LES ANALYSIS ON CYLINDER CASCADE FLOW BASED ON ENERGY RATIO COEFFICIENT

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ABSTRACT

The flow field around the cylinder cascade is widely used to analyze the interaction of vortex shedding and the information on heat transfer. Large eddy simulation (LES) can be used to get the turbulent flow information in detail. The resolved large-scale structures are determined by the size of the grid, and the turbulent vortex dissipation is modeled with a subgrid scale model. Whereas there is no accurate criterion to provide the subgrid scale with the physical meaning. Based on turbulent energy ratio coefficient and numerical simulation results with turbulent model, the subgrid was generated for the incompressible fluid flowing around a column of cylinder cascade with a gap-to-diameter ratio of 2. Smagorinsky-Lily (SM) model was applied to LES analysis. The turbulent flow information was compared with the experimental data by PIV. Two cases with different Reynolds numbers were studied. When the turbulent energy ratio coefficient reached to 30%-40%, the turbulent dissipation could be captured by LES method with less grid number. The large scale vortex interaction behind the cylinder cascade was analyzed further. It is verified that LES method can be used for engineering based on the turbulent energy ratio coefficient with acceptable computational cost.

INTRODUCTION

Circular cylinder arrays are widely used in heat exchanger by the interaction of the vortex shedding. Short circular cylinder arrays, the span width is comparable to the cylinder diameter, are usually used in the confined spaces, such as the blade cooling system in the gas turbine. The cylinder span width is limited by the blade width in high pressure stages. The vortex shedding behind a cylinder is one of the classic problems in fluid dynamics. The large eddy simulation (LES) method is one of the options to study the flow recently, as there are the interactions of flow separation and the coherent structures in the turbulent flow.

Breuer[1] found that the low-order upwind scheme could not obtain the recirculation zone and the separated angle, the high-order upwind scheme had strong numerical dissipation, but the second-order central-difference scheme was with high precision. He also suggested that the subgrid models had no significant effect on the numerical results. It might be the reason that Smagorinsky-Lily (SM) mode is used universally with the simple form and low computational cost. When there are more than one cylinder in the flow field, the flow structures vary with the gap-to-diameter ratio (S_T/D) and Reynolds number ($Re=U_{in}D/\nu$). In the turbulent flow field with two circular cylinders arranged side by side, the flow structure is decided by S_T/D. There is only one pair of vortex shedding behind the cylinders in the condition $S_T/D<1.2$. It could be regarded as the flow field with single circular cylinder. Xu[2] found that the boundary layer between cylinders would not separate because of the wall effect, the boundary layer without the impact of the other cylinder would shed from the solid wall. The vortex shedding frequency was about half of that of the single cylinder. The gap flow swung between the two cylinders, forming a narrow and a broad vortex in the condition $1.2<S_T/D<2.0$. Sumner[3] pointed out that the flow deviation angle and the vortex shedding frequency between two cylinders reduced accordingly with the increment of S_T/D. Brun[4] showed that the frequency of the alternating deflection was accelerated with the increase of Reynolds number. Two pairs of parallel vortex street would be observed with the same vortex shedding frequency in the condition $2.0<S_T/D<6.0$. Sumner[5] showed that the two vortex streets are not independent and the vortex shedding had time synchronization, vortex shedding was in the same direction reverse state in space. Two same phase vortex streets would merge into one vortex street spreading to the downstream eventually, and contrary-phase vortex street remained to develop in time history. The flow field with more cylinders side by side, named as cylinder cascade here, is the common structure in engineering. There are more complicated interaction of the vortex mixing. Therefore, it is necessary to study it further by LES method to get enough turbulence information.

According to the basic idea of LES, turbulent flow structures can be separated into the resolved scales and the subgrid scales. The resolved large-scale structures are determined by the size of the grid, and the turbulent vortex dissipation is modelled with a subgrid scale model. In order to achieve the accurate numerical simulation, proper subgrid model and the corresponding subgrid scale should be set up. Commonly, the subgrid scales in different turbulent flows were determined by the research experiences. Zhang [6] suggested that the subgrid scale could be determined by the turbulent energy ratio from numerical results with turbulent model. Considering about the turbulent energy ratio as 20%, he got the turbulent structure in the backward step flow and compared it with that by direct numerical simulation (DNS). However, the computational cost of LES method is still high. The value of the turbulent energy ratio is also a limitation to extend the application in the complex flows, such as the turbulent flow field with circular cylinder arrays. This paper is an attempt of LES grid refinement based on the turbulent vortex energy scale.

