# SIMULTANEOUS MEASUREMENTS OF TURBULENT ENERGY AND TEMPERATURE DISSIPATION RATES IN THE FAR FIELD OF A SLIGHTLY HEATED CYLINDER WAKE

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## ABSTRACT

Using a multi-hot (cold) wire probe consisting of four "X-wires" and two pairs of parallel cold wires, the full turbulent energy and temperature dissipation rates, denoted as $\varepsilon_f$ and $\chi_f$, were measured simultaneously in the far region of a slightly heated circular cylinder wake. The performance of the probe in measuring the three components of the velocity and vorticity vectors was examined first. The mean values of the turbulent energy and temperature dissipation rates were evaluated based on both the full expressions and the isotropic relations. The differences between the present measurements of the mean turbulent energy and temperature dissipation rates and those reported previously are then analysed.

## INTRODUCTION

Turbulence is one of the most challenging problems in physics, and various methods have been employed to examine the insight nature of this phenomenon. Among the statistical properties of turbulent flows, the turbulent energy dissipation rate $\varepsilon_f$ $(\equiv 2\nu s_{ij}s_{ij})$ (unless specially noted, the summation convention applies), temperature dissipation rate $\chi_f (\equiv k\theta_{,i}\theta_{,i})$ and the enstrophy $\omega^2(\equiv \omega_i\omega_i)$ are important characteristics of the small-scale properties of turbulence, where $s_{ij}$ $[\equiv (u_{i,j}+u_{j,i})/2]$ is the rate of strain; $\nu$ is the kinematic viscosity and $u_{i,j} \equiv \partial u_i/\partial x_j$; $k$ is the thermal diffusivity of the fluid, $\theta_{,i} \equiv \partial\theta/\partial x_i$ and $\omega_i$ ($i$=1, 2, and 3) are the vorticity components; the subscript $f$ denotes the "full" expressions of the two dissipation rates. While the measurements of either energy or temperature dissipation rate have been made by a number of researchers in different turbulent flows using various types of hot/cold wire probes (e.g. [1-3]), few studies were conducted on the simultaneous measurements of these dissipation rates except the study by Zhou et al [4] in a decaying grid turbulent flow and Gulitski et al. [5] in the atmospheric surface layer. The most common methods used to measure the two dissipation rates are employing a single hot wire and a single cold wire by assuming isotropy of the flow and invoking Taylor's hypothesis i.e.

$$\varepsilon_{iso} = 15\nu u^2_{1,1} \tag{1}$$

and

$$\chi_{iso} = 3k\theta_{,1}^2 \tag{2}$$

where the subscript "iso" represents isotropy.

However, significant differences of the above substitutions from their full expressions have been reported (Zhou and Antonia [6]; Hao et al. [7], Gulitski et al. [5]). In the study by Hao et al. [7], a six-wire probe, consisting of four hot-wires and two cold wires, was used to approximate the turbulent energy and temperature dissipation rates in the far field of a circular cylinder wake at a Taylor microscale Reynolds number $R_\lambda$ ($\equiv u_1'\lambda/\nu$, where the Taylor microscale $\lambda$ is defined as $u_1'/u_{1,1}'$ and the superscript prime denotes root-mean-square (RMS) values) of about 60. The authors found that the use of $\varepsilon_{iso}$ and $\chi_{iso}$ as substitutes of $\varepsilon_f$ and $\chi_f$ gave totally different spectral distributions for these two quantities.

Without invoking Taylor's hypothesis, Gulitski et al. [5] measured simultaneously all three components of the velocity vector $u_i$, all nine components of the spatial velocity gradients $\partial u_i/\partial x_j$ and the time derivatives, $\partial u_i/\partial_t$, with synchronous data of temperature $\theta$, its spatial gradients, $\partial\theta/\partial x_j$ and temporal derivative $\partial\theta/\partial t$, along with the corresponding data on the mean flow in an atmospheric surface layer with $R_\lambda$ in the range from $1.6 \times 10^3$ to $6.6 \times 10^3$. They found that the true production of the temperature gradient was far from being fully represented by its surrogate. Also, Gulitski et al. [5] argued that the production of the temperature gradients was much more intensive in regions dominated by the strain, whereas this production was practically independent of (or weakly dependent on) the magnitude of vorticity (i.e. enstrophy). This is in contrast to the predictions of Gonzalez & Paranthoën [8] that vorticity influences essentially the statistics of passive scalars.

