A NUMERICAL STUDY OF NATURAL CONVECTIVE HEAT TRANSFER FROM UPWARD FACING RECESSED AND PROTRUDING HEATED HORIZONTAL ISOTHERMAL CIRCULAR ELEMENTS

Oosthuizen, P.H.
Department of Mechanical and Materials Engineering,
Queen's University,
Kingston, Ontario K7L 3N6
Canada
E-mail: oosthuiz@queensu.ca

ABSTRACT

A numerical study of natural convective heat transfer from a horizontal upward facing circular isothermal heated element that is imbedded in a large flat adiabatic surrounding surface has been undertaken. The heated element is either recessed by a small amount into the surrounding flat horizontal adiabatic surface or it protrudes by a small amount out of the surrounding flat horizontal adiabatic surface. The element is at a higher temperature than the surrounding fluid and attention has been restricted to the case where the element is facing upward. The range of conditions considered is such that laminar, transitional, and turbulent flows can occur. The flow has been assumed to be two-dimensional and steady and in dealing with the buoyancy forces the Boussinesq approach has been adopted. The numerical solution has been obtained using the commercial CFD solver ANSYS FLUENT©. The *k*-epsilon turbulence model with account being taken of buoyancy force effects has been used. The heat transfer rate from the heated element expressed in terms of the Nusselt number is dependent on the Rayleigh number, on whether the heated element is recessed into or protrudes from the surrounding adiabatic surface, on the dimensionless height the element is recessed into or protrudes from the surrounding adiabatic surface, and on the Prandtl number. Results have been obtained for a Prandtl number of 0.74, i.e., effectively the value for air. The effect of the dimensionless height that the element is recessed or protrudes from the surrounding adiabatic surface on the variation of the Nusselt number with Rayleigh number has been studied. The diameter of the circular heated element has been used as the reference length scale and recess and protrusion height-to-diameter ratios of between 0 and 0.25 have been considered.

INTRODUCTION

A numerical study of natural convective heat transfer from a horizontal upward facing circular isothermal heated element that is imbedded in a large flat adiabatic surrounding surface has been undertaken. The heated element is either recessed by a relatively small amount into the surrounding flat horizontal adiabatic surface or protrudes by a relatively small amount out of the surrounding flat horizontal adiabatic surface.

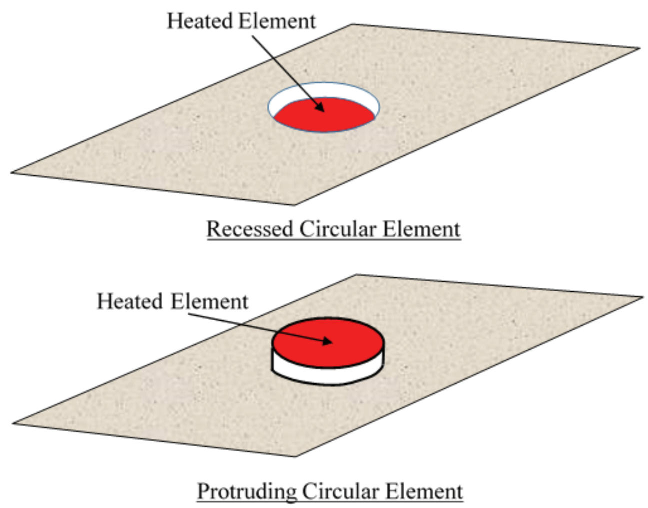

Figure 1 Flow situations considered

These alignments are illustrated in Fig. 1. The element is at a higher temperature than that of the surrounding fluid and attention has been restricted to the case where the element is facing upward. Here, the range of conditions considered is such that laminar, transitional, and turbulent flows can occur. A detailed study of the effect of the dimensionless height that the element is recessed or protrudes from the surrounding adiabatic surface (see Fig. 2) on the heat transfer rate from the isothermal circular element has been undertaken here.

Many studies of natural convective heat transfer from heated horizontal elements have been undertaken, e.g., see [1-24]. However, most of these studies have only considered conditions in which laminar flow exists and almost all have only dealt with the case where the heated element is in the same plane as the surface of the adiabatic surrounding material.

This paper presents a study that is part of a larger overall investigation of natural convective heat transfer from horizontal and near horizontal heated elements for conditions under which laminar, transitional, and turbulent flow occurs. Other representative studies included in this overall investigation are described in [25-33].