NOMENCLATURE

C	[-]	Coefficient
D	[m]	Cylinder diameter
E	[m^{2}s^2]	Turbulent kinetic energy in spectral space
f	[-]	Viscosity damping function
H	[m]	Cylinder span width
K	[m^{2}s^2]	Turbulent kinetic energy in physical space
k	[-]	Wave number in spectral space, von Karman constant
l	[m]	The length scale of the unresolved motion
S	[s^{-1}]	Velocity strain tensor
U	[m/s]	Velocity
x, y, z	[m]	Cartesian axis direction
Special characters		
α	[-]	Kolmogorov constant
Δ	[m]	Grid scale
ε	[m^{2}s^3]	Turbulence kinetic energy dissipation rate
η	[-]	The ratio of the summation of inertial turbulent kinetic energy and dissipative turbulent kinetic energy in the total turbulent kinetic energy
μ	[Pa.s]	Dynamic viscosity
ν	[m^2/s]	Kinematic viscosity
Subscripts		
d		Demarcation of inertial sub-range and dissipation range
D		Dissipation range
L		Energy containing range
S		Inertial sub-rang
s		Energy ratio coefficient
SGS		Subgrid scale

SUBGRID SCALE ANALYSIS

Energy Ratio Coefficient

According to the turbulent energy spectrum distribution[7], simplified in Figure 1, the local turbulent signal could be divided into large-scale and small-scale fluctuations by the spectrum filtering. With the increment of dimensionless wave number k/k_d, turbulent kinetic energy spectrum $E(k)$ distributes in three ranges, i.e. energy-containing range, inertial sub-range and dissipation range. k_d is the maximum wave number corresponding to dissipation range. The energy spectrum of energy-containing range was fitted based on Pope's work[8] and that of inertial sub-range and dissipation range is defined as[9]

$$E(k)[\varepsilon v^5]^{-1/4} = \alpha \cdot (k/k_d)^{-5/3} \exp\left[-\frac{3}{2}\alpha \cdot (k/k_d)^{4/3}\right] \qquad (1)$$

Where, α is Kolmogorov constant. The turbulent energy spectrum distribution in Figure 1 can be integrated along the wave number. Compared with the total energy, the ratio of the summation of inertial turbulent kinetic energy and dissipative turbulent kinetic energy in the total turbulent kinetic energy is approximately 70%, the ratio of dissipative turbulent kinetic energy is about 20%. In LES method the energy ratio coefficient η_s is in the range,

$$\frac{E_D}{E_L + E_S + E_D} \le \eta_s \le \frac{E_D + E_S}{E_L + E_S + E_D} \qquad (2).$$

Proper local filtering grid scale is in the local inertial sub-range. Therefore, η_s is in the range from 20% to 70%. With the consideration of the computational cost and the accuracy of LES method, the value from 30% to 40% is suggested.

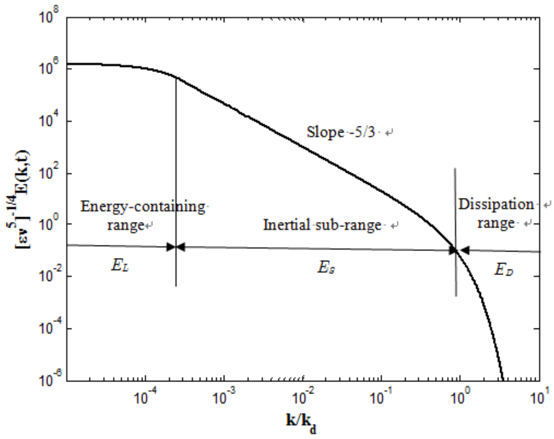


Figure 1 Sketch of turbulent Energy spectrum

Subgrid Scale(SGS)

The turbulent models were studied for century, and could be used to get the turbulent information to some extent.

The Smagorinsky model is an algebraic model for the SGS viscosity. Based on Prandtl mixing length model, the SGS viscosity can be expressed as

$$v_{sgs} = (l_{sgs})^2 \sqrt{2\overline{S}_{ij}\overline{S}_{ij}} \qquad (3)$$

$$l_{sgs} = \min(ky_{wall}, f_\mu C_s \Delta) \qquad (4)$$

Where, l_{sgs} is the mixing length for SGS viscosity. k is von Karman constant, k=0.4. y_{wall} is defined as a function of the calculated wall distance. f_μ is viscosity damping function.

$$f_\mu = 1 - \exp(-y^+ / A^+)$$

Where, A^+=26, y^+ is the dimensionless wall distance, C_s is Smagorinsky constant, 0.1～0.2, Δ is the filtering grid scale. Therefore, the SGS is derived by the turbulent model.

