# MODELLING OF FIBROUS MATERIALS USED IN HIGH TEMPERATURE APPLICATIONS

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## ABSTRACT

The purpose of this work is to develop two different ways to investigate the parameters of the fibrous insulation materials from the point of ability of blocking steady-state radiative heat transfer separately and compare them with each other in terms of advantages and disadvantages. In-house developed code is generated in MATLAB®, based on Monte Carlo Ray Tracing (MCRT) technique and ANSYS Fluent® is used as commercial code. Fiber diameter, number of fibers in per row and Solid Volume Fraction (SVF) are used as input parameters of the fibrous insulation material and fibers are assumed as opaque fibers. Fiber diameters are arranged relatively large compared to the wavelength of the radiation to enable to use the rules of geometric scattering. It has been found that radiative heat flux can be decreased by increasing SVF and thickness of the insulation. It is also found that although commercial code requires shorter solution and setup time, our in-house developed code can provide more flexible investigation. The results obtained from this study are useful for designing fibrous insulation materials.

## INTRODUCTION

Heat transfer in fibrous insulation is a substantial topic due to its wide usage in residential and industrial applications. Conduction, convection, radiation or any combination of these three mechanisms can cause heat transfer. However, due to the suppression of air motion between the fibers, convective heat transfer can be neglected [1]. Radiative heat transfer is the main mechanism in fibrous materials used in high temperature applications. It is important to model radiative heat transfer between fibers in fibrous materials to improve the design and obtain effective insulation materials. Fiber diameter and temperature are the two most important parameters to model radiative heat transfer between fibers.

Radiative heat transfer depends on also parameters such as fiber orientation, and optical specifications of fibers. In our study, those parameters are investigated separately, by using an in-house developed 2D Monte Carlo Ray Tracing Method developed via MATLAB, and a commercial program ANSYS Fluent results.

## NOMENCLATURE

| | | |
|---|---|---|
| $F$ | [-] | View factor from surface to surface |
| $T$ | [K] | Temperature |
| $R$ | [-] | Random number |
| $P(x)$ | [-] | Probability distribution function |
| $D$ | [-] | Dimension |
| $d$ | [μm] | Mean fiber diameter |
| SVF | [-] | Solid volume fraction |
| MCRT | [-] | Monte Carlo Ray Tracing |

**Special characters**

| | | |
|---|---|---|
| $\Psi$ | [rad] | Size parameter |
| $\Theta$ | [rad] | Polar angle |
| $X$ | [-] | Size parameter |
| $\alpha'$ | [rad] | Angle of a ray in 2-D |
| $\alpha$ | [-] | Absorption coefficient |
| $\lambda$ | [μm] | Wavelength of incoming radiation |

**Subscripts**

| | |
|---|---|
| $i$ | ith fiber |
| $j$-$k$ | From fiber j to fiber k |
| $jsource$ | From fiber j to source plate |
| $j$-$sink$ | From fiber j to sink plate |
| $h$ | Hot plate |
| c | Cold plate |
| min | Minimum |
| max | Maximum |

The Monte Carlo method is one of the numerical techniques which has the statistical characteristics in nature. Using the Monte Carlo method in radiative heat transfer is reviewed by Howell [2]. Monte Carlo Ray Tracing (MCRT) method is a solution method which has exhibited general success [3-5]. The general procedure in the MCRT method is as the following: large number of energy bundles emitting from randomly selected locations with selected angles are being traced to other points. These points collide with a boundary or a fiber surface. After the reflections, bundles follow new paths and this tracing continues until the bundles reach the sink or source.

