HEAT TRANSPORT THROUGH THE ACTIVE LAYER OF THE MOVING BED IN ROTARY DRUMS

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ABSTRACT

The heat transfer at the freeboard surface of the solid bed was experimentally investigated in a batch rotary drum with a diameter of 0.6 m and a length of 0.45 m. The change of temperature at the surface and inside the bed was measured with Type-K thermocouples and an infrared camera. The thermocouples are specifically arranged at different radial distances from the inner wall. Experiments have been done with a variation of the operational conditions rotational speed from 1 to 6 rpm and filling degree from 5% to 15%. As test materials glass beads, quartz sand, steel spheres and expanded clay were used, whereas the impact of particle size and thermo-physical properties on the heat transport through free bed surface was analyzed. A characteristic effective thermal conductivity was defined for the active layer to represent the heat transport from the free bed surface.

INTRODUCTION

Rotary drums are processing apparatus used for the thermal treatment of bulk materials for processes such as drying, cooling, calcining or sintering in a variety of industries. The process depends on a variety of parameters including rotational speed, filling degree, diameter, length and inclination angle of the drum. These parameters will affect the solid bed motion behavior and the heat transfer as well. The dominant mechanism of movement is rolling motion. A rolling bed is characterized by the continuous flow of particles with a nearly constant slope on the bed surface. The bed can be divided into two zones: a stagnant zone where the particles are transported as a rigid body with the rotation speed of the drum wall and the mixing zone where the particles flow downwards with relatively high velocities. In the stagnant zone the heat is transferred by the contact between the covered wall and solid bed whereas in the mixing zone the heat is transferred by radiation and convection from the gas to the freeboard surface of the solid bed. Therefore, high temperature gradients inside the bed can occur, which will lead to inhomogeneous quality of the product. In order to avoid this behaviour, the heat transport in the different zones of the bed should be analysed.

NOMENCLATURE

A_S	[m²]	Heat transfer area of free solid surface
$c_{p,S}$	[J/kg/K]	Solid specific heat capacity
d_P	[m]	Particle diameter
dT_{AL}/dr_{AL}	[K/m]	Temperature gradient in the active layer
h	[m]	Bed height
L	[m]	Length of the drum
M_S	[kg]	Mass of the solid bed
R	[m]	Drum radius
$t_{contact}$	[s]	Contact time
T	[°C]	Temperature
$\lambda_{eff,AL}$	[W/m/K]	Effective thermal conductivity of active layer
$\dot{q}$	[W/m²]	Heat flux
$\dot{Q}$	[W]	Heat flow
λ_S	[W/m/K]	Solid bed conductivity
ρ_S	[kg/m³]	Solid bed density

Special characters

F	[%]	Filling degree
n	[rpm]	Rotational speed
s_n	[mm]	Thermocouple distance
γ	[rad]	Filling angle

Subscripts

AL	Active layer
eff	Effective
GS	Gas to solid
GW	Gas to wall
P	Particle
R	Regenerative
S	Solid
S,m	Mean solid bed
W	Wall
WS	Wall to solid

The heat transport through the free bed surface can be characterized by the effective thermal conductivity of the moving bed. Many previous works on the thermal conductivity of the packed bed [1-18] were done numerically and experimentally. However, only two investigations were done on the effective thermal conductivity of moving bed [19, 20]. In addition, these two models are not well validated with experimental values and the influence of process parameters such as rotational speed and filling degree was not evaluated. Hence, it is important to study the effective thermal conductivity of moving bed by evaluating the influencing parameters such as rotational speed, filling degree, material and particle sizes.