With the consideration of the dimensions of turbulent kinetic energy K and dissipation ratio ε, local subgrid scale Δ_s [m], with length unit, can be obtained from turbulent energy ratio η_s and the numerical results with turbulent model[6], is defined as

$$\Delta_s = \frac{\pi}{(1.5\alpha)^{1.5}} \cdot \eta_s^{1.5} \cdot \frac{K^{1.5}}{\varepsilon} \qquad (5)$$

Where, α is Kolmogorov constant, K , ε are got from unsteady Reynolds averaged Navier-Stokes equations simulation (URANS) results. In the averaged computational grid scale $\overline{\Delta} = \sqrt[3]{\Delta_x \Delta_y \Delta_z}$, the grid ratio coefficient is

$$c_t = \frac{\Delta_s}{\overline{\Delta}} \tag{6}$$

Here, energy ratio coefficient is the ratio of the sum of inertial turbulent kinetic energy and dissipative turbulent kinetic energy in the total turbulent kinetic energy, its value varies with Reynolds number and the actual flow field.

LES MEHOD ON THE CYLINDER CASCADE FLOW

Flow Field Model

The geometric parameters and the computational domain were shown in Figure 2, where the gap-to-diameter ratio (S_T/D) of the circular cylinder cascade was 2. The cylinders were installed in two transparent parallel walls, the span width of the short cylinder was H=D. The flow field behind the cylinder cascade was tested by PIV technique[10]. The middle plane along z direction was the tested plane, the tested window by PIV technique was 0<X/D<2.2, as the shadow area in Figure 2. The flow parameters from LES results, URANS results and PIV data would be compared on the plane of the tested window.

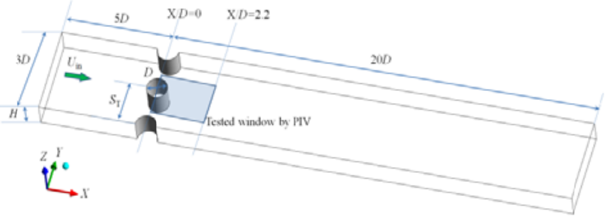


Figure 2 Computational domain

LES was applied in the flow field by ANSYS CFX. In order to get fully developed turbulent flow at the outlet of the flow model, the upstream and downstream of the cylinder were extended respectively in x direction. At the inlet, a constant velocity was set, and the outlet boundary was defined as a constant pressure condition with the pressure set to 1atm. The upper and lower sections in y direction were set as translational periodic conditions, and all solid walls were modelled as no slip and adiabatic.

Numerical simulation on URANS equations with turbulent model was the first step to get the initial c_t. 3D structured grid was generated by ICEM module. Transient $k-\omega$ turbulent model was selected, the dimensionless wall distance y+ was kept lower than 5 and the grid expansion ratio from the wall was 1.2. The total grid number was about 600,000 for the computation model to get the grid independency. The convergence was achieved when the residual level decreased to a given level (10^{-4}). Smagorinsky-Lily (SM) model was selected as the subgrid scale model in LES method.

Flow Structure On Low Reynolds Number Re=3,000

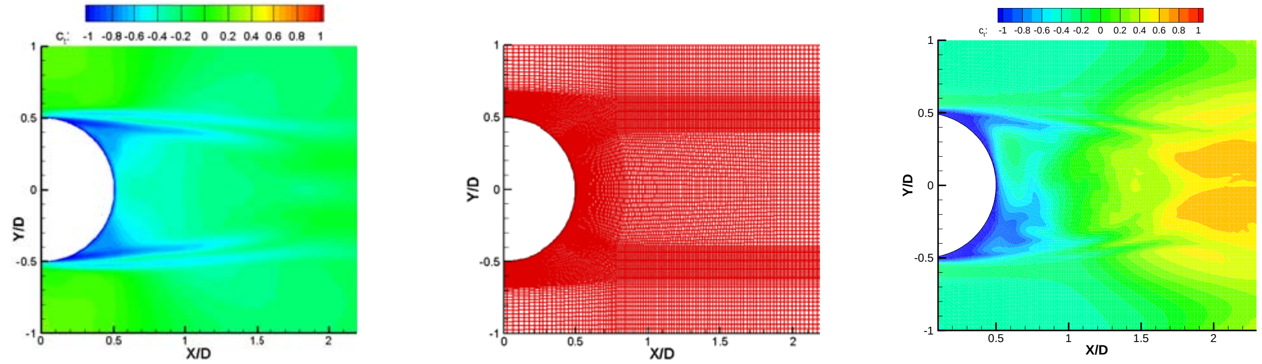
Compared with the experimental conditions, the flow field with Reynolds number 3000 was studied firstly, corresponding to the inlet velocity of 0.373m/s.