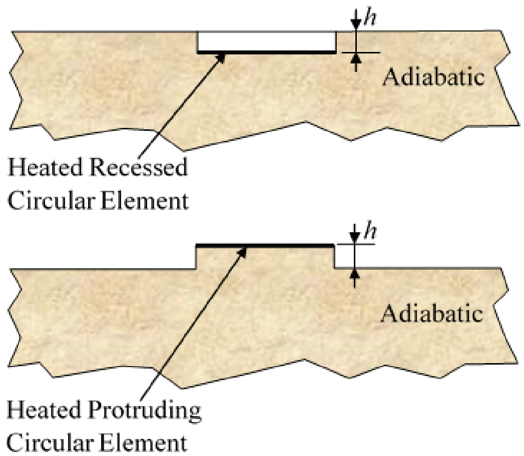

Figure 2 Recess depth and protrusion height, *h*.

NOMENCLATURE

A	[m^2]	Surface area of heated element
d	[m]	Diameter of circular element
g	[m/s^2]	Gravitational acceleration
h	[m]	Recess depth or protrusion height
H	[-]	Dimensionless recess depth or protrusion height, *h/d*
k	[W/mK]	Thermal conductivity
Nu	[-]	Nusselt number
Pr	[-]	Prandtl number
Q'	[W]	Total heat transfer rate from element
R	[-]	Dimensionless radial distance
Ra	[-]	Rayleigh number
T_f	[K]	Undisturbed fluid temperature
T_w	[K]	Element surface temperature

Greek symbols

α	[m^2/s]	Thermal diffusivity
β	[1/K]	Bulk coefficient of thermal expansion
ν	[m^2/s]	Kinematic viscosity

SOLUTION PROCEDURE

The flow has been assumed to be steady and the fluid properties are assumed constant except for the density change with temperature giving rise to the buoyancy forces. This has been dealt with using the Boussinesq approximation. Only the case where the heated elements are facing upward has been considered. Radiant heat transfer effects have been neglected. It has been assumed that the flow is axisymmetric about the vertical center-line of the element. The basic *k*-epsilon turbulence model with standard wall functions and with account being taken of buoyancy force effects has been used. This turbulence model is applied under all conditions and is used to predict when transition occurs. Many previous studies, e.g., [34-40], have demonstrated that this approach provides relatively good predictions of the conditions under which turbulence develops in flows of the type being considered in this study. The commercial CFD solver ANSYS FLUENT© has been employed to numerically solve the governing equations based on the application of the assumptions discussed above and subject to the boundary conditions.

Grid independence and convergence-criteria independence testing was undertaken. These test results demonstrated that the heat transfer results, i.e., the derived Nusselt number values, are grid- and convergence criteria independent to within approximately one per cent with the meshes used in obtaining the results presented here.

RESULTS

The solution has the following governing parameters:

- the Rayleigh number, Ra, based on the diameter, d, of the circular element and on the difference between the temperature of the surface of the heated element, T_w , and the temperature of the undisturbed fluid that is well away from the system, T_f, i.e.:

$$Ra = \frac{\beta g d^3 (T_w - T_f)}{\nu\alpha} \quad (1)$$

- the dimensionless distance to which the element protrudes or to which it is recessed, i.e.:

$$H = \frac{h}{d} \quad (2)$$

- whether the element is recessed or protrudes
- the Prandtl number, Pr.

Only a Prandtl number of 0.74, i.e., effectively for the value of Pr for air, has been considered due to the applications that were the motivation for this study. Rayleigh numbers of between approximately 10^5 and 10^{16} have been considered.

The total heat transfer rate from the heated circular element, Q', has been expressed in terms of a mean Nusselt number based on the diameter, d, of the circular element and on the difference between the temperature of the surface of the heated element and the temperature of the undisturbed fluid well away from the system, i.e.:

$$Nu = \frac{Q' d}{kA(T_w - T_f)} \quad (3)$$

where $A = \pi d^2/4$ is the surface area of the heated circular element. Since a fixed value of Pr is being considered, Nu is a function of Ra and of the dimensionless distance to which the element is recessed or to which it protrudes, H.

