RAYLEIGH-BÉNARD CONVECTION OF Cu-WATER AND CuO-WATER NANOFLUIDS IN RECTANGULAR ENCLOSURE

Belaid F.*, Polidori G., Khezzar L. and Benhamza E.H.
*Author for correspondence
Department of Process Engineering,
University of Guelma,
Guelma, 24000,
Algeria,
E-mail: bel_fair@hotmail.com

ABSTRACT

The heat transfer by natural convection of nanofluid inside a horizontal cavity heated from below (Rayleigh–Bénard problem) was numerically investigated. Two different nanofluids are considered: Cu-Water and CuO-Water nanofluids, for which viscosity and thermal conductivity were determined using Brownian motion models. We supposed that nanofluid is mono-constituent fluid. In this work, simulations have been carried out for different cavity aspect ratios (width/height) and Rayleigh number and nanoparticle volume fraction are taken up to 0.04 to ensure a Newtonian behavior of the mixture.

It is found that the presence of nanoparticles affects the flow and thermal boundary layer and that is due to the high viscosity and thermal conductivity in the nanofluids. The nanofluid Nusselt number exhibits a slight increase as function of aspect ratio comparing to that in pure fluid. That enhancement is strongly influenced by Rayleigh number and the cavity aspect ratio.

NOMENCLATURE

Ar		Aspect ratio ($= L/H$)
Cp	(J/kgK)	Heat capacitance
d	[m]	Diameter
H	(m)	Height
h		Heat transfer coefficient
g	(m/s^2)	Gravitational constant
k	(W/mK)	Thermal conductivity
L	(m)	Length
Nu		Nusselt number
P	[N/m²]	Pressure
Pr		Prantdl number
Ra		Rayleigh number
Re		Reynolds number
T	[K]	Temperature
$\vec{U}$		Velocity field
x, y		Cartesian coordinates
Greek symbol		
α	(m^2/s)	Fluid diffusivity
β	(1/K)	Thermal expansion
μ	(Ns/m²)	Viscosity
ν	(m^2/s)	Kinematic viscosity
ρ	(kg/m^3)	Density
φ		Particle volume fraction
Subscript		
c		Cold
f		Base fluid ($\varphi = 0$)
h		Hot
nf		Nanofluid
p		Particle

INTRODUCTION

Nanofluids are defined as a dispersed and suspended nanoparticles in a base fluid. It is shown that the presence of nanosized solid particles in fluids can enhance its thermal conductivity [1-3]. Great challenges are expected in the potential of enhancing heat transfer by using nanofluid in engineering applications and thermal science. Initial works have been dedicated to predict the effective thermal conductivity of nanofluid [4-6]. Reviews on nanofluid characteristics was as well as published [7, 8]. Since the enhancement of thermal conductivity of nanofluid is well established, there are number of questions which remain unclear concerning heat transfer enhancement using nanofluid.

The past years have witnessed several studies of convective heat transfer in nanofluid. Many authors show that the presence of nanoparticles in a fluid alters the flow structure and increases the natural convection heat transfer [9]. The results presented by Khanafer et al. [10] illustrated that nanofluid substantially increased the heat transfer rate for any given Grashof number when increasing the volume fraction of nanoparticles. Many other studies presented disagreement as they reported a degradation of heat transfer with nanofluids. Santra et al. [11] numerically studied the effect of copper–water nanofluid on the heat transfer due to natural convection in a differentially heated square cavity treating the nanofluid as non-Newtonian. They found that the heat transfer decreases with increase in particle volume fraction for a particular Rayleigh number. Ho et al. [12] conducted a study on natural convection of nanofluid in a square enclosure. They studied the effects of uncertainties due to adopting various formulas for effective thermal conductivity and dynamic viscosity on the heat transfer characteristics. They concluded that enhancement in the dynamic viscosity of nanofluid, counteracting that in the thermal conductivity; these results can thus play as a crucial factor and should be taken into account when assessing heat transfer efficiency for natural convection in enclosures. Putra et al. [13] presented an experimental investigation of the natural convection of water-based Al_2O_3 nanofluids. A paradoxical behavior of heat transfer deterioration was observed in their experiment. They reported that the presence of nanoparticles of Al_2O_3 in base fluid reduced the natural convective heat transfer when increasing the particle concentration for a given Rayleigh number. Corcione [14] investigated theoretically the natural convection features of nanofluid inside rectangular enclosures differentially heated at the vertical walls. He found the existence of an optimal particle loading for maximum heat transfer. In addition that optimum is found to increase with decreasing the nanoparticle size, and to increase with increasing both the nanofluid average temperature and the slenderness of the enclosure. Heat transfer in free convection using different nanofluids taking in account Brownian nanoparticles motion were conducted by Popa et al. [15, 16]. They studied the heat transfer enhancement comparing uniform wall temperature and uniform heat flux thermal boundary conditions. They concluded that the use of nanofluids in heat transfer enhancement purposes doesn’t seem feasible in case of uniform heat flux condition in free convection flow.

