INFLUENCES OF SIDE WALL ANGLE ON HEAT TRANSFER OF POWER-LAW FLUIDS IN TRAPEZOIDAL ENCLOSURES

Çuhadaroğlu B.* and Vakhshouri M.E.
*Author for correspondence
Department of Mechanical Engineering,
Karadeniz Technical University,
Trabzon, 61080,
Turkey,
E-mail: burhan@ktu.edu.tr

ABSTRACT

In recent years, there is an important increase in technological applications of non-Newtonian fluids (*NNF*). *NNF*s are preferred over Newtonian fluids (*NF*) because of their superior hydrodynamic and thermal properties. *NNF*s are particularly used as damping fluid in shock absorbers, raw material for making of armors in defense industry and insulator in thermal systems. The use of *NNF* has become widespread in thermal systems in order to prevent over-heating problem which affects the efficiency. This study presents a numerical analysis for the natural convection in a two dimensional trapezoidal (isosceles trapezoid) enclosure filled with power-law *NNF*. The effects of various parameters are investigated on heat transfer on the bottom wall by developing a two dimensional model of such a cell. The bottom edge of the trapezoidal enclosure is considered as hot, top edge as cold while the side walls are considered as adiabatic. The considered parameters are power-law index (n) and Rayleigh number (Ra) and also the trapezoid side wall angle altering in the range of $0^o \leq \theta \leq 20^o$. The power-law index has been varied in the range of $0.6 \leq n \leq 1.8$ and Rayleigh number in the range of $10^3 \leq Ra \leq 10^5$ while Prandtl number has been kept constant as 1000. The results reveal that the mean Nusselt number ($\overline{Nu}$) on bottom wall of trapezoid increases by increasing trapezoid angle and decreasing power-law index. According to evidences of the study, it may be suggested that the use of power-law *NNF*s may contribute to increase efficiency by averting the over-heating problems in trapezoidal thermal systems which are regarded as a significant application field in green and renewable energy systems.

INTRODUCTION

The analysis of natural convection in enclosures as a heat transfer mechanism has been an attractive problem, in recent years. In some cases, using non-Newtonian fluids, which have some thermal advantages, provide better performance for devices. For instance, concentrated photovoltaic (*CPV*) cells, which expose to overheating, operate at low efficiency. The main design parameter of *CPV*'s is cooling however the best medium and encapsulation configuration need to be identified for the best performance. In order to solve this problem, the method of immersing solar-cells into non-Newtonian fluids seems to be encouraging. It has been found that the silicon oil as a *NNF* ensures the best performance and good stability for the encapsulation of the solar cells [1, 2].

NOMENCLATURE

CPV	[-]	Concentrated Photovoltaic
c_p	[J/kgK]	Specific heat at constant pressure
e_{ij}	[s^{-1}]	Rate of strain tensor
g	[m/s^2]	Gravitational acceleration
h	[W/m^2K]	Heat transfer coefficient
H	[m]	Enclosure height
k	[W/mK]	Thermal conductivity
K	[$N.s^n/m^2$]	Consistency
L	[m]	Horizontal length of the enclosure
n	[-]	Power-law index
Nu	[-]	Nusselt number
$\overline{Nu}$	[-]	Mean Nusselt number
Pr	[-]	Prandtl number
Ra	[-]	Rayleigh number
T	[K]	Temperature
u	[m/s]	Horizontal velocity
v	[m/s]	Vertical velocity
u_i	[m/s]	*i*th velocity component
u_j	[m/s]	*j*th velocity component
x_i	[m]	Coordinate in *i*th direction
x_j	[m]	Coordinate in *j*th direction
α	[m^2/s]	Thermal diffusivity
β	[1/K]	Coefficient of thermal expansion
δ_{ij}	[-]	Kronecker delta
$\dot{\gamma}$	[1/s]	Shear rate
θ	[°]	Inclination angle of trapezoidal enclosure
ν	[m^2/s]	Kinematic viscosity
ρ	[kg/m^3]	Density
τ_{ij} (τ)	[Pa]	Stress tensor

