THE USE OF TFBGF METHOD WITH A 3D TRANSIENT ANALYTICAL SOLUTION TO SOLVE AN INVERSE HEAT CONDUCTION PROBLEM IN THE PRESENCE OF A MOVING HEAT SOURCE

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ABSTRACT

Moving heat source are present in numerous problem pratical in ingeenring. For example, machining process as the Gas tungsten arc welding (GTAW) , laser welding, friction stirwleing process or milding problem. Moving heat source are also present in biological heating as the metabolism or in heat thermal treatment. All this case, the heat input identification is a complex task and represents an important factor in the optimization of the process. The aim of this work is to investigate both the temperature field as the heat flux delivered to a piece during a process with moving heat source. The temperature measurements are obtained using thermocouples at accessible regions of the workpiece surface while the theoretical temperatures are calculated from a 3D transient heat conduction thermal model with a moving heat source. The thermal model solution is obtained analytically (direct problem). The inverse problem, it means, the estimation of the moving heat source, uses the Transfer Function Based on Green's Function (TFBGF) method. This method is based on Green's function and in the equivalence between thermal and dynamic systems. The technique is a simple approach without iterative processes, and therefore extremely fast. From the knowledge of both the temperature profile (hypothetical or experimental temperature far from the heat source) and of the transfer function it is possible to estimate the heat flux by an inverse procedure of the Fast Fourier Transform (IFFT) . The TFBGF is, then, adapted to solve an inverse heat conduction problem with a moving heat source. Simulated and experimental test are used for estimating the heat source delivered to the piece. The estimation of the moving heat source without use of minimization least square, or optimization technique is the great advantages of the technique proposed here. The moving heat source can, then, be obtained directly from the temperature measured since the 3D transient analytical solution is obtained and the TFBGF can be applied in that solution.

NOMENCLATURE

α	[m²/s]	Thermal difusivity
β		Eigen value
δ	[1/m]	Delta function
q_g	[W/m³]	Volumetric heat generation
k	[W/mK]	Thermal conductivity
h	[W/m²K]	Heat transfer coefficient
H	[K W/m³]	Transfer fucntion
G	[C]	Green's fucntion
t	[s]	time
τ	[s]	time delay
T	[C]	Temperature
u	[m/s]	velocity
P	[m]	Location of heat source
x	[m]	Cartesian axis direction
y	[m]	Cartesian axis direction
z	[m]	Cartesian axis direction
W	[C]	Auxiliar temperature

Subscripts

1, 2, 3	related to the x, y, z directions respectively
m, n, p	summation subscripts

INTRODUCTION

Moving heat source are present in numerous problem pratical in ingeenring. For example, machining process as cutting, milling or welding process. Moving heat source are also present in biological heating as the metabolism or in heat thermal treatment. All this case, the heat input identification is a complex task and represents an important factor in the optimization of the process.

Many investigators have studied welding heat flow problems, analytically, numerically, and experimentally [1-4].In the majority of theses work, however, the values of the heat flux input is assumed to be known, taken from literature or determined by using calorimetric techniques. In fact, the heat flux that goes to the workpiece is unknown and its determination represents an inverse problem. In this sense, the use of techniques found in inverse heat conduction problems can represent an alternative way to obtain the heat flux that goes to the workpiece.

The estimation of the moving heat source without use of minimization least square, or optimization technique is the great advantages of the technique proposed here. The moving heat

source can, then, be obtained directly from the temperature measured since the 3D transient analytical solution is obtained and the TFBGF [5] can be applied in that solution.

The temperature measurements are obtained using thermocouples at accessible regions of the workpiece surface while the theoretical temperatures are calculated from a 3D transient heat conduction thermal model with a moving heat source. The thermal model solution is obtained analytically (direct problem). The inverse problem, it means, the estimation of the moving heat source, uses the Transfer Function Based on Green's Function (TFBGF) method [5]. This method is based on Green's function and in the equivalence between thermal and dynamic systems. The technique is a simple approach without iterative processes, and therefore extremely fast. From the knowledge of both the temperature profile (hypothetical or experimental temperature far from the heat source) and of the transfer function it is possible to estimate the heat flux by an inverse procedure of the Fast Fourier Transform (IFFT) . The TFBGF is, then, adapted to solve an inverse heat conduction problem with a moving heat source. Simulated and experimental test are used for estimating the heat source delivered to the workpiece.

