

**OPTIMAL ENERGY-WATER NEXUS MANAGEMENT IN RESIDENTIAL BUILDINGS  
INCORPORATING RENEWABLE ENERGY, EFFICIENT DEVICES AND WATER  
RECYCLING**

by

**Evan Wanjiru**

Submitted in partial fulfilment of the requirements for the degree  
Philosophiae Doctor (Electrical)

in the

Department of Electrical, Electronic and Computer Engineering  
Faculty of Engineering, Built Environment and Information Technology

UNIVERSITY OF PRETORIA

April 2017

## SUMMARY

---

### **OPTIMAL ENERGY-WATER NEXUS MANAGEMENT IN RESIDENTIAL BUILDINGS INCORPORATING RENEWABLE ENERGY, EFFICIENT DEVICES AND WATER RECYCLING**

by

**Evan Wanjiru**

Promoter(s): Prof. Xiaohua Xia  
Department: Electrical, Electronic and Computer Engineering  
University: University of Pretoria  
Degree: Philosophiae Doctor (Electrical)  
Keywords: Demand side management, energy efficiency, energy-water nexus, green buildings, grey water recycling, heat pump water heater, model predictive control, optimal control, renewable energy, rooftop rain water harvesting, time-of-use tariff, water conservation

Developing nations face insurmountable challenges to reliably and sustainably provide energy and water to the population. These resources are intricately entwined such that decisions on the use of one affects the other (energy-water nexus). Inadequate and ageing infrastructure, increased population and connectivity, urbanization, improved standards of living and spatially uneven rainfall are some of the reasons causing this insecurity. Expanding and developing new supply infrastructure is not sustainable due to sky high costs and negative environmental impact such as increased greenhouse gas emissions and over extraction of surface water. The exponentially increasing demand, way above the capacity of supply infrastructure in most developing countries, requires urgent mitigation strategies through demand side management (DSM). The DSM strategies seek to increase efficiency of use of available resources and reducing demand from utilities in the short, medium and long term. Renewable energy, rooftop rain water harvesting, pump-storage scheme and grey water recycling are some alternatives being used to curb the insecurity. However, renewable energy and rooftop water

harvesting are spasmodic in nature hampering their adoption as the sole supply options for energy and water respectively.

The built environment is one of the largest energy and water consuming sectors in the world presenting a huge potential towards conserving and increasing efficiency of these resources. For this reason, coupled with the 1970s energy challenges, the concept of green buildings seeking to, among other factors, reduce the consumption of energy and water sprung up. Conventionally, policy makers, industry players and researchers have made decisions on either resource independently, with little knowledge on the effect it would have on the other. It is therefore imperative that optimal integration of alternative sources and resource efficient technologies are implemented and analysed jointly in order to achieve maximum benefits. This is a step closer to achieving green buildings while also improving energy and water security.

A multifaceted approach to save energy and water should integrate appropriate resource efficient technology, alternative source and an advanced and reliable control system to coordinate their operation. In a typical South African urban residential house, water heating is one of the most energy and water intensive end uses while lawn irrigation is the highest water intensive end use occasioned by low rainfall and high evaporation. Therefore, seamless integration of these alternative supply and most resource intensive end uses provides the highest potential towards resource conservation. This thesis introduces the first practical and economical attempt to integrate various alternative energy and water supply options with efficient devices. The multifaceted approach used in this research has proven that optimal control strategy can significantly reduce the cost of these resources, bring in revenue through renewable energy sales, reuse waste water and reduce the demand for grid energy, water and waste water services.

This thesis is generally divided into cold and hot water categories; both of which energy-water nexus DSM is carried out. Open-loop optimal and closed-loop model predictive (MPC) control strategies that minimize the objective while meeting present technical and operational constraints are designed. In cold water systems, open-loop optimal and MPC strategies are designed to improve water reliability through a pump storage system. Energy efficiency (EE) of the pump is achieved through optimally shifting the load to off-peak period of the time-of-use (TOU) tariff in South Africa. Thereafter, an open-loop optimal control strategy is developed for rooftop rain water harvesting for lawn irrigation. The controller ensures water is conserved by using the stored rain water and ensuring only the required

amount of water is used for irrigation. Further, EE is achieved through load shifting of the pump subject to the TOU tariff. The two control strategies are then developed to operate a grey water recycling system that is useful in meeting non-potable water demand such as toilet flushing and lawn irrigation and EE is achieved through shifting of pump's load. Finally, the two control strategies are designed for an integrated rain and grey water recycling for a residential house, whose life cycle cost (LCC) analysis is carried out. The hot water category is more energy intensive, and therefore, the open-loop optimal control strategy is developed to control a heat pump water heater (HPWH) and an instantaneous shower, both powered by grid-tied renewable energy systems. Solar and wind energy are used due to their abundance in South Africa. Thereafter, the MPC strategy is developed to power same devices with renewable energy systems. In both strategies, energy is saved through the use of renewable energy sources, that also bring in revenue through sale of excess power back to the grid. In addition, water is conserved through heating the cold water in the pipes using the instantaneous shower rather than running it down the drain while waiting for hot water to arrive. LCC analysis is also carried out for this strategy.

Each of the two control strategies has its strengths. The open loop optimal control is easier and cheaper to implement but is only suitable in cases where uncertainties and disturbances affecting the system do not alter the demand pattern for water in a major way. Conversely, the closed-loop MPC strategy is more complicated and costly to implement due to additional components like sensors, but comes with great robustness against uncertainties and disturbances. Both strategies are beneficial in ensuring security and reliability of energy and water is achieved. Importantly, technology alone cannot have sustainable DSM impact. Public education and awareness on importance of energy and water savings, improved efficiency and effect on supply infrastructure and greenhouse gas emissions are essential. Awareness is also important in enabling the acceptance of these technological advancements by the society.

## DEDICATION

To

The Almighty God;

For the breath of life, strength, wisdom and blessings.

My Single Mother, Elizabeth Murimi;

For all the sacrifices you made. All the love, support and prayers.

My Grandmother, Agnes Murimi;

This is your dream come true.

My Siblings, James, Agnes and Victor;

The sky is the limit.

To my sweet, gorgeous Fiancée, Keziah Khalinditsa;

Thanks for the love and patience. I look forward to life's thrill ahead.

## ACKNOWLEDGEMENT

This research has been conducted at the National Hub for Energy Efficiency and Demand Side Management (EEDSM), University of Pretoria, South Africa. The financial support by University of Pretoria and the National Hub for EEDSM is acknowledged and greatly appreciated. I further acknowledge Jomo Kenyatta University of Agriculture and Technology (JKUAT), Kenya, for granting me with a study leave to pursue this Doctoral degree.

I am greatly honoured and moved by the overwhelming support I have received during the period of my Doctoral studies.

Special appreciation, respect and indebtedness goes to my Supervisor, Prof. Xiaohua Xia, a world renown expert in Energy Management. Your advice, guidance and inspiration opened my eyes to the challenges facing the world. You are not only an academic advisor, but also gave me the opportunity to practically learn and understand how to sustainably manage energy and water. I will forever be grateful to you.

To my senior colleagues who finished before me, I really thank you for holding my hand when I joined the EEDSM Hub. Dr. T. Mathaba, Dr. P. Numbi, Dr. L. Zhang, Dr. S. Sichilalu, Dr. H. Tazvinga and Dr. D. Setlhaolo; thank you for supporting and showing me the way. Dr. X. Ye, Mr. N. Wang, and Ms. Y. Fan, thank you for the practical training you have always offered to me. Mr. C. Kagiri, I appreciate the time we have spent together in South Africa. Lastly but no the least, to the other colleagues in the Hub, thanks for all the moments we have shared in this academic journey.

To my family back in Kenya, I really appreciate your prayers, love, encouragement and emotional support, even when things looked tough, being away from home. And to my Fiancée, thanks for the love and emotional support.

Yes! I have finally finished. To God be the Glory.

## PUBLICATIONS

1. E. M. Wanjiru, X. Xia, “Energy-water optimization model incorporating rooftop water harvesting for lawn irrigation,” *Applied Energy* 160 (2015): 521-531.
2. E. M. Wanjiru, X. Xia, “Optimal energy-water management at household level,” *10<sup>th</sup> South African Energy Efficiency Convention*, November 11-12, 2015, Emperor’s Palace Hotel, Johannesburg, South Africa.
3. E. M. Wanjiru, L. Zhang, X. Xia, “Model predictive control strategy of energy-water management in urban households,” *Applied Energy* 179 (2016): 821-831.
4. E. M. Wanjiru, S. M. Sichilalu, X. Xia, “Optimal control of heat pump water heater-instantaneous shower using integrated renewable-grid energy systems” *Applied Energy* 201 (2017): 332-342.
5. E. M. Wanjiru, S. M. Sichilalu, X. Xia, “Optimal Integrated Diesel Grid-renewable Energy System for Hot Water Devices,” *Energy Procedia* 103 (2016): 117-122.
6. E. M. Wanjiru, S. M. Sichilalu, X. Xia, “Optimal integrated diesel grid-renewable energy system for hot water devices,” *Applied Energy Symposium and Forum, REM2016*, April 19-21, 2016, Bandos Island, Maldives.
7. E. M. Wanjiru, S. M. Sichilalu, X. Xia, “Optimal operation of integrated heat pump-instant water heaters with renewable energy,” *The 8<sup>th</sup> International Conference on Applied Energy - ICAE2016*, October 8-11, 2016, Beijing , China.
8. E. Wanjiru, X. Xia, “Optimal energy-water management in urban residential buildings through grey water recycling,” *Sustainable Cities and Society* 32 (2017): 654-668.
9. E. M. Wanjiru, S. M. Sichilalu, X. Xia, “Model predictive control of heat pump water heater-instantaneous shower powered with integrated renewable-grid energy systems,” *Applied Energy*

(2017): <https://doi.org/10.1016/j.apenergy.2017.05.033>

10. E. M. Wanjiru, S. M. Sichilalu, X. Xia, “Optimal operation of integrated heat pump-instant water heaters with renewable energy,” *Energy Procedia* 105 (2017): 2151-2156.
11. E. Wanjiru, X. Xia, “Optimal management of a grey and rain water recycling system for residential houses,” *Journal of Cleaner Production* (Accepted in July 2017 subject to revisions).



## LIST OF ABBREVIATIONS

COP	Coefficient of performance
DSM	Demand side management
DWAF	Department of Water Affairs and Forestry, South Africa
EE	Energy efficiency
FC	Field capacity
GHG	Greenhouse gas
HPWH	Heat pump water heater
LCC	Life cycle cost
MAD	Maximum allowable depletion
MIMO	Multiple input-multiple output
MPC	Model predictive control
PID	Proportional-integral-derivative
POET	Performance, operation, equipment, technology efficiency
PV	Photovoltaic solar
PWP	Permanent wilting point
RAW	Readily available water
RWH	Rooftop water harvesting
SCIP	Solving constraint integer programs
SISO	Single input-single output
TOU	Time-of-use tariff
UV	Ultraviolet
WC	Water conservation

# TABLE OF CONTENTS

<b>CHAPTER 1</b>	<b>INTRODUCTION</b>	<b>1</b>
1.1	DEMAND SIDE MANAGEMENT	1
1.2	OPTIMAL CONTROL STRATEGIES	4
1.2.1	Implementation	5
1.3	CONTRIBUTION AND RESEARCH OBJECTIVES	6
1.4	THESIS LAYOUT	7
<b>CHAPTER 2</b>	<b>ENERGY-WATER DEMAND SIDE MANAGEMENT: LITERATURE REVIEW</b>	<b>10</b>
2.1	INTRODUCTION	10
2.2	ENERGY DEMAND MANAGEMENT	10
2.3	WATER DEMAND MANAGEMENT	12
2.4	ENERGY-WATER NEXUS DEMAND MANAGEMENT	13
2.5	CONTROL METHODS FOR ENERGY-WATER SYSTEMS	15
<b>CHAPTER 3</b>	<b>OPTIMAL ENERGY-WATER MANAGEMENT IN HOUSEHOLDS WITH PUMP-STORAGE SCHEME</b>	<b>17</b>
3.1	INTRODUCTION	17
3.2	LAYOUT AND FORMULATION	19
3.2.1	Schematic layout	19
3.2.2	Open-loop optimal control system	20
3.2.3	Algorithm for solving the open-loop optimization problem	23
3.2.4	Closed-loop MPC system	25
3.3	GENERAL DATA	29
3.3.1	Case study	29
3.3.2	Time-of-use electricity tariff	31

3.4	SIMULATION RESULTS AND DISCUSSION . . . . .	31
3.4.1	Control systems without disturbance . . . . .	31
3.4.2	Control systems with disturbance . . . . .	33
3.4.3	Discussion . . . . .	36
3.5	SYNOPSIS . . . . .	39
<b>CHAPTER 4</b>	<b>ENERGY-WATER OPTIMIZATION OF ROOFTOP WATER HARVESTING FOR LAWN IRRIGATION . . . . .</b>	<b>41</b>
4.1	INTRODUCTION . . . . .	41
4.2	LAYOUT AND FORMULATION . . . . .	43
4.2.1	Schematic layout . . . . .	43
4.2.2	Optimal scheduling without rooftop water harvesting . . . . .	44
4.2.3	Optimal scheduling with rooftop water harvesting . . . . .	48
4.2.4	The Pretoria method to reduce pump maintenance cost . . . . .	52
4.3	GENERAL DATA . . . . .	53
4.3.1	Case study . . . . .	53
4.3.2	Time-of-use electricity tariff . . . . .	54
4.3.3	Water tariffs . . . . .	54
4.3.4	Water tank and pump . . . . .	55
4.4	SIMULATION RESULTS AND DISCUSSION . . . . .	55
4.4.1	Optimal scheduling without rooftop water harvesting . . . . .	55
4.4.2	Comparison of the switching strategies . . . . .	56
4.4.3	Optimal scheduling with rooftop water harvesting . . . . .	57
4.4.4	Discussion . . . . .	61
4.5	SYNOPSIS . . . . .	63
<b>CHAPTER 5</b>	<b>OPTIMAL ENERGY-WATER MANAGEMENT IN URBAN RESIDENTIAL BUILDINGS THROUGH GREY WATER RECYCLING . . . . .</b>	<b>64</b>
5.1	INTRODUCTION . . . . .	64
5.2	CONTROLLER DEVELOPMENT . . . . .	66
5.2.1	Schematic layout . . . . .	66
5.2.2	Dynamics of water flow . . . . .	68
5.2.3	Open-loop controller model . . . . .	70
5.2.4	Closed-loop control model . . . . .	75

5.2.5	Effect of monthly water block tariff . . . . .	78
5.3	GENERAL DATA . . . . .	78
5.3.1	Case study . . . . .	78
5.3.2	Time-of-use electricity tariff . . . . .	80
5.3.3	Water tariffs . . . . .	80
5.3.4	Uncertainty analysis . . . . .	81
5.4	SIMULATION RESULTS AND DISCUSSION . . . . .	81
5.4.1	Open-loop optimal control system . . . . .	81
5.4.2	Closed-loop MPC system . . . . .	85
5.4.3	Effect of monthly water block tariff . . . . .	88
5.4.4	Discussion . . . . .	90
5.4.5	Adoption of water recycling . . . . .	92
5.5	SYNOPSIS . . . . .	94
 <b>CHAPTER 6 OPTIMAL MANAGEMENT OF A GREY &amp; RAIN WATER RECYCLING SYSTEM FOR RESIDENTIAL HOUSES . . . . .</b>		<b>95</b>
6.1	INTRODUCTION . . . . .	95
6.2	SYSTEM DEVELOPMENT . . . . .	97
6.2.1	Schematic layout . . . . .	97
6.2.2	Potable water tank . . . . .	99
6.2.3	Grey water tank . . . . .	100
6.2.4	Holding tank . . . . .	100
6.3	CONTROLLER DESIGN . . . . .	101
6.3.1	Open-loop optimal controller . . . . .	101
6.3.2	Closed-loop MPC control . . . . .	106
6.4	PERTINENT INFORMATION . . . . .	109
6.4.1	Case study . . . . .	109
6.4.2	Time-of-use electricity tariff . . . . .	110
6.4.3	Potable and waste water tariffs . . . . .	111
6.5	ANALYSIS OF OPTIMAL RESULTS . . . . .	112
6.5.1	Open-loop optimal control strategy . . . . .	112
6.5.2	Closed-loop MPC strategy . . . . .	114
6.5.3	Analysis and discussion . . . . .	116

6.5.4	Life cycle cost analysis . . . . .	120
6.6	SYNOPSIS . . . . .	122
<b>CHAPTER 7</b>	<b>OPTIMAL CONTROL OF HEAT PUMP WATER HEATER - INSTANTANEOUS SHOWER USING INTEGRATED RENEWABLE - GRID ENERGY SYSTEMS . . . . .</b>	<b>124</b>
7.1	INTRODUCTION . . . . .	124
7.2	CONTROLLER FORMULATION . . . . .	126
7.2.1	Schematic layout . . . . .	126
7.2.2	Wind energy . . . . .	127
7.2.3	Photovoltaic solar energy . . . . .	128
7.2.4	Grid energy . . . . .	128
7.2.5	Heat pump water heater . . . . .	129
7.2.6	Instantaneous shower . . . . .	131
7.2.7	Optimization problem . . . . .	133
7.3	GENERAL DATA . . . . .	137
7.3.1	Case study . . . . .	137
7.4	SIMULATION RESULTS AND DISCUSSION . . . . .	139
7.4.1	Optimal operation of HPWH . . . . .	139
7.4.2	Optimal operation of instantaneous shower . . . . .	140
7.4.3	Discussion . . . . .	143
7.5	SYNOPSIS . . . . .	145
<b>CHAPTER 8</b>	<b>MODEL PREDICTIVE CONTROL OF HEAT PUMP WATER HEATER - INSTANTANEOUS SHOWER POWERED WITH INTEGRATED RENEWABLE - GRID ENERGY SYSTEMS . . . . .</b>	<b>146</b>
8.1	INTRODUCTION . . . . .	146
8.2	CONTROLLER DESIGN . . . . .	149
8.2.1	Schematic layout . . . . .	149
8.2.2	Wind energy . . . . .	150
8.2.3	Photovoltaic solar energy . . . . .	151
8.2.4	Grid energy . . . . .	151
8.2.5	Heat pump water heater . . . . .	152
8.2.6	Instantaneous shower . . . . .	154

8.2.7	MPC optimization problem . . . . .	156
8.3	GENERAL DATA . . . . .	159
8.3.1	Case study . . . . .	159
8.3.2	Renewable energy . . . . .	160
8.3.3	Uncertainty analysis of the measured data . . . . .	161
8.4	MPC RESULTS AND DISCUSSION . . . . .	161
8.4.1	Optimal operation of HPWH . . . . .	161
8.4.2	Optimal operation of instantaneous shower . . . . .	162
8.4.3	Discussion . . . . .	166
8.5	ECONOMIC ANALYSIS . . . . .	169
8.6	SYNOPSIS . . . . .	171
<b>CHAPTER 9</b>	<b>COMPARISON OF LIFE CYCLE COST ANALYSIS . . . . .</b>	<b>173</b>
9.1	INTRODUCTION . . . . .	173
9.2	LCC COMPARISON IN COLD WATER CATEGORY . . . . .	173
9.3	LCC COMPARISON IN HOT WATER CATEGORY . . . . .	174
9.4	SYNOPSIS . . . . .	175
<b>CHAPTER 10</b>	<b>CONCLUSIONS . . . . .</b>	<b>176</b>
10.1	SUMMARY . . . . .	177
10.2	RECOMMENDATIONS AND FUTURE WORK . . . . .	178
<b>REFERENCES</b>	<b>. . . . .</b>	<b>180</b>
<b>CHAPTER A</b>	<b>LCC ANALYSIS OF COLD WATER INTERVENTIONS . . . . .</b>	<b>213</b>
<b>CHAPTER B</b>	<b>LCC ANALYSIS OF HOT WATER INTERVENTIONS . . . . .</b>	<b>217</b>

## LIST OF TABLES

3.1	Comparison of the cost of pumping energy incurred . . . . .	38
4.1	City of Tshwane water tariff for 2014/2015 . . . . .	54
4.2	Comparison of irrigation and energy amounts and the associated cost . . . . .	62
5.1	Dimensions and capacity of the tanks. . . . .	80
5.2	City of Tshwane water tariff for 2014/2015 . . . . .	80
5.3	Comparison of weekly water consumption. . . . .	89
5.4	Water and energy consumption using open-loop and MPC controllers . . . . .	90
5.5	Uncertainty of the performance index . . . . .	91
6.1	Dimensions and capacity of the tanks. . . . .	110
6.2	City of Tshwane tariff for 2014/2015 . . . . .	111
6.3	Comparison of weekly water consumption. . . . .	117
6.4	Water and energy consumption . . . . .	119
7.1	Parameters of the HPWH . . . . .	137
7.2	Daily energy consumption and savings . . . . .	144
8.1	Parameters of the HPWH . . . . .	159
8.2	Energy consumption, sales, savings and associated costs. . . . .	167
8.3	Uncertainty of the performance index . . . . .	168
A.1	LCC analysis of MPC strategy on integrated water recycling system. . . . .	213
A.2	LCC of open-loop optimal control strategy on integrated water recycling system. . . . .	215
B.1	Life cycle cost analysis of the water heating strategy using MPC strategy. . . . .	217
B.2	Life cycle cost analysis of the water heating strategy using open-loop strategy. . . . .	219

## LIST OF FIGURES

3.1	Schematic of domestic water supply system with pumping and storage. . . . .	20
3.2	Baseline average hourly water demand. . . . .	30
3.3	Comparison of open-loop and MPC switching. . . . .	32
3.4	Water level in the tank without disturbance. . . . .	32
3.5	Water level in the tank with random disturbances. . . . .	34
3.6	Water level in the tank with sudden increase in demand between 18:00-20:00 hours. . . . .	35
3.7	MPC switching and water level variation for 3 days. . . . .	37
4.1	Schematic of directly connected irrigation . . . . .	43
4.2	Schematic of RWH for lawn irrigation . . . . .	43
4.3	Optimal schedule and variation of the state variable. . . . .	56
4.4	Comparison of the switching minimization strategies. . . . .	57
4.5	Optimal schedules of the valve and pump. . . . .	58
4.6	Variation of the water in the tank with and without precipitation. . . . .	59
4.7	Soil water balance with and without precipitation. . . . .	59
5.1	Schematic of water pumping and grey water recycling system . . . . .	67
5.2	Hourly water profile for a typical week day and weekend. . . . .	79
5.3	Open-loop optimal switching for a weekday. . . . .	82
5.4	Open-loop optimal switching for a weekend. . . . .	83
5.5	Water height variation in respective tanks with open-loop controller. . . . .	84
5.6	Week day closed-loop optimal switching. . . . .	85
5.7	Closed-loop optimal switching for a weekend. . . . .	86
5.8	Water height variation using MPC controller. . . . .	87
5.9	Optimal use of potable water in grey uses in a month. . . . .	88



6.1	Schematic of grey and rain water recycling system. . . . .	98
6.2	Hourly water profile for a typical week day and weekend. . . . .	109
6.3	Optimal operation of pumps and valves by open-loop controller. . . . .	112
6.4	Variation of water level in respective tanks with open-loop controller. . . . .	113
6.5	Optimal operation of pumps and valves using MPC strategy. . . . .	114
6.6	Variation of water level in respective tanks with MPC. . . . .	115
7.1	Schematic layout of the energy and hot water flow . . . . .	127
7.2	Overall and shower hot water demand pattern. . . . .	138
7.3	Optimal switching of HPWH. . . . .	139
7.4	Optimal switching of instantaneous shower. . . . .	140
7.5	Optimal grid power flow. . . . .	141
7.6	Variation of temperature . . . . .	142
7.7	The flow of power during the 24-h horizon . . . . .	143
8.1	Schematic layout of the energy and hot water flow . . . . .	150
8.2	Overall and shower hot water demand pattern. . . . .	160
8.3	Optimal operation HPWH to meet overall hot water demand. . . . .	162
8.4	Optimal operation of instantaneous shower. . . . .	162
8.5	Optimal grid power flow. . . . .	163
8.6	Variation of temperature of hot water in both devices . . . . .	164
8.7	Temperature variation for three days . . . . .	165
8.8	Power flow throughout the 24-h operating cycle . . . . .	166

# CHAPTER 1 INTRODUCTION

Africa is reported to bear the brunt of climate change that is worsening the already precarious situation of reliably providing energy and water to the population [1]. The two resources are essential in every corner of life including human survival, social equity, sustainable ecosystem and economic well being. These resources are fundamentally interlinked; energy is required to extract, treat, and deliver water while water is used to develop, generate and transmit energy, hence the name energy-water nexus. Despite the interdependence, concerned stakeholders continue to make decisions that impact on one with little or no understanding on scientific or policy complexities of the other. This prevents both energy and water sectors from fully accounting for the economic, environmental or social impacts they have on each other [2]. Traditionally, there was no need to improve efficiency and conservation of the two resources as supply exceeded demand in areas that were connected to the supply. However climate change, increased population, urbanization, increased connectivity, pollution and market volatility have led to insecurity of these resources, especially in Africa. Expansion or development of new supply infrastructure comes at an exorbitantly high cost because, in most cases, nearby sources have been harnessed to capacity and it could come at negative environmental impact [3]. Unavoidably, alternative management options are required. The 1970s energy crisis coupled with water scarcity have led to research activities geared towards demand side management (DSM) of the two resources with the intention of enhancing efficiency, conservation, reliability and sustainable use.

## 1.1 DEMAND SIDE MANAGEMENT

Energy and water share several attributes in the ways humans have developed them to provide for the livelihoods. Globally, buildings, both residential and commercial, account for about 20% of global energy [4], producing about 17% CO<sub>2</sub> [5]. The sector also uses about 8% of global water, after agricultural and industrial use. However, water consumption highly varies with geographical locations,

as less than 10 sovereign nations possess over 60% of fresh water supply in the world. They are; Brazil, Russia, China, Canada, Indonesia, U.S., India, Columbia and the Democratic Republic of Congo [6]. In South Africa, buildings are responsible for about 28% of the country's energy, coming second after the industry sector, with 40.8% [7], and 30% of water consumption, second after agriculture (irrigation). This sector is touted to grow faster than other sectors as the economy grows [8], increasing the domestic demand for water by 219% from 1996 to 2030 [9]. This continued growth will further stress the available energy and water resources, presenting enormous potential for DSM interventions. The advancement in energy DSM strategies has formed a perfect basis for developing appropriate water DSM strategies [10]. In general, energy-water nexus DSM seeks to increase energy and water security, make the two resources affordable, reduce greenhouse gas emissions and reduce losses and wastage [10, 11].

South Africa suffered from power crisis in 2008 that led to economic losses due to several factors including coal availability, maintenance requirements and unplanned outages. Further, the country is one of the top 20 greenhouse gas (GHG) emitting countries in the world, the largest emitting country in Africa, and worse still, it emits more than most emerging economies such as China and Brazil [12]. The power crisis and the desideratum to reduce the GHG emissions led to an urgent need to implement economical, easily scalable and sustainable energy efficiency and demand side management (EEDSM) measures. The government through ESKOM, the public utility, envisaged a change in energy mix from a majorly coal dependent energy sector to 48% from renewable energy, 23% nuclear, 15% gas and only 15% coal. Following the vision, ESKOM rolled out various DSM programmes in 2003 that saved about 2.8 GW by 2010. These initiatives included energy efficient lighting, heat pump water heaters (HPWHs), solar water heating and industrial energy efficiency measures. Further incentive and funding mechanisms were developed to assist in the uptake of the energy efficient technologies [13]. In 2009, the energy regulator approved the first feed-in tariff programme making South Africa the largest growing renewable energy market among the G20 economies in 2012 [14]. The World Bank states that despite the success of renewable energy and DSM activities, there still lies a lot of opportunities to increase energy efficiency (EE) in residential buildings to help the country achieve climate change goals more cost effectively [13].

As if energy challenges are not monumental enough, South Africa is 40-60% water stressed with low, unevenly distributed and seasonal annual rainfall (500 mm) and high annual evaporation rates (1700 mm) [15]. Traditionally, water service providers opted to look for new sources of fresh water

whenever demand came close to supply. However, this is unsustainable as most fresh water sources have been developed to capacity. Water insecurity has been worsened by frequent droughts, increased population and increased demand arising from more population being connected to the municipal supply. In order for water supply to be sustainable in South Africa, water conservation (WC) and water DSM are necessary to control the demand by fostering efficient use of available water resources. WC and water DSM encompasses two strategies; to reduce the growth of water demand and secondly to minimize losses or wastage. Benefits accruing from WC and water DSM include more efficient use of water resource, cost saving to customer, reliability in water supply, distribution and waste water treatment, energy saving in previously mentioned processes, and promoting social-political equity across the country [16]. The National Water Conservation and Demand Management Strategy [17] identifies the following opportunities for enhancing WC and water DSM. Reduction in distribution and plumbing leaks through various technologies like pressure management, pressure detection, public education and repairing visible leaks. It is estimated that plumbing leaks wastes about 20% of total indoor household use. The second opportunity is retrofitting with water efficient appliances, which can reduce the built area water consumption by 40%. Thirdly, increasing the efficiency of gardening water use can reduce the water used for gardening by up to 30%. Water-wise plants, mulching, efficient irrigation system, irrigation scheduling, rain water harvesting and recycling of grey water can lead to reduced garden water demand. Further, household water demand can be reduced through effective billing system, awareness campaigns, legislation and offering incentives to home owners.

Following the challenges the country faces in energy and water security, a number of research institutions and industry players are developing eco-innovative programmes involving EE, renewable energy, water and waste water management [14]. Various technologies have been developed to conserve and efficiently use of both energy and water with the goal of achieving green buildings. The future therefore lies in optimally integrating these technologies, with opportunities including operation of energy and water efficient devices, integrated renewable energy, rain water harvesting and grey water recycling system. This forms the major motivation of the work presented in this thesis; to provide optimal, economical, practical and sustainable solutions to ensure efficient use of energy and water.

## 1.2 OPTIMAL CONTROL STRATEGIES

In designing an effective resource efficient optimal control system, an accurate estimation of energy and water consumption is mandatory. Integration of efficient devices with alternative sources of energy and water such as renewable energy, rain water harvesting and grey water recycling maximizes energy and water savings achieved in a building. Automatic control of these systems is necessary in not only maximizing energy, water and economic benefits but also minimizing the inconvenience subjected to end users. Classical control techniques are commonly used but they essentially use a trial and error process where various methods are iteratively used to determine design variables of an acceptable system. For instance, proportional-integral-derivative (PID) controllers normally work on systems with single input-single output (SISO) by minimizing the gain error in relation to set targets. Presence of uncertainties and disturbances also make the control system unstable [18]. However, modern complex multiple input-multiple output (MIMO) systems demand more accurate and reliable control systems. Optimal control seeks to determine the control actions that will operate a process within physical and operational constraints while simultaneously minimizing or maximizing a given performance criteria [19]. In lieu of PID, optimal controllers can effectively, robustly and accurately handle multi-variable dynamic systems through minimizing the cost function. They can be implemented as open-loop or closed-loop controllers. Open-loop optimal controllers use the feed forward principle in that measurements of the plant are used by the controller to predict the future behaviour of the system throughout the controller's operating cycle. On the other hand, closed-loop model predictive control (MPC) uses both feed forward and feedback measurements of the plant to compute the control law on-line [20]. The superiority of optimal controllers, whether open-loop or closed-loop MPC, make them suitable for use in this research and have therefore been designed in this thesis.

Although optimal control is suitable for handling energy and water conservation initiatives in buildings, no research has been undertaken towards the same. Most research in optimally operating energy and water systems has focused on achieving efficiency and conservation of either energy [21–25] or water independently [26–29]. This could hamper adoption of the environment conserving measures; for instance, market penetration of HPWHs in South Africa stands at 16% [30], while rooftop water harvesting in Australia stands at 34% [31]. This informs further motivation for this research. Besides, the current control systems used in various devices used in buildings are insensitive to dynamic changes in availability and demand for energy and water. For instance, thermostat and floating switch devices

used in HPWHs and pump storage, respectively, do not consider the cost of energy especially when subjected to the time-of-use electricity tariff (TOU) thereby ending up using unnecessary energy. A thermostat has to continuously heat water to the set point even in times when there is no demand for water, leading to energy loss to the environment. These control methods are also unable to predict the demand and they normally operate even during the undesirable and expensive peak period of the TOU tariff. This research, therefore, proposes optimal control strategy, to control various energy and water devices in a house consequently minimizing the cost through shifting the electric load to the off-peak period of the TOU tariff while also maximizing the use of alternative supply sources namely; renewable energy, rain and grey water. This optimal control strategy is a huge step towards achieving green buildings in South Africa.

### 1.2.1 Implementation

The open-loop optimal and closed-loop MPC controllers discussed in this thesis are designed and simulated using MATLAB. To implement the controllers in a real world scenario, the MATLAB models need to be translated to C and then compiled for a specific hardware while verifying in each step<sup>1</sup>. In the cold water category, values for water demand pattern for the entire control horizon are input into open-loop optimal controller which in return solves the optimization problem and feeds appropriate optimal commands to various pumps and valves. The MPC strategy requires level sensors to measure and feed back the level of water in various tanks to the controller in real time. Using the measurements, the controller then goes ahead to solve an open-loop optimization problem during the current time step and implements only the first element of the control vector to the pumps and valves. During the next step, the process of measuring the tanks and solving the open-loop optimization problem is repeated until the prescribed end of the control horizon. Similarly, in the hot water category, hot water demand is input into the open-loop optimal controller which solves the optimization problem for the entire control horizon and then sends optimal commands to power the hot water devices either using grid or renewable energy. The MPC strategy would require temperature sensors to feed back the temperature of water in both devices to the controller during every iteration. The controller uses the current temperature to predict the future behaviour of the devices.

<sup>1</sup>[www.mathworks.com/company/newsletters/articles/from-matlab-to-embedded-c.html](http://www.mathworks.com/company/newsletters/articles/from-matlab-to-embedded-c.html)

### 1.3 CONTRIBUTION AND RESEARCH OBJECTIVES

Energy in buildings comprises of various sources such as electrical, gas, renewable and fuel oil that meet the demand from various components in the building such as space heating, ventilation and air conditioning, lighting, water heating, plug in equipment and others. The term ‘energy’ in this thesis refers to electrical energy used by pumping or heating water in a residential house. To achieve resource efficient green buildings while considering the energy-water nexus, a multifaceted approach is necessary. Various supply alternatives are optimally and seamlessly integrated with the existing system using optimal controllers. These alternatives include renewable energy, that is, solar photovoltaic and wind energy, rooftop rain water harvesting, grey water recycling and pump-storage scheme. More to that, some of a residential building’s resource intensive end uses are identified for improving their resource efficiency since they would have the greatest impact towards achieving green buildings. For instance, garden irrigation in South Africa uses up to 60% of the total household water demand [32]. The shower, bath, toilet, and washing machine contribute to an average of 70%-90% of indoor water use, albeit varying with geographic location [33]. Further, domestic water heating is the fourth largest energy end use in commercial buildings, and third largest in residential buildings worldwide [34]. In South Africa, water heating is the largest energy user in the domestic sector with 40-50% of the monthly electricity use [35]. Therefore, besides efficiently providing hot water in a house, the shower end use is a perfect candidate to further enhance the efficiency of both resources.

This thesis presents the first multifaceted approach to optimally integrate alternative resources as well as operate various efficient devices. This leads to conservation and enhanced efficiency of both resources, cost savings and milestone closer to achieving green buildings. The work presented in this thesis effectively overcomes the existing technical and operational challenges of incorporating these technologies in buildings and maximizing their benefits. To achieve the optimal strategies, the following objectives have been achieved;

1. To design and compare the performance of open-loop and closed-loop model predictive controllers that ensure energy efficiency is achieved in a pumping-storage scheme used in areas with unreliable water supply. The two controllers minimize the cost of pumping energy as well as the maintenance cost of the pump. The suitability of using either of the two control strategies in various conditions is further recommended.

2. To optimally reduce energy and water consumption of a rooftop water harvesting system used for lawn irrigation. This is achieved by optimally scheduling a water pump used to take the stored rain water to the lawn automatically using the open-loop controller.
3. To develop open-loop and closed-loop MPC controllers that optimally operate a grey water recycling system. Each controller is used to reduce cost of energy consumed in pumping both potable and grey water while maximizing the use of treated grey water in their respective end uses.
4. To design and analyse open-loop and closed-loop MPC controllers for operating an integrated rooftop water harvesting and grey water recycling system. Both controllers seek to reduce the cost of energy consumed in pumping while maximizing the use of treated rain and grey water. Further, the economic analysis of the integrated system is carried out.
5. To optimally control a heat pump water heater (HPWH) and an instantaneous shower with the aim of reducing the cost of energy used to heat water as well as water wasted due to cooling in the pipes. Further, the devices are powered using integrated renewable energy systems to save grid energy as well as sell power back to the grid using a feed-in tariff. The feed forward controller also maximizes the use of renewable energy sources.
6. To design a closed-loop model predictive controller to operate the HPWH with an instantaneous shower to ensure energy and water efficiency are achieved. More to that, the two devices are also powered using integrated renewable energy systems, and the controller maximizes their energy. Thereafter, a life cycle cost analysis is carried out to analyse the feasibility of the control strategy.

## 1.4 THESIS LAYOUT

In general, the work in this thesis is presented in two broad categories: cold and hot water management strategies. The structure is as follows.



**Chapter 1** is the introduction of this research. The chapter provides relevant background information, highlighting the problem and the current research gap. Then, the motivation, contributions and objectives of the research are described.

**Chapter 2** offers a comprehensive literature study of energy and water management strategies and the various control methods used for energy-water systems. The chapter also highlights the weakness in managing energy and water independently, and hence the importance of energy-water nexus management.

**Chapter 3** begins the cold water management strategies with two control strategies, open and closed-loop, developed and their performance in controlling a pump-storage scheme of supplying water in buildings analysed. This scheme is common in areas with unreliable water supply, whether water rationing or insufficient supply pressure.

**Chapter 4** contains the design and testing of an open-loop optimal controller being used to control the operation of a rooftop rain water harvesting scheme that is used for lawn irrigation. The controller seeks to reduce the amount of water used in irrigating the lawn while minimizing the cost of pumping energy.

**Chapter 5** has a typical domestic grey water recycling system optimally operated to meet the household water demand. The performance of open-loop and closed-loop MPC strategies is compared in terms of reliability and savings achieved, with the recommendations on the use of either controller given.

**Chapter 6** is an advancement of Chapters 3 - 5. An integrated grey water and rooftop rain water harvesting system for residential houses. The performance of two controllers discussed earlier is analysed in this chapter, and recommendations on the use of either given. More to that, a life cycle cost analysis is carried out and discussed.

**Chapter 7** begins the hot water management techniques with designing an open-loop optimal controller to operate a HPWH and an instantaneous shower connected at the shower, which is used whenever the temperature of water from the HPWH is lower than required. Further the controller

operates integrated renewable energy systems that are used to power the two hot water devices with the aim of achieving energy and water efficiency and conservation.

**Chapter 8** is a continuation of Chapter 7 whereby, a closed-loop MPC is developed for the same devices and the performance analysed in relation to energy and water savings while meeting the hot water demand. A life cycle cost analysis is also carried for the two control systems.

**Chapter 10** contains a summary of the work done in the previous chapters. Thereafter, recommendations and areas for further research are outlined.

# **CHAPTER 2 ENERGY-WATER DEMAND SIDE MANAGEMENT: LITERATURE REVIEW**

## **2.1 INTRODUCTION**

Energy and water are inherently connected on both supply and demand side. The supply side includes electricity generation, water and waste water facilities while the demand side includes various sectors such as residential, commercial, industrial and agricultural sectors that require both energy and water in their activities. Traditionally, energy and water practitioners opted for supply based management techniques to address energy and water challenges. However, this method is not sustainable due to astronomical cost of implementation especially because the easily accessible options have already been fully developed and new ones are difficult to find. Further, expansion of existing supply resources has negative impact to the environment [3]. Demand side management (DSM) programs seek to reduce the gap between supply and demand through various activities such as conservation, load management, fuel substitution, self generation [36], recycling, storm water capture, reduction of unaccounted water and consumer education [37].

## **2.2 ENERGY DEMAND MANAGEMENT**

The global energy crisis experienced in the 1970s was the impetus behind energy efficiency and conservation methods being developed [38]. In South Africa, insufficient power supply led to serious power shortages in 2008 with economic losses of up to USD 282 million [39]. The country's electricity sector faces three main challenges; Firstly, supply cannot meet demand leading to narrow reserve

margin and even power shortages [40]. Secondly, the public utility, ESKOM, is severely underfunded to expand the existing supply infrastructure and thirdly, the economy has high emission intensity, as 86% of the country's power is generated from coal, resulting in high greenhouse gas emission levels [39]. To deal with these challenges, the government introduced both supply side and demand side management measures. Supply side interventions involved increasing the generation capacity by building two new coal-fired power stations, return to service three coal power stations and explore co-generation and renewable energy options. Increasing coal generating capacity comes at monumentally high cost as well as negative environmental impact due to greenhouse gas emissions [41]. Demand side interventions included mass installation of energy efficient technologies such as solar water heaters and energy efficient bulbs, installation of smart meters and quota allocations with penalties and positive incentives. These interventions were meant to reduce the demand in short, medium and long term measures, by 3000 MW in 2012 and a further 5000 MW by 2025 [39].

The Centre of New Energy Systems at University of Pretoria, South Africa, proposed a unified classification of efficiencies in an energy system [36,42,43]. The concept classifies energy efficiency (EE) of a system in terms of Performance, Operation, Equipment, and Technology (POET) efficiencies. This framework is useful in systematically identifying EE opportunities in an energy system [42]. Performance efficiency is a measure of EE determined by external but deterministic system indicators such as cost, sources of energy, environmental factors, technical factors among others. In some instances, performance indicators contradict each other and therefore, to maximize EE, trade-off is necessary among concerned indicators [44]. Operation efficiency considers coordination of various system's components. In this case, indicators include sizing, matching, time control and human coordination [45]. In equipment efficiency, energy output of an isolated equipment is measured while considering its capacity, specification, constraints and maintenance. Equipment efficiency seeks to minimize deviation of the measured parameters from design specifications. Finally, technology efficiency is achieved through new and improved energy conversion, processing, transmission and consumption method. It is typified by novelty and optimality in searching for superior technologies that may seek to expand scientific limits. Indicators include feasibility, life-cycle cost analysis, return on investment and coefficients in energy conversion/process/transmission rates [36].

Industrial, transport and residential sectors are the largest energy consumers in South Africa and are therefore the best suited sectors for DSM initiatives [46]. Using POET framework, various studies have been undertaken in these sectors. Some of the studies aiming to reduce energy consumption in the

industrial sector include; industrial pumping systems [47], pump-storage systems in coal beneficiation plants [48], coal washing processes [49, 50], water pumping station [51], operation control of conveyor belts in mining industry [52–54], coal crushers [55–57], rock winders [58], ventilation systems [59], to name but a few. In the transport sector, operation of heavy haul trains was modelled [60], and then optimised to improve EE [61–63]. In residential sector, DSM interventions include solar water heating [64], optimal operation of heat pump water heaters (HPWHs) powered using integrated renewable energy systems [65, 66], with battery [67, 68] or fuel cell storage [69]. Other interventions include demand response [70, 71], building envelope [25], lighting [72–74], air conditioning [75, 76] and maintenance and retrofitting [24, 77, 78]. In addition to above energy DSM activities, research has been carried out to address the problem of optimal energy feed/mix [22], and switching in the residential sector [79]. This includes photovoltaic-diesel-battery system for remote consumers [80, 81], photovoltaic-wind-diesel-battery hybrid power system [23], and optimal dispatch for a microgrid with renewable energy systems [82]. All these initiatives have continually reduced the pressure being experienced by the national grid as well as lowered the cost of energy being incurred by the consumer.

### 2.3 WATER DEMAND MANAGEMENT

To ensure reliable and sustainable water provision over time, it is important to consider and match supply and demand. Since expansion of existing centralised water infrastructure is undesirable, available alternative water supply options, including desalination, waste water reuse and rain water harvesting, are suitable for increasing reliability of water supply. Managing demand is also important in not only to reducing the demand but also taking care of unmet demand that is likely to further stress the resource in future. The grim water situation is not limited to South Africa as global water consumption is predicted to rise by 50% in 2025 as compared to consumption in 1995. This increase is attributed to rapid population increase and climate change that could lead to drought in some areas [83].

South Africa is a semi-arid country with 40%-60% water stress and an average rainfall of about 500 *mm* per annum [84]. Water availability across the country is faced by three major challenges. One, the country experiences uneven spatial and seasonal rainfall whereby 43% of the rain falls on 13% of the land. Two, there is relatively low stream flow in rivers most of the time, limiting the proportion of stream flow that can be relied upon and lastly, most urban and industrial developments are located

far away from the country's large water courses necessitating bulk transfer of water. Agricultural and residential (urban and rural) sectors are the largest water consumers, with 60% and 30% of the national water consumption respectively [85]. By 2000, eleven out of the nineteen water catchment areas were facing water deficit as the demand was more than they could supply [86]. Although expanding water supply could alleviate water scarcity, water demand management is better and cheaper in enhancing efficiency and conservation in existing uses [3].

The Department of Water Affairs and Forestry (DWAf) acknowledges that water scarcity in South Africa is an impediment to socio-economic development [87]. In the recent past, the scarcity has been worsened by frequent droughts, increasing demand and economic growth, quickly leading to full utilisation of available water resources. Augmentation of water schemes is extortionately expensive and deleterious to the environment. Therefore, water has to be efficiently used before consideration of development new supply development. DWAf defines water demand management as: "The adaptation and implementation of a strategy (policies and initiatives) by a water institution to influence the water demand and usage of water in order to meet economic efficiency, social development, social equity, environmental protection, sustainability of water supply and services, and political acceptability" [17]. Water demand management can be achieved through various initiatives. Technical initiatives involve implementation of measures like retrofitting with water saving devices, water metering, leak detection, water recycling systems, desalination and rain water harvesting systems. Economic initiatives include water pricing, abstraction taxes, new allocation methods and rebates. In most countries, water services are currently provided at prices well below the total cost (including full cost of provision, social and environmental costs) incurred by public providers such as municipalities [88]. Information initiatives and particularly public awareness campaigns are necessary to encourage the public to change their water use behavioural pattern and also readily accept new measures meant to conserve water. Regulatory initiatives are also important in specifying limits for water abstraction or surface water flow threshold [89]. In extreme cases like in developing nations, intermittent water supply is also used to manage demand [90].

## **2.4 ENERGY-WATER NEXUS DEMAND MANAGEMENT**

The interconnectivity of energy and water systems is such that on one hand, water is used in almost all stages of energy and electricity generation, from irrigating crops for biofuels, hydraulic fracturing

to cooling thermoelectric power plants. On the other hand, energy is first required to extract, treat, and deliver water of allowable quality for diverse human uses, and then required to treat waste water before it is allowed back to the environment. Historically, the synergy between the two resources has been minimal. The per capita water resources in most countries were in abundance and the economic sensitivity of variations in natural systems was low [91]. In the recent past, limitations and vulnerabilities associated with the two resources around the world have become prominent. In any country, energy-water nexus is affected by various decision makers and stakeholders such as; state planners, electricity utilities, water utilities, plant operators, environmental regulators, water resource managers, refineries, oil and gas producers and citizens. Energy and water systems are normally managed and regulated independently with subsequent activities affecting both resources. Consequently, energy-water nexus is increasingly being highlighted as an important element for future planning and policy consideration [91]. In United States, the Department of Energy has identified opportunities in technology research, development and deployment that would highly impact energy-water nexus. These opportunities include; optimizing water efficiency in energy production and generation to the end use systems, optimizing energy efficiency of water extraction, treatment, conveyance to the end use systems, increase safe and productive use of non-traditional sources of water and finally exploit sustainable synergies between energy and water systems [92].

Globally, developed nations have appreciated the importance of energy-water nexus, with research cutting across the design, operation, policy and planning ambit. The importance of the deep connection between energy and water was first reported in United States (US) by Gleick [93]. This led to several states recognizing the fact that sustainable management of either resource requires consideration of the other<sup>1</sup>. In Spain, a study was carried out to calculate the energy used in the water sector as well as water used in the energy sector. It was recommended that research on more efficient use of farm water, urban waste water treatment and desalinated water must include the energy component [94]. In Australia, information on the interdependence of the two resources, and the effect climate change has on them is limited. This could have led to development of policies having unintended outcomes on either of the resources. Therefore, better cross-sectoral information is necessary for policy makers, technologies that have dual benefit should be adopted and integrated governance institutions should be formed [95]. Far afield, China's economic growth led to the need to understand the effect of resource scarcity on the economy to enhance better policy making [96]. Although water prices are low, the

<sup>1</sup><http://www.ncsl.org/research/environment-and-natural-resources/overviewofthewaterenergynexusintheus.aspx>

high energy intensity of water treatment facilities that need to recover costs should be appreciated more by policy makers in designing pricing and conservation methods that involve both resources [97]. Developing nations are also acknowledging the importance of energy-water nexus. For instance, a study on integrated on-site water and sewage management strategies in low income houses in Brazil showed that sustainability of water services, while considering energy management, is dependent on reduced effluents to centralised systems [98]. In India, strategies used in urban and irrigation sectors to cope with the shortages and unreliability of both energy and water was assessed. Both short and long term policies and measures are required in order to reliably and sustainably narrow the gap between supply and demand and consequently improve the quality of life [99]. South Africa leads the research in energy-water nexus in Africa. Most research has been in urban water systems due to increased urbanisation. In centralised urban water systems, losses must be minimised as they lead to significant energy losses incurred during extraction, treatment and transportation [100]. Similarly, waste water recycling is proposed as the most sustainable alternative water supply by South African municipalities [101]. However, these centralised supply and waste water treatment systems are very expensive to implement, have negative environmental impact and waste water plants are very energy intensive.

A paradigm shift in urban energy-water management is therefore required. Decentralised energy and water systems, that is, those that can be managed as stand alone or integrated with centralised systems are suitable for providing alternative source of water and energy. Decentralised systems not only seek to augment the existing energy and water supply but also seek to reduce greenhouse gas emissions [102]. Decentralised water systems provide an opportunity to maximize utilisation of resources and adoption of alternative water supply options is important in urban water services. Such alternatives include rain water harvesting [103], grey water recycling [104, 105], or a combination of both [106]. Renewable energy, on the other hand, offer decentralised alternatives to grid supply. In South Africa, substantial research has been carried out on stand-alone systems [80] or grid tied systems [69].

## **2.5 CONTROL METHODS FOR ENERGY-WATER SYSTEMS**

Automatic control has been used for a long time with, for instance, an extensive range of thermostat devices being commercially available in the 19<sup>th</sup> century [107]. These thermostat devices are still applied in various devices, like storage water heaters, despite their inability to predict the desired output



and demand over time. The need by governments to launch, manoeuvre and track missiles and space vehicles and the advent of digital computer led to accelerated development of modern control methods in the 1950s [108]. Such control methods include proportional-integral-derivative (PID) controllers, which have been employed in a wide range of applications owing to better accuracy and feedback than thermostat devices. Although PID controllers have advanced over time, they struggle to accurately tune the controller's gain, especially in the presence of disturbances and uncertainties, resulting in the plant being unstable and are mostly suitable for single input-single output (SISO) systems [109].

The inadequacies of PID controller and the need for higher accuracy and precision led to development and rapid adoption of advanced adaptive and optimal control strategies [110]. The advanced optimal control techniques are able to achieve optimal output, predict future behaviour of the plant [20], handle multiple input-multiple output (MIMO) systems [111] while simultaneously handling plant constraints and disturbances on-line [112]. In fact, closed-loop model predictive control (MPC) strategy is superior to PID in terms of Rise time, Settling time and maximum overshoot [113]. The optimal control strategy minimizes controlled variables to global or local minimum of the objective function, within a set of constraints. Optimal control can be simulated using various optimization algorithms commonly available in several software such as MATLAB [114]. These control strategies are slowly replacing PID controllers both in industrial plants and operation of energy and water systems [115].