Special characters

ΔT	[K]	difference between hot and cold wall temperature

Subscripts

C	*Cold wall*
H	*Hot wall*
ow	Opposite wall
w	*Wall*

An experimental study on the effects of thickness of silicon layer on the *CPV*'s performance reported that 1.5 mm provides 8.5-15.2% increase in efficiency [2]. In order to prevent overheating in *CPV*'s some design techniques are also studied numerically as well as experimentally. A CFD analysis of heat removing through silicon sealed solar cell array by different coolant dielectric liquids show that de-ionized water provides lowest temperature among others such as isopropyl alcohol, ethyl acetate and dimethyl silicon oil [3]. It is analyzed numerically that using proper number of fins at the base of the encapsulated solar cell reduces the rise temperature [4].

The natural convection in a rectangular cavity (high aspect ratio) filled with a power-law fluid and subjected to a horizontal temperature gradient has been numerically and analytically investigated and a correlation function of $Nu = (-0.08n^2 + 0.264n + 0.155)(\frac{Ra}{n})^{2/(5n+4)}$ at the mid vertical axes which was obtained by the scale analysis is proposed [5]. It also reported that heat transfer at the mid vertical axes of rectangular cavity is very sensitive to the power-law index, in other words shear-thinning fluids (*n*<1) enhances heat transfer rate while shear-thickening fluids (*n* >1) resists to heat transfer. The laminar natural convection of power-law fluids in square enclosures with differentially heated horizontal wall (side walls are adiabatic) was analyzed and the effects of constant wall temperature and constant wall heat flux boundary conditions on thermal characteristics were revealed in detail [6]. It was also reported that the $\overline{Nu}$ increases with increasing values of *Ra* number while the power-law index decreases for both boundary conditions [6].

As one of the widespread applications, natural convection of Newtonian fluids in trapezoidal enclosures have been studied by some researches [7-9]. The boundary conditions of linearly heated left wall and cold right wall with uniformly heated bottom wall and adiabatic top wall were used and it was found that overall heat transfer rates are larger for square enclosure compared to trapezoidal enclosures at *Pr*=1000 for all *Ra* numbers [9].

Although majority of the studies on natural convection in square and rectangular enclosures subjected to *NNF'*s as well as natural convection of *NF*'s in trapezoidal enclosures have been studied by many investigators, natural convection of *NNF*'s in trapezoidal geometry has not been studied in detail. Here, the cooling problem of a *CPV* filled with *NNF* is modelled as a trapezoidal enclosure filled with *NNF* and solar cell at the base of enclosure. The influences of power-law index, trapezoidal angle and *Pr* and *Ra* numbers on isotherms, streamlines and local *Nu* number have been studied numerically in a trapezoidal cavity with differentially heated side walls [10]. Following the studies in the literature, we attempt to analyze the problem of natural convection of power-law fluids in a trapezoidal enclosure having different top and bottom wall temperatures with the parameters of side wall angle and power-law index.

OUTLINE OF THE PROBLEM

The governing equations of laminar natural convection are the coupled conservation equations of mass, momentum and energy as follows:

$$\frac{\partial u_i}{\partial x_i} = 0 \qquad (1)$$

$$\rho u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \rho g \delta_{ij} \beta (T - T_{ref}) + \frac{\partial \tau_{ij}}{\partial x_j} \qquad (2)$$

$$\rho u_j c_p \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_j}\left(k \frac{\partial T}{\partial x_j}\right) \qquad (3)$$