## MONTE CARLO METHOD IN RADIATION HEAT TRANSFER

Monte Carlo Method is used in radiation heat transfer with principle of tracing of a ray.
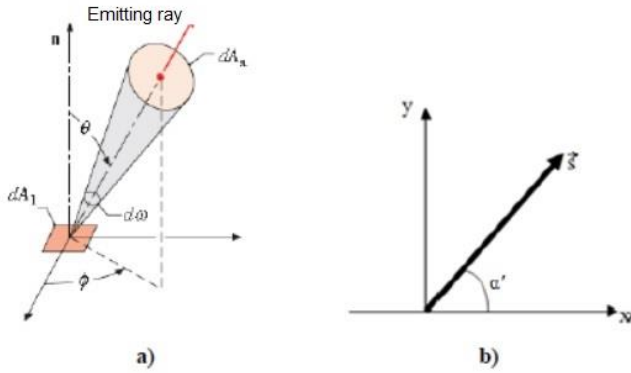


**Figure 1** Scattering point and direction of a ray

Scattering point and direction of a ray scattering from a surface is specified using two coordinates and two angles (polar $\theta$ and azimuthal $\psi$).

$$0 < \theta < \pi/2 \tag{1}$$
$$0 < \psi < 2\pi \tag{2}$$

On the other hand, scattering point and direction of a ray scattering from a volume i.e. fiber surface;

$$0 < \theta < \pi \tag{3}$$
$$0 < \psi < 2\pi \tag{4}$$

Firstly, probability distribution of a surface which emitting radiative rays is found as shown in below.

$$R(\theta) = \frac{2\pi \int_0^\theta i(\theta^*)\sin\theta^* d\theta^*}{2\pi \int_0^{\pi/2} i(\theta^*)\sin\theta^* d\theta^*} \tag{5}$$

R represents the probability of emission at any wavelength in the direction interval between 0 and $\theta$. Its value is between 0 and 1, depending on the value of $\theta$. The strategy for determining the polar angle $\theta$ for emission is to generate a uniformly distributed random number R between 0 and 1, and then set it equal to R and solve for $\theta$.

$$\theta = \sin^{-1}\sqrt{R(\theta)} \tag{6}$$

Probability distribution in azimuthal direction can be found similarly as shown in below.

$$R(\psi) = \frac{\int_0^\psi \int_0^{\pi/2} i(\psi^*)\sin\theta^* d\theta^* d\psi^*}{\int_0^{2\pi} \int_0^{\pi/2} i(\psi^*)\sin\theta^* d\theta^* d\psi^*} \tag{7}$$
$$\Psi = 2.\Pi.R(\Psi) \tag{8}$$

For isotropic radiation from volume element

$$R(\psi) = \frac{\int_0^\psi \int_0^\pi i(\psi^*)\sin\theta^* d\theta^* d\psi^*}{\int_0^{2\pi} \int_0^\pi i(\psi^*)\sin\theta^* d\theta^* d\psi^*} \tag{9}$$
$$\theta = \cos^{-1}(1 - 2R(\theta)) \tag{10}$$

Scattering point is specified by using random numbers:

$$R_x = \frac{\int_{x_{min}}^x P(x^*)dx^*}{\int_{x_{min}}^{x_{max}} P(x^*)dx^*} \tag{11}$$

$$x_i = R_x(x_{max} - x_{min}) + x_{min} \tag{12}$$

To obtain emitting point in y-direction, same treatment should be applied:

$$y_i = R_y(y_{max} - y_{min}) + y_{min} \tag{13}$$

where $x_{max}$, $y_{max}$, $x_{min}$, $y_{min}$ are boundary of scattering surfaces and $R_x$, $R_y$ are random numbers between 0 and 1.

According to Arambakam et al [3] 2-D and 3-D simulations give similar results and in 2-D configurations there is no impact on results.

Finally, angle of slope which is the only angle to trace a ray in 2D configuration, can be found by using polar and azimuthal angles, by the help of Eq. 12.