In addition, Gulitski et al. [5] suggested that their high-Reynolds number results were qualitatively, if not quantitatively, the same as previous low-Reynolds-number results, i.e. it is not always necessary to have high Reynolds numbers in order to study the basic physics of turbulence. This means that the important concepts such as inertial range were probably overstressed. Thus, it will not always be necessary to push to higher Reynolds numbers for experiments and direct numerical simulation (DNS). As pointed out by these authors, the results of Gulitski et al. [5] were obtained with cold wires of 2.5 μm in diameter, which was too big to resolve the smallest scale turbulent structures. In order to obtain adequate resolution in the smallest scales, the cold wires should be at least about 1 $\mu m$ in diameter to allow addressing a number of issues concerning the small scales of the temperature.

The objective of this paper is to measure the full turbulent energy and temperature dissipation rates simultaneously in a cylinder wake by using a multiple hot and cold wire probe (Figure 1), which consists of four X-wires (i.e. eight hot wires) and four cold wires. The performance of the probe will be assessed by comparing the present results with those obtained preciously using various approximations to the two dissipation rates so that appropriateness of these approximations can be assessed.

## EXPERIMENTAL SET UP

The experiments were conducted in a wind tunnel with a cross-section of 1.2m (width) × 0.8m (height) and 2 m long. The free stream velocity $U_\infty$ is about 3.6 m/s, corresponding to a Reynolds number Re $(=U_\infty d/\nu)$ of 1520, or a Taylor microscale Reynolds number $R_\lambda$ of about 50, where $d$ = 6.35mm is the diameter of the brass cylinder. The measurement location is at $x/d$ = 240. At this location, the Kolmogorov length scale η $[\equiv(\nu^3/<\varepsilon>)^{1/4}]$ is about 0.66 mm. The local turbulence intensity $u_i'/U_\infty$ is about 2%, favouring the use of Taylor's hypothesis. A heating wire with diameter of about 0.5 mm was wrapped and inserted into a ceramic tube, which was put inside the brass cylinder as a heating element. The resistance of the heating wire is about 30Ω. The wire was heated electrically with a power consumption of about 200W. The mean temperature increase $\Delta T$ on the centerline of the wake at the measurement location, relative to the ambient, is about 1.0°C. This value is small enough to avoid any buoyancy effect and large enough to assure high signal-to-noise ratio.

A probe consisting of four X-wires and two pairs of parallel cold wires (Figure 1) was used to measure the turbulent energy and temperature dissipation rates simultaneously. The four wires also allow the calculation of all components of the vorticity vector using the measured velocity signals $u_1$, $u_2$, and $u_3$. With the two pairs of parallel cold wires, the three temperature derivatives $\partial\theta/\partial x_i (i=1,\ 2\ \text{and}\ 3)$ can be measured simultaneously. The separations $\Delta x_2$ and $\Delta x_3$ of the two X-wires in the opposite direction are about 2mm and 2.7mm, respectively, and the separations $\Delta x_{2c}$ and $\Delta x_{3c}$ between the opposite two cold wires are about 2.5mm and 2.2 mm. As a result, the outer dimension of the probe is about 3mm × 3mm.

The hot and cold wires were etched from Wollaston (Pt-10%Rh) wires. The active lengths are about $200d_w$ and $800d_w$ for the hot and cold wires, respectively (where $d_w$ is the diameter of the wires and equals to 2.5 μm for the hot wires and 1.27 μm for the cold wires). The hot wires were operated with in-house constant temperature circuits at an overheat ratio of 1.5. The cold wires were operated with constant current (0.1 mA) circuits. The hot-wires were calibrated at the centerline of the tunnel against a Pitot-static tube connected to a MKS Baratron pressure transducer and the cold-wire was calibrated using a thermocouple (type K) with HP-34970A data acquisition system. As the hot-wire probe involved only four X-wires, the calibration of the 3D vorticity probe was the same as that for a single standard X-wire. The included angles of the X-wires are about 100°. The hot-wire yaw angle calibration was performed over ±20°. The output signals from the anemometers were passed through buck and gain circuits and low-pass filtered at a frequency $f_c$ of 800 Hz, which is close to the Kolmogorov frequency $f_K$ (≡U/2πη). The filtered signals were subsequently sampled at a frequency $f_s = 2f_c$ using a 16 bit A/D converter. The record duration was about 60 s.