The conditions under which the flow is laminar, transitional and fully turbulent can be determined from the numerically calculated variations of Nusselt number with Rayleigh number. These variations clearly show three relatively distinct flow regions, e.g., see Figs. 3 and 5 below. For example, for the case where $H = 0$ it will be seen that laminar flow exists up to a Rayleigh number of approximately 10^7 and that fully-turbulent flow exists for Rayleigh numbers above approximately 10^{11}. The transitional region therefore exists when the Rayleigh number is between 10^7 and 10^{11}.

The case where the heated element is recessed will first be considered. Typical variations of the mean Nusselt number with Rayleigh number for various values of the dimensionless distance to which the element is recessed, H, are shown in Fig. 3. From the results given in Fig. 5 it will be seen that the dimensionless recess depth has a significant influence on the

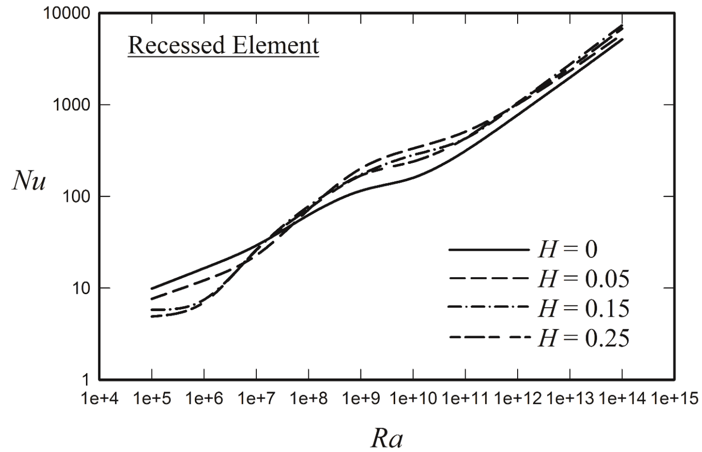

Figure 3 Variation of Nusselt number with Rayleigh number for a recessed element for various values of the dimensionless recess depth.

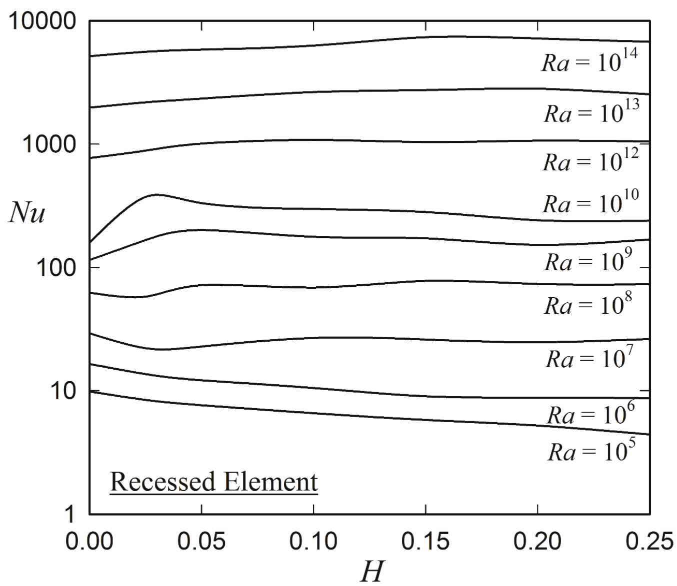

Figure 4 Variation of Nusselt number with dimensionless recess depth for various Rayleigh number values for a recessed element.

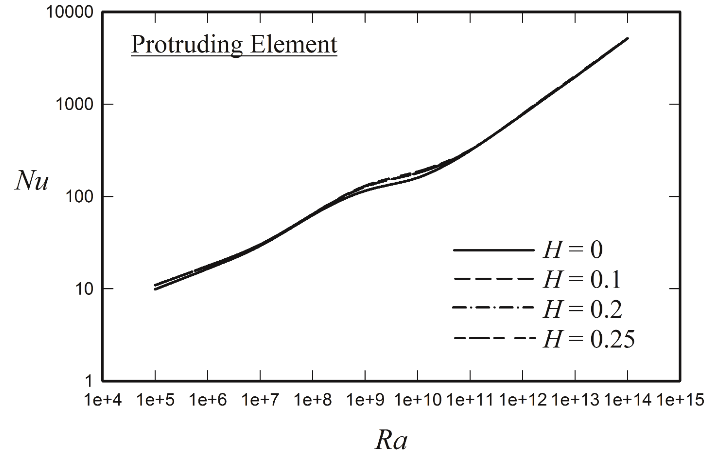

Figure 5 Variation of Nusselt number with Rayleigh number for a protruding element for various values of the dimensionless protrusion height.