Direct Problem

Figure (1) presents a 3D-transient thermal problem with heat source moving with a uniform and gaussian distribution, located initically at the postion $(0,02,\ 0,05,\ L_3)$ and moving at a constant velocity of $v = 0,0001 ms^{-1}$ in foward at direction x. since the plate has large dimensions, as superfices is supposed to keep the temperature constant and equal to $T_o(x,y,z) = 0C$ and the upper susrface is exposed to the enviromenntal medium wiht the heat transfer coefficient $h = 20 W m^{-2} K^{-1}$. Before the heating moving suource be applied the plate is initially on $T_o(x,y,z) = 0C$.
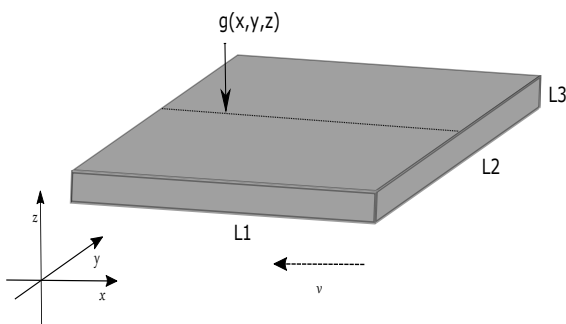

Figure 1. 3D-transient thermal problem with a heat source moving

Figure (2) presents the heat source distribution anaylsed here.
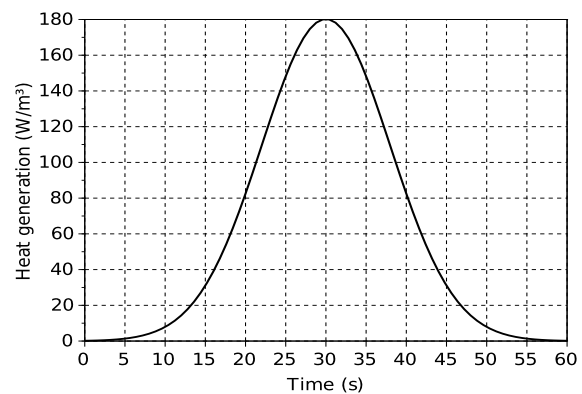

Figure 2. Heat source generation with an gaussian distribuition

There are two manner of considering the effect of the heat source moving. One is to consider the heat fux releases at surface and in this sense the heat flux need to be consdered as a boundary condition. The second hypoteses, used here, is to consider the heat flux as a heat sourge generation. This assuptions is a better approximation for pratical engineering application as engineering as machining, grinding, cutting, and sliding of surfaces, that has energy generated as a result of friction heating in a area with depth penetrarion, it means a volume heat source. In this sense, in this work, the heat source moving will be considered a point heat source of constant strength delta(W), releasing its energy continuously over time while moving along the x axis in the positive x direction with a constant velocity u, in a stationary medium that is initially at zero temperature.

The thermal problem shown in (Fig. 1) can be described by the 3D heat conduction equation in the fixed x, y, z coordinate system, assuming constant properties, as

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{k}\delta(x-vt)\delta(y-P_y)\delta(z-P_z) = \frac{1}{\alpha}\frac{\partial T}{\partial t} - u\frac{\partial T}{\partial x} \tag{1}$$

sbjected to the boundary conditions:

$$T(0,y,z,t) = T(L_1,y,z,t) = T_0 \tag{2}$$
$$T(x,y,z,t) = T(x,L_2,z,t) = T_0$$

and the initial condition

$$T(x,y,z,0) = T_0 \tag{3}$$

A tranformation is proposed [6] in order to remove the last term from the Eq.(1). It means, a new variable $W(x,y,z,t)$ is introduced as

$$T(x,y,z,t) = W(x,y,z,t)exp\left(\frac{ux}{2\alpha} - \frac{u^2t}{4\alpha}\right) \quad (4)$$