$$\alpha' = tan^{-1}(tan\theta.cos\Psi) \tag{14}$$

## SIMULATION DOMAIN AND RAY TRACING

Establishing the simulation domain which represents the microstructure of the fibrous insulation in 2-D is the starting point. As can be seen in Figure 2, fibers are placed between two plates. One of these plates is hot plate (source) and other one is cold plate (sink).
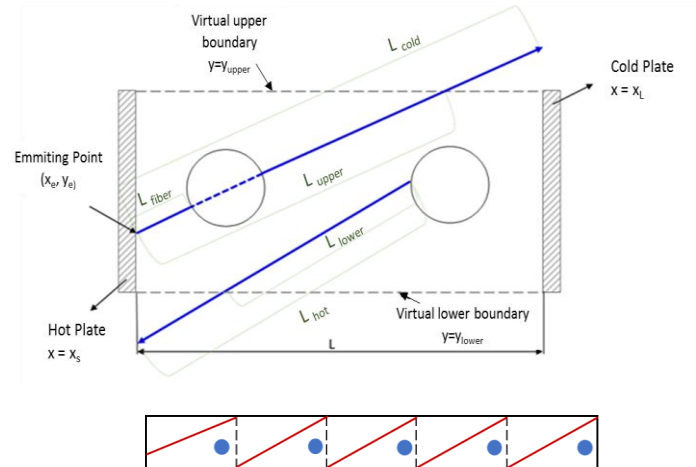


**Figure 2 a)** Possible surfaces that the ray could hit **b)** depiction of using periodic boundary condition as virtual boundary condition

There are five surfaces that the ray scattering from a surface could hit: virtual upper boundary, virtual lower boundary, cold

plate, hot plate and fiber surfaces. Periodic boundary condition is used for virtual upper and lower boundaries which is depicted in Figure 2b.

MATLAB® code developed to simulate radiation on the domain depicted above uses fiber diameter, Solid Volume Fraction (SVF), absorptivity of fibers, and number of the fibers in one row as input parameters. By means of this code, effect of the input parameters on the insulation material performance to blocking heat transfer can be investigated separately. Thus, new ideas to develop better insulation materials can be proposed.

Fiber diameter and temperature of fiber which are the two main parameters for modelling of radiation heat transfer in fibrous materials are used to specify another parameter, called "size parameter":

$$\chi = \frac{\pi d_f}{\lambda} \tag{15}$$

where $d_f$ and $\lambda$ are mean diameter of fiber diameter in fibrous material and wave length of ray respectively. "Size parameter" is used to determine mode of scattering of a ray after it interacts with fibrous material. Table 1 gives these modelling techniques:

**Table 1** Scattering techniques

| Size parameter | Modelling | Assumption |
|---|---|---|
| $\chi > 10$ | Geometric Scattering | Ray wave length rank is smaller than fiber diameter. |
| $0.1 < \chi < 10$ | Mie Scattering | Ray wave length has the same rank with fiber diameter. |
| $\chi < 0.1$ | Rayleigh Scattering | Ray wave length rank is bigger than fiber diameter. |

In geometric scattering region, when an energy bundle collides with a fiber, it is reflected as if reflected from a mirror [6].

We consider only geometric scattering in this paper and assume source and sink temperatures as 1200 K and 300 K, respectively. Both of source and sink are considered as perfectly emitters and absorbers (black body surface). Fiber diameter is assumed as 30 μm in order to use geometric scattering (Eq. 15).

Our in-house developed code calculates also steady-state energy transmittance across the simulation domain, in addition to fiber-to-source, fiber-to-sink and fiber-to-fiber view factors. When a ray emitted from the source, our in-house developed code calculates the next point collided with a boundary or fiber. In view factor calculations when the ray collided with a fiber or a boundary, ray tracing is terminated and a new ray is emitted. However, in energy transmittance calculation, the ray is traced until to collide with a boundary (sink or source). These

algorithms are explained in detail, for view factor calculations and energy transmittance in Figure 3, 4, respectively.

Steady-state fiber temperatures are obtained by solving the conservation of energy equation after the view factors are calculated. For a fiber subscripted i

$$\sum_{j=1}^{n} F_{i,j}\left(T_i^4 - T_j^4\right) + F_{i,h}\left(T_i^4 - T_j^4\right) + F_{i,c}\left(T_i^4 - T_c^4\right) = 0 \tag{16}$$