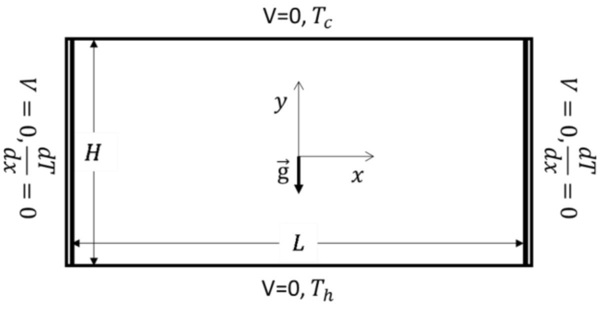

Figure 1 Set of geometry and boundary conditions

Buoyancy-driven heat transfer of nanofluids in a rectangular cavity (Rayleigh-Bénard configuration) was studied by Hwang et al. [17] where: the effects of the volume fraction, the size of nanoparticles, and the average temperature of nanofluid was theoretically presented. They showed that nanofluid is more stable than base fluid in a rectangular cavity heated from below as the volume fraction of nanoparticles increases, the size of nanoparticles decreases, or the average temperature of nanofluid increases. In addition, they theoretically found that the ratio of heat transfer coefficient of nanofluid to that of base fluid decreased as the size of nanoparticles increased, or the average temperature of nanofluid is decreased. Corcione [18] assigned theoretically that, for Rayleigh-Bénard convection of nanoparticles suspension, the optimal volume fraction has a peak for each Rayleigh number that depends on both the average temperature of the nanofluid and the diameter of the suspended nanoparticles. Elhajjar et al. [19] studied by direct numerical simulation the heat transfer in Rayleigh–Bénard convection. They used two relations for specific heat capacity and thermal expansion coefficient that are in agreement with the laws of thermodynamics. They found that the onset of convection is delayed with nanofluid. Contrary to what is argued by many authors, they showed the use of nanofluid can reduce heat transfer instead of increasing it. Eyad Abu Nada [20] showed deterioration in heat transfer at high Rayleigh number by increasing the volume fraction of nanoparticles [21].

It appears from this literature review, that the use of nanofluid to improve natural convection heat transfer remains an open problem. In the present study we examine the effect of two nanofluids Cu-Water and CuO-Water, on the conduction and convection heat transfer rate in a rectangular cavity heated from below (Rayleigh–Bénard configuration). In this work we try to show the effects of the aspect ratio of the cavity.

MATHEMATICAL FORMULATION

Rayleigh-Bénard convection in a horizontal enclosures heated from below are studied in this paper. Four aspect ratios are taken for this enclosure and the lower plate is at higher temperature. Figure 1 shows the schematic of the configuration. For this study, the aspect ratio Ar is equal to the ratio of the horizontal wall length to the vertical wall height L/H.

Thermophysical properties modelling

According to the assumption that the nanoparticles and the base fluid are in thermal equilibrium and that the nanoparticles are uniformly dispersed within the base fluid.

Table 1 Thermophysical properties at reference temperature

Properties	Water	Cu	CuO
$\rho\ [kg/m^3]$	998.3	8954	6450
$\mu\ [Ns/m^2]$	0.001	/	/
$Cp\ [J/kgK]$	4070	385	561
$k\ [W/mK]$	0.6	401	20
$\beta \times 10^{-6}\ [1/K]$	206.0	16.7	18.0

The thermophysical properties of the nanofluid, namely the density, specific heat capacity and volumetric thermal expansion coefficient, were calculated from nanoparticles and base fluid properties at ambient temperature (Table 1) using the following formulas as mentioned in several papers [11, 14]

$$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_p \tag{1}$$

Where the subscripts (p) and (f), denote respectively the nanoparticles and the base fluid.
The volumetric thermal expansion coefficient for nanofluid β_{nf} is given by

$$\beta_{nf} = (1-\varphi)\beta_f + \varphi\beta_p \tag{2}$$

The heat capacitance of the nanofluid is expressed as [14]:

$$(\rho Cp)_{nf} = (1-\varphi)(\rho Cp)_f + \varphi(\rho Cp)_p \tag{3}$$

The dynamic viscosity of the nanofluid is obtained from the second order correlation proposed by davalos-orozco and Del castillo based on semi-dilute suspension nanoparticles and Brownian motion effect. μ_{nf} is given by:

$$\mu_{nf} = \mu_f\left((5.2 + 0.97)\varphi^2 + 2.5\varphi + 1\right) \tag{4}$$

The effective thermal conductivity of the nanofluid is approximated by semi-empirical model in sort of taking into account a possible effect of the brownian motion

$$\frac{k_{nf}}{k_f} = 1 + $$

$$+64.7\varphi^{0.746}\left(\frac{d_f}{d_p}\right)^{0.369}\left(\frac{k_p}{k_f}\right)^{0.7476} Pr_f^{0.9955} Re^{1.232} \tag{5}$$

where the reynolds number is based on the brownian velocity (V_{Br}) of the nanoparticles, which is defined in [15]:

$$Re = \frac{\rho_f V_{Br} d_p}{\mu_f} = \frac{\rho_f K_B T}{3\pi \ell_f \left(\mu_f\right)^2}$$

where ℓ is the mean free-path and K_B is the Boltzman constant.