HEAT TRANSFER MECHANISM AND ROLLING MOTION

The heat transfer mechanism in rotary drums involves many mechanisms including conduction, convection and radiation. Figure 1 shows the schematic heat transfer in a direct heated rotary drum. Heat from flame and combustion products inside the drum is transferred to free solid bed surface and free wall surface by convection and radiation. The combination of convection and radiation results in an additional heat flow from the gas to the solid bed surface, $\dot{Q}_{GS,eff}$ and an additive heat flow from the freeboard gas to the free wall surface, $\dot{Q}_{GW,eff}$. A part of the heat transferred to the free wall surface is absorbed by the wall and the other part is reflected to the free solid bed surface, $\dot{Q}_{WS,\varepsilon}$. A part of the absorbed heat in the wall is conducted through the wall, $\dot{Q}_W$ and proceeds as the heat loss to surrounding by convection and radiation, $\dot{Q}_{Shell,loss}$. The other part is stored in the wall, $\dot{H}_W$ and transported through the rotation of the wall to the contact region between the inner wall and the solid bed, serving as regenerative heat transfer, $\dot{Q}_R$, where the heat is then conducted from the covered wall surface to the covered solid bed, $\dot{Q}_{WS,\lambda}$.

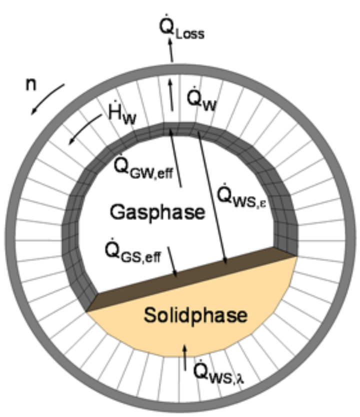


Figure 1 Schematic of the heat transfer in the cross section of a directly heated rotary drum

The free and covered surface of the drum wall and solid bed depends on the filling degree. The filling degree can be defined as the ratio of the cross sectional area of the solid bed to the total cross sectional area of the drum as shown in Figure 2. The geometric relation can be expressed as

$$F = \frac{A_{\text{solid bed}}}{A_{\text{drum}}} = \frac{\gamma - \sin\gamma\cos\gamma}{\pi} \quad \text{with } \gamma = \arccos\left(1 - \frac{h}{R}\right) \qquad (1)$$

with the filling angle γ and the solid bed height h. In rotary drum processes, rolling motion is desirable due to efficient mixing of particles in the solid bed. The rolling motion is characterized by the continuous flow of particles along the surface with a nearly constant slope, defined as the dynamic angle of repose, Θ. This slope is dependent on the material properties of solid bed and the wall friction. The rolling bed can be divided into two zones, namely the stagnant zone (passive layer) and the mixing zone (active layer). These zones are separated by a fictitious boundary line ACB (dotted line), which is approximately symmetrical over the vortex point C. Initially, the particles in the stagnant zone below the bed are transported in the radial direction as a rigid body with the rotational speed of the wall from point B to A. There is no particle mixing in this zone. The solid bed in this zone has no direct contact with the gas/ flame, but it has direct contact with the covered wall surface. Therefore, the heat is transferred through direct contact with the covered wall. After reaching the upper point A, the particles flow downwards parallel to the free bed surface in a thin layer (active layer) with relatively high velocities from point A to B. This is the particle mixing zone where the particles present radial and axial velocity components. Within this layer, the particles are transported axially through the drum length. Extensive experimental studies and modeling of rolling motion were completed by Henein et al. [21, 22], Liu et al. [23] and Mellmann [24].

