, cf. Dongarra et.[39].

NUMERICAL RESULTS

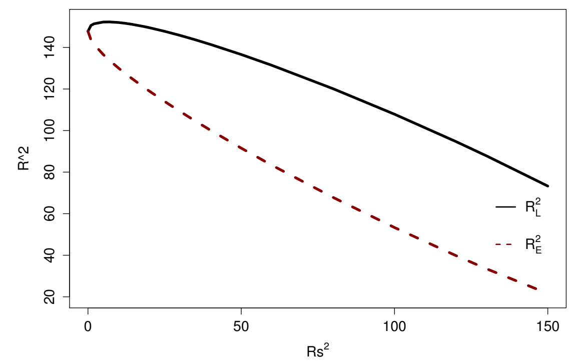
Salted above system

In this article, we are presenting the interpretation of the heated below salted above system, while the interpretation of the heated below salted below system is presented in depth by Al-Sulaimi [40]. Numerically, the results show the coincidence of the linear instability boundary and the energy stability boundary for different values of the anisotropy parameters when there is no reaction, $h = \eta = 0$, as fig.2(a) shows, there is no region of potential subcritical instabilities. To investigate the effect of increasing the reaction rates, different values of h *and* η are implemented for $\alpha = k_{Tx}/k_{Tz} = 0.5$ and $\chi = K_z/K_x = 10$. Increasing the reaction rates, see fig.2(b), results in a wider gap between the linear instability and nonlinear energy stability boundaries; therefore, there is a wider space of potential subcritical instability. To study the effect of each of h and η on the onset of convection, a large difference between their values is implemented for different values of α and χ. For all chosen values of α and χ, when η is larger than h the two boundaries coincide, which is expected from system $(6)_4$ as $-\eta\phi$ is a stabilizing term but the region of stability varies due to the effect of the anisotropy parameters α and χ. On the other hand, implementing larger values of h than η for different cases of α and χ, reveals regions of potential subcritical instability. This is a result of a divergence of the energy stability boundaries(dashed lines) from the linear instability boundaries(continuous lines), which is also expected from system $(6)_4$ as $+h\theta$ is a destabilizing term.

The effect of the thermal anisotropic parameter $\alpha = k_{Tx}/k_{Tz}$ and the mechanical anisotropic parameter $\chi = K_z/K_x$ may be interpreted as follows: When $\chi < 1$, keeping the vertical permeability constant $K_z = 1$ and decreasing the horizontal permeability K_x, lowers the the energy stability boundary and the linear instability boundary indicating that the effect is destabilizing as fig.$5(a,b)$ shows. When $\chi > 1$, keeping the horizontal permeability constant $K_x = 1$ and increasing the vertical permeability K_z, shifts the two boundaries to higher positions indicating that the effect is stabilizing, as is clear in fig.$5(c,d)$.



a) $h = 0$ *and* $\eta = 0$



b) $h = 5$ *and* $\eta = 3$

Figure 2. Linear instability and energy stability boundaries for the salted above Darcy convection problem with anisotropic effect for $\alpha = 0.5$, $\chi = 10$ and for different values of the reaction rates h and η. The figure shows the effect of increasing the reaction rates.

Fig.6 indicates the effect of the thermal anisotropy parameter $\alpha = k_{Tx}/k_{Tz} \leq 1$ for fixed values of the mechanical anisotropy parameter χ and reaction rates h and η which can be interpreted as follows. Keeping the horizontal thermal diffusivity constant, $k_{Tx} = 1$, and increasing the vertical thermal diffusivity, k_{Tz}, lowering the two boundaries which results in smaller definite stable space below the energy stability boundary(dashed lines), see fig.$6(a,b)$, as an indication of a destabilization effect. Note that the effect of the thermal anisotropy parameter α is opposite to that of the mechanical anisotropy parameter χ when $\chi > 1$. This result agrees with the findings of Malashetty and Biradar [1], Gaikwad et al. [28], and Malashetty and Swamy [41].