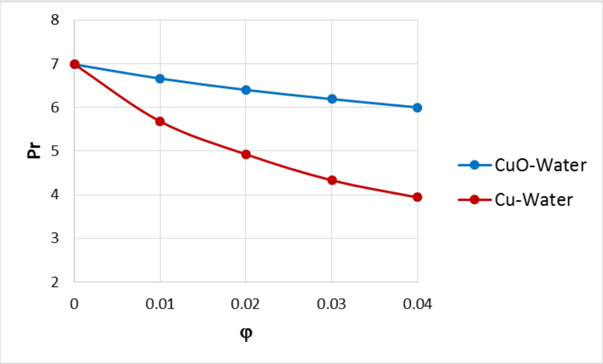

Figure 2 Prandtl number variation with volume fraction of the nanofluids

Governing equations

The nanofluid in the enclosure is Newtonian, incompressible, and laminar. The nanoparticles are assumed to have a uniform shape and size. A single-phase model is assumed where both the fluid phase and the solid particles are in thermal equilibrium state and flow with the same local velocity. The thermo physical properties of the nanofluid are considered constant except the density ρ in the buoyancy term which varies linearly with the temperature which is based on the Boussinesq approximation, thus:

$$\rho_{nf} = (\rho_0)_{nf}\left(1 - \beta_{nf}\Delta T\right) \tag{6}$$

The governing equations for natural convection in an enclosure are continuity, momentum, and energy equations. Taking into the account the above-mentioned assumptions, the equations are given in dimensionless form by:

$$\vec{\nabla}\cdot\vec{U} = 0 \tag{7}$$

$$\left(\vec{U}\cdot\vec{\nabla}\right)\vec{U} = -\vec{\nabla}P + Pr_{nf}Ra_{nf}T\vec{e} + Pr_{nf}\nabla^2\vec{U} \tag{8}$$

$$\vec{U}\cdot\vec{\nabla}T = \nabla^2 T \tag{9}$$

where U is the velocity, P is the pressure and T is the temperature. The dimensionlesse parameters governing this problem are the Rayleigh number and the Prandtl number for the nanofluid, defined by:

$$Ra_{nf} = \frac{\rho_{nf}\beta_{nf}Cp_{nf}g(T_h - T_c)H^3}{\mu_{nf}k_{nf}} \tag{10}$$

And

$$Pr_{nf} = \left(\frac{\mu Cp}{k}\right)_{nf} \tag{11}$$

where g is the gravitational acceleration, β_{nf} is the volumetric thermal expansion coefficient, ρ_{nf} is the nanofluid density, μ_{nf} is the dynamic viscosity, $(Cp)_{nf}$ is the specific heat capacity and k_{nf} is the thermal conductivity. The Rayleigh number Ra denotes the ratio of the strengths of thermal transport due to buoyancy force to that due to thermal conduction. The Prandtl number represents the ratio of momentum diffusion to thermal diffusion. Alternatively, Pr can outlook the ratios of viscous boundary layer to thermal boundary layer thicknesses. The effects of the nanoparticle material and the base fluid on Pr_{nf} are highlighted in Figure 2. Since the Cu-water nanofluid provides a higher thermal conductivity comparing to that in CuO-water nanofluids, Pr decreases as the nanoparticle volume fraction increases.

In the current study, the Rayleigh–Bénard (RB) heat transfer in nanofluid is calculated for fixed Rayleigh number as well as in water, i.e. $Ra_f(\varphi = 0) = Ra_{nf}(\varphi > 0)$, we obtain:

$$Nu_{nf} = \frac{h_{nf}H_{nf}}{k_{nf}} \tag{12}$$

The Nusselt number is directly determined by calculation as: $Nu(x) = -(\partial T/\partial y)$ thus the average Nusselt number is obtained from the hot surface integral as:

$$Nu = \frac{1}{Ar}\int_0^{Ar} Nu(x)dx \tag{13}$$

Numerical methods

The range of Rayleigh number and volume fraction of nanoparticles are taken $10^3 \leq Ra_{nf} \leq 10^5$ and $0 \leq \varphi \leq 0.04$, respectively.The bottom wall temperature is maintained at the reference temperature i.e., T_h, whereas the temperature at the top wall is T_c. The calculation is carried out for cavities Ar varying from 1 to 8.