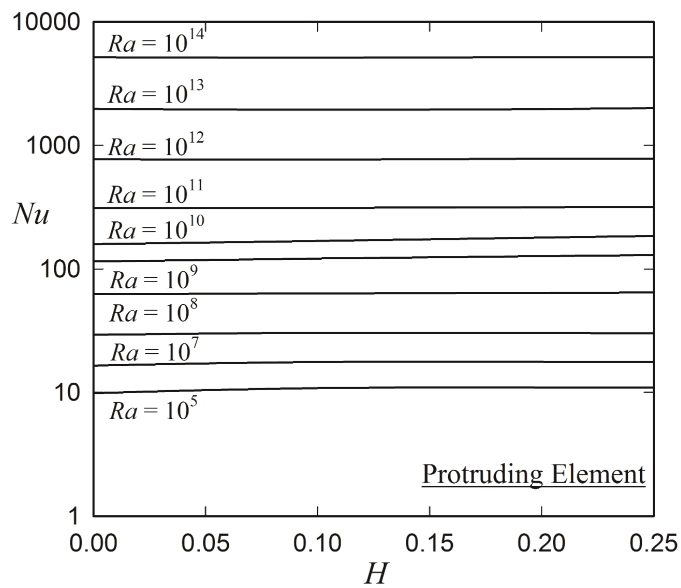

Figure 6 Variation of Nusselt number with dimensionless protrusion height for various Rayleigh number values for a protruding element.

heat transfer rate at low and intermediate values of the Rayleigh number when the flow is in the laminar and transitional regions. The effect of the dimensionless recess depth on the heat transfer rate in the fully turbulent flow region is, however, significantly less than in laminar and transitional flow regions. It will also be noted from the results given in Fig. 3 that at low Rayleigh numbers the recessing of the element leads to a decrease in the heat transfer rate whereas at higher Rayleigh numbers the recessing of the element leads to an increase in the heat transfer rate.

The effect of the dimensionless recess depth is further illustrated by the results presented in Fig. 4 which show the variations of the mean Nusselt number with dimensionless recess depth for various Rayleigh number values. The results presented in Fig. 4 show how the form of the variation of Nusselt number with the dimensionless recess depth varies quite significantly with Rayleigh number. The changes in the form of the Nusselt number variation with dimensionless recess depth is, of course, the result of the changes in the flow pattern over the heated element.

Attention will next be turned to the case where the heated element protrudes from the surrounding adiabatic surface. Typical variations of the mean Nusselt number with Rayleigh number for various values of the dimensionless distance to which the element protrudes, *H,* are shown in Fig. 5. It will be observed from the results given in Fig. 5 that in the case of a protruding element the dimensionless protrusion height has a very small effect on the heat transfer rate. The effect of the

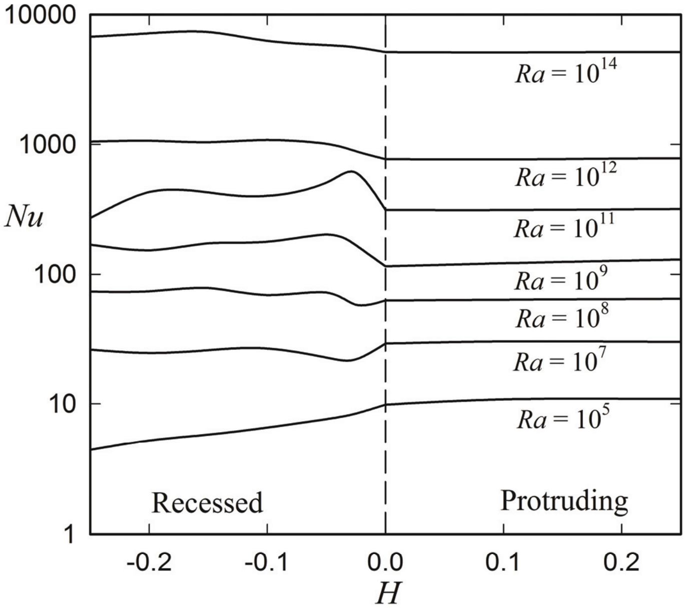

Figure 7 Variation of Nusselt number with dimensionless recess and protrusion height for various Rayleigh number values for recessed and protruding elements.