Optimal control strategy, whether open-loop or closed-loop, has widely been used in operating distributed renewable energy systems [23, 65, 66, 116] and water systems [117–119] to enhance energy and water efficiency respectively. Further, the strategy has recently been used to achieve energy-water efficiency in the supply side [120, 121]. However, little work has been carried out on optimal control strategy to enhance energy and water efficiency as a means of demand side management. This thesis seeks to design both open-loop and closed-loop optimal controllers for practical control of energy and water efficient devices, renewable energy systems and water recycling systems, solving one of technical challenges involved with adoption of these technologies.

# **CHAPTER 3 OPTIMAL ENERGY-WATER MANAGEMENT IN HOUSEHOLDS WITH PUMP-STORAGE SCHEME**

## **3.1 INTRODUCTION**

The rising global population is increasingly putting pressure on the limited source of potable water [122], with 60% of the global demand estimated to be met by 2030 [123]. Actually, the world bank estimates that global water demand will increase at a projected rate ranging from 43% in North America to 283% in Sub-Saharan Africa from 2005 to 2030 [124]. The impact of water insecurity is higher in the developing nations, like South Africa, due to high rate of economic development and subsequent rise in living standards [125]. Management of water demand in buildings through efficient technology and behavioural changes has strong entailment in reducing the demand for energy as well as conserving potable water supplies. Demand side management in buildings has mainly focused on energy, such as demand response [71], energy efficient building retrofitting [126, 127], renewable energy utilization [116] and control of efficient hot water systems [67]. Research has started to focus on the importance of demand management of energy-water nexus such as system-based framework for assessing the nexus in cities [128], nexus at a micro-component level using rain water tank [129], conservation in a building [130] and even in industries [96]. Although water supply in developed nations is reliable [131], the supply in developing nations is quite unreliable, and in some instances haphazard, where the end users are forced to rely on other forms of supply like trucks [132]. This is the case in some cities in Nigeria, Ghana, Mexico and Indonesia [133].

South Africa, a semi-arid nation, is not only included in the worldwide trend of inefficient management of both energy and water but also has her demand far higher than the supply [41, 134]. The popula-

tion growth rates and trends in socio-economic development indicate that South Africa's freshwater resources cannot sustain the current patterns of water consumption and discharge. At present, multiple regions in the country are experiencing water deficit [135], like KwaZulu-Natal province, whose severe water shortage has forced it to implement water rationing<sup>1</sup>. Similarly, the energy deficit has forced the power utility, ESKOM, to implement national load shedding program<sup>2</sup>. Despite this dire situation, there is little information on household water consumption according to the department of water and sanitation [87]. A study by Jacobs *et.al.* [136, 137] found out that for the urban families with lawns and gardens, the most significant water end uses were garden irrigation (37%), toilet (21%), shower/bath (12%) and clothes washing machine (9%) of the total water consumption. This research compares well with other studies conducted in United States [138], Branz, New Zealand [139], Perth [140] and Melbourne in Australia [141].

The continual urbanization is increasing the adoption of decentralized water systems although there is still technological challenges in their operation [102]. Previous studies show that the problem with water supply in developing nations is forcing end users to pump and store their water. Malik [99] looked at various coping strategies adopted by end users in dealing with the uncertainties, unreliability and shortages of water supply in India. End users are forced to have various water storage means such as overhead tanks and having containers in the house. They are also forced to change their water use pattern highly inconveniencing them. Vieira and Ghisi [98] conducted a study on energy-water nexus for low income houses in Brazil and found out that the dis-economies of scale associated with pumping and storage increased the energy intensity for water services. Another study on residential rain water tanks showed that pumps increase the energy intensity and the consequent bills to the end users [129]. Research on pump scheduling has mostly focused on industrial [47], agricultural [142] and large scale municipal pumping [143]. However, for domestic users, pumping takes place with no consideration to the peak power consumption negatively affecting the grid and increasing electricity bills incurred by the end users.

This introductory chapter reports the first attempt to design novel, practical and economically attractive open-loop optimal control and closed-loop model predictive control (MPC) strategies for pumping and storing water in a tank to meet the hourly water demand in a house subject to the time-of-use (TOU) electricity tariff. These control systems ensure energy efficiency in cases where either there

---

<sup>1</sup>Department of Water and Sanitation [www.dwa.gov.za/default.aspx](http://www.dwa.gov.za/default.aspx)

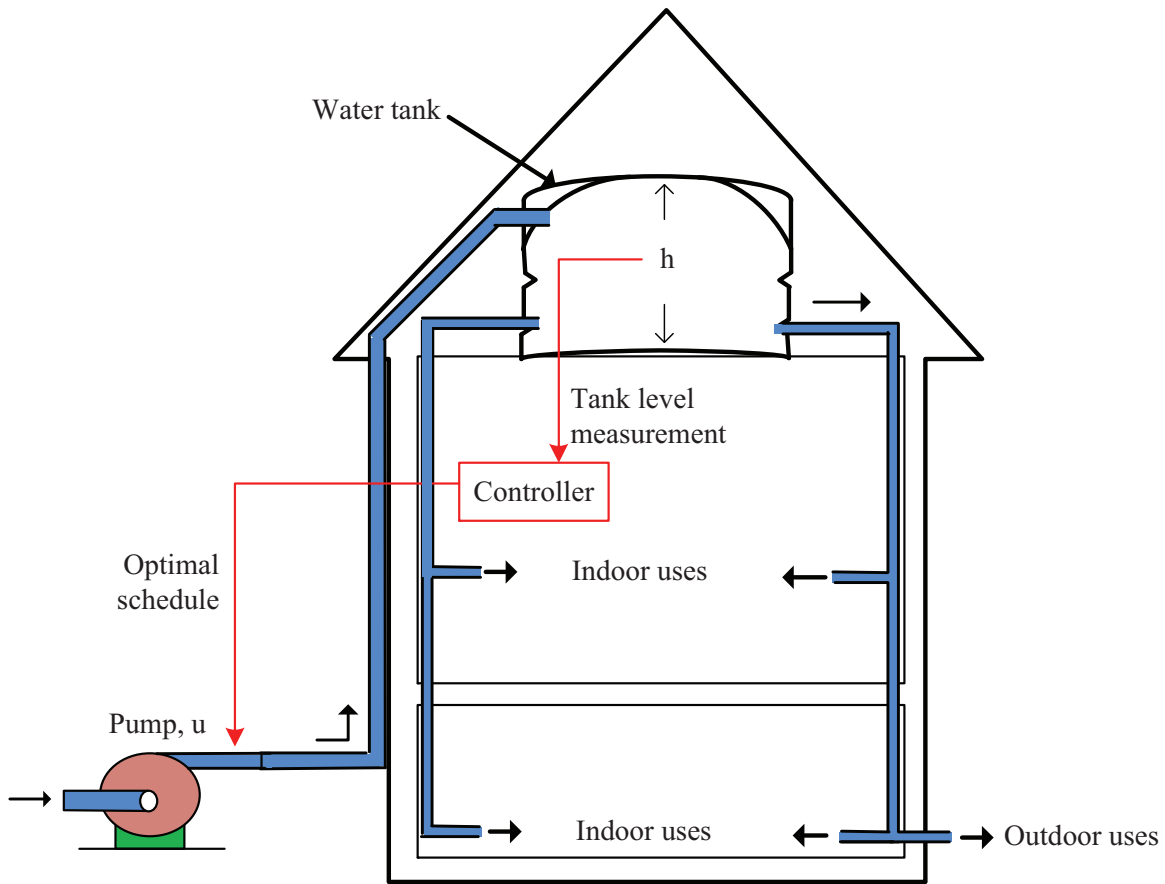
<sup>2</sup><http://loadshedding.eskom.co.za/>

is water rationing or the water supply is always there but it has low pressure and cannot therefore be used directly in the house. We consider the latter case in this chapter. The controls are superior to the classical control methods such as proportional-integral-derivative (PID) controls. PID controllers have low accuracy in processes which are either non-linear or have a large time delay [18]. Further, PID controllers only handle effectively single input-single output (SISO) systems. However, MPC can handle multiple input-multiple output (MIMO) systems, deal with constraints [111], has higher accuracy, robustness against disturbances and has the ability to predict the future behaviour of the plant [20]. It nonetheless comes at a higher computational cost [144]. The application of MPC and PID to water pumping and level control systems have been studied by many researches, such as [145] and [146], who concluded that MPC is better than PID in terms of Rise time, Settling time and maximum overshoot. It also intrinsically and quickly compensates for disturbances in the system [113]. The aim of the controls presented in this study is to optimally operate the pump in the pump-storage scheme such that the customer's water demand can be satisfied with minimum electricity cost and maintenance cost of the pump. Two controllers, namely open-loop control and closed-loop MPC control, are introduced to cater for different application requirements. The open-loop control is easy to implement and more cost effective and is more suitable for applications where the water demand pattern is known to be relatively stable. However, in cases where the demand pattern is known to be fluctuating in a way that is difficult to predict and/or the external disturbances to the implementation of the control system is of significant impact, the closed-loop control must be adopted. Although it has the ability to robustly control the system under the aforementioned fluctuating demand and disturbances, it requires installation of additional monitoring devices to the system such as water level measurement of the tank, which increases the cost and complexity of the control system.

## 3.2 LAYOUT AND FORMULATION

### 3.2.1 Schematic layout

The increasing water demand due to increasing population and urbanization is causing inadequate supply to the end users. The situation is worsened by the fact that the existing supply infrastructure is not expanding proportionately to the demand forcing end users to resolve to alternative measures like pumping and storing the water [147]. Figure 3.1 shows the schematic diagram of the water supplying system in a house where water has to be pumped and stored for later use. A fixed speed pump with a



**Figure 3.1.** Schematic of domestic water supply system with pumping and storage.

switch ( $u$ ) pumps the water to a rooftop storage tank whose size is restricted to the space available. From the tank, water flows by gravity with the required pressure to various end uses. Two pump controllers are considered in this chapter. First, an open-loop control system is developed by using the predicted diurnal water demand. Secondly, a closed-loop MPC control system is developed whereby, the level of water in the tank is measured using level sensors. This is used as the feedback signal to the controller, which then optimises the schedule for pumping to meet the demand.

### 3.2.2 Open-loop optimal control system

This control system seeks to minimize the cost of energy used in pumping water to the tank while simultaneously minimizing the maintenance cost of the pump and effectively maximizing its life cycle. It is assumed that water is always flowing in the municipal pipe albeit at low pressure and there is no water rationing. In case of water rationing, the same formulation is applicable, but the controller would

have the information on when the water will be present in order to start pumping. In this chapter, we consider an evaluation period of one day, or a full operation cycle of 24 h, from 0 to hour 24 with a sampling period,  $t_s$ , of 10 min. This leads to a total number of samples  $N = \frac{24}{t_s} = 144$ . An operating cycle of one day is suitable as normally the hourly water consumption pattern is the same for different days. The sampling period is chosen as a trade-off between accuracy and computational capacity of the controllers. A larger sampling period could lead to large inaccuracies such as spilling the water or running the tank dry. Therefore, to minimize cost of energy used in pumping the water, the objective function is,

$$J = \sum_{j=1}^N p_m t_s p_e(j) u(j), \quad (3.1)$$

where  $p_m$  is the pump rating,  $p_e(j)$  is the cost of electricity in the  $j^{\text{th}}$  sampling interval using the time-of-use tariff and  $u(j)$  is the on/off status of the pump during the  $j^{\text{th}}$  sampling interval. This objective is subject to the constraints discussed below.

### 3.2.2.1 Capacity of the water tank

The dynamics of the volume of water in the tank can be expressed in discrete-time domain by a first order differential equation as follows:

$$V(j+1) = V(j) + Q t_s u(j) - D_{tot}(j), \quad (3.2)$$

where  $Q$  is the flow rate of the water through the pump in  $m^3/h$  and  $D_{tot}(j)$  is the total water demand in the  $j^{\text{th}}$  sampling interval in  $m^3$ . Expressing this volume in terms of the initial volume  $V(0)$  in the tank using recurrence manipulation gives,

$$V(j) = V(0) + \sum_{i=1}^j \left( Q t_s u(i) - D_{tot}(i) \right), \quad (1 \leq j \leq N). \quad (3.3)$$

The water pumped into the tank must not spill from the tank as it would lead to wastage of water and damage the house's ceiling. Similarly, the tank should not be completely empty in order to prevent air from getting into the pipes [148], as well as to avoid inconveniencing the end users [149]. Therefore, the amount of water in the tank is restricted by the tank's dimensions as follows

$$V_{min} \leq V(0) + \sum_{i=1}^j \left( Q t_s u(i) - D_{tot}(i) \right) \leq V_{max}, \quad (1 \leq j \leq N), \quad (3.4)$$

where  $V_{min}$  and  $V_{max}$  are the minimum and maximum allowable volumes of the water in the tank respectively.

### 3.2.2.2 Tank's terminal constraint

It is desired that at the end of the horizon, a certain volume,  $V_f$ , of water is left in the tank. From the state equation (3.2), then the volume of water in the tank during the last sampling interval is;

$$V_f = V(N) = V(0) + \sum_{j=1}^N \left( Q_{ts}u(j) - D_{tot}(j) \right). \quad (3.5)$$

### 3.2.2.3 Pump maintenance cost

Minimum energy cost may be achieved by frequently switching the pump on and off during the control period. Unfortunately, this frequent switching of the pump increases the pump maintenance cost due to high wear [117]. The number of pump switching can be used as an alternative variable for measuring the pump maintenance cost [143].

Even though maintenance cost has not been considered in some studies [150], common methods used to minimize it are; switching the pump for a pre-set minimum duration [151], and restricting the maximum number of times a pump can switch during the control period [152]. These methods, nonetheless, do not optimally control the pump while minimizing the maintenance cost. Actually, if the water demand rises, the optimal solution would easily become infeasible as the controller is restricted on the number of times it can switch instead of adapting accordingly. To overcome this, the Pretoria method by Mathaba et al. [153] introduces an auxiliary variable  $s(j)$  represented by a value 1 whenever the pump's state is changed from off to on. This auxiliary variable is optimally determined by the following optimization problem.

$$\min \sum_{j=1}^N s(j), \quad (3.6)$$

constrained by

$$u(1) - s(1) \leq 0, \quad (3.7a)$$

$$u(j) - u(j-1) - s(j) \leq 0, \quad (3.7b)$$

with  $s(j) \in \{0, 1\}$ . The inequality (3.7a) initialises the auxiliary variable as the initial status of  $u$  while the inequality (3.7b) favours the control that involves less switching. Using this method, the overall objective that simultaneously minimizes the cost of energy and maintenance is

$$J = \sum_{j=1}^N \left( (1 - \omega) p_{mts} p_e(j) u(j) + \omega s(j) \right), \quad (3.8)$$

where  $\omega$  is a weighting factor.

### 3.2.2.4 Pump's switch

This chapter focuses on fixed speed pumps which are commonly used in water pumping in urban houses. This type of pump is ideal for the current task, unlike the more expensive variable speed drives, because the water just needs to be pumped and stored. Therefore, under optimal operation, the pump should not require additional investments on flow rate adjustment devices such as valves and variable speed drives [47]. The pump controller only switches the pump on/off such that it can be modelled as a switch control problem, where

$$u(j) \in \{0, 1\}, \quad (1 \leq j \leq N). \quad (3.9)$$

### 3.2.3 Algorithm for solving the open-loop optimization problem

The objective function (3.8) is solved using the following canonical form [56].

$$\min f^T X \quad (3.10)$$

subject to

$$\begin{cases} AX \leq b \text{ (linear inequality constraint),} \\ A_{eq}X = b_{eq} \text{ (linear equality constraint),} \\ L_B \leq X \leq U_B \text{ (lower and upper bounds).} \end{cases} \quad (3.11)$$

Here, vector  $X$  contains the control variables, which are the pump switch  $u(j)$  and the auxiliary variable  $s(j)$  used to minimize the frequency of pump switching. Thus,

$$X = \left[ u(1), \dots, u(N), s(1), \dots, s(N) \right]_{2N \times 1}^T. \quad (3.12)$$

The vector  $f^T$  in the canonical form (3.10) can be obtained from the objective function (3.8) as

$$f^T = \left[ (1 - \omega)p_{mts}p_e(1) \dots (1 - \omega)p_{mts}p_e(N) \quad \omega \dots \omega \right]_{1 \times 2N}. \quad (3.13)$$

The linear inequality constraint (3.4) can be transformed into

$$\begin{aligned} A_1 X &\leq b_1, \\ -A_1 X &\leq b_2, \end{aligned} \quad (3.14)$$



where,

$$A_1 = \begin{bmatrix} -t_s Q & 0 & \dots & 0 & 0 & \dots & 0 \\ -t_s Q & -t_s Q & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ -t_s Q & -t_s Q & \dots & -t_s Q & 0 & \dots & 0 \end{bmatrix}_{N \times 2N}, \quad (3.15)$$

$$b_1 = \begin{bmatrix} -D_{tot}(1) - (V_{min} - V(0)) \\ -(D_{tot}(1) + D_{tot}(2)) - (V_{min} - V(0)) \\ \vdots \\ -(D_{tot}(1) + D_{tot}(2) + \dots + D_{tot}(N)) - (V_{min} - V(0)) \end{bmatrix}_{N \times 1} \quad (3.16)$$

and

$$b_2 = \begin{bmatrix} D_{tot}(1) + (V_{min} - V(0)) \\ (D_{tot}(1) + D_{tot}(2)) + (V_{min} - V(0)) \\ \vdots \\ (D_{tot}(1) + D_{tot}(2) + \dots + D_{tot}(N)) + (V_{min} - V(0)) \end{bmatrix}_{N \times 1}. \quad (3.17)$$

Again, the linear inequalities (3.7a) and (3.7b) can be represented by

$$A_3 X \leq b_3, \quad (3.18)$$

where

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 & -1 & 0 & 0 & \dots & 0 \\ -1 & 1 & 0 & \dots & 0 & 0 & 0 & -1 & 0 & \dots & 0 \\ 0 & -1 & 1 & \dots & 0 & 0 & 0 & 0 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & -1 & 1 & 0 & 0 & 0 & \dots & -1 \end{bmatrix}_{N \times 2N}, \quad (3.19)$$

$$b_3 = [0 \ 0 \ 0 \ \dots \ 0]_{1 \times N}^T.$$

Then, the canonical linear inequality constraint in (3.11) becomes

$$\begin{bmatrix} A_1 \\ -A_1 \\ A_3 \end{bmatrix}_{3N \times 2N} X \leq \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}_{3N \times 1}. \quad (3.20)$$

In similar veins, linear equality constraint (3.5) can be written as follows;

$$A_{eq} X = b_{eq}, \quad (3.21)$$

where

$$A_{eq} = \begin{bmatrix} 0 & \dots & 0 & | & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & | & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & | & 0 & \dots & 0 \\ \hline t_s Q & \dots & t_s Q & | & 0 & \dots & 0 \end{bmatrix}_{N \times 2N} \quad (3.22)$$

and

$$b_{eq} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \hline (V_f - V(0) + (D_{tot}(1) + \dots + D_{tot}(N))) \end{bmatrix}_{N \times 1} \quad (3.23)$$

Finally, the canonical lower and upper bounds in (3.11) are written as

$$L_B = [0, \dots, 0, 0, \dots, 0]_{2N \times 1}^T \quad U_B = [1, \dots, 1, 1, \dots, 1]_{2N \times 1}^T \quad (3.24)$$

This binary linear optimization problem is solved using the SCIP solver, available in the Matlab interface OPTI toolbox. SCIP (Solving Constraint Integer Programs) is currently one of the fastest non-commercial solvers for mixed integer (linear and non-linear) programming [154]. Its framework developed by Zuse Institute Berlin for mixed integer linear and nonlinear programming allowing for total control of the solution process [155].

The open-loop controller is able to provide the optimal pump control schedule in vector  $X$  throughout the whole control horizon using the predicted water demand. The results are discussed later in the chapter.

### 3.2.4 Closed-loop MPC system

Closed-loop MPC method uses the explicit model of the plant to optimize the future plant behaviour [156]. The current control action is obtained by solving on-line a finite open-loop optimal control problem using the current state of the plant as the initial state. The optimization produces an optimal control sequence, but only the first control step is applied to the plant [112]. This ability to compute the control law on-line is a huge benefit of MPC over the conventional control in instances where off line computation of the control law is difficult [20].

Taking measurements and feeding them back to the controller provides stability and robustness against disturbances and inaccurate system modelling [157]. In this system, volume of water in the tank is

measured using sensors, and then fed back to the controller. The tank considered is assumed to be cylindrical, such that level sensors, which are the most economical and easy to implement, can be used to provide the height of the water in the tank. The tank dynamic equation (3.2) can then be modified to give the height of the water,  $h$ , in the tank as;

$$h(j+1) = h(j) + \frac{1}{A_{tank}} \left( Q_{ts}u(j) - D_{tot}(j) \right). \quad (3.25)$$

Following the idea of MPC, the objective function (3.8), can now be modified to,

$$J_{mpc} = \sum_{j=k}^{k+N_c-1} \left( (1-\omega)p_{mts}p_e(j)u(j|k) + \omega s(j|k) \right), \quad (3.26)$$

where  $N_c$  is the control horizon,  $u(j|k)$  and  $s(j|k)$  are the optimized control action and auxiliary values, respectively, at  $j^{th}$  interval based on the most recent measurement at time  $k$ . Although common MPC optimization problems include both the predicting ( $N_p$ ) and control ( $N_c$ ) horizons, such that  $N_c \leq N_p$ , this MPC problem does not include  $N_p$  because the above objective function does not have the state variable,  $h(j)$ , included.

In the optimization algorithm, the control vector,  $X^{mpc}$  will still contain the pump switch  $u(j)$  and the auxiliary variable  $s(j)$  such that

$$X^{mpc} = \left[ u(k|k), u(k+1|k), \dots, u(k+N_c-1), s(k|k), s(k+1|k), \dots, s(k+N_c-1) \right]_{2N_c \times 1}^T. \quad (3.27)$$

Therefore, the vector  $f^T$  in the objective function's canonical form (3.10) can be derived from objective function (3.26) as,

$$f^T = \left[ (1-\omega)p_{mts}p_e(k), (1-\omega)p_{mts}p_e(k+1), \dots, (1-\omega)p_{mts}p_e(k+N_c-1), \right. \\ \left. \omega, \dots, \omega \right]_{1 \times 2N_c}. \quad (3.28)$$

This objective function is minimized subject to the same constraints modelled in section 3.2.2 albeit with the following adjustments.

1. *Tank's capacity:* The volume of the water in the tank within the control horizon,  $N_c$ , is constrained by the tank's capacity between the maximum and minimum allowable water volume. Since the tank is assumed to be cylindrical, and the level sensor will give the height of water in the tank, then,

$$h_{min} \leq h(k) + \sum_{i=k}^j \frac{1}{A_{tank}} \left( Q_{ts}u(i|k) - D_{tot}(i) \right) \leq h_{max}, \quad k \leq j \leq k+N_c-1, \quad (3.29)$$

where  $h(k)$  is the measured height of the water in the tank at time  $k$  having a cross-sectional area  $A_{tank}$  while,  $h_{min}$  and  $h_{max}$  are the minimum and maximum allowed water heights in the tank

respectively such that  $h_{min} = \frac{V_{min}}{A_{tank}}$  and  $h_{max} = \frac{V_{max}}{A_{tank}}$ . In the algorithm, the linear inequality can be transformed into,

$$\begin{aligned} A_1^{mpc} X^{mpc} &\leq b_1^{mpc}, \\ -A_1^{mpc} X^{mpc} &\leq b_2^{mpc}, \end{aligned} \quad (3.30)$$

where,

$$A_1^{mpc} = \begin{bmatrix} -t_s Q & 0 & \dots & 0 & 0 & \dots & 0 \\ -t_s Q & -t_s Q & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ -t_s Q & -t_s Q & \dots & -t_s Q & 0 & \dots & 0 \end{bmatrix}_{N_c \times 2N_c}, \quad (3.31)$$

$$b_1^{mpc} = \begin{bmatrix} -D_{tot}(k) - A_{tank} \left( h_{min} - h(k) \right) \\ - \left( D_{tot}(k) + D_{tot}(k+1) \right) - A_{tank} \left( h_{min} - h(k) \right) \\ \vdots \\ - \left( D_{tot}(k) + D_{tot}(k+1) + \dots + D_{tot}(k+N_c-1) \right) - A_{tank} \left( h_{min} - h(k) \right) \end{bmatrix}_{N_c \times 1} \quad (3.32)$$

and

$$b_2^{mpc} = \begin{bmatrix} D_{tot}(k) + A_{tank} \left( h_{max} - h(k) \right) \\ \left( D_{tot}(k) + D_{tot}(k+1) \right) + A_{tank} \left( h_{max} - h(k) \right) \\ \vdots \\ \left( D_{tot}(k) + D_{tot}(k+1) + \dots + D_{tot}(k+N_c-1) \right) + A_{tank} \left( h_{max} - h(k) \right) \end{bmatrix}_{N_c \times 1} \quad (3.33)$$

2. *Pump switching constraints:* The constraint minimizing the switching frequency of the pump now becomes,

$$u(1|k) - s(1|k) \leq 0, \quad (3.34a)$$

$$u(j|k) - u(j-1|k) - s(j|k) \leq 0, \quad (3.34b)$$

with  $s(j|k) \in \{0, 1\}$ . The inequality constraint (3.34a) initialises the auxiliary variable as the initial status of  $u$  based on information available at time  $k$  while the inequality (3.34b) favours the control involving less switching between adjacent sampling intervals based on information available at time  $k$ . The constraints can be represented by,

$$A_3^{mpc} X^{mpc} \leq b_3^{mpc} \quad (3.35)$$

where

$$A_3^{mpc} = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 & -1 & 0 & 0 & \dots & 0 \\ -1 & 1 & 0 & \dots & 0 & 0 & 0 & -1 & 0 & \dots & 0 \\ 0 & -1 & 1 & \dots & 0 & 0 & 0 & 0 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & -1 & 1 & 0 & 0 & 0 & \dots & -1 \end{bmatrix}_{N_c \times 2N_c}, \quad (3.36)$$

$$b_3^{mpc} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \end{bmatrix}_{N_c \times 1}^T.$$

Therefore, the linear inequality constraint (3.11) now becomes

$$\begin{bmatrix} A_1^{mpc} \\ -A_1^{mpc} \\ A_3^{mpc} \end{bmatrix}_{3N_c \times 2N_c} X^{mpc} \leq \begin{bmatrix} b_1^{mpc} \\ b_2^{mpc} \\ b_3^{mpc} \end{bmatrix}_{3N \times 1}. \quad (3.37)$$

3. *Terminal constraint:* At the end of the 24-h horizon, the height of water in the tank should be  $h_f$  such that,

$$h_f = h(k) + \sum_{j=k}^{N-k+1} \frac{1}{A_{tank}} \left( Q_{ts} u(j|k) - D_{tot}(j) \right). \quad (3.38)$$

In the algorithm, this constraint is written as the canonical linear equality constraint in (3.11) as,

$$A_{eq}^{mpc} X^{mpc} = b_{eq}^{mpc}, \quad (3.39)$$

where

$$A_{eq}^{mpc} = \begin{bmatrix} 0 & \dots & 0 & | & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & | & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & | & 0 & \dots & 0 \\ \hline t_s Q & \dots & t_s Q & | & 0 & \dots & 0 \end{bmatrix}_{N_c \times 2N_c} \quad (3.40)$$

and

$$b_{eq}^{mpc} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \hline (h_f - h(k) + (D_{tot}(k) + \dots + D_{tot}(N-1))) \end{bmatrix}_{N_c \times 1}. \quad (3.41)$$

In this case,  $N_c$  is evolving all the time as  $N_c = N - k + 1$ .

4. *Upper and lower bounds*: The on/off control of the pump is still modelled as switch control problem such that,

$$u(j|k) \in \{0, 1\}. \quad (3.42)$$

In the canonical lower and upper bounds in (3.11), they are written as

$$\begin{aligned} L_B^{mpc} &= \left[ 0, \dots, 0, 0, \dots, 0 \right]_{2N_c \times 1}^T, \\ U_B^{mpc} &= \left[ 1, \dots, 1, 1, \dots, 1 \right]_{2N_c \times 1}^T. \end{aligned} \quad (3.43)$$

The open-loop optimal control problem (3.26) is solved, using SCIP solver in the Matlab interface OPTI toolbox, during each iteration over the finite 24-h horizon. Although, in MPC, the optimal vector,  $X$  contains the controls, using the principle of the receding horizon control, only the first element in the control vector  $X^{mpc}$  is implemented after each iteration, ignoring the rest of the elements [158]. The state of the plant (water level in the tank,  $h(j)$ ) is also measured. At the next iteration,  $k + 1$ , the objective function and the constraints are updated while  $h(k)$  is taken as the initial state and the process of optimization is carried out in real time over the new control horizon ( $N_c = N - k + 1$ ) to give the receding horizon control law.

### 3.3 GENERAL DATA

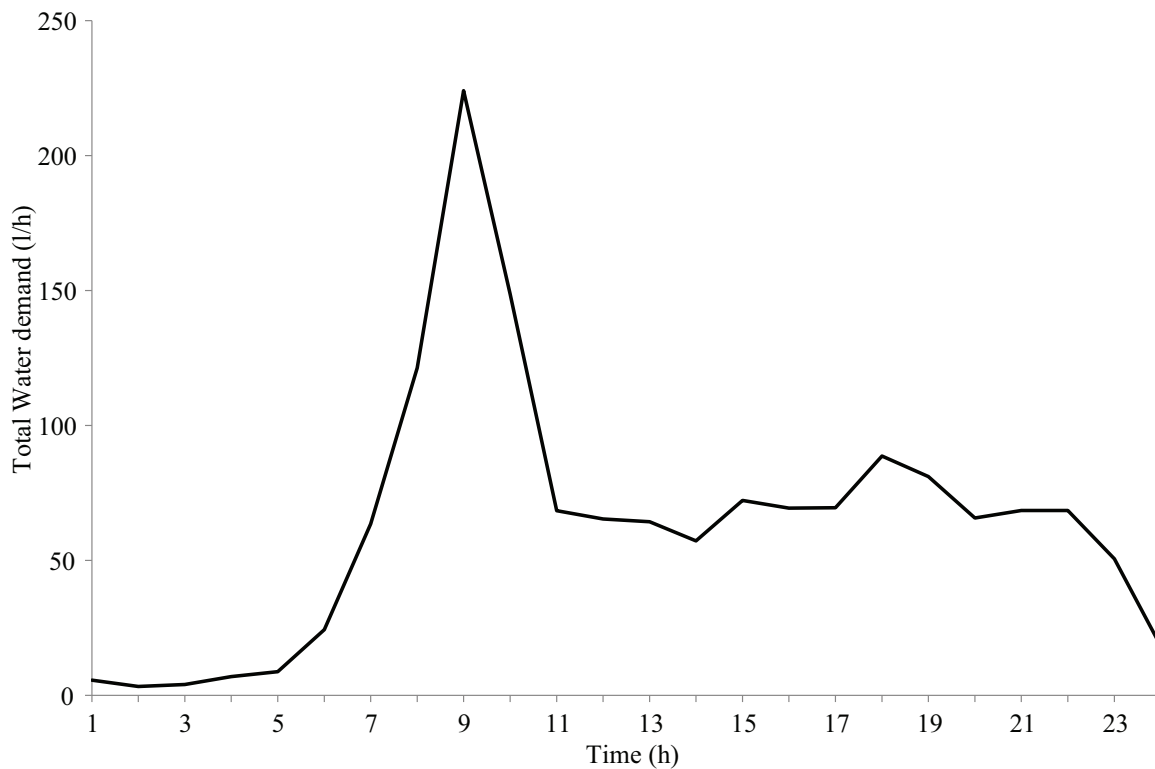
#### 3.3.1 Case study

A maisonette house in a gated community in Tshwane, South Africa was chosen for this study. The house, with five occupants, has the following end uses; two showers, a bath tub, two toilets, two hand taps, a kitchen tap, a clothes washing machine, a dish washer, a grass lawn and an electric water heater for all the hot water demand. The water from the municipal supply requires pumping and storage due to its low pressure. The pump is currently controlled by two sensors in the tank such that when the water level is low, the pump is switched on until the tank is full regardless of the cost of electricity using the TOU tariff. This is used as the baseline in this chapter. The water flows from the storage tank to the various end uses by gravity. Jojo's<sup>3</sup> 1000 l cylindrical water tank with a diameter and height of 1.1 and 1.3 m respectively is used and the lower and upper levels of the water in the tank are set as 0.12 and 1 m, respectively to avoid spilling the water from the tank as well as running it completely

<sup>3</sup>Water tank [www.jojotanks.co.za/](http://www.jojotanks.co.za/)

empty. In addition, Grundfos Leader EBS 800 pump<sup>4</sup> rated at 0.8 kW with a flow rate of 0.9 m<sup>3</sup>/h at a maximum head of about 30 m is used. This pump is specifically designed for pressurised water supply in domestic and other small scale applications.

The water demand in the house was measured every hour by placing a digital flow meter connected to a data logger, for a period of one week. From the measurements carried out, the average hourly water demand is shown in Figure 3.2. The demand has the highest peak in the morning between 7



**Figure 3.2.** Baseline average hourly water demand.

and 10 AM. There is also another peak in the evening from 5 to around 9 PM. The morning peak is attributed to people waking up and preparing to go to work or school with the highest end use demand being showering and the clothes washer. Likewise, the evening demand is attributed to people getting back home from work. The consumption during the day is attributed to those at home during the day. Although this hourly demand is similar to the one obtained by Willis et al. [159], the extensive conservation awareness and measures carried out in their study area made the daily water consumption to be less than what was obtained in our study.

<sup>4</sup>[www.davisandshirtiliff.com/categories/product/104-eps-800](http://www.davisandshirtiliff.com/categories/product/104-eps-800)

### 3.3.2 Time-of-use electricity tariff

The TOU tariff is commonly used globally [48] and it can vary by time of day, day of week and season [160]. Eskom's TOU Homeflex structure<sup>5</sup> for residential consumers given below is used for the house.

$$p_e(t) = \begin{cases} p_{off} = 0.5510 \text{ R/Kwh} & \text{if } t \in [0, 6] \cup [10, 18] \cup [20, 24], \\ p_{peak} = 1.7487 \text{ R/Kwh} & \text{if } t \in [7, 10] \cup [18, 20], \end{cases} \quad (3.44)$$

where  $p_e(t)$  is the hourly price of electricity,  $p_{off}$  is the off peak price,  $p_{peak}$  is the peak time price,  $R$  is the South African currency, Rand, and  $t$  is the time of day in hours.

## 3.4 SIMULATION RESULTS AND DISCUSSION

### 3.4.1 Control systems without disturbance

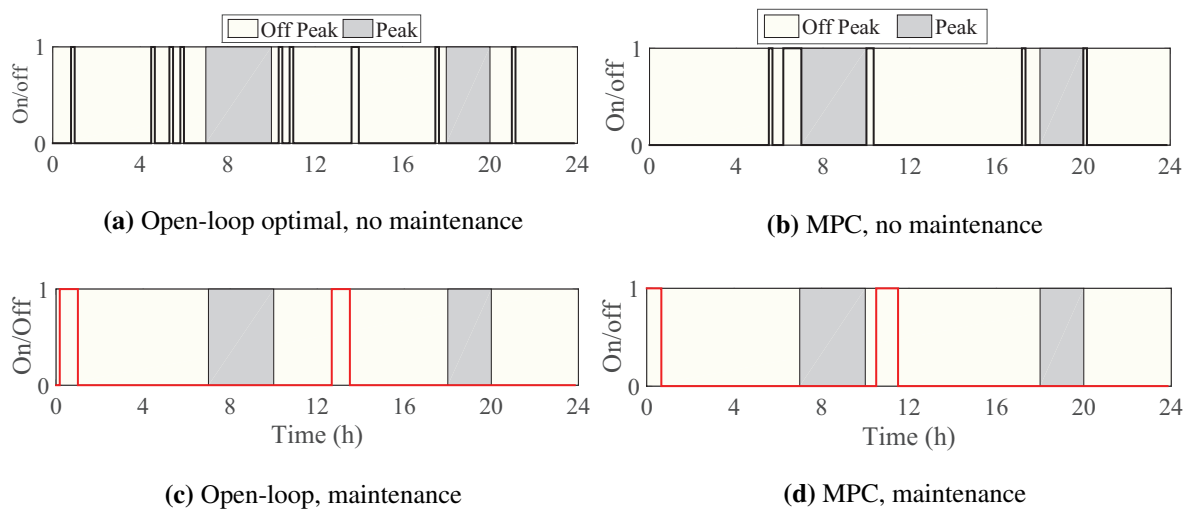
This section compares both the open and closed-loop controllers with the intention of showing that the closed-loop MPC system is just as effective as the open-loop optimal control system when there are no disturbances, system inaccuracies or plant failures. In addition, the legend representing peak time and off-peak time for the TOU tariff is the same throughout the chapter.

It can be seen from Figure 3.3 that all the pump schedules take place during the off-peak period, effectively shifting the load from the peak time. Figures 3.3(a) and 3.3(b) show the results without considering the pump's maintenance cost. The frequent switching of the pump is undesired, as it leads to high wear and tear of the pump's motor as it tries to overcome the dead weight caused by the stationary load (water), hence increasing the maintenance cost. Although not accounted for in this control system, the frequent starting of the pump could also lead to higher energy costs due to the high start-up current required in overcoming the dead weight. Therefore, the strategy employed to minimize the switching frequency and effectively minimize the maintenance cost works both in open-loop controller (Figure 3.3(c)) as well as the MPC (Figure 3.3(d)). All the four control systems, however, incur the same cost of energy proving that both the open-loop and MPC controllers are effective with no disturbances.

---

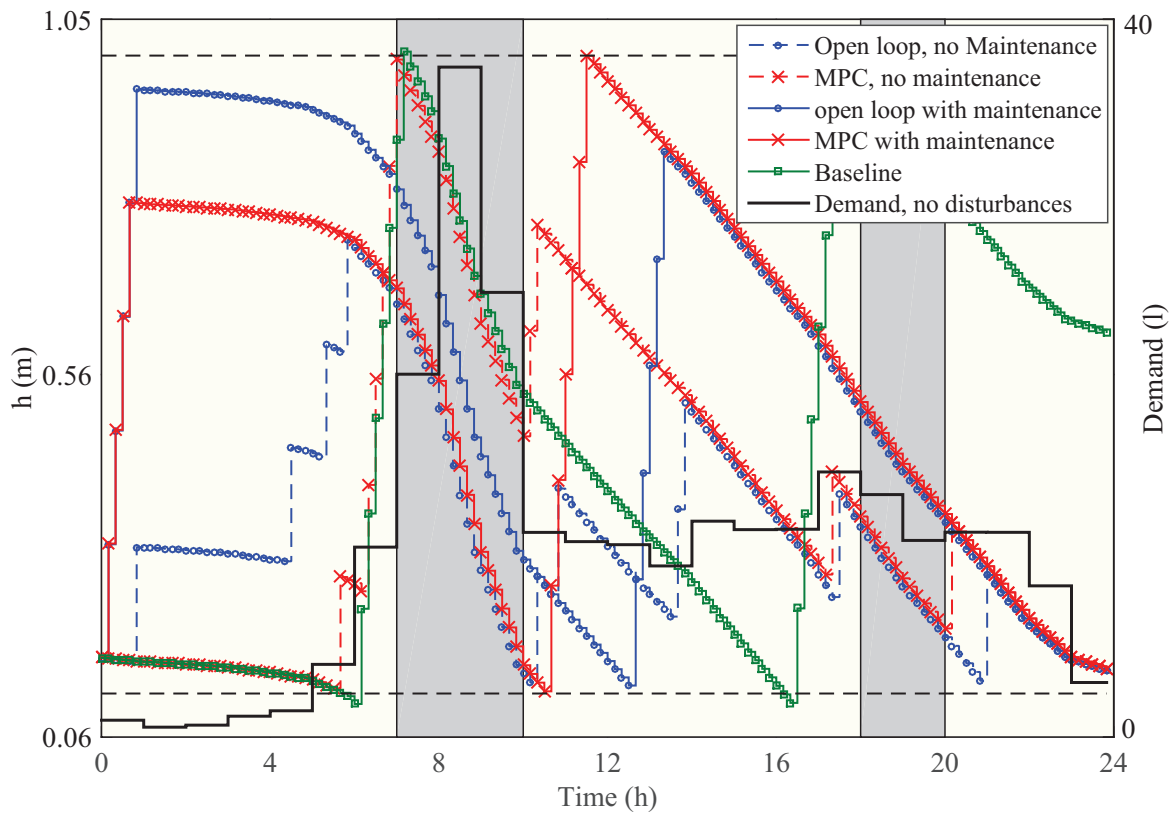
<sup>5</sup>Eskom tariffs and charges booklet 2011/2012. [www.eskom.co.za](http://www.eskom.co.za)





**Figure 3.3.** Comparison of open-loop and MPC switching.

The variation of the state variable, that is, water level in the tank ( $h(j)$ ) due to optimal operation in Figure 3.3 is shown in Figure 3.4. It can be seen that none of the controllers violated the constraints



**Figure 3.4.** Water level in the tank without disturbance.

for the tank. In the optimal open-loop system without consideration of the maintenance cost (Figure 3.3(a)), the pump switches at 00:50 the first time for a duration of 10 minutes only, causing the water level in the tank to increase in the tank. The pump switches on frequently between 04:30-06:00 hours causing another rise of the water level in the tank to a height of 0.75 m. The pump is then switched off through the peak time where the water level in the tank declines to 0.135 m due to the water demand in the house. The controller then senses that the water level is declining and switches on the pump at 10:20, again frequently with the longest duration being 20 min causing the water level in the tank to vary. Similarly, the MPC schedule with no maintenance cost (Figure 3.3(b)) frequently switches the pump on starting at 05:30. This is after the water level in the tank declined to 0.13 m due to the early demand in the house. The pump is switched on twice before the peak time where the tank is filled. This water is used up during the peak time with the level declining to 0.47 m before the pump is switched on at 10:00 hours, just after peak time, for a duration of 10 min. Thereafter, the pump is switched on twice, for a duration of 10 min each, causing the rise of water level in the tank at 17:20 and 20:10 hours. Unfortunately, these two schedules are undesirable for the operation of the pump.

When the open-loop optimal controller with maintenance (Figure 3.3(c)) is considered, the two switching regimes at 00:10 and 12:40 hours, each lasting 50 minutes lead to the increase in the water level in the tank to 0.95 m and 0.87 m respectively. This water is sufficient to meet the demand in the house without violating the constraints. Identically, the closed-loop MPC system (Figure 3.3(d)) has two switching regimes at 00:00 and 10:30 hours, 40 minutes and 1 hour long respectively. They lead to a water rise in the tank to 0.80 and 1 m respectively. Thereafter, when the pump is off, this water level declines due to the demand in the house. It is also important to note that all the schedules were terminally constrained to also ease in comparison as seen in the graph.

### 3.4.2 Control systems with disturbance

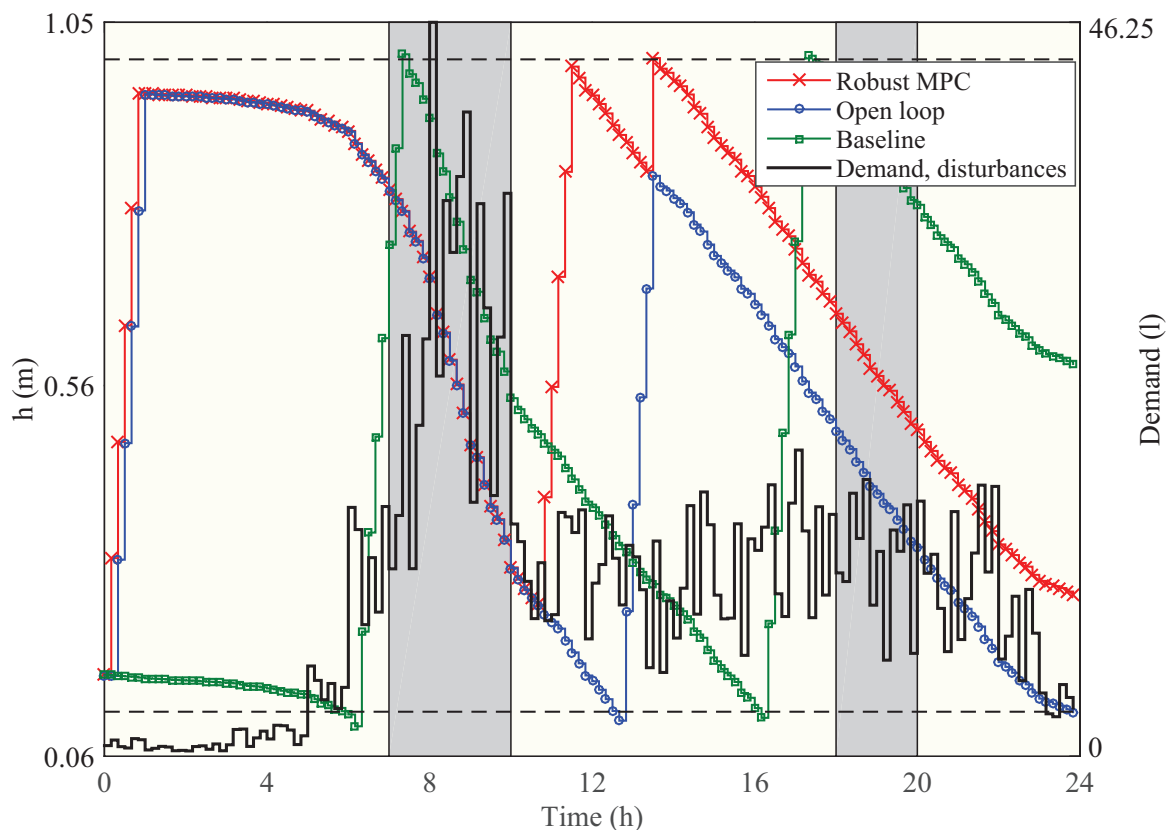
The controllers are tested for robustness using two likely disturbances to be experienced in dealing with water demand. First, a random disturbance throughout the horizon is applied and then a sudden spike in water demand is used.

### 3.4.2.1 Random disturbance

Random disturbance can be caused by errors in accurately predicting the water demand, measuring the water level in the tank or inaccuracies of the pump's flow rate. By applying the random error signal,  $\varepsilon(j)$ , affecting the demand such that the new demand is,

$$D_{tot}^d(j) = D_{tot}(j) + \varepsilon(j) \quad (3.45)$$

This demand in turn, affects the state equation (3.25). In this chapter, the error signal is randomly generated as  $\pm 50\%$  of the demand, signifying a very inaccurate system. Figure 3.5 shows the



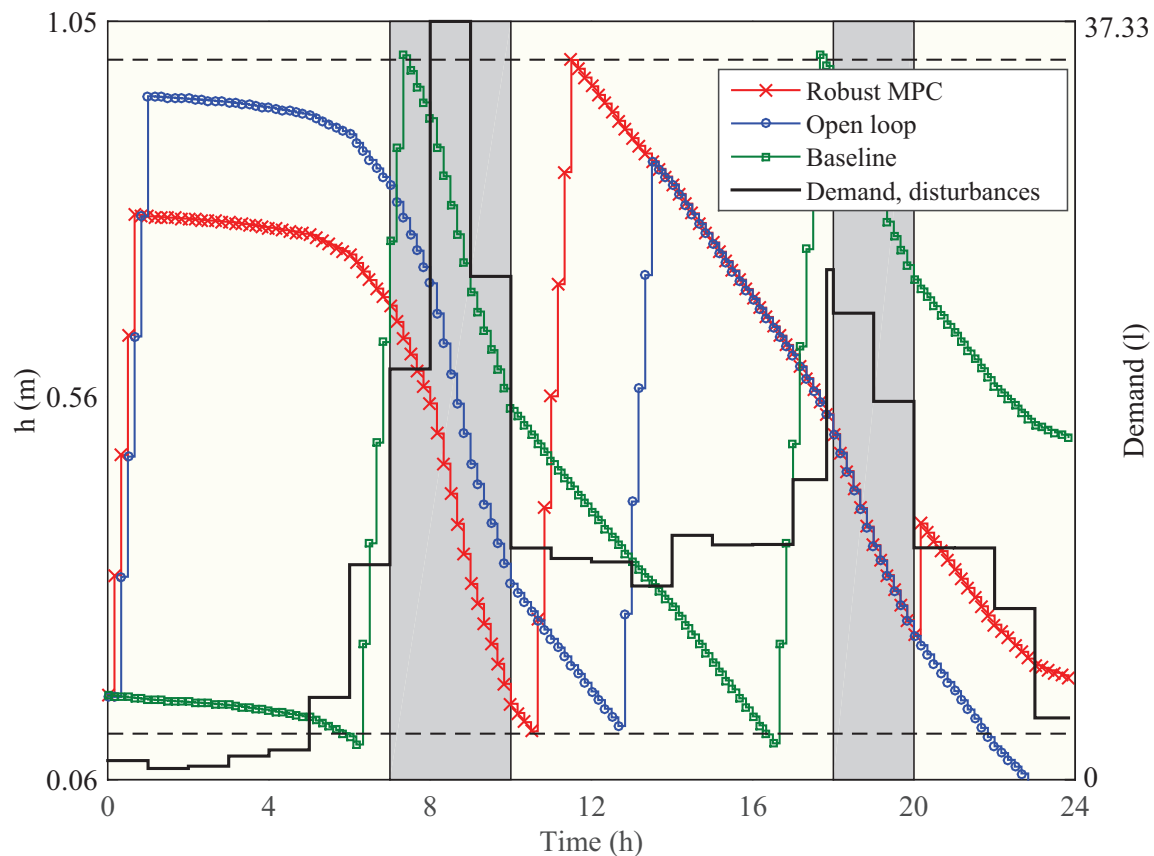
**Figure 3.5.** Water level in the tank with random disturbances.

comparison of the water level in the tank with such a disturbance. Since the disturbance signal is randomly generated, the figure shows the average variation of the water level while using the two controllers. The open-loop controller does not violate the boundary constraints in most cases. Even in the cases where the open-loop controller violated the constraints, the violation was not very severe. Further, the MPC controller switches on the pump at 10:00 hours for 10 minutes in order to deal with the disturbances. This extra switching also causes more water to be left in the tank. In order to

ensure the MPC system remained feasible, the terminal constraint is implemented as a soft constraint taking note of the increase of the height of water in the tank when the pump is switched on during one sampling interval. The open-loop and the MPC controllers incur pumping cost of 0.84 and 0.92 Rands respectively. This means that, in dealing with random disturbances arising from system inaccuracies, the open-loop optimal controller is better than the MPC, as it is cheaper to implement and also incurs less running costs.