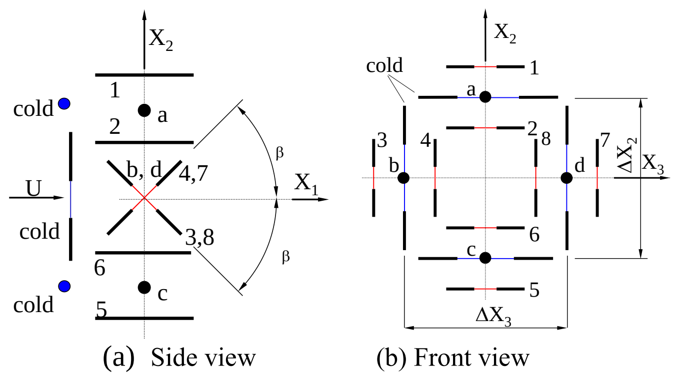



**Figure 1.** Sketches of the multi–wire probe. $X_1$ is in the flow direction. The sensors in red represent the hot wires and those in blue represent the cold wires. (a) Side view; (b) Front view.

Using the measured velocity and temperature fluctuation signals, the turbulent energy and temperature dissipation rates can be quantified. The full expression for the mean turbulent energy dissipation rate can be written as

$$\begin{aligned}\langle\varepsilon_f\rangle = \nu\{&2\langle u_{1,1}^2\rangle + 2\langle u_{2,2}^2\rangle + 2\langle u_{3,3}^2\rangle \\ &+\langle u_{1,2}^2\rangle + \langle u_{2,1}^2\rangle + \langle u_{1,3}^2\rangle + \langle u_{3,1}^2\rangle + \langle u_{2,3}^2\rangle + \langle u_{3,2}^2\rangle \\ &+2\langle u_{1,2}u_{2,1}\rangle + 2\langle u_{1,3}u_{3,1}\rangle + 2\langle u_{2,3}u_{3,2}\rangle\ \}.\end{aligned} \tag{3}$$

Assuming homogeneity, Eq. (3) can be simplified as:

$$\begin{aligned}\langle\varepsilon_f\rangle = \nu\{&4\langle u_{1,1}^2\rangle + \langle u_{1,2}^2\rangle + \langle u_{2,1}^2\rangle + \langle u_{1,3}^2\rangle \\ &+\langle u_{3,1}^2\rangle + \langle u_{2,3}^2\rangle + \langle u_{3,2}^2\rangle + 2\langle u_{1,2}u_{2,1}\rangle \\ &+2\langle u_{1,3}u_{3,1}\rangle - 2\langle u_{2,3}u_{3,2}\rangle\ \}.\end{aligned} \tag{4}$$

With the four X-wires the three vorticity components can also be obtained from the measured velocity signals, viz.

$$\omega_1 = u_{3,2} - u_{2,3} \cong \frac{\Delta u_3}{\Delta x_2} - \frac{\Delta u_2}{\Delta x_3} \tag{5}$$

$$\omega_2 = u_{1,3} - u_{3,1} \cong \frac{\Delta u_1}{\Delta x_3} - \frac{\Delta u_3}{\Delta x_1} \tag{6}$$

$$\omega_3 = u_{2,1} - u_{1,2} \cong \frac{\Delta u_2}{\Delta x_1} - \frac{\Delta u_1}{\Delta x_2}\,, \qquad (7)$$

where $\Delta u_3$ and $\Delta u_1$ in Eqs. (5) and (7) are velocity differences between X-wires a and c, respectively (Figure 1); $\Delta u_2$ and $\Delta u_1$ in Eqs. (5) and (6) are velocity differences between X-wires b and d, respectively. Derivatives in the $x_1$ direction were estimated using Taylor's hypothesis, i.e. $U_1\partial/\partial x_1 = -\partial/\partial t$. With the two pairs of parallel cold wires (Figure 1), the three temperature derivatives $\partial\theta/\partial x_i (i = 1,\ 2,\ 3)$ can be quantified. The full temperature dissipation rate $\langle\chi\rangle$ can be obtained, viz.