where $T_h$, $T_c$, and $T_i$ stand for temperature of the hot plate, cold plate, and the fibers, respectively. If conservation of energy equation is written for each fiber, $n$ equations for $n$ unknowns (fiber temperatures) are calculated [3].
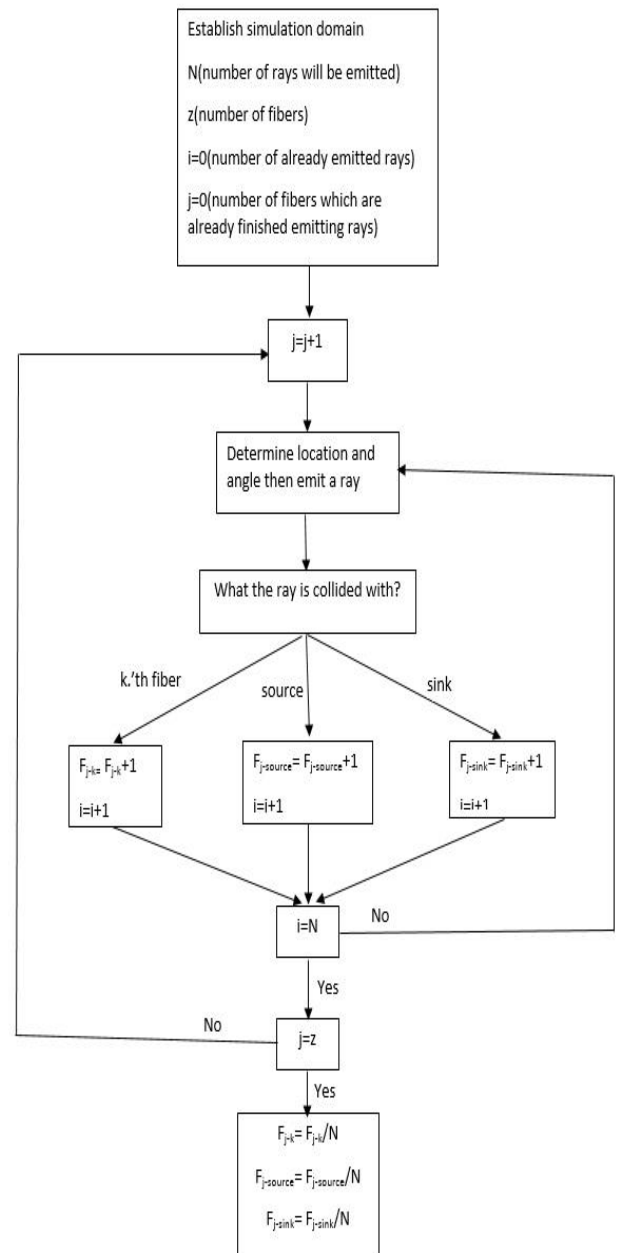


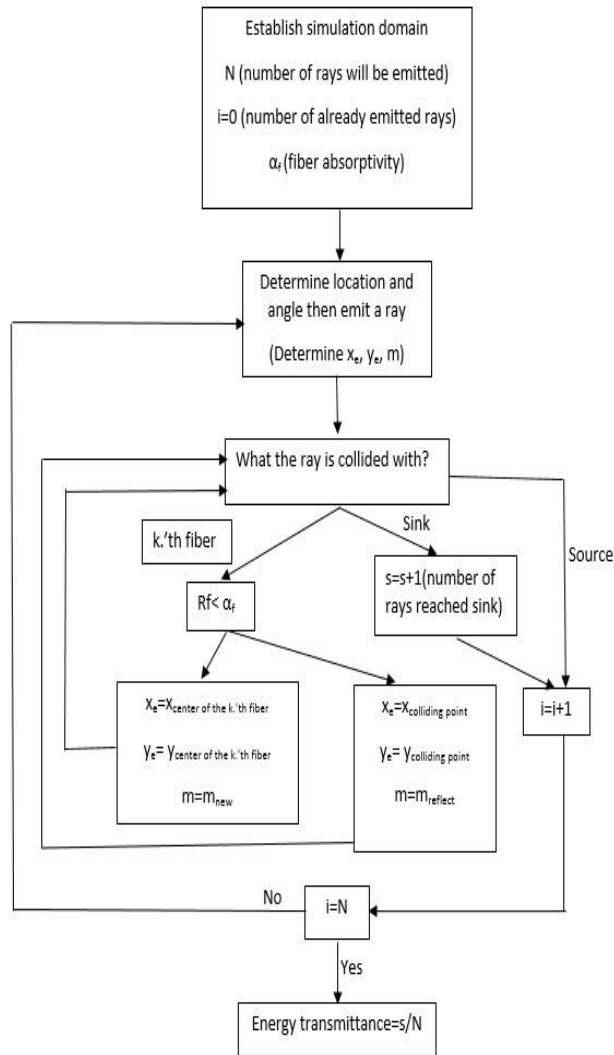**Figure 3** Determination of view factors algorithm

