HEAT EXCHANGER NETWORKS USING HYBRID META-HEURISTIC APPROACH

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ABSTRACT

Meta-heuristic approaches have been used to achieve good solutions in the heat exchanger network (HEN) synthesis task. Several meta-heuristic approaches have been proposed in the literature. Two of the most important techniques are Simulated Annealing (SA) and Particle Swarm Optimization (PSO). In general, SA is able to provide good solutions, but with large computational efforts. PSO is faster than SA in finding good solutions, but it is not capable of handling discrete variables properly. In the present work, a bi-level HEN synthesis approach is presented. SA is used to a single heat exchanger addition, along with group optimizations to improve PSO performance. A parallel processing technique is also presented in order to improve local searching performance. The method was tested in 3 literature case studies and results were compared to literature solutions. The solutions presented have lower Total Annual Costs (TAC) when compared to other HEN. The proposed method is able to present near-optimal solutions by more efficiently exploring the search space and using simple moves for local searches.

INTRODUCTION

The Heat Exchanger Network (HEN) problem has been studied by various researchers through the last decades. Even in the most simplified formulations, it is a sort of problem which requires elaborate methods to obtain even satisfactory solutions. Global minima are rather difficult to obtain, given the non-convexities in the objective function and nonlinearities in its constraints. Even with such difficulties, obtaining optimal solutions for HEN synthesis problem is a rewarding task, given the magnitude of the potential reduction in both utilities and capital costs, as well as in greenhouse gases emissions.

In order to achieve better solutions, a wide range of methods have been proposed in the literature. These methods may vary from heuristics and thermodynamic approaches, mathematical programming formulations and solving methods and purely computational stochastic and meta-heuristic methods. Most of these meta-heuristic approaches do not require elaborate mathematical concepts such as derivatives. Such characteristics make such approaches attractive. Although in some cases much computational effort may be required, these methods have proved reliable in solving HEN synthesis problems.

NOMENCLATURE

A	[m^2]	Heat exchanger area
Acu	[m^2]	Cooler area
Ahu	[m^2]	Heater area
B	[\$]	Equipment fixed cost
C	[\$/m^{2}y]	Annualized capital cost
Ccu	[\$/W]	Cold utility cost
Chu	[\$/W]	Hot utility cost
CP	[W/m^{2}K]	Stream heat load
Q	[W]	Heat exchanger heat load
Qcu	[W]	Cooler area
Qhu	[W]	Heater area
Tin	[K]	Inlet temperature
$Tout$	[K]	Outlet temperature
w	[-]	HE presence binary 2-D matrix
z	[-]	HE presence binary 1-D matrix
zcu	[-]	Cooler presence binary vector
zhu	[-]	Heater presence binary vector

Special characters

β	[-]	Capital cost exponent
θ	[K]	Exchanger approach temperature

Subscripts

m	Exchanger number, from left to right
i	Hot stream
j	Cold stream
k	Stage number

Many authors have proposed different algorithms using meta-heuristics on HEN synthesis and some works must be highlighted. Simulated Annealing was applied and has achieved good solutions in the works of Dolan [1] and Athier et al. [2]. The concept of two-level HEN design, which will be presented further, was proposed in the work by Lewin [3], where Genetic Algorithms (GA) was used in both levels. Yerramsety and Murty [4] used Differential Evolution (DE) in both HEN synthesis levels. Ravagnani et al. [5] combined GA with Pinch Technology [6] heuristics. Silva et al. [7] first applied Particle Swarm Optimization (PSO) to HEN synthesis. Huo et al. [8] used a hybrid two-levels GA/PSO method. Khorasany and Fesanghary [9] hybridized Harmony Search (HS) and Sequential Quadratic Programming (SQP).

In this work a stochastic hybrid method is proposed. Its main features are the simplicity of both its formulation and solution strategy. The ease of implementation is also worth noting.

The method uses a no-split formulation, which is more efficient in regards to piping complexity and costs. The two-level solution approach uses both combinatory and continuous methods. Simulated Annealing (SA), an essentially combinatorial method is applied to an empty HEN solution. The only neighbourhood exploring move is adding a new heat exchanger. However, this is only the combinatory stage of the problem. Particle Swarm Optimization (PSO), which is mainly used for continuous optimization, is then applied to each match combination that SA proposes in order to find the best heat exchanger heat loads and areas for that neighbourhood move.