The turbulent kinetic energy and dissipation rate could be got directly by the numerical simulation with the turbulent model. The grid ratio coefficient c_t on the tested plane was plotted by 10-logarithm law in Figure 3a. The regions with the value below 0 should be refined. They were near wall behind the cylinder, shear layer and recirculation zone.

The refined LES grid distribution was shown in Figure 3b. The grid distribution along the cylinder axis was non-uniform. Along z direction, $\Delta z^+_{\max} = (U_{in}\Delta z)/\nu$ =60 was located in the middle plane. Based on the energy ratio coefficient of 30%, the total grid number was 2,000,000, while the total grid number would be 8,000,000 according to conventional method. It was obviously that the value of the energy ratio coefficient decided the grid number more directly.

URANS equations were simulated with central-difference scheme, and the time step was 0.0029s. While there was oscillation of vortex shedding behind the cylinders, about 20 periods data (1640 steps) were saved.

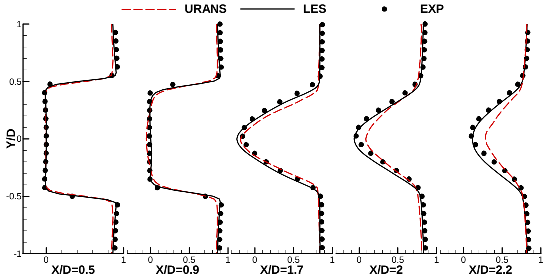
Grid ratio coefficient based on LES result was revealed in Figure 3c. Although the grid was increased a lot near the surface of the cylinder in Figure 3b, the region near the cylinder surface should be refined further to satisfy c_t>1 . The grid scale satisfied the energy ratio coefficient criterion in the cylinder cascade downstream position x/D >1.5.



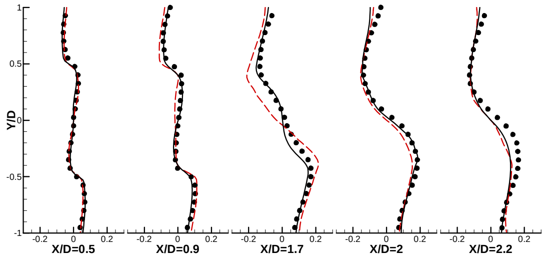
(a) original c_t by turbulent model (b) refined grid for LES (c) c_t by LES

Figure 3 Grid ratio coefficient c_t and refined grid on the tested plane

As for the cylinder cascade flow, the separation characters of fluid would be observed on account of Reynolds number. The shear layer generated from the cylinder narrowest position and the alternating vortex shedding near the upper and lower surfaces. The region of 0<X/D<2 was recirculation zone, its length was as the point along the wake centreline (Y/D=0) where the time averaged x-component velocity was zero[11].

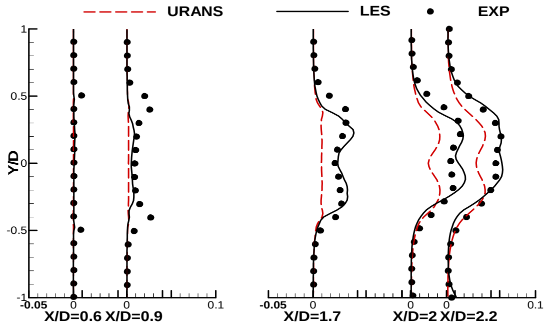


(a) x-component velocity $\overline{u}$

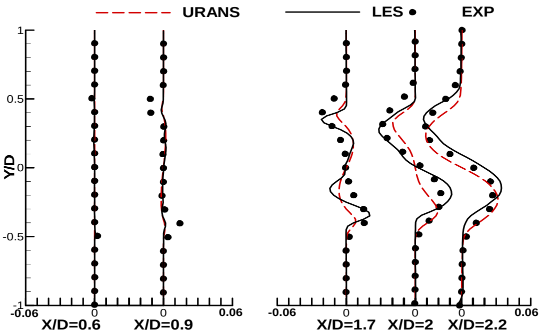


(b) y-component velocity $\overline{v}$

Figure 4 Time averaged velocity on the tested plane



(a) Turbulent kinetic energy $(\overline{u'u'}+\overline{v'v'})/2$



(b) Reynolds stress $\overline{u'v'}$

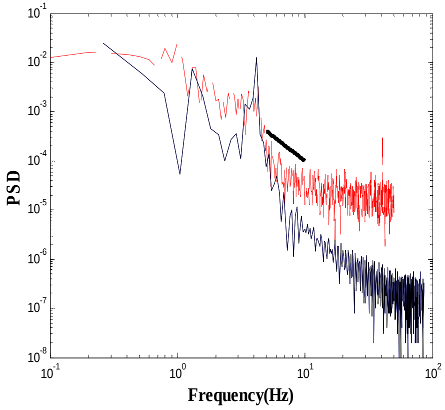
Figure 5 Time averaged turbulent parameter on the tested plane

The time averaged data were compared in Figure 4 and Figure 5 in the tested plane. The data from URANS equations simulation, LES and experiments were listed behind the cylinder cascade at different downstream positions.