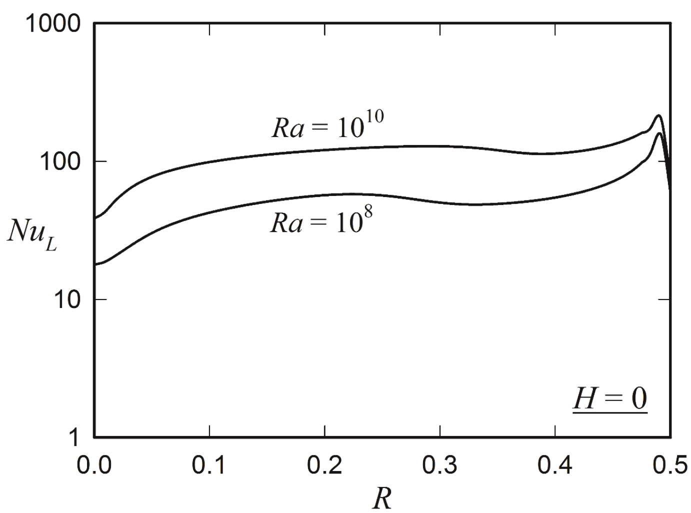

Figure 8 Variation of local Nusselt number with dimensionless radius for two Rayleigh numbers for an element that is in the same plane as the surface of the surrounding adiabatic material.

dimensionless protrusion height is further illustrated by the results presented in Fig. 6 which show the variations of the mean Nusselt number with dimensionless protrusion height for various Rayleigh number values. The results given in Fig. 6 also show that in the case of a protruding element the dimensionless protrusion height has a very small effect on the heat transfer rate. The difference between the forms of the variations of the mean Nusselt number with Rayleigh number for various values of the dimensionless recess depths and

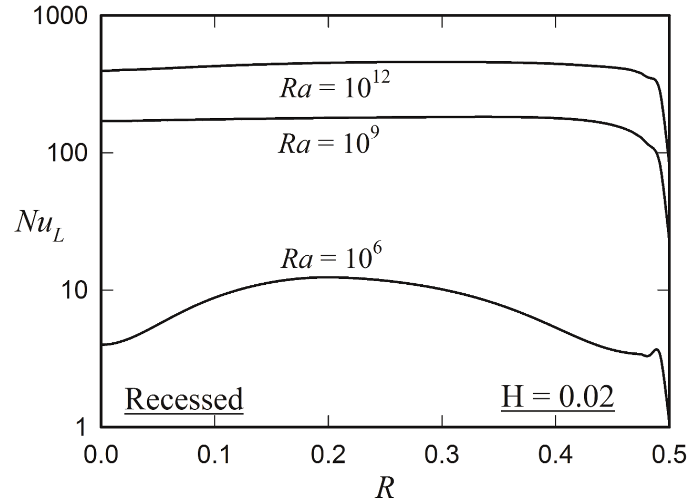

Figure 9 Variation of local Nusselt number with dimensionless radius for three Rayleigh numbers for a recessed element for a dimensionless recess depth of 0.02.

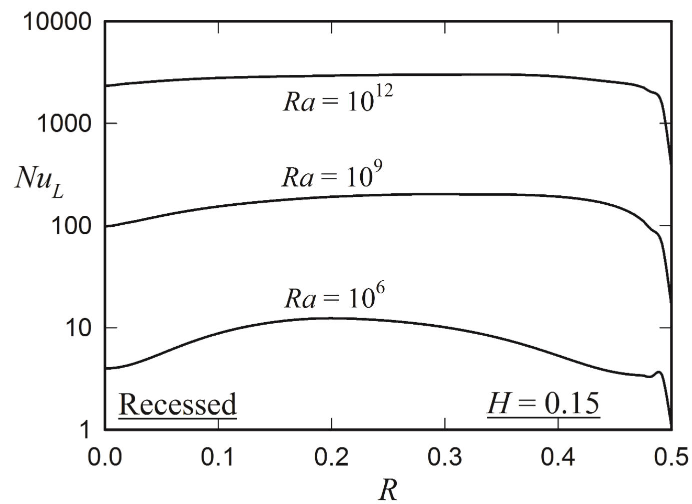

Figure 10 Variation of local Nusselt number with dimensionless radius for three Rayleigh numbers for a recessed element for a dimensionless recess depth of 0.15.

protrusion heights for recessed and protruding elements is illustrated by the results shown in Fig. 7. This figure shows typical variations of Nusselt number with the dimensionless recess depth for various Rayleigh numbers for both recessed and protruding elements. In Fig. 7 the dimensionless recess depth for recessed elements is shown as a negative quantity.