CONCLUSION

The onset of thermosolutal convection with reaction in anisotropic porous medium of the Darcy type is investigated us-

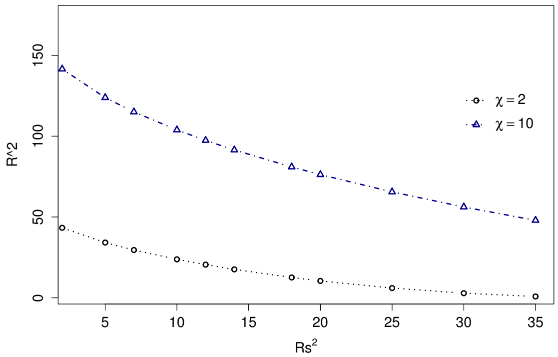


Figure 3. The energy stability boundaries for the salted above Darcy convection problem with anisotropic effect for $\alpha = 1$, $h = 20$, $\eta = 0$. The figure shows the effect of increasing the vertical permeability component, K_z.

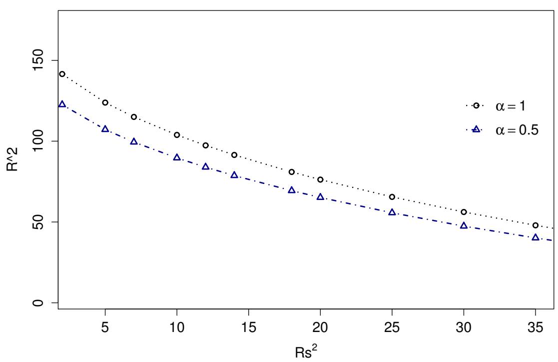


Figure 4. The energy stability boundaries for the salted above Darcy convection problem with anisotropic effect for $\chi = 10$, $h = 20$, $\eta = 0$. The figure shows the effect of increasing the vertical thermal diffusivity component, k_{Tz}.

ing the nonlinear energy stability method. The system of equations with the corresponding boundary conditions is solved by using the D^2 Chebyshev Tau method. The reaction rates may stabilize or destabilize according to the values of each of the reaction terms h and η. h plays the role of destabilizing while η plays the role of stabilizing. When the vertical permeability is high ($\chi > 1$), the system is more stable. While decreasing the horizontal permeability for fixed vertical permeability such that ($\chi < 1$), the system will be more unstable. When the vertical component of the thermal diffusivity is high ($\alpha < 1$), the system is more unstable. While increasing the horizontal component of the thermal diffusivity for fixed vertical component of the themal diffusivity such that ($\alpha < 1$), the system will be more stable. The results reveal the opposite effect of the anisotropic parameters when the vertical components are higher, as fig.(3) and fig.(4) show. This finding agrees with the findings of Malashetty and Biradar [1], Gaikwad et al. [28], and Malashetty and Swamy [41].

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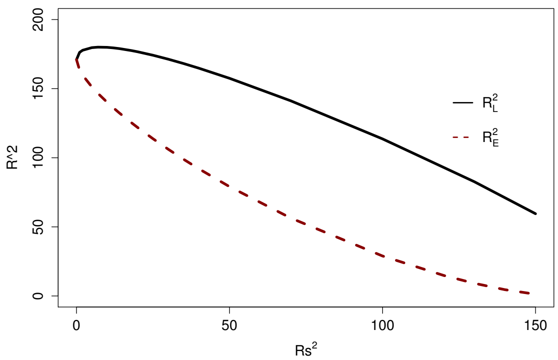
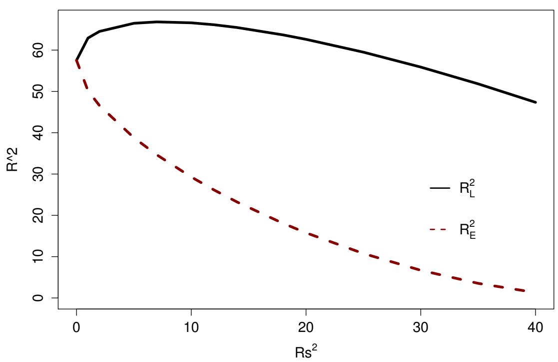
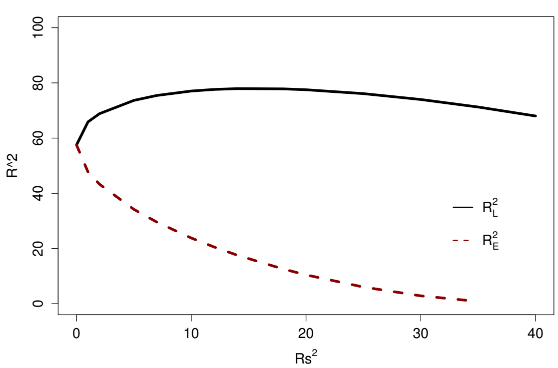