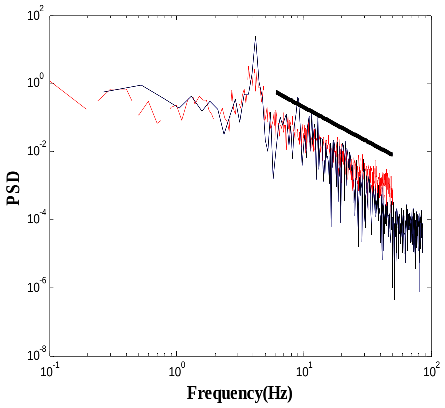
Even the grid was needed to refine, the time averaged velocity components, including URANS results, were good enough to describe the flow behaviour in the position X/D<1, compared with the experimental data. Along x direction, the discrepancies between URANS results and experimental data expanded. LES results were much better than those of URANS.

The time averaged turbulent kinetic energy and Reynolds stress in Figure 5 provided more discrepancies of URANS results and experimental data. Based on the 2D experimental data in the tested plane, the numerical simulation results were transferred to 2D data here. The turbulent information was averaged in the region near the cylinder wall, nearly no variation could be found by URANS results. Owing to the grid scale in the region, LES results just showed a little variation. Along x direction, LES results were gradually close to the experimental data. LES results of turbulent flow information were much better than those of URANS. The time averaged turbulent kinetic energy and Reynolds stress could be captured by LES method with $\eta_s = 30\%$ and $c_t>1$.

It proved that the LES method would replace URANS method to get more accurate turbulent flow information.



(a) u' at X/D=0.6, Y/D=0



(b) u' at X/D=2, Y/D=0

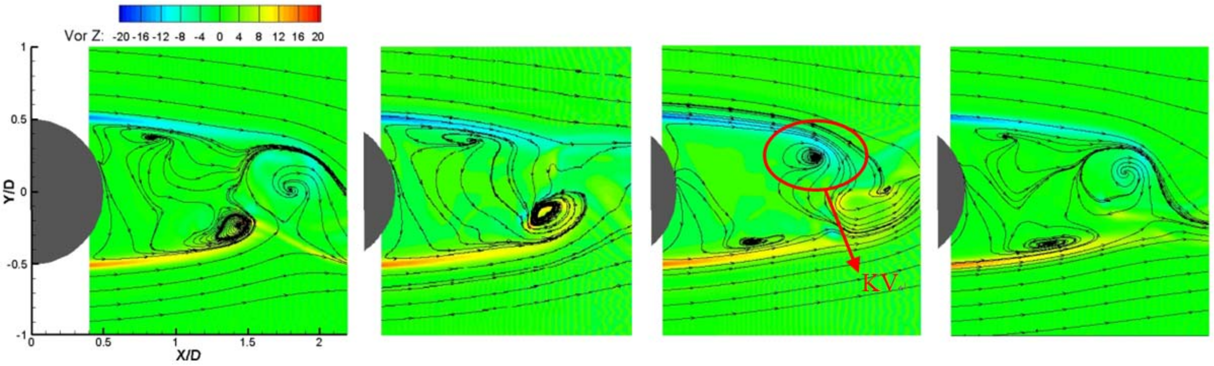
Figure 6 Power Spectrum of Monitor points

（Black line: LES result. Red line: experimental data)

In addition, the time dependent information at the centreline (Y/D = 0) was studied. The u' spectrum, by experiment and LES method, was plotted in Figure 6 respectively.