The fact that the dimensionless protrusion height has a very small effect on the heat transfer rate results arises, of course, from the fact that with a protruding element the flow pattern over the element is essentially not affected by the height to which the element protrudes whereas with a recessed element the presence of the side recess wall causes the flow pattern over the element to change significantly with changes in the dimensionless recess depth. The flow pattern changes cause changes in the form of the local Nusselt number variation with radius across the element surface and this is illustrated by the results given in Figs. 8, 9, and 10. Figure 8 presents results for the case of an element that is neither recessed nor protruding, i.e., for an element that lies in the same plane as the surface of the surrounding adiabatic material. It will be seen from Fig. 8 that the local Nusselt number variations for this case have the same form at the two Rayleigh numbers considered. This form of local Nusselt number variation is essentially the same as that existing over the surface of a protruding element at all Rayleigh numbers considered.

Figures 9 and 10 show local Nusselt number variations for two dimensionless recess depths. From these results it will be seen that there are significant changes in the form of local Nusselt number variation for different Rayleigh numbers. The changes in the form of the variation are particularly significant at low Rayleigh number values. This agrees with the conclusions that can be drawn from the results given in Fig. 3 which also show that the relative changes in the Nusselt number values with changes in the Rayleigh number and the dimensionless recess depth H are greatest at the lower Rayleigh number considered.

CONCLUSIONS

The results obtained in the present study show that:

1. In the case of a recessed circular element the dimensionless recess depth has a strong influence on the heat transfer rate at low and intermediate values of the Rayleigh number, when the flow is in the laminar and transitional regions, but the effect of the dimensionless recess depth on the heat transfer rate in the fully turbulent flow region is significantly less than in laminar and transitional flow regions.
2. In the case of a recessed circular element at low Rayleigh numbers the recessing of the element leads to a decrease in the heat transfer rate whereas at higher Rayleigh numbers the recessing of the element leads to an increase in the heat transfer rate.
3. In the case of a protruding element the dimensionless protrusion height has a very small effect on the heat transfer rate.
4. Changes in the form of the local Nusselt number variation across the element surface with the dimensionless radius are minor for a protruding element but are quite large for a recessed element this particularly being the case for lower values of the Rayleigh number.

ACKNOWLEDGEMENTS

This work was supported by the Natural Sciences and Engineering Research Council of Canada (NSERC) through its Discovery Grant Program (RGPIN-2015-06444).

REFERENCES

[1] Aihara, T., Yamada, Y., and Endö, S., Free convection along the downward-facing surface of a heated horizontal plate, *International Journal of Heat and Mass Transfer*, Vol. 15, No. 12, 1972, pp. 2535-2538.

[2] Al-Arabi, M., and El-Riedy, M.K., Natural convection heat transfer from isothermal horizontal plates of different shapes, *International Journal of Heat and Mass Transfer*, Vol. 19, No. 12, 1976, pp. 1399-1404.

[3] Chambers, B.B., and Lee, T.T., A numerical study of local and average natural convection Nusselt numbers for simultaneous convection above and below a uniformly heated horizontal thin plate, *Journal of Heat Transfer*, Vol. 119(1), 1997, pp. 102-108. doi:10.1115/1.2824074.

[4] Clifton, J.V., and Chapman, A.J., Natural-convection on a finite-size horizontal plate, *International Journal of Heat and Mass Transfer*, Vol. 12, No. 12, 1969, pp. 1573-1584.

[5] Goldstein, R.J., Sparrow, E.M., and Jones, D.C., Natural convection mass transfer adjacent to horizontal plates, *International Journal of Heat and Mass Transfer*, Vol. 16, No. 5, 1973, pp. 1025-1035.

[6] Hassan, K.-E., and Mohamed, S.A., Natural convection from isothermal flat surfaces, *International Journal of Heat and Mass Transfer*, Vol. 13, No. 12, 1970, pp. 1873-1886.

[7] Hatfield, D.W., and Edwards, D.K., Edge and aspect ratio effects on natural convection from the horizontal heated plate facing downwards, *International Journal of Heat and Mass Transfer*, Vol. 24, No. 6, 1981, pp. 1019-1024.