SUPERSTRUCTURE AND OBJECTIVE FUNCTION

The stage-wise superstructure (SWS) for HEN synthesis, originally proposed by Yee and Grossmann [10], and its variants is used by many authors because of its simplicity. When a reliable solution method is applied, this formulation usually leads to good HEN designs.

In this work, the super-structure is modified. In its original form, the SWS includes all possible matches in a single stage, each in a single stream branch after a splitter. Here, all matches are also possible, but in series, with no splits. The SWS used is presented in Figure 1.

HEN with no splits are efficient, simpler to implement and cheaper regarding piping costs. Another advantage is that this model requires less computational effort to be solved, and is suitable to solve large HEN synthesis problems.

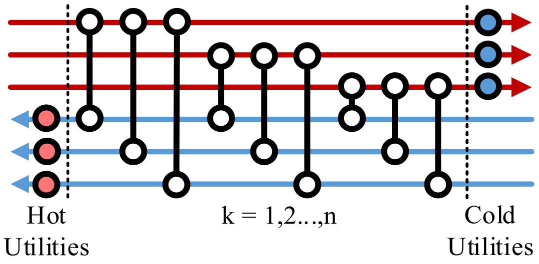

Figure 1 Stage-Wise Superstructure

In general, SWS HEN formulations use a three-dimension matrix to represent variables regarding to each possible match, where usually *i* stands for hot streams, *j* for cold streams and *k* for the stage number. However, for large HEN synthesis problems, this may generate large sparse matrixes which consume much computer memory and increase computation time. In this work, when the topology is defined, the variables are set in two matrixes with two dimensions, i.e. $Qh_{m,j}$ and $Qc_{m,j}$ instead of $Q_{i,j,k}$, where *m* is the heat exchanger number, from left to right. The proposed objective function is presented in Eq. 1.

$$\begin{aligned} min:\ & \sum_i C_{cu}\cdot Qcu_i + \sum_j C_{hu}\cdot Qhu_j \\ & + \sum_m z_m\cdot\left(B + C\cdot A_m{}^{\beta}\right) \\ & + \sum_i zcu_i\cdot\left(B + C\cdot Acu_i{}^{\beta}\right) \\ & + \sum_j zhu_j\cdot\left(B + C\cdot Ahu_j{}^{\beta}\right), \\ & m\in N_{HE}, i\in N_H, j\in N_C \end{aligned} \tag{1}$$

With this approach, it is also possible to explicitly calculate outlet temperatures for each stream with the energy balances in Eqs. (2) to (5).

$$Tin_{m,i} = Tout_{m-1,i}, \qquad m\in N_{HE}, i\in N_H \tag{2}$$

$$Tout_{m,i} = Tin_{m,i} + w_{m,i}\frac{Q_{m,i}}{CP_i}, \qquad m\in N_{HE}, i\in N_H \tag{3}$$

$$Tin_{m-1,j} = Tout_{m,j}, \qquad m\in N_{HE}, j\in N_C \tag{4}$$

$$Tout_{m-1,j} = Tin_{m,j} + w_{m,j}\frac{Q_{m,j}}{CP_i}, m\in N_{HE}, j\in N_C \tag{5}$$

With the calculated outlet temperatures, it is possible to calculate the *LMTD* and the areas with Eqs. 6 and 7. Chen's [11] approximation is used for the *LMTD* because it led to a slightly better PSO performance.

$$LMTD_m = \left(\frac{\theta_{1,m}\theta_{2,m}\left(\theta_{1,m}+\theta_{2,m}\right)}{2}\right)^{\frac{1}{3}}, \quad m\in N_{HE} \tag{6}$$

$$A_m = \frac{z_m Q_m}{U_m\, LMTD_m}, \qquad m\in N_{HE} \tag{7}$$

There are also infeasibility constraints regarding the Second Law of Thermodynamics (Eqs. 8 and 9), and maximum heat for each heat exchanger and heater/cooler (Eqs. 10 to 13).