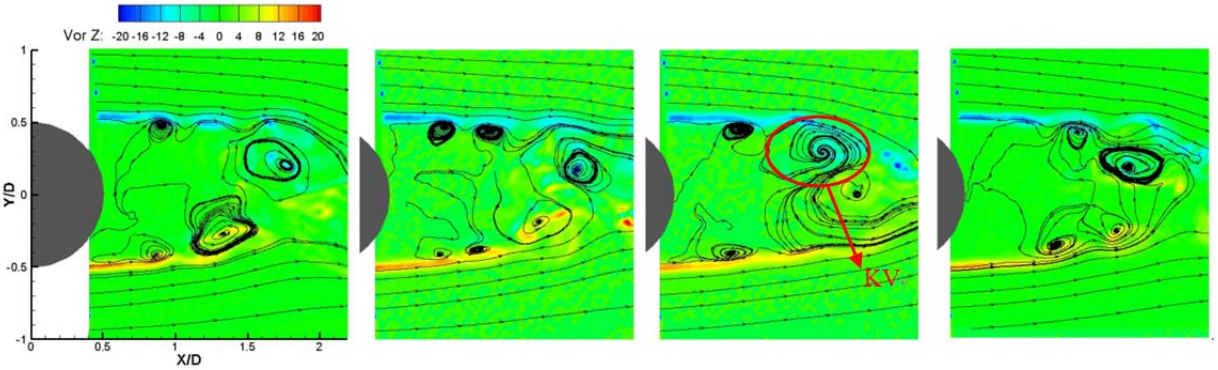
The accuracy of turbulent energy spectrum of the flow field was related with the local grid ratio. It was found that the LES result far from cylinder wall was more accurate than that near the cylinder back side in Figure 6. There existed obvious difference between the experimental data and the LES result near the cylinder back side in Figure 6a. The time averaged velocity could not provide the necessary turbulent information here. The LES result was coincident with experimental data at the position X/D =2 in frequency domain, especially in the range that the power spectrum lower than 20Hz. The vortex shedding frequency of experimental result was 4.3Hz, while the LES result was 4.2 Hz in Figure 6(b). The grid ratio here was proper to get the information of turbulent flow. At the point far from cylinder wall, the corresponding turbulent energy ratio was 18.7%, whereas the total turbulent energy ratio of the summation of that in inertial sub-range and dissipation range was 43% by energy integration. According to the vortex shedding frequency, Strouhal number $S_{tr} = fD/U_{in}$ was 0.732. It was different from that of single cylinder flow and the long two circular cylinders arranged side by side. It suggested that the vortex shedding frequency would be increased by the interaction of the cylinder cascade wakes. It might be one of the reasons that the heat transfer intensity was higher than that in single cylinder flow field.

The time development of the flow vorticity and the shape of Karman vortex (KV) in a period were shown in Figure 7 and Figure 8. Compared with the experimental data, the oscillated behaviour of Karman vortex could be captured by LES method, too. The strength of the vortices in the shear layer was more than that in the cylinder wake area. The vortices generated alternately in the shear layers and moved downstream. Mixing with the low momentum fluid on the cylinder backside, Karman vortex developed and then dissipated quickly. The thickness of the shear layer was about 0.1D, and the scale of Karman vortex was about 1D, which were consistent with the experimental data. Although the LES result could not capture the energy evolution law near the solid wall, it predicted subsequent turbulent vortex development where the subgrid scale was generated as the energy ratio of 30%.



(a) t=0, 4T/4 (b) t= T/4 (c) t=2T/4 (d) t=3T/4

Figure 7 Streamline and vorticity contour of LES result in the tested plane

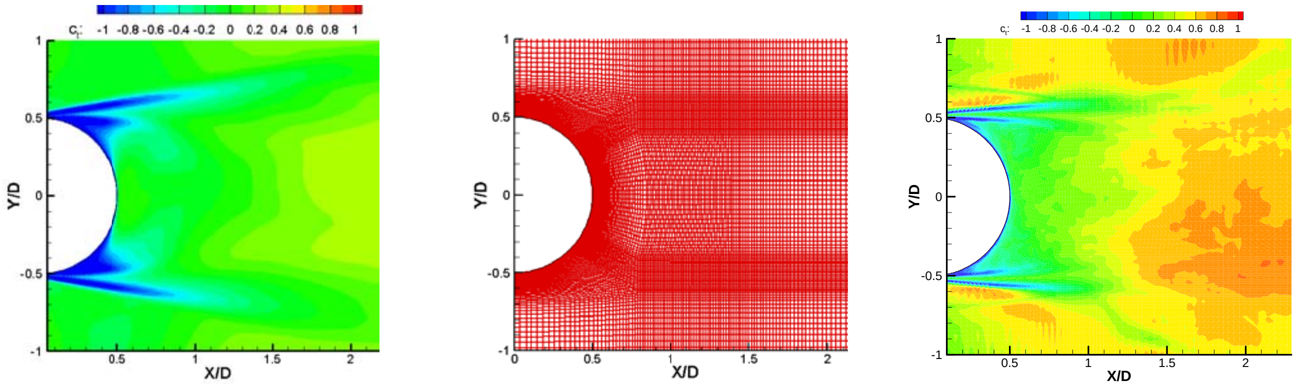


(a) t=0, 4T/4 (b) t= T/4 (c) t=2T/4 (d) t=3T/4

Figure 8 Streamline and vorticity contour of experimental data in the tested plane