$$\langle\chi_f\rangle = k\left\{\langle(\partial\theta/\partial x_1)\rangle^2 + \langle(\partial\theta/\partial x_2)\rangle^2 + \langle(\partial\theta/\partial x_3)\rangle^2\right\}. \qquad (8)$$

Just like $\partial u_i/\partial x_1$, $\partial\theta/\partial x_1$ can be determined using Taylor's hypothesis.

## RESULTS AND DISCUSSION

### Basic performance checks of the probe

Measurements of $\varepsilon$ and $\chi$ are not straightforward since the velocity and temperature derivatives have to be resolved adequately by using finite differences. It is therefore important to ensure the performance of the probe before any further analysis can be conducted. Figure 2 shows the distributions of the root-mean-square (RMS) values (indicated by a prime) of $u_1$ across the wake normalized by the maximum velocity deficit $U_0$ and the wake half-width $L$. The values of $u_1'$ from the four different X-wires agree very well with each other and also show the same level of agreement with the results by Hao et al. [7] measured using 2 X-wire probes (results are not shown here).

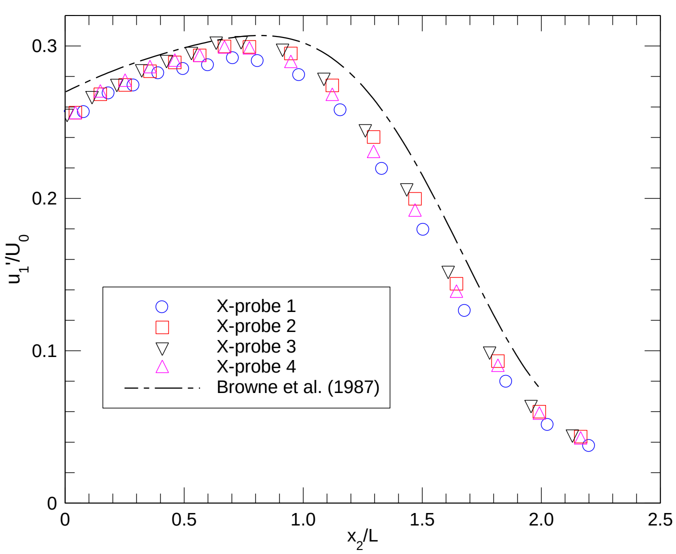



Fig. 2. Comparison of the RMS values of the velocity fluctuations measured using four X-wires across the wake and compared with those reported by Browne et al. (1987).

Figure 3 shows the distributions of $\theta'$ normalized by the temperature increase $\Delta T_0$ at the centerline across the wake from the four cold wires. The differences of the RMS values of temperature fluctuations measured by the cold wires are small and should be within the experimental uncertainty ($\pm 4\%$).

The measured values of the vorticity and the turbulent energy and temperature dissipation rates have to be corrected due to the effect of imperfect spatial resolution of the probe on the measurement of the velocity and temperature derivatives. The spectra of the velocity derivatives and the vorticity will be attenuated due to finite wire length and wire separation. This attenuation can be corrected based on local isotropy assumption. Details of the correction procedures can be found in Zhu et al. [3]. As a brief check on the performance of the probe, Figure 4 shows the comparison of the measured vorticity spectra with those calculated based on isotropic assumption. For isotropic turbulence, the spectra $\phi_{\omega_i}(k_1)$ can be calculated from $\phi_{u_{1,1}}$ (Antonia et al. [9]):

$$\phi_{\omega_1}^{cal}(k_1) = \phi_{u_{1,1}}(k_1) + 4\int_{k_1}^{\infty} \frac{\phi_{u_{1,1}}(k)}{k}\,dk \qquad (9)$$

$$\phi_{\omega_2}^{cal}(k_1) = \phi_{\omega_3}^{cal}(k_1) = \frac{5}{2}\phi_{u_{1,1}}(k_1) - \frac{k_1}{2}\frac{\partial\phi_{u_{1,1}}(k_1)}{\partial k_1} + 2\int_{k_1}^{\infty} \frac{\phi_{u_{1,1}}(k_1)}{k}\,dk \qquad (10)$$