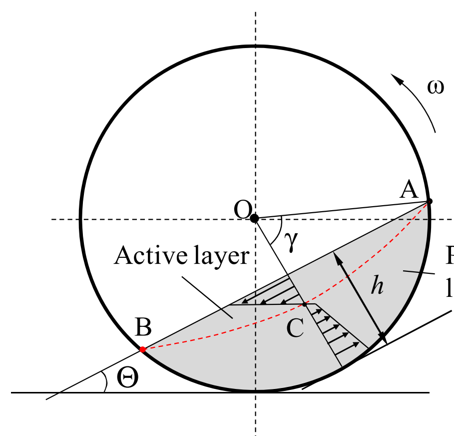


Figure 2 Schematic of the rolling motion in a rotary drum

The active layer predominantly influences mixing inside the bed [26]. With higher rotational speed, *n,* and bed depth, *h*, the absolute thickness of active layer increases, providing higher particle velocities and thus enhancing the particle mixing in this layer. The rolling motion is desirable in industrial applications because of the efficient mixing of the particles in the solid bed. The continuous circulation of the particles in the active layer promotes good mixing and results in a uniform temperature distribution inside the bed. As better heat transfer inside the bed means that a product with high quality can be expected. Furthermore, in order to predict the heat transport in the bed, information on the active layer thickness is necessary. Some models from literature can be used to calculate the active layer thickness [23, 25, 27-29]. The analytical model by Liu et al. [23] is suitable for practical use as no fitting parameter is needed for the calculation.

EXPERIMENTS

Experimental Setup and Test Materials

The heat transport from the free bed surface was investigated in a batch rotary drum as illustrated in Figure 3. The cylindrical drum was made of stainless steel with a wall thickness of 2 mm, an inner diameter of 600 mm and a length of 450 mm. The drum was partially closed on both sides to prevent spillage of material. The drum was heated directly using three ceramic black glaze super high temperature heaters (SHTS), installed in series inside the drum and positioned parallel to the free bed surface. The total capacity of the heaters is 2.47 kW, sufficient to reach a maximum temperature of 700ºC. The test material was heated up from ambient temperature to steady-state condition using a constant heat flux from the internal heating system.

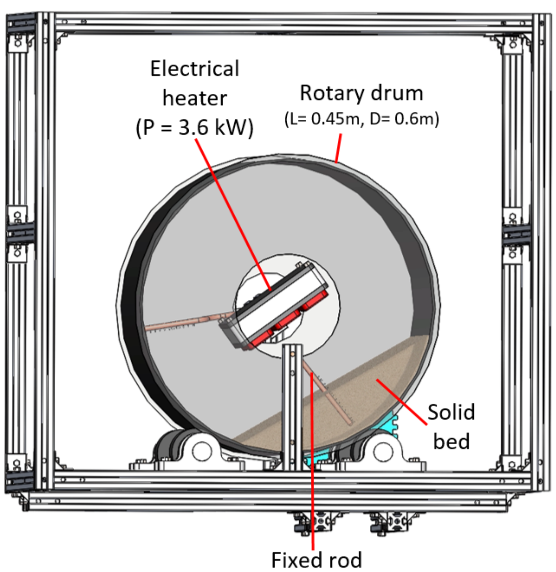
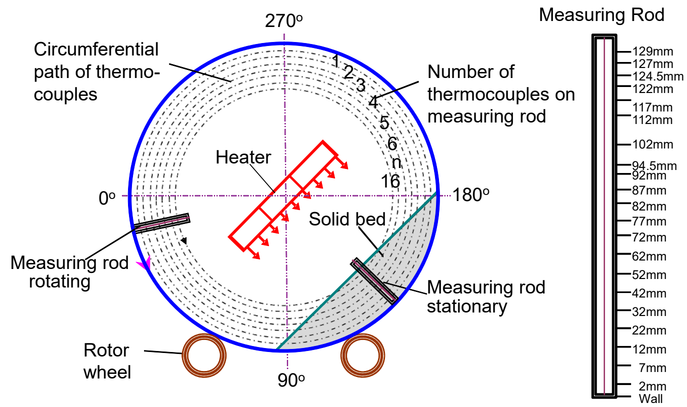


Figure 3 Batch rotary drum for the investigation of heat transport from the free bed surface

The temperatures of the inner wall, air and solid bed were measured by 16 thermocouples attached to two different rods, one rotating and one stationary, which are installed inside the drum. These thermocouples are NiCr-Ni based (Type K), with a diameter of 0.5 mm. They were arranged at specific distances from the inner drum wall as shown in Figure 4(b). The distances of the thermocouples at the upper part of the measuring rod are smaller (2.5 and 5 mm). This adjustment is necessary to detect the temperature gradient in the thin active layer. In relation to the active layer, the passive layer is bigger so that the distance between the thermocouple are 10 mm. With the rotating rod, the radial and circumferential temperature profile of the solid bed could be measured simultaneously. One thermocouple was also installed directly on the inner wall surface of the drum to measure the wall temperature. The stationary rod was installed and positioned in the middle of the solid bed at a 130º circumferential position. This stationary measuring rod was used to assess the delay of the thermocouples at the rotating rod. Fifteen thermocouples were attached to this rod with the same radial spacing of the rotating rod. Figure 4(a) shows the schematic of the batch rotary drum with the internal heater and the thermocouples arrangement.