$$Tin_{m,i} \geq Tout_{m,j} + EMAT, \quad m\in N_{HE}, i\in N_H, j\in N_C \tag{8}$$

$$Tout_{m,i} \geq Tin_{m,j} + EMAT, \quad m\in N_{HE}, i\in N_H, j\in N_C \tag{9}$$

$$0 \leq Q_{m,i} \leq CP_i\cdot\Delta T_i, \qquad m\in N_{HE}, i\in N_H \tag{10}$$

$$0 \leq Q_{m,j} \leq CP_j\cdot\Delta T_j, \qquad m\in N_{HE}, j\in N_C \tag{11}$$

$$0 \leq Qu_i \leq \sum CP_i\cdot\Delta T_i, \qquad i\in N_H \tag{12}$$

$$0 \leq Qu_j \leq \sum CP_j \cdot \Delta T_j\,, \qquad j \in N_C \tag{13}$$

HYBRID META-HEURISTIC APPROACH

Upper-level combinatorial optimization

In the upper level, a combinatorial method is used to define what heat exchangers from the SWS are present. In this work, Simulated Annealing is used.

SA was developed by Kirkpatrick et al. [12] to solve combinatorial problems with the Metropolis algorithm [13]. It is an analogy with annealing processes, where a solid has its internal energy minimized through slow cooling. Large temperatures mean chaotic atomic configurations. In optimization cases, a higher Temperature (T) parameter means that in every new solution tested, there are higher chances of accepting worst results. Such “uphill” moves are necessary in order to avoid local minima. A better solution is automatically accepted. The probability of acceptance function for HEN synthesis is calculated with Eq. (14).

$$PoA(\Delta TAC, T) = e^{\frac{-\Delta TAC}{T}} \tag{14}$$

For each Temperature, there are a number of moves to be performed. This number is called Temperature Length (*TL*). After these moves are performed, temperature is reduced with Eq. (15).

$$T_{k+1} = \alpha\, T_k \tag{15}$$

Lower-level continuous optimization

In the lower level, a continuous meta-heuristic approach must be applied in order to find optimal values for the heat loads of the exchangers proposed by upper level algorithm. To do such, Particle Swarm Optimization is used.

PSO was developed by Kennedy and Eberhart [14] and has a natural analogy with the behaviour of animals seeking for resources. When a particle finds a better spot, i.e. a better solution, it becomes the leader and the other particles have their behaviour influenced by it.

Initially, a population of particles is generated in random positions within the searching space. Their velocities and positions are updated every iteration with Eqs. 16 and 17 and the ω parameter is updated with Eq. 18, as proposed by Shi and Eberhart [15].

$$\begin{aligned} v_{k+1}^{(i)} = \omega_k v_{k+1}^{(i)} &+ c_1 r_1\left(p_k^{(i)} - x_k^{(i)}\right) \\ &+ c_2 r_2\left(p_k^{global} - x_k^{(i)}\right) \end{aligned} \tag{16}$$

$$x_{k+1}^{(i)} = x_k^{(i)} - v_{k+1}^{(i)} \tag{17}$$

$$\omega_{k+1} = \omega_{max} - \frac{\omega_{max} - \omega_{min}}{K}(k+1) \tag{18}$$

PSO also needed improvements in order to better avoid local minima and handle with constraints. A simple condition was added to the code to reset particles’ velocities when the global best is not improved by a given number of iterations, repelling the particles when they are stagnated on local minima. Regarding to constraints handling, penalty functions were added. Since they grow as particles go further from feasible range, the solutions are able to easily be guided back to the valid areas. The penalty function generalized form is presented in Eq. 19.

$$pen(X) = A + B \cdot \left(X^{Bound} - X\right)^2 \tag{19}$$

Where X is a generic variable, X^{Bound} is the maximum or minimum value it can assume, and A and B are constant parameters.