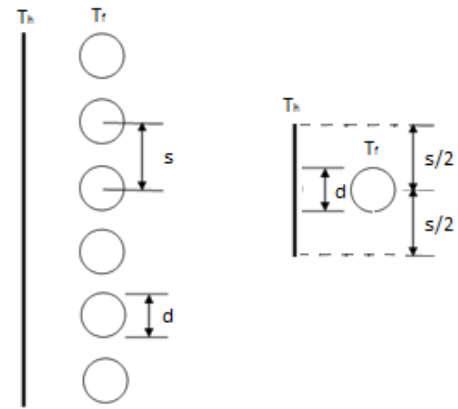**Figure 4** Determination of energy transmittance algorithm



**Figure 5** Validation geometry

## CFD ANALYSIS

Simulation domain and boundary conditions are shown in Figure 6. Source and sink surface temperatures are 1200 K and 300 K, respectively and they are considered as perfect emitters and absorbers (black body surface). Fiber diameter is assumed as 30 μm. Surface to surface (S2S) radiation model is used which ignore any absorption, emission or scattering of radiation by the medium and considers only "surface-to-surface" radiation [7]. In order to consider only radiation heat transfer mode between fibers, vacuum condition is built. To imply this, extreme thermal conductivity and specific heat values are used for air properties.
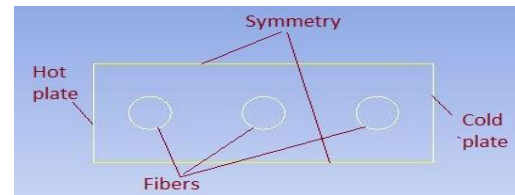


**Figure 6** Simulation domain

Fibers are placed in simulation domain according to three different SVF cases 5 %, 10% and 15 %.

## RESULTS

Temperature distribution along the thickness is given in Figure 7. It can be seen from the figure that with increasing SVF, temperature decreases more rapidly. Results of our in-house developed code are compared to the ones of the commercial program. It can be said that with increasing SVF, our in-house developed code agrees better with commercial program. The difference in low SVF between two results might be due to existence of air between fibers in the commercial program even if extreme values of air properties have been implied. As a conclusion, it can be said that to obtain fiber temperatures, commercial code gives better solution from the viewpoints of easiness to establish simulation domain and solution time. Solution time of in-house developed code is approximately takes 10 minutes, whereas it takes a few seconds with commercial code.
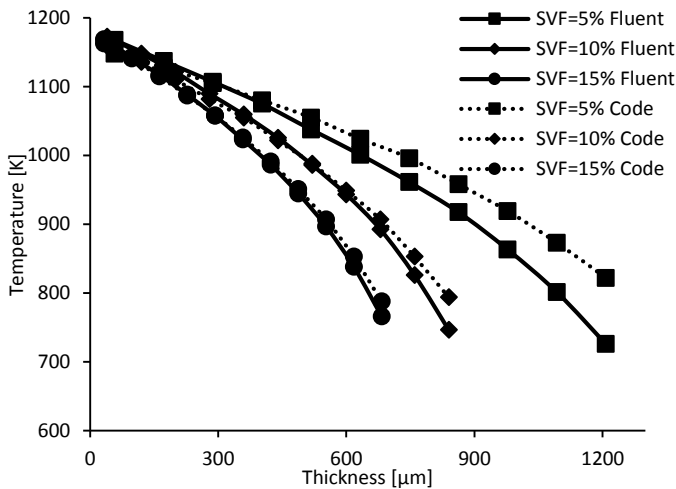
## VALIDATION

We compared our view factor calculations with a case which can be solved analytically to see how accurate our results were.