In addition, in the solution of moving heat source problems, it is convenient to let the coordinate system move with the source [7]. This is achieved by introducing a new coordinate ξ defined by

$$\xi = x - ut \quad (5)$$

Thus, considering Eqs. (4) and (5) the governing equation is obtaned as

$$\frac{\partial^2 W}{\partial \xi^2} + \frac{\partial^2 W}{\partial y^2} + \frac{\partial^2 W}{\partial z^2} + \frac{q_g}{k}\delta(\xi - P_\xi)\delta(y - P_y)\delta(z - P_z) = \frac{1}{\alpha}\frac{\partial W}{\partial t} \quad (6)$$

subjected to the boundary conditions

$$W(0,\gamma,z,t) = W(L_1,y,z,t) = 0 \quad (7)$$
$$W(\xi,0,z,t) = W(\xi,L_2,z,t) = 0 \quad (8)$$
$$W(\xi,y,0,t) = W(\xi,y,L_3,t) = 0 \quad (9)$$

and

$$k\frac{\partial W}{\partial z}\bigg|_{z=L_3} + W(L_3,t)\left(h + \frac{ku}{2\alpha}\right) = hW_\infty e^{\frac{-uL_3}{2\alpha} - \frac{u^2t}{4\alpha}} \quad (10)$$

where

$$h + \frac{ku}{2\alpha} = h_{eff} \quad (11)$$

The term h_{eff} in Eq.(11) is the effective heat convection coefficien [6].

Equation (6) can then be solved by Greens function method [6] as

$$W = \int_{\tau=0}^{t}\int_{\xi'=0}^{L_1}\int_{y'=0}^{L_2}\int_{z'=0}^{L_3} G^*(t-\tau)q_g(\tau)dz'dy'd\xi'd\tau \quad (12)$$

where

$$G^*(\xi,y,z,t|\xi',y',z',t-\tau) = G(t-\tau)\delta(\xi' - P_\xi)\delta(y' - P_y)\delta(z' - P_z)$$
$$\times e^{\frac{uP_\xi}{2\alpha} - \frac{u^2\tau}{4\alpha}} \quad (13)$$

and the Greens fucntion for this problem can be obtained by [6]

$$G(\xi,y,z,t|\xi',y',z',t-\tau) = \frac{8}{L_1L_2L_3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty} e^{-\beta_m^2 u}e^{-\beta_n^2 u}e^{-\beta_p^2 u}$$
$$\times sen(\beta_m x)sen(\beta_m x')sen(\beta_n y) \quad (14)$$
$$\times sen(\beta_n y')sen(\beta_p z)sen(\beta_p z')\frac{\beta_p^2 + B^2}{\beta_p^2 + B^2 + B}$$

Where $\beta_m = \frac{m\pi}{L_1}$, $\beta_n = \frac{n\pi}{L_2}$, $-B = \beta_p cot(\beta_p)$, $B = \frac{hL_3}{k}$ e $u = \alpha(t - \tau)$.

Original temperature can then be recorved by Eq.(4), it means

$$T(x,y,z,t) = W(x,y,z,t)exp\left(\frac{ux}{2\alpha} - \frac{u^2t}{4\alpha}\right) \quad (15)$$

Figure (3) presents the temperatures generated by analytical solution using gaussian heat source, shown in figure (2) for the point $(0.02, 0.05, L_3)$. And Fig. (4) presents the temperatures in any times for the same point.
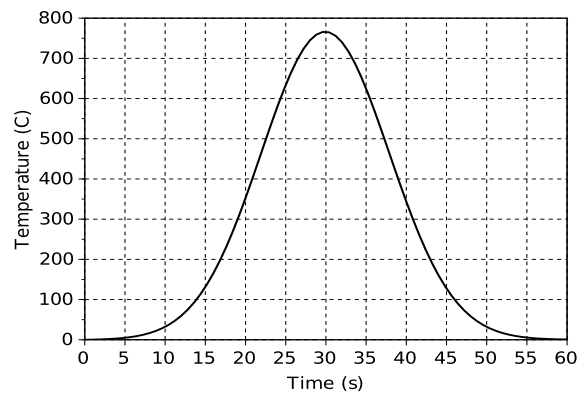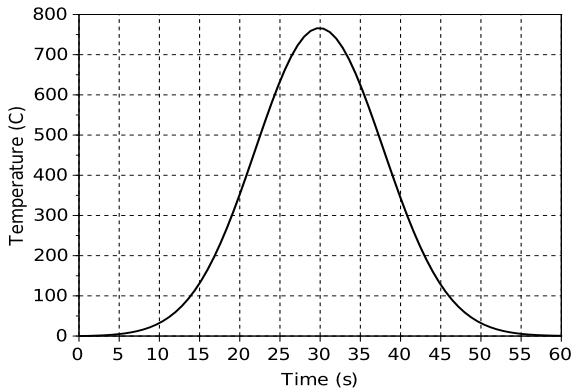
Figure 3. Temperatures