### 3.4.2.2 Sudden spike in water demand

A sudden increase in water demand could be caused either by more people in the house during some period, or some unforeseen demand for water. In this chapter, we assumed that the water demand suddenly increased by 70% between 18:00-20:00 hours. Figure 3.6 shows the comparison of the robust



**Figure 3.6.** Water level in the tank with sudden increase in demand between 18:00-20:00 hours.

MPC and the open-loop optimal control in the presence of disturbances. The open-loop controller does not react to the unforeseen increase in demand. This leads to the emptying of the tank towards the

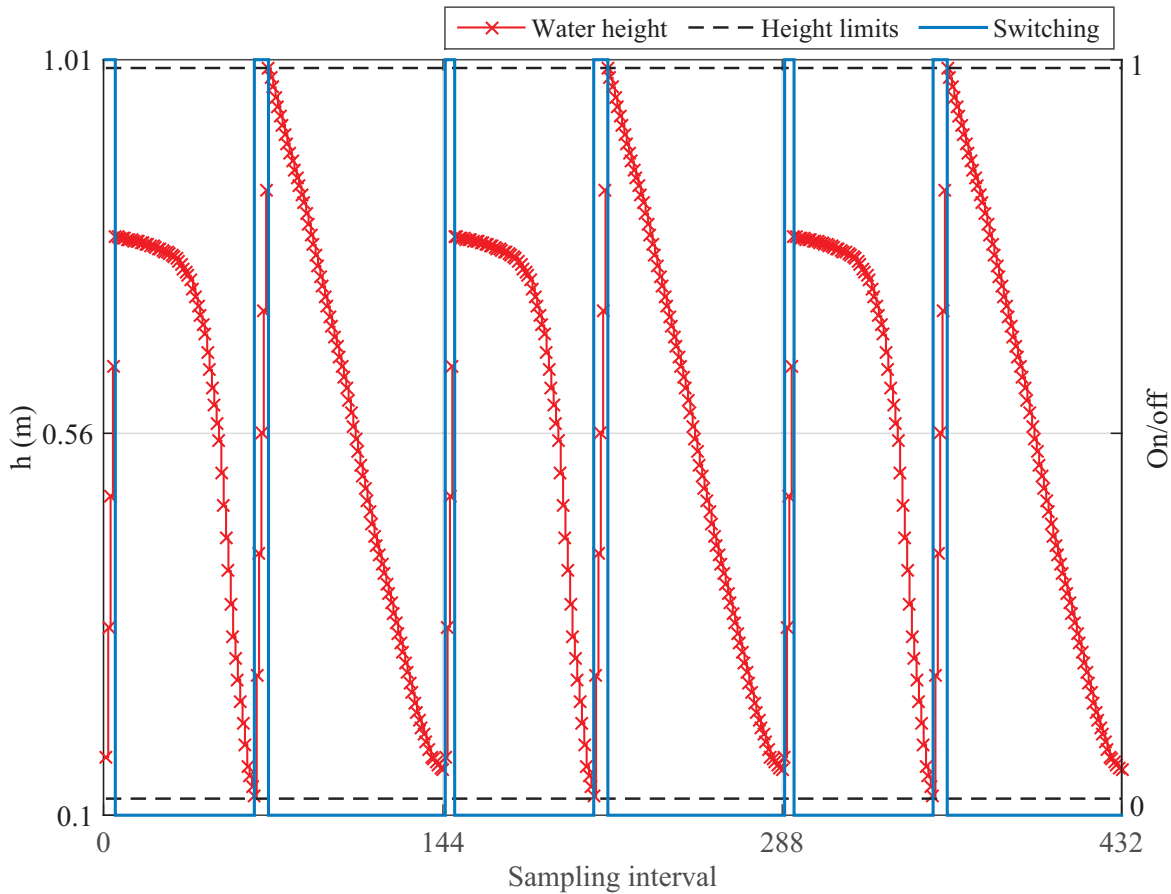
end of the day effectively meaning the end users would not have water to use. On the contrary, the MPC controller reacts to the increased demand by switching on the pump at 20:10 hours for just 10 minutes. The extra water pumped in this duration means that the tank has enough water to meet the rest of the demand. The MPC controller, however, incurs a pumping cost of 0.92 Rands against 0.84 Rands incurred by the open-loop optimal controller. Although the cost of using MPC is slightly higher, it is more robust in dealing with sudden disturbances and it is more reliable in ensuring that the end users always have water.

### 3.4.2.3 Turnpike phenomenon

During implementation, the open-loop optimal controller does not guarantee proper operation in subsequent days as the water level in the tank at the end of the control horizon, which will be the initial state for the following day, is not always the same as  $h(0)$  as seen in Figure 3.4. This problem is called the turnpike phenomenon [161]. Turnpike property has been described by Faulwasser et al. [162] as the phenomenon where the optimal solution in many finite-horizon optimal control problems for different initial conditions approach the purview of the best steady state but might leave it towards the end of the control horizon. This phenomenon has been observed in optimization problems with and without terminal constraints. The closed-loop MPC automatically corrects this problem over several days as it uses the previous state in the tank instead of the initial state. Since the sampling in this chapter is done every 10 *min*, one day has 144 samples. Therefore, from Figure 3.7, the water level in the tank at the end of each day is the same, at a height of 0.15 *m*. The advantages of the MPC to handle constraints and disturbances while possessing closed-loop stability and robustness makes it suitable for use in practical problems [163].

### 3.4.3 Discussion

The open-loop and MPC controllers considering the maintenance cost of the pump are suitable for optimally controlling the pump to meet the household water demand. Both can save up to 48.5% of the pumping energy cost with respect to the baseline assuming there are no disturbances. However, this is never the case in reality. Open-loop optimal controller is suitable in cases where the only disturbances are due to the measurements errors, such that demand pattern doesn't change significantly. This is



**Figure 3.7.** MPC switching and water level variation for 3 days.

because while it still efficiently controls the pump, it incurs 8.7% less cost than the MPC controller and it is cheaper to implement.

However, in reality, disturbances due to measurements uncertainties are not the only disturbances. There are instances when the house will either have more or less occupants, or need more water to perform some chores that are not performed daily, hence it is not possible to predict. The open-loop controller is unable to respond to such kind of a disturbance, leading to emptying of the tank even before the horizon is over. This controller also runs at the danger of spilling the water if the demand unexpectedly goes down when it was already pumping the water. To mitigate against such disturbances, the MPC controller has proved its robustness by adapting accordingly while ensuring that none of the constraints are violated. This robustness is at the expense of 9.5% more cost of energy than the open-loop controller, though it can still save 43.6% with respect to the baseline. Therefore, the robustness of the MPC controller makes it more suitable in pumping the water to meet household water

demand. In South Africa, the TOU period for different seasons was reviewed in 2015 where winter peak periods start one hour earlier<sup>6</sup>. The MPC controller, when tested in both summer and winter periods, yields the same results indeed showing its robustness. If a different tariff is used, the MPC would automatically adopt to it to ensure the pump is efficiently operated.

Table 3.1 shows the comparison of the energy costs incurred by the two control systems as compared to the baseline. The cost incurred in the baseline is constant throughout because for the day chosen, the

**Table 3.1.** Comparison of the cost of pumping energy incurred

	Pumping energy cost (Rands/day)		
	Baseline	Open-loop	Robust MPC
No disturbance	1.63	0.84	0.84
Random disturbance	1.63	0.84	0.92
Spike disturbance	1.63	0.84	0.92

baseline does not need to pump extra water whether there are disturbances or not, since enough water is left in the tank at the end of the day. Similarly, since the open-loop control system is unable to predict and adapt to the disturbances, it also incurs the same energy cost in the three cases. However, the robust MPC system's cost of pumping energy increases by 9.5% due to the disturbances that necessitate the controller to switch on the pump for an extra sampling interval in response to the disturbances. It is also noted that the extra switching pumps enough water to counter both types of disturbances, hence the same cost in the presence of disturbances.

Both the open-loop optimal and the robust MPC controllers can potentially lead to a cost saving of 48.5% and 43.6% respectively. The open-loop controller therefore leads to higher cost saving, and is suitable in situations where only random disturbances are present. However, in practical situations, the demand for water in the house is bound to change, either rise or drop, without prediction. The ability of the robust MPC controller to adapt to such a disturbance, unlike the open-loop controller, makes it superior although at a slightly higher cost.

The two control control systems are applicable in situations with unreliable municipal water supply that forces end users to pump and store water such as cities in developing nations. This would enhance

<sup>6</sup>[www.eskom.co.za](http://www.eskom.co.za)

energy efficiency through load shifting translating to lower electricity cost to the end users while ensuring reliability and convenience. The control systems are also useful for end users using boreholes as their source of water. In the case where water rationing exists, the controllers would be fed with the time water is available so as to optimally control the pump. In scenarios where end users have more than one source of water, for example, stored rain water or bore hole and municipal sources. In such cases, the controllers can be modified to accommodate the extra sources of water in ensuring energy efficiency and reliability by means of properly modelling the new system with extra components.

### 3.5 SYNOPSIS

The increasing population and urbanization in developing nations is increasing pressure on existing energy and water infrastructure causing insufficient and unreliable supply to end users. Some of these end users are forced to install water tanks in their houses to pump and store the water for use. The pumping increases the load on the power utilities in the same economies where the energy security is very low. In South Africa, for instance, the energy demand management has steadily been developing, while leaving out the energy-water demand management among domestic end users. This chapter, therefore, presents the introductory research for energy-water nexus demand management in urban houses through optimal operation.

Open-loop optimal and closed-loop MPC controllers are developed to meet the water demand in the house while minimizing the energy cost for pumping the water in the house. The controllers are developed using the TOU tariff in South Africa. Both controllers can potentially lower the energy cost by upto 48.5% when the demand is correctly predicted with no disturbances. This is never the case in reality and therefore the robust MPC controller proves suitable in dealing with all the disturbances, while still saving upto 43.6% of the energy cost. The open-loop controller can still be used in cases where the only disturbances present are due to random errors arising from measurements and the demand profile doesn't change that much. If widely adopted, the control systems would lead to lower energy costs to the consumers through lower electricity bills. Further, shifting the peak load would improve the stability of the grid by shaving the peak. This in turn would mean that some black outs experienced during peak time would cease as the demand from the grid would not be more than its capacity.



## CHAPTER 3 OPTIMAL ENERGY-WATER MANAGEMENT ON PUMP-STORAGE SCHEME

---

Further research will involve optimal controls for water conservation strategies within households. The research can also be furthered to commercial buildings in developing nations where the water needs to be pumped and stored. Energy efficiency can further be enhanced through the incorporation of renewable energy to power the pump. This will lead to more economic benefits to the end users.

# **CHAPTER 4 ENERGY-WATER OPTIMIZATION OF ROOFTOP WATER HARVESTING FOR LAWN IRRIGATION**

## **4.1 INTRODUCTION**

Water and energy are vital resources for human for survival facing immense challenges [164, 165]. Existing potable water supplies are fast reaching their limit whereas water demand is rapidly increasing [166]. Worse still, rapid urbanization is increasing the strain to the water and electricity utilities especially in developing nations [167, 168], like South Africa, which is a water-scarce country [169]. In reality, the demand for both potable water and energy in South Africa far outweighs the supply [127, 170].

The water-energy nexus is receiving increased attention [171]. However, domestic consumers are hardly aware of the direct and indirect benefits of the water-energy nexus savings at home [130], such as; energy savings for purifying water, utility pumping and corresponding CO<sub>2</sub> reduction [172], energy savings from the waste water purification and lower cost for potable water and waste water management [173]. Whilst most research in South Africa has concentrated on energy efficiency [24, 67, 174] and electrical demand side management [58, 59, 81], little attention has been given to water demand management and its effect on energy consumption [175, 176].

According to the Department of Water Affairs and Forestry (DWAf), although there is no consolidated database of information for water use from water utilities in South Africa, the national urban household water use is roughly estimated at 50% of the total water demand [87]. The outdoor water demand, mainly garden irrigation for those households with urban gardens [136], is estimated to contribute

40-60% of the total household water demand [32]. Therefore, by increasing the efficiency of gardening water use, the consumption can be reduced by 6% to 30% of the total gardening water use<sup>1</sup>. The potential means of reducing this amount are: using water-wise plants, mulching, efficient irrigation systems, irrigation scheduling, rain water harvesting and recycling waste water [87].

Research on irrigation scheduling in urban lawns has mainly concentrated on water conservation while rooftop water harvesting (RWH) of rain water has concentrated on making the water safe and reliable to the rural areas of South Africa [177, 178]. However, another benefit of RWH is improving water quality through runoff reduction [179]. Evapotranspiration based controllers are superior to time-based controllers in irrigation scheduling [180]. However, in case of precipitation, they are programmed by the manufacturers to pause for a certain period of days before resuming irrigation irrespective of whether the precipitation was sufficient or not [181]. In addition, the scheduling considered so far makes use of water solely from the utility [182]. Bocanegra-Martínez et al [183] designed an optimal rainwater collecting system in residential areas and showed its viability to use to meet certain residential water demands. However, the system was not optimized to specific end uses.

This chapter reports the first attempt to design a practical and economically attractive optimal irrigation scheduling control system using the harvested rain water and the TOU electricity price tariff. The harvested water from the rooftop is stored in a reservoir which only gets the potable utility water whenever the stored water has been depleted through the irrigation scheduling. There has been several studies seeking to address the reliability of RWH systems meant for supplying water to various demands like lawn irrigation and toilet flushing using solely the harvested water [184, 185]. The municipal water sources are used as a back up to improve the reliability of the system and ensure that the tank never runs dry. Further, optimal control strategy has the ability to predict the dynamics of water in the tank effectively minimizing the chance of spillage taking place. This chapter presents an optimal scheduling breakthrough that can reduce both water and energy consumption leading towards achieving more sustainable buildings. Furthermore, the optimal control system with RWH is useful in developing the nations' cities where utility potable water is unreliable due to the high demand that surpasses the existing supply infrastructure. This optimal system, if widely adopted, would reduce the demand for potable water and energy from the utilities, lower waste water drainage and purification cost and at the same time lowering the bills associated with both resources.

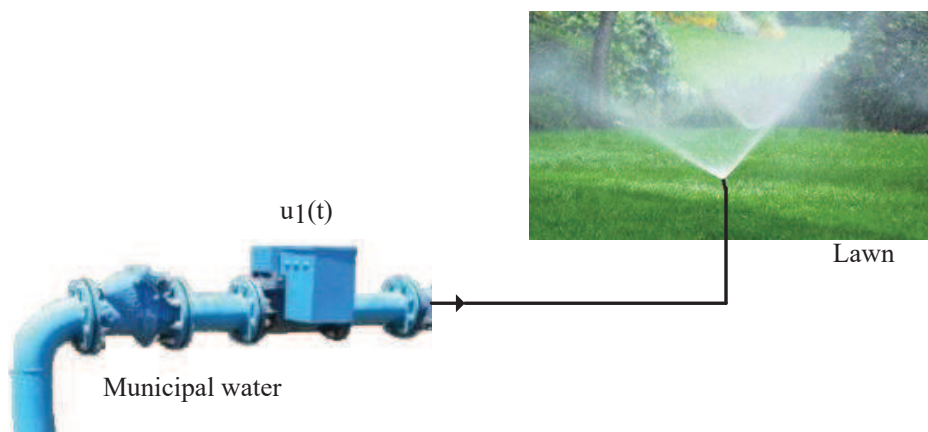
---

<sup>1</sup>Water conservation and water demand management [www.dwa.gov.za](http://www.dwa.gov.za)

## 4.2 LAYOUT AND FORMULATION

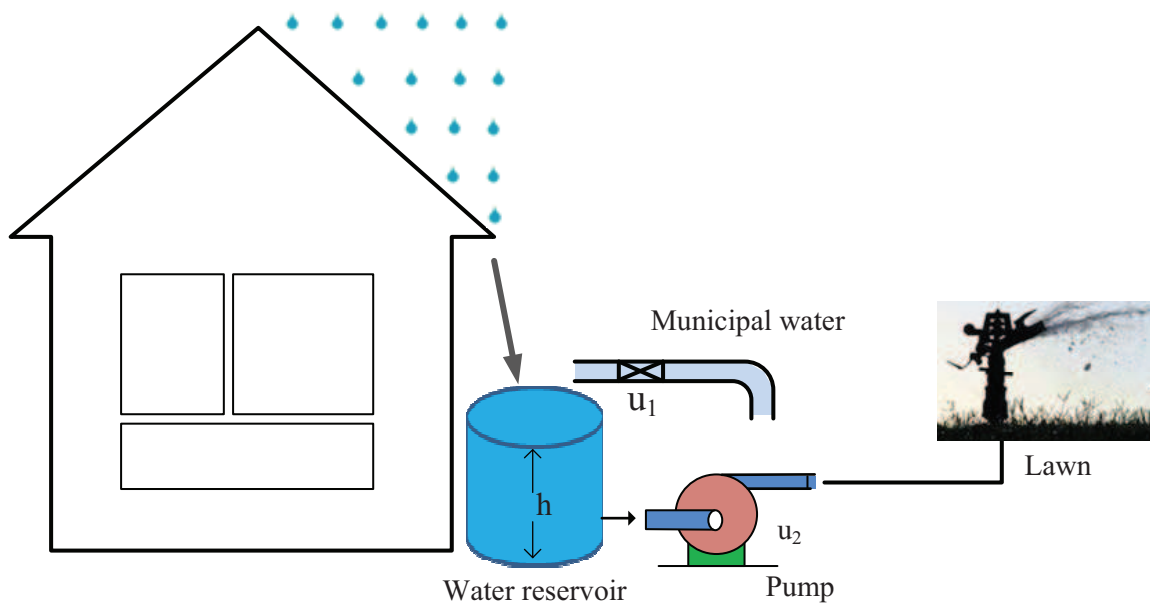
### 4.2.1 Schematic layout

The first system constitutes the sprinkler directly connected to the utility water as schematically shown in Figure 4.1. In this case, the irrigation is controlled using the utility switch  $u_1$ . The schematic layout



**Figure 4.1.** Schematic of directly connected irrigation

of the system for RWH system is shown in Figure 4.2. When the rain falls on the rooftop of a house,



**Figure 4.2.** Schematic of RWH for lawn irrigation

it is directed and stored in a tank reservoir placed on the ground. However, since it only rains for a certain period, the tank is also fed by utility water to supplement the harvested water if the stored water nears depletion and there is demand from the lawn vegetation.

The main operational energy consumption for RWH systems come from ultraviolet (UV) disinfection and pumping water from the tank to the end use [186]. Only the pumping energy is considered in this chapter as the intended water use doesn't require treatment but has low flow rate and head. The energy consumed by this pump is a function of the water consumed by the end uses it supplies [129]. Here, both the tank filling with municipal water and irrigation are controlled through switch  $u_1$  and pump switch  $u_2$ .

## 4.2.2 Optimal scheduling without rooftop water harvesting

The control system involves scheduling for lawn irrigation making use of the utility water directly assuming that the water is always reliable and has the correct flow rate and feed to operate the sprinklers.

### 4.2.2.1 Objective function

The objective in this system is to minimize the cost of the water used to irrigate the lawn in order to maintain the water level in the soil within the required limits over the given control horizon [119]. In this chapter, we consider an evaluation period of one day, or a full operation cycle of 24 hours, from 0 to hour 24 with a sampling period,  $t_s$ , of 15 minutes. This leads to a total number of samples  $N = \frac{24}{t_s} = 96$ . Hence, the objective function,  $J$ , is,

$$J = \sum_{j=1}^N p_w Q_1 u_1(j), \quad (4.1)$$

where  $Q_1$  ( $m^3$ ) is the volume of utility water flowing in a sampling interval,  $p_w$  ( $R/m^3$ ) is the cost of water charged by the water utility and  $u_1$  is the state of the solenoid valve used to switch on or off the flow of the water to irrigation. It is a binary control variable that assumes the values of 0 or 1 representing off or on states respectively. The authors acknowledge that  $Q_1$  maybe practically variable. However, due to its complexity, we have, in this work, assumed that it is constant.

### 4.2.2.2 Constraints

These are the limits that affect the operation of the irrigation system.

**SOIL WATER BALANCE** The FAO-56 Penman-Monteith equation is the standard technique for obtaining reference evapotranspiration,  $ET_0$ , whose data is obtained from weather stations<sup>2</sup>. Crop evapotranspiration,  $ET_c$ , obtained from  $ET_0$ , results from the various water inefficiencies in various crops [187]. It is obtained as

$$ET_c = K_c ET_0, \quad (4.2)$$

where  $K_c$  is the crop coefficient. Irrigation scheduling is meant to ensure that the soil does not dry beyond a certain threshold [182]. The amount of water held in the root zone available to the plant,  $S_{AW}$  in (m), is

$$S_{AW} = (FC - PWP) \times R_z, \quad (4.3)$$

where  $FC$  (volume %) is the field capacity,  $PWP$  (volume %) is the permanent wilting point and  $R_z$  (m) is the plant root zone. The amount of water allowed to leave the root zone without causing plant stress is the readily available water,  $RAW$  (m), [188]. Irrigation should be applied when the water level drops by a percentage known as the maximum allowable depletion,  $MAD$  (%), which is the amount of water relatively easily extracted by plant without causing plant stress [181]. The two quantities are related as follows:

$$RAW = S_{AW} \times MAD. \quad (4.4)$$

This means that the amount of water that can be extracted from the soil by the plant,  $S$  (m) is such that

$$(FC - RAW) \leq S \leq FC. \quad (4.5)$$

The plant water extraction then leads to a water balance such that

$$S = I + P - ET_c - R_o - D_r. \quad (4.6)$$

In this equation, all units are (m). Irrigation ( $I$ ) aims to match the crop evapotranspiration ( $ET_c$ ) losses and precipitation ( $P$ ) added by adding sufficient water in order to maintain the water content within the acceptable range. In this chapter, the precipitation mainly considered is rainfall. Therefore, the run-off,  $R_o$ , and drainage,  $D_r$ , are assumed to be negligible [182] making equation (4.6) to,

$$S = I + P - ET_c. \quad (4.7)$$

<sup>2</sup>[www.fao.org/docrep/x0490e/x0490e00.htm](http://www.fao.org/docrep/x0490e/x0490e00.htm)

The dynamics of the amount of water in the soil can be expressed in discrete-time domain by a first order difference equation as follows:

$$S(j) = S(j-1) + I(j) + P(j) - ET_c(j), \quad (1 \leq j \leq N). \quad (4.8)$$

But irrigation is provided through pumping the water. Therefore, the above equation can be re-written as

$$S(j) = S(j-1) + \frac{Q_1}{A_{lawn}} u_1(j) + P(j) - ET_c(j), \quad (1 \leq j \leq N), \quad (4.9)$$

where  $A_{lawn}$  is the top area of the lawn being irrigated ( $m^2$ ). The amount of water in one sampling interval,  $Q_1$ , is assumed as constant throughout the control horizon. Therefore, only the status of the solenoid valve,  $u_1$ , is the control variable. Therefore, by recurrence manipulation, the amount of water in the soil at the  $j^{th}$  sampling interval can be expressed in terms of the initial amount of water in the soil  $S(0)$  as follows,

$$S(j) = S(0) + \sum_{i=1}^j \left( \frac{Q_1}{A_{lawn}} u_1(i) + P(i) - ET_c(i) \right), \quad (1 \leq j \leq N). \quad (4.10)$$

The water content must not exceed the  $FC$  and not allow more depletion than the  $MAD$ . By letting  $d_l(m) = FC - RAW$  and  $d_u(m) = RAW$ , inequality (4.5) can be expressed as,

$$d_l \leq S(0) + \sum_{i=1}^j \left( \frac{Q_1}{A_{lawn}} u_1(i) + P(i) - ET_c(i) \right) \leq d_u, \quad (1 \leq j \leq N), \quad (4.11)$$

where  $d_l(m)$  and  $d_u(m)$  are the lower and upper allowable depth of water in the soil respectively.

**WATERING RESTRICTION** High solar illumination coupled with water scarcity in South Africa has made some water utilities to ban garden watering between 11:00-15:00 hours<sup>3</sup> since most of the water would be lost to the high evaporation. This is modeled as below.

$$u_1(i) = 0 \quad \forall i \in \left[ \frac{11}{t_s}, \frac{15}{t_s} \right] \quad (4.12)$$

**BOUNDARIES** The utility water switch is bounded between 0 and 1 representing off and on status respectively.

$$0 \leq u_1(j) \leq 1 \quad (1 \leq j \leq N). \quad (4.13)$$

#### 4.2.2.3 Algorithm

The generalized optimization formulation of a linear problem is to minimize  $f^T X$  subject to inequality constraints ( $AX \leq b$ ), equality constraints ( $A_{eq}X = b_{eq}$ ) and the upper and lower bounds of the control

<sup>3</sup>Umhlathuze Local Municipality Water Services By-laws 2010 [www.umhlathuze.gov.za](http://www.umhlathuze.gov.za)

variable ( $L_B \leq X \leq U_B$ ) [56]. Vector  $X$  contains the variables being controlled, where in this case, it contains the status of the solenoid valve controlling the irrigation events using the municipal water.  $A$  and  $A_{eq}$  are matrices while  $f$ ,  $b$ ,  $L_B$  and  $U_B$  are vectors obtained as follows.

$$X = \left[ u_1(1) \quad u_1(2) \quad u_1(3) \quad \dots \quad u_1(N) \right]_{N \times 1}^T, \quad (4.14)$$

and from objective function (4.1),

$$f^T = \left[ p_w Q_1 \quad p_w Q_1 \quad \dots \quad p_w Q_1 \right]_{1 \times N}. \quad (4.15)$$

If we denote

$$A_1 = \begin{bmatrix} -\frac{Q_1}{A_{lawn}} & 0 & \dots & 0 \\ -\frac{Q_1}{A_{lawn}} & -\frac{Q_1}{A_{lawn}} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{Q_1}{A_{lawn}} & -\frac{Q_1}{A_{lawn}} & \dots & -\frac{Q_1}{A_{lawn}} \end{bmatrix}_{N \times N}, \quad (4.16)$$

$$b_1 = \begin{bmatrix} -d_l + S(0) + P(1) - ET_c(1) \\ -d_l + S(0) + (P(1) + P(2)) - (ET_c(1) + ET_c(2)) \\ \vdots \\ -d_l + S(0) + (P(1) + \dots + P(N)) - (ET_c(1) + \dots + ET_c(N)) \end{bmatrix}_{N \times 1} \quad (4.17)$$

and

$$b_2 = \begin{bmatrix} d_u - S(0) - P(1) + ET_c(1) \\ d_u - S(0) - (P(1) + P(2)) + (ET_c(1) + ET_c(2)) \\ \vdots \\ d_u - S(0) - (P(1) + \dots + P(N)) + (ET_c(1) + \dots + ET_c(N)) \end{bmatrix}_{N \times 1} \quad (4.18)$$

then, inequality constraints (4.11) can be written in the standard form of matrix  $A$  and vector  $b$  as,

$$A = \begin{bmatrix} A_1 \\ -A_1 \end{bmatrix}_{2N \times N}, \quad b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}_{2N \times 1}. \quad (4.19)$$



Similarly, linear equality constraint (4.12) where irrigation is banned is between 11<sup>th</sup> hour and 15<sup>th</sup> hour can be represented by

$$A_{eq} = \begin{bmatrix} 0 & \dots & 0 & \frac{11}{T_s} & \dots & \frac{15}{T_s} & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \hline 0 & \dots & 0 & 1 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 0 & \dots & 1 & 0 & \dots & 0 \\ \hline 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 \end{bmatrix}_{N \times N} \quad \text{and} \quad b_{eq} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}_{N \times 1}, \quad (4.20)$$

while boundary conditions are represented by low and upper bounds:

$$L_B = \begin{bmatrix} 0 & \dots & 0 \end{bmatrix}_{N \times 1}^T, \quad U_B = \begin{bmatrix} 1 & \dots & 1 \end{bmatrix}_{N \times 1}^T. \quad (4.21)$$

This optimization problem is solved using the COIN Branch and Cut (Cbc) solver<sup>4</sup>, in the Matlab interface OPTI toolbox<sup>5</sup> preferred for its high solving speed.

### 4.2.3 Optimal scheduling with rooftop water harvesting

Unlike the previous case, this system incorporates RWH involving an energy model of the pump subjected to the TOU tariff. The controller optimally schedules the on/off status of the utility valve and the pump based on the water restriction periods and the TOU tariff in order to minimize the cost of potable water and energy used. Hence, the water cost is conserved through using less water as well as shifting the irrigation to the low solar illumination periods. Likewise, cost of energy is reduced through using less energy to pump the water as well as load shifting by the TOU tariff.

This control system assumes that the utility water is not reliable to irrigate directly hence the need to store it too, as is the case in many developing nations.

<sup>4</sup>Cbc <https://projects.coin-or.org/Cbc>

<sup>5</sup>OPTI Toolbox <http://www.i2c2.aut.ac.nz/Wiki/OPTI/>

### 4.2.3.1 Objective function

Here, the objective is to minimize the cost of water and energy consumed during the irrigation. The objective function,  $J$ , is therefore,

$$J = \alpha_1 \sum_{j=1}^N p_w Q_1 u_1(j) + \alpha_2 \sum_{j=1}^N p_m p_e(j) u_2(j), \quad (4.22)$$

where  $\alpha_1$  and  $\alpha_2$  are dimensionless weight factors,  $p_m$  is the pump's rating and  $p_e(j)$  is the price of electricity in a sampling interval. The weighting factors are chosen such that  $\sum \alpha = 1$  and all the weights are positive leading to a convex combination of the objectives. The values assigned to these weights are chosen relative to the importance of each objective function [189].

### 4.2.3.2 Constraints

**SOIL WATER BALANCE** The soil water content in constraint (4.11) now becomes

$$d_l \leq S(0) + \sum_{i=1}^j \left( \frac{Q_2}{A_{lawn}} u_2(i) + P(i) - ET_c(i) \right) \leq d_u, \quad (1 \leq j \leq N). \quad (4.23)$$

**TANK'S CAPACITY** The volume of the water stored in the tank in the  $j^{th}$  sampling interval is,

$$V(j) = V(j-1) + Q_1 u_1(j) - Q_2 u_2(j) + A_{rt} P(j). \quad (4.24)$$

The quantity  $A_{rt} P(j)$  is the volume of the harvested rain water in the tank with  $A_{rt}$  being the area of the rooftop. Expressing this volume in terms of the initial volume  $V(0)$  in the tank using recurrence manipulation gives,

$$V(j) = V(0) + \sum_{i=1}^j \left( Q_1 u_1(i) - Q_2 u_2(i) + A_{rt} P(i) \right) \quad (1 \leq j \leq N). \quad (4.25)$$

This volume is restricted by the tank's dimensions as follows

$$V_l \leq V(0) + \sum_{i=1}^j \left( Q_1 u_1(i) - Q_2 u_2(i) + A_{rt} P(i) \right) \leq V_u, \quad (4.26)$$

where  $V_l$  and  $V_u$  are the minimum and maximum volumes of the water in the tank respectively.

**PUMP MAINTENANCE COST** Although the pump maintenance cost cannot be easily quantified, the total number of its switching is used to estimate the cost. Frequent switching (on/off) increases the mechanical stress induced in the pump [54] causing more wear and tear thereby increasing the

maintenance cost while reducing the pump's life [150]. It is therefore necessary to minimize the interruption of the pump while in operation [153] by allowing a wider operation band [157].

In this chapter, we propose two methods to deal with the issue. We called the first method the constraint method, which restricts the pump to a maximum number of switching on,  $s_{max}$ , during the control horizon.

$$\sum_{j=1}^{N-1} \left( u_2(j+1) - u_2(j) \right)^2 \leq 2s_{max}. \quad (4.27)$$

In this equation, the status of the pump at two adjacent sampling intervals is compared throughout the control horizon. The sum of the squares of this comparison is set to a maximum switching of  $2s_{max}$  in order to effectively minimize the maintenance cost.

The second method is described in section 4.2.4.

**WATERING RESTRICTION** The watering restriction constraint (4.12) now becomes.

$$u_2(i) = 0 \quad \forall i \in \left[ \frac{11}{t_s}, \frac{15}{t_s} \right]. \quad (4.28)$$

**BOUNDARIES** In addition to bounds (4.13), the pump switch is bounded as

$$0 \leq u_2(j) \leq 1 \quad (1 \leq j \leq N). \quad (4.29)$$

### 4.2.3.3 Algorithm

The generalized optimization formulation is similar to section 4.2.2.3 with the exception of the non-linear inequality constraint notation  $C(X) \leq d$ . Vector  $X$  contains the control variables which are the status of the solenoid valve,  $u_1$ , controlling the flow of the municipal water into the tank and the status of the switch controlling the pump,  $u_2$ . Therefore,

$$X = [u_1(1), \dots, u_1(N), u_2(1), \dots, u_2(N)]_{2N \times 1}^T. \quad (4.30)$$

From objective function (4.22),

$$f^T = \left[ \alpha_1 Q_1 \quad \dots \quad \alpha_1 Q_1 p_w \quad \alpha_2 p_m p_e(1) \quad \dots \quad \alpha_2 p_m p_e(N) \right]_{1 \times 2N}. \quad (4.31)$$

If we denote

$$A'_1 = \begin{bmatrix} 0 & A_1 \end{bmatrix}_{N \times 2N}, \quad (4.32)$$

$$A_3 = \begin{bmatrix} -\frac{Q_1}{A_{tank}} & 0 & \dots & 0 & \frac{Q_2}{A_{tank}} & 0 & \dots & 0 \\ -\frac{Q_1}{A_{tank}} & -\frac{Q_1}{A_{tank}} & \dots & 0 & \frac{Q_2}{A_{tank}} & \frac{Q_2}{A_{tank}} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -\frac{Q_1}{A_{tank}} & -\frac{Q_1}{A_{tank}} & \dots & -\frac{Q_1}{A_{tank}} & \frac{Q_2}{A_{tank}} & \frac{Q_2}{A_{tank}} & \dots & \frac{Q_2}{A_{tank}} \end{bmatrix}_{N \times 2N}, \quad (4.33)$$

$$b_3 = \begin{bmatrix} h(0) - h_l + \frac{A_{rt}}{A_{tank}} P(1) \\ h(0) - h_l + \frac{A_{rt}}{A_{tank}} (P(1) + P(2)) \\ \vdots \\ h(0) - h_l + \frac{A_{rt}}{A_{tank}} (P(1) + P(2) + \dots + P(N)) \end{bmatrix}_{N \times 1}, \quad (4.34)$$

and

$$b_4 = \begin{bmatrix} h_u - h(0) - \frac{A_{rt}}{A_{tank}} P(1) \\ h_u - h(0) - \frac{A_{rt}}{A_{tank}} (P(1) + P(2)) \\ \vdots \\ h_u - h(0) - \frac{A_{rt}}{A_{tank}} (P(1) + P(2) + \dots + P(N)) \end{bmatrix}_{N \times 1}, \quad (4.35)$$

where  $h_l = \frac{V_l}{A_{tank}}$  and  $h_u = \frac{V_u}{A_{tank}}$  are the lower and upper water height of the tank, respectively and  $A_{tank}$  is the cross-sectional area of the tank. The quantity  $h(0) = \frac{V(0)}{A_{tank}}$  is the initial height of the water in the tank. Then, the inequality constraints (4.23) and (4.26) can be written in the standard form using matrix  $A$  and vector  $b$  as:

$$A = \begin{bmatrix} A'_1 \\ -A'_1 \\ A_3 \\ -A_3 \end{bmatrix}_{4N \times 2N}, \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}_{4N \times 1}. \quad (4.36)$$

Linear equality constraint (4.28) is modelled similarly to equality constraint (4.20) with the following modifications

$$A'_{eq} = \begin{bmatrix} 0 & A_{eq} \end{bmatrix}_{N \times 2N}, \quad b'_{eq} = \begin{bmatrix} b_{eq} \end{bmatrix}_{N \times 1} \quad (4.37)$$

and boundaries (4.13) and (4.29) become

$$L'_B = \begin{bmatrix} 0 & L_B \end{bmatrix}_{2N \times 1}^T \quad \text{and} \quad U'_B = \begin{bmatrix} 0 & U_B \end{bmatrix}_{2N \times 1}^T. \quad (4.38)$$

Finally, non-linear inequality constraint (4.27) is written in the standard form  $C(X) \leq d$  to become,

$$(X(N+2) - X(N+1))^2 + \dots + (X(2N) - X(2N-1))^2 \leq 2s_{max}. \quad (4.39)$$

This non-linear binary problem is solved using the SCIP<sup>6</sup> optimization solver which is one of the fastest non-commercial solvers for mixed integer linear and non-linear programming available in the

<sup>6</sup>SCIP <http://scip.zib.de/>

Matlab interface OPTI toolbox.

#### 4.2.4 The Pretoria method to reduce pump maintenance cost

The maintenance cost of a pump, as mentioned earlier, is reduced by minimizing the switching times of the pump. The Pretoria method developed by Mathaba et al. [153] introduces an auxiliary variable  $s(j)$  represented by a value 1 whenever a start-up occurs. This changes the objective function (4.22) to

$$J = \alpha_1 \sum_{j=1}^N p_w Q_1 u_1(j) + \alpha_2 \sum_{j=1}^N p_m p_e(j) u_2(j) + \alpha_3 \sum_{j=1}^N s(j), \quad (4.40)$$

where  $\alpha_3$  is a weight factor. The objective is subject to constraints in Section 4.2.3.2 and

$$u_2(1) - s(1) \leq 0, \quad (4.41)$$

$$u_2(j) - u_2(j-1) - s(j) \leq 0,$$

with  $s(j) \in \{0, 1\}$ . The first inequality in (4.41) initialises the auxiliary variable as the initial status of  $u_2$  while the second favours the control that involves less switching.

In similar veins, the optimization can be re-written in the standard linear form as

$$X = \left[ u_1(1) \quad \dots \quad u_1(N) \quad u_2(1) \quad \dots \quad u_2(N) \quad s(1) \quad \dots \quad s(N) \right]_{3N \times 1}^T \quad (4.42)$$

and

$$f^T = \left[ \alpha_1 Q_1 \quad \dots \quad \alpha_1 Q_1 p_w \quad \alpha_2 p_m p_e(1) \quad \dots \quad \alpha_2 p_m p_e(N) \quad 1 \quad \dots \quad 1 \right]_{1 \times 3N}. \quad (4.43)$$

If we denote

$$A_1'' = \left[ A_1' \quad 0 \right]_{N \times 3N}, \quad A_3' = \left[ A_3 \quad 0 \right]_{N \times 3N}, \quad (4.44)$$

$$A_5 = \begin{bmatrix} 0 & \dots & 0 & 1 & 0 & 0 & \dots & 0 & -1 & 0 & 0 & \dots & 0 \\ 0 & \dots & 0 & -1 & 1 & 0 & \dots & 0 & 0 & -1 & 0 & \dots & 0 \\ 0 & \dots & 0 & 0 & -1 & 1 & \dots & 0 & 0 & 0 & -1 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 0 & 0 & \dots & -1 & 1 & 0 & 0 & 0 & \dots & -1 \end{bmatrix}_{N \times 3N} \quad (4.45)$$

and

$$b_5 = \left[ 0 \quad 0 \quad 0 \quad \dots \quad 0 \right]_{N \times 1}^T, \quad (4.46)$$

then, linear inequality constraints (4.23), (4.26) and (4.41) become,

$$A = \begin{bmatrix} A_1'' \\ -A_1'' \\ A_3' \\ -A_3' \\ A_5 \end{bmatrix}_{5N \times 3N} \quad \text{and} \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix}_{5N \times 1}, \quad (4.47)$$

while linear equality constraint (4.28) becomes,

$$A_{eq}'' = \begin{bmatrix} A_{eq}' & 0 \end{bmatrix}_{N \times 3N} \quad \text{and} \quad b_{eq}'' = \begin{bmatrix} b_{eq}' \end{bmatrix}_{N \times 1}. \quad (4.48)$$

Finally, boundaries (4.13) and (4.29) become,

$$L_B'' = \begin{bmatrix} L_B' & 0 \end{bmatrix}_{3N \times 1}^T \quad \text{and} \quad U_B'' = \begin{bmatrix} U_B' & 0 \end{bmatrix}_{3N \times 1}^T \quad (4.49)$$

This binary linear problem is solved using the SCIP solver.

### 4.3 GENERAL DATA

#### 4.3.1 Case study

Since there is no consolidated information on domestic water consumption in South Africa [87], the case used was the Acclima TDT scheduling done by Blonquist Jr. et al. [182], who used the Kentucky bluegrass on a 280  $m^2$  field plot with Millville Silt Loam soil. The estimated rooting depth of the turfgrass was 30 cm with a MAD of 0.5. Further, the average daily evapotranspiration,  $ET_c$ , was 3.77 mm and a precipitation of 1 mm was experienced. A controller was connected to the solenoid valve on the pipe to irrigate the lawn using a gear-driven sprinkler head (Hunter<sup>®</sup> PGP with a #9 in. nozzle) with an approximate flow rate of 0.374  $l/s^7$ .

Since the above case used municipal water directly without storing, a case study was conducted in Pretoria which showed that a typical house has a rooftop area of 120  $m^2$  and lawn irrigation is normally done in the morning between 8 and 10 am when workers report on duty.

Further, in this chapter, the initial value of the soil water content,  $S(0)$ , and water level in the tank,  $h(0)$ , are taken as 29.1 cm and 18 cm respectively. In the system with RWH, equal weighting factors

<sup>7</sup>Hunters product catalogue [www.hunterindustries.com](http://www.hunterindustries.com)

( $\alpha_1 = \alpha_2 = 0.5$ ) are chosen, because most end users prefer both energy and water savings. Similarly, in the Pretoria method,  $\alpha_1 = \alpha_2 = 0.4$ , and the pump maintenance cost weighting factor,  $\alpha_3 = 0.2$  has less weight. Finally, in the constraint method, the maximum allowable number of pump switching,  $s_{max} = 3$ .

### 4.3.2 Time-of-use electricity tariff

The time-of-use (TOU) tariff is commonly used globally [48] and it can vary by time of day, day of week and season [160]. Eskom's TOU Homeflex structure<sup>8</sup> for residential consumers given below is used [57].

$$p_e(t) = \begin{cases} p_{off} = 0.6281 \text{ R/Kwh} & \text{if } t \in [0, 6] \cup [10, 18] \cup [20, 24], \\ p_{peak} = 1.9935 \text{ R/Kwh} & \text{if } t \in [7, 10] \cup [18, 20], \end{cases} \quad (4.50)$$

where  $p_{off}$  is the off peak price,  $p_{peak}$  is the peak time price,  $R$  is the South African currency, Rand, and  $t$  is the time of day in hours. The tariff has five charge components as service charge, network charge, environmental levy, peak charge and off-peak charges [154]

### 4.3.3 Water tariffs

The city of Tshwane<sup>9</sup> has various water tariffs for different classes of consumers and the domestic consumers are charged using the rates in Table 4.1. The monthly amount of water used for irrigation is

**Table 4.1.** City of Tshwane water tariff for 2014/2015

Volume ( $m^3/month$ )	0-6	7-12	13-18	19-24	25-30	31-42	43-72	>72
Rates ( $R/m^3$ )	6.81	9.72	12.77	14.77	16.89	18.25	19.53	20.91

assumed to be less than  $6 m^3$  meaning the unit price of water used  $p_w = 6.81 (R/m^3)$ .

<sup>8</sup>Eskom tariffs and charges booklet 2011/2012. [www.eskom.co.za](http://www.eskom.co.za)

<sup>9</sup>Tshwane Municipal Tariffs 2014/2015 [www.tshwane.gov.za](http://www.tshwane.gov.za)

#### 4.3.4 Water tank and pump

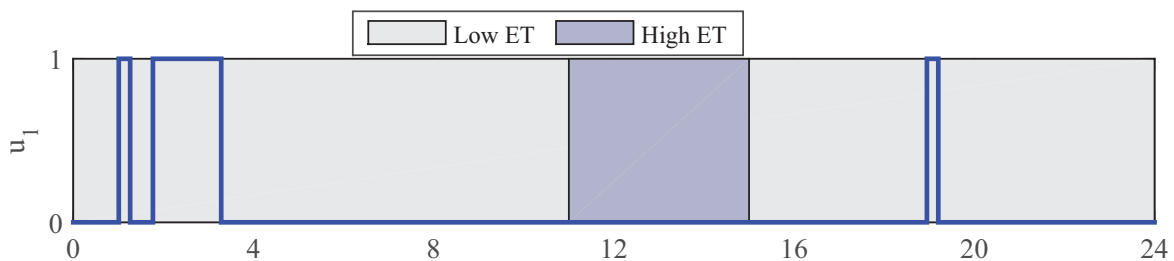
There are various companies supplying water tanks in South Africa. Jojo's<sup>10</sup> 1000 l cylindrical water tank with a diameter and height of 1.1 and 1 m respectively is used chosen. The lower and upper levels of the water in the tank are set as 0.12 and 1 m, respectively to avoid spilling the water from the tank as well as running it completely empty.

The pump chosen for this system is the Al-Ko HW 3000 Classic<sup>11</sup>, with a power rating of 650 W and a flow rate of 3.1 m<sup>3</sup>/h. It is ideal for small sprinklers and domestic applications.

### 4.4 SIMULATION RESULTS AND DISCUSSION

#### 4.4.1 Optimal scheduling without rooftop water harvesting

Figure 4.3(a) shows the optimal irrigation schedule. The schedule has three switching regimes with two taking place early in the morning. Another schedule takes place in the evening taking advantage of the low evapotranspiration rates as well as obey the by-laws. The water content in the soil, shown in Figure 4.3(b),

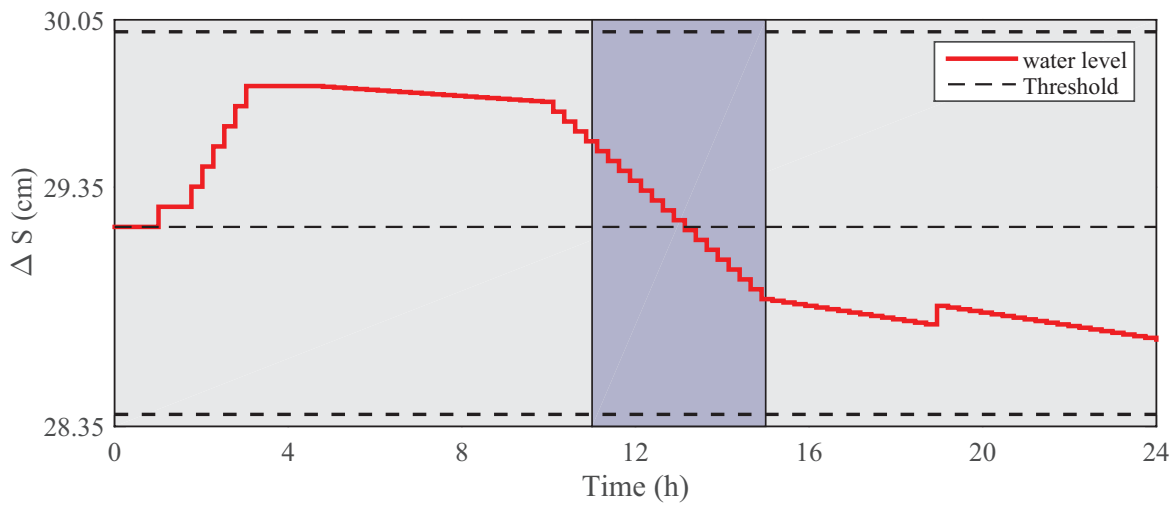


(a) Optimal schedule

<sup>10</sup>Water tank [www.jojo.co.za](http://www.jojo.co.za)

<sup>11</sup>[www.urbanrainsystems.co.za/accessories/accessories.asp](http://www.urbanrainsystems.co.za/accessories/accessories.asp)





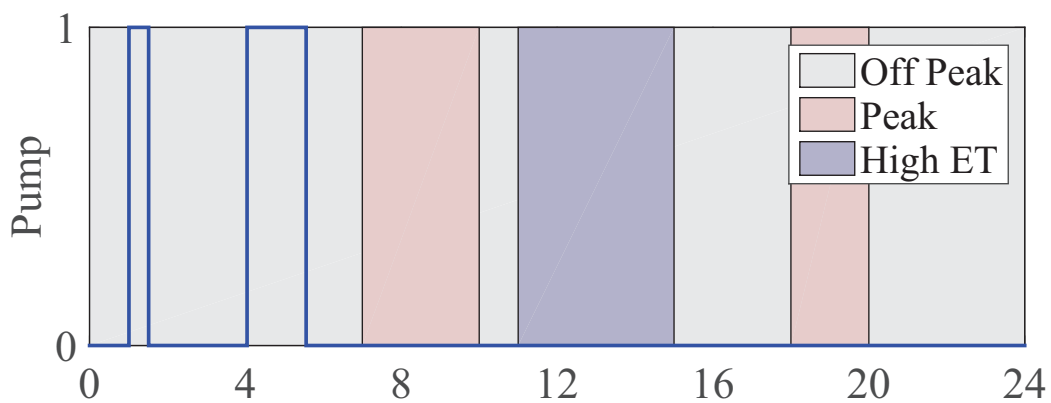
(b) Soil water variation

Figure 4.3. Optimal schedule and variation of the state variable.

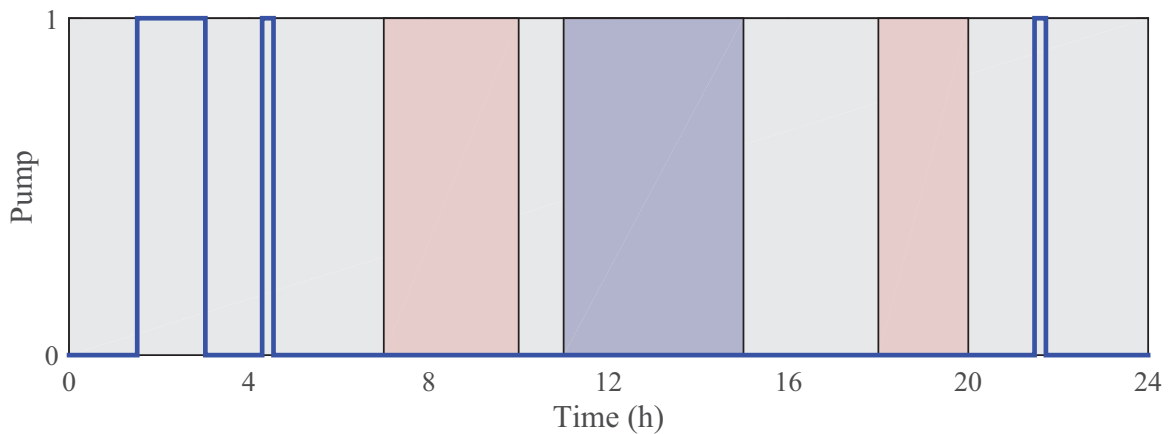
is maintained within the required range, ensuring that there is no drainage from the soil as well as the soil doesn't become too dry.

#### 4.4.2 Comparison of the switching strategies

Figure 4.4 shows the comparison of the two switching minimization strategies used to reduce the pump switching frequency.