where $\phi_{u_{1,1}}$ is the spectrum of the streamwise velocity derivative $\partial u_1/\partial x_1$. The agreement between the measured and calculated spectra is very good, especially for wavenumbers larger than 0.05 except for $\phi_{\omega_1}(k_1)$ over the wavenumber range of 0.01–0.05. In Figure 4, the superscript asterisk denotes normalization by the Kolmogorov length scale $\eta$ and the velocity scale $u_K \equiv \nu/\eta$.

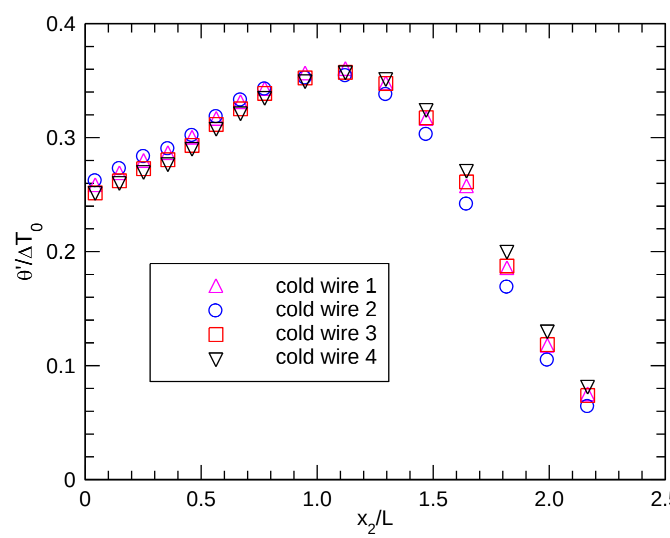



Fig3. Comparison of RMS values of the temperature fluctuations measured using the four cold wires.

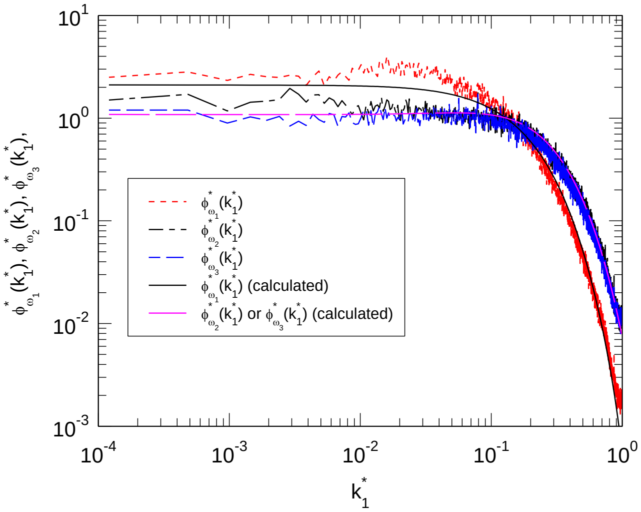
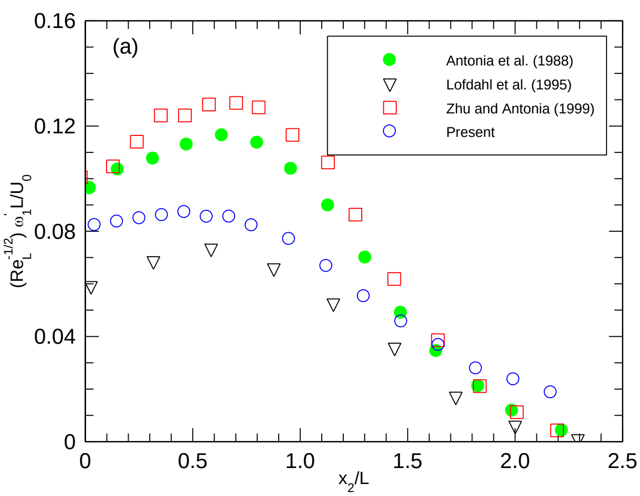
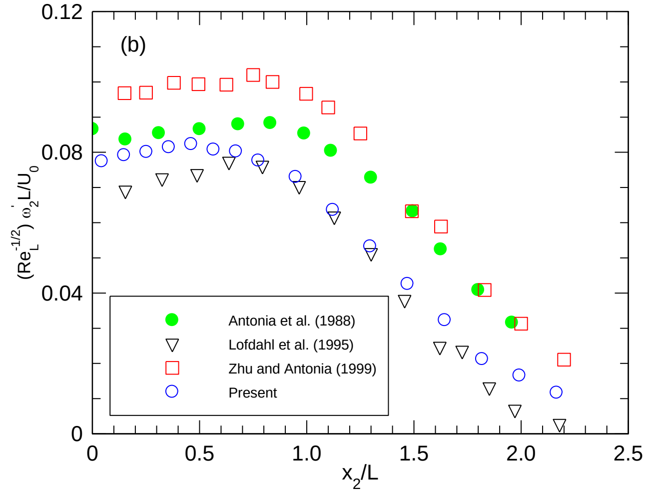