The above sets of equation (7-9) are solved numerically using the finite volume technique on a Cartesian grid with grid sizes that ensure grid-independent solutions. The Simple algorithm, the Quick scheme, and Standard descritization are used for the velocity-pressure coupling, convective terms discretization, and pressure interpolation respectively. Convergence is assumed when the normalized residuals reached a value of 10^{-9} in monitoring appropriate field variables. The numerical code was validated against the results of other benchmark studies for RB convection in an enclosure. To check the sensitivity of the code to the grid size, an extensive mesh testing procedure was conducted to guarantee a grid independent solution. Five different mesh combinations varying from 41×41 to 121×121. a grid independence study was performed using water. It was confirmed that the grid size (61×61) ensures a grid independent solution for square cavity i.e., $Ar = 1$. In the cases of higher aspect ratio cavities i.e., $Ar = 2, 4$ and 8, grids of $(61 \times Ar \cdot 61)$ are assumed. The calculated *Nu* values agree very well with the calculated values obtained in the literature [20].

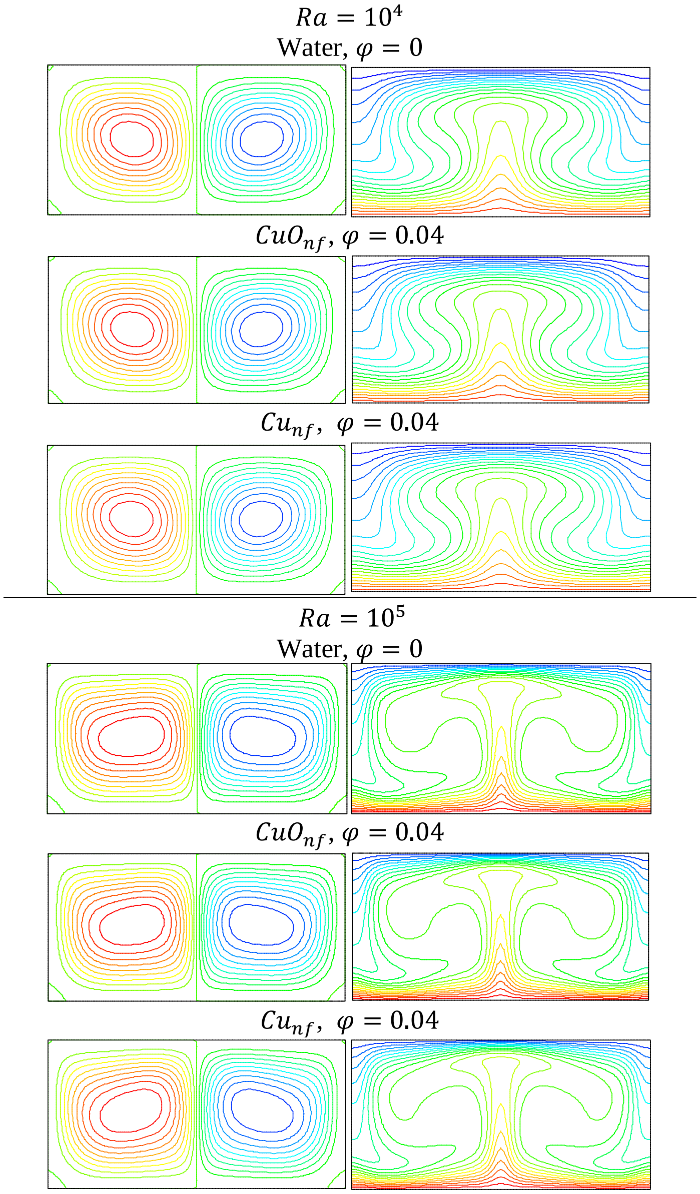

Figure 3 Stream Function contours (left) and isotherms (right) for $Ar = 2$

RESULTS AND DISCUSSION

Rayleigh–Bénard (RB) convection represents a fundamental process for heat transfer in fluids. For low Rayleigh number, i.e., $Ra = 10^3$ the heat transfer is dominated by conduction and Nu values remains equal to one in all cases. The set of convection is established when the critical Rayleigh number, $Ra_c \approx 1708$ is reached [20]**.** Thermal convective fluid motion is initiated between the plates and the heat transfer is no longer dominated by conduction. This convective motion develops a thermal plume with adjacent fluid rolls rotating in opposite directions as depicted in **Figure 3** and **Figure 4**.