According to Ostwald-De Waele model the viscous stress tensor $\tau_{ij}$is defined as:

$$\tau_{ij} = \mu_a e_{ij} = K(e_{kl}e_{kl}/2)^{(n-1)/2} e_{ij} \qquad (4)$$

Where $e_{ij} = (\partial u_i/\partial x_j + \partial u_j/\partial x_i)$ is the rate of strain tensor, while the apparent viscosity is given by:

$$\mu_a = K(e_{kl}e_{kl}/2)^{(n-1)/2} \qquad (5)$$

The dimensionless numbers of Rayleigh, Prandtl and Nusselt considered in the present study are defined as [6]:

$$Ra = g\beta \Delta T L^{2n+1}/(\alpha^n (K/\rho))$$

$$Pr = (K/\rho)\alpha^{n-2}H^{2-2n} \qquad (6)$$

$$\boldsymbol{Nu = hL/k}$$

Where the heat transfer coefficient *h* is defined as:

$$h = \left|-k\frac{\partial T}{\partial x}\right|_w \times \left|\frac{1}{T_w - T_{Ow}}\right| \qquad (7)$$

Where subscript "*w*" is used to refer the wall, while "*ow*" refers to opposite wall. The boundary conditions of the problem which is sketched in Figure 1 are defined as:

$u = v = 0;\ \frac{\partial T}{\partial n} = 0$ on the side walls

$u = v = 0;\ T = T_H$ for $y = 0$ and $0 < x < L_H$

$u = v = 0;\ T = T_C$ for $y = H$ and $0 < x < L_C$

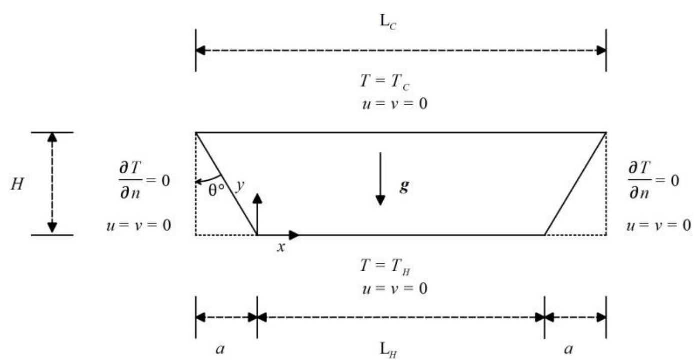

Figure 1 Schematic view of the boundary conditions

NUMERICAL PROCEDURE

In the numerical analysis a commercial code *ANSYS-Fluent* is used to solve the conservation equations. A common procedure of schemes for the diffusive and convective terms are used namely, the second-order central differencing scheme and second order upwind scheme respectively, while the well-known *SIMPLE* algorithm is used for coupling of the pressure and velocity. The discretization of pressure is achieved by *PRESTO*. The convergence criteria were set to 10^{-6} for all the residuals. The grid independency of the results has been established using three different non-uniform meshes (100×80), (220×180) and (240×200). The mesh face elements are Quad and mesh face type is Map. It has been noted that the deviation of $\overline{Nu}$ number between the meshes was less than 1% and the results of the mesh of (240×200) are presented here.

RESULTS AND DISCUSSIONS

All computations were conducted for aspect ratio (H/L_c) of 0.25 and Prandtl number of 1000, while the Ra number and the angle of inclination vary as $10^2 \leq Ra \leq 10^5$ and $0^o \leq \theta \leq 20^o$, respectively. In some computations, converged solutions could not be obtained due to the numerical restrictions, particularly for the cases of shear-thinning fluids with low *n's* such as n=≤0.7 for θ=0^o and θ=10^o at $Ra>10^4$

Effects of the Rayleigh Number

The results of the numerical analysis show that increasing Ra number increases $\overline{Nu}$ on the bottom (hot) wall for the all angle of inclinations of trapezoidal enclosure. The $\overline{Nu}$ drastically increases for the shear-thinning fluids, while the rate of increase of $\overline{Nu}$ gets lower values for the shear-thickening fluids at $Ra \leq 10^4$ as illustrated in Figures 2-4. It also has been found that the $\overline{Nu}$ slightly increases for the all power-law fluids at the range of $10^4 \leq Ra \leq 10^5$. An overview of the $\overline{Nu} - Ra$ variations provided in Figures 2-4, showing that the $\overline{Nu}$ increases with increasing Ra number for the all power-law fluids which is an expected characteristic, namely, increasing buoyancy enhances heat transfer on the bottom wall.