In addition to the measurement using the thermocouples, the temperatures of the free bed surface, the outer wall and front sides of the drum were measured. For this purpose, the infrared camera Flir SC 3000 and infrared pyrometer Voltcraft IR 260-85 were used. The measured data from Infrared camera Flir SC 3000 was analyzed with ThermaCAM Researcher 2001 analysis software.



(a) (b)

Figure 4 (a) Schematic cross section of the batch rotary drum and (b) the thermocouples arrangement

Table 1 Operating parameters and test materials for the experimental investigation of the heat transport from free bed surface

Parameter	Parameter variation
Rotational speed	1 rpm; 2 rpm; 3 rpm; 4 rpm; 5 rpm; 6 rpm
Filling degree	7.5 %, 10 %; 12 %, 15 %, 20%
Solid material	Quartz sand d_P = 0.2 mm; $\lambda_{S,eff}$ = 0.10 W/m/K; $c_{p,s}$ = 1.08 kJ/kg/K; ρ_S = 1.64 g/cm³ Glass beads d_P = 1.3; 2.0; 3.0; 4.0; 5.0 mm; $\lambda_{S,eff}$ = 0.25 W/m/K; $c_{p,s}$ = 0.80 kJ/kg/K; ρ_S = 1.68 g/cm³ Expanded clay d_P = 3.0 mm; $\lambda_{S,eff}$ = 0.11 W/m/K; $c_{p,s}$ = 0.77 kJ/kg/K; ρ_S = 0.43 g/cm³ Steel spheres d_P = 2 mm; $\lambda_{S,eff}$ = 1.37 W/m/K; $c_{p,s}$ = 0.46 kJ/kg/K; ρ_S = 4.82 g/cm³

The experiments were performed by varying rotational speed and filling degree. Different materials such as quartz sand, glass beads, expanded clay and steel spheres, which were different in particle size and penetration depth, were used as test material.

The experimental parameters and effective thermos-physical properties of the test materials are listed in Table 1.

Experimental Analysis

The heat transport inside the solid bed depends on the mixing in the active layer. Hence, the heat diffusion through this layer is determined by the effective thermal conductivity of the active layer, $\lambda_{eff,AL}$. It is represented by

$$\dot{q} = \frac{\dot{Q}_{Bed}}{A_S} = -\lambda_{eff,AL} \cdot \frac{dT_{AL}}{dr_{AL}} \tag{2}$$

and

$$\dot{Q}_{Bed} = d\dot{H}_S = m_S \cdot c_{p,S} \cdot \frac{dT_S}{dt} \tag{3}$$

where A_S is the heat transfer area of free solid surface and dT_{AL}/dr_{AL} is the temperature gradient across the active layer from the free bed surface to the vortex point, which is the middle point of the boundary line as given by Liu et al. [23]. Radial temperature profiles for the total bed depth are required to evaluate the heat transport inside the bed as shown in Figure 5.