(a) Pretoria method



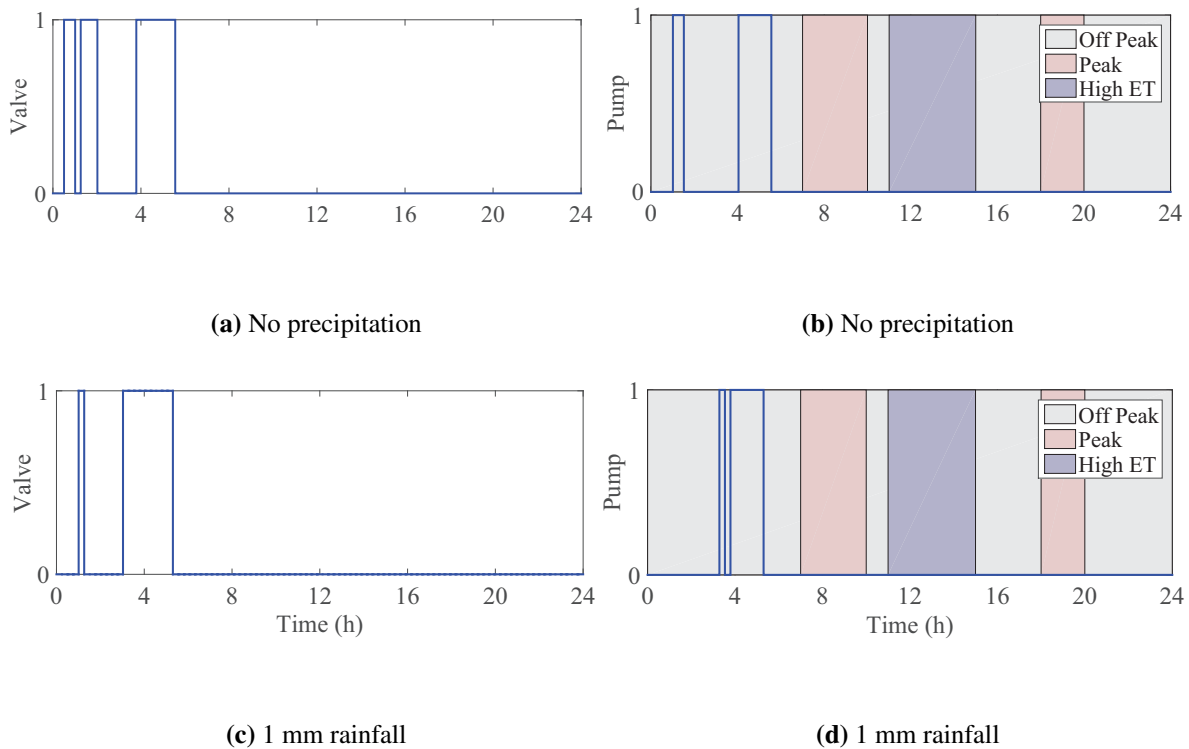
(b) Constraint method

**Figure 4.4.** Comparison of the switching minimization strategies.

The legend used is the same throughout this chapter. The Pretoria method (Figure 4.4(a)) optimally reduces the switching to the minimum feasible times. In this case, the strategy reduces the switching to two times during the entire control horizon. On the contrary, the constraint method (Figure 4.4(b)), which explicitly sets the maximum number of possible switching ( $s_{max} = 3$ ), actually switches the pump 3 times during the control horizon. The constraint method however runs at a risk of infeasibility if the irrigation demand increases to a level where the pump must switch on more than the set  $s_{max}$ . This shows that the Pretoria method is more effective in reducing the maintenance cost than the constraint method and it is therefore used in the later sections. It is important to note that both strategies incur the same cost of energy during the period, as the same amount of water is pumped for irrigation. Further, although in one day, the extra switching regime in the constraint method may not have very high difference in maintenance cost, over a long period of time, the extra switching will affect the maintenance cost by lowering the life cycle of the pump.

#### 4.4.3 Optimal scheduling with rooftop water harvesting

Two scenarios are analysed when incorporating rooftop water harvesting. The optimal schedules of the valve and the pump are shown in Figure 4.5 in cases where there is no precipitation and when 1-mm of precipitation event occurs. The valve controlling the municipal water into the tank (Figures 4.5(a) and



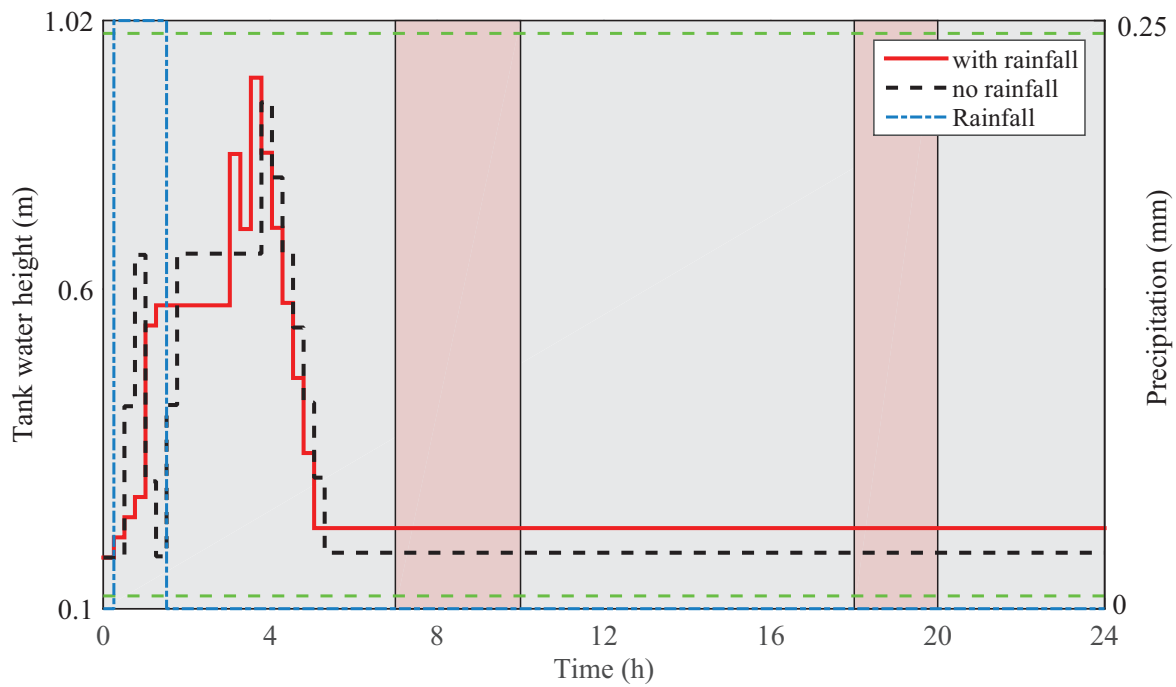
**Figure 4.5.** Optimal schedules of the valve and pump.

4.5(c)) uses negligible amount of power, hence it is allowed to operate throughout the control horizon irrespective of whether it is peak or off-peak in the TOU tariff.

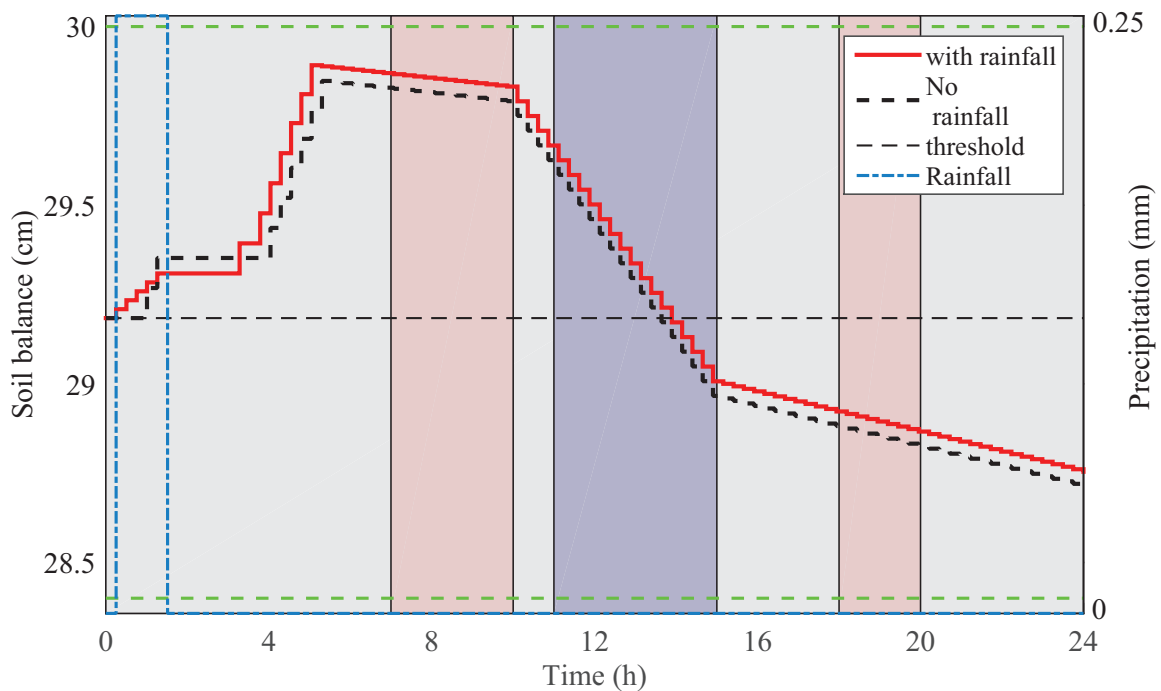
This system has two state variables; height of water in the tank and depth of water in the soil. The height of water in the tank,  $h(j) = \frac{v(j)}{A_{tank}}$ , is obtained using equation (4.25) while the soil water variation is obtained using equation (4.10). The variation of these variables during the control horizon is shown in Figures 4.6 and 4.7 respectively. In both figures, left vertical axis represents the water level either in the tank or soil while the rainfall event is shown by the right axis.

#### 4.4.3.1 Scenario 1: Optimal schedule with no precipitation

With no precipitation, or zero rainfall recorded, the tank is purely filled with utility water with the valve optimally operating as shown in Figure 4.5(a). This results from the demand of water in the lawn which is met by optimally operating the pump as shown in Figure 4.5(b). The optimal schedule (Figures 4.5(a) and 4.5(b)) have three tank filling and two irrigation regimes to ensure that the water



**Figure 4.6.** Variation of the water in the tank with and without precipitation.



**Figure 4.7.** Soil water balance with and without precipitation.

level in the tank and the soil remain within the required height and depth respectively. The solenoid valve switches on at 00:30 causing the water level in the tank to rise to 0.65 m. The pump then switches

on between 01:00 to 01:30 leading to the water height in the tank dropping to 0.18 *m* while the water level in the soil rises to 29.35 *cm*. The water height in the tank then rises again to 0.66 *m* until the solenoid valve switches off at 02:15 hours. The water level then remains constant both in the tank and the soil since both solenoid valve and pump are off and since it is early morning, the evapotranspiration losses are assumed as negligible. The controller then predicts that another irrigation is scheduled to start at 04:00 and opens the solenoid valve at 03:45. For the 15 minutes that only the solenoid valve is open, the water height in the tank further rises to 0.89 *m* and thereafter starts dropping when irrigation starts until when the pump switches off at 05:30 hours, where it remains constant for the rest of the day. This last irrigation event also causes the water level in the soil to rise to 29.85 *cm*, and thereafter the water content drops due to the evapotranspiration over the control horizon.

Even though this scenario has no rain harvested, the irrigation schedule differs with the schedule in section 4.4.1 because of addition of the pump and water storage tank. However, about 6.7 *mm* of water is used for irrigation in both cases.

#### 4.4.3.2 Scenario 2: Optimal schedule with precipitation

The optimal schedules for the valve and pump obtained assuming about 1-*mm* of precipitation event between midnight and 1 AM are shown in figures 4.5(c) and 4.5(d). Since the rain is assumed to fall both on the rooftop and the lawn, both the valve and pump optimal schedules switch for less duration than when there is no precipitation taking place. The solenoid valve switches on at 01:00 hours but since there is precipitation taking place, it switches off in the next sampling interval. The harvesting of rain water and opening of the valve cause the height of the water in the tank to rise to 0.57 *m*. This precipitation event also causes the water level in the soil to rise to 29.31 *cm*. Thereafter the water level remains constant in both the tank and soil until 03:00 for the tank when the solenoid next switches on and 03:15 when the pump switches on. The pump switches off at 03:30 for 15 minutes enabling the water height in the tank to reach a peak of 0.93 *m*. Thereafter the pump switches on, while the solenoid valve is still filling the tank, until 05.15, where they both switch off. During this duration, the water height in the tank drops to 0.23 *m* while the water level in the soil rises to 29.89 *cm*. Thereafter, both the solenoid valve and the pump remain off for the rest of the control horizon. Therefore, the water height in the tank remain constant at 0.23 *m* while the water level in the soil drops due to evapotranspiration losses to 28.73 *cm*. About 5.9 *mm* of water is pumped for irrigation in this scenario.

With the 1-*mm* precipitation event, about 120 *l* of rain water is harvested from the rooftop and stored in the tank. Even though the same 1-*mm* rain is assumed to have fallen on the lawn, it is not sufficient to maintain the soil water content as required during the control horizon leading to about 5.9 *mm* of irrigation water being applied, which is less than 6.7 *mm* applied when no precipitation takes place. This leads to about 0.4 *mm* more water content being left in the soil at the end of the control horizon as seen in Figure 4.7. Part of this 5.9 *mm* of irrigation water is met by harvested water stored in the tank effectively leading to conservation of 120 *l* of municipal water during this day. Over a long time, with more precipitation events, more water would be harvested leading to even higher water conservation.

#### 4.4.4 Discussion

The optimal irrigation scheduling without RWH is applicable if the water is always reliable and no rain water can be harvested from the building. In cases where the utility water is not reliable, like in developing nations, the water needs to be stored and pumped. The control system incorporating RWH becomes useful.

Table 4.2 shows the comparison of the two optimal irrigation scheduling and the techniques used by Blonquist Jr. et al. [182]. The Acclima TDT sensor was connected to the the Acclima CS3500 controller and the irrigation results compared with the regular irrigation practice in the area [182]. The amount of irrigation water used in both control systems, with and without RWH is the same. When the rain falls on the lawn, less water will be required to irrigate in order to meet the required level making the amount used in this scenario same in both systems. Given the optimal controller with no RWH is directly coupled to the municipal water, there are no associated direct energy costs. However, the control system results in 14.7% of water conserved in relation to the Acclima TDT system, leading to a similar cost saving in a day. The water conservation is even higher; 17.4% with a similar cost saving, in relation to the regular water application. Unlike the other methods, the controller reduces the amount irrigated even when a small amount of rainfall is experienced.

The optimal controller with RWH introduces energy costs from the pumping. Assuming there is no harvested water in the tank, the operational cost of the system with RWH is about 13% higher than the system with no RWH. However, with 1 *mm* precipitation, the system with RWH exhibits the lowest

**Table 4.2.** Comparison of irrigation and energy amounts and the associated cost

Mode	Irrigation water		Pumping Energy		Total
	Amount (mm/day)	Cost (R/day)	Amount (Kwh/day)	Cost (R/day)	Cost (R/day)
Optimal(No RWH)					
P=0 mm	6.73	12.90	-	-	12.90
P=1 mm	5.90	11.30	-	-	11.30
Acclima TDT	6.92	13.20	-	-	13.20
Regular	7.14	13.70	-	-	13.70
Optimal(RWH)					
P=0 mm	6.73	12.90	1.3000	0.82	13.72
P=1 mm	5.90	10.50 <sup>a</sup>	1.1375	0.72	11.22
Acclima TDT	6.92	13.20	1.337 <sup>b</sup>	0.84	14.04
Regular	7.14	13.70	1.379 <sup>b</sup>	2.75 <sup>c</sup>	16.45

<sup>a</sup> This cost was obtained after reduction of 120 l harvested water from the total water irrigated.

<sup>b</sup> Pumping assumed to be done with the energy proportional to the amount of water irrigated.

<sup>c</sup> Irrigation normally carried out in the morning during peak time.

operational cost. It leads to about 7.1% reduction on the cost of water used for irrigation with respect to the optimal schedule with no RWH with the precipitation. There is a further 20.5% and 23.4% savings in the cost of water relative to the Acclima TDT and the regular water application respectively. In addition, the controller results in 14.3% and 73.8% savings in energy cost relative to the Acclima TDT scheduling and regular irrigation practice respectively. The high savings in energy cost when compared to the regular irrigation occur because regular irrigation normally takes place during peak time.

It is important to note that although RWH introduces energy costs, the optimal irrigation scheduling with RWH, greatly reduces the operational cost by saving both the water and energy.

## 4.5 SYNOPSIS

Optimal irrigation scheduling in urban lawns can have direct and indirect benefits. This chapter shows the potential direct benefits including significant savings in the cost water by about 14.7% and 17.4% relative to the Acclima TDT scheduling and regular irrigation, respectively, when optimally scheduling using municipal water directly. Optimal scheduling with RWH lowers this cost even further by 20.5% and 23.4% relative to the Acclima TDT scheduling and regular irrigation respectively. Following implementation of water conservation measures, the consequent potential savings in energy cost in the RWH system are 14.3% and 73.8% relative to the Acclima TDT and regular irrigation respectively. These savings result from the load shifting to off-peak times as well as just the necessary amount of water is pumped to the lawn. Furthermore, the Pretoria method maximizes the pump life by minimizing the maintenance cost resulting from switching frequency better.

The optimal control systems can potentially lead to improved economic efficiency through cost savings of both water and energy. In addition, the optimal controller with RWH best suits application in developing nations where water demand far surpasses the supply requiring water storage. It not only leads to water conservation but also reduces the demand of potable water from municipal sources as well as shifting and reducing the load on the power utility.



# **CHAPTER 5 OPTIMAL ENERGY-WATER MANAGEMENT IN URBAN RESIDENTIAL BUILDINGS THROUGH GREY WATER RECYCLING**

## **5.1 INTRODUCTION**

Energy and water are intricately entwined resources (energy-water nexus) that are vital for human survival and economic progress of any nation [190]. Increasing global population is putting the two resources under colossal pressure. By 2050, insecurity of these resources will mostly be felt in urban areas resulting from continued increase in urban population. On one hand, urbanization increases the demand for these resources in urban areas, while on the other hand, urban centres are hotspots for innovation on their sustainable consumption [191]. In developing nations, growing cities face a momentous task of providing energy and water as utilities have limited or no capacity to adequately respond to growing demand. These challenges are further aggravated by demographic shifts, changing lifestyles, thriving middle class and the growing impact of climate change on demand and supply chains of the two resources [192]. For instance, the World Bank estimates the urban population in most African cities will double by 2030 [193]. Whereas developed nations have embarked on researching on energy-water nexus in urban areas [2, 194, 195], developing nations have been concentrating on either energy demand management [67, 70, 71, 196, 197], or water demand management [176, 198–200], until recently when energy water nexus in buildings started gaining research interest [201, 202]. Coordinated management of the two resources has the benefit of minimizing unaccounted indirect impact of one resource on the other [203].

Cities and urban areas are large consumers of energy and water in many countries [204]. For instance, they accounted for 95% of water consumption growth in United States between 1985 and 2005 [122]. Therefore, conservation of water and associated energy in buildings is a huge opportunity in realizing savings of these resources, while at the same time improving their security [205], with the goal of achieving green buildings [206]. Conventional management of water resources concentrates on large [207], expensive centralized water supply and sewage disposal systems that have negative environmental impact and are incompatible with modern requirements, especially in growing cities [208]. Therefore, a paradigm shift seeking to minimize amount of pollution generated and discharged, using and reusing water very near to the point of origin as well as treating water to the required quality is required [209]. This shift is leading to adoption of decentralized solutions such as water recycling and rain water harvesting [210], further accelerated by advanced water treatment technologies [211], change of attitude of end users and increased awareness on the need to conserve fresh water [212]. In general, decentralized systems present numerous benefits, depending on the geographical area, including; cost reduction, resource efficiency, improved resource security, reduction of system failure, economic empowerment of the local community and environmental benefits. The guidelines for safe use of wastewater, excreta and greywater provided by the World Health Organization (WHO) accentuate on the prominence of grey water as an alternative water resource [213]. This is because grey water constitutes a significant volume of the waste flow from households, has nutrients that can be beneficial for irrigation, has low pathogen content and can therefore be used to reduce the demand for potable water. Water can be recycled for potable or non-potable uses. Recycling for direct potable use is rare but has been implemented in Windhoek, Namibia, where low precipitation and high evaporation necessitated augmentation of water supply with reclaimed water [214]. Indirect potable reuse can either be planned or unplanned. Planned indirect potable reuse utilizes an environmental buffer to provide further treatment and retention time such as in California and Florida. Unplanned potable reuse takes place through discharging treated waste water into the environment which is subsequently abstracted for potable use [211]. Non-potable reuse is the most commonly applied decentralized water recycling system in urban areas. Such networks are well established in Tokyo and Fukuoka in Japan [215], as well as Queensland in Australia [216]. A study conducted among university students in Pretoria revealed their alacrity to adopt recycled water systems for non-potable uses, especially if it would also lead to less bills [217].

Although various studies have shown that grey water recycling in residential buildings is possible [218, 219], and of utmost importance [220, 221], there has been little research considering energy and

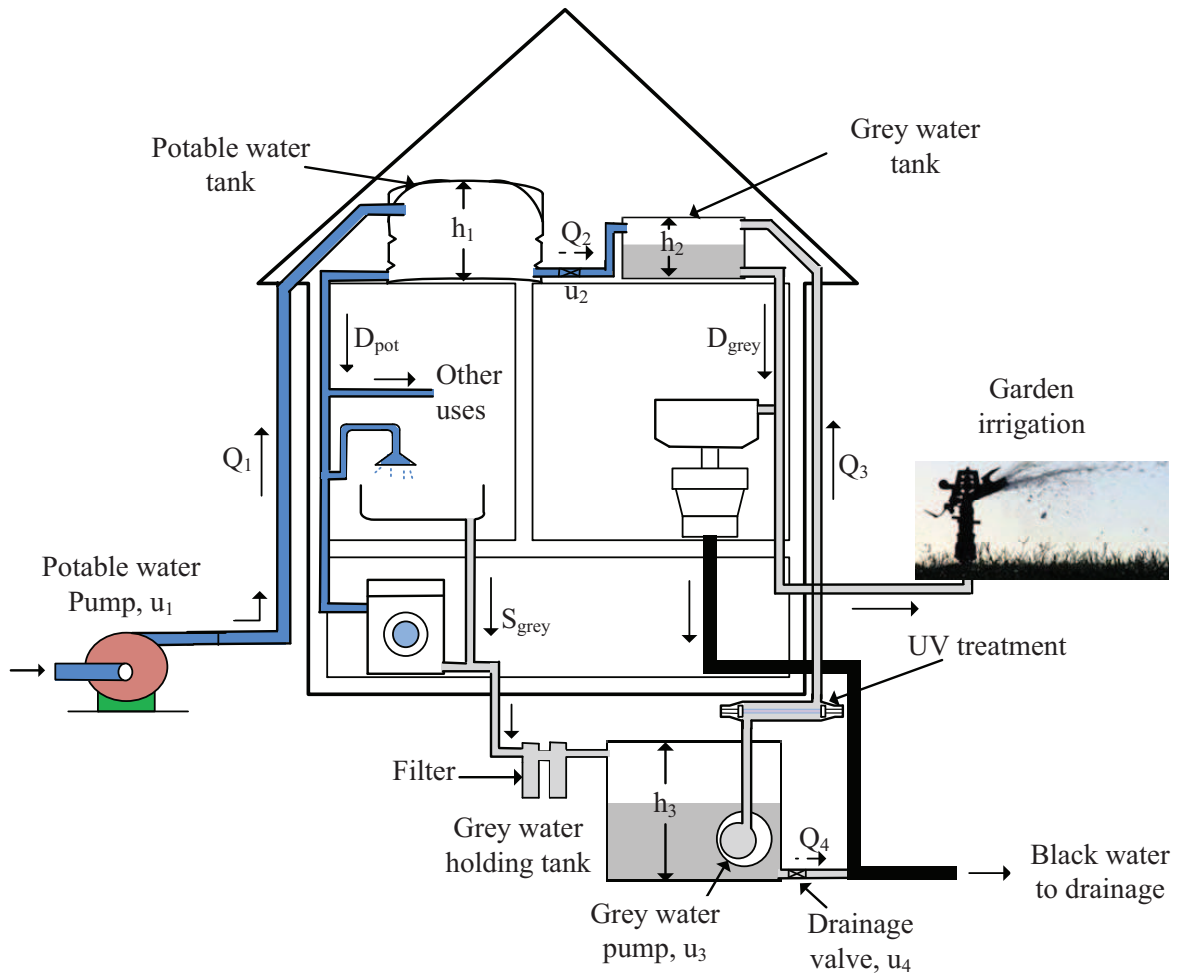
water management involving grey water. In fact, a study conducted in a small scale water recycling plant in England showed that the plant's operational cost was 20 times higher than in large scale recycling plants. The high cost was attributed to operational inefficiencies whereby staff and energy accounted for 51% and 27% respectively [211]. It is therefore important improve the operational efficiency of such plants by developing technologies that optimally operate the system by ensuring energy efficiency and minimal labour cost through autonomous operation are achieved. Unfortunately, the initial cost of implementation of grey water recycling system is high in most places [222], so government intervention is needed such as offering incentives to encourage the uptake of these solutions [223, 224].

This chapter introduces the first attempt to design novel, cost effective and advanced optimal controllers to operate a grey water recycling system in residential areas. The open-loop optimal and closed-loop model predictive control (MPC) controllers are designed to ensure that water is conserved while the energy associated with the system is used efficiently. The two control strategies are designed to meet the hourly water demand for a house. Although the open-loop control is more cost effective and easy to implement, it is suitable where the water demand is known to be relative stable. However, in cases where the water demand fluctuates such that it is difficult to accurately predict and the system is susceptible to external disturbances that significantly affect the grey water system, the closed-loop MPC should be espoused. It, however, requires installation of additional monitoring devices such as level monitoring of the tanks thereby increasing the cost and complexity of the control system. The two optimal controllers, if widely adopted, would reduce the demand for potable water, energy and waste flow from utilities and municipalities. They would further lower the cost of water and sewage purification leading to lower bills incurred by the end user associated to both resources and waste water disposal.

## 5.2 CONTROLLER DEVELOPMENT

### 5.2.1 Schematic layout

Figure 5.1 shows the schematic layout of the grey water recycling system for a typical house. Two scenarios are considered in this chapter. Firstly, supply from water utility is reliable meaning pumping and storing is not required. Hence, potable water pump and tank are not required in the grey water recycling system for this scenario. Water flows directly from the municipal supply pipe to potable end



**Figure 5.1.** Schematic of water pumping and grey water recycling system

uses, and through potable valve,  $u_2$ , when required. Secondly, water pumping and storage is required in a building with unreliable water supply either due to low water pressure or water rationing taking place in the area. Potable water pump, which is a fixed speed pump whose state is represented as  $u_1$  is required to pump the water to the roof storage tank. It then flows from this tank to various end uses by gravity. In both scenarios, grey water from some end uses, such as, shower and washing machine, can be treated and re-used for end uses that do not require potable water such as garden irrigation and toilet flushing. Therefore, grey water is collected, filtered to remove physical impurities and stored in a holding tank, which can either be placed underground or at the back of the building. This tank must be emptied using the drainage solenoid valve, whose state is represented as  $u_4$ , within 24-h, to prevent formation of bacteria that produce foul smell. Grey water must be treated, and in this system, ultra violet (UV) treatment is chosen for its adaptability, low space requirement and low power consumption. Therefore, grey water is pumped from the holding tank using the grey water pump, a fixed speed pump

whose state is represented as  $u_3$ . It passes through the UV treatment chamber to the grey water tank at the roof of the building, next to the potable water tank. The treated grey water also flows by gravity to the suitable end uses. There are instances, however, when there won't be sufficient grey water to meet the grey water demand. The potable water valve,  $u_2$ , allows potable water to flow to the grey water tank to assist in meeting the demand. Black water, that is, water that cannot easily be recycled (e.g. water from the toilet), is allowed to flow to the drainage directly. The aim, therefore, is to control the potable and grey water pumps, drainage and potable water valves to ensure that water supply in the house is reliable while enhancing energy and water efficiency.

## 5.2.2 Dynamics of water flow

When potable water supply is reliable, only grey and holding tanks would be required. However, if the supply is unreliable, water is stored in three tanks to meet the overall demand in the house. It is assumed that all the tanks used in this chapter have uniform cross-sectional area. The dynamics of water flow in each tank is mathematically modelled below.

### 5.2.2.1 Potable water tank

When potable water supply is unreliable, it has to be pumped by potable water pump into the potable water tank, to meet the overall potable water demand and supplement grey water whenever needed to. The dynamics of volume of water in this tank,  $\dot{V}_1$  ( $m^3/h$ ), is,

$$\dot{V}_1 = A_1^t \dot{h}_1 = Q_1 u_1 - Q_2 u_2 - \dot{D}_{pot}, \quad (5.1)$$

where  $A_1^t$  is the cross-sectional area of the tank ( $m^2$ ) while  $\dot{h}_1$  is the rate of change of the height of water in the tank ( $m/h$ ).  $\dot{D}_{pot}$  is the potable water demand ( $m^3/h$ ) in the house while  $Q_1$  and  $Q_2$  are the flow rates ( $m^3/h$ ) of potable water pump and solenoid valve respectively. Dynamic equation (5.1) can be expressed in discrete-time domain by a first order difference equation as follows;

$$h_1(j+1) = h_1(j) + \frac{1}{A_1^t} \left[ t_s Q_1 u_1(j) - t_s Q_2 u_2(j) - D_{pot}(j) \right], \quad (5.2)$$

where  $j$  is the sampling interval and  $t_s$  is the sampling period during a full operating cycle of 24-h. The equation is modelled in terms of the height of water in the tank in a sampling interval,  $h_1(j)$ , because level sensors are the most economical and easy to use in measuring the volume of water in a tank. Since the tanks have been assumed to have a uniform cross-section, then the height reading will be

converted to the volume by the controller. Through recurrence manipulation, equation (5.2) can further be modelled as,

$$h_1(j) = h_1(0) + \frac{t_s}{A_1^t} \sum_{i=1}^j \left[ Q_1 u_1(i) - Q_2 u_2(i) \right] - \frac{1}{A_1^t} \sum_{i=1}^j D_{pot}(i) \quad 1 \leq j \leq N, \quad (5.3)$$

where  $N$  is the total number of cycles during the full operating cycle given as  $N = \frac{24}{t_s}$ .

### 5.2.2.2 Grey water tank

This tank receives treated grey water and stores it for use by non-potable water end uses. In case there is no grey water available, potable water is allowed to flow from the potable water tank to supplement the grey water. Therefore, the volumetric rate of change,  $\dot{V}_2$  ( $m^3/h$ ), of the water in the tank is,

$$\dot{V}_2 = A_2^t \dot{h}_2 = Q_2 u_2 + Q_3 u_3 - \dot{D}_{grey}, \quad (5.4)$$

where  $A_2^t$  is the cross-sectional area ( $m^2$ ) of the tank,  $\dot{h}_2$  is the rate of change of the height of water ( $m/h$ ) in the tank,  $\dot{D}_{grey}$  is the grey water demand ( $m^3/h$ ) while  $Q_3$  is the water flow rate ( $m^3/h$ ) through the grey water pump. The equation can be expressed in the discrete-time domain as

$$h_2(j+1) = h_2(j) + \frac{1}{A_2^t} \left[ t_s Q_2 u_2(j) + t_s Q_3 u_3(j) - D_{grey}(j) \right], \quad (5.5)$$

which can further be expressed as

$$h_2(j) = h_2(0) + \frac{t_s}{A_2^t} \sum_{i=1}^j \left[ Q_2 u_2(i) + Q_3 u_3(i) \right] - \frac{1}{A_2^t} \sum_{i=1}^j D_{grey}(i) \quad 1 \leq j \leq N. \quad (5.6)$$

### 5.2.2.3 Holding tank

The holding tank temporarily stores filtered grey water that is collected from the recyclable potable water end uses. The tank acts as a temporary reservoir where grey water is collected from recyclable end uses and later pumped and treated to grey water tank when required. To avoid foul smell from developing, this tank must be emptied at least every 24 hours. Therefore, the dynamics of the volume of water in the holding tank,  $\dot{V}_3$  ( $m^3/h$ ), is,

$$\dot{V}_3 = A_3^t \dot{h}_3 = \dot{S}_{grey} - Q_3 u_3 - Q_4 u_4, \quad (5.7)$$

where  $A_3^t$  is the cross-sectional area ( $m^2$ ) of the tank,  $\dot{h}_3$  is the rate of change of the height ( $m/h$ ) of water in the tank,  $\dot{S}_{grey}$  is the amount ( $m^3/h$ ) of water supplied from the recyclable potable water end uses in an hour and  $Q_4$  is the flow rate ( $m^3/h$ ) of the grey water through the drainage valve. Equation

(5.7) can be written in discrete-time domain as,

$$h_3(j+1) = h_3(j) + \frac{1}{A_3^t} \left[ S_{grey}(j) - t_s Q_3 u_3(j) - t_s Q_4 u_4(j) \right], \quad (5.8)$$

which transforms to,

$$h_3(j) = h_3(0) + \frac{1}{A_3^t} \sum_{i=1}^j S_{grey}(i) - \frac{t_s}{A_3^t} \sum_{i=1}^j \left[ Q_3 u_3(i) + Q_4 u_4(i) \right] \quad 1 \leq j \leq N. \quad (5.9)$$

The dynamic equations are used in designing two model based controllers using advanced optimal control concept to ensure that water demand in the house is reliably met through efficient and optimal operation of the grey water system. The performance indicators of the control systems are;

- the cost of pumping energy of both potable and grey water,
- the maintenance cost of the pumps, and
- consumption of potable water in the house.

The open-loop controller and closed-loop MPC are designed and their performance compared in the subsequent sections.

### 5.2.3 Open-loop controller model

The open-loop controller uses a feed forward principle whereby, measurements of water demand are used by the controller to predict the future behaviour of the system in meeting the demand throughout the operating cycle. Therefore, the previously mentioned performance indicators are modelled in the following objective function,

$$J = \sum_{j=1}^N \left[ \alpha_1 t_s p_e(j) P_1^m u_1(j) + \alpha_2 t_s Q_2 p_w(j) u_2(j) + \alpha_3 t_s p_e(j) P_3^m u_3(j) \right] + \alpha_4 \sum_{j=1}^N \left[ s_1(j) + s_3(j) \right], \quad (5.10)$$

where  $P_1^m$  (kW) and  $P_3^m$  (kW) are potable and grey water pump's power consumption respectively, while  $p_w$ ,  $p_e$  and  $t_s$  are the cost of water, electricity during the  $j^{th}$  sampling interval and the sampling time respectively.  $s_1(j)$  and  $s_3(j)$  are auxiliary variables used to minimize the maintenance cost for potable and grey water pumps respectively. Each auxiliary variable is represented by a value 1 whenever a pump's state changes from off to on [147, 153]. Weights  $\alpha_1$  to  $\alpha_4$  are used to tune the controller

according to user's preference. The first and third terms in equation (5.10) minimize cost of energy consumed by the pumps, the second term minimizes the cost of potable water consumed by grey water end uses and the fourth term minimizes the maintenance cost of the pumps.

The objective function is subject to physical and operational constraints. The constraints are represented mathematically as;

$$h_1^{min} \leq h_1(j) \leq h_1^{max}, \quad (5.11)$$

$$h_2^{min} \leq h_2(j) \leq h_2^{max}, \quad (5.12)$$

$$h_3^{min} \leq h_3(j) \leq h_3^{max}, \quad (5.13)$$

$$h_3(N) = h_3^f, \quad (5.14)$$

$$u_1(1) - s_1(1) \leq 0, \quad (5.15)$$

$$u_1(j) - u_1(j-1) - s_1(j) \leq 0, \quad (5.16)$$

$$u_3(1) - s_3(1) \leq 0, \quad (5.17)$$

$$u_3(j) - u_3(j-1) - s_3(j) \leq 0, \quad (5.18)$$

$$u_m(j) \in \{0, 1\} \text{ where } m = 1, 2, 3, 4, \quad (5.19)$$

$$s_1(j), s_3(j) \in \{0, 1\}. \quad (5.20)$$

Inequalities (5.11), (5.12) and (5.13) limit the state variables, that is, height of water in respective tanks between minimum and maximum allowable levels. Potable and grey water tanks are set never to run completely empty during the full operating cycle. However, holding tank must be emptied within the 24-h operating cycle, in order to avoid formation of bacteria producing foul smell. This is given by equation (5.14) where  $h_3^f$  is the final water level in the tank. Inequalities (5.15) and (5.17) initialize the auxiliary variables as the initial state of the respective  $u$  while inequalities (5.16) and (5.18) favour the control that involve less switching frequency of the respective pumps. Finally equations (5.19) and (5.20) are bounds for the control variables, that is, the status of the pumps and switches and the auxiliary variables respectively.



### 5.2.3.1 Open-loop algorithm

In the open-loop control model, the objective function and constraints are solved using the following canonical form [56,57],

$$\min f^T X \quad (5.21)$$

subject to

$$\begin{cases} AX \leq b \text{ (linear inequality constraint),} \\ A_{eq}X = b_{eq} \text{ (linear equality constraint),} \\ L_B \leq X \leq U_B \text{ (lower and upper bounds).} \end{cases} \quad (5.22)$$

Vector  $X$  consists all the control variables in the optimization problem. That is,

$$X = \begin{bmatrix} u_1(1), \dots, u_1(N), u_2(1), \dots, u_2(N), u_3(1), \dots, u_3(N), u_4(1), \dots, u_4(N), \\ s_1(1), \dots, s_1(N), s_3(1), \dots, s_3(N) \end{bmatrix}_{6N \times 1}^T. \quad (5.23)$$

This means that vector  $f^T$  in the canonical form (5.21) can be obtained from the objective function (5.10) as,

$$f^T = \begin{bmatrix} \alpha_1 t_s P_1^m p_e(1), \dots, \alpha_1 t_s P_1^m p_e(N), \alpha_2 t_s Q_2, \dots, \alpha_2 t_s Q_2, \alpha_3 t_s P_3^m p_e(1), \dots, \alpha_3 t_s P_3^m p_e(N), \\ 0, \dots, 0, \alpha_4, \dots, \alpha_4, \alpha_4, \dots, \alpha_4 \end{bmatrix}_{1 \times 6N}. \quad (5.24)$$

The linear inequality constraint (5.11) is transformed to

$$\begin{aligned} A_1 X &\leq b_1, \\ -A_1 X &\leq b_2, \end{aligned} \quad (5.25)$$

such that

$$A_1 = \begin{bmatrix} -Q_1 t_s & 0 & \dots & 0 & Q_2 t_s & 0 & \dots & 0 & 0 & \dots & \dots & 0 \\ -Q_1 t_s & -Q_1 t_s & \dots & 0 & Q_2 t_s & Q_2 t_s & \dots & 0 & 0 & \dots & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\ -Q_1 t_s & -Q_1 t_s & \dots & -Q_1 t_s & Q_2 t_s & Q_2 t_s & \dots & Q_2 t_s & 0 & \dots & \dots & 0 \end{bmatrix}_{N \times 6N}, \quad (5.26)$$

$$b_1 = \begin{bmatrix} -D_{pot}(1) - A_1^t \{h_1^{min} - h_1(0)\} \\ -\{D_{pot}(1) + D_{pot}(2)\} - A_1^t \{h_1^{min} - h_1(0)\} \\ \vdots \\ -\{D_{pot}(1) + \dots + D_{pot}(N)\} - A_1^t \{h_1^{min} - h_1(0)\} \end{bmatrix}_{N \times 1}, \quad (5.27)$$

and

$$b_2 = \begin{bmatrix} D_{pot}(1) + A_1^t \{h_1^{max} - h_1(0)\} \\ \{D_{pot}(1) + D_{pot}(2)\} + A_1^t \{h_1^{max} - h_1(0)\} \\ \vdots \\ \{D_{pot}(1) + \dots + D_{pot}(N)\} + A_1^t \{h_1^{max} - h_1(0)\} \end{bmatrix}_{N \times 1}. \quad (5.28)$$

Similarly, inequality constraint (5.12) is transformed to

$$A_2 X \leq b_3, \quad (5.29)$$

$$-A_2 X \leq b_4,$$

such that

$$A_2 = \begin{bmatrix} 0 & \dots & 0 & -Q_2 t_s & 0 & \dots & 0 & -Q_3 t_s & 0 & \dots & 0 & 0 & \dots & \dots & 0 \\ 0 & \dots & 0 & -Q_2 t_s & -Q_2 t_s & \dots & 0 & -Q_3 t_s & -Q_3 t_s & \dots & 0 & 0 & \dots & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & -Q_2 t_s & -Q_2 t_s & \dots & -Q_2 t_s & -Q_3 t_s & -Q_3 t_s & \dots & -Q_3 t_s & 0 & \dots & \dots & 0 \end{bmatrix}, \quad (5.30)$$

$$b_3 = \begin{bmatrix} -D_{grey}(1) - A_2^t \{h_2^{min} - h_2(0)\} \\ -\{D_{grey}(1) + D_{grey}(2)\} - A_2^t \{h_2^{min} - h_2(0)\} \\ \vdots \\ -\{D_{grey}(1) + \dots + D_{grey}(N)\} - A_2^t \{h_2^{min} - h_2(0)\} \end{bmatrix}, \quad (5.31)$$

and

$$b_4 = \begin{bmatrix} D_{grey}(1) + A_2^t \{h_2^{max} - h_2(0)\} \\ \{D_{grey}(1) + D_{grey}(2)\} + A_2^t \{h_2^{max} - h_2(0)\} \\ \vdots \\ \{D_{grey}(1) + \dots + D_{grey}(N)\} + A_2^t \{h_2^{max} - h_2(0)\} \end{bmatrix}, \quad (5.32)$$

while inequality (5.13) is remodelled to,

$$A_3 X \leq b_5, \quad (5.33)$$

$$-A_3 X \leq b_6,$$

such that,

$$A_3 = \begin{bmatrix} 0 & \dots & \dots & 0 & Q_3 t_s & 0 & \dots & 0 & Q_4 t_s & 0 & \dots & 0 & 0 & \dots & \dots & 0 \\ 0 & \dots & \dots & 0 & Q_3 t_s & Q_3 t_s & \dots & 0 & Q_4 t_s & Q_4 t_s & \dots & 0 & 0 & \dots & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & 0 & Q_3 t_s & Q_3 t_s & \dots & Q_3 t_s & Q_4 t_s & Q_4 t_s & \dots & Q_4 t_s & 0 & \dots & \dots & 0 \end{bmatrix}, \quad (5.34)$$

$$b_5 = \begin{bmatrix} S_{grey}(1) - A_3^t \{h_3^{min} - h_3(0)\} \\ \{S_{grey}(1) + S_{grey}(2)\} - A_3^t \{h_3^{min} - h_3(0)\} \\ \vdots \\ \{S_{grey}(1) + \dots + S_{grey}(N)\} - A_3^t \{h_3^{min} - h_3(0)\} \end{bmatrix}, \quad (5.35)$$

and

$$b_6 = \begin{bmatrix} -S_{grey}(1) + A_3^t \{h_3^{max} - h_3(0)\} \\ -\{S_{grey}(1) + S_{grey}(2)\} + A_3^t \{h_3^{max} - h_3(0)\} \\ \vdots \\ -\{S_{grey}(1) + \dots + S_{grey}(N)\} + A_3^t \{h_3^{max} - h_3(0)\} \end{bmatrix}. \quad (5.36)$$

Finally, auxiliary variables in inequalities (5.15)-(5.18) are remodelled as

$$A_4 X \leq b_7 \quad (5.37)$$

where,

$A_4 =$

$$\begin{bmatrix} 1 & 0 & \dots & 0 & 0 & | & 0 & \dots & 0 & 1 & 0 & \dots & 0 & 0 & | & 0 & \dots & 0 & -1 & 0 & \dots & 0 & | & -1 & 0 & \dots & 0 \\ -1 & 1 & \dots & 0 & 0 & | & 0 & \dots & 0 & -1 & 1 & \dots & 0 & 0 & | & 0 & \dots & 0 & 0 & -1 & \dots & 0 & | & 0 & -1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & | & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & | & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & | & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & -1 & 1 & | & 0 & \dots & 0 & 0 & 0 & \dots & -1 & 1 & | & 0 & \dots & 0 & 0 & 0 & \dots & -1 & | & 0 & 0 & \dots & -1 \end{bmatrix}, \quad (5.38)$$

and

$$b_7 = \begin{bmatrix} 0 & \dots & 0 \end{bmatrix}^T. \quad (5.39)$$

Matrices  $A_1$  to  $A_4$  have  $(N \times 6N)$  dimension while vectors  $b_1$  to  $b_7$  have a dimension of  $(N \times 1)$ .

Therefore, linear inequality in the canonical form (5.22) becomes,

$$\begin{bmatrix} A_1 \\ -A_1 \\ A_2 \\ -A_2 \\ A_3 \\ -A_3 \\ A_4 \end{bmatrix}_{7N \times 6N} X = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix}_{7N \times 1}. \quad (5.40)$$

In the same manner, linear equality constraint (5.14) becomes,

$$A_{eq} X = b_{eq}, \quad (5.41)$$

where

$$A_{eq} = \begin{bmatrix} 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 & | & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & | & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 & | & 0 & \dots & 0 \\ 0 & \dots & 0 & Q_{3t_s} & \dots & Q_{3t_s} & Q_{4t_s} & \dots & Q_{4t_s} & 0 & \dots & 0 & | & 0 & \dots & 0 \end{bmatrix}_{N \times 6N}, \quad (5.42)$$

and

$$b_{eq} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ A_3^t \{h_3(0) - h_3^f\} + \{S_{grey}(1) + \dots + S_{grey}(j)\} \end{bmatrix}_{N \times 1}, \quad (5.43)$$

while the bounds given in equations (5.19) and (5.20) become,

$$L_B = \begin{bmatrix} 0 & \dots & 0 \end{bmatrix}_{6N \times 1}^T \quad \text{and} \quad U_B = \begin{bmatrix} 1 & \dots & 1 \end{bmatrix}_{6N \times 1}^T. \quad (5.44)$$

This binary integer optimization problem is solved using the SCIP solver in OPTI toolbox, a free Matlab optimization toolbox. This solver is used as it is reported as the fastest non-commercial optimization solver [70, 71].

#### 5.2.4 Closed-loop control model

The closed-loop model predictive control (MPC) strategy is formulated in this chapter due to its predictive nature, ability to cope with constraints in the design process and the ability to deal with disturbances that are always there in any system, whether external or errors within the system [225]. The closed-loop MPC uses both the feed forward and feed back measurements from the system to compute the control law on-line [20].

The control and state variables in this strategy are the same as those for the open-loop controller. Denoting the control variables as  $u_m$ , with  $m = 1, 2, 3, 4$ ,  $s_1$  and  $s_3$ , the objective function encompassing the previously listed performance index for the closed-loop controller,  $J_{mpc}$ , is derived from the open-loop objective (5.10) as,

$$J_{mpc} = \sum_{j=k}^{k+N_c-1} \left[ \alpha_1 t_s p_e(j) P_1^m u_1(j|k) + \alpha_2 t_s Q_2 u_2(j|k) + \alpha_3 t_s p_e(j) P_3^m u_3(j|k) \right] + \alpha_4 \sum_{j=k}^{k+N_c-1} \left[ s_1(j|k) + s_3(j|k) \right], \quad (5.45)$$

where  $N_c$  is the control horizon,  $u_1(j|k)$ ,  $u_2(j|k)$  and  $u_3(j|k)$  are optimized control actions while  $s_1(j|k)$  and  $s_3(j|k)$  are auxiliary values at  $j^{th}$  sampling interval according to the most recent measurement at time  $k$ . Normally, MPC problems include both predicting,  $N_p$ , and control,  $N_c$ , horizons. However, since none of the state variables (height of water in the tank) is included in the objective function, this MPC problem does not include the predicting horizon,  $N_p$ . The control horizon is therefore given

as

$$N_c = N - k + 1. \quad (5.46)$$

The state equations are modified from equations (5.3), (5.6) and (5.9) to,

$$h_1(j|k) = h_1(k) + \frac{t_s}{A_1^t} \sum_{i=k}^j \left[ Q_1 u_1(i|k) - Q_2 u_2(i|k) \right] - \frac{1}{A_1^t} \sum_{i=k}^j D_{pot}(i), \quad (5.47)$$

$$h_2(j|k) = h_2(k) + \frac{t_s}{A_2^t} \sum_{i=k}^j \left[ Q_2 u_2(i|k) + Q_3 u_3(i|k) \right] - \frac{1}{A_2^t} \sum_{i=k}^j D_{grey}(i), \quad (5.48)$$

$$h_3(j|k) = h_3(k) + \frac{1}{A_3^t} \sum_{i=k}^j S_{grey}(i) - \frac{t_s}{A_3^t} \sum_{i=k}^j \left[ Q_3 u_3(i|k) + Q_4 u_4(i|k) \right], \quad (5.49)$$

$$k \leq j \leq k + N_c - 1.$$

Here,  $h_1(j|k)$ ,  $h_2(j|k)$  and  $h_3(j|k)$  are the predicted values of the height of water in respective tanks at  $j^{th}$  sampling interval based on information available at time  $k$ . Additionally, physical and operational constraints are similar to those discussed and mathematically modelled in constraints and equations (5.11)-(5.20), with the following modifications,

$$h_1^{min} \leq h_1(j|k) \leq h_1^{max}, \quad (5.50)$$

$$h_2^{min} \leq h_2(j|k) \leq h_2^{max}, \quad (5.51)$$

$$h_3^{min} \leq h_3(j|k) \leq h_3^{max}, \quad (5.52)$$

$$h_3(N) = h_3^f, \quad (5.53)$$

$$u_1(1|k) - s_1(1|k) \leq 0, \quad (5.54)$$

$$u_1(j|k) - u_1(j-1|k) - s_1(j|k) \leq 0, \quad (5.55)$$

$$u_3(1|k) - s_3(1|k) \leq 0, \quad (5.56)$$

$$u_3(j|k) - u_3(j-1|k) - s_3(j|k) \leq 0, \quad (5.57)$$

$$u_{m,c}(j|k) \in \{0, 1\} \text{ where } m = 1, 2, 3, 4, \quad (5.58)$$

$$s_1(j|k), s_3(j|k) \in \{0, 1\}. \quad (5.59)$$

At a particular time,  $k$ , the controller solves an open-loop optimization problem for  $N_c$  horizon. Only the first element of each of the control variables  $u_m(j|k)$ ,  $s_1(k|k)$  and  $s_3(j|k)$  is implemented to the plant. The states (heights of water in respective tanks,  $h_m(j|k)$ ) is measured and fed back to the controller to be used as the initial heights during the next time step,  $k+1$ . Other input variables are also updated and the optimization continues up to a predetermined operating cycle.

### 5.2.4.1 Closed-loop algorithm

Closed-loop MPC obtains the current control action by solving, in each sampling time, a finite horizon open-loop optimal control problem using the current state of the plant as the initial state. The optimization yields an optimal control sequence and the first control in this sequence is applied to the plant. This process is repeated throughout the entire control period [20]. Using the principle of the receding horizon control in closed-loop MPC, only the first element in the control vector  $X^{mpc}$  is implemented after each iteration, ignoring the rest of the elements [158]. The state of the plant (water level in the tanks) is measured. During the next iteration,  $k + 1$ , the objective function and the constraints are updated while taking the previous state of the tanks (water level at sampling time  $k$ ) as the initial state. The process of optimization is carried out in real time over the new control horizon ( $N_c = N - k + 1$ ) to give the receding horizon control law. Similar to the open-loop control algorithm, the control vector,  $X^{mpc}$ , contains the control variables such that,

$$\begin{aligned}
 X^{mpc} = & \\
 & \left[ u_1(k|k), u_1(k+1|k), \dots, u_1(k+N_c-1|k), u_2(k|k), u_2(k+1|k), \dots, u_2(k+N_c-1|k), \right. \\
 & u_3(k|k), u_3(k+1|k), \dots, u_3(k+N_c-1|k), u_4(k|k), u_4(k+1|k), \dots, u_4(k+N_c-1|k), \\
 & \left. s_1(k|k), s_1(k+1|k), \dots, s_1(k+N_c-1|k), s_3(k|k), s_3(k+1|k), \dots, s_3(k+N_c-1|k) \right]_{6N \times 1}^T.
 \end{aligned} \tag{5.60}$$

The work flow of the MPC controller is as follows [197];

1. For time,  $k$ , find the control horizon ( $N_c(k)$ ) using equation (5.46).
2. *Optimization*: Find the optimal solution within the control horizon;

minimize objective function (5.45),

subject to constraints (5.50)-(5.59).

3. From the optimal solution, implement  $[u_1(k|k), u_2(k|k), u_3(k|k), u_4(k|k)]^T$  to the plant.
4. *Feed back*: Measure state variables  $h_1(k+1)$ ,  $h_2(k+1)$  and  $h_3(k+1)$ .

5. Set  $k = k + 1$  and update system states and inputs and outputs.
6. Repeat steps 1-5 until  $k$  reaches a predefined value.

This binary integer optimization problem solved using the SCIP solver in OPTI toolbox.

