

# A NOVEL EXTREMUM SEEKING SCHEME FOR CLOSED-LOOP JET CONTROL

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## ABSTRACT

Jet control is of great interest to a wide range of industrial and environmental applications. This work presents an investigation on the closed-loop control of a turbulent round jet for enhancing the jet mixing. The Reynolds number  $Re_D$  range is 7333 to 9600 based on the jet exit diameter  $D$  and the exit velocity  $U_e$ . A single unsteady minijet is deployed upstream of the jet exit, which is produced through the on-off state of an electromagnetic valve. The open-loop control is investigated first. It has been found that, given the duty cycle  $a$  and volumetric flow rate ratio  $C_m$  of the minijet to that of the main jet, the decay rate  $K$  of jet centerline mean-velocity exhibits a maximum at the frequency ratio  $f_e/f_0 \approx 0.5$ , where  $f_e$  and  $f_0$  are the excitation frequency of the minijet and the preferred mode frequency of the natural main jet, respectively. A novel extremum seeking closed-loop scheme is then investigated, with a view to achieving the optimal jet control performance automatically. The feedback signal is provided by the jet decay rate  $K_I$  of jet centerline running-mean velocity measured by a hot-wire, based on which the output voltage signal controls the frequency of the electromagnetic valve. Comparing with classical extremum seeking schemes, the present scheme introduces an input estimation error to adjust adaptively the excitation frequency, thus reducing greatly the steady-state oscillation and hence enhancing  $K_I$ . Furthermore, this scheme is robust when  $Re_D$  and initial  $f_e$  are separately changed.

## INTRODUCTION

As one of typical basic shear flows, jet mixing is widely seen in engineering, e.g. in aero and automobile engines, combustion, heat transfer and chemical reactors. Naturally, jet mixing enhancement has received large numbers of attention in literature.

Passive techniques were firstly applied to enhance jet mixing, such as deploying tabs [1] and using noncircular nozzles at the nozzle exit [2]. Azimuthal non-uniformities at the jet exit may have a dramatic impact on the evolution of shear layers, because the three-dimensional jet development is particularly sensitive to initial conditions. Noncircular jets are naturally more unstable than the circular counterpart [3], which exhibit shorter potential core length due to faster velocity decay. In practice, the operation state of the jet often vary, e.g., burners used in the process industries. The operation state of

the actuator is difficult to change arbitrarily by passive control according to the variation of initial conditions. In order to overcome the drawback of the passive control, some active control techniques were proposed, including acoustic excitation [4], synthetic jet actuators [5] and plasma actuators [6]. For some applications, the pulsed or unsteady jet may be used to improve the actuator efficiency [7]. Zhou et al. [8] use unsteady minijets to enhance jet mixing of a round jet, finding the quasi-steady longitudinal counter-rotating vortex pairs induced by the unsteady minijets could enhance the centreline velocity decay rate. However, most jet mixing studies were performed in open-loop. Thus, it is inefficient to remove perturbations induced by external sources, e.g., Reynolds number change. The sensor-feedback or closed-loop may treat effectively the random-phase problem in turbulence dynamics [9] and allow flexibly controlled jet mixing [10]. The latter is important since the optimal control configuration of jet mixing varies with the operating conditions such as Mach number and temperature ratio.

The well-known extremum seeking is a model-free closed-loop control schemes. The objective of these controllers is to maximize or minimize the output of a flow system via online optimization of actuator parameters. When the steady-state input-output map has a well-defined extremum, the extremum seeking controller can be implemented to achieve the optimal control performance [11]. When the control performance is characterized by a saturated plateau in the input-output map, the slope-seeking controller was more suitable than the extremum seeking controller [12]. Wang et al. [13] propose a novel extremum seeking scheme to eliminate the steady-state oscillation by using a variable perturbation amplitude in the extremum seeking scheme [13]. The simulation results indicate this scheme could greatly improve the dynamic and static performance of the extremum seeking scheme.

This work aims to study a novel extremum seeking control system for jet flow control, where the jet centreline velocity decay rate  $K$  and the excitation frequency of minijet  $f_e$  are used as the feedback signal and the control signal, respectively. The novel extremum seeking is investigated and compared with the classical extremum seeking in the steady-state error of  $K$  and  $f_e$ . The robustness at different Reynolds number  $Re_D$  (range from 7333 to 9600), and adaptability at different initial excitation frequency  $f_{e,i}$  of the novel control system are also investigated.