View factor between the surfaces given in Figure 5a is calculated analytically according to:

$$F_{h,f} = \left(\frac{d}{s}\right) . cos^{-1}\left(\frac{d}{s}\right) + 1 - (1 - (d/s)^2)^{0,5} \qquad (17)$$

Where fiber diameter d=10 μm and s=20 μm. From the above equation view factor from hot plate to fiber calculated as $F_{h,f} = 0{,}6576$ whereas our program found as $F_{h,f} = 0{,}6578$. As can be seen in Figure 5b one fiber and periodic boundary condition are used in calculation.

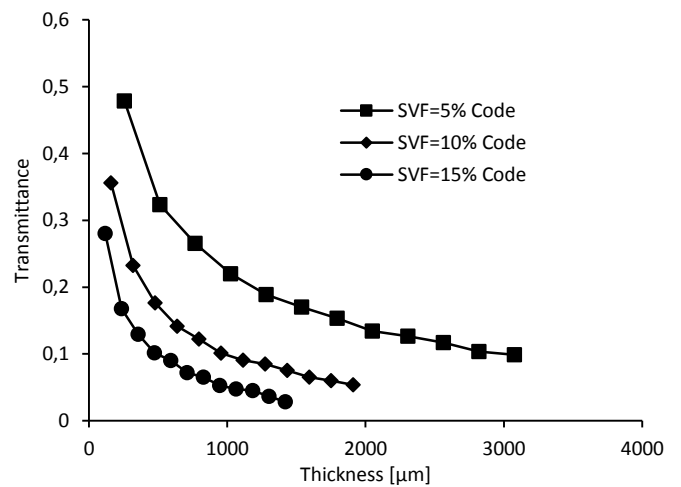**Figure 7** Fiber temperatures versus thickness



**Figure 8b** Transmittance versus thickness for various SVF

Figure 8a compare the steady-state energy transmittance values which are obtained from our in-house developed code and commercial program for different SVF and thickness values. It can be seen from the Figure 8a, our in-house developed code and commercial program give similar results. It is easy to say that transmittance is decreasing with increasing thickness. Results of simulation of energy transmittance for 10% SVF are compared to the ones of the similar study by Arambakam et al [3] in literature. Figure 8a shows that our in-house developed code simulates energy transmittance across the media more accurate than the commercial code. In Figure 8b, relation between steady state energy transmittance across the media and increasing SVF and thickness is indicated. It can be understood from the figure that increasing SVF causes decreasing energy transmittance like increasing thickness. The reason why increasing SVF or thickness results in decreasing energy transmittance can be explained as that when increasing the thickness or SVF, probability to encounter a ray with fiber resulting in to turn it towards the sink increases. As a conclusion, our in-house developed code and commercial program give similar results and represent the nature of the energy transmittance in relation to SVF and thickness. But in our in-house developed code changing the simulation domain with respect to thickness or SVF is easier than the commercial program.

Figure 9a indicates the dependence of the steady-state energy transmittance on fiber diameter with respect to SVF. It is easily concluded that energy transmittance is independent from the fiber diameter and decreasing with increasing SVF as said previously. Using commercial code fiber averaged temperature versus fiber diameter is obtained for different SVF in Figure 9b. From both of the figures, it is concluded that changing fiber diameter has no impact on temperature and transmittance. Note that our in-house developed code provides advantage to change in simulation domain in terms of changing fiber diameter and SVF.
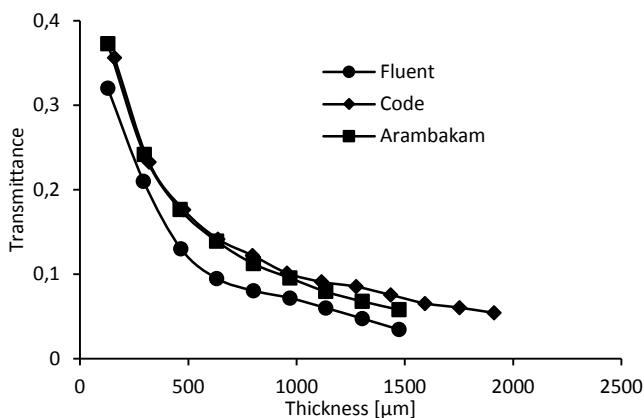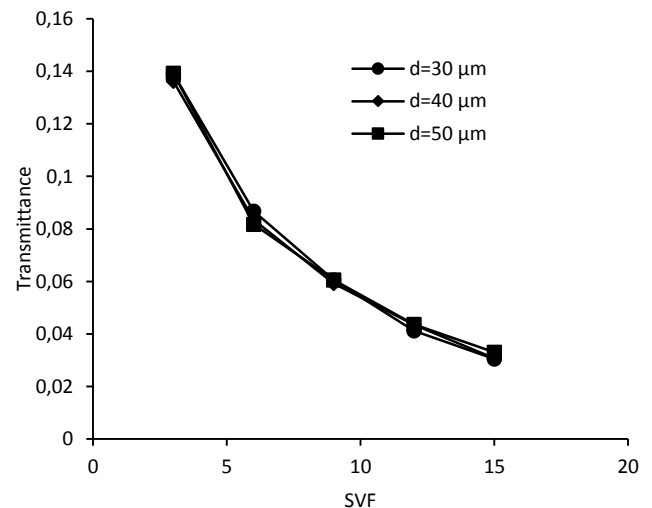