### 5.2.5 Effect of monthly water block tariff

The control systems are run over a 24-h operating cycle as the demand pattern is assumed to be repeated over this cycle, though different for week days and days of the weekend. However, since water is priced monthly using the block tariff in Table 5.2, it is important to investigate the effect increasing price of water has on the optimal operation of the system. Both open-loop and MPC controllers are run for a month, by considering each week to have 5 weekdays and 2 days of the weekend. In this study, the weekday water demand profile,  $D_{pot}(weekday)$ , is assumed to be the same for all the 5 week days, while the weekend demand profile,  $D_{pot}(weekend)$ , is also same for the 2 days of the weekend. Further, the first day of the month is taken as a Monday, and the month has exactly four weeks (28 days). Therefore, the cumulative volume of water consumed up to a certain weekday,  $D_{pot,wkdy}$ , or a weekend,  $D_{pot,wknd}$ , is obtained as;

$$D_{pot,wkdy} = (5q)D_{pot}(weekday) + (2q - 2)D_{pot}(weekend), \quad (5.61)$$

$$D_{pot,wknd} = (5q)D_{pot}(weekday) + (2q - 1)D_{pot}(weekend),$$

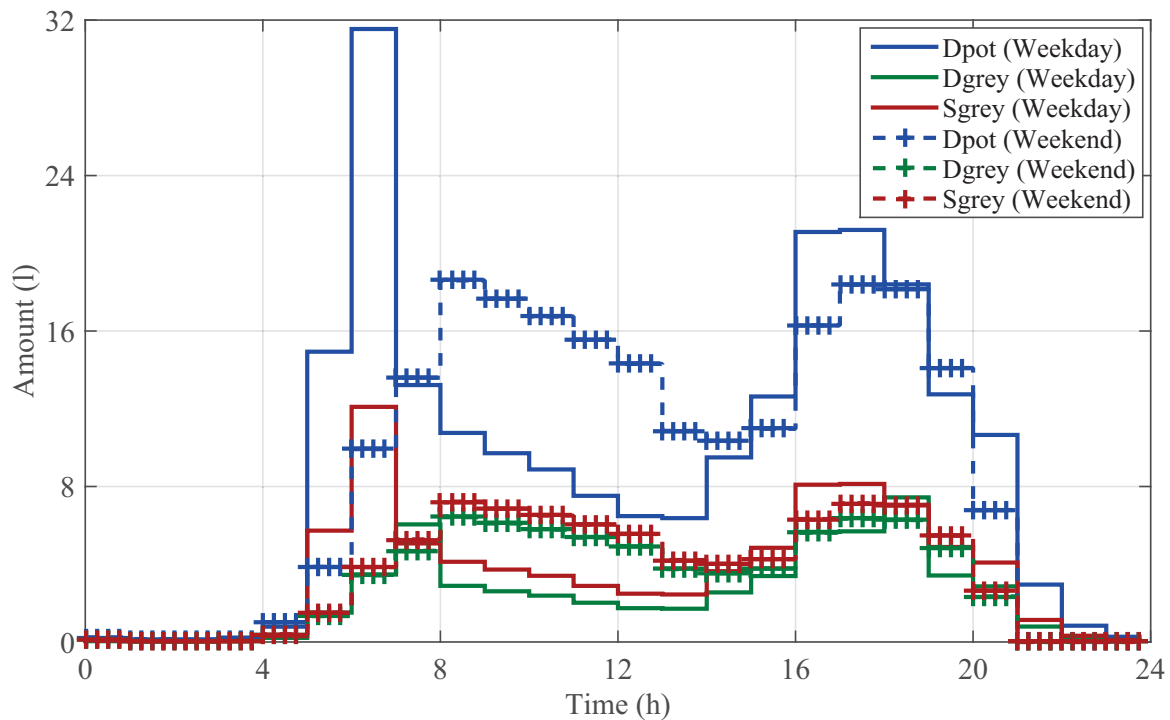
where  $q$  is the number of the week in the month ( $q = 1, 2, 3, 4$ ). This volume is then used to compute the cost of water.

## 5.3 GENERAL DATA

### 5.3.1 Case study

A house in Pretoria, which is forced to pump and store the water due to the unreliability of municipal water supply was studied. The water consumption and energy associated with pumping water in this house is used as the baseline, as the house uses only potable water for all its end uses. The pump, rated as 0.8 kW and 0.75 m<sup>3</sup>/h, is controlled by level switches that just detect empty and full levels.

Whenever the tank is empty, the pump switches on until the tank is full, regardless of TOU period. Water then flows from the potable water tank to the end uses through gravity. Various end uses were categorized to enable identify those that could be used for water recycling and those that could use treated grey water. Hourly water demand pattern of these uses was measured using digital flow meters connected with data loggers. Therefore, the hourly water demand for a typical week day and a weekend is shown in Figure 5.2. The week day water demand has a high peak in the morning and evening



**Figure 5.2.** Hourly water profile for a typical week day and weekend.

caused by the house occupants preparing to leave the house and coming back respectively. However, during the weekend, the peak demand occurs later than during week days, and remains relatively high during day time as occupants carry on with weekend's chores throughout the day time. It can be seen from the curves that grey water supply,  $S_{grey}$ , is always less than potable water demand,  $D_{pot}$ , as some of this potable water qualifies to be recycled. On the contrary, grey water demand,  $D_{grey}$ , does not necessarily follow the others, as this demand entirely depends on occupants' behaviour.

The cylindrical potable water tank has the dimensions given in Table 5.1. In order to incorporate grey water recycling, two tanks; grey and holding water tanks would be required and their dimensions and capacity constraints are given in Table 5.1. In this chapter, level sensors would be placed in all tanks to limit the water level between minimum and maximum water levels shown in Table 5.1. This is meant



**Table 5.1.** Dimensions and capacity of the tanks.

Tank	Radius (m)	Height (m)	Min	Max
Potable	0.55	1.2	0.1	1.0
Grey	0.36	1.0	0.1	0.8
Holding	0.30	0.6	0	0.5

to avoid either running the tank completely empty or spilling the water in the tank hence damaging the roof of the house. Only the holding tank is allowed to run completely empty so as to prevent formation of foul smell. The grey water pump to be incorporated would be rated at 650 W with flow rate of  $0.35 \text{ m}^3/h$ .

### 5.3.2 Time-of-use electricity tariff

The time-of-use (TOU) electricity tariff is commonly used globally to encourage shifting of peak load [48] and it can vary by time of day, day of week and season [160]. Eskom's TOU Homeflex structure for residential consumers given below is used [70].

$$p_e(t) = \begin{cases} p_{off} = 0.5510 \text{ R/Kwh} & \text{if } t \in [0, 6] \cup [10, 18] \cup [20, 24,] \\ p_{peak} = 1.748 \text{ R/Kwh} & \text{if } t \in [7, 10] \cup [18, 20], \end{cases} \quad (5.62)$$

where  $p_{off}$  is the off peak price,  $p_{peak}$  is the peak time price,  $R$  is the South African currency, Rand, and  $t$  is the time of day in hours.

### 5.3.3 Water tariffs

Table 5.2 shows the water tariffs for domestic consumers in the city of Tshwane [147].

**Table 5.2.** City of Tshwane water tariff for 2014/2015

Volume ( $\text{m}^3/\text{month}$ )	0-6	7-12	13-18	19-24	25-30	31-42	43-72	>72
Rates ( $R/\text{m}^3$ )	6.81	9.72	12.77	14.77	16.89	18.25	19.53	20.91

### 5.3.4 Uncertainty analysis

Uncertainty or error analysis is required to determine the confidence level of measurements carried out. In this case, the analysis of measured water demand data is carried out using the approach taken by Sichilalu and Xia [68]. Random and instrument's error are assumed to affect the measurements. Random errors are generated in MATLAB software with a distribution mean and standard deviation of 0 and 1 respectively while the instrument's absolute uncertainty of  $\pm 0.01$  is provided by the manufacturer. The measured value,  $S_{meas}$ , is therefore given as,

$$S_{meas} = S_{actual} + (Err_{random} \times Err_{inst}), \quad (5.63)$$

where  $Err_{random}$  and  $Err_{inst}$  are the random and instrument errors respectively, while  $S_{actual}$  is the true value. The relative error,  $Err_{relative}$ , is then obtained as,

$$Err_{relative} = \frac{Err_{eff}}{S_{meas}} \%. \quad (5.64)$$

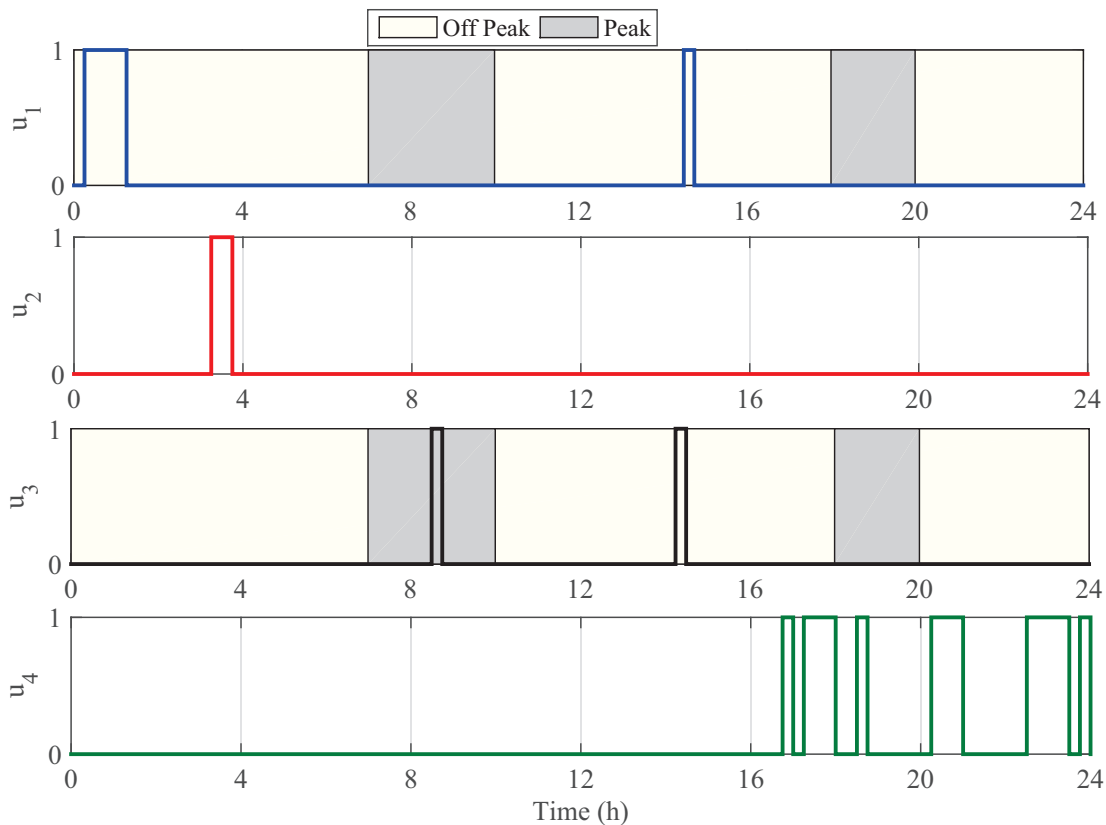
From rule of the weakest link, the measurement with the largest relative error is used to determine the final absolute error of the performance index [226], which is the cost of energy and water in this case.

## 5.4 SIMULATION RESULTS AND DISCUSSION

The simulation results for the two control strategies are discussed below. Simulations for both open-loop and closed-loop MPC control systems are done over a 24-h operating cycle, with the sampling period,  $t_s = 15$  minutes. For both controllers, simulations are done for a weekday, a weekend and finally for one month to investigate the effect of increasing water tariff.

### 5.4.1 Open-loop optimal control system

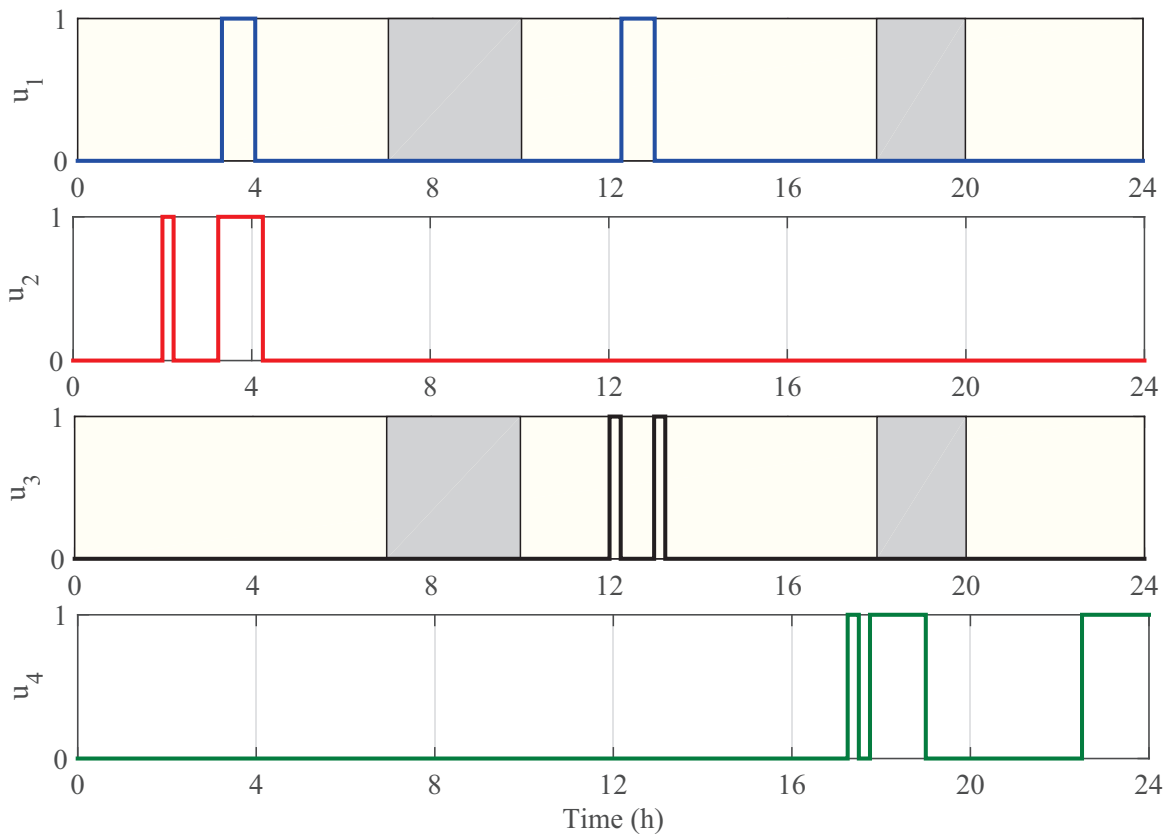
The open-loop optimal operation of the grey water recycling system in a weekday, which mainly involves switching of pumps and solenoid valves is shown in Figure 5.3. The legend showing peak and off peak periods of the TOU tariff is used throughout the chapter. Moreover, only pumps, whose status are represented by  $u_1$  and  $u_3$ , are considered to consume power hence subjected to the TOU tariff as solenoid valves  $u_2$  and  $u_4$  consume negligible amount of energy. It can be seen that the optimal controller seeks to operate both pumps during off-peak TOU period, effectively shifting the load as



**Figure 5.3.** Open-loop optimal switching for a weekday.

desired, while meeting the household potable and grey water demand. However, grey water pump is operated during the morning TOU peak for 15 minutes, due to increased grey water demand in the same period and there is sufficient grey water collected in the holding tank. The open-loop controller switches both pumps twice during the 24-h operating cycle. This is in line with the objective that seeks to also minimize the maintenance cost of the pumps, which is normally represented as the number of pumps' switching taking place. Frequent switching destroys a pump's motor as it tries to overcome dead load (water) while changing from off to on status. Valves are however allowed to switch any number of times. The open-loop controller allows the use of potable water for grey uses early in the morning by opening the potable water valve,  $u_2$ , as it awaits more grey water to be collected so that it can be pumped into the grey water tank.

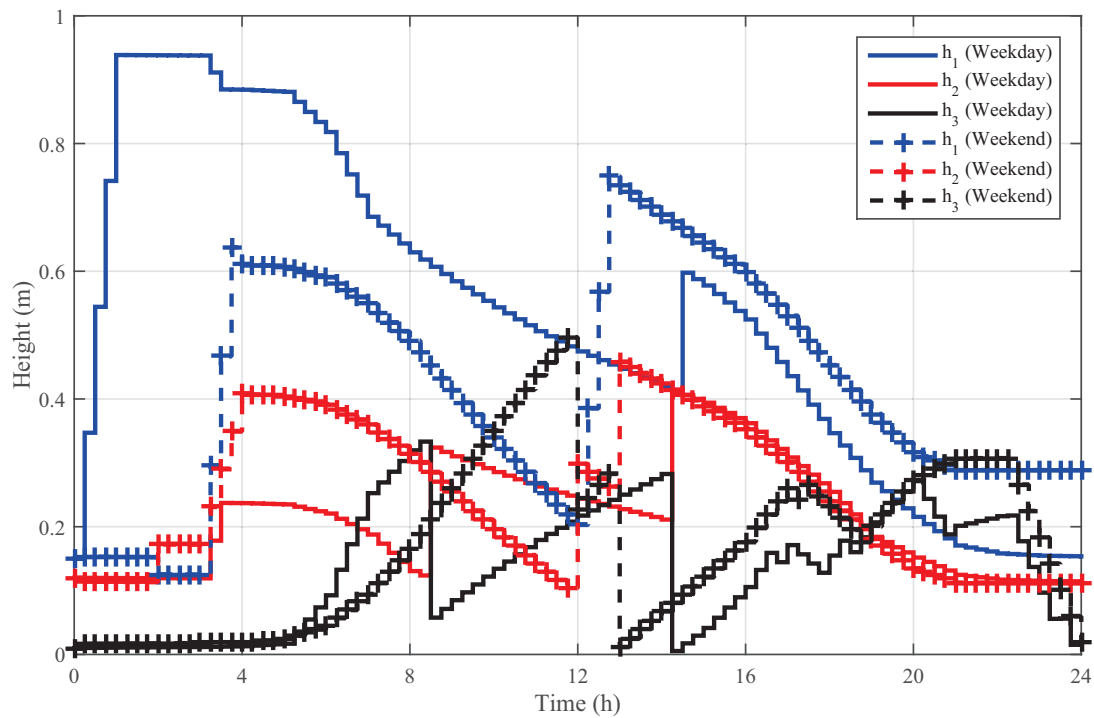
The optimal operation for a weekend is shown in Figure 5.4. Unlike in the weekday optimal operation, the controller manages to operate the pumps during the cheaper off-peak TOU period. Since insufficient grey water has been collected by early morning, the controller is forced to use potable water, through valve  $u_2$ , to ensure that grey water demand is met. Thereafter, the required grey water is pumped into



**Figure 5.4.** Open-loop optimal switching for a weekend.

the grey water tank to meet the demand. Both pumps are only switched twice throughout the operating cycle as desired.

Optimal operation of the pumps and valves in a weekday and weekend leads to water variation in the respective tanks as shown in Figure 5.5. In all tanks, none of the constraints is violated throughout the 24-h operating cycle. During the weekday, the controller predicts that water available in the potable water tank isn't sufficient to meet potable water demand, necessitating switching on potable water pump for 1-h in the beginning of the operating cycle. This leads to a rise in water level,  $h_1$ , in the potable water. Thereafter, the level of this water drops to meet the potable water demand in the house until 14:30 h when the pump switches on again for 15 minutes. This amount of water is sufficient to meet the potable water demand for the remaining period of the 24-h operating cycle. In the same day, the height of water in the grey water tank,  $h_2$ , is mainly reducing due the grey water demand in the house. The water level rises at 03:15 when the potable water valve,  $u_2$ , is opened for 30 minutes. Thereafter, the level declines as grey water demand increases until 08:30 when the controller realises



**Figure 5.5.** Water height variation in respective tanks with open-loop controller.

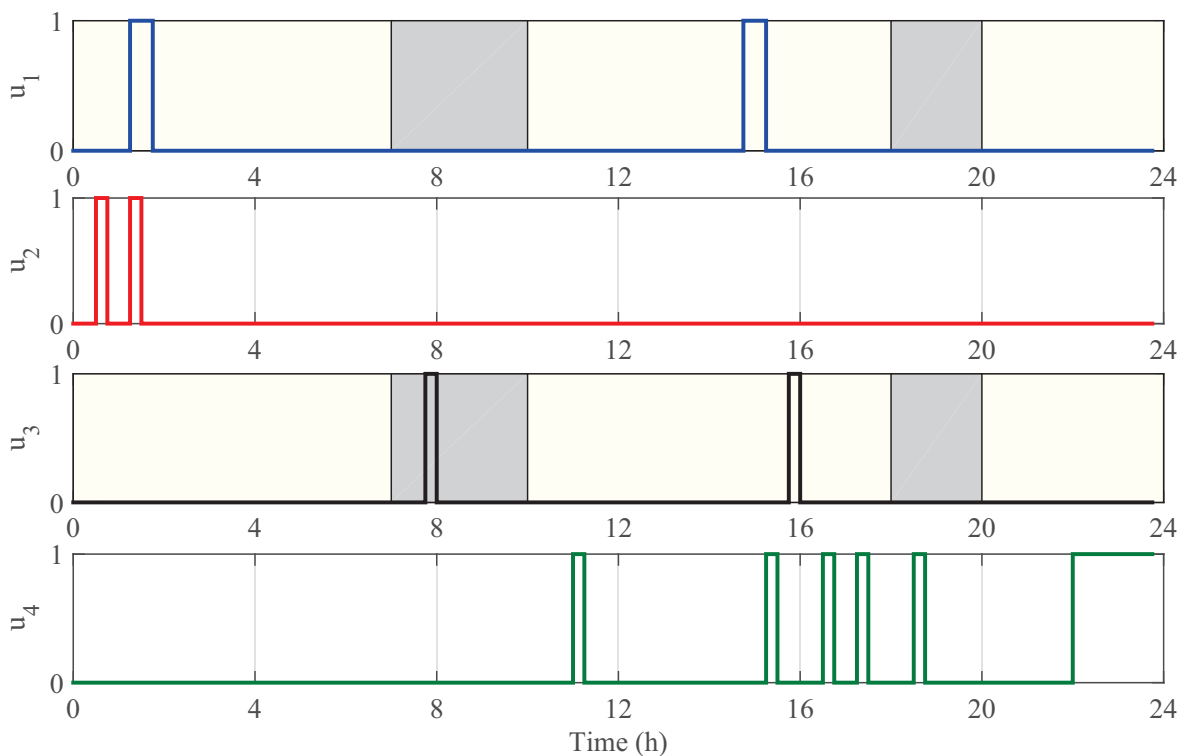
that holding tank has enough grey water collected while the grey water tank is at the risk of running dry. Therefore, grey water pump is switched on for 15 minutes leading to simultaneous rise in water level  $h_2$  and drop in  $h_3$ . This is only experienced again at 14:15 when grey water tank needs more grey water, which is sufficient to meet the remaining duration of the operating cycle. Once grey water tank has sufficient water, the controller then opens drainage valve,  $u_4$  to release grey water being collected, so as to ensure the tank is emptied by the end of the 24-h operating cycle.

In a weekend, the optimal controller ensures that both pumps are operated during the off-peak TOU period. Water level in potable tank,  $h_1$  rises at 03:15 when the potable water pump is switched on for 45 minutes. Subsequently, this water level starts to decline as the water is used to meet potable water demand until 12:15 when the pump is again switched on for another 45 minutes. This water is sufficient to meet the remaining potable water demand. During the weekend, however, the grey water demand,  $D_{grey}$ , increases almost at the same rate as the amount of grey water being collected,  $S_{grey}$ . The controller is therefore forced to use potable water through valve,  $u_2$ , between 02:00-04:15 to meet the grey water demand. This causes a rise in the water level  $h_2$  in the grey water tank which is used to meet grey water demand until 12:00. The controller then operates grey water pump twice

for 15 minutes each at 12:00 and 13:00 causing a simultaneous rise in  $h_2$  and drop in  $h_3$  as water is transferred from holding to grey water tank to meet the demand for the remaining duration of operating cycle. It is not possible switch on grey pump just once. The controller switches it on when grey tank has minimum amount of water but before it is full, the holding tank is emptied necessitating switching off of the pump. When more water is collected, only sufficient amount is pumped to meet the rest of the demand. At 17:15, the controller opens drainage valve  $u_4$  severally to ensure the grey water being collected is allowed to flow to the drainage as it is no longer required.

### 5.4.2 Closed-loop MPC system

Optimal operation of the grey water recycling system obtained while using the closed-loop MPC for a typical weekday is shown in Figure 5.6. Similar to the weekday open-loop operation, the pumps

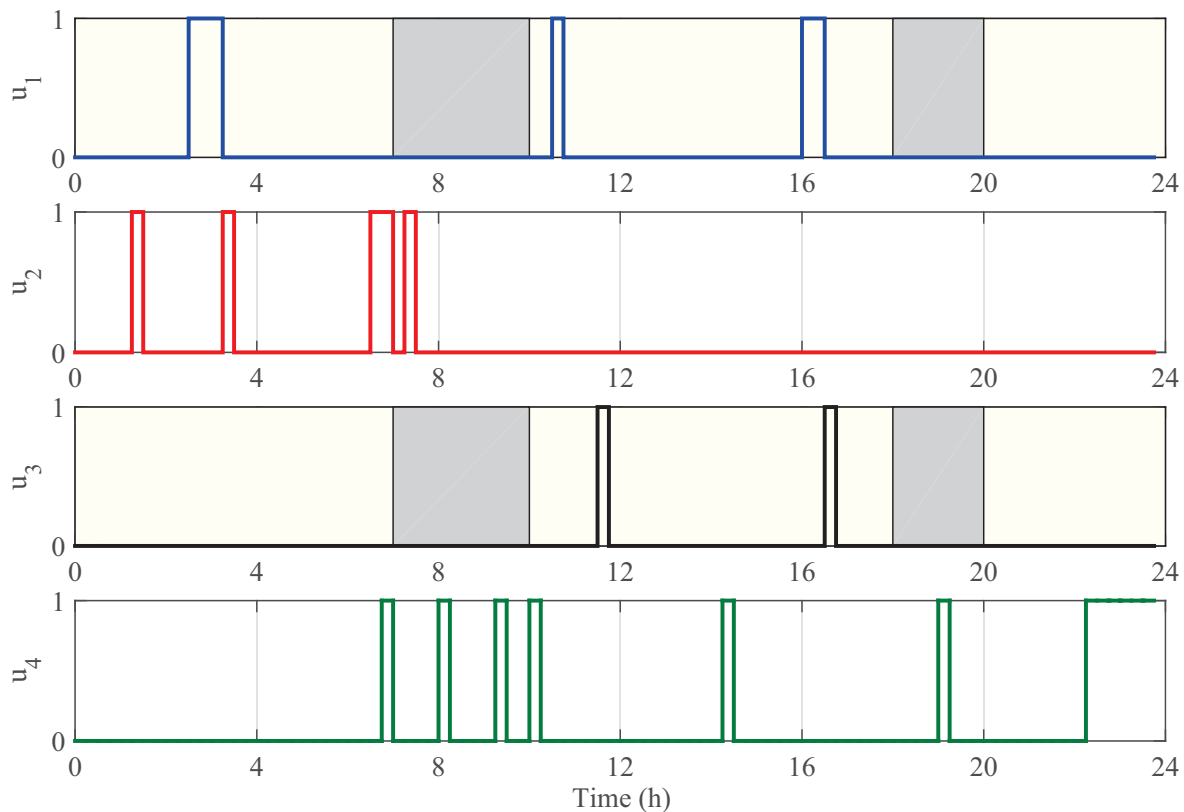


**Figure 5.6.** Week day closed-loop optimal switching.

operate twice each throughout the 24-h operating cycle. Potable water pump operates in the off-peak TOU periods in meeting the potable water demand while grey water pump operates in the morning TOU peak for 15 minutes due to rising grey water demand and sufficient amount of grey water has been collected in the holding tank. Before operating the grey water pump in the morning peak, the

controller allows potable water to be used in grey water uses while waiting for sufficient grey water to be collected in the holding tank. Further, the controller operates the drainage valve severally in the afternoon in order to ensure that the holding tank is emptied by the end of the operating cycle.

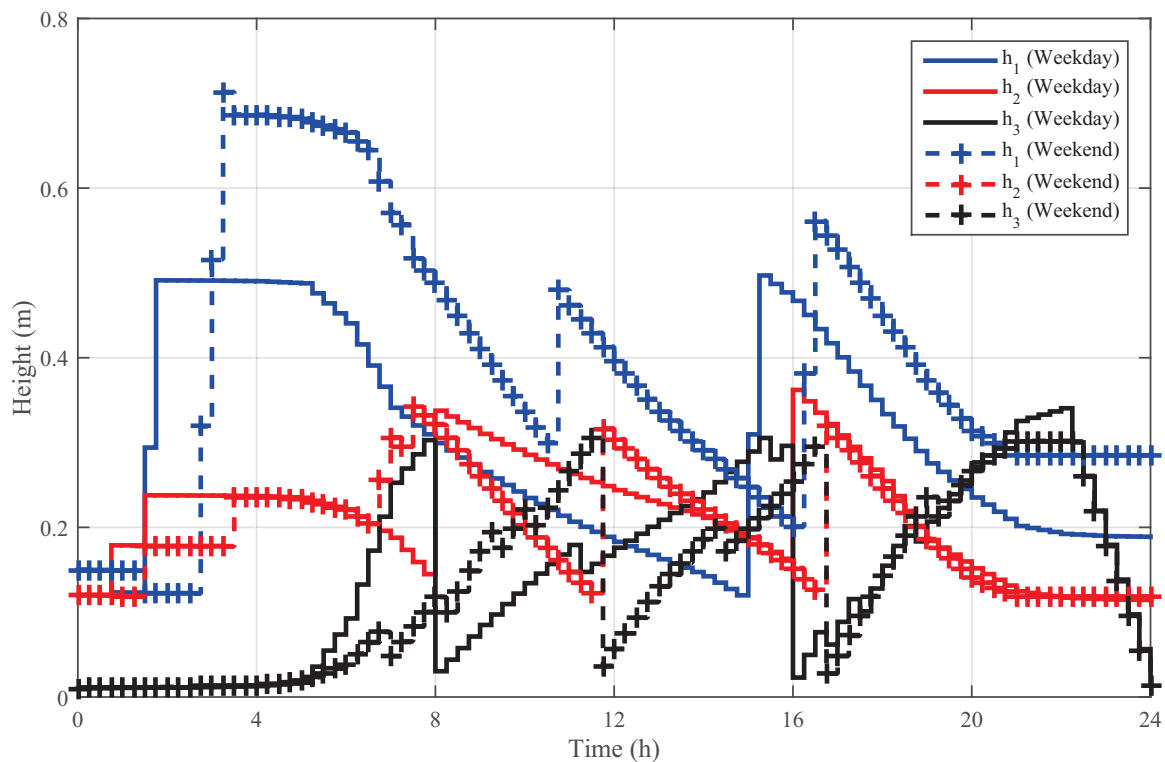
Optimal operation of the system for a typical weekend is shown in Figure 5.7. The controller operates



**Figure 5.7.** Closed-loop optimal switching for a weekend.

both pumps in the off-peak TOU periods as desired. Potable water pump is switched on three times while grey water pump operates twice. Just like the open-loop controller during the weekend, the MPC uses potable water for grey water purposes in the morning as it awaits sufficient water to be collected in the holding tank. Further the controller ensures that the holding tank is empty by the end of the control horizon by opening drainage valve,  $u_4$ , whenever necessary.

The operation of the system using MPC controller for both weekday and weekend leads to variation of water levels in various tanks as shown in Figure 5.8. Water levels are maintained within the prescribed maximum and minimum levels in all tanks. In the weekday, water level in potable water tank,  $h_1$ , rises at 01:15 when the controller switches on the potable water pump for 30 minutes. The level,



**Figure 5.8.** Water height variation using MPC controller.

subsequently, decreases while meeting the potable water demand until 14:45 when the pump is again switched on for another 30 minutes. This water in the tank is sufficient to meet potable water demand for the remaining duration. Since the holding tank is empty, the controller opens potable water valve,  $u_2$ , at 00:30 and 01:15 for 15 minutes each to meet the expected grey water demand. Consequently, water level,  $h_2$ , rises and this water is used to meet grey water demand until 07:45 when the holding tank has sufficient water. This makes the controller switch on the grey water pump for 15 minutes, resulting in simultaneous increase and drop of water levels  $h_2$  and  $h_3$  respectively. This event only happens again at 15:45 when the grey water pump is operated again for another 15 minutes, and thereafter, the controller predicts that treated grey water in the grey water tank is sufficient to meet the remaining grey water demand. Since more grey water is collected, the controller keeps opening the drainage water valve, especially towards the end of the operating cycle in order to ensure the holding tank.

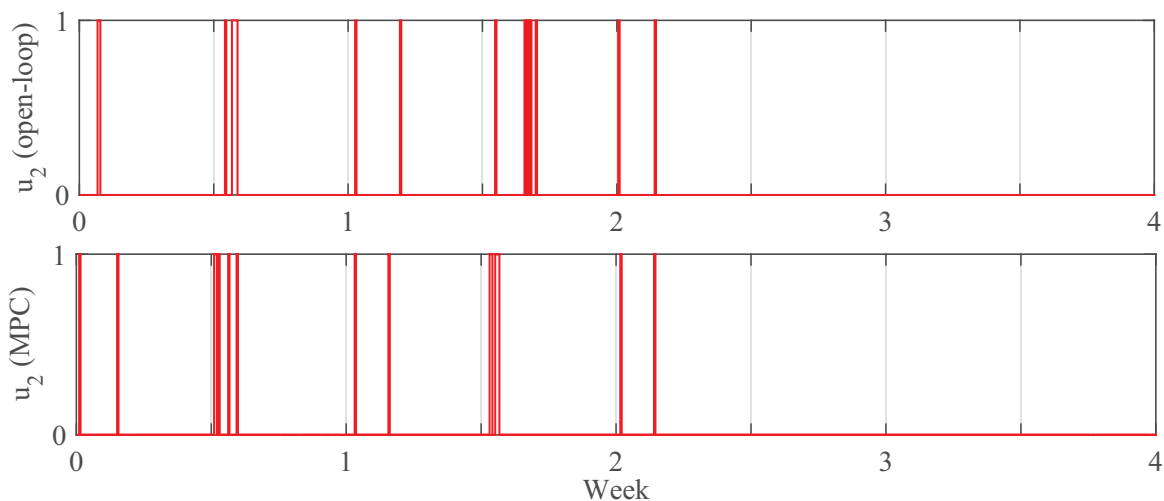
During the weekend, water level in potable water tank,  $h_1$ , rises when the MPC controller switches on the potable water pump at 02:30 for 45 minutes. This water then declines as it is used up in meeting potable water demand until 10:30 when the pump is again switched on for 15 minutes. This causes an increase in the water level, which is used up until 16:00 when the pump is switched on for 30



minutes making the water level to rise again. The controller predicts that this water is sufficient for the remaining period of the operating cycle. Moreover, the MPC controller has to use potable water through potable water valve,  $u_2$ , in the early morning so as to ensure grey water tank has sufficient water to meet grey water demand, while waiting for grey water to be collected in the holding tank. This leads to the increase in water level in grey water tank between 01:15-07:30, which is sufficient to meet grey water demand until 11:30. Afterwards, grey water pump is operated for 15 minutes leading to a simultaneous rise and drop in  $h_2$  and  $h_3$  respectively. This occurs again at 16:30 for another 15 minutes, and thereafter, the controller predicts that the water in grey water tank is sufficient to meet grey water demand for the remaining period. The MPC controller keeps opening the drainage water valve for short intervals, which causes the water level,  $h_3$ , to fall but it eventually opens the valve at 22.15 till the end to ensure the tank is empty as required.

### 5.4.3 Effect of monthly water block tariff

The open-loop and MPC optimal schedules of potable water flowing through valve  $u_2$ , for a month are shown in Figure 5.9. Both open-loop and MPC controllers open  $u_2$  for the first two weeks, and



**Figure 5.9.** Optimal use of potable water in grey uses in a month.

the weekday of the third week. In the first and second week, both controllers open  $u_2$  twice and 5 times during the weekday and weekend respectively. However, in the third week, they open  $u_2$  twice during the weekday but none on the weekend and following weeks.  $u_2$  remains off due to a very high weight being given to the cost of water minimizing term in the objective function which was

increasing as the cost of water increased. Table 5.3 shows the consumption of potable water in the house during the month, as well as use of potable water for grey water uses. The baseline and potable

**Table 5.3.** Comparison of weekly water consumption.

Wk	Day	Baseline		Potable water		$\sum Q_{2t_s} u_2 (m^3)$		Grey water ( $m^3$ )	
		Amount ( $m^3$ )	Price( $R/m^3$ )	Amount ( $m^3$ )	Price( $R/m^3$ )	Open-loop	MPC	Open-loop	MPC
1	Weekday	5.80	6.81	4.47	6.81	0.05	0.05	0.18	0.18
	Weekend	8.14	9.72	6.34	9.72	0.13	0.13	0.18	0.18
2	Weekday	13.94	12.77	10.81	9.72	0.05	0.05	0.18	0.18
	Weekend	16.29	12.77	12.68	12.77	0.13	0.13	0.18	0.18
3	Weekday	22.09	14.77	17.15	12.77	0.05	0.05	0.18	0.18
	Weekend	24.43	16.89	18.89	14.77	0	0	0.26	0.26
4	Weekday	30.23	18.25	23.31	14.77	0	0	0.26	0.26
	Weekend	31.61	18.25	24.18	16.89	0	0	0.26	0.26

water columns show the cumulative amount of water used and the unit price during the month. The weekday cumulative potable water is the amount of potable water used in the house at the end of 5 week days, while the weekend's is the amount consumed at the end of the 2 days of the weekend. Further,  $\sum Q_{2t_s} u_2 (m^3)$  is the amount of potable water used to supplement the grey water uses through valve  $u_2$  in each period. Since more potable water is used in the baseline than while using grey water with control strategies, its unit cost increases faster as weeks go by. By the end of the month, the consumer pays  $18.25 R/m^3$  as opposed to  $16.89 R/m^3$  charged while recycling grey water. This means that consumers will have the added benefit of lower cost of potable water in addition to conserving it. Grey water recycling system operated by either open-loop or MPC control strategy use  $0.05 m^3$  and  $0.13 m^3$  of potable water for grey uses in a week day and weekend, respectively, during the first two weeks. Thereafter,  $0.05 m^3$  is used during the week day of the third week. Up to this point, the cost of water has risen to  $12.77 R/m^3$ . However, when the price reaches  $14.77 R/m^3$ , during the weekend of the third week, both controllers do not use potable water for grey end uses. This is caused by weight of the term responsible for minimizing the cost of water in objective functions (5.10) and (5.45) increasing significantly, such that both controllers give this term more preference as compared to other terms. Finally, the increasing weighting factor leads to an increase in use of grey water from  $0.18 m^3$  to  $0.26 m^3$  for both controllers.

#### 5.4.4 Discussion

Table 5.4 shows monthly water and associated energy consumption together with their associated costs. Given that the baseline has unreliable potable water which is used for all household end uses,

**Table 5.4.** Water and energy consumption using open-loop and MPC controllers

	Baseline	Open-loop	MPC
<b>Potable water</b>			
Amount ( $m^3/month$ )	31.61	24.18	24.18
Cost ( $R/month$ )	395.15	267.46	267.46
<b>Potable pump</b>			
Energy ( $kWh/month$ )	13.04	8.00	7.80
Cost ( $R/month$ )	14.33	5.84	6.54
<b>Grey pump</b>			
Energy( $kWh/month$ )	0 <sup>a</sup>	3.25	3.25
Cost ( $R/month$ )	0 <sup>a</sup>	3.37	3.37
<b>Total cost (<math>R/month</math>)<sup>b</sup></b>	<b>409.48</b>	<b>276.67</b>	<b>277.37</b>

<sup>a</sup> The household was only using potable water.

<sup>b</sup> Cost of water and pumping energy.

about  $31.61 m^3/month$  is required to meet the overall demand, costing  $409.48 R/month$  (inclusive of pumping energy cost). This amount meets both potable and grey water demands, meaning that recyclable water is effectively wasted. With grey water recycling system in place, which is operated by either open-loop or MPC controllers, the amount of potable water used in a month reduces by about 23.5%. At the same time, open-loop and MPC controllers can save the cost of pumping energy by up to 59.2% and 54.3% respectively, through load shifting. The energy consumed by the pumps while using both controllers reduced by 38.7%, which is attributed to the use of a lower power rated grey water pump that ends up using less power than the potable water pump used in the baseline case. Therefore, open-loop and MPC controllers lead to overall cost saving of 32.5% and 32.3% respectively. The MPC

controller incurs slightly more cost of energy than open-loop controller as MPC controller does not give the solution at global minimum as open-loop optimal controller does.

With reliable potable water supply, the baseline would not require pumping and storing hence the total cost would be 395.15 *R/month*. Therefore, the grey water system with both controllers in such a house would incur a total cost (potable water and grey water pumping) of 270.83 *R/month*, which is a cost saving of 31.5%. This means that optimal operation of grey water recycling has both conservation and economic benefits whether there is reliable potable water supply or not. If widely adopted, these savings would be of immense benefit to both energy utilities and municipal companies over a long time. In addition, recycled water leads to less water going down the drain, which leads to less costs incurred by domestic users and municipalities in transporting and purifying waste water.

Uncertainty analysis for measurements done is conducted for a typical weekday. A maximum relative error,  $Err_{relative} = 13.6\%$  is obtained, whose effect on the performance index is shown in Table 5.5. The performance index when actual values are used in both open-loop and MPC controllers leads to

**Table 5.5.** Uncertainty of the performance index

	Cost ( <i>R/day</i> )		
	Baseline	Open-loop	MPC
Measured	8.46	7.44	7.44
Actual	8.46	6.04	6.04

28.61% cost savings compared to the baseline. Therefore, final relative error of the performance index of both controllers is  $(7.44 - 6.04)/7.44 = 18.82\%$ .

During implementation, all systems have disturbances. It has been shown by Wanjiru et al. [225] that closed-loop MPC is more robust and superior than open-loop controller in dealing with disturbances. This however comes at a higher cost and more complexity as it would need extra components to enable the feedback of height of water in the tanks to take place. It is therefore recommended that each controller is adopted depending on the nature of each application. If the demand pattern does not change significantly, then the open-loop controller is suitable. However, if disturbances affecting the system cause the demand pattern to change, closed-loop MPC is suitable.

The two control strategies for operating a domestic grey water recycling system aim at reducing water and energy demand. In order to determine the period taken by an investor to recover the investment, simple payback method is used as it is commonly used for estimating the economic potential of a project [227]. This method, however, does not take into account the time value of money and long term inflows and therefore provides a hypothetical payback period [228]. The simple payback period (*SPP*) is given as,

$$SPP = \frac{I_0}{S}, \quad (5.65)$$

where  $I_0$  is the initial investment while  $S$  is the annual savings achieved from using the grey water system using either controllers. Using the prevailing market cost of grey water recycling systems in South Africa, the grey water recycling system operated by open-loop optimal or MPC controller has a payback period of 15 and 16 years respectively. Despite the technological and conservation benefits of the proposed interventions, the two strategies would take a long time to recover the capital investment. This cannot motivate home owners to invest in such systems unless policies encouraging the same with monetary benefits are implemented in the country. In comparison, a study done in two universities in South Africa [229], as well as in other parts of the world such as Ireland [230], Greece [231] and Austria [232] have found that such systems do not necessarily pay back within their lifetime. Low water tariffs significantly influence end users' willingness to embrace water recycling. Therefore, government subsidies are necessary in order to create the market for these technologies that will help in preventing water insecurity around the country and the region [15]. In cities with intermittent or no supply infrastructure, the cost of buying potable water as well as waste disposal could be much higher, and the system would make much more economic sense besides ensuring security of water supply.

#### 5.4.5 Adoption of water recycling

Integrated urban water management (IUWM) seeks to achieve a more sustainable solution for water and sewage systems, with a trade-off among water, energy and land use. The optimal solution is a balance between energy intensive technologies and land intensive forms of water supply and treatment [233]. In this regard, water recycling provides an opportunity to increase the available water for consumption at a lower cost and sustainable environmental and social outcomes. In any country, or city, there exists localised and complex relationship between political, social, economic, environmental and technological factors that affect decision and policy making for urban water recycling. Although this

study provides a technological solution to reliably operate decentralized water recycling systems, a lot of effort is required to deal with social, political and economic factors as they could hinder the uptake of the systems.

Adoption of decentralized water recycling systems in cities with existing and functional centralized water and waste water systems is low. As a matter of necessity, development and implementation of such systems requires financial incentives to enable market penetration. Besides, public perception is one of the main hindrances [234], and therefore, public participation, education and adoption of publicly visible standards on water recycling should be carried out. Such initiatives can lead to widespread acceptance of these systems as is the case in the USA [235]. Government policies can dictate market behaviour as seen in Japan, where acceptable policies led to development and adoption of water recycling systems. Without effective policies, there is no motivation for home owners and developers to invest in and adopt these alternatives or augmenting systems, considering the centralized systems are still functional. In South Africa, where cities are rapidly expanding and the infrastructure is ageing, abstraction of fresh water will soon go beyond their hydrological limits. Therefore, there needs a paradigm shift in policy making to incorporate water recycling in buildings. Such policies with incentives have successfully been implemented in the energy sector in the country, and with water becoming more scarce, water recycling policies should be in place before water insecurity becomes irreversible.

In cities with intermittent or no centralized water supply and waste water systems such as Nairobi in Kenya, Jakarta in Indonesia and Lima in Peru, home owners have to rely on water vendors to augment water supply and septic tanks for sanitation. In fact, it is estimated that 25% of the population in cities in developing nations buy water from vendors at exorbitant price of up to 20 times higher than the utility supply [236]. Worse still, continued urbanization is increasing the demand for housing requiring water and sanitation infrastructure. It is therefore prudent for government in such countries to develop proper and acceptable policies that would increase water and sanitation security. Water recycling, such as the one developed in this chapter, would be a relief to such residents, as it would greatly increase the efficiency of water usage at reduced cost of water, energy and operation. In addition, less water would go down the drain meaning that it would take a longer period before the septic tank requires emptying.

## 5.5 SYNOPSIS

The incessant strain on energy and water resources in developing nations, arising from urbanization and increasing population, is causing further energy and water insecurity. Resource conservation through recycling and efficient use in urban areas are important in improving the security of these resources. This chapter presents two optimal controllers that enable efficient operation of water recycling in a house. The controllers are designed using the TOU electricity tariff in South Africa, where a case was considered. Grey water recycling can conserve about 23.5% of water while leading to 32.3% cost savings on water in a month. Both open-loop and closed-loop MPC controllers can potentially lead to 59.2% and 54.3% energy cost savings respectively. Although open-loop controller is easier and more cost effective to implement, closed-loop MPC is more robust and reliable in dealing with most disturbances. Importantly, both controllers adapt well when subjected to the monthly block water tariff that increases as more potable water is consumed.

Such a grey water recycling system has a huge potential to conserve water while ensuring efficient use of energy when either control strategies is employed. If widely adopted, the environmental impact would be significant as demand for energy and water from the utilities and municipalities would reduce. In addition, the stress put in the sewerage system would greatly reduce as less water would be running down the drain. The system would also reduce the costs incurred by end users for energy, water and sewage, making significant savings. These benefits would go a long way in improving the security of both resources in countries facing challenges of sufficiently providing the resources such as South Africa. The systems' payback period could discourage home owners from adopting them besides their benefit. Government intervention is therefore necessary to provide conducive policies and incentives that will create the market for the grey water systems. In cities where water supply is intermittent or non existent, the optimal grey water recycling system is necessary so as to reduce cost incurred while buying water from vendors while ensuring resource efficiency and security. The systems are expected to make more economic sense in such cities, where they could be adopted as a vital and cheap alternative water supply for non potable uses. Public perception is normally a huge barrier against implementation of such initiatives. Lack of public awareness as well as lack of evidence of such working systems can lead to general objection of such systems by the public. Governments should formulate and implement policies that are clear to the public, which can easily change the perception by providing clear safety standards. Establishment of such regulations could encourage adoption grey water recycling systems especially for non potable uses.

# CHAPTER 6 OPTIMAL MANAGEMENT OF A GREY & RAIN WATER RECYCLING SYSTEM FOR RESIDENTIAL HOUSES

## 6.1 INTRODUCTION

Most developing nations are struggling to provide water and energy, two resources that are greatly connected, hence the name energy-water nexus [201]. In South Africa, various factors such as economic growth [40], improved standards of living, population increase [237], and rural-urban migration have increased the demand for both resources [238]. Following these challenges, various demand side management (DSM) initiatives that seek to bridge the gap between supply and demand have been taking place across various sectors [36]. In commercial and industrial sector, energy DSM initiatives include energy efficiency in coal plants [48], conveyor belts [52], rock winders [58], coal crushers [55], ventilation systems [59], pumping stations [47, 160], dynamic power dispatch by utilities [239], efficient and optimal operation of trains [240]. Other energy DSM initiatives have involved optimisation of hybrid energy systems [80], with energy storage systems [23, 116]. In residential buildings, the initiatives include optimal energy-efficient building retrofitting decision model [77, 126], demand response [70], building envelope [25], efficient hot water supply using heat pump water heaters powered by hybrid energy systems [66, 67] and solar water heating [64]. South Africa has a constant water stress of about 40% to 60% resulting from low amount of rainfall averaging at about 500 *mm* per annum [84], and high evaporation of about 1700 *mm* per annum [15]. In addition, water use in the country is nearing yield point and with continuing pollution of surface and ground water resources, municipalities are forced to explore alternative means of supplying and managing water consumption [241]. Various water DSM initiatives being implemented include water restrictions, pressure management [242], monitoring water consumption pattern, management of meters, installation and retrofitting with efficient devices,



planting of water efficient vegetation [15], communication and education [243], and promotion of waste water reuse [244]. Most DSM initiatives have focused on either energy or water separately despite them being heavily intertwined both in production and consumption levels [128]. It is therefore important to consider energy-water nexus DSM in order to bridge the gap between supply and demand through conservation and efficient utilisation of both resources. In residential buildings, energy- water DSM activities include optimal hot water supply using heat pump water heaters and instantaneous heaters with integrated renewable energy systems [201, 202, 245, 246], rain water harvesting [147], pump-storage systems [225], and grey water recycling [105].

Domestic grey water recycling and rain water harvesting systems not only have environmental benefits but also economic benefits to end users and the country at large [230]. Most research has mainly focused on separate rain water harvesting systems [178] or grey water recycling systems [104]. However, combined grey water recycling and rain water harvesting systems could have more benefits than exclusive water recycling systems [247]. Ghisi and de Oliveira [248] looked into the potential of grey water recycling and rain water harvesting system in Brazil. First, the performance and payback period of the grey water and rain water harvesting systems was analysed separately, and later, the combined grey and rain water system's performance was analysed. It was found out that though the three systems have huge potential in conserving water, they had a high payback period of more than 20 years. Another study in Beijing revealed that grey water recycling is more suitable in the area than rain water harvesting due to severe pollution in the city that could reduce the quality of rain water [28]. A localized water recycling and rain water harvesting scheme was designed and the analysis showed that although both rain water harvesting and grey water recycling lead to water conservation, schemes based on recycling grey water are less susceptible to climatic changes, while those based on rain water harvesting are more susceptible to changes [249]. Therefore, a combined grey water recycling and rain water harvesting is suitable for a country like South Africa with low rainfall, in order to achieve maximum benefits and reliability. Three land uses, that is, single-family house, apartment cluster and mixed use site, were analysed for viability of use of grey water recycling and rain water harvesting systems as an alternative source of water supply for non-potable uses. Though water is conserved in all the three uses, the largest impediment to their adoption is cost. In fact, the cost is much higher for single-family house, but the discrepancy between cost and savings levels out at higher densities [250]. Despite the important benefits that such systems have on the environment, and the municipal systems, there are still technological challenges on optimal operation of such systems so as to ensure both water and energy efficiency are achieved. Whilst research has focused on designing grey and rain

water systems, little attention has been given to optimal, reliable and autonomous operation of such systems.