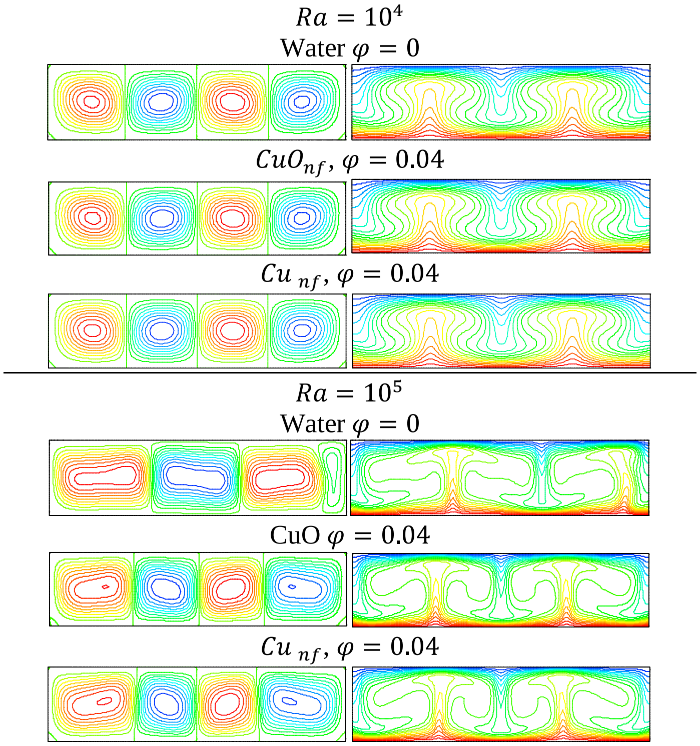

Figure 4 Stream Function contours (left) and isotherms (right) for $Ra = 10^4$ and $Ar = 4$

It is clear that Ar has a significant effect on the flow patterns, temperature distributions and the heat transfer rates. The increase in the number of roll cells as Ar increases is explained through the progressive breakdown of the density stratification in the fluid layers adjacent to the top and bottom walls affecting the formation of hot and cold fluid streams moving upward and downward across the cavity and thereby shaping the temperature distribution [21] and [22]. The isotherms and stream function contours of **Figure 3** and **Figure 4** show alternating compression and spreading of isotherms near the bottom and top conducting walls implying a varying heat flux on these walls. Besides, it is shown that next to the top and bottom plates the temperature isotherms are almost horizontal, which demonstrates the dominance of the conduction heat transfer whereas in the region between the plates the temperature isotherms are no longer horizontal due to the dominance of convection.

The presence of nanoparticle in the base fluid seems to affect somehow on the thermal boundary layer. For $Ra = 10^5$ and $Ar = 4$, it is noticed an asymmetric distribution in isotherm pattern and streamlines, in the case of pure fluid, i.e. $\varphi = 0$. A symmetric pattern is reestablished progressively with increasing φ, refer to . It could be said that the natural convection is more stable in nanofluid than in a pure fluid [18, 20].

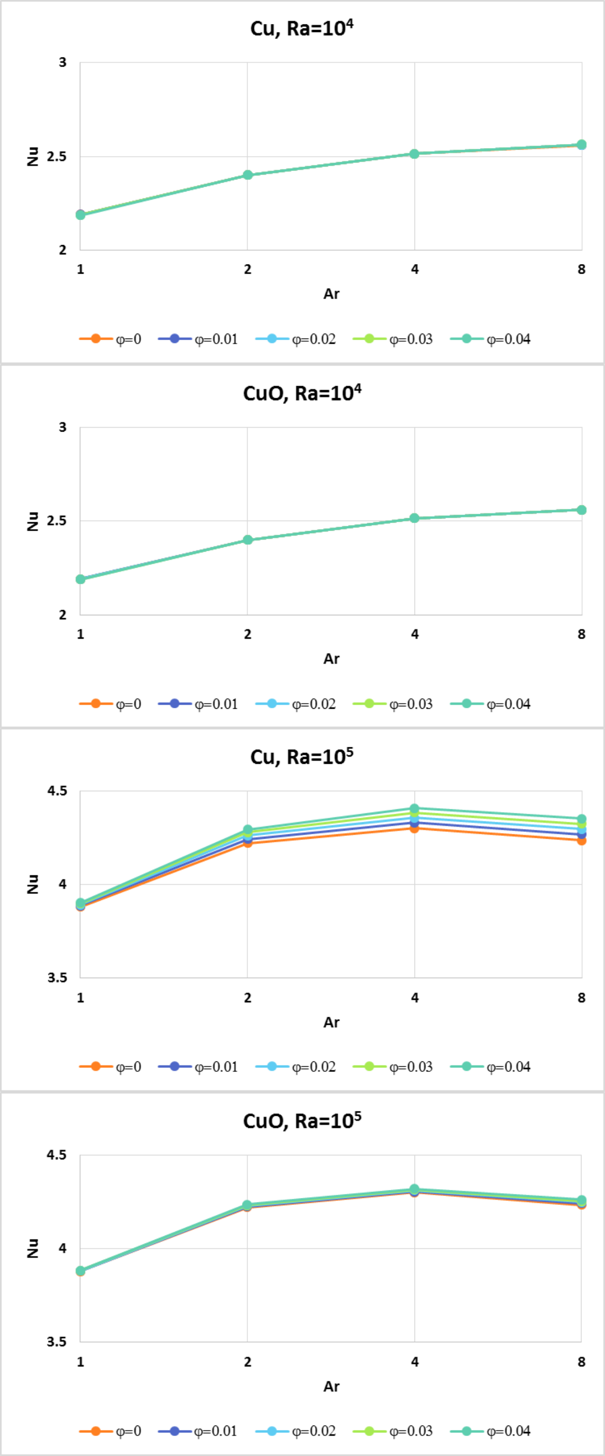

Figure 5 Nusselt number Nu variation with Aspect ratio Ar

Figure 5 presents Nusselt number versus Aspect ratio for both Cu-water/CuO-water nanofluids and $Ra = 10^4$ and 10^5at various volume fractions of nanoparticles.