Figure 4. Temperatures

Heat source idenftification using TFBGF method

For any dynamic system (Fig. 5), the relation between input $X(t)$ and output $Y(t)$ can be given by the convolution equation
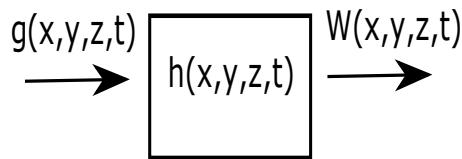

Figure 5. Diagram for dynamical systems

$$Y(t) = h(t) * X(t) = \int_{-\infty}^{+\infty} h(t-\tau) X(\tau) d\tau \qquad (16)$$

That in Laplace domain is expressed by the multiplication

$$Y(x,s) = H(x,s) \cdot X(s). \qquad (17)$$

In this sense, input X in domain s can be calculated by

$$X(s) = (1/H(x,s)) \cdot Y(x,s). \qquad (18)$$

or in the time domain by the deconvolution

$$X(t) = \mathcal{L}^{-1}\{1/h(x,t)\} * Y(x,t). \qquad (19)$$

By comparing Eq.(13) and (18) it can be identified an equivalent thermal systemw where the output of the system can be represented by the auxiliar temperature W, the transfer function by the modified Green's function G^* and heat source moving q_g can be considered the input of the system. It means,

$$G^*(\xi, y, z, t | \xi', y', z', t-\tau) = h(t-\tau)$$

and

$$q_g(t) = X(t) \qquad (20)$$

Identification of the analytical impulse response

The proposed methodology for identification of the analytical impulse response is based on the theory of dynamical systems of one input and one output.

It can be observed that for any input $X(t)$ and the output $Y(t)$, the transfer fucntion, $H(t)$, remains the same. In this case, if the input $X(\tau) = \delta(\tau)$ is applied, the tranfer function, then, can be obtained by the auxiliar temperature distribution, $W(t)$. It means, $Y(t) = W(t) = h(t) = G^*(t)$.

Therefore, by comparing Eq.(13) and Eq.(16), and using transformation Eq. (4). In this case we use $q_g(\tau) = \delta(\tau)$. After the integral performing the Transfer function can then be calculated as

$$h(\xi, y, z, t) = \frac{\alpha}{k} \frac{8}{L_1 L_2 L_3} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{p=1}^{\infty} e^{-(\beta_m^2 + \beta_n^2 + \beta_p^2)\alpha t}$$
$$\times sen(\beta_m \xi) sen(\beta_m P_\xi) sen(\beta_n y) sen(\beta_n P_y) \qquad (21)$$
$$\times sen(\beta_p z) sen(\beta_p P_z) e^{\frac{u P_\xi}{2\alpha}} \left(\frac{\beta_p^2 + B^2}{\beta_p^2 + B^2 + B} \right)$$

$$T(s) = q_g(s) \cdot H(s) \implies q_g(s) = \frac{T(s)}{H(s)} \qquad (22)$$

Thus, the solution of the inverse problem (in original variable T) is obtained by means of one of Eq. (22) in the Laplace domain or, in the time domain by Eq. (19) [5].

In this case, the input, the heat source $q_g(s)$, that can be estimated from the impulse response $H(s)$ and from the temperature measured at any position of the system. As mentioned, heat

source estimation can be done in the Laplace domain or in time domain using the comercial code Matlab with functions deconvolution (*deconv*), Fourier transform (FFT) and inverse Fourier transform ($IFFT$) as the following expressions

$$q_g(s) = (1/H(x,s)) \cdot W(x,s). \qquad (23)$$

and

$$q_g(t) = \{1/h(x,t)\} * W(x,t). \qquad (24)$$

1 Results

The three-dimensional case described in (Fig. 1) is analyzed in this section. Temperature distributions for the direct problem are generated using the solution of Eq. (15) considering a known heat source moving $q_g(t)$ as shown in (Fig.2). Random errors are then added to these temperatures. The temperatures with error are then used in the inverse algorithm to reconstruct the imposed heat source moving. The simulated temperatures are calculated from the following equation

$$Y(t) = T(x,y,z,t) + \varepsilon \qquad (25)$$

With the hypothetical temperatures (without added noise) calculated using Eq. (15) and the impulse response of the system (Fig. 6) calculated using Eq. (22) we obtain the temperature that will be used for estimations of the heat moving source using Eqs. (26 or 27).