[8] Hossain, M.A., and Takhar, H.S., Thermal radiation effects on the natural convection flow over an isothermal horizontal plate, *Journal of Heat and Mass Transfer*, Vol. 35, No. 4,1999, pp. 321-326.

[9] Kitamura, K., and Kimura, F., Heat transfer and fluid flow of natural convection adjacent to upward-facing horizontal plates, *International Journal of Heat and Mass Transfer*, Vol. 38, No. 17, 1995, pp. 3149-3159.

[10] Kozanoglu, B., and Lopez, J., Thermal boundary layer and the characteristic length on natural convection over a horizontal plate, *Journal of Heat and Mass Transfer*, Vol. 43, No. 4, 2007, pp. 333-339.

[11] Kwak, C.E., and Song, T.H., Natural convection around horizontal downward-facing plate with rectangular grooves: experiments and numerical simulations, *International Journal of Heat and Mass Transfer*, Vol., 43, No. 5, 2000, pp. 825-838.

[12] Lewandowski, W.M., Radziemska, E., Buzuk, M., and Bieszk, H., Free convection heat transfer and fluid flow above horizontal rectangular plates, Appl. Energ., Vol. 66, No. 2, 2000, pp. 177-197.

[13] Lloyd, J.R., and Moran, W.R., Natural convection adjacent to horizontal surface of various planforms, *Journal of Heat Transfer*, Vol. 96, No. 4, 1974, pp. 443-447.

[14] Martorell, I., Herrero, J., and Grau, F.X., Natural convection from narrow horizontal plates at moderate Rayleigh numbers, *International Journal of Heat and Mass Transfer*, Vol. 46, No. 13, 2003, pp. 2389-2402.

[15] Pretot, S., Zeghmati, B., and Le Palec, G., Theoretical and experimental study of natural convection on a horizontal plate, *Applied Thermal Engineering*, Vol. 20, No. 10, 2000, pp. 873-891.

[16] Prétot, S., Zeghmati, B., and Caminat, Ph., Influence of surface roughness on natural convection above a horizontal plate, *Advances in Engineering. Software*, Vol. 31, No. 10, 2000, pp. 793-801.

[17] Radziemska, E., and Lewandowski, W.M., The effect of plate size on the natural convective heat transfer intensity of horizontal surfaces, *Heat Transfer Engineering*, Vol. 26, No. 2, 2005, pp 50-53.

[18] Restrepo, F., and Glicksman, L.R., The effect of edge conditions on natural convection from a horizontal plate, *International Journal of Heat and Mass Transfer*, Vol. 17, No. 1, 1974, pp. 135-142.

[19] Rotem, Z., and Claassen, L. Natural convection above unconfined horizontal surfaces, *Journal of Fluid Mechanics*, Vol. 38, No. 1, 1969, pp. 173-192.

[20] Tetsu, F., Hiroshi, H., and Itsuki, M., A theoretical study of natural convection heat transfer from downward-facing horizontal surfaces with uniform heat flux, *International Journal of Heat and Mass Transfer*, Vol. 16, No. 3, 1973, pp. 611-627.

[21] Wei, J.J., Yu, B., Wang, H.S., and Tao, W.Q., Numerical study of simultaneous natural convection heat transfer from both surfaces of a uniformly heated thin plate with arbitrary inclination, *Journal of Heat and Mass Transfer*, Vol. 38, Nos. 4-5, 2002, pp. 309-317.

[22] Yousef, W.W., Tarasuk, J.P., and McKeen, W.J. Free convection heat transfer from upward-facing isothermal horizontal surfaces, *Journal of Heat Transfer*, Vol. 104, 1982, pp. 493-500.

[23] Kobus, C.J., and Wedekind, G.L., An experimental investigation into natural convection heat transfer from horizontal isothermal circular disks, *International Journal of Heat and Mass Transfer*, Vol.44, 2001, pp. 3381-3384.

[24] Kitamura, K., and Kimura, F., Fluid flow and heat transfer of natural convection over upward-facing, horizontal heated circular disks, *Heat Transfer-Asian Research*, Vol. 37, No. 6, 2008, pp. 339-351.

[25] Oosthuizen, P.H., A numerical study of natural convective heat transfer from horizontal isothermal heated elements of complex shape, *Proceedings of the 1st Thermal and Fluids Engineering Summer Conference TFESC 2015*, 2015, Paper TFESC-12863.