Overall observation indicates that an increase in the Ar leads to increase the Nusselt number for $Ra = 10^4$ and 10^5. However, for $Ra = 10^4$ the major increase is observed at $Ar = 8$ for both Nanofluid. For $a = 10^3$, no significant effect of the Ar neither nanoparticle volume fraction was noticed on Nu.

In fact, the presence of multiple thermal plumes at high aspect ratio leads to increase the temperature gradient in the thermal boundary layer. However, the presence of the nanoparticle leads to decrease the Prandtl number (**Figure 2**). Therefore, for $Ra = 10^5$, the thermo-convective transport contribution on enhancing the heat transfer is more pronounced at high Ar as the nanoparticle volume fraction increased.

For the range of $Ra = [10^4, 10^5]$, an increase in nanofluid Nusselt number occurs when increasing φ and Ar comparing to that obtained in water. In this range $Nu_{nf}/Nu_f > 1$. A maximum enhancement is noticed for $Ar = 8$ for both $Ra = 10^4$ and 10^5 and both Cu-water and CuO-water nanofluids.

It is also observed that, the variation of heat transfer enhancement E defined as:

$$E = \frac{Nu_{nf}}{Nu_f} - 1 \tag{14}$$

with nanoparticle volume fraction is more sensitive to Ra number than that to Ar. These results are in contrast with the results added by Abu Nada [21] where the set of the boundary condition and the thermophysical models are not the same in this work.

The effects of aspect ratio and volume fraction on heat transfer enhancement (or weakening) are portrayed on **Figure 6** and **Figure 7.** It is shown that for the cases of $Ra = 10^4$ except for $Ar = 8$, a decrease in Nusselt number ratio (E) occurs for an increase in volume fraction of nanoparticles. It is also worth mentioning that the Cu-water nanofluid experiences more deterioration in Nusselt number when compared to CuO-water nanofluid Nusselt values. On the other hand, for CuO-water nanofluid, the thermal conductivity is lower than that for Cu-water nanofluid. Hence, by adding more nanoparticles, the conduction is primarily enhanced due to the high thermal conductivity of nanoparticles, and consequently the Nusselt number is deteriorated according to equation **(12)**.

For large aspect ratio and higher Rayleigh number where the convection is dominant, an increase in nanofluid Nusselt ratio Nu_{nf}/Nu_f is obtained as the nanoparticle volume fraction is increased. The enhancement rate is primary affected by increasing Ra than that by increasing Ar. For $Ra = 10^4\ and\ 10^5$, smaller enhancement in Nu_{nf}/Nu_f is obtained as the Ar increases for CuO-water nanofluid compared to that obtained in the case of Cu-water nanofluid.