## NOMENCLATURE

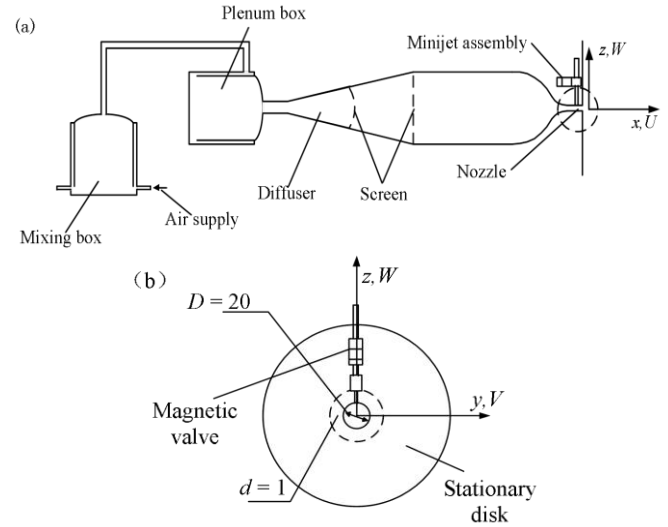
$a$	[Hz]	Amplitude of sinusoidal perturbation signal
$C_m$	[-]	Volumetric flow rate ratio of minijet to that of the main jet
$D$	[mm]	Exit diameter of the nozzle extension
$f_e$	[Hz]	Excitation frequency of minijet
$f_{e,i}$	[Hz]	Initial excitation frequency
$f_{e,optimal}$	[Hz]	Optimal value of excitation frequency
$f_0$	[Hz]	Preferred-mode frequency in the uncontrolled jet with $C_m = 0$ and $f_e = 0$
$f_p$	[Hz]	Perturbation frequency
$K$	[-]	Jet centerline velocity decay rate
$k$	[-]	Gain
$Re$	[-]	Reynolds number
$U$	[m/s]	Streamwise velocity component along $x$ -axis of Cartesian coordinates
$U_e$	[m/s]	Jet centerline velocity at the exit of the nozzle extension
$x$		$x$ axis of Cartesian coordinates
$y$		$y$ axis of Cartesian coordinates
$z$		$z$ axis of Cartesian coordinates
$HP$		High-Pass filter
$LP$		Low-Pass filter
$r$	[-]	The gain for adjusting the amplitude $a$ of the novel extremum seeking scheme
$F$		A function of $\langle K \rangle$ and $f_e$
$\phi$		Phase
Special characters		
$\langle \rangle$	[-]	Averaged quantity using 0.4-sec-long running window
$\Delta$	[-]	Correction term
Subscripts		
$e$		Exit of the nozzle extension
$c$		Jet centerline
$3D$		$x = 3D$

## JET FACILITY AND ACTUATOR SYSTEM

All experiments were performed in a round air facility. The jet facility is composed of main-jet and minijet assemblies. The minijet includes a stationary disk and an electromagnetic valve. Figure 1 shows the schematic diagram of the jet facility. Compressed air pass through a tube, a plenum chamber, a 300-mm-long diffuser of  $15^\circ$  in half angle, two fine screens (7mesh/cm) and a cylindrical settling chamber of 400 mm in length and 114 mm in inner diameter. The nozzle contraction followed a contour specified by equation  $R = 57 - 47\sin^{1.5}(90 - 9x/8)$ , as used by [14]. The contraction ratio was 32.5 with an exit diameter  $D$  of 20 mm. The nozzle was extended by a 47-mm-long smooth tube of the same diameter  $D$ . The exit Reynolds number  $Re_D = U_e D / \nu$  of the main jet was fixed to 8000 mostly, where  $U_e$  (6 m/s) is the jet centerline velocity measured at the exit of the nozzle extension and  $\nu$  is the kinematic viscosity.

Figure 1(b) shows the minijet assembly. A new unsteady minijet actuator is applied. The stationary disk was drilled with one orifice of 4 mm in diameter along the horizontal direction. The orifice was connected via short plastic hose to a constant-pressure chamber. The unsteady minijet actuator is actually controlled by an electromagnetic valve (Koganei K2-100SF-09-LL) with maximum frequency of 1 kHz. The contraction nozzle was drilled with one orifice of 1 mm in diameter, and located at 17 mm upstream of the extension exit. The electromagnetic is driven by the modified voltage signal (0-5V square wave signal) that may act to adjust the frequency of air injection with a

maximum of 450 Hz, exceeding  $3f_0$  as is evident in Figure 2(a, b), where  $f_0$  is the preferred mode frequency of the natural main jet. The volumetric flow rates of both main jet and minijet were changed and measured respectively by two flow meters, whose experimental uncertainty was no more than 1%. The volumetric flow rate required by one minijet was further quantified using a dimensionless volumetric flow rate ratio  $C_m$  of the minijet to that of the main jet.



**Figure 1** Schematic of experimental setup: (a) main-jet assembly; (b) minijet assembly.

The coordinate system is defined such that its origin is at the center of the nozzle extension exit, with the  $x$  axis along the streamwise direction, the  $z$  axis along the radial minijet and the  $y$  axis along the direction normal to  $(x, z)$  plane, following the right-hand system. The  $(x, z)$  and  $(x, y)$  planes are referred to as the injection and non-injection planes, respectively. Following reference [8], the jet centerline decay rate  $K$  is used to evaluate main jet mixing, given by  $K = (\overline{U_e} - \overline{U_{5D}}) / \overline{U_e}$ , where  $\overline{U_e}$  and  $\overline{U_{5D}}$  are the mean jet centerline velocity at  $x/D = 0.05$  and at  $x/D = 5$ , respectively.

## HOTWIRE MEASUREMENTS AND CLOSED-LOOP CONTROL FACILITIES

A single hot-wire was connected to the hot-wire anemometers (Dantec Streamline), operating on a constant temperature circuit at an overheat ratio of 0.6, to measure the streamwise fluctuating velocity at  $x/D < 10$ . The voltage signal from the hot-wire was filtered at a cut-off frequency of 3 kHz, amplified 8 times and then digitized using a 16-bit A/D board (NI USB-6361) at a sampling frequency of 6 kHz. The sampling number of 480,000 (acquisition time is 80 sec) is used for the open-loop control experiments. The uncertainty of the hot-wire measurement has been estimated as less than 2%.

A NI USB-6361 system, connected to a personal computer, was used for closed-loop control study. The system contains 16 analog inputs (16 bit) and 2 analog outputs (16 bit). The former

is used for acquiring the signal from hot wire as mentioned earlier and the latter is used for generating the control signal for minijet actuators at a rate of 6 kHz. The loop time is 0.4 seconds. The variation of  $f_e$  was realized by changing the frequency of the output square wave signal from the NI system output board. The Labview 2011 software was used to establish the link between the computer and the NI USB-6361 hardware. The interface for supervising information in real-time was designed with the Labview 2011 software.