Algorithm Description

The method starts with an empty HEN solution. SA proposes the addition of a random HE. This solution is divided among the processing cores, and each will perform an independent exploration. The new topology needs to be optimized, which is done with PSO. If PSO succeeds in finding a solution with lower TAC than the previous, SA accepts that solution. Otherwise, it is accepted according to the probability of acceptance function. In the developed approach, the only possible SA move is adding a heat exchanger. There is no need for deletion, since PSO will eventually find solutions where a heat exchanger heat load is zero. When this occurs, that match is deleted from the topology. PSO may also fail to converge to an optimal solution. Premature PSO convergence to exchangers with zero heat loads sometimes is, in fact, beneficial. In such situations, SA may accept the worse solution and escape from being stuck in a local minima neighbourhood. For these reasons, no deletion moves were needed in the proposed method. When all the moves allowed by the Temperature Length parameter are performed, temperature is reduced. The best current solution among the processing cores is then copied and will be used by all cores in the next temperature. If T is lower than a given value, the algorithm returns the best solution found. Two flowcharts are used to present the method. In Figure 1, the main process is depicted, while Figure 2 shows the procedure that is carried out in each processor core during a whole temperature length.

CASE STUDIES

In order to test the proposed procedure performance and reliability, it was applied to three case studies from literature cases. Final solutions are compared to the ones provided by previous authors’ methods both for HEN with and without splits. Results are presented at the end of each case. Experiments were carried out in a 4 cores 3.5GHz Intel i5 4690 computer with 8GB of RAM.

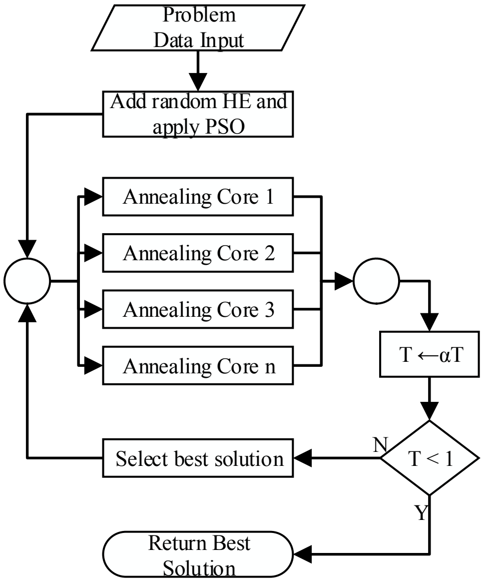

Figure 2 Main process

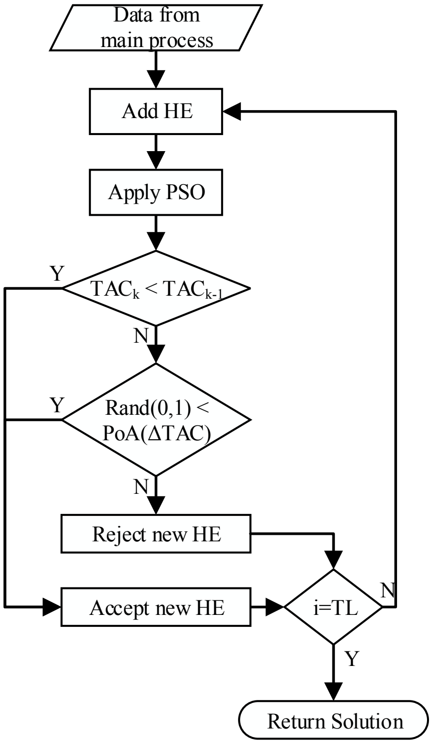

Figure 3 Procedure in each core

Case Study 1

Case study 1 is the classic 9 streams aromatics plant problem, proposed by Linnhoff and Ahmad [16] and investigated by numerous authors. Problem data is presented in Table 1. The solution with best TAC so far has stream splits and was obtained by Petterson [17] by using a slightly different HEN synthesis formulation, which is not based in SWS models, and linearized sub-problems. The authors also changed the final hot stream temperature. Moreover, his solution has splits, as well as series of HE on single branches of split streams.