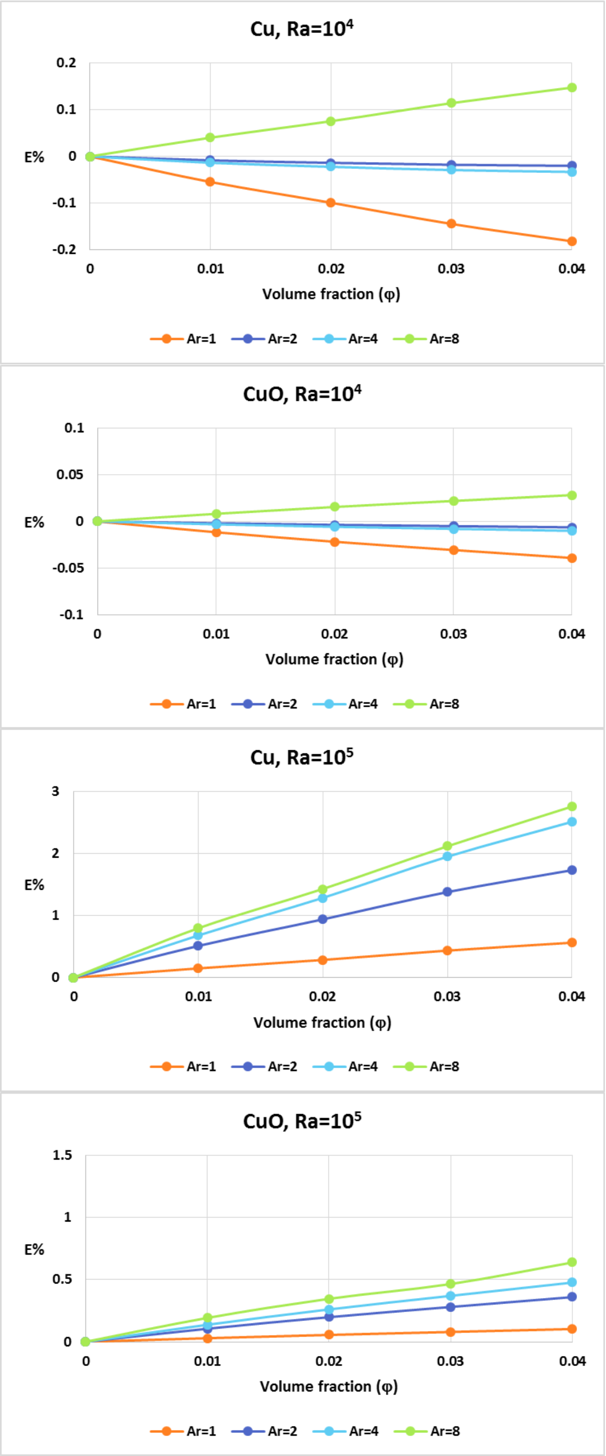

Figure 6 Heat transfer enhancement function of nanoparticle volume fraction

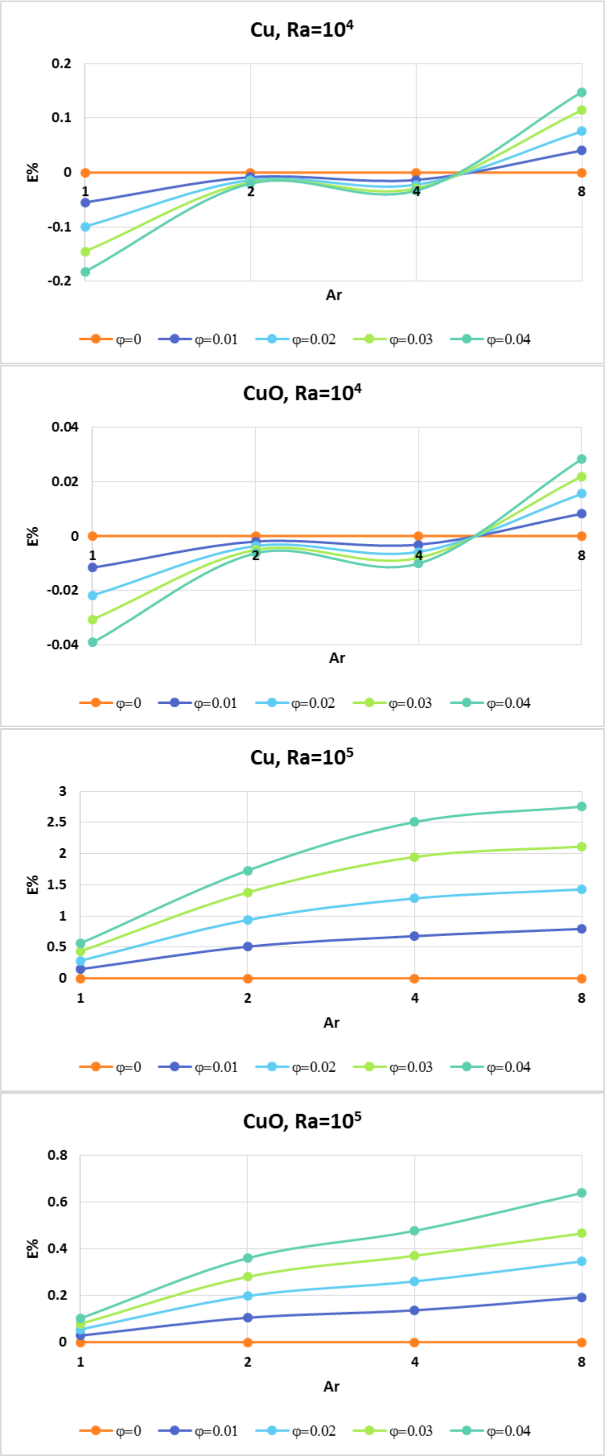

Figure 7 Heat transfer enhancement function of nanoparticle aspect ratio

CONCLUSION

In this paper, we investigate natural convective instability and heat transfer characteristics of water-based Cu/CuO nanofluids in rectangular cavity heated from below. The effects of cavity aspect ratios and the nanoparticles nature are numerically presented. Brownian motion was taken into account for calculating the effective thermophysical properties of nanofluids.

The total heat transfer rate in a rectangular cavity increases with increasing Rayleigh number and aspect ratio, though, the rate of increase depends on the nanoparticle material and nanoparticle volume fraction. For the same Rayleigh number, the flow stability is improved with the presence of nanoparticle in base fluid by improving a symmetrical behavior in flow patterns and thermal distribution.

In nanofluids, tThe heat transfer enhancement increases with increasing the nanoparticle volume fraction up to an optimal combination of Rayleigh number, Prandtl number and the aspect ratio of the cavity. The increase or decrease in the nanofluid Nusselt as the nanoparticle volume fraction increased, is more sensitive to the nanofluids nature and that enhancement is increased or decreased for large or small aspect ratio of the cavity respectively.