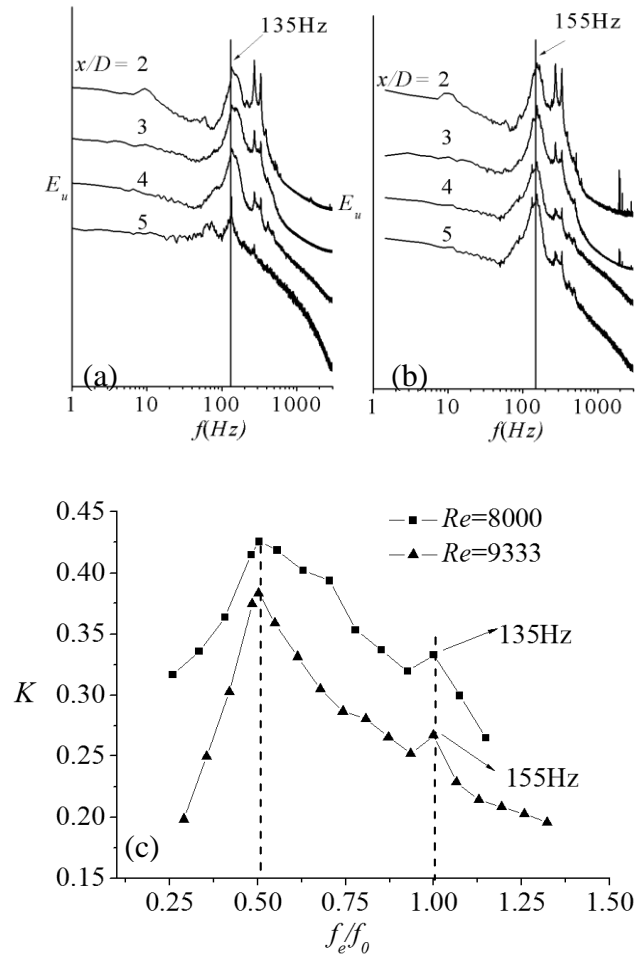
## FLOW VISUALIZATION SYSTEM

A planar PIV (Dantec SpeedSense90C10) system was deployed for flow visualization measurements in the injection ( $x, z$ ) plane of the one minijet and the orthogonal non-injection ( $x, y$ ) plane. A TSI oil droplet generator (TSI MCM-30) is used to generate fog for the seeding of flow. The particles are supplied into the mixing chamber (Figure 1a), which mix with air and fully spread throughout the main jet. Flow illumination was provided by two standard pulsed laser (Beam Tech 500-10) sources with 532 nm in wavelength and a maximum energy output of 120 mJ per pulse. Particle images were captured at a sampling rate of 10 Hz. The synchronization of flow illumination and image capturing was controlled by Dynamic studio v3.41. The captured images covered an area of  $x/D = 0 \sim 7$  and  $y/D$  or  $z/D = -3 \sim +5$  in the ( $x, y$ ) and ( $x, z$ ) planes. The uncertainty of the position of the traversing system is about 0.1mm.

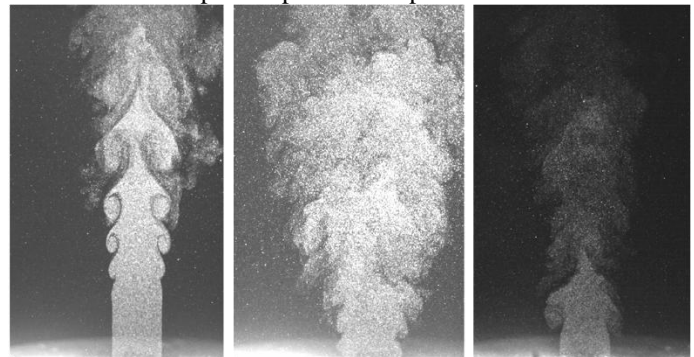
## OPEN-LOOP CONTROL RESULTS

The effect of the  $f_e$  on the  $K$  is firstly investigated at  $Re_D = 8000$  and  $9333$  with a constant volumetric flow rate of the minijet, i.e.  $m = 2.51/\text{min}$ . One objective of the open-loop control experiments is to provide the steady-state of input-output map between  $f_e$  and  $K$  in order to evaluate the feasibility of the closed-loop control. Figure 2(b) presents the measured  $K$  at different  $f_e$  for two different Reynolds numbers. At  $Re_D = 8000$ , the  $K$  strongly depends on  $f_e$ , showing a twin-peak value, with a global maximum ( $K = 0.425$ ) at  $f_e/f_0 \approx 0.5$  and the minor local maximum at  $f_e/f_0 \approx 1.0$ . The former is the optimal excitation frequency  $f_{e,optimal}$  at  $Re_D = 8000$  in this paper, leading to an increment in  $K$  by 818% compared with the natural jet. That is,  $f_{e,optimal}$  is actually equal to one-half the frequency of the preferred-mode vortices. In order to illustrate the profound impact of control at  $f_{e,optimal}$  on the main jet, we show typical photographs (Figure 3) in both injection ( $x, y$ ) and non-injection ( $x, z$ ) planes from flow visualization. There is a distinctly difference in the flow structure with and without control. Without control, the main jet remains laminar near the nozzle exit, as is evident in Figure 3. The potential core decrease significantly and the shear layer rollup occurs early under control and vortex pairing occur early in both planes than the uncontrolled jet. The control result produces a vigorous entrainment of ambient fluid (dark-coloured around the vortex in the injection plane) into the jet. On the other hand, smoke-marked fluid (white-colored in the non-injection plane) is widely spread from the region where almost near the exit of the

jet, producing an extensive diffusion than the uncontrolled case. When the  $Re_D$  is increase to 9333, the trends of the  $f_e/f_0$ - $K$  curve are similar with that at  $Re_D$  of 8000. Note that, the  $K$  at  $Re_D$  of 9333 is less than that at  $Re_D$  of 8000, which is due to the  $C_m$  at  $Re_D$  of 9333 is smaller than that at  $Re_D$  of 8000.