Fig4. Comparison of the measured vorticity spectra with their counterparts calculated based on the isotropic assumption.

Figure 5 shows the RMS values of $\omega_1$, $\omega_2$ and $\omega_3$ obtained in the present study with those obtained previously for the wake flow. These values are corrected for the spatial resolution of the probe based on the corrected spectra shown in Figure 4 using $<\omega_i^{*c2}>=\int_0^\infty \phi^*_{\omega_i}(k_1^*)dk_1^*$, where the superscript "$c$" represents "corrected". The RMS values of $\omega_i$ are normalized based on $L$ and $U_0$. To account for the Reynolds number effect, Antonia et al. [10] suggested that $Re_L$ ($\equiv U_0L/\nu$) should be included, i.e. $(Re_L^{-1/2})\omega_i' L/U_0$. The present RMS values of the three vorticity components after correction for spatial resolution are significantly smaller than those obtained by Antonia et al. [11] and Zhu et al. [3]. The present values of $\omega_i'$ agree more favorably with those reported by Löfdahl et al. [12].



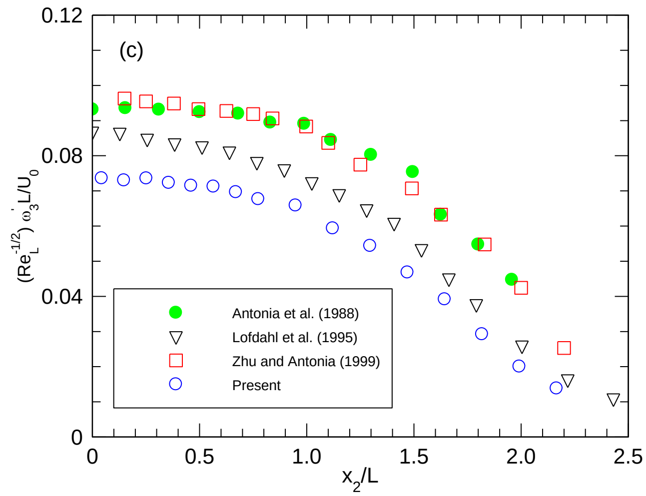




Figure 5. Comparison of the measured RMS values of the three vorticity components with those reported previously. (a) $\omega_1$; (b) $\omega_2$; (c) $\omega_3$.

**Statistics of the turbulent energy and temperature dissipation rates**

The mean turbulent energy dissipation rates obtained using Eqs. (1) and (4) are compared in Figure 6. All the velocity derivatives have been corrected according to the procedures outlined by Zhu et al. [3] to account for the effect of spatial resolution. Since more derivative correlations are included in Eq. (4) than that in Eq. (1), where isotropy is assumed, it is expected that the former should provide more reasonable values for $\varepsilon$ than that when $\varepsilon_{iso}$ (Eq. 1) is used. Figure 6 shows that the magnitude of $\langle\varepsilon_f\rangle$ is about 10% larger than that of $\langle\varepsilon_{iso}\rangle$ over the range of $y/L < 0.8$. For $y/L > 0.8$, the agreement between $\langle\varepsilon_f\rangle$ and $\langle\varepsilon_{iso}\rangle$ is satisfactory, indicating that the one-dimensional surrogate can represent the full mean energy dissipation rate reasonably well, at least for the purpose of estimating the mean values. The present values of $\langle\varepsilon_f\rangle$ and $\langle\varepsilon_{iso}\rangle$ are about 20~50% smaller than those reported by Zhu et al. [3], Lofdahl et al [12] and Browne et al. [13]. These differences need to be further examined. One possibility could be due to different Reynolds numbers for different experiments conducted. Another possibility may be related with the heating of the flow in the present study. Even though the temperature increase on the centerline at the measurement location is small (1°C relative to ambient temperature), the increased temperature on the cylinder surface may result in some influences to the flow in the far wake.