REFERENCES

[1] Choi S.U.S., Eastman J.A., “Enhancing thermal conductivity of fluids with nanoparticles,” 1995. In: Siginer, D.A. and Wang, H.P. Ed. “Developments and Application of Non-Newtonian Flows,” ASME, New York, FED-Vol. 231/MD- vol. 66, pp. 99–103.

[2] Lee S., Choi S.U.S., Li S., Eastman J.A., “Measuring thermal conductivity of fluids containing oxide nanoparticles,” J. Heat Transfer, ASME 121, pp. 280-289, 1999.

[3] Eastman J. A., Choi S.U.S., Li S., Yu W., Thomson L.J., “Anomalously increased effective thermal conductivities of ethylene glycol-based nanofluids containing copper nanoparticles,” App. Physics Letter, vol. 78, pp. 718-720, 2001.

[4] Li C.H., Peterson G.P., “Experimental investigation and volume fraction variations on the effective thermal conductivity of nanoparticle suspensions (nanofluids),” J. App. Phys., 99 (8), 084314, 2006.

[5] Buongiorno J., et al. “A benchmark study on the thermal conductivity of nanofluids,” J. Appl. Phys. 106, 094312, 2009.

[6] Das S.K., Putra N., Thiesen P., Roetzel W., “Temperature dependence of thermal conductivity enhancement for nanofluids,” J. Heat Transfer, Vol. 125, pp. 567-574, 2003

[7] Wang X.Q., Mujumdar A.S., “Heat transfer characteristics of nanofluids: a review,” Int. J. Therm. Sci. 46, pp. 1–19, 2007.

[8] Kakaç S., Pramuanjaroenkij A., “Review of convective heat transfer enhancement with nanofluids,” Int. J. Heat and Mass Transfer, Vol. 52, pp. 3187–3196, 2009.

[9] Xuan Y., Roetzel W., “Conceptions for heat transfer correlation of nanofluids,” Int. J. Heat and Mass Transfer, Vol. 43, pp. 3701-3707, 2007.

[10] Polidori G., Fohanno S., Nguyen C.T., A note on heat transfer modeling of Newtonian nanofluids in laminar free convection, Int. Jornal of thermal sciences, vol. 46, pp. 739-744, 2006.

[11] Khanafer K., Vafai K., Lightstone M., “Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids,” Int. J. Heat and Mass Transfer, vol. 46, pp. 3639-3653, 2003.

[12] Santra K., Sen S., Chakraborty N., “Study of heat transfer augmentation in a differentially heated square cavity using copper water nanofluid,” Int. J. Thermal Science, vol. 47, pp. 1113-1122, 2008.

[13] Ho C.J., Chen M.W., Li Z.W., Numerical simulation of natural convection of nanofluid in a square enclosure: Effects due to uncertainties of viscosity and thermal conductivity” Int. J. Heat and Mass Transfer, vol. 51, pp. 4506–4516, 2008.

[14] Putra N., Roetzel W., Das S.K., “Natural convection of nanofluids,” Heat Mass Transfer, vol. 39, pp. 775–784, 2003.

[15] Corcione M., “Heat transfer features of buoyancy-driven nanofluids inside rectangular enclosures differentially heated at the sidewalls,” Int. J. Thermal Sciences, vol. 49, pp. 1536-1546, 2010.

[16] Popa C.V., Fohano S., Nguyen C.T., Polidori G., On Heat transfer in external natural convection using two nanofluids, Int. Journal of thermal sciences, vol.49, pp. 901-908, 2010.

[17] Popa C.V., Kasaeian A.B., Nasiri Sh., Korichi A., Polidori G., Natural convection heat end mass transfer modeling for Cu/water and CuO/water nanofluids, Advances in mechanical engineering, vol. 2013, Article ID 863935, 7 pages.

[18] Hwang K.S., Lee Ji-H., Jang S. P., Hwang K. S., Lee J. H., Jang S.P., “Buoyancy-driven heat transfer of water-based Al_2O_3 nanofluids in a rectangular cavity,” Int. J. Heat Mass Transfer, vol. 50, pp. 4003–4010, 2007.

[19] Corcione M., “Rayleigh-Bénard convection heat transfer in nanoparticle suspensions,” Int. J. Heat Fluid Flow, 2010

[20] Elhajjar B., Bachir G., Mojtabi A., Fakih C., Charrier-Mojtabi M. C., “Modeling of Rayleigh–Bénard natural convection heat transfer in nanofluids,” C. R. Mécanique, vol. 338, pp. 350–354, 2010.

[21] Abu-Nada E., “Rayleigh-Bénard convection in nanofluids: Effect of temperature dependent properties,” Int. J. Heat Fluid Flow, vol. 30, pp. 669–678, 2009.

[22] Abouali O., Ahmadi G., “Computer simulations of natural convection of single phase nanofluids in simple enclosures: A critical review,” Applied Thermal Engineering, vol. 36, pp. 1-13, 2012.