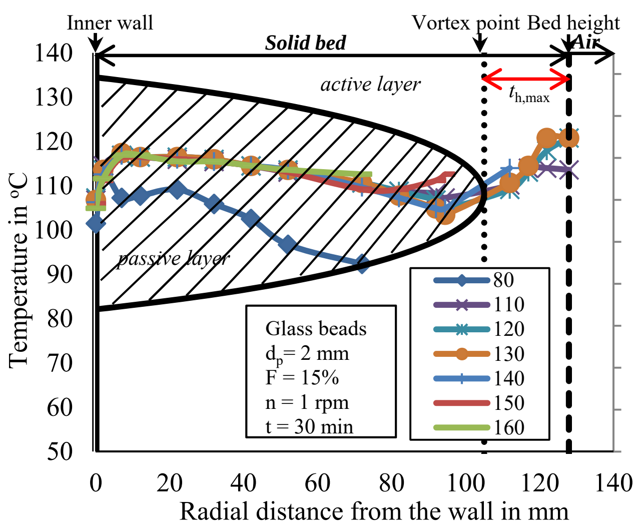


Figure 5 Radial temperature profiles for different circumferential positions

Figure 5 shows the radial temperature profiles of glass beads with a rotational speed of 1 rpm and a filling degree of 15%, 30 minutes after the start of the experiment. The temperature profiles correspond to the solid bed region between 80° to 171° circumferential position. It can be seen that the temperature is highest at the bed surface and decreases towards the vortex point. From the vortex point, the temperature increases until the region near the inner drum wall and then decreases. This is due to the contact heat transfer from the covered solid surface to the covered inner wall as well as heat losses to the surrounding air.

The point between the vortex point and the maximum bed height is the maximum thickness of the active layer. The maximum thickness of the active layer and the temperature differences in this layer are necessary for the calculation of effective thermal conductivity by using Eq. (2).

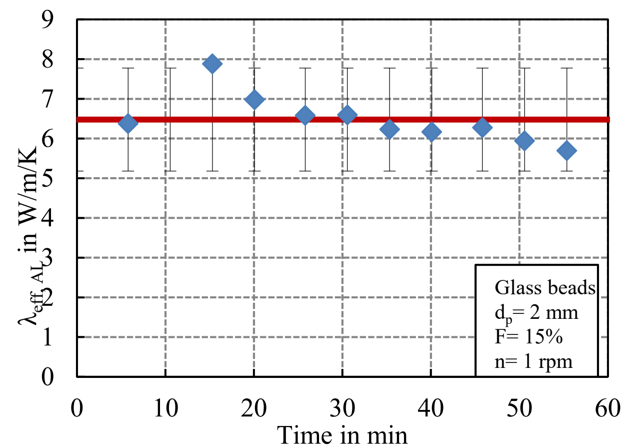


Figure 6 Time curve and regression deviation of the effective thermal conductivity during the experiment.

Figure 6 shows the calculated effective thermal conductivity of the active layer during the experimental time for glass beads with a filling degree of 15% and a rotational speed of 1 rpm. These values are in the range of 20% error from the mean values. It can be seen that the measured values fluctuate. This is due to the oscillation of solid bed temperature as well as the temperature of the free bed surface. Both of these temperatures are very sensitive and have direct influence on the calculation of effective thermal conductivity of active layer as previously shown in Eq. (2).

RESULTS AND DISCUSSION

The influence of rotational speed on the effective thermal conductivity is shown in Figure 7 for quartz sand material with a constant filling degree of 15%. It is apparent that the effective thermal conductivity increases with higher rotational speed. With higher drum rotation, the velocity of particles in the active layer increases. Hence, the intensity of mixing inside the active layer increases and more particle-particle interaction and heat diffusion occur inside the layer. Consequently, a higher effective thermal conductivity can be obtained with higher rotational speed. The measured values are in the range of 20% error from the regression line values. However, these measured values seem quite high. At this time, the measurements by the pyrometer were failed and thus the values only based on the surface temperature measured by the thermocouples located near the free bed surface. Hence, due to the drum rotation and not exactly flat surface of the bed, the thermocouples may not exactly measure the bed surface, thus resulting in higher effective thermal conductivities than is actually the case.