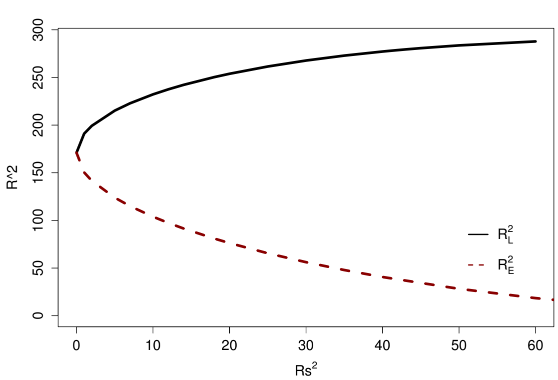
a) $h = 20$ *and* $\eta = 0$, $\chi = 0.1$



b) $h = 20$ *and* $\eta = 0$, $\chi = 0.5$

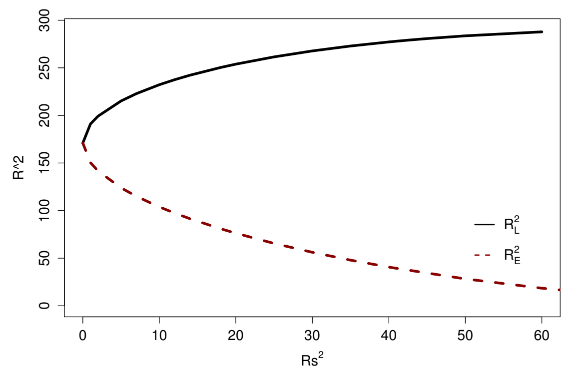


c) $h = 20$ *and* $\eta = 0$, $\chi = 2$

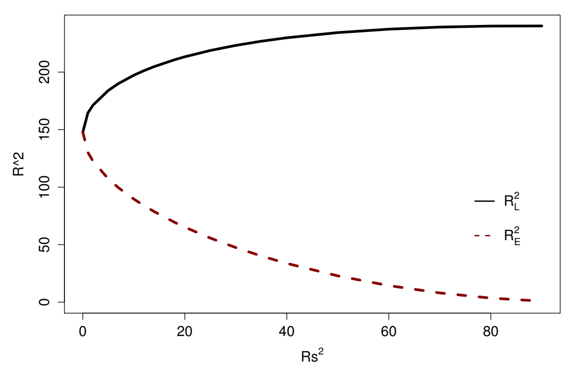


d) $h = 20$ *and* $\eta = 0$, $\chi = 10$

Figure 5. Linear instability and energy stability boundaries for the salted above Darcy convection problem with anisotropic effect for $\alpha = 1$. The figure represents the effect of different values of the permeability tensor, $\chi = \frac{K_z}{K_x}$.



a) $h = 20$ *and* $\eta = 0$, $\alpha = 1$



b) $h = 20$ *and* $\eta = 0$, $\alpha = 0.5$

Figure 6. Linear instability and energy stability boundaries for the salted above Darcy convection problem with anisotropic effect for $\chi = 10$. The figure represents the effect of different values of the thermal diffusivity tensor, $\alpha = \frac{k_{Tx}}{k_{Tz}}$.