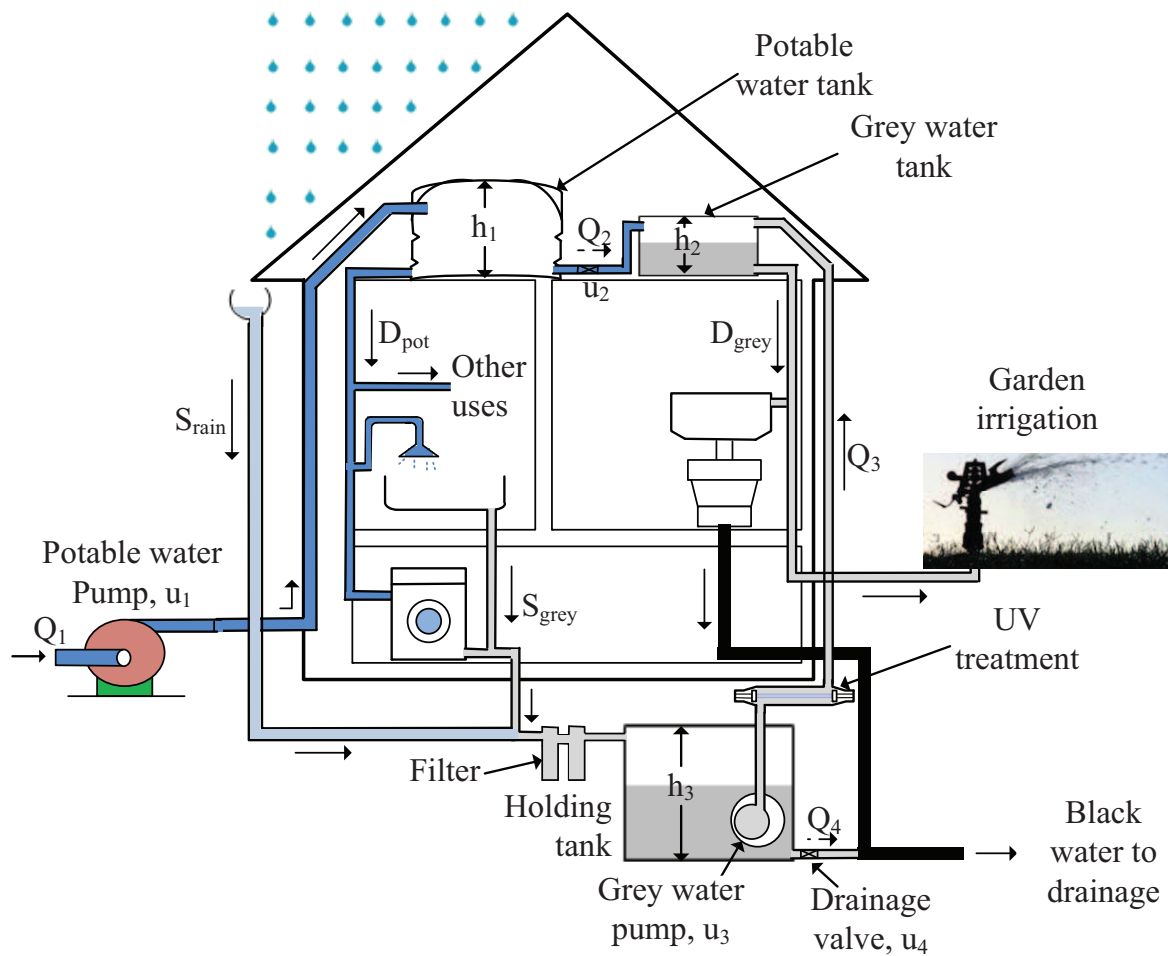
This chapter introduces the first attempt to design novel, economical and advanced optimal controllers to operate the grey water recycling and rain water harvesting system in residential areas. Open-loop optimal control and closed-loop model predictive control (MPC) systems are designed to meet hourly potable and non-potable water demand for a house leading to water conservation and energy efficiency. The control strategies are designed to cater for different application requirements. Although open-loop control is more cost effective and easy to implement, it is suitable where the water demand is known to be relatively stable. However, in cases where it is difficult to accurately predict water demand and the system is susceptible to external disturbances that significantly affect the demand pattern, the closed-loop MPC should be adopted. It, however, requires installation of additional monitoring devices to the system such as level monitoring of the tanks thereby increasing the cost and complexity of the control system. The proposed system, if widely adopted, would reduce the demand for potable water, energy and sewage services from the utilities and municipalities, leading to lower cost of potable and waste water which corresponds to lower bills paid by the end user associated to both resources. However, it is important for subsidies and rebates to be offered by the government to lower the cost of implementing such systems in individual houses.

This chapter is outlined as follows: Section 6.2 shows how the model for the proposed grey water recycling and rain water harvesting is developed. Section 6.3 discusses the design of the controllers, Section 6.4 provides information about the case used to test the control systems and other information necessary for implementing the proposed strategy. Section 6.5 discusses the results while Section 6.6 gives the conclusion and recommendations.

## **6.2 SYSTEM DEVELOPMENT**

### **6.2.1 Schematic layout**

A typical grey and rain water recycling system for a stand alone house is shown in Figure 6.1. Two scenarios motivated by the water situation in South Africa are considered. Firstly, the house is considered to have reliable municipal water supply such that pumping and storage is not required



**Figure 6.1.** Schematic of grey and rain water water recycling system.

as water flows to various end uses in the house and through valve  $u_2$  when necessary. Secondly, water supply is unreliable, either because of low water pressure or water rationing taking place in the area. Therefore, water pumping and storage is necessary to improve the reliability and convenience of potable water supply to the house occupants. A fixed speed potable water pump whose state is represented as  $u_1$  pumps water to the potable storage water tank from where it flows by gravity to various end uses in the house. Some end uses such as shower and washing machine produce grey water that can easily be treated for non-potable end uses such as toilet and garden irrigation. Further, rain water can also be harvested from the roof and used for the same non-potable end uses. Recycling grey water and harvesting rain water would lead to water conservation as well as reducing demand for potable water and sewage services in both scenarios. Both grey and rain water pass through a filter to remove particles and then flows to the holding tank for temporary storage. This tank must be emptied, through the drainage valve represented as  $u_4$ , every 24-h to prevent formation of bacteria

responsible for producing foul smell. Collected grey water is then pumped through an ultraviolet (UV) water purifier and stored in a rooftop grey water tank, from where it flows by gravity to non-potable end uses. In instances where grey water tank has insufficient treated water, potable water is allowed to flow through potable water valve  $u_2$  to assist in meeting the demand. Finally, black water, which cannot easily be recycled, is allowed to flow to the drainage. Therefore, the aim is to control pumps and valves to ensure convenient and reliable water supply that ensures both energy-water efficiency and conservation are achieved.

### 6.2.2 Potable water tank

In the scenario where municipal water supply is reliable, potable water tank is not required. On the contrary, if municipal water supply is unreliable, water has to be pumped by potable water pump into the potable water tank for storage, where it flows by gravity to various end uses. Assuming that all tanks in this study have uniform cross-sectional area, the volume of water in this tank,  $V_1$  ( $m^3$ ), can be modelled as,

$$\dot{V}_1 = A_1^t \dot{h}_1 = Q_1 u_1 - Q_2 u_2 - \dot{D}_{pot}, \quad (6.1)$$

where  $A_1^t$  is the cross-sectional area of the tank ( $m^2$ ) while  $h_1$  is the height of water in the tank ( $m$ ).  $D_{pot}$  is the potable water demand ( $m^3$ ) in the house while  $Q_1$  and  $Q_2$  are the flow rates ( $m^3/h$ ) of potable water pump and solenoid valve respectively. Differential equation (6.1) can be expressed in discrete-time domain by a first order difference equation as follows;

$$h_1(j+1) = h_1(j) + \frac{1}{A_1^t} \left[ t_s Q_1 u_1(j) - t_s Q_2 u_2(j) - D_{pot}(j) \right], \quad (6.2)$$

where  $j$  the sampling interval and  $t_s$  is the sampling period during a full operating cycle of 24-h. Level sensors are economical and easy to use in measuring the volume of water in tanks with uniform cross-sectional area [251]. Therefore, equation (6.2) can be modelled in terms of the water level in a sampling interval,  $h_1(j)$ , which the controller would use to convert to volume. Through recurrence manipulation, the equation becomes,

$$h_1(j) = h_1(0) + \frac{t_s}{A_1^t} \sum_{i=1}^j \left[ Q_1 u_1(i) - Q_2 u_2(i) \right] - \frac{1}{A_1^t} \sum_{i=1}^j D_{pot}(i) \quad 1 \leq j \leq N, \quad (6.3)$$

where  $N$  is the total number of cycles during the full 24-h operating cycle, obtained as  $N = \frac{24}{t_s}$ .

### 6.2.3 Grey water tank

Treated grey water is stored in this tank for future use by non-potable water end uses. If the tank has no water, potable water is allowed into this tank through valve  $u_2$  and then flows to meet the required demand. Therefore, volume,  $V_2$  ( $m^3$ ), of water in this tank is,

$$\dot{V}_2 = A_2^t \dot{h}_2 = Q_2 u_2 + Q_3 u_3 - \dot{D}_{grey}, \quad (6.4)$$

where  $A_2^t$  is the cross-sectional area ( $m^2$ ) of the tank,  $h_2$  is the height of water ( $m$ ) in the tank,  $D_{grey}$  is the grey water demand ( $m^3$ ) while  $Q_3$  is the water flow rate ( $m^3/h$ ) through the grey water pump. Expressing equation (6.4) in discrete-time domain yields,

$$h_2(j+1) = h_2(j) + \frac{1}{A_2^t} \left[ t_s Q_2 u_2(j) + t_s Q_3 u_3(j) - D_{grey}(j) \right], \quad (6.5)$$

which can further be expressed as

$$h_2(j) = h_2(0) + \frac{t_s}{A_2^t} \sum_{i=1}^j \left[ Q_2 u_2(i) + Q_3 u_3(i) \right] - \frac{1}{A_2^t} \sum_{i=1}^j D_{grey}(i) \quad 1 \leq j \leq N. \quad (6.6)$$

### 6.2.4 Holding tank

Untreated grey and rain water flows through filters to remove physical impurities to temporary storage in the holding tank. Whenever treated grey water is required in the grey water tank, the collected water is pumped by grey water pump through the UV purifier. It is important to empty the holding tank every 24 hours to prevent formation of bacteria that cause foul smell. Consequently, the volume,  $V_3$  ( $m^3$ ), of water in this tank can be modelled as,

$$\dot{V}_3 = A_3^t \dot{h}_3 = \dot{S}_{grey} + \dot{S}_{rain} - Q_3 u_3 - Q_4 u_4, \quad (6.7)$$

where  $A_3^t$  is the cross-sectional area ( $m^2$ ) of the tank,  $h_3$  is the height ( $m$ ) of water in the tank.  $S_{grey}$  and  $S_{rain}$  are the volume ( $m^3$ ) of water supplied from the recyclable potable water end uses and rain water harvesting respectively, while  $Q_4$  is the flow rate ( $m^3/h$ ) of untreated grey water through the drainage valve. Expressing equation (6.7) in discrete-time domain leads to,

$$h_3(j+1) = h_3(j) + \frac{1}{A_3^t} \left[ S_{grey}(j) + S_{rain}(j) - t_s Q_3 u_3(j) - t_s Q_4 u_4(j) \right], \quad (6.8)$$

which transforms to,

$$h_3(j) = h_3(0) + \frac{1}{A_3^t} \sum_{i=1}^j \left[ S_{grey}(i) + S_{rain}(i) \right] - \frac{t_s}{A_3^t} \sum_{i=1}^j \left[ Q_3 u_3(i) + Q_4 u_4(i) \right] \quad 1 \leq j \leq N. \quad (6.9)$$

Dynamic equations (6.3), (6.6) and (6.9) are used in designing the two controllers that optimally operate the proposed grey and rain water recycling system.

### 6.3 CONTROLLER DESIGN

In this study, two model based controllers that use advanced optimal control concept are designed. The controllers seek to minimize cost of pumping energy of potable and collected grey water, minimize consumption of potable water in the house and finally maximize the life of these pumps through minimizing the maintenance cost normally represented as the number of times a pump is switched on and off during the operating cycle.

#### 6.3.1 Open-loop optimal controller

The open-loop optimal controller uses the feed forward principle in that hourly water demand in the house is measured prior to running the controller. This demand pattern is used by the controller to predict the future behaviour of the system throughout the full operating cycle. As previously stated, the open-loop controller seeks to minimize the cost of pumping energy, consumption of potable water in the house and maximize the life of the pumps. These performance indicators can be modelled to form the following objective function,

$$J = \sum_{j=1}^N \left[ \alpha_1 t_s p_e(j) P_1^m u_1(j) + \alpha_2 t_s Q_2 u_2(j) + \alpha_3 t_s p_e(j) P_3^m u_3(j) \right] + \alpha_4 \sum_{j=1}^N \left[ s_1(j) + s_3(j) \right], \quad (6.10)$$

where  $P_1^m$  (kW) and  $P_3^m$  (kW) are potable and grey water power pump's power consumption respectively, while  $p_e$  and  $t_s$  are cost of electricity using the TOU tariff during the  $j^{th}$  sampling interval and the sampling time respectively.  $s_1(j)$  and  $s_3(j)$  are auxiliary variables used to minimize the switching frequency of potable and grey water pumps respectively. Each auxiliary variable is represented by a value 1 whenever a pump's state changes from off to on [153]. Weights  $\alpha_1$  to  $\alpha_4$  are used to tune the controller according to user's preference. First and third terms in equation (6.10) minimize the cost of energy consumed by the pumps, the second term minimizes the consumption of potable water by grey water end uses while the fourth term is responsible for minimizing the switching frequency the two pumps.

Every system functions within certain physical and operational constraints for safe and reliable operation. Constraints present in this system are mathematically modelled as follows;

$$h_1^{\min} \leq h_1(j) \leq h_1^{\max}, \quad (6.11)$$

$$h_2^{\min} \leq h_2(j) \leq h_2^{\max}, \quad (6.12)$$

$$h_3^{\min} \leq h_3(j) \leq h_3^{\max}, \quad (6.13)$$

$$h_3(N) = h_3^f, \quad (6.14)$$

$$u_1(1) - s_1(1) \leq 0, \quad (6.15)$$

$$u_1(j) - u_1(j-1) - s_1(j) \leq 0, \quad (6.16)$$

$$u_3(1) - s_3(1) \leq 0, \quad (6.17)$$

$$u_3(j) - u_3(j-1) - s_3(j) \leq 0, \quad (6.18)$$

$$u_m(j) \in \{0, 1\} \text{ where } m = 1, 2, 3, 4, \quad (6.19)$$

$$s_1(j), s_3(j) \in \{0, 1\}. \quad (6.20)$$

Various tank capacities are the physical constraints while emptying of the holding tank and switching frequency of the pumps are the main operational constraints affecting the system. Therefore, the tanks are modelled in inequalities (6.11), (6.12) and (6.13) to have the level of water maintained between set minimum and maximum levels. Potable and grey water tanks should never be emptied whereas the holding tank must be emptied within the 24-h operating cycle. This is given by equation (6.14), where  $h_3^f$  is the final water level in the tank. Inequalities (6.15) and (6.17) initialize the auxiliary variables as the initial state of the respective  $u$  while inequalities (6.16) and (6.18) favour the control that involves less switching frequency of the respective pumps. Finally equations (6.19) and (6.20) are bounds for the control variables that is, the status of the pumps and switches as well as the auxiliary variables respectively.

### 6.3.1.1 Open-loop control algorithm

The objective function and constraints are solved using the following canonical form [57],

$$\min f^T X \quad (6.21)$$

subject to

$$\begin{cases} AX \leq b \text{ (linear inequality constraint),} \\ A_{eq}X = b_{eq} \text{ (linear equality constraint),} \\ L_B \leq X \leq U_B \text{ (lower and upper bounds).} \end{cases} \quad (6.22)$$

Here, vector  $X$  consists of all the control variables in the optimization problem, that is,

$$X = \begin{bmatrix} u_1(1), \dots, u_1(N), u_2(1), \dots, u_2(N), u_3(1), \dots, u_3(N), u_4(1), \dots, u_4(N), \\ s_1(1), \dots, s_1(N), s_3(1), \dots, s_3(N) \end{bmatrix}_{6N \times 1}^T, \quad (6.23)$$

while elements of vector  $f^T$  are obtained from objective function (6.10) as,

$$f^T = \begin{bmatrix} \alpha_{1t_s} P_1^m p_e(1), \dots, \alpha_{1t_s} P_1^m p_e(N), \alpha_{2t_s} Q_2 \dots, \alpha_{2t_s} Q_2, \alpha_{3t_s} P_3^m p_e(1), \dots, \alpha_{3t_s} P_3^m p_e(N), \\ 0, \dots, 0, \alpha_4, \dots, \alpha_4, \alpha_4, \dots, \alpha_4 \end{bmatrix}_{1 \times 6N}. \quad (6.24)$$

Since there are several linear inequalities, each is modelled separately and later combined into the canonical linear inequality form ( $AX \leq b$ ). First, linear inequality constraint (6.11) is modelled to,

$$\begin{aligned} A_1 X &\leq b_1, \\ -A_1 X &\leq b_2, \end{aligned} \quad (6.25)$$

where

$$A_1 = \begin{bmatrix} -Q_{1t_s} & 0 & \dots & 0 & Q_{2t_s} & 0 & \dots & 0 & 0 & \dots & \dots & 0 \\ -Q_{1t_s} & -Q_{1t_s} & \dots & 0 & Q_{2t_s} & Q_{2t_s} & \dots & 0 & 0 & \dots & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\ -Q_{1t_s} & -Q_{1t_s} & \dots & -Q_{1t_s} & Q_{2t_s} & Q_{2t_s} & \dots & Q_{2t_s} & 0 & \dots & \dots & 0 \end{bmatrix}_{N \times 6N}, \quad (6.26)$$

$$b_1 = \begin{bmatrix} -D_{pot}(1) - A_1^t \{h_1^{min} - h_1(0)\} \\ -\{D_{pot}(1) + D_{pot}(2)\} - A_1^t \{h_1^{min} - h_1(0)\} \\ \vdots \\ -\{D_{pot}(1) + \dots + D_{pot}(N)\} - A_1^t \{h_1^{min} - h_1(0)\} \end{bmatrix}_{N \times 1} \quad (6.27)$$

and

$$b_2 = \begin{bmatrix} D_{pot}(1) + A_1^t \{h_1^{max} - h_1(0)\} \\ \{D_{pot}(1) + D_{pot}(2)\} + A_1^t \{h_1^{max} - h_1(0)\} \\ \vdots \\ \{D_{pot}(1) + \dots + D_{pot}(N)\} + A_1^t \{h_1^{max} - h_1(0)\} \end{bmatrix}_{N \times 1}. \quad (6.28)$$



Then, inequality constraint (6.12) becomes,

$$A_2X \leq b_3, \quad (6.29)$$

$$-A_2X \leq b_4,$$

where

$$A_2 = \begin{bmatrix} 0 & \dots & 0 & -Q_2t_s & 0 & \dots & 0 & -Q_3t_s & 0 & \dots & 0 & 0 & \dots & \dots & 0 \\ 0 & \dots & 0 & -Q_2t_s & -Q_2t_s & \dots & 0 & -Q_3t_s & -Q_3t_s & \dots & 0 & 0 & \dots & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & -Q_2t_s & -Q_2t_s & \dots & -Q_2t_s & -Q_3t_s & -Q_3t_s & \dots & -Q_3t_s & 0 & \dots & \dots & 0 \end{bmatrix}, \quad (6.30)$$

$$b_3 = \begin{bmatrix} -D_{grey}(1) - A_2^t \{h_2^{min} - h_2(0)\} \\ -\{D_{grey}(1) + D_{grey}(2)\} - A_2^t \{h_2^{min} - h_2(0)\} \\ \vdots \\ -\{D_{grey}(1) + \dots + D_{grey}(N)\} - A_2^t \{h_2^{min} - h_2(0)\} \end{bmatrix} \quad (6.31)$$

and

$$b_4 = \begin{bmatrix} D_{grey}(1) + A_2^t \{h_2^{max} - h_2(0)\} \\ \{D_{grey}(1) + D_{grey}(2)\} + A_2^t \{h_2^{max} - h_2(0)\} \\ \vdots \\ \{D_{grey}(1) + \dots + D_{grey}(N)\} + A_2^t \{h_2^{max} - h_2(0)\} \end{bmatrix}, \quad (6.32)$$

while inequality (6.13) can be remodelled to,

$$A_3X \leq b_5, \quad (6.33)$$

$$-A_3X \leq b_6,$$

where

$$A_3 = \begin{bmatrix} 0 & \dots & \dots & 0 & Q_3t_s & 0 & \dots & 0 & Q_4t_s & 0 & \dots & 0 & 0 & \dots & \dots & 0 \\ 0 & \dots & \dots & 0 & Q_3t_s & Q_3t_s & \dots & 0 & Q_4t_s & Q_4t_s & \dots & 0 & 0 & \dots & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & 0 & Q_3t_s & Q_3t_s & \dots & Q_3t_s & Q_4t_s & Q_4t_s & \dots & Q_4t_s & 0 & \dots & \dots & 0 \end{bmatrix}, \quad (6.34)$$

$$b_5 = \begin{bmatrix} S_{grey}(1) + S_{rain}(1) - A_3^t \{h_3^{min} - h_3(0)\} \\ \{S_{grey}(1) + S_{grey}(2) + S_{rain}(1) + S_{rain}(2)\} - A_3^t \{h_3^{min} - h_3(0)\} \\ \vdots \\ \{S_{grey}(1) + \dots + S_{grey}(N) + S_{rain}(1) + \dots + S_{rain}(N)\} - A_3^t \{h_3^{min} - h_3(0)\} \end{bmatrix} \quad (6.35)$$

and

$$b_6 = \begin{bmatrix} -\{S_{grey}(1) + S_{rain}(1)\} + A_3^t \{h_3^{max} - h_3(0)\} \\ -\{S_{grey}(1) + S_{grey}(2) + S_{rain}(1) + S_{rain}(2)\} + A_3^t \{h_3^{max} - h_3(0)\} \\ \vdots \\ -\{S_{grey}(1) + \dots + S_{grey}(N) + S_{rain}(1) + \dots + S_{rain}(N)\} + A_3^t \{h_3^{max} - h_3(0)\} \end{bmatrix}. \quad (6.36)$$

Lastly, auxiliary variables in inequalities (6.15)-(6.18) are remodelled as

$$A_4X \leq b_7, \quad (6.37)$$

where

$$A_4 = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 & | & 0 \dots 0 & 1 & 0 & \dots & 0 & 0 & | & 0 \dots 0 & -1 & 0 & \dots & 0 & | & -1 & 0 & \dots & 0 \\ -1 & 1 & \dots & 0 & 0 & | & 0 \dots 0 & -1 & 1 & \dots & 0 & 0 & | & 0 \dots 0 & 0 & -1 & \dots & 0 & | & 0 & -1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & | & \vdots \ddots \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & | & \vdots \ddots \vdots & \vdots & \vdots & \ddots & \vdots & | & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & -1 & 1 & | & 0 \dots 0 & 0 & 0 & \dots & -1 & 1 & | & 0 \dots 0 & 0 & 0 & \dots & -1 & | & 0 & 0 & \dots & -1 \end{bmatrix} \quad (6.38)$$

and

$$b_7 = \begin{bmatrix} 0 & \dots & 0 \end{bmatrix}^T. \quad (6.39)$$

Matrices  $A_1$  to  $A_4$  have  $(N \times 6N)$  dimension while vectors  $b_1$  to  $b_7$  have a dimension of  $(N \times 1)$ .

Therefore, linear inequality in the canonical form (6.22) becomes,

$$A = \begin{bmatrix} A_1 \\ -A_1 \\ A_2 \\ -A_2 \\ A_3 \\ -A_3 \\ A_4 \end{bmatrix}_{7N \times 6N} \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix}_{7N \times 1}. \quad (6.40)$$

In the same degree, linear equality constraint (6.14) becomes,

$$A_{eq}X = b_{eq}, \quad (6.41)$$

where

$$A_{eq} = \begin{bmatrix} 0 & \dots & 0 & | & 0 & \dots & 0 & 0 & \dots & 0 & | & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & | & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & | & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & | & 0 & \dots & 0 & 0 & \dots & 0 & | & 0 & \dots & 0 \\ 0 & \dots & 0 & | & Q_{3t_s} & \dots & Q_{3t_s} & Q_{4t_s} & \dots & Q_{4t_s} & | & 0 & \dots & 0 \end{bmatrix}_{N \times 6N} \quad (6.42)$$

and

$$b_{eq} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ A_3^t \{h_3(0) - h_3^f\} + \{S_{grey}(1) + \dots + S_{grey}(N) + S_{rain}(1) + \dots + S_{rain}(N)\} \end{bmatrix}_{N \times 1}, \quad (6.43)$$

while the bounds given in equations (6.19) and (6.20) become,

$$L_B = \begin{bmatrix} 0 & \dots & 0 \end{bmatrix}_{6N \times 1}^T \quad \text{and} \quad U_B = \begin{bmatrix} 1 & \dots & 1 \end{bmatrix}_{6N \times 1}^T. \quad (6.44)$$

This binary integer optimization problem is solved using the SCIP solver in OPTI toolbox, a free Matlab optimization toolbox. This solver is the fastest non-commercial optimization solver [71].

### 6.3.2 Closed-loop MPC control

Closed-loop model predictive control (MPC) has the ability to predict the future behaviour of the system, cope with constraints in the design process and robustly deal with disturbances present in the system [225]. MPC makes use of both feed forward and feed back measurements from the system to compute the control law on-line [20]. It obtains the current control response by solving an open-loop optimal control optimization problem using the current state of the plant as the initial state in each sampling time. From the optimal sequence generated, only the first control is implemented [158]. The state of the plant (water level in the tanks) is measured. During the next iteration,  $k + 1$ , objective function and constraints are updated while taking the previous state of the tanks (water level at sampling time  $k$ ) as the initial state. The process of optimization is carried out in real time over the new control horizon ( $N_c = N - k + 1$ ) to give the receding horizon control law. This process is repeated throughout the entire operating cycle [252].

The objective function,  $J_{mpc}$ , can be derived from the open-loop objective (6.10) as,

$$J_{mpc} = \sum_{j=k}^{k+N_c-1} \left[ \alpha_1 t_s p_e(j) P_1^m u_1(j|k) + \alpha_2 t_s Q_2 u_2(j|k) + \alpha_3 t_s p_e(j) P_3^m u_3(j|k) \right] + \alpha_4 \sum_{j=k}^{k+N_c-1} \left[ s_1(j|k) + s_3(j|k) \right], \quad (6.45)$$

where  $N_c$  is the control horizon,  $u_1(j|k)$ ,  $u_2(j|k)$  and  $u_3(j|k)$  are optimized control actions while  $s_1(j|k)$  and  $s_3(j|k)$  are auxiliary values at  $j^{th}$  sampling interval based on most recent measurements carried out at time  $k$ . Although MPC problems normally have predicting,  $N_p$ , and control,  $N_c$ , horizons, only the control horizon,  $N_c$ , is included in this optimization problem since none of the state variables (height of water in the tank) is present in the objective function. Therefore, the control horizon can be given as

$$N_c = N - k + 1. \quad (6.46)$$

State equations are modified from equations (6.3), (6.6) and (6.9) to,

$$h_1(j|k) = h_1(k) + \frac{t_s}{A_1^t} \sum_{i=k}^j \left[ Q_1 u_1(i|k) - Q_2 u_2(i|k) \right] - \frac{1}{A_1^t} \sum_{i=k}^j D_{pot}(i), \quad (6.47)$$

$$h_2(j|k) = h_2(k) + \frac{t_s}{A_2^t} \sum_{i=k}^j \left[ Q_2 u_2(i|k) + Q_3 u_3(i|k) \right] - \frac{1}{A_2^t} \sum_{i=k}^j D_{grey}(i), \quad (6.48)$$

$$h_3(j|k) = h_3(k) + \frac{1}{A_3^t} \sum_{i=k}^j \left[ S_{grey}(i) + S_{rain}(i) \right] - \frac{t_s}{A_3^t} \sum_{i=k}^j \left[ Q_3 u_3(i|k) + Q_4 u_4(i|k) \right], \quad (6.49)$$

$$k \leq j \leq k + N_c - 1,$$

where  $h_1(j|k)$ ,  $h_2(j|k)$ , and  $h_3(j|k)$  are predicted levels of water in the respective tanks at  $j^{th}$  sampling interval based on information available at time  $k$ . Moreover, the system experiences the same physical and operational constraints as the open-loop control system. With feedback of water level measurements in various tanks at every iteration, constraints and equations (6.11)-(6.20), are modified to,

$$h_1^{min} \leq h_1(j|k) \leq h_1^{max}, \quad (6.50)$$

$$h_2^{min} \leq h_2(j|k) \leq h_2^{max}, \quad (6.51)$$

$$h_3^{min} \leq h_3(j|k) \leq h_3^{max}, \quad (6.52)$$

$$h_3(N) = h_3^f, \quad (6.53)$$

$$u_1(1|k) - s_1(1|k) \leq 0, \quad (6.54)$$

$$u_1(j|k) - u_1(j-1|k) - s_1(j|k) \leq 0, \quad (6.55)$$

$$u_3(1|k) - s_3(1|k) \leq 0, \quad (6.56)$$

$$u_3(j|k) - u_3(j-1|k) - s_3(j|k) \leq 0, \quad (6.57)$$

$$u_{m,c}(j|k) \in \{0, 1\} \text{ where } m = 1, 2, 3, 4, \quad (6.58)$$

$$s_1(j|k), s_3(j|k) \in \{0, 1\}. \quad (6.59)$$

### 6.3.2.1 MPC algorithm

Similar to the open-loop control algorithm, control vector, represented as  $X_{mpc}$ , contains the control variables such that,

$$\begin{aligned}
 X_{mpc} = & \\
 & \left[ u_1(k|k), u_1(k+1|k), \dots, u_1(k+N_c-1|k), u_2(k|k), u_2(k+1|k), \dots, u_2(k+N_c-1|k), \right. \\
 & u_3(k|k), u_3(k+1|k), \dots, u_3(k+N_c-1|k), u_4(k|k), u_4(k+1|k), \dots, u_4(k+N_c-1|k), \\
 & \left. s_1(k|k), s_1(k+1|k), \dots, s_1(k+N_c-1|k), s_3(k|k), s_3(k+1|k), \dots, s_3(k+N_c-1|k) \right]_{6N \times 1}^T.
 \end{aligned} \tag{6.60}$$

The work flow of the MPC controller is as follows [163];

1. For time,  $k$ , find the control horizon ( $N_c(k)$ ) using equation (6.46).
2. *Optimization:* Find the optimal solution within the control horizon;

minimize objective function (6.45),  
 subject to constraints (6.50)-(6.59).

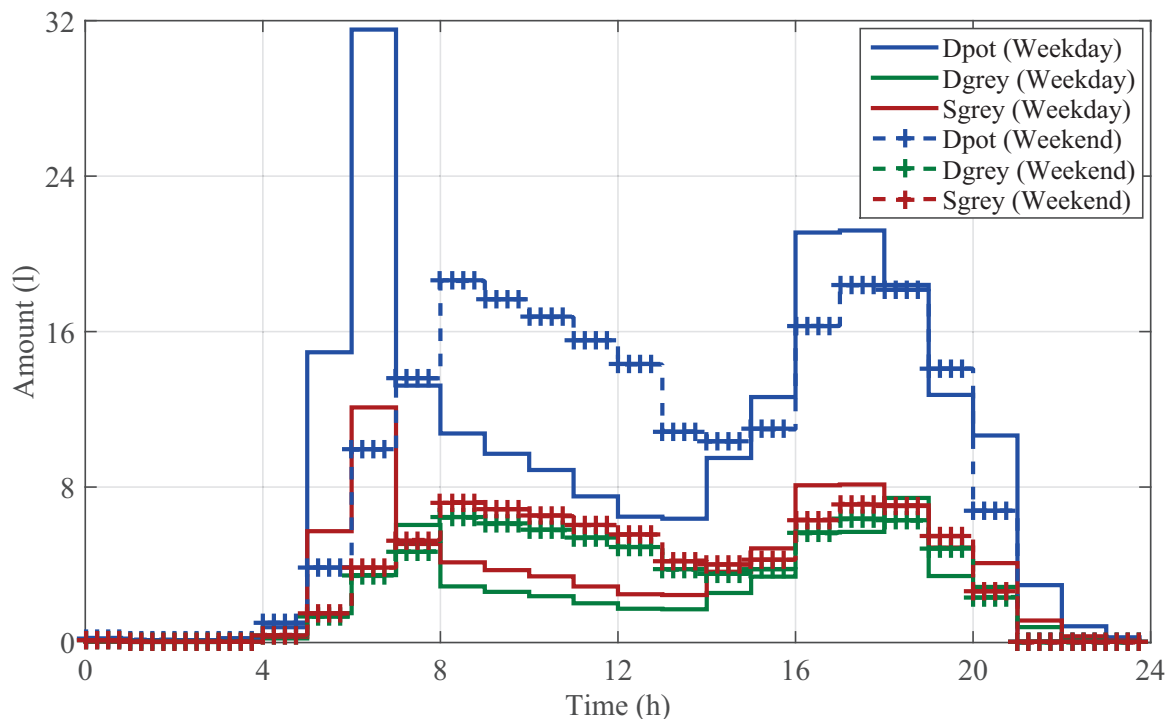
3. From the optimal solution, implement  $[u_1(k|k), u_2(k|k), u_3(k|k), u_4(k|k)]^T$  to the plant.
4. *Feed back:* Measure state variables  $h_1(k+1)$ ,  $h_2(k+1)$  and  $h_3(k+1)$ .
5. Set  $k = k + 1$  and update system states and inputs and outputs.
6. Repeat steps 1-5 until  $k$  reaches a predefined value.

This binary integer optimization problem solved using the SCIP solver in OPTI toolbox.

## 6.4 PERTINENT INFORMATION

### 6.4.1 Case study

A house in Pretoria, South Africa, has unreliable municipal water supply forcing the occupants to pump and store water in a rooftop storage tank, from where it flows by gravity to all end uses. Consumption of potable water and energy associated with pumping in this house is used as the baseline for this study. The 0.8 kW fixed speed pump with a flow rate of  $0.9 \text{ m}^3/h$  is controlled by level switches that just detect empty and full levels. Whenever the tank is empty, water is pumped until the tank is full, regardless of TOU period. The end uses in the house were identified and some had their hourly water use measured using digital flow meters and data loggers while others were estimated after interviewing the occupants. These end uses were categorized as those that must use potable water, those that could use treated grey water and those whose used water is suitable for recycling. The hourly water demand for a typical week day and a weekend in this house is shown in Figure 6.2. It can be seen from the



**Figure 6.2.** Hourly water profile for a typical week day and weekend.

curves that the grey water supply,  $S_{grey}$ , is always less than the potable water demand,  $D_{pot}$ , as some

of this potable water qualifies to be recycled. On the contrary, the grey water demand,  $D_{grey}$ , doesn't necessarily follow the others, as this demand entirely depends on the human behaviour.

The current cylindrical potable water tank has the dimensions given in Table 6.1. In order to incorporate grey and rain water recycling, two tanks; grey and holding water tanks are required. Typical dimensions and capacity constraints of these tanks are given in Table 6.1. Level sensors will be used monitor the

**Table 6.1.** Dimensions and capacity of the tanks.

Tank	Radius (m)	Height (m)	Min	Max
Potable	0.55	1.2	0.1	1.0
Grey	0.36	1.0	0.1	0.8
Holding	0.30	0.6	0	0.5

water level between minimum and maximum levels given in Table 6.1. This is to ensure safe and reliable operation of the system by avoiding either running the tanks completely empty or spilling the water hence damaging the roof of the house. The grey water pump to be incorporated would be rated at 650 W with flow rate of  $0.35 \text{ m}^3/h$ .

To enable rain water harvesting, about  $50 \text{ m}^2$  of the house's roof can easily have rain water directed to the holding tank, through the filters. The area's weather data, that includes the hourly amount of rainfall, is obtained from the Southern African Universities Radiometric Network<sup>1</sup>, University of Pretoria's station.

#### 6.4.2 Time-of-use electricity tariff

Time-of-use (TOU) tariff, commonly used across the world to encourage shifting of peak load [253], can vary by time of day, day of week and season [45]. Eskom's TOU Homeflex<sup>2</sup> structure for residential consumers given below is used.

$$p_e(t) = \begin{cases} p_{off} & = 0.5510 \text{ R/Kwh} & \text{if } t \in [0, 6] \cup [10, 18] \cup [20, 24], \\ p_{peak} & = 1.748 \text{ R/Kwh} & \text{if } t \in [7, 10] \cup [18, 20], \end{cases} \quad (6.61)$$

<sup>1</sup><http://www.sauran.net>

<sup>2</sup><http://www.eskom.co.za/>

where  $p_{off}$  is the off peak price,  $p_{peak}$  is the peak time price,  $R$  is the South African currency, Rand, and  $t$  is the time of day in hours.

### 6.4.3 Potable and waste water tariffs

Table 6.2 shows the water and waste water tariffs for domestic consumers in the City of Tshwane<sup>3</sup>. The

**Table 6.2.** City of Tshwane tariff for 2014/2015<sup>3</sup>

Water ( $m^3/month$ )	0-6	7-12	13-18	19-24	25-30	31-42	43-72	>72
Price ( $R/m^3$ )	6.81	9.72	12.77	14.77	16.89	18.25	19.53	20.91
Discharge ( $\%/month$ )	98	90	75	60	52	10	1	1
Price ( $R/m^3$ )	5.06	6.83	8.81	8.81	8.81	8.81	8.81	8.81

amount of waste water discharged into the drainage system is calculated as a percentage of the amount of potable water consumed in a household per month. Since potable and waste water are charged through an incremental block tariff, it is important to carry out simulations over a month in order to obtain the cost incurred by the end user. It is assumed that the demand pattern repeats itself over the 24-h operating cycle, for weekdays and the two days of the week end. In this study, the weekday water demand profile,  $D_{pot}(weekday)$ , is assumed to be the same for all the 5 week days. Similarly, the weekend demand profile,  $D_{pot}(weekend)$ , is also taken to be the same for the 2 days of the weekend. Therefore, both open-loop and closed-loop control systems are run over the 24-h operating cycle and then repeated over a month. The month is taken to have 4 complete weeks with each week having 5 weekdays and 2 days of the weekend. Taking the first day of the month to be a Monday, the cumulative volume of potable water consumed up to a certain weekday,  $D_{pot,wkdy}$ , or a weekend,  $D_{pot,wknd}$ , is obtained as;

$$D_{pot,wkdy} = (5q)D_{pot}(weekday) + (2q - 2)D_{pot}(weekend), \quad (6.62)$$

$$D_{pot,wknd} = (5q)D_{pot}(weekday) + (2q - 1)D_{pot}(weekend),$$

where  $q$  is the number of the week in the month ( $q = 1, 2, 3, 4$ ). This amount is then used to compute the amount of waste water discharged and eventually the cost of potable and waste water in a month.

<sup>3</sup>[www.tshwane.gov.za](http://www.tshwane.gov.za)

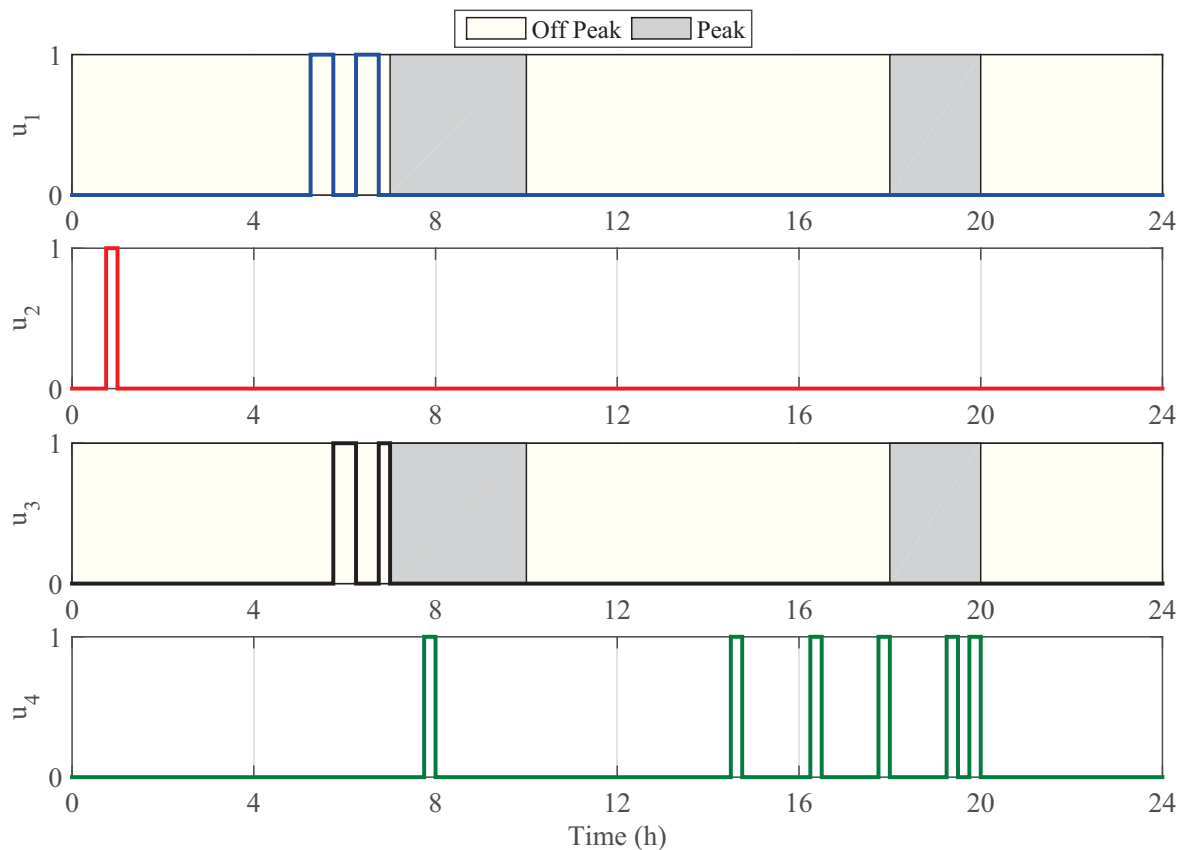


## 6.5 ANALYSIS OF OPTIMAL RESULTS

The two control strategies are run for an operating cycle of 24-h with a sampling period,  $t_s = 15$  minutes. The legend showing peak and off peak periods of the TOU is used throughout the chapter. Moreover, only potable and grey water pumps, whose status are represented by  $u_1$  and  $u_3$  respectively, are considered to consume power hence subjected to the TOU tariff.

### 6.5.1 Open-loop optimal control strategy

The optimal operation of the pumps and valves in the proposed system by using open-loop optimal controller is shown in Figure 6.3. The controller operates both potable and grey water pumps during

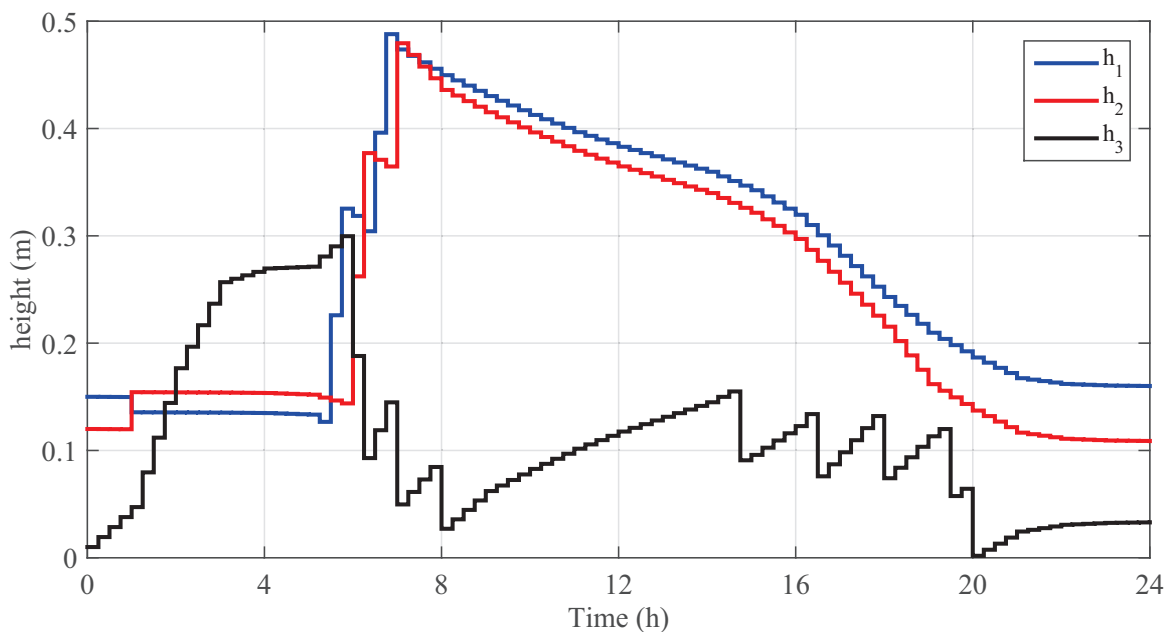


**Figure 6.3.** Optimal operation of pumps and valves by open-loop controller.

the off-peak period of the TOU tariff in meeting the household potable and grey water demand. This effectively shifts the electrical load to the period when the grid experiences less load, hence improving its stability. In addition, the controller switches both pumps only 2 times during the 24-h operating

cycle in line with the objective seeking to minimize the maintenance cost of the pumps. This cost is attributed to frequent switching of a pump that causes wear and tear to its motor as it tries to overcome dead load (water) while changing from off to on status. In addition, the controller operates potable water valve once in early morning to supplement treated grey water. It also operates the drainage valve several times after predicting that collected water is no longer needed for treatment and pumping, and yet the holding tank has to be emptied within the operating cycle.

Optimal operation of the proposed system using the open-loop controller leads to variation of water level in various tanks as shown in Figure 6.4. The controller does not violate any constraints in



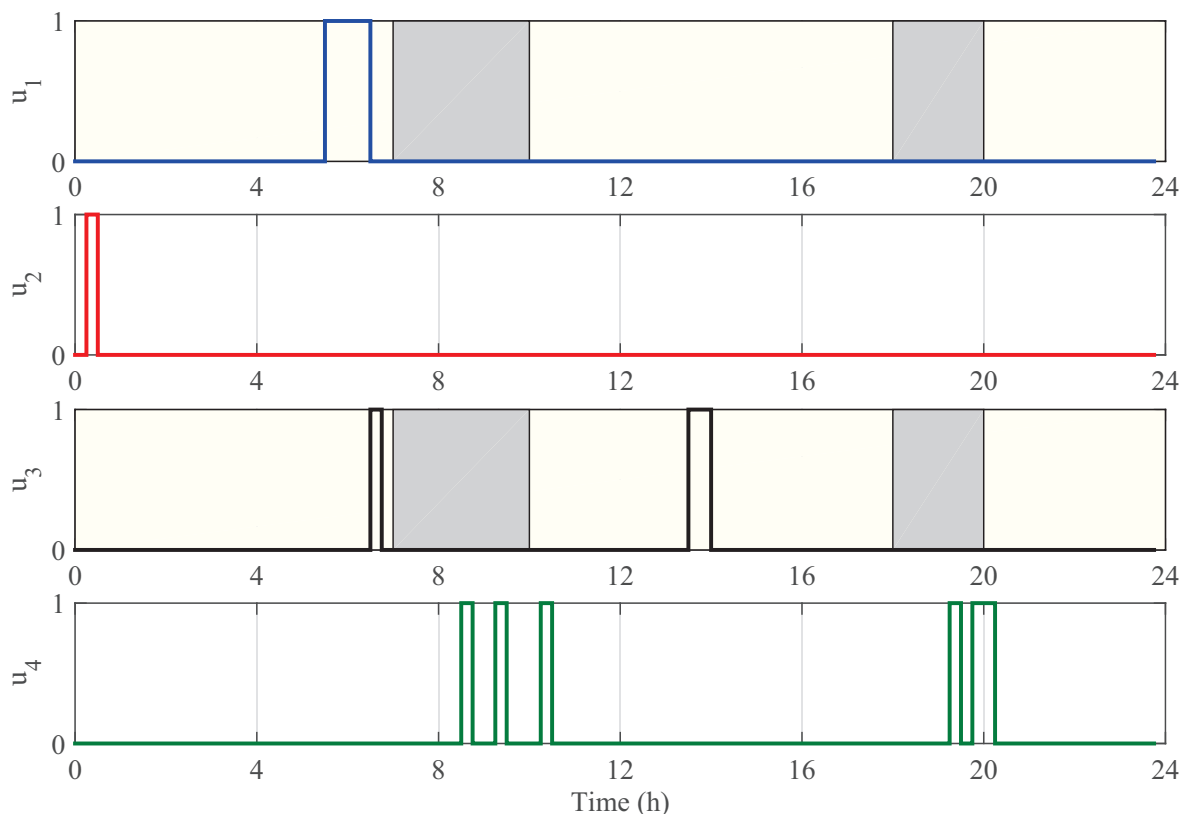
**Figure 6.4.** Variation of water level in respective tanks with open-loop controller.

operating the system throughout the 24-h operating cycle. After a 15 minutes draw by potable water valve, the controller predicts that potable tank does not have sufficient water to take it through the high morning water demand, which coincides with peak TOU period. It, therefore, opts to switch on the pump twice at 05:15 and 06:15 for 30 minutes each, raising the level,  $h_1$ , to 0.49 m, which is sufficient for the remaining period of the operating cycle. After this, water level  $h_1$  keeps dropping while meeting potable water demand to a low of 0.16 m at the end of the operating cycle. At the onset, the holding tank is empty while the treated water level in the grey water tank is almost at the minimum allowable level. For this reason, the controller has to use potable water to meet grey water demand in the early morning leading to the potable water valve being switched on at 00:45 for 15 minutes. A simultaneous rise of water level in grey water tank and drop in potable water tank takes place during

this time. As the day progresses, more water is collected hence there is no need for using potable water for non-potable uses. The controller predicts an increase in grey water demand in the morning hours, which again coincides with the peak TOU period. It consequently operates the grey water pump twice at 05:45 for 30 minutes and 06:45 for 15 minutes leading to a rise in level  $h_2$  to 0.48 m, which is sufficient to meet the grey water demand for the rest of the operating cycle. In addition, the pumping leads to a simultaneous drop in level  $h_3$  to 0.05 m. Since the controller predicts that the treated grey water is sufficient for the rest of the operating cycle, it then keeps draining the collected water to the drainage, and also ensures that the tank is emptied within the operating cycle to avoid formation bacteria responsible for foul smell.