Flow Structure On High Reynolds Number Re=20,000

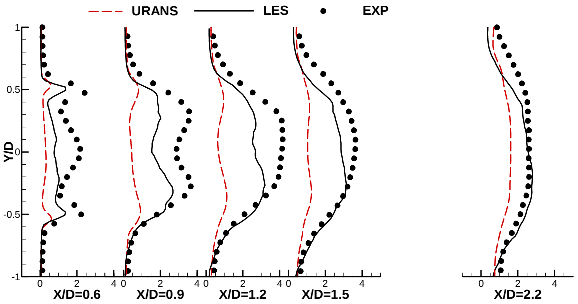
The inlet velocity was increased to 2.487m/s, and the time step was 0.0004s. The other boundaries were as same as those of Re=3,000. Turbulence intensity of the flow field increased with the Reynolds number, which resulted in the refinement of the local subgrid scale by expression (5). The computational time would rise a lot. The energy ratio was increased to 40% to save computational consumption.



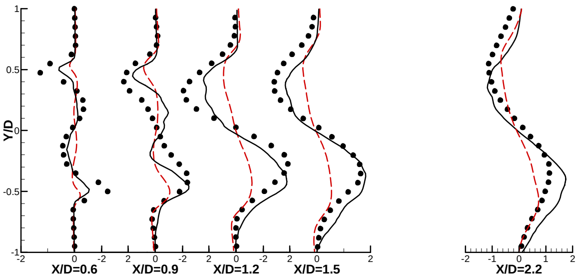
(a) original c_t by URANS (b) refined grid for LES (c) c_t by LES

Figure 9 Grid ratio coefficient and distribution of the grid in the tested plane

Grid ratio coefficient got by URANS method was shown in Figure 9a. Ten-logarithm law was used, once the value of the position was lower than zero, the grid number would be increased base on the selected energy ratio of 40%. In figure 9a, the grid near the wall behind the cylinder, in the shear layer and in the recirculation zone X/D<1, was not fine enough. LES grid was generated with the requirement of the energy ratio and plotted in Figure 9b. The grid scale along the cylinder axis was non-uniform, and the $\Delta z^{+}{}_{\max} = (U_{in}\Delta z)/\nu$ =100 located at the central plane. Based on the energy ratio coefficient, the total grid number was 2,400,000, the corresponding grid ratio was shown in Figure 9c. The grid scale in most area reached the standard of c_t>1, but there still left the region in the shear layer and near cylinder surface needed to be refined.



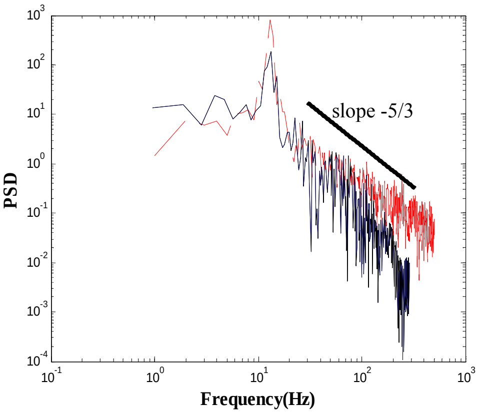
(a) Turbulent kinetic energy $(\overline{u'u'}+\overline{v'v'})/2$



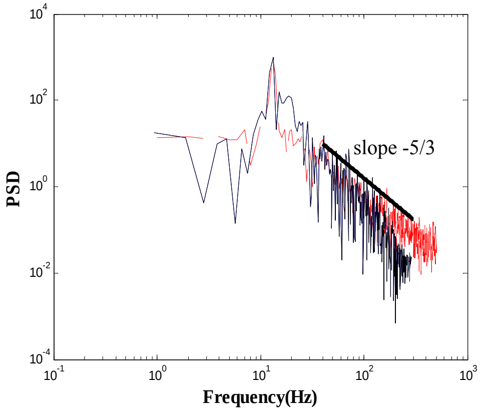
(b) Reynolds stress $\overline{u'v'}$

Figure 10 Turbulent information on the tested plane

As there was not enough turbulent information by time averaged velocity, the parameters of the time averaged turbulence kinetic energy and Reynolds stress were plotted behind the cylinder cascade at different downstream positions in Figure 10. URANS results could not show the enough turbulent intensity, compared with the experimental data. LES results were closer to the experimental data. Therefore, URANS method would not be recommended to get the turbulent information in the high Reynolds cylinder cascade flow field.