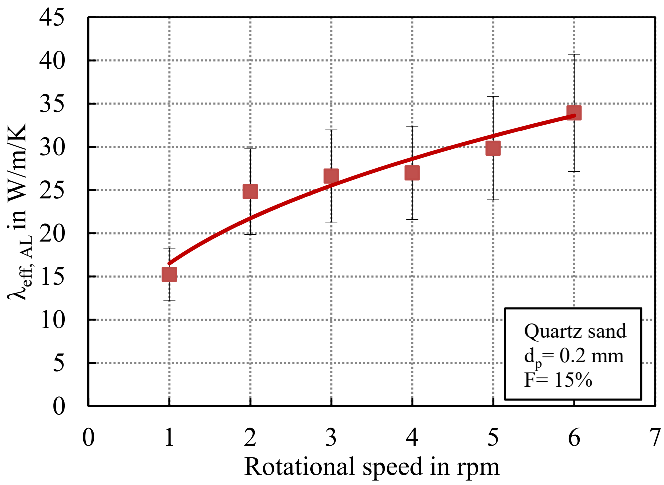


Figure 7 Influence of rotational speed on effective thermal conductivity of the active layer

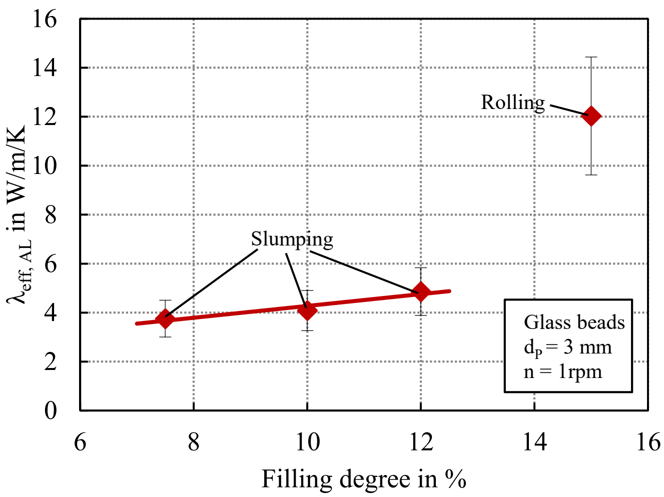


Figure 8 Influence of filling degree on effective thermal conductivity of the active layer

Figure 8 shows the influence of filling degree on the effective thermal conductivity. This figure corresponds to glass beads with particle size d_P = 3 mm with a constant filling degree of F = 15 % and a constant rotational speed of n = 1rpm. It was expected that with higher filling degree, the absolute thickness of the active layer, the number of particles and the mass of the solid bed increase, but the number of bed rotation decreases, thus resulting in lower effective thermal conductivity. However, the effective thermal conductivity from the measurements increases slightly with higher filling degree. One reason for this could be that, with a higher filling degree, the potential energy of the particles on the upper edge of the bed surface increases and the relative thickness of the active layer decreases. Hence, the particles flow down the bed surface with higher velocity and more collisions happen between the particles. This situation leads to higher effective thermal conductivity. Higher particle velocity with an increase in active layer thickness was previously investigated by Liu et al. [23]. It also can be shown that the values for 7.5 to 12 % are very low compared to 15%. This is due to different motion behavior; at 15% the bulk bed moves in rolling motion while the other filling degrees lead to a slumping motion type. Thus, the effective thermal conductivity for 15% is greater as the continuous flow of particles at the free surface enhanced the heat transport inside the bed. As can be seen from this figure, the measured values for slumping motion are in the range of 20% of the regression line values.

Figure 9 shows the influence of the particle size on the effective thermal conductivity of active layer. The values are shown for glass bead material with a constant filling degree of 15 %, a constant rotational speed of 1 rpm and particle sizes from 1.3 mm to 4 mm. It can be seen that the effective thermal conductivity increases with bigger particle size. As particle size increases, the number of particles inside the bed reduces, thus increasing the particle velocity in the active layer. Higher particle velocity provides more particle collisions and increases the intensity of mixing inside the active layer. Hence, better heat transport occurs inside the bed, as shown by higher effective thermal conductivity. All the measured values are in the range of 20% of the regression line.