For the rectangular enclosure (θ=0^o), the trends of $\overline{Nu} - Ra$ variations are qualitatively consistent with previous studies of natural convection of power-law fluids in enclosures for different boundary conditions [5-6]. It may be concluded that for the all boundary condition combinations, natural convection of shear-thinning fluids in diagonal enclosures Ra number is dominant on the heat transfer at all walls in the range of $Ra \leq 5x10^5$.

For the trapezoidal enclosure with the inclination angle of 10^o, increasing ratio of $\overline{Nu}$ with respect to Ra number gets higher values than that of rectangular enclosure and trapezoidal enclosure with θ=20^o for n≥1. For instance, increasing ratio of $\overline{Nu}$ is about 74% between Ra=10^4 and 10^5 for trapezoidal enclosure with θ=10^o filled with n=1 (Figure 3), while the increasing ratio of $\overline{Nu}$ takes the values of 56% and 60% for θ=0^o and θ=20^o (Figures 2 and 4) respectively. In a similar manner, increasing ratio of $\overline{Nu}$ takes the value of 79% between Ra=10^4 and 10^5 for rectangular enclosure filled with n=1.2, while the increasing ratio of $\overline{Nu}$ takes the values of 76% and 58% for θ=0^o and θ=20^o respectively for n=1.2.

The increasing ratio of $\overline{Nu}$ with respect to Ra number gets almost same values for shear-thinning fluids (n<1) according to the available data of the present study. For instance, increasing ratio of $\overline{Nu}$ gets a constant value of 55% between Ra=10^4 and 10^5 for rectangular and both trapezoidal enclosures for n=0.9, while the increase ratio gets about 33% between Ra=10^4 and 5x10^5 for the all enclosures filled with shear-thinning fluid of n=0.8.

All of these results indicate that trapezoidal enclosure with inclination angle of 10^o filled with shear-thickening fluids exhibits some extraordinary behavior in terms of heat transfer.

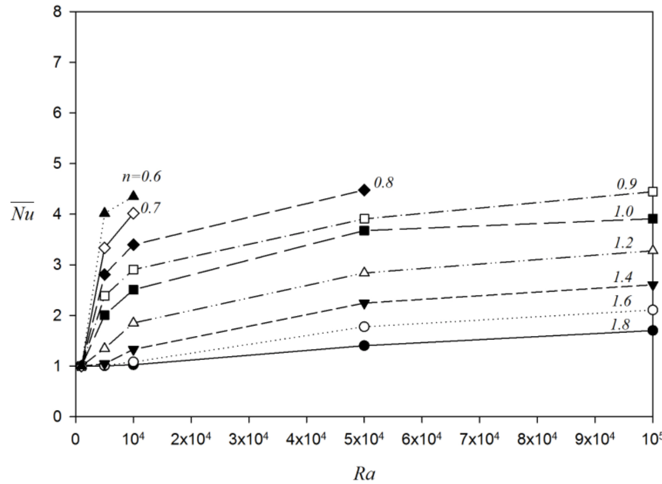

Figure 2 Variations of $\overline{Nu}$ with Ra for θ=0^o.

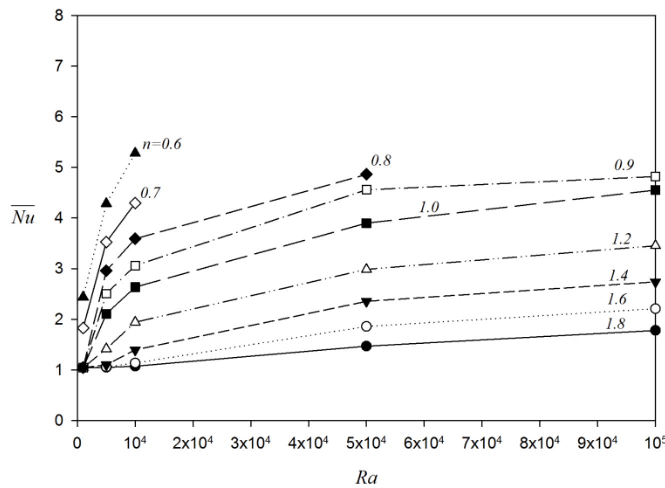

Figure 3 Variations of $\overline{Nu}$ with Ra for θ=10^o.