Table 1. Data for Case Study 1

Stream #	Tin (C)	Tout (C)	CP (kW/K)	h (kW/m^2 K)
H1	327	40	100	0.50
H2	220	160	160	0.40
H3	220	60	60	0.14
H4	160	45	400	0.30
C1	100	300	100	0.35
C2	35	164	70	0.70
C3	85	138	350	0.50
C4	60	170	60	0.14
C5	140	300	200	0.60
HU	350	250		0.50
CU	15	30		0.50
Area Cost		2000+70 A		
Utility Costs		60 HU+6 CU		

As aforementioned, such characteristics increase HEN complexity and piping implementation costs, which are not taken into account when calculating TAC. The solution obtained with the proposed method is much simpler with less units, but with slightly higher TAC. It was obtained in 5,914s. However, when only other solutions with no-splits in literature are considered, the best results were presented by Huo et al. [8]. The developed method is able to lead to lower TAC. The obtained solution is depicted in Figure 4 and is the best reported so far to the aromatics plant when considering only HEN with no splits. The results are compared to literature in Table 2.

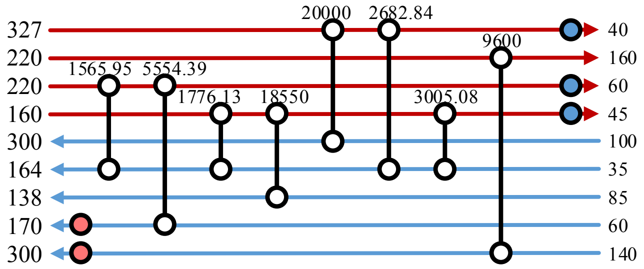

Figure 4. Solution to Case Study 1

Table 2. Results comparison for Case Study 1

	TAC (M$/y)	Units
Linnhoff and Ahmad*[16]	2.93	13
Zhu et al.*[18]	2.98	14
Zhu et al.[18]	2.98	10
Lewin*[3]	2.936	12
Lewin[3]	2.946	11
Pettersson**[17]	2.905	17
Yerramsetty and Murty[4]	2.942	15
Huo et al.*[8]	2.922	13
Huo et al.[8]	2.936	11
This work	2.930479	13

***Works with splits**
****Works with different formulations**

Case Study 2

This is 10 streams problem with 6 hot and 4 cold streams. It was proposed by Ahmad [19]. In its formulation there are no fixed costs for heat exchangers and no area costs exponent. These formulation parameters usually penalize solutions with too many units, as well as small sized heat exchangers. Thus, it is expected that solutions to this problem have a larger number of units, being some with small areas. Although such costs data may not be the most realistic industrial scenario, it is a classic benchmark problem and it is going to be useful for testing the proposed approach efficiency. Problem data is presented in Table 3.

Table 3. Data for Case Study 2

Stream #	Tin (C)	Tout (C)	CP (kW/K)	h (kW/m^2 K)
H1	85	45	156,3	0,05
H2	120	40	50	0,05
H3	125	35	23,9	0,05
H4	56	46	1250	0,05
H5	90	85	1200	0,05
H6	225	75	50	0,05
C1	40	55	466,7	0,05
C2	55	65	600	0,05
C3	65	165	180	0,05
C4	10	170	81,3	0,05
HU	200	199		0,05
CU	15	25		0,05
Area Costs			60 A^1	
Utility Costs			100 UQ+15 UF	

The proposed approach was able to achieve better results than the literature. The optimal solution was found in 1,637s and has 16 units, which is a rather large number for a problem with this size. However, it has less units and lower TAC than both the solutions with and without splits proposed by Huo et al. [8]. The optimal solution is depicted in Figure 5. Results comparison is presented in Table 4.

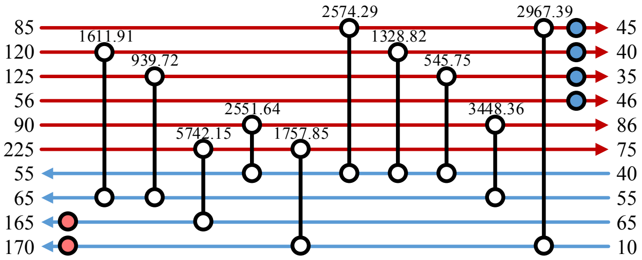

Figure 4. Solution to Case Study 2

Case Study 3

This problem was proposed by Castillo et al. [20] and it regards a nitric acid plant. It has 11 streams, being 6 hot and 4 cold. Data is presented in Table 5. The authors were able to reduce energy costs by using Pinch Analysis in their study. Silva et al. [7] used Particle Swarm Optimization to achieve better results with stream splits.