(a) u' at X/D=0.6, Y/D=0



(b) u' at X/D=2, Y/D=0

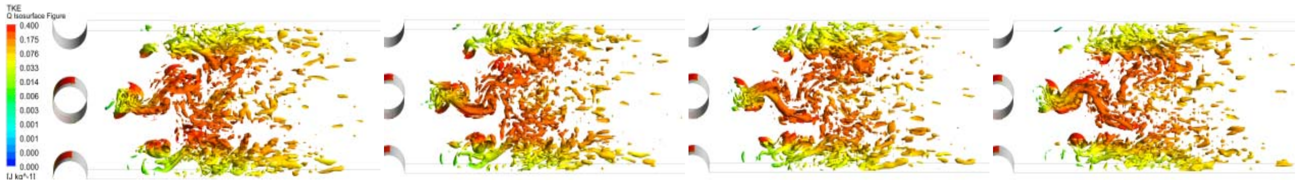
Figure 11 Power Spectrum of Monitor points
（Black line: LES result. Red line: experiment data)

Similarly as the low Reynolds number condition, the time dependent information at the centreline (Y/D = 0) was studied. u' spectrum by experimental and LES method was plotted in Figure 11 respectively. The vortex shedding frequency 13.3Hz was found in Figure 11 for both methods from the cylinder backside to the downstream position X/D=2. The turbulent energy spectrum of the tested data was close the slope -5/3 in the inertia sub-range. LES result was coincident with experimental data at the position X/D =2 in the frequency domain, especially in the inertia sub-range that the power spectrum lower than 100Hz. Even with the insufficient grid number near the cylinder surface, the turbulent energy spectrum was close to the experimental data in the range that less than 70Hz. The corresponding energy ratio above 100Hz was about 32.4%, the total energy ratio of the sum of inertial sub-range and dissipation range is 56.4% with the energy ratio as 40%. It verified the energy ratio was proper to get the turbulent information in the cylinder cascade flow.

Strouhal number was 0.34 in the flow field. It was different from that of the single cylinder flow and varied with Reynolds number. It suggested that the vortex shedding characters would be influenced by the adjacent wake.

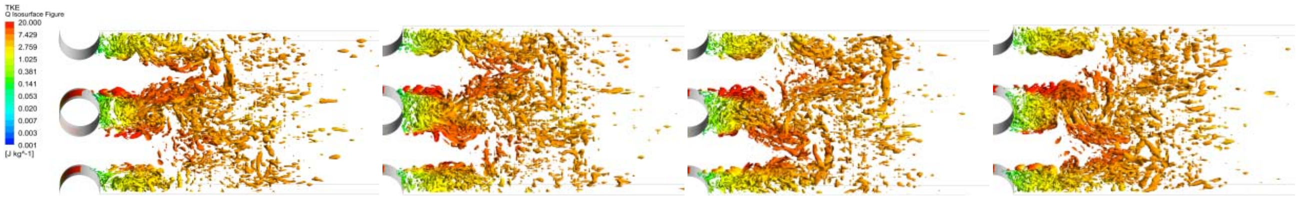
COHERENT STRUCTURE IN THE TURBULENT FLOW

With the application of energy ratio coefficient, the proper subgrid scale could be generated for LES method. It would result in the accurate turbulent energy spectrum below a determined frequency in the inertia sub-range. Therefore, the turbulent coherent structure could be got by the LES method. The 3D vortex shedding processes on the two Reynolds numbers were shown in Figure 12 and Figure 13, respectively. The turbulent kinetic energy contour was plotted by 10-logarithm law on the constant Q-criterion[12]. The oscillated wake of the cylinder mixed with those of the adjacent cylinders, and dissipated downstream quickly. Different from those of the single cylinder, the wake flow would keep in the downstream range 0<X/D<4. Although the turbulent flow was strengthened with the high Reynolds number, the interaction of the cylinder cascade would confine the coherent structure scale.



(a) t=0 or 4T/4 (b) t= T/4 (c) t=2T/4 (d) t=3T/4

Figure 12 Vortex interaction at Re=3,000



(a) t=0 or 4T/4 (b) t= T/4 (c) t=2T/4 (d) t=3T/4

Figure 13 Vortex interaction at Re=20,000

SUMMARY

Cylinder cascade flows were studied by LES method. In order to get the turbulent flow structures efficiently, subgrid scales were figured out by the turbulent energy coefficient. With turbulent energy coefficient of 30% and 40%, the downstream turbulent flow in X/D>2 were coincident with the experimental data. Therefore, LES method here can be used to study the cylinder cascade flow with less computation cost. The vortex shedding frequency of the cylinder cascade flow is related with Reynolds number and is faster than that of the single cylinder flow. In addition, the vortices dissipate quickly with the interaction of the adjacent wakes.

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