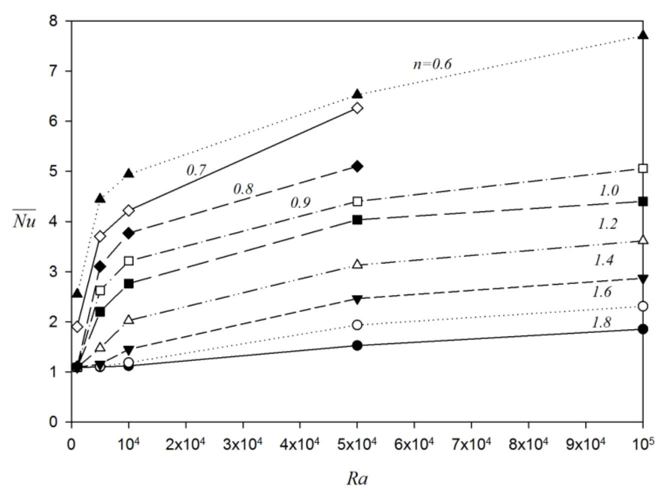

Figure 4 Variations of $\overline{Nu}$ with Ra for θ=20^o.

Effects of the Power-Law Index (*n*)

Regarding the results of *NF*'s (n=1) as a reference data, it is clearly evaluated that shear-thickening fluids (n>1) reduce $\overline{Nu}$ with increasing n, whereas shear-thinning fluids (n<1) increase $\overline{Nu}$ with decreasing n for the all inclination angles (Figures 5-7). For rectangular enclosure (θ=0°), for a given Ra number, a decrease of n leads to enhancing the convection effect except the case of Ra=1000 (Figure 5).

The same trends of $\overline{Nu}$ variations with n are obtained for both trapezoidal enclosures with inclination angles of 10° and 20°, as illustrated in Figures 6 and 7. An overall view of the illustrations of $\overline{Nu} - n$ variations indicate that, for the shear-thinning fluids (n<1) decreasing ratio of $\overline{Nu}$ with respect to n is higher than that of shear-thickening fluids (n>1) for $Ra \geq 5000$. It is also remarkable that $\overline{Nu}$ drastically increases for the both trapezoidal enclosures filled with fluids of n<0.8 at Ra=1000 on account of speed up effect of inclination angle on rolling cells.

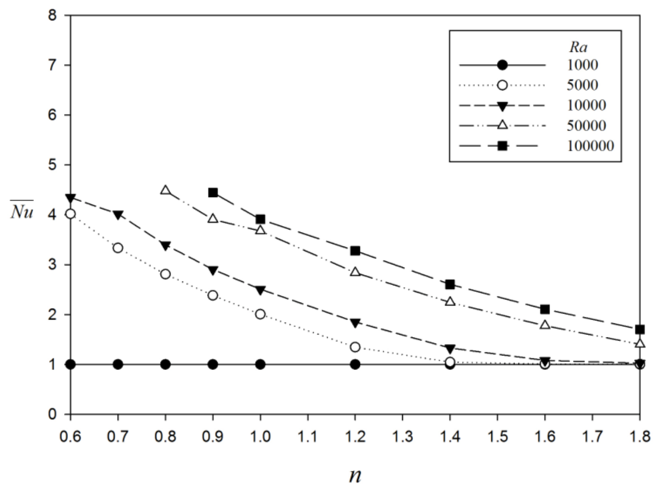

Figure 5 Variations of $\overline{Nu}$ with n for θ=0°.