**Figure 2** Power spectral density function of streamwise fluctuating velocity  $u$  measured on the centerline: (a) at  $Re_D = 8000$ ; (b) at  $Re_D = 9333$ . (c) The relationship of  $K = F(f_e/f_0)$  for  $Re_D = 8000, 9333$  for the open-loop control experiment.



**Figure 3** Photographs of flow visualization. Flow is from the bottom up at  $Re_D = 8000$ . Comparison in the typical flow structure (a) the uncontrolled jet and (b) the non-injection and (c) injection planes of controlled jet ( $C_m = 2.3\%$ ,  $f_e = 68$  Hz), respectively.

## CLASSICAL EXTREMUM SEEKING SCHEME

The schematic diagram of the classical extremum seeking algorithm is represented in Figure 4(a). The basic principle of the classic extremum seeking algorithm is given below. The flow control system is considered as a block with the input  $f_e(t)$  and the output  $\langle K \rangle(t)$ , where  $\langle \rangle$  indicates averaged quantity using 0.4-sec-long running window. A local maximum takes place on curve  $\langle K \rangle = F(f_e)$  when  $f_e = f_{e,optimal}$ , where the  $d\langle K \rangle/df_e = 0$ . The algorithm drives the  $f_e$  automatically approaching  $f_{e,optimal}$  following the feedback law, i.e.,  $df_e/dt = kd\langle K \rangle/df_e$ , where  $k$  is the gain and  $d\langle K \rangle/df_e$  is the local gradient of the input-output map. As such, autonomous adjustment of  $f_e(t)$  from the initial excitation frequency  $f_{e,i}$  to  $f_{e,optimal}$  is achieved

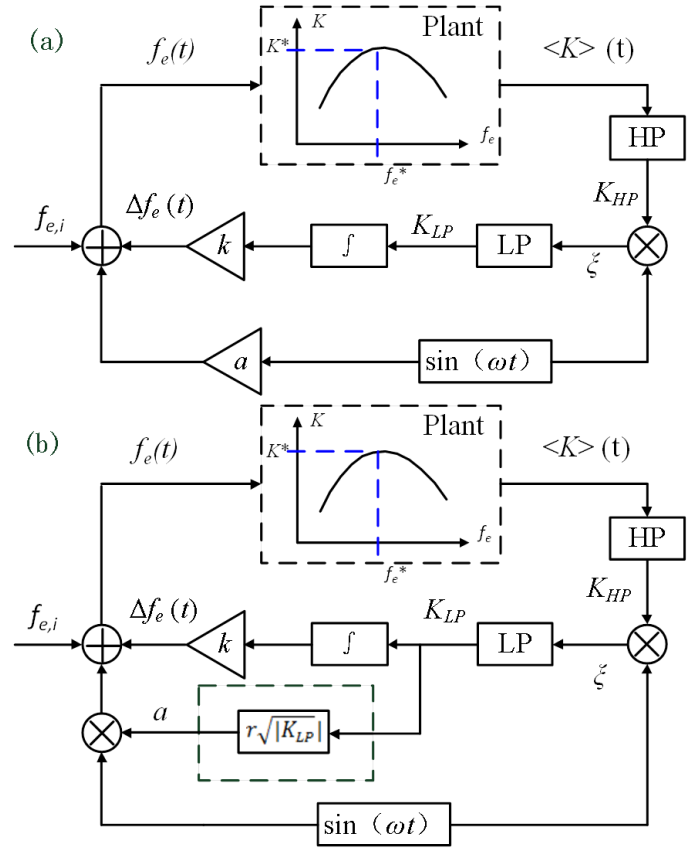
For calculating the  $d\langle K \rangle/df_e$ , a small-amplitude sinusoidal signal  $asin(\omega t)$  is superimposed on the control input  $f_e$ , i.e.,  $f_e(t) = f_{e,i} + \Delta f_e(t) + asin(\omega t)$ , where  $\omega (=2\pi f_p)$  indicates the perturbation frequency and  $\Delta f_e(t)$  is the feedback increment (Figure 4). This input can result in an approximate sinusoidal output in  $\langle K \rangle(t)$ , fluctuating around the mean value  $\langle K_{DC} \rangle$  with the amplitude governed by  $aF'$ . That is,  $\langle K \rangle(t) = K_{DC} + aF'\sin(\omega t)$ , where  $F'$  is the local gradient of the input-output curve. The  $\langle K_{DC} \rangle$  in the output is then removed by a first order high-pass filter (HP), i.e.,  $G_{HP}(j\omega) = j\omega/(j\omega + \omega_{HP})$ , to produce  $K_{HP}(t) = |G_{HP}|aF'$ , where  $|G_{HP}|$  is magnitude of the HP transfer function. The filtered signal  $K_{HP}(t)$  is demodulated by multiplying a zero-mean signal with the perturbation  $\sin(\omega t)$ . The demodulated signal  $\xi (= |G_{HP}|aF'\sin(\omega t))$  contains a constant, i.e.,  $|G_{HP}|(F'a/2)\cos(\phi_{HP})$ , and a time-varying component, i.e.,  $|G_{HP}|(F'a/2)\cos(2\omega t + \phi_{HP})$ , where  $|G_{HP}| = \frac{1}{\sqrt{1+(\omega_{HP}/\omega)^2}}$  and  $\phi_{HP} = \arg(G_{HP}) = \arctan(\omega_{HP}/\omega)$ . The time-varying component in  $y_d$  is then filtered out by a first order low-pass filter (LP). Thus, the output  $K_{LP} = |G_{HP}|(F'a/2)\cos(\phi_{HP})$  is a steady signal proportional to the local gradient  $F'$ . The  $K_{LP}$  signal is then integrated and amplified by a gain  $k$  to obtain the feedback increment  $\Delta f_e(t)$ . In general, the  $\Delta f_e(t)$  is positive for  $f_e < f_{e,optimal}$ , so the  $f_e$  would increase. Otherwise,  $f_e$  would decrease for  $f_e > f_{e,optimal}$ . Eventually,  $f_e$  will converge to  $f_{e,optimal}$ .