Table 4. Results comparison for Case Study 2

	TAC ($/y)	Units
Ahmad [19]	7,074,000	
Ravagnani et al. [5]	5,672,821	13
Yerramsetty and Murty [4]	5,666,756	12
Khorasany and Fesanghary[9]	5,662,366	12
Huo et al. [8]	5,657,486	13
Huo et al. [8]*	5,645,688	18
This work	5,622,043	16

***Works with splits**

However, the authors did not present stream splitting fractions, which makes it impossible to directly recalculate the TAC of their HEN with the reported data.

In order to better compare solutions and TAC achieved by the authors and by this work, a revision method may be applied. It is possible to lock heat load variables and find optimal stream splits for those values. Since there are only 2 splits, with 2 branches each, only 2 independent variables need to be optimized. PSO is reliable in this case. The best split configuration for the HEN proposed by Silva et al.[7] led to TAC slightly higher than those reported. That means there were probably errors in their costs calculation. However, their solution is still better than that presented previously by Castillo et al. [20]. The SA/PSO hybrid method is then applied in order to find better results. The optimal HEN found in this work was achieved in 1,450s and is depicted in Figure 6. Results comparison is presented in Table 6.

Table 5. Data for Case Study 3

Stream #	Tin (K)	Tout (K)	CP (kW/K)	h (kW/m2 K)
H1	1113	313	4,9894	1,5
H2	349	318	4,6840	1,5
H3	323	313	0,7720	1,5
H4	453	350	0,6097	1,5
H5	453	452	292,70	0,8
H6	363	318	3,066	1,5
C1	297	298	329,8	0,8
C2	298	343	0,5383	1,5
C3	308	395	3,7270	1,5
C4	363	453	0,6097	1,5
C5	453	454	2581,1	0,8
HU	503	503		1,5
CU	293	313		0,8
Area Costs			9094+485 $A^{0.81}$	
Utility Costs			110 HU+15 CU	

CONCLUSION

A hybrid meta-heuristic approach for automated HEN synthesis with no stream splits was developed and applied to literature cases. It proved reliable in solving the three examples and led to solutions better than those previously reported by other authors who achieved no-split solutions. In two cases, the TAC achieved were even lower than the those of best solutions with splits. The parallel processing strategy provides the upper level optimization approach (Simulated Annealing) with a wider exploration, hence, better HEN topologies are more likely to be found. Particle Swarm Optimization improvements and penalty functions also made the algorithm more robust in finding feasible optimal solutions efficiently. Simulated Annealing is a time consuming meta-heuristic, however, in the developed method, total processing times were always lower than 2 hours, which is satisfactory. The method presented was reliable and may serve as a basis for further investigation. The algorithm may be expanded to also use stream splits or multi-objective formulations for HEN synthesis.

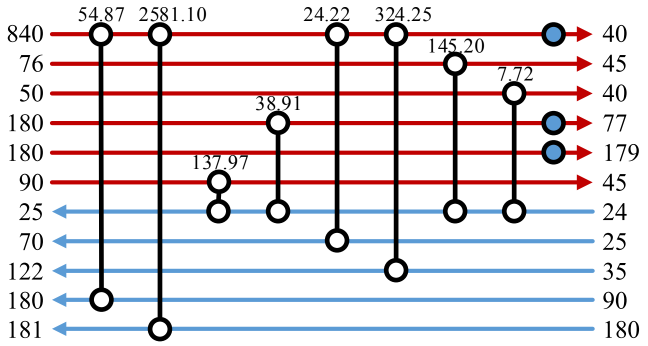

Figure 5. Solution to Case Study 3

Table 6. Results comparison for Case Study 3

	TAC ($/year)	Units
Castillo et al. [20]	141,554	11
Silva et al. *^{a}[7]	140,142	11
This work	139,838	11

***Solutions with stream splits**
^{a}Revised solution

REFERENCES

[1] Dolan, W. B. , Cummings, P. T. , and LeVan, M. D. Process optimization via simulated annealing: Application to network design. *AIChE Journal* 1989, Vol.35, pp. 725–736.