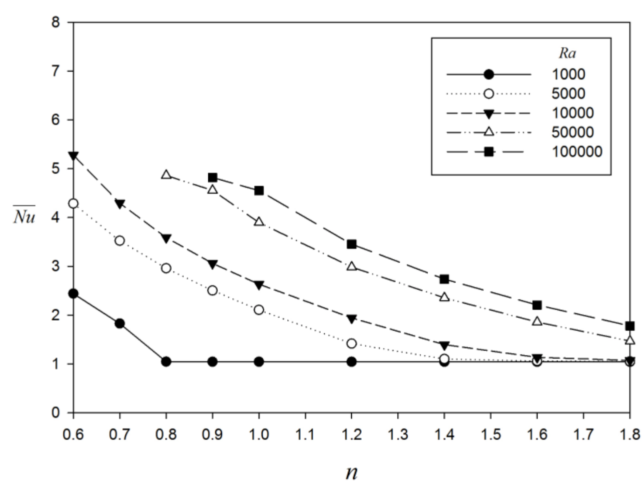

Figure 6 Variations of $\overline{Nu}$ with n for θ=10°.

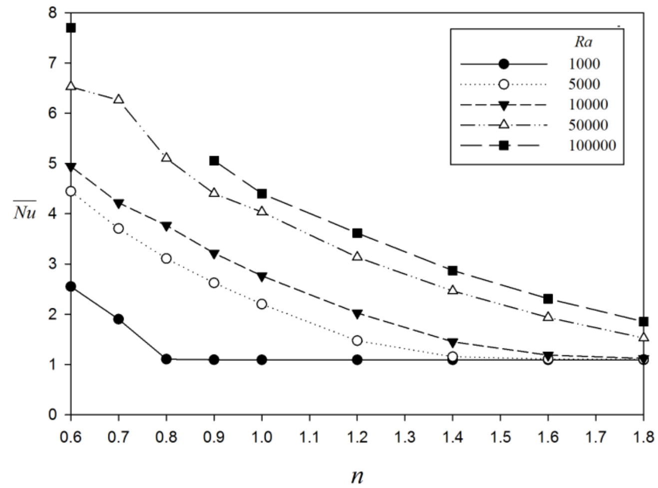

Figure 7 Variations of $\overline{Nu}$ with n for θ=20°.

Effects of the Inclination Angle (θ)

By the increase of inclination angle of side walls toward to 20° (trapezoidal enclosures), $\overline{Nu}$ on the bottom wall gets slightly higher values than that of rectangular enclosure for the all kind of fluids (Figures 8-10). Regarding the fact that increasing angle of inclination corresponds to shorter bottom wall although wall temperature is kept constant, observing a slightly increase of $\overline{Nu}$ may be attributed of increasing intensity of rolling in the enclosures.

It is interesting to observe that a sudden increase of $\overline{Nu}$ at Ra=10^4 in trapezoidal enclosure with the inclination angle of θ=10° for the shear-thinning fluid of n=0.6. This phenomenon is attributed exactly by shifting of the number of roll cells two to four as seen in Figure 8. This also reveals that increase of heat transfer on the bottom wall depends on number of roll cells as well as intensity of rolling.

Another "step-up effect" of $\overline{Nu}$ is also observed at Ra=10^3 in trapezoidal enclosure with inclination angle of θ=10° for the fluid of n=0.6 as seen in Figure 8. In a similar manner, the four roll cells are formed in natural convection of Newtonian fluid (n=1) in trapezoidal enclosure with inclination angle of θ=10° at Ra=10^5 which leads to higher $\overline{Nu}$ than that of other inclination angles (Figure 9). This characteristic indicates that the inclination angle of θ=10° of trapezoidal enclosure has a dominant effect on $\overline{Nu}$ in conjunction with other parameters such as power-law index and Ra number.