## A NOVEL EXTREMUM SEEKING SCHEME

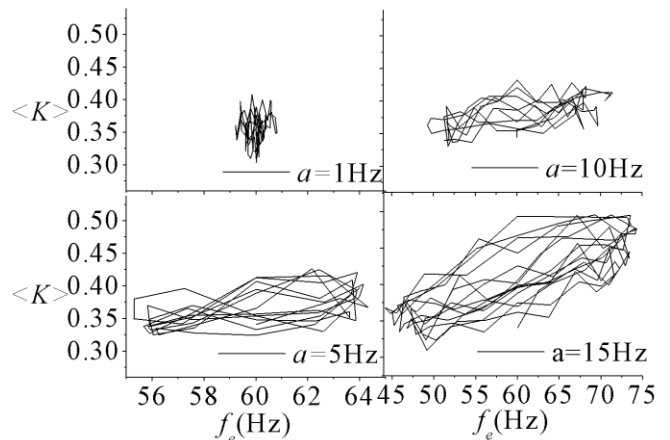
In classical extremum seeking scheme, the dynamic and static performance of the scheme is directly related to the amplitude  $a$  of the excitation sinusoidal signal  $asin(\omega t)$ . As the amplitude  $a$  increasing, the dynamic performance of the scheme becomes more effective and the static performance of the scheme becomes worse, and vice versa [13]. Therefore, on the basis of the classical methods, the novel extremum seeking scheme determine the amplitude  $a$  based on the variation of the estimated gradient, aiming to reduce the steady-state error [13].

As shown in Figure 4(b), the 'a' in the classical extremum seeking is replaced by a variable governed by  $r\sqrt{|K_{LP}|}$  in the novel extremum seeking, i.e.,  $a = r\sqrt{|K_{LP}|}$ , where the  $r$  is a constant factor. Since the  $K_{LP} = |G_{HP}|(F'a/2)\cos(\phi_{HP})$  as discussed earlier in Figure 4(a), so we can get  $a^2 =$

$r^2|G_{HP}(F'a/2)\cos(\phi_{HP})|$ . Therefore, the  $a = |F'r^2|G_{HP}\cos(\phi_{HP})/2|$ , which indicates that the amplitude of the perturbation signal is proportional to the  $|F'|$ . Thus, the amplitude  $a$  will gradually reduce as the  $|F'|$  approaches 0. In other words, the output fluctuation will decrease as the  $f_e$  converges to  $f_{e,optimal}$ .



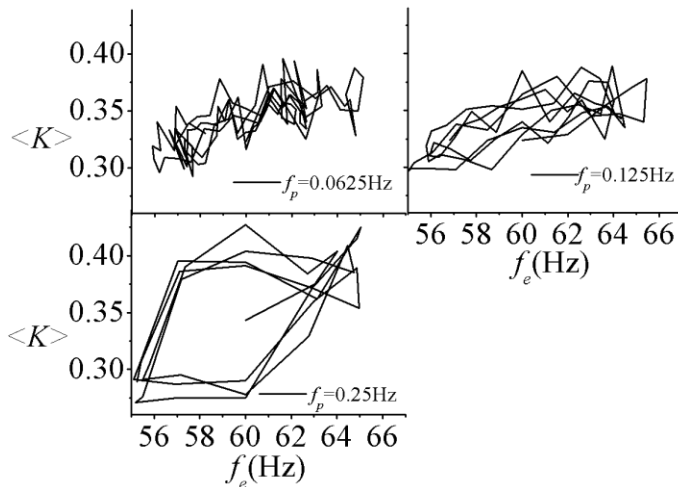
**Figure 4** (a) Block diagram of the classical extremum seeking control scheme; (b) Block diagram of the novel extremum seeking control scheme.



**Figure 5** Effects of the amplitude  $a$  of the applied perturbation on the input-output map ( $k = 0$ ,  $Re_D = 8000$ , perturbation imposed around  $f_e = 60$  Hz).