### 6.5.2 Closed-loop MPC strategy

The closed-loop MPC strategy operates the proposed system by switching the pumps and valves as shown in Figure 6.5. Just like the open-loop controller, the closed-loop controller also operates both

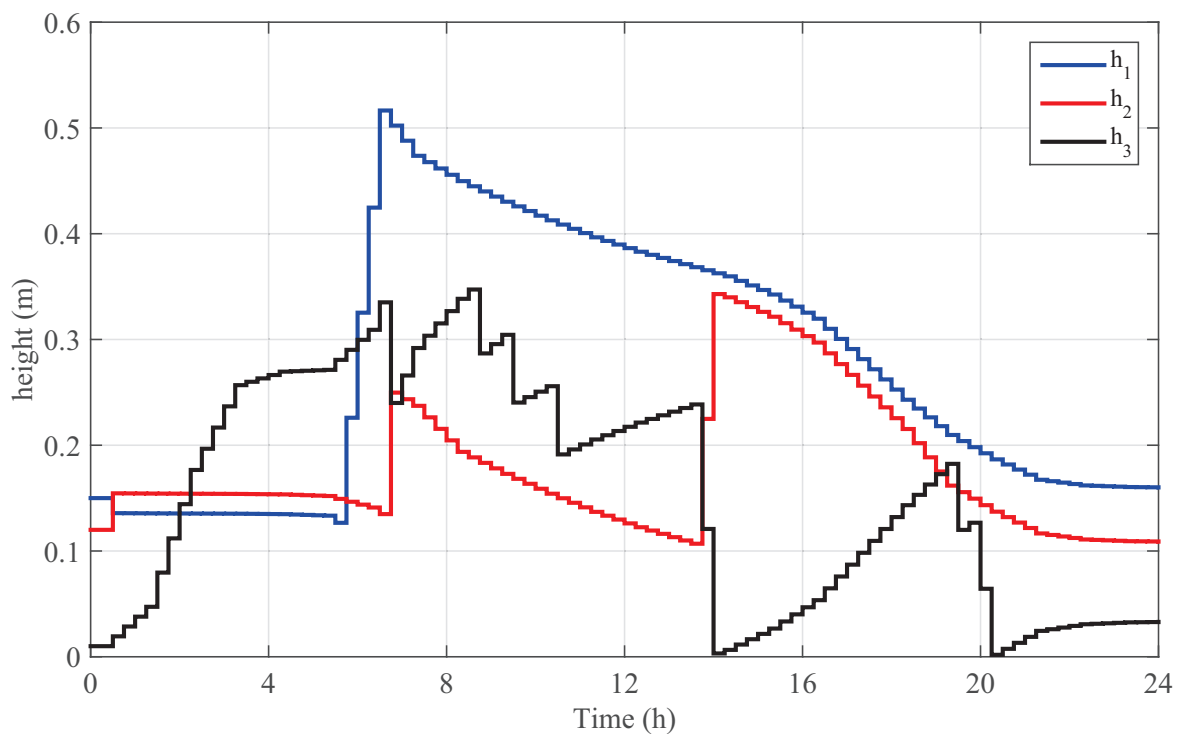


**Figure 6.5.** Optimal operation of pumps and valves using MPC strategy.

pumps during the cheaper off-peak TOU periods, in line with the utility's desire. Additionally, the

closed-loop controller ensures that both pumps are not switched on frequently in order to minimize the maintenance cost. In predicting increasing potable water demand in the same period as the peak TOU period, the closed-loop controller switches the potable water pump once at 05:30 for 1 hour. This water is enough to meet potable water demand in the house for the remaining period of the operating cycle. In addition, the controller operates the grey water pump twice, first at 06:30 for 15 minutes and later at 13:30 for 30 minutes. However, the solenoid valves are switched on at any time since they use negligible amount of power. The controller switches the potable water valve early in the morning at 00:15 for 15 minutes when there is insufficient collected and treated grey water, and yet there is grey water demand to be met. It also switches the drainage valve frequently as more water is collected during the operating cycle to ensure the tank is emptied.

Optimal operation of the pumps and valves leads to variation of water levels in various tanks as shown in Figure 6.6. The closed-loop controller also ensures that none of the constraints is violated. Similar



**Figure 6.6.** Variation of water level in respective tanks with MPC.

to the open-loop controller, the closed-loop controller predicts that the amount of stored potable water is not sufficient to meet the high potable water demand that coincides with the morning TOU peak. This situation is made worse, by insufficient collected and treated grey water in respective tanks forcing a 15 minute draw of potable water to meet grey water demand in the early morning. This prompts

the controller to operate the potable water pump at 05:30-06:30 raising the water level,  $h_1$ , to 0.52 m, which is enough to meet potable water demand for the remaining period of the operating cycle. To meet the early morning grey water demand, the controller has to operate the potable water valve,  $u_2$  for 15 minutes leading to a rise in level  $h_2$  to 0.15 m and a simultaneous drop of  $h_1$  to 0.14 m. By morning hours, enough water has been collected in the morning even though the demand for grey water is increasing during the peak TOU period. Consequently, the controller pumps water from the holding tank at 06:30 raising the water level in the grey tank to 0.25 m while at the same time leading to a drop of water level in the holding tank to 0.24 m. The treated grey water helps in meeting the morning water demand but unfortunately, it is not enough for the rest of the operating cycle. Therefore more water is treated and pumped to the grey tank at 13:30 for 30 minutes raising water level,  $h_2$ , to 0.34 m while also emptying the holding tank, as desired. Thereafter, the closed-loop controller predicts that the water in both storage tanks is sufficient to meet the demand for the remaining period of the operating cycle, hence, no more pumping is required. It therefore keeps operating the drainage valve and empties the tank again in the evening, in line with ensuring that the tank remains healthy and bacteria forms.

### 6.5.3 Analysis and discussion

The performance of the two optimal controllers is compared with the baseline, where potable water is used to meet all end uses in the house, over a period of one month. Table 6.3 shows the weekly water consumption, waste discharge and the associated cost in the baseline and the proposed water recycling and harvesting operated using either control strategies. The consumption of water presented in the table holds for both scenarios with reliable and unreliable municipal water supply. Baseline and proposed strategies columns show the cumulative amount of potable water consumed and waste water discharged from the house together with their respective unit price. The weekday or weekend cumulative water is the amount of either potable or treated grey water used in the house at the end of 5 week days or 2 days of the weekend respectively.

**Table 6.3.** Comparison of weekly water consumption.

Wk	Day	Baseline				Proposed strategies				$\Sigma Q_2 t_s u_2 (m^3)$		Treated water ( $m^3$ )	
		Potable ( $m^3$ )	Cost( $R/m^3$ )	Waste ( $m^3$ )	Cost( $R/m^3$ )	Potable ( $m^3$ )	Cost( $R/m^3$ )	Waste ( $m^3$ )	Cost( $R/m^3$ )	Open-loop	MPC	Open-loop	MPC
1	Weekday	5.80	6.81	5.68	5.06	4.47	6.81	4.38	5.06	0.05	0.05	0.18	0.18
	Weekend	8.14	9.72	7.81	6.83	6.34	9.72	6.19	6.83	0.13	0.13	0.18	0.18
2	Weekday	13.94	12.77	12.89	8.81	10.81	9.72	10.21	6.83	0.05	0.05	0.18	0.18
	Weekend	16.29	12.77	14.65	8.81	12.68	12.77	11.79	8.81	0.13	0.13	0.18	0.18
3	Weekday	22.09	14.77	18.53	8.81	17.15	12.77	15.14	8.81	0.05	0.05	0.18	0.18
	Weekend	24.43	16.89	19.90	8.81	18.89	14.77	16.31	8.81	0	0	0.26	0.26
4	Weekday	30.23	18.25	22.82	8.81	23.31	14.77	18.97	8.81	0	0	0.26	0.26
	Weekend	31.61	18.25	25.98	8.81	24.18	16.89	19.47	8.81	0	0	0.26	0.26

Besides,  $\sum Q_{2t_s} u_2 (m^3)$  is the amount of potable water used to supplement the grey water uses through valve  $u_2$  in each period of the week. It is evident that more potable water is consumed in the baseline than when using the water recycling and harvesting system controlled by either control systems. Consequently, more waste water is discharged from the baseline than from the proposed system. As a result, the household ends up paying for potable water at a maximum unit cost  $18.25 R/m^3$  in the baseline and as opposed to  $16.89 R/m^3$  in the proposed strategies at the end of the month. Similarly, the household currently (baseline) pays for waste water at the maximum  $8.81 R/m^3$  in a month from the second week while the proposed water conservation interventions would lead to the same happening from the weekend of the second week. Therefore, end users, whether with reliable or unreliable water supply, will have the added benefit of lower cost of potable and waste water in addition to conserving it. Operation of the proposed system by either control strategies consumes about  $0.05 m^3$  and  $0.13 m^3$  of potable water for grey uses in a week day and weekend, respectively, during the first two weeks. Thereafter,  $0.05 m^3$  is used during the week day of the third week. Up to this point, the cost of water has risen to  $12.77 R/m^3$ . Nonetheless, during the weekend of the third week, when the unit price rises to  $14.77 R/m^3$ , both controllers do not use potable water for grey end uses. This results from weight of the term responsible for minimizing the cost of water in objective functions (6.10) and (6.45) increasing significantly, making both controllers to give this term more preference as compared to the other terms. Further, the increasing weighting factor leads to an increase in the use of grey water from  $0.18 m^3$  to  $0.26 m^3$  for both controllers.

The monthly water and energy consumption, waste water discharge and the associated costs in the baseline and the two control strategies operating the water recycling and harvesting system are compared as shown in Table 6.4. If municipal water supply is reliable, the baseline water consumption is about  $31.61 m^3$  leading to a discharge of about  $25.98 m^3$  in a month. Therefore, the cost of both water supply and waste water discharge is about  $589.69 R/month$ , as there is no energy cost associated with pumping potable water. Adoption of the proposed system in such a house would reduce the monthly potable water consumption and waste discharge by about 32.3% and 29.5% respectively. This would lead to a total reduction of operational cost by 30.8%, irrespective of the added cost of energy incurred by the grey water pump. In the second scenario where potable water is pumped and stored, the baseline still uses  $31.61 m^3/month$  of potable water, discharges about  $25.98 m^3/month$  but at a higher cost of  $604.02 R/month$ , resulting from cost of pumping potable water to the storage tank. Incorporation of the proposed system would still conserve about 32.3% potable water, reduce discharge by up to 29.5% but lower the monthly cost water and waste water by 31.4%. In addition, the

**Table 6.4.** Water and energy consumption

	Baseline	Open-loop	MPC
<b>Potable water</b>			
Amount ( $m^3/month$ )	31.61	24.18	24.18
Cost ( $R/month$ )	395.15	267.46	267.46
<b>Potable pump</b>			
Energy ( $kWh/month$ )	13.04	8.00	8.00
Cost ( $R/month$ )	14.33	5.84	5.84
<b>Grey pump</b>			
Energy( $kWh/month$ )	0 <sup>a</sup>	3.25	3.25
Cost ( $R/month$ )	0 <sup>a</sup>	3.37	3.37
<b>Total cost (<math>R/month</math>)<sup>b</sup></b>	<b>409.48</b>	<b>276.67</b>	<b>276.67</b>

<sup>a</sup> The household was only using potable water.

<sup>b</sup> Cost of water and pumping energy.

two controllers can save the cost of energy by up to 35.7% through shifting the load to the cheaper off-peak periods of the TOU tariff. Eventually, the proposed water recycling and harvesting system with optimal control would save up to 31.5% of the total operation cost. This shows that the proposed system would conserve water, reduce waste water discharge and lead to economic benefits in both water supply scenarios.

Previous studies have shown that the two control strategies are known to adapt differently. The closed-loop MPC is stable and robust in dealing with disturbances [163], unlike the open-loop controller which can only deal with disturbances that do not largely change the demand pattern [225]. This, however, comes at a higher cost in terms of computation and financial as well as more complexity, since it would require extra components to enable feedback of the height of water in the tanks to take place [144]. Adoption of either controller depends on the nature of each application. The open-loop controller is suitable where the demand pattern does not change significantly, otherwise the closed-loop MPC is suitable.



Collection of grey water is dependent on human behaviour making it a more reliable alternative source of water for non-potable end uses. On the other hand, raining is a natural occurrence making rain water harvesting largely dependent on climatic conditions and weather patterns. Since both rain water and grey water are collected to the same holding tank, rain water would have greatest impact early in the morning when the holding tank is almost empty. This would enable the two controllers to use this other than using potable water, and would lower the cost of operation even further. Wide adoption of the system would greatly and positively influence the environment. The savings out of water conservation, waste water reduction and energy efficiency would immensely benefit municipal companies and energy utilities over a long time. In addition, the reduced demand for lower the pressure being experienced by municipalities and utilities in trying to cope with the huge demand for the two resources.

#### 6.5.4 Life cycle cost analysis

It is necessary to evaluate the feasibility of implementing any project, not only in terms of environmental benefits, but also based on economic effect. The cost effectiveness of implementing this water recycling and harvesting system is based on comprehensive consideration of various cost and revenue components. One effective method is the present worth method that discounts back all future elements of the financial analysis of a project to their present worth, apart from capital costs that are already given in present terms. Thereafter, the positive and negative elements of the cash flow are summed, and if the net present value (NPV) is positive, then the investment is financially attractive [254]. Life cycle cost (LCC) involves carrying out such analysis over the entire life of the project, and therefore has the benefit of capturing all costs and revenue that would take place during operation of the project. In this analysis, it is assumed that interest rate, taken as the inflation, revenue and operation cost are constant throughout the life of the project. Costs included in analysis of LCC include cost of acquisition, operation, maintenance and disposal [255]. Therefore,

$$LCC = C_c + C_o + C_s, \quad (6.63)$$

where  $C_c$  is the capital cost,  $C_o$  is the operation cost and  $C_s$  is the salvage cost at the end of life of the system. Capital cost includes total cost of acquiring and installing the system and labour. In the operation stage, the operation cost includes water, waste water, energy and maintenance cost incurred during the service life. Finally, salvage cost is the cost incurred at the end of system's life including the salvage value of the system, cost of removal and disposal [256]. Equation (6.63) can be written in terms of the discounting factor, that is, the factor by which future cash flows must be multiplied with

to get the present worth, as,

$$LCC = C_c + \sum_{n=1}^m \frac{C_o(n)}{(1+r)^n} + C_s, \quad (6.64)$$

where  $n$  and  $m$  are the number of years and project lifetime respectively while  $r$  is the discounting factor. The costs involved in this study are based on the South African market rates. Some assumptions are made while carrying out the life cycle cost analysis of the proposed system; the discounting factor is taken as South Africa's average inflation rate in 2016. The inflation rate, depreciation, operation and maintenance costs are assumed to be constant throughout the life of the system. The annualized cost and revenue are average from monthly values are obtained when the simulations are carried out over the four seasons in a year having varying demand for water. Table A.1 shows the life cycle cost analysis of the proposed system controlled using the MPC strategy. Expenses are indicated using negative values (brackets) while revenue is indicated as positive values. A discount factor of 6.56%, which is the average inflation rate of South Africa for 2016<sup>4</sup> is used to obtain the time value of money. In this analysis, all capital investment is taken to be done in the beginning of the project, and all components of the system will be operational for the 20 year life of the system. Further, cost of potable and waste water is assumed to remain constant for the entire life of the project. The discounted cash flows continuously increase the cumulative cash flows in each year, and the year which the cumulative cash flows becomes zero is an indicator of the break even point or the payback period. In this case, the proposed water recycling and harvesting system does not pay back in its 20 year life period, attributed to high capital cost coupled with low cost of water in South Africa, even though the country is semi arid. These findings are similar to another study done in two universities in South Africa [229], as well as in other parts of the world such as Ireland [230], Greece [231] and Austria [232]. A further study reveals that the current low water tariffs significantly influence end users' willingness to embrace water recycling. Government subsidies are therefore necessary in order to encourage the uptake of these technologies that will help in preventing water insecurity around the country and the region [15]. Even though the proposed strategy currently looks economically infeasible, it is important to conduct a thorough analysis while considering full cost of water supply and waste water treatment.

South Africa is a semi-arid country that has constantly struggled to provide reliable water supply to the population. In the recent past, the situation has become worse forcing municipalities to implement water restriction in various parts of the nation<sup>5</sup>. In addition, since the demand for water and energy

<sup>4</sup>[www.inflation.eu/inflation-rates/south-africa/historic-inflation/cpi-inflation-south-africa-2016.aspx](http://www.inflation.eu/inflation-rates/south-africa/historic-inflation/cpi-inflation-south-africa-2016.aspx)

<sup>5</sup>[www.droughtsa.org.za/about-the-drought/water-restrictions.html](http://www.droughtsa.org.za/about-the-drought/water-restrictions.html)

is expected to keep growing as the population increases, their will keep increasing and the proposed system could soon become economically feasible. The implication of water scarcity and increased pressure on existing infrastructure is evident in the City of Cape Town<sup>6</sup> where the municipality has opted to increase the cost of water and waste water in an effort to encourage efficient and sustainable use. The proposed water recycling and harvesting system is therefore an important intervention in ensuring reliable water supply, water conservation and energy efficiency are achieved.

## 6.6 SYNOPSIS

Water and energy, two inseparable resources, are vital for sustainable economic development of any country. Supply of these resources is however unreliable in South Africa due to various factors such as climatic factors, population increase, improved standards of living and rapid urbanization. This has led in increased demand surpassing existing supply capacity and various demand management strategies are required. Grey water recycling and rain water harvesting are suitable for conserving water by providing alternative sources hence reducing the demand for water and waste water services from the municipalities. Two controllers are designed in this study to optimally operate the grey water recycling and rain water harvesting system for a house. The open-loop and the closed-loop MPC controllers are designed using the TOU tariff in South Africa, where a case was considered. Although the open-loop controller is easier and more cost effective to implement, the closed-loop MPC is more robust and reliable in controlling the proposed system in domestic houses. The proposed system can potentially reduce potable water consumption by 32.3% and consequent waste water discharge by 29.5%. Optimal operation of the system using either controllers can reduce the cost of energy by 35.7% through load shifting. For the two scenarios considered in this study, that is reliable and unreliable municipal water supply, optimal operation of the proposed system can lead to a total operation cost saving of up to 31.5%. Despite the proposed system having the benefit to conserve water and efficiently use energy, the cost of implementation is high. Further, low water and waste water tariffs are making such systems economically infeasible. However, with increased demand and climate change, it is important for the government to look for ways to encourage adoption of such systems in order to conserve water and environment at large. This can be done through subsidizing and offering incentives to home owners. Wide adoption of the system would have huge benefit to the environment and municipalities that would not require to rapidly expand their existing supply and

---

<sup>6</sup>[www.westerncape.gov.za/general-publication/how-manage-water-restrictions-your-home](http://www.westerncape.gov.za/general-publication/how-manage-water-restrictions-your-home)

drainage infrastructure. This study forms the basis for future research into optimal operation of grey water recycling and rain water harvesting systems in houses to enhance water conservation and energy efficiency.

# **CHAPTER 7 OPTIMAL CONTROL OF HEAT PUMP WATER HEATER - INSTANTANEOUS SHOWER USING INTEGRATED RENEWABLE - GRID ENERGY SYSTEMS**

## **7.1 INTRODUCTION**

Renewable energy is increasingly being adopted by many countries in the world with the aim of reducing over-reliance on fossil fuels. However, remote areas in many developing nations, such as Africa and most islands developing nations are not connected to the grid. They are therefore relying on fossil fuel generators despite their high potential for renewable energy [257]. The negative environmental effect of fossil fuels, coupled with high fuel importation transport cost and dis-economies of scale in electricity production lead to exorbitantly high energy cost and long term financial risks for the economy [258]. Furthermore, increasing population is straining the existing energy infrastructure through increasing energy demand. By 2020, energy use by developing nations will grow at an average annual rate of 3.2% surpassing 1.1% in the developed countries [259]. Renewable energy technologies are a sustainable solution to providing cheaper and cleaner energy in these areas. For instance, in Maldives islands, hybrid solar and wind electricity generation systems have been proven to be financially feasible for supplementing the fossil fuel based grid [260]. Adoption of these hybrid renewable energy systems is faced with the challenge of designing an optimal energy management system that satisfies the load while considering the intermittent nature of renewable energy sources and variations in power demand [23].

Domestic water heating is the fourth largest energy user in commercial buildings, after heating, air conditioning, and lighting, and third largest in residential buildings [34]. Studies from various parts of the world reveal that domestic water heating, when compared with total energy consumption in buildings, is responsible for 18% in the USA [261], 25% in UK [262], 26% in Spain [259], 30% in Australia [263] and 20% in Brazil [264]. This has led to increased effort in improving energy efficiency of domestic hot water systems, whose level determines the energy saving potential [265]. Heat pumps offer economically attractive choices as they recoup heat from various industrial, commercial and residential applications [266]. Heat pump water heaters (HPWHs) operate on the principle of the refrigerant cycle converting one unit of electrical energy to produce three units of thermal energy [267]. Despite the superiority, improvement of its performance, reliability, and environmental impact have been a concern [266], while optimal operation and integration of the technologies remains a challenge [67]. These challenges, coupled with high initial investment cost especially in developing countries have hindered their uptake. For instance, in South Africa the market penetration of HPWH is 16% [268].

Various studies have looked at ways to achieve energy efficiency using HPWHs. Kreuder and Spataru [269] showed that heat pumps can indeed be used to enhance energy efficiency in homes while Aste et al. [270] showed their economic feasibility. Further, various control algorithms have shown their potential in reducing the delivered energy and its cost but predictive control algorithm was the most effective [271]. Integration of distributed renewable energy provides a huge potential in powering HPWHs further increasing the energy savings [65]. Therefore, an optimal controller operating a grid tied photovoltaic (PV) and diesel generator integrated system applicable in areas with intermittent power supply was used to power HPWHs [67]. In another study, an optimal power dispatch model of a grid tied photovoltaic system was used to power HPWH. The control system showed energy savings potential and ability to use the stored energy in the battery in case of power black out, or during peak time [68]. An optimal controller for the HPWH powered using integrated wind generator-photovoltaic-grid system led to energy savings further improving the energy efficiency of the HPWH while feeding back excess power back to the grid through a suitable feed-in tariff [66]. This study was advanced by incorporating a fuel cell which improved the reliability of the intermittent renewable power supplies. The optimal control strategy showed the possibility of integrating renewable energy systems and energy efficient systems [69]. Despite the superiority of HPWHs to other water heating technologies, they have a slow rate of heating water. Consequently, in cases with high demand for hot water, HPWHs are unable to supply it. Further, since HPWHs are normally centrally located in the house, energy

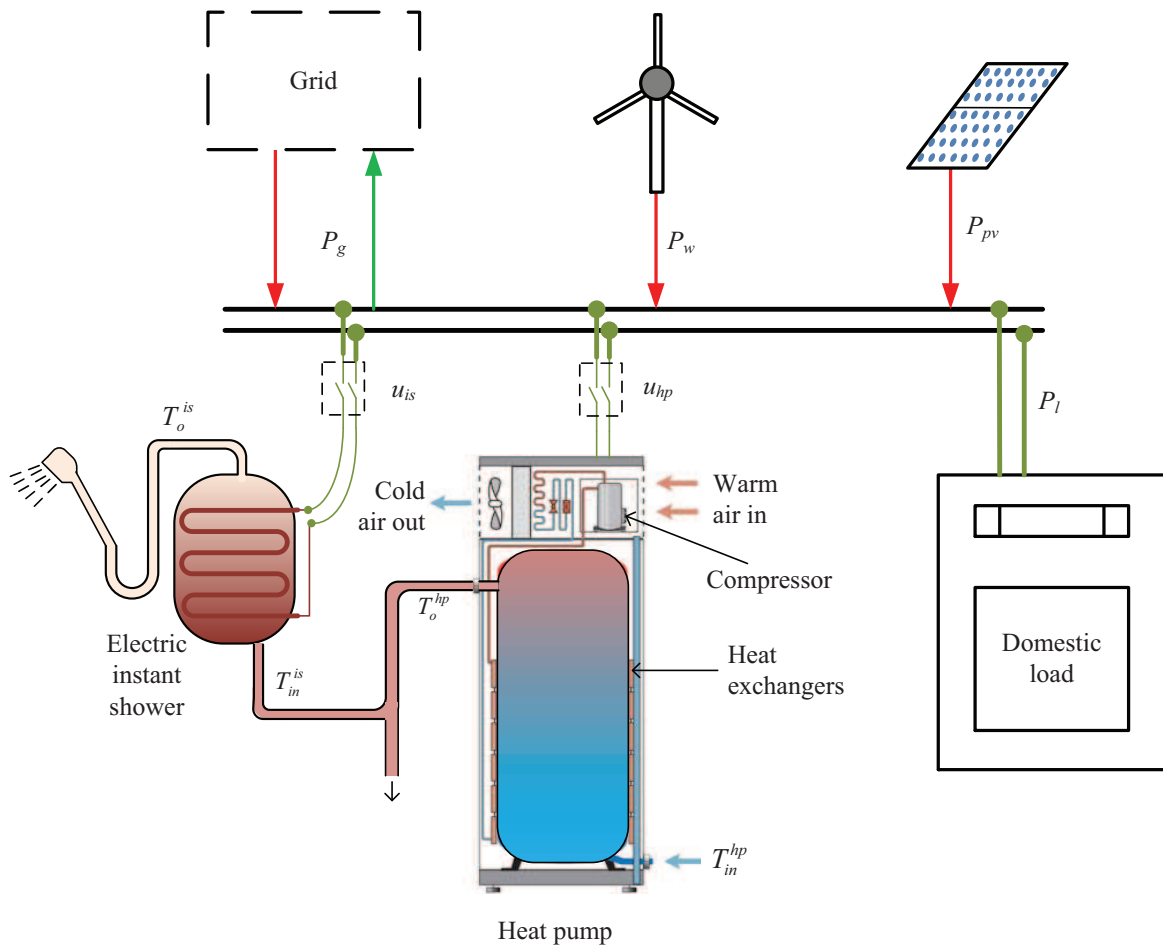
and water losses associated with the hot water conveyance to the consumption point occur as the cold water in the pipe is wasted while waiting for hot water at the point of use. Upon finishing with the use, the remaining water in the pipe quickly cools down [272]. To address these challenges, instantaneous water heaters, placed at the consumption point, can be used [273]. Although previous studies have mainly concentrated on energy demand management, the demand management of energy-water nexus gaining more attention in developing nations [147], with instantaneous heaters proposed as a suitable option [274].

This chapter introduces a novel and economical optimal control strategy that ensures both energy and water are efficiently consumed by using HPWH and instantaneous shower to conveniently meet domestic hot water demand. The shower end use is selected in this study as it is one of the most hot water intensive end uses in homes. The control system consists of integrated wind-photovoltaic renewable energy powering the hot water devices, needing grid power whenever they are insufficient. However, whenever renewable energy is in excess, it is sold back to the grid through an appropriate feed-in tariff. Therefore, the aim of the controls is to optimally operate both hot water devices, maximizing the use of renewable energy effectively ensuring the customer incurs the least cost of electricity. The controls further minimize water loss during conveyance to the shower.

## 7.2 CONTROLLER FORMULATION

### 7.2.1 Schematic layout

Figure 7.1 shows the schematic diagram of the water heating system comprising of the HPWH and the instantaneous shower. The water heating devices are powered using photovoltaic solar,  $P_{pv}$ , and wind generator,  $P_w$ , while grid,  $P_g$ , acts as a back whenever the renewable sources are insufficient. The HPWH, centrally located in the house, meets the total hot water demand. The instantaneous shower is placed in the shower to act as back up whenever the water from the HPWH is not at the required temperature. Switches  $u_{hp}$  and  $u_{is}$  control the power flow to HPWH and instantaneous shower respectively. The grid supplements power from the renewable sources as well as accepting excess power back.



**Figure 7.1.** Schematic layout of the energy and hot water flow

### 7.2.2 Wind energy

Wind energy is one of the integrated renewable energy system used to power the hot water devices. Whenever there is excess wind energy than that required by the hot water devices, it is fed back to the grid using an appropriate feed-in tariff. The power output of a typical wind turbine is proportional to the cubed wind speed as long as this speed is between the cut in wind speed,  $V_i$  and rated wind speed,  $V_N$ . For a simplified model of a wind generator, the power output,  $P_w$ , at the rated wind speed is [275],

$$P_w = 0.5\eta_t\eta_g\rho_a C_p A_w V_r^3, \quad (7.1)$$

where  $\eta_t$  and  $\eta_g$  are the gearbox and generator efficiency respectively.  $\rho_a$  is the density of air ( $kg/m^3$ ),  $C_p$  is the power coefficient of the turbine,  $A_w$  is the sweeping area of the turbine rotor ( $m^2$ ) while  $V_r$  is the speed of wind ( $m/s$ ). Whenever the wind speed is above  $V_N$ , the aerodynamic efficiency is reduced



by pitching the blades so that shaft power remains constant. However, if the wind speed exceeds the pitch control limit, it reaches the cut-out wind speed,  $V_c$ , and power production is stopped [276].

### 7.2.3 Photovoltaic solar energy

A solar cell is the basic component used to directly transform sunlight into electricity. The series connection of several of these cells form a module while series-parallel connection of modules form an array [277]. The power generated by a PV array is given as,

$$P_{pv} = \eta_{pv} A_c I_{pv}, \quad (7.2)$$

where  $A_c$  is the area of the PV array,  $I_{pv}$  is the solar irradiation incident on the PV array ( $kWh/m^2$ ) while  $\eta_{pv}$  is the efficiency of the PV generator, which is dependent on  $I_{pv}$  and ambient temperature,  $T_A$ , [80]. The solar irradiation varies depending on the time of the day such that,

$$I_{pv} = R_B(I_B + I_D) + I_D, \quad (7.3)$$

where  $R_B$  is a geometric factor representing the ratio of beam irradiance incident on a tilted plane to that incident on a horizontal plane.  $I_B$  and  $I_D$  are the hourly global and diffuse irradiation ( $kWh/m^2$ ) respectively, [278]. The PV array is also a part of the integrated renewable energy system supplying power to the hot water devices. Just like the case of wind power, excess PV power is also sold back to the grid.

### 7.2.4 Grid energy

The grid is modelled as an infinite bar capable of supplying power to hot water devices whenever renewable energy is insufficient. It is also capable of accepting excess renewable power to meet other energy demands within its network. In supply mode, the grid electricity's price is structured as a time-of-use (TOU) tariff. In this study, Eskom's TOU Homeflex structure is used [225], The hourly price of electricity,  $p_e(t)$ , being,

$$p_e(t) = \begin{cases} p_{off} = 0.5510 \text{ R/Kwh} & \text{if } t \in [0, 6] \cup [10, 18] \cup [20, 24], \\ p_{peak} = 1.7487 \text{ R/Kwh} & \text{if } t \in [7, 10] \cup [18, 20], \end{cases} \quad (7.4)$$

where  $p_{off}$  is the off peak price,  $p_{peak}$  is the peak price,  $R$  is the South African currency, Rand, and  $t$  is the time of day in hours. The tariff has five charge components as service charge, network charge, environmental levy, peak charge and off-peak charges [70]. The National Energy Regulator of South

Africa (NERSA) revised the renewable energy feed in tariff (REFIT) in 2013 with the prevailing REFIT for wind and solar photovoltaic being 1.25 R/kWh and 3.94 R/kWh respectively [39].

The hourly power balance for meeting the demand of the hot water devices is modelled as,

$$P_{hp}u_{hp}(t) + P_{is}u_{is}(t) + P_l(t) = P_g(t) + P_w(t) + P_{pv}(t), \quad (7.5)$$

where  $P_{hp}$  and  $P_{is}$  are the power rating (kW) of the heat pump and instantaneous shower respectively, whose on/off status are represented as  $u_{hp}(t)$  and  $u_{is}(t)$  respectively.  $P_l(t)$  is the hourly power (kW) demand from other domestic load.

### 7.2.5 Heat pump water heater

A HPWH is generally composed of heat pumping and hot water reservoir parts. Although mechanical and thermal inertia are important in modelling the heat pumping part, HPWHs take much shorter time in stabilizing from mechanical inertia than thermal inertia. This means that the suction and discharge conditions of the heat pump can be used to model the steady state characteristics of the HPWH if the compressor operates in a constant speed condition [279]. The hot water reservoir stores the hot water heated upon being forced to circulate through the condenser absorbing the heat [280]. Whenever hot water flows from the reservoir to meet the demand, the reservoir is re-filled with cold water. In order to simplify the model, energy losses in the evaporator, refrigerant and compressor are neglected. However, the overall efficiency the thermal components are accounted for by the coefficient of performance (COP) obtained in the case study. It is also assumed that the temperature of water throughout the reservoir is uniform. Hence, energy losses considered are standby losses and losses associated to the hot water demand.

Standby power losses ( $Q_l$ ) are thermal losses to the environment due to heat conduction through the surface of the tank and natural convection that transfers heat from the surface of the tank to the environment. These losses can be reduced through the use of improved thermal insulated material [281].

Standby losses are modelled as,

$$Q_l = A_{hp} \left( \frac{T_{hp} - T_a}{R_{hp}} \right), \quad (7.6)$$

where  $A_{hp}$  is the surface area of the HPWH's tank ( $m^2$ ) while  $T_{hp}$  and  $T_a$  are the hot water and ambient temperatures respectively ( $^{\circ}C$ ).  $R_{hp}$  is the thermal resistance of the insulation material ( $m^2K/W$ ),

which can be written as,

$$R = \frac{\Delta x}{k} + \frac{1}{h}, \quad (7.7)$$

where  $\Delta x$  is the thickness of the insulation material ( $m$ ) while  $k$  and  $h$  are the coefficients of thermal conductivity ( $W/mk$ ) and surface heat transfer ( $W/m^2k$ ) respectively [282].

Hot water demand causes hot water to flow out of the tank which is consequently replaced by the same volume of cold water [283]. This flow of water in and out of the tank causes a drop in the average temperature,  $T_{hp}$ , of the hot water in the tank [284]. Thermal power losses due to water flow,  $Q_d$ , can be modelled as,

$$Q_d = c_w D_{tot} (T_{hp} - T_{hp}^{in}), \quad (7.8)$$

where  $c_w$  is the specific heat capacity of water ( $J/(kg \cdot ^\circ C)$ ).  $T_{hp}^{in}$  is the temperature of the cold water into the tank ( $^\circ C$ ) and  $D_{tot}$  is the total hot water demand, given as the mass flow rate of hot water ( $kg/h$ ).

The electrical power required from the HPWH,  $Q_h$ , to maintain water at the required temperature while overcoming the above losses is [285],

$$Q_h = COP \times P_{hp}, \quad (7.9)$$

where  $COP$  is the coefficient of performance. In meeting the hot water demand, the dynamic model of HPWH is based on the open energy balance [286]. The resulting differential equation describing the average thermal response of water in the tank is [287],

$$c_w m_{hp} \frac{dT_{hp}}{dt} = Q_h - Q_l - Q_d, \quad (7.10)$$

where  $m_{hp}$  is the mass of water ( $kg$ ) in HPWH's storage tank. By substituting for  $Q_h$ ,  $Q_l$  and  $Q_d$ , equation (7.10) now becomes,

$$c_w m_{hp} \frac{dT_{hp}}{dt} = (COP)P_{hp}u_{hp}(t) - A_{hp} \left( \frac{T_{hp}(t) - T_a}{R} \right) - c_w D_{tot}(t) (T_{hp}(t) - T_{hp}^{in}(t)). \quad (7.11)$$

In order to simplify the modelling process, the above differential equation can be written as

$$\frac{dT_{hp}}{dt} = -\alpha(t)T_{hp}(t) + \beta u_{hp}(t) + \gamma(t), \quad (7.12)$$

where

$$\alpha(t) = \frac{A_{hp}}{c_w m_{hp} R} + \frac{D_{tot}(t)}{m_{hp}}, \quad (7.13a)$$

$$\beta = \frac{(COP)P_{hp}}{c_w m_{hp}}, \quad (7.13b)$$

$$\gamma(t) = \frac{A_{hp}T_a}{c_w m_{hp} R} + \frac{D_{tot}(t)T_{hp}^{in}(t)}{m_{hp}}. \quad (7.13c)$$

Differential equation (7.12) can be expressed in discrete-time domain such that the temperature of water in HPWH's storage at  $j^{th}$  sampling interval becomes,

$$T_{hp}(j+1) = (1 - t_s \alpha(j))T_{hp}(j) + t_s \beta u_{hp}(j) + t_s \gamma(j), \quad (7.14)$$

where  $t_s$  is the sampling period ( $h$ ). The status of the HPWH's switch,  $u_{hp}(j)$ , is such that,

$$u_{hp}(j) \in \{0, 1\} \quad 1 \leq j \leq N, \quad (7.15)$$

where  $N$  is the total number of samples in a 24-h operating cycle such that  $N = \frac{24}{t_s}$ . Through recurrence manipulation, equation (7.14) can be expressed in terms of the initial temperature,  $T_{hp}(0)$ , as,

$$T_{hp}(j) = T_{hp}(0) \prod_{i=1}^j (1 - t_s \alpha(i)) + \beta t_s \sum_{i=1}^j u_{hp}(i) \prod_{k=i+1}^j (1 - t_s \alpha(k)) + t_s \sum_{i=1}^j \gamma(i) \prod_{k=i+1}^j (1 - t_s \alpha(k)),$$

$$1 \leq j \leq N. \quad (7.16)$$

## 7.2.6 Instantaneous shower

Instantaneous water heaters, also called demand or tank-less water heaters, have heating elements that are activated by the flow of water thereby heating the water instantly as it passes through [288]. By virtue of heating water on demand, they require larger power input than the storage water heaters [289]. They are either electric, gas or propane powered [290], though this chapter focuses on electric powered instantaneous showers. Central storage water heaters lead to energy and water wastage in supplying remote end uses such as shower. The tap has to run until hot water arrives and the remaining hot water in the pipe after the tap is closed becomes cold quickly. In such a case, instantaneous shower would improve the efficiency of these resources by nearly eliminating distribution and standby losses. They also offer a perfect candidate to support central storage hot water systems that cannot conveniently meet the hot water demand such as central solar tanks and HPWHs [273]. The instantaneous shower is modelled in two states;

### 7.2.6.1 Active state

Whenever there is demand for shower water, hot water from the HPWH is flowing. In this study, we assume there is negligible heat loss as the water flows along the pipes to the instantaneous shower. Similar to the HPWH, the energy balance of the instantaneous shower in active state can be represented

by a first order differential equation as,

$$c_w m_{is} \frac{dT_{is}}{dt} = \eta_{is} P_{is} u_{is}(t) - A_{is} \left( \frac{T_{is}(t) - T_a}{R_{is}} \right) - c_w D_{is}(t) (T_{is}(t) - T_{is}^{in}(t)), \quad (7.17)$$

where  $m_{is}$  is the mass of water inside the instantaneous shower's chamber ( $kg$ ),  $\eta_{is}$  is the efficiency of the heating element rated at  $P_{is}$  in  $kW$ .  $A_{is}$  is the surface area of the instantaneous shower while  $R_{is}$  is the thermal resistance of the shower's material and  $T_{is}^{in}$  is the temperature of water ( $^{\circ}C$ ) into the instantaneous shower.  $u_{is}(t)$  is the state of the instantaneous shower's switch at time,  $t$ , while  $D_{is}(t)$  is the hot water demand in the shower ( $kg/hr$ ). Just like in equation (7.11), the second and third terms of the right hand side of equation (7.17) represent the standby and water usage thermal losses respectively [291]. Differential equation (7.17) can be simplified to,

$$\frac{dT_{is}}{dt} = -\phi(t)T_{is}(t) + \lambda u_{is}(t) + \zeta(t), \quad (7.18)$$

where

$$\phi(t) = \frac{A_{is}}{c_w m_{is} R_{is}} + \frac{D_{is}(t)}{m_{is}}, \quad (7.19a)$$

$$\lambda = \frac{\eta P_{is}}{c_w m_{is}}, \quad (7.19b)$$

$$\zeta(t) = \frac{A_{is} T_a}{c_w m_{is} R_{is}} + \frac{D_{is}(t) T_{is}^{in}(t)}{m_{is}}. \quad (7.19c)$$

Discretizing equation (7.18) yields,

$$T_{is}(j+1) = (1 - t_s \phi(j))T_{is}(j) + t_s \lambda u_{is}(j) + t_s \zeta(j), \quad (7.20)$$

where the status of the instantaneous shower's switch,  $u_{is}(j)$ , is such that,

$$u_{is}(j) \in \{0, 1\} \quad 1 \leq j \leq N. \quad (7.21)$$

Expressing equation (7.20) in terms of the initial temperature,  $T_{is}(0)$ , of water in the instantaneous shower becomes,

$$T_{is}(j) = T_{is}(0) \prod_{i=1}^j (1 - t_s \phi(i)) + \lambda t_s \sum_{i=1}^j u_{is}(i) \prod_{k=i+1}^j (1 - t_s \phi(k)) + t_s \sum_{i=1}^j \zeta(i) \prod_{k=i+1}^j (1 - t_s \phi(k)), \quad (7.22)$$

$$1 \leq j \leq N.$$

### 7.2.6.2 Idle state

The idle state occurs whenever the instantaneous shower is not in use. Unlike the HPWH that takes the inlet water at the ambient temperature, the water into the instantaneous shower is already heated by the HPWH. However, in the idle state, there is no demand for the shower implying that the water is stagnant in the pipes between HPWH and instantaneous shower as well as in the instantaneous shower's small reservoir. Assuming that the water in the pipe is losing heat at the same rate as that in

the instantaneous shower, the differential equation governing the behaviour of water in the instantaneous shower can be modelled as [292],

$$c_w m_{is} \frac{dT_{is}}{dt} + A_{is} \left( \frac{T_{is}(t) - T_a}{R_{is}} \right) = 0. \quad (7.23)$$

### 7.2.7 Optimization problem

The aim of the controller is to minimize the cost of grid power consumed as well the use of instantaneous shower in meeting the hot water demand. In this chapter, we consider an evaluation period of a 24-h operating cycle from 0 to hour 24 with a sampling period  $t_s = 15$  min. The objective function,  $J$ , can therefore be expressed as,

$$J = \omega \sum_{j=1}^N t_s p_e(j) P_g(j) + (1 - \omega) \sum_{j=1}^N t_s P_{is} u_{is}(j) \quad (7.24)$$

where  $\omega$  is a weighting factor chosen to indicate the relative importance of minimizing each term in the objective function [293]. The first term seeks to minimize the cost of grid power consumed by the hot water devices while the second term minimizes the use of instantaneous shower. The objective function is subject to the following technical and operational constraints,

$$P_w(j) + p_g(j) + P_{pv}(j) = P_{hp} u_{hp}(j) + P_{is} u_{is}(j) + P_l(j), \quad (7.25)$$

$$T_{hp}^{min} \leq T_{hp}(j) \leq T^{max}, \quad (7.26)$$

$$T_{is}^{min} \leq T_{is}(j) \leq T^{max}, \quad (7.27)$$

$$-\infty \leq P_g(j) \leq \infty, \quad (7.28)$$

$$u_{hp}(j) \in \{0, 1\}, \quad (7.29)$$

$$u_{is}(j) \in \{0, 1\}, \quad (7.30)$$

where  $T_{hp}^{min}$  and  $T_{is}^{min}$  are the minimum allowable temperature ( $^{\circ}C$ ) of the HPWH and the instantaneous shower respectively, while  $T^{max}$  is the maximum temperature ( $^{\circ}C$ ) which is the same for both devices. Equality constraint (7.25) represents the power balance in the system, whereby, power from renewable energy sources and grid are used to meet power demand from hot water devices as well as domestic load at every sampling interval,  $j$ . Constraints (7.26) and (7.27) show the state variables, that is, temperature of water from HPWH and instantaneous shower respectively, must be between set minimum and maximum acceptable temperature in every sampling interval,  $j$ . Constraints (7.28)-(7.30) show how control variables are bound. Grid power,  $P_g$ , is bound such that, even though it provides power, it can

also accept power back from the renewable sources. Finally, the status of the HPWH and instantaneous shower switches can take the values of either 0 or 1 representing off and on status respectively.

### 7.2.7.1 Algorithm

Objective function (7.24) is solved using the canonical form [56].

$$\min f^T X \quad (7.31)$$

subject to

$$\begin{cases} AX \leq b \text{ (linear inequality constraint),} \\ A_{eq}X = b_{eq} \text{ (linear equality constraint),} \\ L_B \leq X \leq U_B \text{ (lower and upper bounds).} \end{cases} \quad (7.32)$$

Vector  $X$  contains all control variables such that,

$$X = [u_{hp}(1) \ \dots \ u_{hp}(N) \ u_{is}(1) \ \dots \ u_{is}(N) \ P_g(1) \ \dots \ P_g(N)]_{3N \times 1}^T, \quad (7.33)$$

meaning

$$f^T = [0, \ \dots, \ 0, \ (1-\omega)t_s P_{is} \ \dots \ (1-\omega)t_s P_{is} \ \omega t_s p_e(1) \ \dots \ \omega t_s p_e(N)]_{1 \times 3N}. \quad (7.34)$$

From the power balance equality constraint (7.28), matrix  $A_{eq}$  becomes,

$$A_{eq} = \left[ \begin{array}{cccc|cccc|cccc} P_{hp} & 0 & \dots & 0 & P_{is} & 0 & \dots & 0 & -1 & 0 & \dots & 0 \\ 0 & P_{hp} & \dots & 0 & 0 & P_{is} & \dots & 0 & 0 & -1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & P_{hp} & 0 & 0 & \dots & P_{is} & 0 & 0 & \dots & -1 \end{array} \right]_{N \times 3N}, \quad (7.35)$$

and vector  $b_{eq}$ ,

$$b_{eq} = \begin{bmatrix} P_w(1) + P_{pv}(1) - P_l(1) \\ \vdots \\ P_w(N) + P_{pv}(N) - P_l(N) \end{bmatrix}_{N \times 1}. \quad (7.36)$$

The temperature of hot water from HPWH modelled in inequality constraint (7.26) can be written as,

$$\begin{aligned} A_1 X &\leq b_1, \\ -A_1 X &\leq b_2. \end{aligned} \quad (7.37)$$

By letting  $(\sigma(j) = 1 - t_s \alpha(j))$ , matrix  $A_1$ , with a dimension of  $(N \times 3N)$  is,

$$A_1 = -\beta t_s \times \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \sigma(1) & 1 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \sigma(2)\sigma(1) & \sigma(2) & 1 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \{\sigma(N-2) \times \dots \times \sigma(1)\} & \{\sigma(N-2) \times \dots \times \sigma(2)\} & \dots & \dots & 1 & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \{\sigma(N-1) \times \dots \times \sigma(1)\} & \{\sigma(N-1) \times \dots \times \sigma(2)\} & \dots & \dots & \sigma(N-1) & 1 & 0 & \dots & 0 & 0 & \dots & 0 \end{bmatrix}. \quad (7.38)$$

Further, vector  $b_1$  ( $N \times 1$ ) can be written as,

$$b_1 = -b_{1,1} + b_{1,2} + b_{1,3}, \quad (7.39)$$

where,

$$b_{1,1} = \begin{bmatrix} T_{hp}^{min} & \dots & T_{hp}^{min} \end{bmatrix}_{N \times 1}^T, \quad (7.40)$$

$$b_{1,2} = T_{hp}(0) \begin{bmatrix} \sigma(1) \\ \sigma(2)\sigma(1) \\ \sigma(3)\sigma(2)\sigma(1) \\ \vdots \\ \sigma(N)\sigma(N-1)\dots\sigma(1) \end{bmatrix}_{N \times 1} \quad (7.41)$$

and

$$b_{1,3} = t_s \begin{bmatrix} \gamma(1) \\ \sigma(2)\gamma(1) + \gamma(2) \\ \sigma(3)\sigma(2)\gamma(1) + \sigma(3)\gamma(2) + \gamma(3) \\ \vdots \\ \{\sigma(N)\sigma(N-1)\dots\sigma(2)\}\gamma(1) + \dots + \sigma(N-2)\gamma(N-1) + \gamma(N) \end{bmatrix}_{N \times 1}, \quad (7.42)$$

while vector  $b_2$  ( $N \times 1$ ) becomes,

$$b_2 = b_{2,1} - b_{1,2} - b_{1,3}, \quad (7.43)$$

where,

$$b_{2,1} = \begin{bmatrix} T_{max} & \dots & T_{max} \end{bmatrix}_{N \times 1}^T. \quad (7.44)$$

Similarly, the temperature of hot water from the instantaneous shower in inequality (7.27) becomes,

$$\begin{aligned} A_2 X &\leq b_3, \\ -A_2 X &\leq b_4. \end{aligned} \quad (7.45)$$



By letting  $(\psi = 1 - t_s \phi(j))$ , matrix  $A_2$  ( $N \times 3N$ ) becomes,

$$A_2 = -\lambda t_s \times \begin{bmatrix} 0 & \dots & 0 & | & 1 & & 0 & & 0 & \dots & 0 & & 0 & & 0 & & 0 & & 0 & \dots & 0 \\ 0 & \dots & 0 & | & \psi(1) & & 1 & & 0 & \dots & 0 & & 0 & & 0 & & 0 & & 0 & \dots & 0 \\ 0 & \dots & 0 & | & \psi(2)\psi(1) & & \psi(2) & & 1 & \dots & 0 & & 0 & & 0 & & 0 & & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & | & \vdots & & \vdots & & \vdots & \ddots & \vdots & & \vdots & & \vdots & & \vdots & & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & | & \{\psi(N-2) \times \dots \times \psi(1)\} & & \{\psi(N-2) \times \dots \times \psi(2)\} & & \dots & \dots & 1 & & 0 & & 0 & & 0 & & 0 & \dots & 0 \\ 0 & \dots & 0 & | & \{\psi(N-1) \times \dots \times \psi(1)\} & & \{\psi(N-1) \times \dots \times \psi(2)\} & & \dots & \dots & \psi(N-1) & & 1 & & 0 & & 0 & & 0 & \dots & 0 \end{bmatrix}. \quad (7.46)$$

Vector  $b_3$  ( $N \times 1$ ) now becomes,

$$b_3 = -b_{3,1} + b_{3,2} + b_{3,3}, \quad (7.47)$$

where,

$$b_{3,1} = \begin{bmatrix} T_{is}^{min} & \dots & T_{is}^{min} \end{bmatrix}_{N \times 1}^T, \quad (7.48)$$

$$b_{3,2} = T_{is}(0) \begin{bmatrix} \psi(1) \\ \psi(2)\psi(1) \\ \psi(3)\psi(2)\psi(1) \\ \vdots \\ \psi(N)\psi(N-1)\dots\psi(1) \end{bmatrix}_{N \times 1} \quad (7.49)$$

and

$$b_{3,3} = t_s \begin{bmatrix} \zeta(1) \\ \psi(2)\zeta(1) + \zeta(2) \\ \psi(3)\psi(2)\zeta(1) + \psi(3)\zeta(2) + \zeta(3) \\ \vdots \\ \{\psi(N)\psi(N-1)\dots\psi(2)\}\zeta(1) + \dots + \psi(N-2)\zeta(N-1) + \zeta(N) \end{bmatrix}_{N \times 1}. \quad (7.50)$$

while vector  $b_4$  ( $N \times 1$ ) becomes,

$$b_4 = b_{2,1} - b_{3,2} - b_{3,3}. \quad (7.51)$$

Therefore, matrix  $A$  and vector  $b$  in canonical form (7.32) are,

$$A = \begin{bmatrix} A_1 \\ -A_1 \\ A_2 \\ -A_2 \end{bmatrix}_{4N \times 3N}, \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}_{4N \times 1}. \quad (7.52)$$

Finally, the bounds of vector  $X$  given in canonical form (7.32) become,

$$L_B = \begin{bmatrix} 0 & \dots & 0 & | & 0 & \dots & 0 & | & -\infty & \dots & -\infty \end{bmatrix}_{3N \times 1}^T, \quad (7.53)$$

and

$$U_B = \left[ 1 \quad \dots \quad 1 \quad | \quad 1 \quad \dots \quad 1 \quad | \quad \infty \quad \dots \quad \infty \right]_{3N \times 1}^T. \quad (7.54)$$

This optimization problem is solved using the SCIP solver in optimization interface (OPTI) toolbox, which is a Matlab toolbox for solving optimization problems.