The shear-thickening fluid of n=1.8 which is an extreme sample of the present study exhibits a pseudo-fluid behavior more than a fluid as seen in Figure 10. The $\overline{Nu}$ variations of the bottom wall with the inclination angle reveal that shear-thickening fluid of n=1.8 is insensible to the inclination angle of trapezoidal enclosure.

As a result of $\overline{Nu} - \theta$ variations, it may be stated that the inclination angle of θ=10° of trapezoidal enclosure has an attractive characteristic to be focused in heat transfer problem of natural convection with shear-thinning fluids.

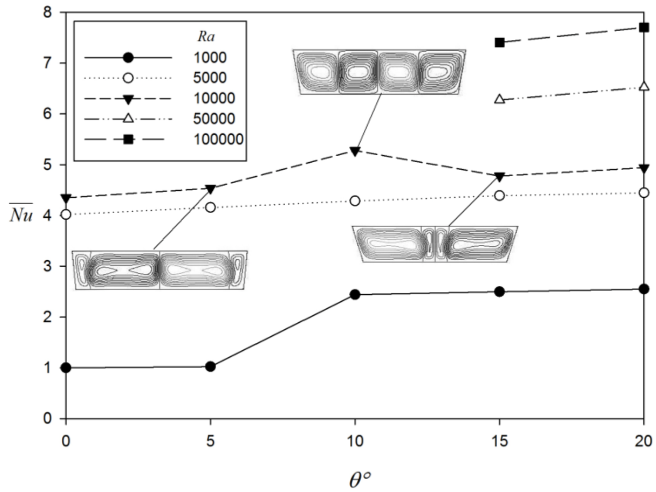

Figure 8 Variations of $\overline{Nu}$ with θ for *n*=0.6.

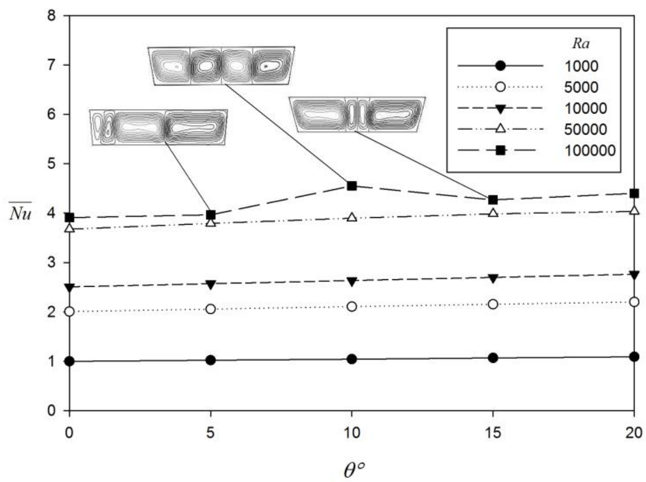

Figure 9 Variations of $\overline{Nu}$ with θ for *n*=1.0

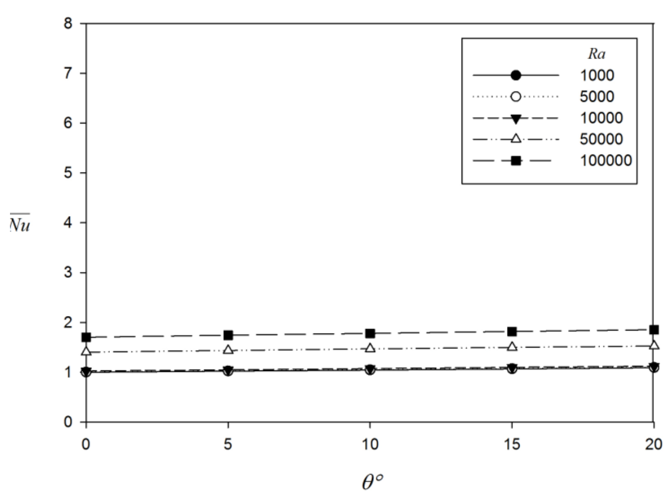

Figure 10 Variations of $\overline{Nu}$ with θ for *n*=1.8.