## IMPLEMENTATION AND PARAMETERIZATION OF THE NOVEL EXTREMUM SEEKING SCHEME

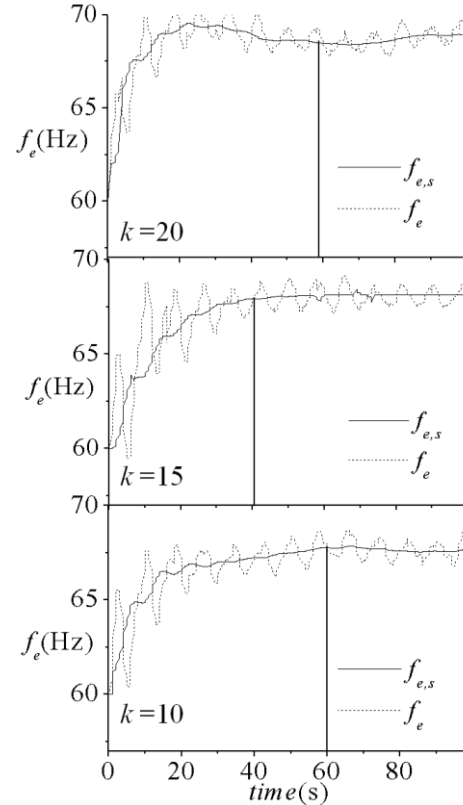
In most active flow control applications, the plant exhibits a hysteresis between the input and output signals, which may adversely affect the gradient estimation for the novel extremum seeking scheme. Using a sinusoidal input in the novel extremum seeking scheme may result in a phase shift of the output, however, appropriate setting of the perturbation parameters, i.e.  $f_p$  and  $a$ , of the controller could decrease the influence of hysteresis. The parameters of the ES controller are determined experimentally at  $Re_D = 8000$  with the volumetric flow rate of one minijet fixed at  $C_m = 2.3\%$ . The selection of key parameters in the control algorithm was similar to that used in [15]. A sinusoidal perturbation around the  $f_{e,i}$  ( $= 60$  Hz), i.e.  $f_e(t) = f_{e,i} + a \sin(\omega t)$ , is applied to the input signal to study the impact of different  $a$  and  $f_p$  on the  $K$ . Note that the  $k$  of 0 is used for the parameterization experiments. The amplitude  $a$  and  $f_p$  are investigated individually by fixing one of the two parameters and adjusting the other one. As such, we may determine the optimal perturbation parameters of the controller such that the output  $\langle K \rangle$  is in phase with the  $f_e(t)$ . Figure 5 shows the comparison between the responses of  $\langle K \rangle$  measured with  $a = 1, 5, 10$  and  $15$  Hz ( $k = 0, Re_D = 8000, f_{e,i} = 60$  Hz,  $f_p = 0.125$  Hz). The relation of  $\langle K \rangle$  and  $f_e$  appears approximately a linear trend. At  $a = 1$  Hz, the range of the sinusoidal perturbation yields a minor change in the  $\langle K \rangle$  compared with that at  $a = 5$  Hz,  $10$  Hz and  $15$  Hz, which means the amplitude  $a$  is too small. When  $a = 10$  Hz and  $15$  Hz, the  $f_e(t)$  could induce a conspicuous phase lag between  $\langle K \rangle$  and  $f_e$ . As such, we choose  $a = 5$  Hz, which is adequately large to influence the  $\langle K \rangle$  signal but does not produce a significant hysteresis in the  $f_e$ - $\langle K \rangle$  map.



**Figure 6** Effect of the frequency  $f_p$  of the applied perturbation on the input-output map ( $k = 0, Re_D = 8000$ , perturbation imposed around  $f_e = 60$  Hz)

Figure 6 presents the response of  $\langle K \rangle$  to the variation in  $f_p$  from  $0.0625$  Hz to  $0.25$  Hz. Firstly, the irregular changes in  $\langle K \rangle$  occurs more obviously at  $f_p = 0.0625$  Hz (Figure 6) compared with that at  $f_p = 0.125, 0.25$  Hz. The reason may be due to the change in  $f_e$  is too slow at  $f_p = 0.0625$  Hz. Thus,  $\langle K \rangle$

is more easily disturbed by the external flow, adversely influencing the accurate gradient estimation. Secondly, the  $f_e$ - $\langle K \rangle$  map exhibits an extreme hysteresis at  $f_p = 0.25$ . Therefore,  $f_p = 0.125$  Hz was selected for the controller, since only a small phase lag occurs in relationship between the  $f_e$  and  $\langle K \rangle$  at  $f_p = 0.125$  Hz. In addition,  $0.1$  times cutoff frequency is used for the  $HP$  and  $LP$  filters, i.e.  $\omega_{HP} = \omega_{LP} = 0.1(2\pi f_p)$ .

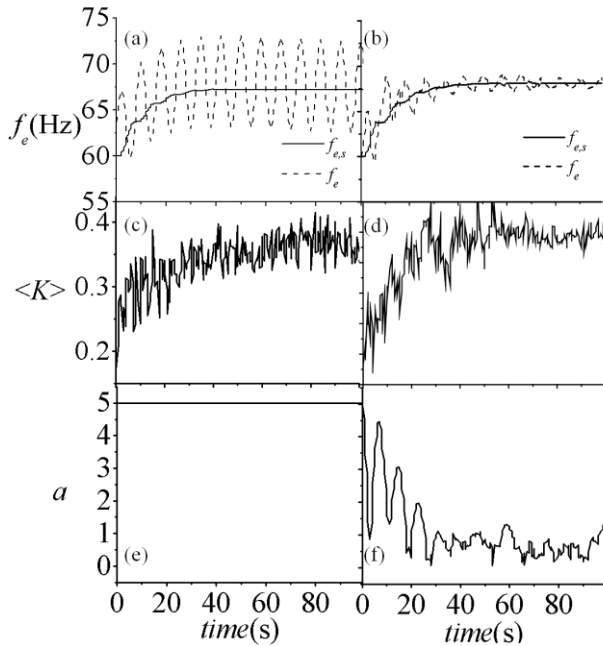


**Figure 7** The  $f_e$  response of the controller at different gain  $k$  ( $Re_D = 8000$ , the feedback increment  $\Delta f_e$  is added  $t = 0$  sec)