[2] Athier, G. , Floquet, P. , Pibouleau, L. , and Domenech, S. Process optimization by simulated annealing and NLP procedures. Application to heat exchanger network synthesis. *Computers & Chemical Engineering* 1997, Vol.21, pp. S475–S480.

[3] Lewin, D. R. A generalized method for HEN synthesis using stochastic optimization - II. The synthesis of cost-optimal networks. *Computers and Chemical Engineering* 1998, Vol.22, pp. 1387–1405.

[4] Yerramsetty, K. M. and Murty, C. V. S. Synthesis of cost-optimal heat exchanger networks using differential evolution. *Computers & Chemical Engineering* 2008, Vol.32, pp. 1861–1876.

[5] Ravagnani, M. A. S. S. , Silva, A. P. , Arroyo, P. A. , and Constantino, A. A. Heat exchanger network synthesis and optimisation using genetic algorithm. *Applied Thermal Engineering* 2005, Vol.25, pp. 1003–1017.

[6] Linnhoff, B. and Hindmarsh, E. The pinch design method for heat exchanger networks. *Chemical Engineering Science* 1983, Vol.38, pp. 745–763.

[7] Silva, A. P. , Ravagnani, M. A. S. S. , Biscaia, E. C. , and Caballero, J. A. Optimal heat exchanger network synthesis using particle swarm optimization. *Optimization and Engineering* 2009, Vol.11, pp. 459–470.

[8] Huo, Z. , Zhao, L. , Yin, H. , and Ye, J. Simultaneous synthesis of structural-constrained heat exchanger networks with and without stream splits. *The Canadian Journal of Chemical Engineering* 2013, Vol.91, pp. 830–842.

[9] Khorasany, R. M. and Fesanghary, M. A novel approach for synthesis of cost-optimal heat exchanger networks. *Computers & Chemical Engineering* 2009, Vol.33, pp. 1363–1370.

[10] Yee, T. F. and Grossmann, I. E. Simultaneous optimization models for heat integration—II. Heat exchanger network synthesis. *Computers & Chemical Engineering* 1990, Vol.14, pp. 1165–1184.

[11] Chen, J. J. J. Comments on improvements on a replacement for the logarithmic mean. *Chemical Engineering Science* 1987, Vol.42, pp. 2488–2489.

[12] Kirkpatrick, S. , Gelatt, C. D. , and Vecchi, M. P. Optimization by simulated annealing. *Science (New York, N.Y.)* 1983, Vol.220, pp. 671–80.

[13] Metropolis, N. , Rosenbluth, A. W. , Rosenbluth, M. N. , Teller, A. H. , and Teller, E. Equation of State Calculations by Fast Computing Machines. *The Journal of Chemical Physics* 1953, Vol.21, pp. 1087.

[14] Kennedy, J. and Eberhart, R. Particle swarm optimization, *Proceedings of IEEE International Conference on Neural Networks,* 1995, Vol. 4, pp. 1942-1948.

[15] Shi, Y. and Eberhart, R. C. Parameter selection in particle swarm optimization. *In Evolutionary Programming VII SE - 57*; 1998; Vol. 1447, pp. 591–600.

[16] Linnhoff, B. and Ahmad, S. Cost optimum heat exchanger networks—1. Minimum energy and capital using simple models for capital cost. *Computers & Chemical Engineering* 1990, Vol.14, pp. 729–750.

[17] Pettersson, F. Synthesis of large-scale heat exchanger networks using a sequential match reduction approach. *Computers & Chemical Engineering* 2005, Vol.29, pp. 993–1007.

[18] Zhu, X.X., O'neill, B.K., Roach, J.R., and Wood, R.M. Area-targeting methods for the direct synthesis of heat exchanger networks with unequal film coefficients. *Computers & Chemical Engineering*. , 19(2) , 223–239. (1995).

[19] Ahmad, S. Heat Exchanger Networks, Cost Tradeoffs in Energy and Capital, 1985.

[20] Castillo, E. F. , Acevedo, L. , and Reverberi, A. P. Cleaner Production of Nitric Acid by Heat Transfer Optimization: A Case Study. *Chemical and Biochemical Engineering Quarterly* 1998, Vol.12, pp. 157–163.