CONCLUSION

The numerical analysis has been performed for the natural convection of power-law *NNF* within a trapezoidal enclosure with hot bottom and cold top walls. The effects of some parameters are investigated on heat transfer on the bottom wall by a two dimensional model of such an enclosure. The bottom wall of the trapezoidal enclosure is considered as hot, top wall as cold while the side walls are considered as adiabatic. The parameters considered are power-law index (*n*) and Rayleigh number (*Ra*) and also the trapezoid side wall angle altering in the range of $0^o \leq \theta \leq 20^o$. The power-law index has been varied in the range of $0.6 \leq n \leq 1.8$ and Rayleigh number in the range of $10^3 \leq Ra \leq 10^5$ while Prandtl number has been kept constant as 1000.

The heat transfer on the bottom wall of trapezoidal enclosure filled with shear-thinning fluids depends on inclination angle as well as other parameters. The forming of more number of roll cells in the enclosure has dominant effect on $\overline{Nu}$ in the case of θ=10^o. According to this evidence, it may be suggested that the use of shear-thinning fluids in the enclosures may contribute to increase efficiency by averting the over-heating problems in trapezoidal thermal systems which are regarded as a significant application field in green and renewable energy systems.

REFERENCES

[1] Wang Y., Zhenlei F., Zhu L., Huang Q., Zhang Y., and Zhang Z., The performance of silicon solar cells operated in liquids, *Applied Energy*, Vol. 86, 2009, pp. 1037-1042.

[2] Han X., Wang Y., and Zhu L., The performance and long-term stability of silicon concentrator solar cells immersed in dielectric liquids, *Energy Conversion and Management*, Vol. 66, 2013, pp. 189-198.

[3] Han X., Wang Y., and Zhu L., Electrical and thermal performance of silicon concentrator solar cells immersed in dielectric liquids, *Applied Energy*, Vol. 88, 2011, pp. 4481-4489.

[4] Natarajan S.K., Mallick T.K., Katz M., and Weingaertner S., Numerical investigations of solar cell temperature for photovoltaic concentrator system with and without passive cooling arrangements, *International Journal of Thermal Sciences*, Vol. 50, 2011, pp. 2514-2521.

[5] Lamsaadi M., Naimi M., Hasnaoui M., and Mamou M., Natural convection in a vertical rectangular cavity filled with a non-Newtonian power law fluid and subjected to a horizontal temperature gradient, *Numerical Heat Transfer, Part A*, Vol.49, 2006, pp. 969-990.

[6] Turan O., Fotso-Choupe F., Lai J., Poole R.J., and Chakraborty N., Boundary condition effects on laminar natural convection of power-law fluids in a square enclosure heated from below with differentially heated horizontal walls, *Industrial & Engineering Chemistry Research*, Vol. 53, 2014, pp. 456-473.

[7] Natarajan E., Basak T., and Roy S., Natural convection flows in a trapezoidal enclosure with uniform an non-uniform heating of bottom wall, *International Journal of Heat and Mass Transfer*, Vol.51, 2008, pp. 747-756.

[8] Basak T., Roy S., Singh A., and Pandey B.D., Natural convection flow simulation for various angles in a trapezoidal enclosure with linearly heated side wall(s), *International Journal of Heat and Mass Transfer*, Vol.52, 2008, pp. 4413-4425.

[9] Basak T., Ramakrishna D., Roy S., Matta A., and Pop I., A comprehensive heatline based approach for natural convection flows in trapezoidal enclosures: Effect of various walls heating, *International Journal of Thermal Sciences*, Vol.50, 2011, pp. 1385-1404.
[10] Sojoudi A., Saha, S.C., Gu, Y.T., and Hossain, M.A, Steady natural convection of non-Newtonian power-law fluid in a trapezoidal enclosure, *Advances in Mechanical Engineering*,Vol.5, 2013, pp.1-8.