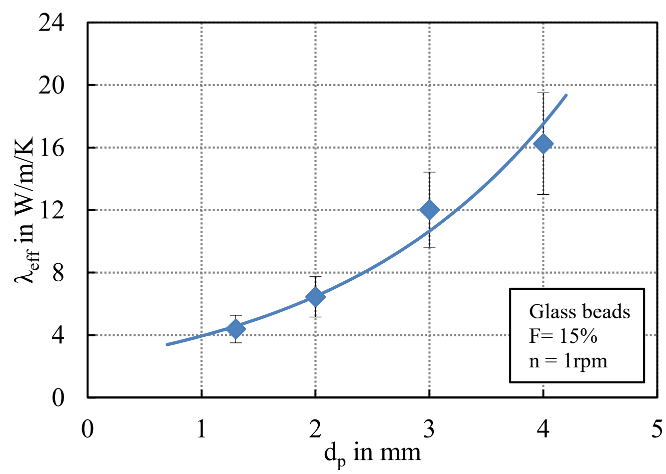


Figure 9 Influence of particle size on effective thermal conductivity of the active layer

The influence of packed bed thermal properties in terms of penetration depth, $\sqrt{\lambda_S \rho_S c_{p,S}}$ on the effective thermal conductivity of the active layer is shown in Figure 10. The values are depicted for expanded clay (d_P = 3 mm), glass beads (d_P = 2 mm) and steel spheres (d_P = 2 mm) for a constant filling degree of 15% and 2 rotational speeds (1 and 6 rpm). These materials have wide range of thermal properties. From this figure, it is clear that the effective thermal conductivity increases with the rotational speed as discussed before. In addition, the effective thermal conductivity is higher with higher penetration depth. This is due to better heat penetration and heat conduction through the active layer. Therefore, higher effective thermal conductivity of moving bed was obtained with higher penetration depth.

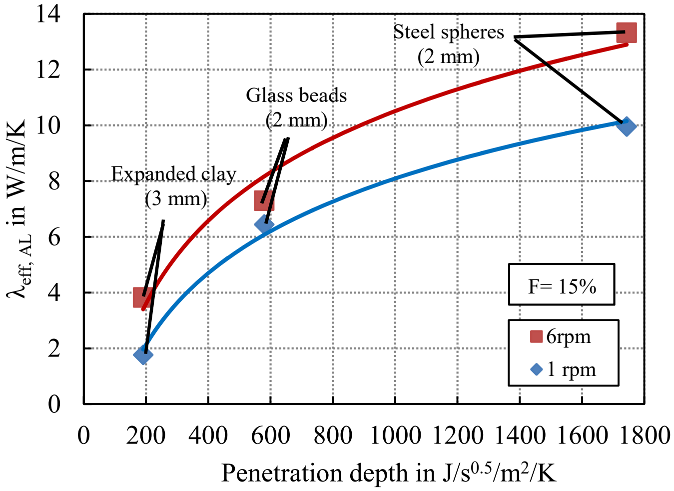


Figure 10 Influence of material thermal properties on the effective thermal conductivity of the active layer

CONCLUSION

The heat transport through the free bed surface was experimentally investigated in a direct heated batch rotary drum. A characteristic effective thermal conductivity of the active layer was defined. A variety of materials such as glass beads, steel spheres, quartz sand and expanded clay were used as test material. The particle size, filling degree and rotational speed was varied to show the influence of these parameters on heat transport from the free bed surface. It is shown that the effective thermal conductivity increases with higher rotational speed due to the higher amount of bed circulation and the intensity of mixing inside the bed. In addition, due to the higher rotational speed, particles inside the active layer will have higher velocity, thus increasing particle-particle interaction and heat diffusion and resulting in higher effective thermal conductivity. Higher filling degree results in higher effective thermal conductivity as the potential energy of the particles at the upper edge of free bed surface increases, leading to more collisions between the particles. With bigger particle size, the number of particles inside the active layer is reduced, thus leading to more particle-particle interaction in the active layer and to higher effective thermal conductivity. Furthermore, the effective thermal conductivity of the active layer increases with higher thermos-physical properties of the solid bed since the heat penetration and heat conduction through the active layer was improved.

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