The mean temperature dissipation rates obtained using Eqs. (2) and (5 ) are compared in Figure 7. The magnitudes of $\langle \chi_{iso} \rangle$ and $\langle \chi_f \rangle$ collapse across the whole wake, indicating that $\langle \chi_{iso} \rangle$ is a good approximation to $\langle \chi_f \rangle$, at least in terms of the mean values. This result seems in agreement with that reported previously by Hao et al. [7] using other simpler approximations.

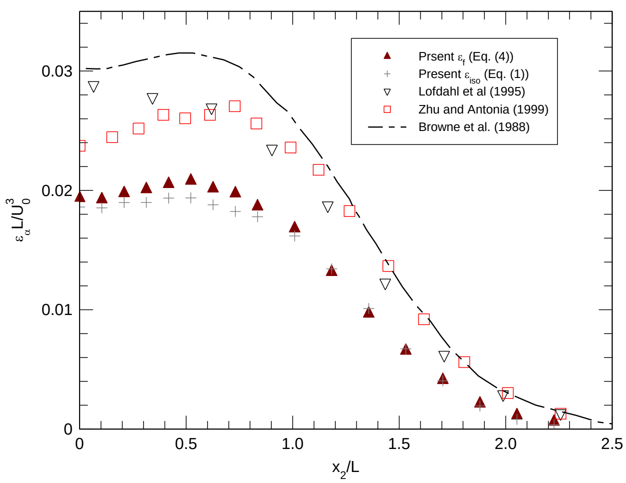



Figure 6. Comparison of the mean turbulent energy dissipation rates with those obtained previously.

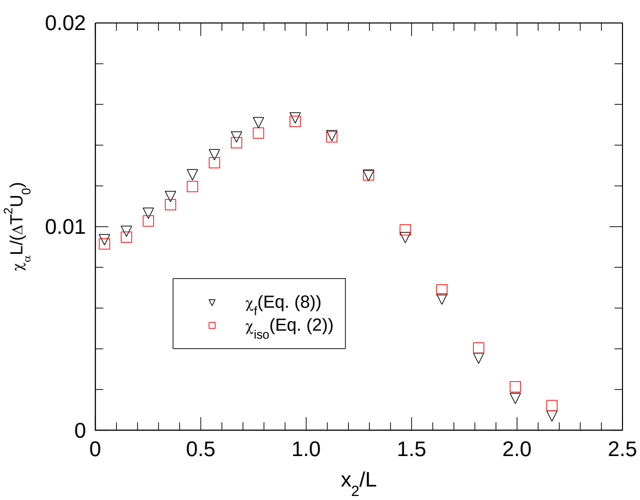



Figure 7. Comparison of the temperature dissipation rates between the full expression (Eq. (8)) and the isotropic relationship (Eq. (2)).

## CONCLUSIONS

A twelve-wire probe consisting of eight-hot wires and four-cold wires was used to measure the full turbulent energy and temperature dissipation rates simultaneously in a slightly heated wake flow. The performance of the probe in measuring the two dissipation rates and also the three velocity and vorticity components has been examined. The present study shows that the relations based on the isotropic assumption, i.e. ($\varepsilon_{iso}, \chi_{iso}$), can evaluate the mean turbulent energy and temperature dissipation rates, i.e. ($\varepsilon_f, \chi_f$), reasonably well. Further examinations on the adequacy of using the instantaneous signals of ($\varepsilon_{iso}, \chi_{iso}$) to represent the full dissipation rates ($\varepsilon_f, \chi_f$) will need to be conducted based on the experimental data acquired so far.

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