## RESPONSE OF THE NOVEL EXTREMUM SEEKING SCHEME

Experiments are performed at  $Re_D = 8000$  and  $C_m = 2.3\%$  to investigate the performance of the novel extremum seeking algorithm not only in the search of the optimal  $f_e$  but also to choose the optimal gain  $k$ . Figure 7 presents the time domain response of the closed-loop control system for different gains  $k = 10, 15, 20$ , when  $a = 5$  Hz,  $f_p = 0.125$ . Initially, the  $f_{e,i}$  of  $60$  Hz is applied on the minijet at  $t = 0$  sec, which is less than  $f_{e,optimal}$  in the open-loop case (Figure 2(c)). The  $f_e$  increases immediately once the feedback signal is feedback to the controller at  $t > 0$  sec. As indicated in Figure 7, the gain  $k$  has a direct impact on the feedback increment  $\Delta f_e$ , which may effect on the system stability and convergence time. For  $k = 10$ , it takes about  $60$  sec to converge the  $f_{e,s}$  to near the  $f_{e,optimal}$ , where  $f_{e,s} = f_{e,i} + \Delta f_e(t)$  is the quasi-steady component of  $f_e$ . As a result, the  $\langle K \rangle$  converges to  $0.41$  after  $60$  sec. As the  $k$  increases, the change of  $\Delta f_e$  will be enlarged. Consequently, the convergence time reduces, about  $40$  sec, at  $k = 15$ . However, when the  $k$

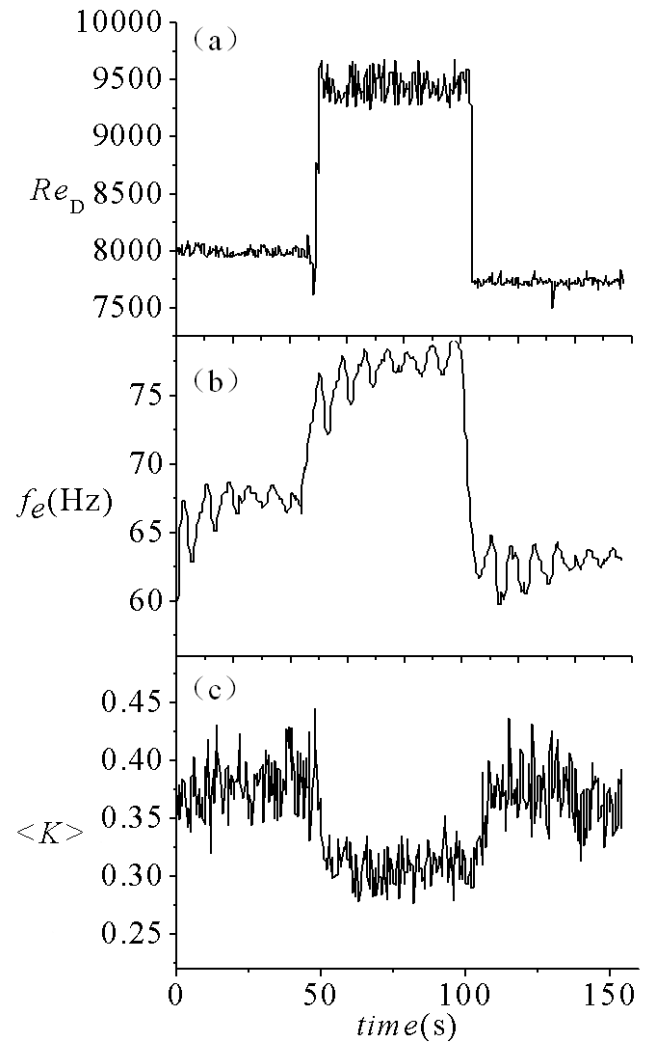
further increase to 20, the enlarged change in  $\Delta f_e$  may lead to an overshoot in  $f_e$ , thus increasing the fluctuation in  $\langle K \rangle$  and weaken the system stability. In conclusion,  $k = 15$  exhibits the shortest convergence time, and the good stability of the closed-loop control system.



**Figure 8** Compare the classical extremum seeking scheme and the novel extremum seeking scheme with  $f_e$ ,  $\langle K \rangle$  and  $a$

For a better understanding of the novel extremum seeking scheme, we compared the results of the novel extremum seeking method with that of the classical one. As shown in Figure 8, the performance of the two methods show distinctly different under the same parameters, i.e.  $k = 15$ ,  $f_{e,i} = 60$  Hz,  $f_p = 0.125$  Hz and  $a = 5$ . For the classical extremum seeking scheme, the  $f_e$  gradually growth with a fixed perturbation amplitude due to the using of a fixed  $a = 5$  Hz. After about 40 sec, the controller is stabilized with  $f_e$  fluctuating within 63-73 Hz (Figure 8a). Meanwhile, the  $K$  increases from 0.26 to 0.36 and is stabilized within 0.32 to 0.38 eventually (Figure 8c). On the other hand, the  $f_e$  in the novel extremum seeking begins with a large perturbation amplitude (Figure 8b). However, the perturbation amplitude decreases as the  $f_e$  approaching the  $f_{e,optimal}$  and is stabilized at a range of 66.5 - 68.5 Hz. Correspondingly, the  $K$  increases from 0.26 to 0.40, stabilized within a range of 0.36 ~ 0.42 (Figure 8d). The reason is mainly due to the perturbation amplitude in the novel extremum seeking is  $|F'|r^2|G_{HP}\cos(\phi_{HP})/2|$ , which is directly proportional to  $|F'|$  since the value of  $r$ ,  $G_{HP}$ ,  $\cos(\phi_{HP})$  are constant. The  $|F'|$  tends to zero as time in the novel extremum seeking. Thus the amplitude  $a$  also tends to zero as shown in Figure 8 (f), however, the amplitude  $a$  in the classical extremum seeking is a constant (Figure 8e). Therefore, the amplitude  $a$  of the novel scheme will less than the amplitude  $a$  of the classical scheme when the control schemes are stable. Accordingly, the change of  $a$  in the novel scheme could lead to a reduction in the steady-