**Figure 9a** Transmittance versus SVF for various fiber diameters



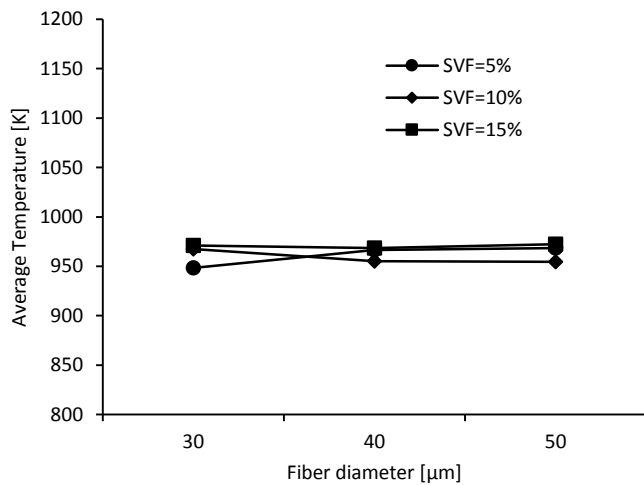**Figure 8a** Transmittance versus thickness for 10% SVF

**Figure 9b** Average fiber temperatures versus fiber diameter for various SVF

## CONCLUSION

The purpose of this work was to investigate the parameters of the insulation materials which are Solid Volume Fraction (SVF), fiber diameter and number of fibers in per row to block the heat transfer between surfaces at different temperatures. This is done by two ways: our in-house developed code and commercial code. Our in-house developed code provides wider investigation perspective in terms of the parameters of the insulation materials and easier to change simulation domain. On the other hand, commercial code gives the opportunity of shorter solution time and easier to establish simulation domain.

## ACKNOWLEDGEMENT

## REFERENCES

[1] Arambakam R., Modeling effect of microstructure on the performance of fibrous insulation, *PhD Thesis, Virginia Commonwealth University*

[2] Howell J.R., The Monte Carlo method in radiative heat, *Journal of Heat Transfer*, Vol. 120, 1998, pp. 547-560

[3] Arambakam R.,Hosseini S.A., Vahedi Tafreshi H., Pourdeyhimi B., A Monte Carlo simulation of radiative heat through fibrous media: Effects of boundary conditions and microstructural parameters, *International Journal of Thermal Sciences*, Vol. 50, 2011, pp. 935-941

[4] Kalos, M. H., Whitlock P. A., *Monte Carlo Methods Volume I: Busics.* 1986, New York: Wiley.

[5] Haji-Shiekh A., Monte Carlo methods in *Handbook of Numerical Transfer,* 1988, *pp.* 673-717, New York: Wiley.

[6] Nisipeanu E., Jones P.D., Monte Carlo simulation of radiative heat transfer in coarse fibrous media, ASME Journal of Heat Transfer, Vol. 125, 2003, pp. 748-752

[7] ANSYS Fluent User's Guide, *ANSYS Inc. Southpointe 275 Technology Drive Canonsburg, PA 15317,* Release 15.0, November 2013