[26] Oosthuizen, P.H., Natural convective heat transfer from a horizontal rectangular isothermal element imbedded in a plane adiabatic surface with a parallel adiabatic covering surface, *Proceedings of the ASME 2014 International Mechanical Engineering Congress and Exposition 2014 ASME IMECE*, 2014, Paper IMECE2014-36780.

[27] Oosthuizen, P.H., and Paul, J.T., Numerical study of the effect of vent positioning on the natural convective heat transfer rate from a horizontal isothermal heated surface mounted in a flat adiabatic base and surrounded by a protective surface, *Proceedings of the 13th UK Heat Transfer Conference UKHTC2013*, 2013, Paper UKHTC2013/66.

[28] Oosthuizen P.H., Natural convective heat transfer from an inclined isothermal square flat element mounted in a flat adiabatic surrounding surface, *Proceedings of the 15th International Heat Transfer Conference IHTC-15, 2014*, Paper IHTC15-8499.

[29] Oosthuizen, P.H., 2015, Laminar, transitional and turbulent natural convective heat transfer from a horizontal rectangular isothermal element imbedded in a flat adiabatic surrounding surface, *Proceedings of the 6th ICHMT International Symposium on Advances in Computational Heat Transfer CHT-15, 2015, Paper CHT-15-145*.

[30] Oosthuizen, P.H., A numerical study of natural convective heat transfer from a horizontal isothermal square element with an unheated square adiabatic inner section, *Proceedings of the 10th Int. Conf. on Heat Transfer, Fluid Mechanics and Thermodynamics HEFAT 2015,* 2015, Paper 570075655.

[31] Oosthuizen, P.H., A numerical study of natural convective heat transfer from a horizontal isothermal square element imbedded in an adiabatic surface with a parallel adiabatic covering surface. *Proceedings of the 10th International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics HEFAT 2014*, 2014, pp. 192-199.

[32] Oosthuizen, P.H., Natural convective heat transfer from a horizontal isothermal circular element imbedded in a flat adiabatic surface with a parallel adiabatic covering surface, *Proceedings of the 11th AIAA/ASME Joint Thermophysics and Heat Transfer Conference*, 2014, Paper AIAA 2014-3357.

[33] Oosthuizen, P.H., and Kalendar, A.Y., A numerical study of natural convective heat transfer from a pair of adjacent horizontal isothermal square elements embedded in an adiabatic surface – effect of element spacing on heat transfer rate, *Proceedings of the 11th International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics (HEFAT 2015)*, 2015, Paper 1570075659.

[34] Savill, A.M., Evaluating turbulence model predictions of transition. An ERCOFTAC special interest group project, *Applied Scientific Research*, Vol. 51, 1993, pp. 55-562.

[35] Schmidt, R.C., and Patankar, S.V., Simulating boundary layer transition with low-Reynolds-number *k-ε* turbulence models: Part 1- An evaluation of prediction characteristics, *Journal of Turbomachinery*, Vol. 113, No. 1, 1991, pp. 10-17.

[36] Plumb, O.A., and Kennedy, L.A., Application of a *k-e* turbulence model to natural convection from a vertical isothermal surface, *Journal of Heat Transfer*, Vol. 99, No. 1, 1977, pp. 79-85.

[37] Zheng, X., Liu, C., Liu, F., and Yang, C.-I., Turbulent transition simulation using the k-ω model, *International Journal for Numerical Methods in Engineering*, Vol. 42, No. 5, 1998, pp. 907-926.

[38] Albets-Chico, X., Oliva, A., and Perez-Segarra, C.D., Numerical experiments in turbulent natural convection using two-equation eddy-viscosity models, *Journal of Heat Transfer*, Vol. 130, No. 7, 2008, pp. 072501-1–072401-11.

[39] Oosthuizen, P.H., and Naylor, D., A numerical study of laminar-to-turbulent transition in the flow over a simple recessed window-plane blind system, *Proceedings of the 4th Canadian Solar Buildings Conference*, 2009, Toronto, M. Stylianou, ed., The Solar Buildings Research Network, Montreal.

[40] Xamán, J., Álvarez, G., Lira, L., and Estrada, C., Numerical study of heat transfer by laminar and turbulent natural convection in tall cavities of façade elements, *Energy and Buildings*, Vol. 37, No. 7, 2005, pp. 787-794.