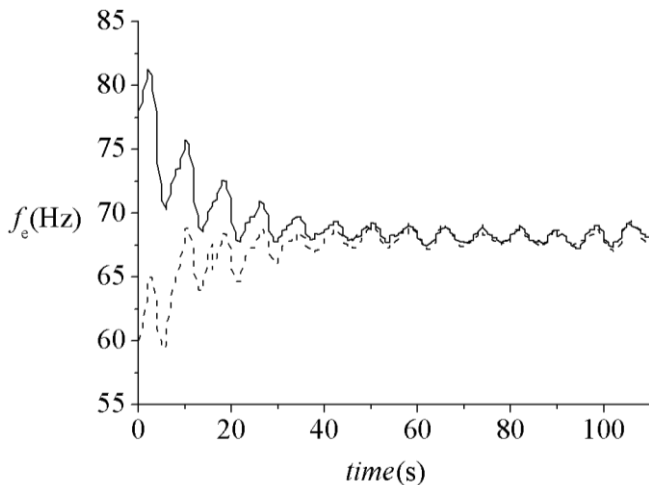
state error of  $f_e$  by 80%. More importantly, the mean value of the stabilized  $\langle K \rangle$  increased by 11.11% compared classical scheme (from 0.36 to 0.40). The mean value of the stabilized  $\langle K \rangle$  is averaged from the 40 to 100 sec corresponding to the  $\langle K \rangle$  in Figure 8c and 8d.



**Figure 9** Responses of the novel extremum seeking scheme to varying  $Re_D$ : (1) from  $Re_D = 8000$  to 9600 at 45 sec. (2) from  $Re_D = 9600$  to 7333 at 104 sec,  $C_m = 2.3\%$

Robustness is an important aspect for a closed-loop control system and is one of the important indicators for the overall control performance. The robustness is examined when  $Re_D$  is changed suddenly at a fixed  $C_m$  and  $k = 15$ . At  $Re_D = 8000$ , the controller is applied at  $t = 0$  sec and the control system takes about 40 sec to increase  $f_e$  from  $f_{e,i}$  to  $f_{e,optimal} = 67.5$  Hz (Figure 9b). The  $\langle K \rangle$  signal increases from around 0.015 to 0.40 accordingly (Figure 9c). The results are consistent with that achieved in Figure 8, indicating a good repeatability of the control system. As  $Re_D$  is suddenly increased from 8000 to 9600 at  $t = 45$  sec (Figure 9a), the  $f_e$ - $K$  map changes and  $f_{e,optimal}$  rises. The gradient of the  $f_e$ - $\langle K \rangle$  curve becomes positive, resulting in a positive  $\Delta f_e$ . The controller increases  $f_e$  until the maximum  $\langle K \rangle$  is reached, with  $f_{e,optimal} = 78$ Hz. Note that

$f_{e,optimal} = 78$  Hz is slightly higher than the optimal  $f_e$  ( $= 77.5$  Hz) achieved at  $Re_D = 9600$ , due to a slightly higher  $Re_D$ . On the other hand, as  $Re_D$  is decreased from 9600 to 7333 at 104 sec, the controller could also maintain a stable  $f_e$  at  $f_{e,optimal}$  and  $\langle K \rangle = 0.38$  (Figure 9c), indicating an excellent robustness. The adaptation of the controller is studied by applying two different initial excitation frequencies, i.e.,  $f_{e,i} = 60$  Hz and 75 Hz, at  $Re_D = 8000$ . When  $f_{e,i}$  is less than  $f_{e,optimal}$  ( $= 67.5$  Hz), the gradient of the  $f_e$ - $\langle K \rangle$  curve (Figure 2) is positive, corresponding to a positive  $\Delta f_e$ . On the other hand, when  $f_{e,i}$  is larger than  $f_{e,optimal}$ , the gradient is negative; so is  $\Delta f_e$ . As shown in Figure 10, in all the cases, the controller adjusts  $f_e$  [ $= f_{e,i}(t) + \Delta f_e(t) + a \sin(\omega t)$ ] to  $f_{e,optimal}$ , where the gradient of the  $f_e$ - $\langle K \rangle$  curve is zero, suggesting an excellent adaptation.



**Figure 10** The  $f_e$  response of the novel extremum seeking scheme at different  $f_{e,i}$

## CONCLUSION

The closed-loop control of a turbulent round jet has been experimentally investigated based on a single unsteady radial minijet.

1. The open-loop control, with  $C_m$  fixed at 2.3%, is highly effective in enhancing the jet mixing performance at  $Re_D = 8000$ , producing an increase in  $K$  by 818% at  $f_{e,optimal} = 0.5f_0$ , compared with the natural jet. Accordingly, the flow structure exhibits a drastic change.
2. The novel extremum seeking feedback control has been developed without a priori knowledge of relevant fluid dynamics. The system output  $\langle K \rangle$  is obtained online. Given a fixed  $C_m$ , the novel extremum seeking controller succeeds in finding autonomously  $f_{e,optimal}$  and hence the maximum  $\langle K \rangle$ , as obtained in the open-loop control. This control technique is also found to be robust and adaptable when  $Re_D$  and  $f_{e,i}$  are separately changed. More importantly, the novel scheme can reduce the steady-state error of  $f_e$  by 80% and meanwhile increase the mean  $\langle K \rangle$  by 11%, compared with the classical scheme. The choice of the parameters in the controller is crucial for the desired control performance. The right amplitude  $a$  and frequency  $f_p$  of the applied sinusoidal perturbation may lead to a

minimum phase lag between the input and output signal, thus ensuring accurate gradient estimation. On the other hand, the proper selection of gain  $k$  may promote the stability of the system.

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