Eqs. (15) or (16) can also be used to estimate the heat moving source using temperatures at each position x; y; z The respective temperature evolution for each position heat source is presented in (Fig. 3).

Figure (6) presents the tranfer fucntion relative to the thermocuple position that will be used to estimate the heat source moving. Any par Temperature/heat transfer fucntion can be used to the estiamtion.
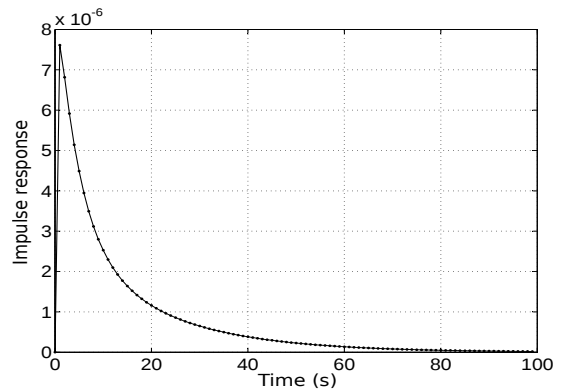

Figure 6. Impulse response

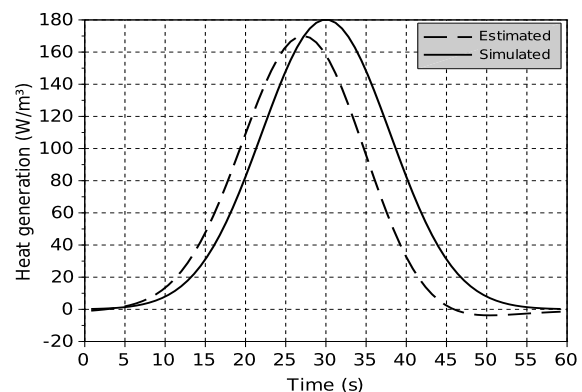Figure (8) presents the heat source estimated using $(0.02, 0.04, L_3)$ position

Figure 7. Estimated heat source and used heat source

It can be observed that the true value was obtained with a dispersion of ....%

Since the heat source moving has been obtained, temperature can be calculated for the region. In order to compare the results, Fig. (7) preesnts the temperature calculated with the heat flux simulated and the tmeperature measured in other position, for example in postition $x = 0.02$; $y = 0.04$; $z = L_3$.
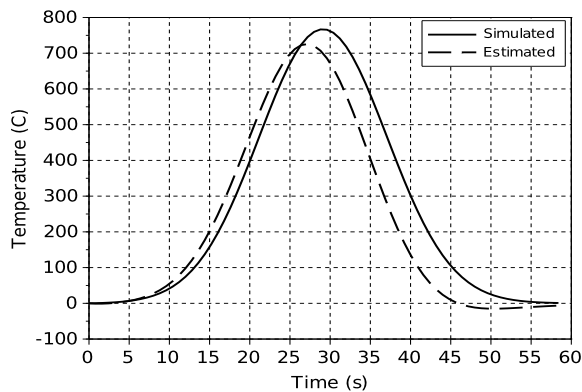
Figure 8. Temperatures

CONCLUSION

The moving heat source has been obtained directly from the temperature measured since the 3D transient analytical solution is obtained and the TFBGF can be applied in that solution.

Use of the Transfer Function Based on Green's Function (TF-BGF) method allows the estimation of heat source without iterative processes, and therefore extremely fast.

The temperature measurements are obtained using thermocouples at accessible regions of the workpiece surface while the theoretical temperatures are calculated from a 3D transient heat conduction thermal model with a moving heat source.

From the knowledge of both the temperature profile (hypo-thetical or experimental temperature far from the heat source) and of the transfer function it is possible to estimate the heat flux by an inverse procedure of the Fast Fourier Transform (IFFT) .

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