## 7.3 GENERAL DATA

### 7.3.1 Case study

A case of a farm house in Port Elizabeth, South Africa, where an air source HPWH is used to provide hot water, whose temperature is controlled using a thermostat, is considered in this chapter. The parameters of this HPWH are shown in Table 7.1. In order to satisfy the demand for hot water, the

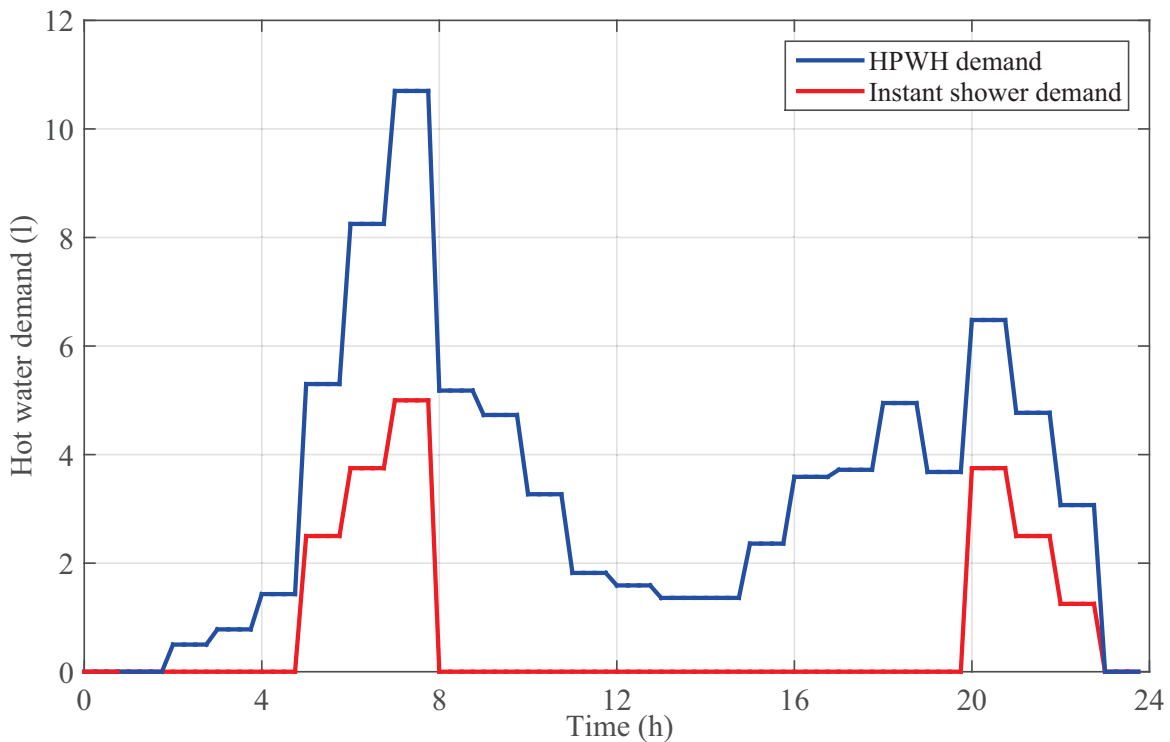
**Table 7.1.** Parameters of the HPWH

Power (kW)	COP	Volume (l)	$\Delta x$ (m)	k (W/mK)	h (W/m <sup>2</sup> K)
6	3.8	260	0.035	0.055	6.3

thermostat has been set at 50 °C. Further, the plumbing pipes for transporting the hot water in the house are 1/2 inch in diameter, with the length of the pipes from the location of the HPWH to the shower being about 25 m. In this study, Stiebel Eltron IS 60E<sup>1</sup> instantaneous shower, rated at 8.5 kW, is used. Since the shower is the end use requiring hottest water from the HPWH, the instantaneous shower is set to have the temperature of the hot water such that  $47 \leq T_{is} \leq 50$  while the HPWH can be set at  $45 \leq T_{hp} \leq 50$ .

The hourly hot water demand for the house was measured, with Figure 7.2 showing the demand pattern for a period of one day. Further, the occupants provided the information on their showering behaviour in a day, that is, duration in the shower and the average time of the day they take their shower. Using the average flow rate of the low flow shower head already installed in the house, the shower demand in a day was obtained as shown in Figure 7.2. It can be seen that the demand for hot water is highest in the morning, between 06:00-10:00 which is normally caused by the occupants waking up and preparing to go to work or school. The demand also peaks in the evening hours between 16:00-20:00 caused by

<sup>1</sup><http://www.stiebel-eltron.co.za/is60e.html>



**Figure 7.2.** Overall and shower hot water demand pattern.

the occupants who are back in the house. The demand for hot water in the shower also has a pattern similar to the overall hot water demand.

The hourly temperature variation, together with the ambient temperature for this region was obtained from the Southern African Universities Radiometric Network<sup>2</sup>.

### 7.3.1.1 Renewable energy

Wind power is part of the input power into this controller. Therefore, a 3.5 kW Raam wind turbine<sup>3</sup> is chosen as the wind generator. The turbine has a rated wind speed of 11 m/s, with the cut-in and cut-out wind speeds being 2.8 m/s and 22 m/s respectively while the rotor blades have a sweeping area of 12.6 m<sup>2</sup>. The gearbox and generator efficiency are taken as 0.9 and 0.8 respectively. The average hourly wind speed pattern for a typical day in Port Elizabeth is given by the Southern African Universities Radiometric Network<sup>2</sup>.

<sup>2</sup><http://www.sauran.net/>

<sup>3</sup>[www.raumenergy.com](http://www.raumenergy.com)

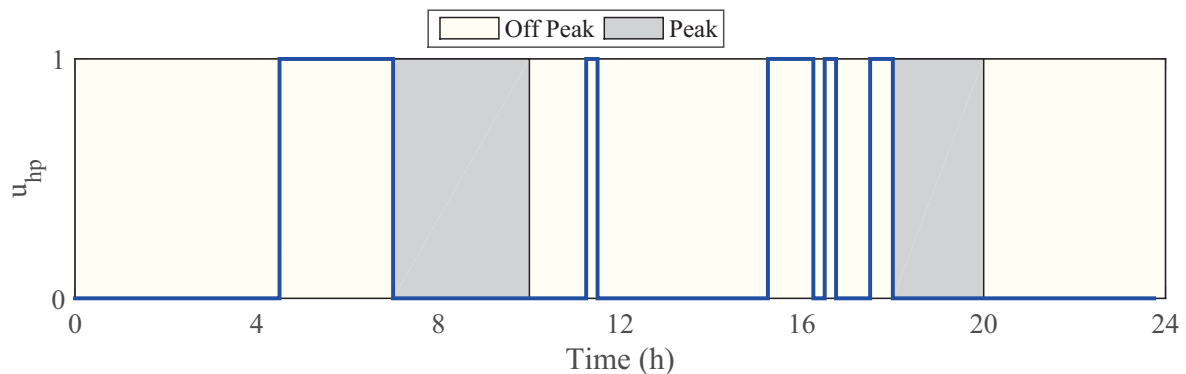
Photovoltaic power is also part of the input parameters into the controller, whose parameters are taken from [66].

## 7.4 SIMULATION RESULTS AND DISCUSSION

This section includes the results of optimal control of hot water devices, optimal consumption of grid power and the effect on the temperature of hot water from both devices. The legend showing peak and off-peak periods of the TOU tariff is the same throughout this chapter.

### 7.4.1 Optimal operation of HPWH

In meeting the overall hot water demand in the house, the HPWH is optimally operated as shown in Figure 7.3. The optimal controller operates the HPWH during the cheaper off-peak TOU period hence

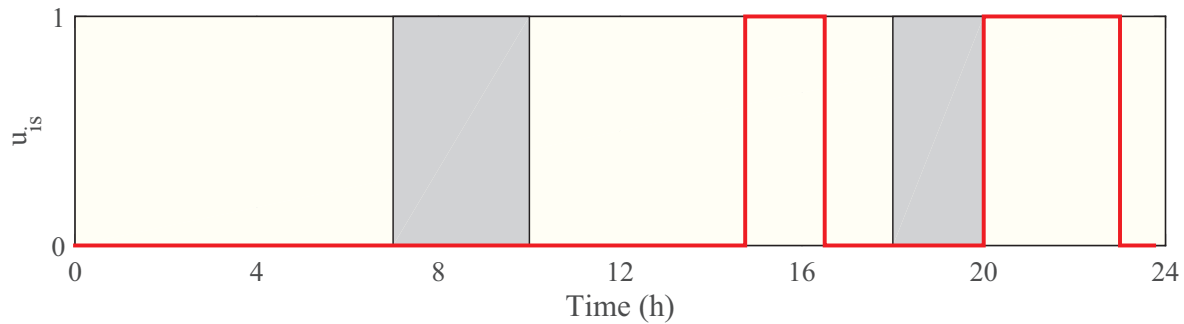


**Figure 7.3.** Optimal switching of HPWH.

incurring minimal cost. This load shifting is in line with the country's desire to minimize the load during the peak time so as to improve the quality of the grid. The controller switches on the HPWH at 04:30 on predicting that overall hot water demand is rapidly increasing, until 07:00 to avoid operating during the TOU peak period. The controller thereafter switches on the HPWH at 11:30 for 30 minutes to ensure water temperature is within the required temperature during this period of low hot water demand. As the hot water demand increases in the afternoon going to evening, the HPWH is switched on at 15:15-16:15, 16:30-16:45 and 17:30-18:00, after which it is switched off to avoid the evening TOU peak period. The controller predicts that this hot water will be sufficient to meet the demand for the remaining period of the 24-h operating cycle.

### 7.4.2 Optimal operation of instantaneous shower

Figure 7.4 shows the optimal operation of instantaneous shower. Similar to the HPWH optimal

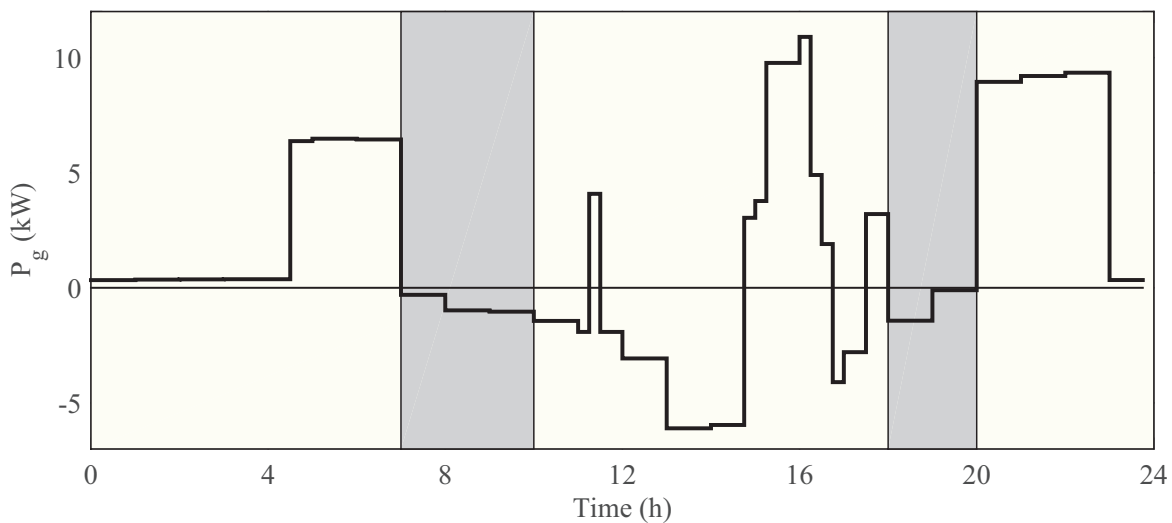


**Figure 7.4.** Optimal switching of instantaneous shower.

operation, the optimal controller also switches the instantaneous shower during the cheaper off-peak periods hence minimizing the cost of electricity while shifting the load. During the morning hours, the controller determines that hot water from HPWH is sufficiently hot to meet the morning hot water demand at the shower. There is therefore no need to switch on the instantaneous shower. However, as the demand from the shower is predicted to rise in the evening, the optimal controller switches on the instantaneous shower on between 14:45-16:30, while switching it off during the evening TOU peak. Thereafter, it is switched on again from 20:00-23:00 when the demand for hot water in the shower ends.

#### 7.4.2.1 Optimal grid power consumption

The consequent optimal power flow in the grid following optimal operation of the hot water devices as well as powering the rest of the domestic load is shown in Figure 7.5. The grid provides minimal power of 0.33 kW from 00:00-04:30 to cater for the domestic load, in the absence of renewable energy. The operation of the HPWH from 04:30-07:00 increases grid power consumption which reduces after it is switched off. This increase is attributed to the absence of renewable energy during these hours. During the morning TOU peak, both devices are off and the presence of solar power leads to excess power being sold back to the grid. This is actually useful in providing power to other households that need power during the peak TOU period. The amount of excess power to the grid continues until 14:45, except at 11:15 when HPWH was switched on for 15 minutes. Grid power consumption is



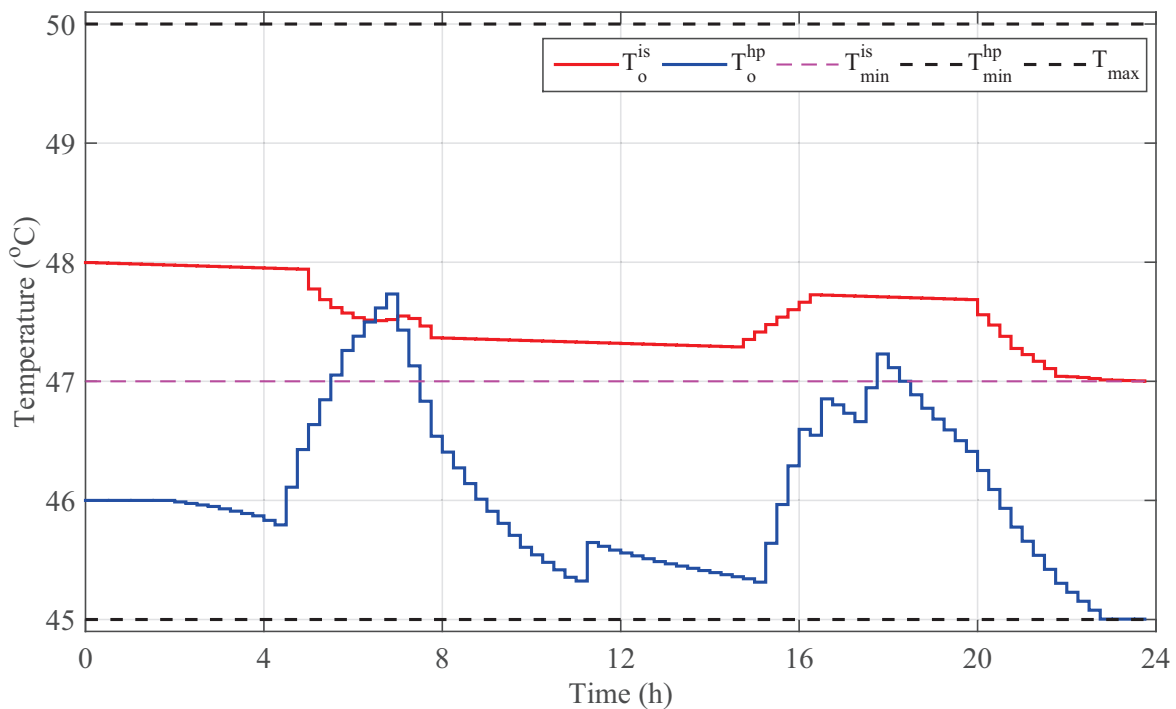
**Figure 7.5.** Optimal grid power flow.

used to supplement the renewable energy at 14:45-18:00 when both devices are switched on, except at 16:45-17:30 when they are off. Again, during the evening peak, both devices are off and there is excess renewable power which is sold back to the grid. Thereafter, there is a steep decline in renewable energy sources forcing the grid supply most of the energy from 20:00-23:00 when the instantaneous shower is in operation. It is important to note that the energy sold to the grid appears sporadic as the controller assumes that there is no energy storage.

#### 7.4.2.2 Hot water temperature

During optimal operation of both HPWH and instantaneous shower, the temperature of water varies as shown in Figure 7.6 throughout the 24-h operating cycle. The controller does not violate any constraints as it meets the required conditions for hot water in the whole house as well as the shower. The temperature of hot water stored in HPWH rises from 46-48.7 °C between 02:00-04:30 when the controller switches on the HPWH. Thereafter, the temperature starts to fall even though HPWH is on, caused by heat loss as hot water consumption increases. The temperature rises again at 11:00 for 30 minutes and finally between 16:00-18:00 and 20:30-22:00 when HPWH is on.

The temperature of water in instantaneous shower keeps dropping due to standby losses whenever it is idle, like at 00:00-03:30 and 07:30-14:30. However, in its active status, that is, whenever there

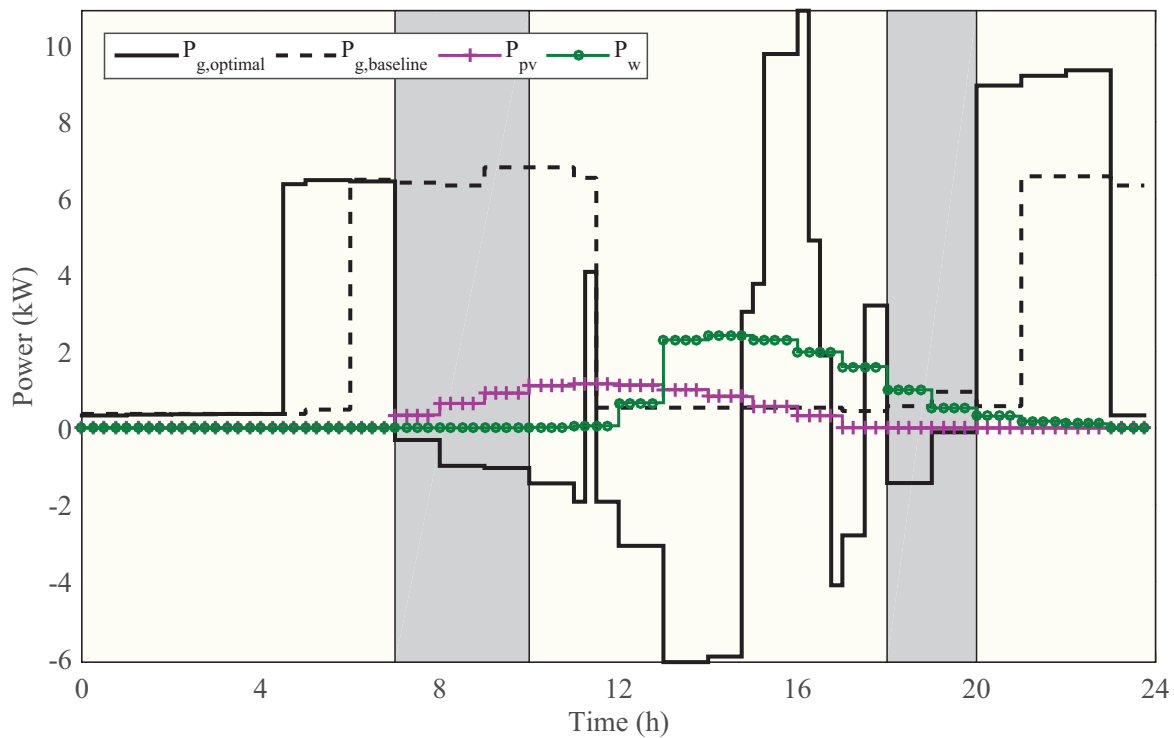


**Figure 7.6.** Variation of temperature

is demand of hot water in the shower, the temperature variation is caused either by switching on the instantaneous shower if the hot water coming from the HPWH is not sufficiently hot. In the morning, between 04:00-07:00, the rise of temperature of water in the instantaneous shower is caused by the rising temperature of water in the HPWH. There is therefore no need of switching on the instantaneous shower during this period. On the contrary, in the evening shower demand, that is, from 20:00-23:00, the temperature of water rises from 15:00 to 18:00. Since the instantaneous shower is off during the evening TOU peak, the temperature keeps falling even when its switched on between 20:00-23:00.

### 7.4.2.3 Power flow

Figure 7.7 shows the flow of power in the system throughout the operating cycle whereby,  $P_{g,optimal}$  is the optimal grid power flow resulting from the optimal control strategy.  $P_{g,baseline}$  is the grid power consumption when the HPWH is controlled by a thermostat and solely powered by the grid in the case study.  $P_{pv}$  and  $P_w$  are the renewable energy generated through PV and wind sources respectively. It can be seen that renewable sources are available during the day, with PV generation beginning at



**Figure 7.7.** The flow of power during the 24-h horizon

around 07:00 while wind begins at around 12:00, while they end at 17:00 and 23:00 respectively. As the amount of renewable energy increases during the day time, more of it is sold back to the grid (appearing as negative values of  $P_{g,optimal}$ ) whenever the controller switches off both devices. The consumption of power from the grid,  $P_{g,optimal}$ , reaches maximum in the evening when both HPWH and instantaneous shower are switched on. This is because the renewable energy being generated in this period is not sufficient to meet the power demand from both devices. The baseline power flow  $P_{g,baseline}$  is operated throughout the morning TOU peak to ensure the water remains hot. This not only leads to higher energy costs being incurred by the end user but it is also against the desire of the power utility to shift load to off-peak periods in order improve the quality of the grid.

### 7.4.3 Discussion

Table 7.2 shows the total daily energy consumption and the savings obtained from use of optimal control strategy with renewable energy sources. The baseline energy is the current scenario in the case study, where the HPWH is controlled by a digital thermostat and it is powered by grid power alone. It is in addition operated at the temperature range acceptable in the shower, despite the other hot



**Table 7.2.** Daily energy consumption and savings

Baseline		Optimal		Feed-in	Savings	
Energy (kWh/day)	Cost (R/day)	Energy (kWh/day)	Cost (R/day)	energy (kWh/day)	Energy (%)	Cost (%)
51.00	56.61	38.83	24.40	22.80	23.4	56.9

water demands requiring less hot water. Therefore, the baseline cost is the cost incurred by the HPWH under the TOU electricity tariff. The optimal energy is the grid energy consumed upon using optimal controller and renewable energy sources, with the cost obtained from the TOU electricity tariff. The energy and cost savings are obtained from the difference between the baseline and optimal energy consumption and cost respectively. Finally, feed-in energy is the excess solar and wind energy fed back to the grid. In this chapter, it is assumed there is no storage and therefore, whenever the renewable energy is excess, it is fed back to the grid through an appropriate feed-in tariff. The optimal controller with both HPWH and instantaneous shower has the potential to save 23.4% grid energy in a day when compared to the baseline thermostat control of the HPWH. These energy savings occur because of the use of renewable energy as well as operating the HPWH at a slightly lower minimum temperature acceptable in other end uses in the house. The instantaneous shower is used to heat water to higher temperature required in the shower. This control strategy can lead to a cost saving of up to 56.9% in a day when compared to the baseline thermostat controlled case.

In the case considered, where the length of the 1/2 inch diameter plumbing system from the HPWH storage to the shower is about 25 m, about 3.2 l of water is always stored in these pipes. This water had been heated at one point and remained in the pipes after the previous draw where it cooled off. For a house with four occupants, this translates to about 12.8 l if each of them takes just one shower in a day, and up to 19.2 l if two occupants shower twice in a day. Therefore, before each shower event in the case study, about 3.2 l of water and the associated heating energy is lost as the end user awaits arrival of hot water. With the use of instantaneous heater, the water stored in the pipe is not wasted as it is heated by the instantaneous shower. Saving this water is important for a dry and developing country like South Africa which is facing water crisis. The results of optimal control of the system incorporating renewable energy systems are in line with similar research [66]. Further, incorporation of instantaneous shower shows the potential in eliminating cold water wastage as hot water is being conveyed to the end use. Although it is an extra energy consuming device in the house, its efficient

operation through optimal control strategy ensures that both energy and water are saved in line with Luo's [274] recommendation.

It has been demonstrated that the combination of HPWH and instantaneous shower powered using renewable energy sources can operate efficiently through the optimal control strategy. This leads to energy, water and associated cost savings. Further, through the sale of excess renewable energy back to the grid, the end users can indeed experience further financial benefits, while also edging closer to achieving near net-zero buildings. If widely adopted, the benefits of this system are important for developing nations as they strive towards energy and water security. The strategy would go a long way in reducing the greenhouse gas emissions while also improving the quality of grid system available. In remote areas with no grid connectivity, the control system can be used with diesel generators being used to supplement renewable energy. However in such a case, an energy storage mechanism should be included.

## 7.5 SYNOPSIS

The optimal control strategy of both hot water devices has the potential to save energy cost. Further, incorporation of integrated renewable energy sources with optimal control has shown the potential to save energy-not-delivered from the grid, as well as make some money by selling excess energy back to the grid. The inclusion of instantaneous shower allows the HPWH to be operated at a slightly lower temperature hence consuming less energy. It also leads to potential water savings that cools in the pipes and is normally drained as end users wait for hot water to arrive. This control system has the potential to save up to 23.4% of energy in a day while also avoiding wastage of up to 19 l of water in a day. It also has the potential of reducing energy and water bills. The strategy provides a practical means of integrating of renewable energy sources into homes with the ability of energy trade-off, marking a step closer to zero-energy buildings. This control system is suitable for peri-urban and rural home owners who intend to integrate renewable energy to supply energy-efficient equipment. There is need for further research to incorporate energy storage systems such as batteries and fuel cells. In addition, the control strategy should be advanced to deal with disturbances which are always present in any plant.

# **CHAPTER 8 MODEL PREDICTIVE CONTROL OF HEAT PUMP WATER HEATER - INSTANTANEOUS SHOWER POWERED WITH INTEGRATED RENEWABLE - GRID ENERGY SYSTEMS**

## **8.1 INTRODUCTION**

In most developing nations, such as African countries, increasing population has the proclivity of concentrating in urban areas and cities. Many nations in Sub-Saharan Africa have been experiencing rapid urban expansion averaging at 5% per annum [294]. The rapid growth has many social, economic and physical repercussions including increased demand for key services such as energy and water. These factors have made Sub-Saharan Africa to be the most energy insecure region in the world with the average urban and rural electrification standing at 59% and 17% respectively [295], while over 40% of the population do not have safe clean water. This is despite renewable energy having the potential to increase the energy capacity through micro grids, combined heat and power systems and production of bio fuels. Tapping into this potential would increase electrification, improve grid quality, also lower the cost of electricity which eventually would lead to improved quality of life. In South Africa, access to electricity increased from 35% to 84% between 1990 and 2011. The increased demand led to a very narrow reserve margin in the grid eventually causing power shortages (black outs) and load shedding from 2008, with huge negative economic ramifications [39]. In addition, electricity is mainly generated from coal leading to a very high carbon footprint, and the government is considering introduction of carbon tax [296]. To deal with these challenges, the government introduced both

supply and demand side management initiatives. In supply side, the government sought to increase the generation capacity through building of new coal power stations, return to service of some coal power stations and explore co-generation and renewable energy options [41]. The existing coal power plants have become outdated, while the coal reserves are dwindling, making construction of new plants not only environmentally hazardous but also prohibitively expensive to implement. Therefore, the only viable option in supply side is co-generation and renewable energy options. The demand side management (DSM) measures introduced sought to reduce the demand for power by up to 5000 MW by 2025 [39]. DSM seeks to reduce the gap between supply and demand through improving energy efficiency (EE) as well as load management (LM) [36]. LM is tailored to reduce the demand for electricity during peak period by offering incentives to shift load to off-peak periods. This is normally done through the use of time-of-use tariffs or demand response programs [297]. Following the above reasons, this study seeks to consider a more grid independent system using available renewable energy sources while also ensuring EE takes place.

Energy efficiency and DSM have also become very attractive research topics [44]. Areas of interests and applications have been in industrial systems [23, 81, 116, 160, 298, 299], power systems [239, 300–302], building energy systems [70, 77, 127], and the eventual measurement and verification [303]. Water heating is one of the most important energy intensive components in the building energy systems. In a typical South African residence, water heating leads to 40–60% of total energy consumption [304] There is therefore a huge potential for EE and energy conservation measures for water heating especially in South Africa. One such way is through the use of efficient technologies such as HPWHs [269]. They have a high coefficient of performance making them a suitable alternative to electric storage water heaters (geysers) in reducing the monthly peak electricity demand charges. Despite their superiority and government intervention, their market penetration is still low standing at about 16% [268]. Coupled with high investment cost, there are technological challenges in HPWH's optimal operation, sizing and integration [67].

HPWHs are not only ideal in enhancing EE for domestic hot water systems [269], but have also been proven to be economically feasible [270]. Further, it is possible to shift the load using HPWHs increasing the prospect of integrating it with renewable energy sources [305]. Various control algorithms aiming to reduce energy consumption and its associated cost have been developed. A feed forward artificial neural network (ANN) was designed to control HPWHs [306]. However, of the control algorithms tested, including proportional-integral-derivative (PID) controllers, predictive

control algorithms proved to be most effective [271]. Use of renewable energy systems to power HPWHs and other domestic loads has a huge potential to save more energy, cost and reducing greenhouse gas emissions [65]. Various open loop predictive control algorithms for controlling HPWHs with distributed renewable energy systems have been designed. An optimal controller operating a HPWH powered using grid tied photovoltaic (PV) and diesel generator integrated system was developed for application in areas with intermittent power supply [67]. An optimal power dispatch model of a grid tied photovoltaic system was used to power HPWH. The cost of grid energy was structured as a time-of-use (TOU) tariff and the system not only showed the potential to save energy but also the ability to use the energy stored in the battery in case of either power black out or during peak time [67]. In addition, an optimal controller was designed to operate a HPWH powered using integrated wind generator-photovoltaic-grid system. This controller led to energy and cost savings from renewable energy systems. The grid was designed such that it could accept power back from renewable energy systems whenever it was not required [66]. The optimal controller was advanced by incorporating a fuel cell storage system that improved the reliability of the intermittent renewable power supplies [69].

All the above open-loop control algorithms can only deal with disturbances and uncertainties that are almost predictable or known in advance as open-loop optimal control assumes a perfectly predictable system behaviour. However, in cases where random disturbances such as sudden hot water draws that really affect the operation of the HPWH are present, closed-loop model predictive control (MPC) robustly deals with them through feedback. For effective control, it is more desirable to pro-actively respond to upcoming draw events than reactively turn on the HPWH when the temperature is already too low. In this regard, the volume of hot water to be drawn can be estimated using the past hot water consumption and incorporated into a closed-loop MPC framework which would predict the future behaviour on-line [307]. In addition, HPWHs have a slow rate of heating water such that they cannot provide hot water in case of high demand [201]. Previous studies have also mainly concentrated on improving EE, though it is important to look at the energy-water nexus demand management [147]. For instance, HPWHs are normally located in a central place in the house, and then distributes hot water to various end uses. There are consequent energy and water losses from hot water conveyance to the consumption point. This arises as the previously heated water stored in the pipes cools down before the next use takes place. Normally, end users open the shower and allow the cold water to run until hot water arrives. Worse still, upon finishing with the use, the remaining water in the pipe quickly cools down [272]. Therefore an instantaneous heater, placed at the point of consumption can be used

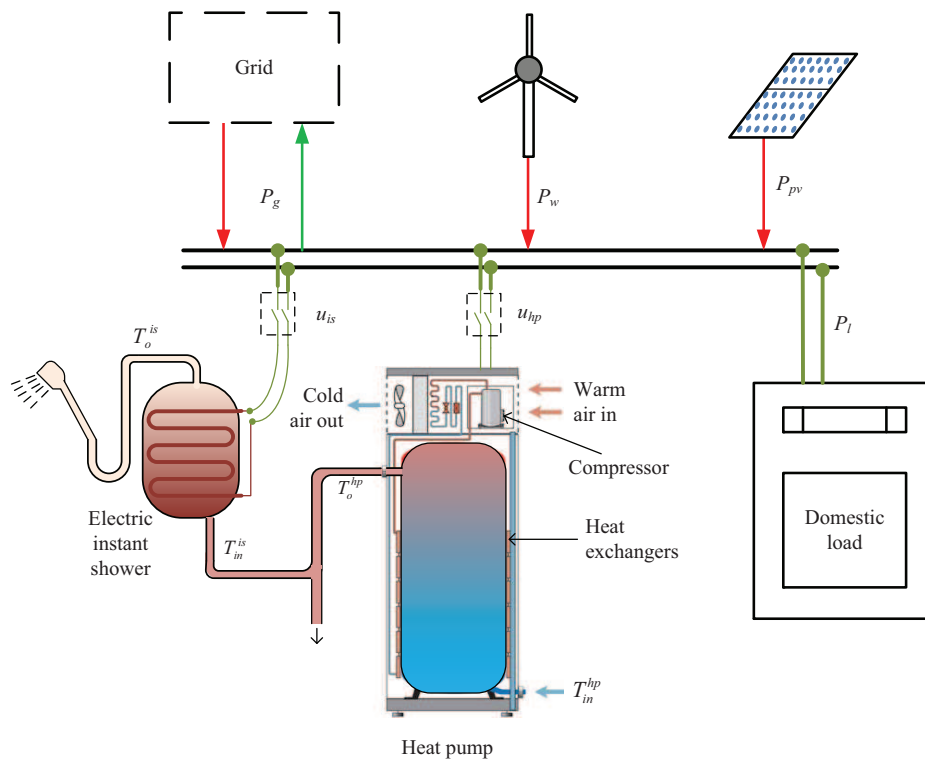
to eliminate water wastage [273]. Since shower is one of the most intensive hot water end use in a domestic house [308], an instantaneous shower is suitable to eliminate this water wastage and provide hot water in the shower when there is high demand [202].

This chapter is an advancement of previously developed open-loop optimal controller used to operate both HPWH and instantaneous shower powered using integrated renewable energy systems [201]. A robust and economical closed-loop MPC strategy is introduced. The MPC strategy ensures both energy and water are efficiently consumed by using HPWH and instantaneous shower to conveniently meet domestic hot water demand. Integrated renewable energy sources comprising of wind and photovoltaic (PV) solar are used to power the hot water devices, while grid power is only used as back up. Importantly, the grid is designed such that it can accept excess renewable energy through an appropriate feed-in tariff. This control strategy seeks to optimally operate both hot water devices, maximize the use of renewable energy and minimize water loss during conveyance to the shower. Unlike the previously designed open-loop control strategy, the closed loop MPC has feedback, that is, temperature of water in both hot water devices at the current instant, that is used to predict the future behaviour of both devices in each iteration. This greatly improves the robustness of the controller in dealing with random disturbances that could occur. In addition, life cycle cost (LCC) analysis is carried out over the expected lifetime of the system in order to determine the economic feasibility of the system. The proposed interventions provide sustainable measures to achieve EE and energy-water conservation in a convenient and affordable manner.

## 8.2 CONTROLLER DESIGN

### 8.2.1 Schematic layout

Figure 8.1 shows the schematic diagram of the water heating system comprising of HPWH and instantaneous shower powered by photovoltaic solar,  $P_{pv}$ , and wind generator,  $P_w$ , with grid,  $P_g$ , acting as back up whenever the renewable sources are insufficient. The HPWH, centrally located in the house, meets the total hot water demand while the instantaneous shower is placed in the shower to act as back up whenever water from the HPWH is not at the required temperature. Switches  $u_{hp}$  and  $u_{is}$  control the power flow to the HPWH and instantaneous shower respectively. Since the house under consideration in this study is currently connected to the grid, power from the renewable sources is mainly used in



**Figure 8.1.** Schematic layout of the energy and hot water flow

the house. However, if renewable power is insufficient, grid power supplements power, while it also accepts excess renewable energy to be used by other users in the network.

### 8.2.2 Wind energy

Wind energy is one of the integrated renewable energy system used to power the hot water devices and whenever it is more than the energy required by the hot water devices, it is fed back to the grid. For wind speed between cut in,  $V_i$ , and rated,  $V_N$ , the power output of a typical wind turbine is proportional to the cubed wind speed of the turbine. The power output,  $P_w$ , of a simplified model of a wind generator at the rated wind speed is [275],

$$P_w = 0.5\eta_t \eta_g \rho_a C_p A_w V_r^3, \quad (8.1)$$

where  $\eta_t$  and  $\eta_g$  are the gearbox and generator efficiency respectively.  $\rho_a$  is the density of air ( $kg/m^3$ ),  $C_p$  is the power coefficient of the turbine,  $A_w$  is the sweeping area of the turbine rotor ( $m^2$ ) while  $V_r$  is the speed of wind ( $m/s$ ). Whenever the wind speed is above  $V_N$ , aerodynamic efficiency is reduced by pitching the blades so that shaft power remains constant. However, if the wind speed exceeds the pitch

control limit, it reaches the cut-out wind speed,  $V_c$ , and power production is stopped [276].

### 8.2.3 Photovoltaic solar energy

The power generated by a PV array, which is the series-parallel connection of solar modules [277], is given as,

$$P_{pv} = \eta_{pv} A_c I_{pv}, \quad (8.2)$$

where  $A_c$  is the area of the PV array,  $I_{pv}$  is the solar irradiation incident on the PV array ( $kWh/m^2$ ) and it depends on the time of the day.  $\eta_{pv}$  is the efficiency of the PV generator which is dependent on  $I_{pv}$  and ambient temperature,  $T_A$ , [80]. The PV array is also a part of the integrated renewable energy system supplying power to the hot water devices, while excess PV power is also sold back to the grid.

### 8.2.4 Grid energy

The grid is modelled as an infinite bar capable of either supplying power to hot water devices whenever renewable energy is insufficient or accepting excess renewable power to meet other energy demands within its network. In supply mode, the grid electricity's price is structured as a time-of-use (TOU) tariff. In this study, Eskom's TOU Homeflex structure is used [225], whose hourly price of electricity,  $p_e(t)$ , is,

$$p_e(t) = \begin{cases} p_{off} = 0.5510 \text{ R/Kwh} & \text{if } t \in [0, 6] \cup [10, 18] \cup [20, 24], \\ p_{peak} = 1.7487 \text{ R/Kwh} & \text{if } t \in [7, 10] \cup [18, 20], \end{cases} \quad (8.3)$$

where  $p_{off}$  is the off peak price,  $p_{peak}$  is the peak price,  $R$  is the South African currency, Rand, and  $t$  is the time of day in hours [70]. The National Energy Regulator of South Africa (NERSA) has developed the renewable energy feed-in tariff (REFIT) [39]. Municipalities are allowed to buy electricity from small scale embedded systems producing below 100 kW at the same price as Eskom's retail price. So far, a few municipalities have started buying electricity from residential customers like the City of Cape Town and eThekweni municipality [309].

The hourly power balance for meeting the demand of the hot water devices is modelled as,

$$P_{hp} u_{hp}(t) + P_{is} u_{is}(t) + P_l(t) = P_g(t) + P_w(t) + P_{pv}(t), \quad (8.4)$$



where  $P_{hp}$  and  $P_{is}$  are the power rating ( $kW$ ) of the HPWH and instantaneous shower respectively, whose on/off status are represented as  $u_{hp}(t)$  and  $u_{is}(t)$  respectively.  $P_l(t)$  is the hourly power ( $kW$ ) demand from other domestic load.

### 8.2.5 Heat pump water heater

Unlike a refrigerator that moves heat from an enclosed box to the surrounding air, HPWHs operate in reverse refrigeration process by taking heat from the surrounding air and transferring it to water in an enclosed reservoir. A HPWH is generally composed of the heat pumping and hot water reservoir parts. Although mechanical and thermal inertia are important in modelling the heat pumping part, HPWHs take much shorter time in stabilizing from mechanical inertia than thermal inertia. In this chapter, the compressor of the HPWH is taken to be operating at constant speed condition, and therefore, suction and discharge are used to model the steady state characteristics of the HPWH. [279]. The hot water reservoir stores the hot water heated upon being forced to circulate through the condenser absorbing the heat [280]. The overall efficiency of the HPWH's thermal components, mainly evaporator, refrigerant and compressor, are accounted for by the coefficient of performance (COP) obtained in the case study. Therefore, energy losses in these components have been neglected during modelling. Further, it is assumed that the temperature of water throughout the reservoir is uniform. Hence, the energy losses considered in this study are standby losses and losses associated to the hot water demand.

Whenever hot water is stored in the HPWH's reservoir, there are thermal losses to the environment resulting from heat conduction through the surface of the tank and natural convection that transfer heat from the surface of the tank to the environment. These losses can be reduced through the use of improved thermal insulated material [281]. Standby losses are modelled as,

$$Q_l = A_{hp} \left( \frac{T_{hp} - T_a}{R_{hp}} \right), \quad (8.5)$$

where  $A_{hp}$  is the surface area of the HPWH's tank ( $m^2$ ) while  $T_{hp}$  and  $T_a$  are the hot water and ambient temperatures ( $^{\circ}C$ ) respectively.  $R_{hp}$  is the thermal resistance of the insulation material ( $m^2K/W$ ) which can be written as,

$$R = \frac{\Delta x}{k} + \frac{1}{h}, \quad (8.6)$$

where  $\Delta x$  is thickness of insulation material ( $m$ ) while  $k$  and  $h$  are the coefficients of thermal conductivity ( $W/mk$ ) and surface heat transfer ( $W/m^2k$ ) respectively [282].

Demand for hot water from the HPWH causes hot water to flow out of the reservoir which is consequently replaced by the same volume of cold water [283]. The flow of hot and cold water through the HPWH's reservoir leads to a drop in the average temperature,  $T_{hp}$ , of hot water in the reservoir [284]. Thermal power losses due to water flow,  $Q_d$ , can be modelled as,

$$Q_d = c_w D_{tot} (T_{hp} - T_{hp}^{in}), \quad (8.7)$$

where  $c_w$  is the specific heat capacity of water ( $J/(kg \cdot ^\circ C)$ ).  $T_{hp}^{in}$  is the temperature of the cold water into the tank ( $^\circ C$ ) and  $D_{tot}$  is the total hot water demand, given as mass flow rate of hot water ( $kg/h$ ).

Water is maintained at the required temperature by overcoming above losses. Electrical power required from the HPWH,  $Q_h$ , is [285],

$$Q_h = COP \times P_{hp}, \quad (8.8)$$

where  $COP$  is the coefficient of performance. In meeting hot water demand, the dynamic model of the HPWH is based on open energy balance [286], that leads to a differential equation that describes the average thermal response of water in the tank as [287],

$$c_w m_{hp} \frac{dT_{hp}}{dt} = Q_h - Q_l - Q_d, \quad (8.9)$$

where  $m_{hp}$  is the mass of water ( $kg$ ) in the HPWH's storage tank. Substituting for  $Q_h$ ,  $Q_l$  and  $Q_d$  in equation (8.9) leads to,

$$c_w m_{hp} \frac{dT_{hp}}{dt} = (COP)P_{hp}u_{hp}(t) - A_{hp} \left( \frac{T_{hp}(t) - T_a}{R} \right) - c_w D_{tot}(t) (T_{hp}(t) - T_{hp}^{in}(t)). \quad (8.10)$$

To simplify the modelling process, the differential equation can be written as

$$\frac{dT_{hp}}{dt} = -\alpha(t)T_{hp}(t) + \beta u_{hp}(t) + \gamma(t), \quad (8.11)$$

where

$$\alpha(t) = \frac{A_{hp}}{c_w m_{hp} R} + \frac{D_{tot}(t)}{m_{hp}}, \quad (8.12a)$$

$$\beta = \frac{(COP)P_{hp}}{c_w m_{hp}}, \quad (8.12b)$$

$$\gamma(t) = \frac{A_{hp}T_a}{c_w m_{hp} R} + \frac{D_{tot}(t)T_{hp}^{in}(t)}{m_{hp}}. \quad (8.12c)$$

Differential equation (8.11) can be expressed in discrete-time domain such that temperature of water in the HPWH's storage at the  $j^{th}$  sampling interval becomes,

$$T_{hp}(j+1) = (1 - t_s \alpha(j))T_{hp}(j) + t_s \beta u_{hp}(j) + t_s \gamma(j), \quad (8.13)$$

where  $t_s$  is the sampling period ( $h$ ). The status of the HPWH's switch,  $u_{hp}(j)$ , is such that,

$$u_{hp}(j) \in \{0, 1\} \quad 1 \leq j \leq N, \quad (8.14)$$

where  $N$  is the total number of samples in a 24-h operating cycle ( $N = \frac{24}{t_s}$ ). Through recurrence manipulation, equation (8.13) can be expressed in terms of the initial temperature,  $T_{hp}(0)$ , as,

$$T_{hp}(j) = T_{hp}(0) \prod_{i=1}^j (1 - t_s \alpha(i)) + \beta t_s \sum_{i=1}^j u_{hp}(i) \prod_{k=i+1}^j (1 - t_s \alpha(k)) + t_s \sum_{i=1}^j \gamma(i) \prod_{k=i+1}^j (1 - t_s \alpha(k)),$$

$$1 \leq j \leq N.$$
(8.15)

### 8.2.6 Instantaneous shower

Instantaneous, demand or tank-less water heaters have heating elements that are activated by the flow of water thereby heating the water instantly as it passes through [288]. Although they can be electric, gas or propane powered [290], this chapter focuses on the electric powered instantaneous showers. Storage water heaters, that are normally centrally located in a house, lead to water wastage in supplying remote end uses such as shower. This arises as end users have to open the tap/shower to pour the cold water in the pipe until hot water arrives and the remaining hot water in the pipe after the tap/shower is closed becomes cold quickly. In the case of shower end use, an instantaneous shower would improve the efficiency of these resources by nearly eliminating distribution losses. They also offer a perfect candidate to support central storage hot water systems that cannot conveniently meet hot water demand such as central solar tanks and HPWHs [273]. Assuming that the water in the pipe is losing heat at the same rate as that in the instantaneous shower, the instantaneous shower can be modelled in two states [292];

#### 8.2.6.1 Active state

Whenever there is demand for shower water, hot water from the HPWH is flowing. Assuming negligible heat loss as water flows along the pipes to the shower, the energy balance of the instantaneous shower in active state can be represented by a first order differential equation as,

$$c_w m_{is} \frac{dT_{is}}{dt} = \eta_{is} P_{is} u_{is}(t) - A_{is} \left( \frac{T_{is}(t) - T_a}{R_{is}} \right) - c_w D_{is}(t) (T_{is}(t) - T_{is}^{in}(t)), \quad (8.16)$$

where  $m_{is}$  is the mass of water inside the instantaneous shower's chamber (kg),  $\eta_{is}$  is the efficiency of the heating element rated at  $P_{is}$  in kW.  $A_{is}$  is the surface area of the instantaneous shower while  $R_{is}$  is the thermal resistance of the shower's material and  $T_{is}^{in}$  is the temperature of water ( $^{\circ}\text{C}$ ) flowing into the instantaneous shower.  $u_{is}(t)$  is the state of the instantaneous shower's switch at time,  $t$ , while

$D_{is}(t)$  is the hot water demand in the shower ( $kg/hr$ ). Just like in equation (8.10), the second and third terms of the right hand side of equation (8.16) represent the standby and water usage thermal losses respectively [291]. Differential equation (8.16) can be simplified to,

$$\frac{dT_{is}}{dt} = -\phi(t)T_{is}(t) + \lambda u_{is}(t) + \zeta(t), \quad (8.17)$$

where

$$\phi(t) = \frac{A_{is}}{c_w m_{is} R_{is}} + \frac{D_{is}(t)}{m_{is}}, \quad (8.18a)$$

$$\lambda = \frac{\eta P_{is}}{c_w m_{is}}, \quad (8.18b)$$

$$\zeta(t) = \frac{A_{is} T_a}{c_w m_{is} R_{is}} + \frac{D_{is}(t) T_{is}^{in}(t)}{m_{is}}. \quad (8.18c)$$

Discretizing equation (8.17) yields,

$$T_{is}(j+1) = (1 - t_s \phi(j))T_{is}(j) + t_s \lambda u_{is}(j) + t_s \zeta(j), \quad (8.19)$$

where the status of the instantaneous shower's switch,  $u_{is}(j)$ , is such that,

$$u_{is}(j) \in \{0, 1\} \quad 1 \leq j \leq N, \quad (8.20)$$

Expressing equation (8.19) in terms of the initial temperature,  $T_{is}(0)$ , of water in the instantaneous shower becomes,

$$T_{is}(j) = T_{is}(0) \prod_{i=1}^j (1 - t_s \phi(i)) + \lambda t_s \sum_{i=1}^j u_{is}(i) \prod_{k=i+1}^j (1 - t_s \phi(k)) + t_s \sum_{i=1}^j \zeta(i) \prod_{k=i+1}^j (1 - t_s \phi(k)), \quad (8.21)$$

$$1 \leq j \leq N.$$

### 8.2.6.2 Idle state

The idle status takes place whenever the instantaneous shower is not in use implying that previously hot water is stagnant in the pipes between HPWH and instantaneous shower as well as in the instantaneous shower's small reservoir. Assuming that this stagnant water in the pipes is losing heat at the same rate as that in the instantaneous shower, the differential equation governing the behaviour of water in the instantaneous shower can be modelled as [292],

$$c_w m_{is} \frac{dT_{is}}{dt} + A_{is} \left( \frac{T_{is}(t) - T_a}{R_{is}} \right) = 0. \quad (8.22)$$

### 8.2.7 MPC optimization problem

Closed-loop MPC strategy employs explicit plant's (HPWH and instantaneous shower) model to predict the future behaviour [112]. Using the principle of receding horizon control, only the first element of the control vector is implemented after each iteration, ignoring the rest of the elements [225]. The temperature of hot water in both HPWH and instantaneous shower is measured during each time step and fed back to the controller in order to provide stability and robustness against uncertainties and disturbances present in the plant [157]. This temperature is used as the initial temperature during the next time step, such that the state equations (8.15) and (8.21) are modified as follows,

$$T_{hp}(j|k) = T_{hp}(k) \prod_{i=k}^j (1 - t_s \alpha(i)) + \beta t_s \sum_{i=k}^j u_{hp}(i|k) \prod_{m=i+1}^j (1 - t_s \alpha(m)) + t_s \sum_{i=k}^j \gamma(i) \prod_{m=i+1}^j (1 - t_s \alpha(m)), \quad (8.23)$$

$$T_{is}(j|k) = T_{is}(k) \prod_{i=k}^j (1 - t_s \phi(i)) + \lambda t_s \sum_{i=k}^j u_{is}(i|k) \prod_{m=i+1}^j (1 - t_s \phi(m)) + t_s \sum_{i=k}^j \zeta(i) \prod_{m=i+1}^j (1 - t_s \phi(m)), \quad (8.24)$$

$$k \leq j \leq k + N_c - 1.$$

where  $T_{hp}(j|k)$  and  $T_{is}(j|k)$  are predicted temperature of water in the HPWH and instantaneous shower, respectively, at the  $j^{th}$  sampling interval based in information measured at time  $k$ .

The controller, firstly, seeks to minimize the cost of grid power consumed. This minimization aims at using the devices during cheaper off-peak TOU periods and only use them in peak period only if it is inevitable. It further ensures that renewable energy, which is treated as free energy during consumption, is used first and only use grid energy if renewable energy is insufficient. Secondly, the use of instantaneous shower is minimized in order to ensure that HPWH, which is more efficient, is used to heat water while the instantaneous shower is used if the HPWH is unable to meet the demand. In this chapter, we consider an evaluation period of a 24-h operating cycle from 0 to hour 24. A sampling period  $t_s = 5$  min is chosen bearing in mind that a shower on average takes about six minutes [310]. The objective function,  $J$ , is modelled as,

$$J = \omega \sum_{j=k}^{k+N_c-1} t_s p_e(j) P_g(j|k) + (1 - \omega) \sum_{j=k}^{k+N_c-1} t_s P_{is} u_{is}(j|k), \quad (8.25)$$

where  $\omega$  is a weighting factor used as indicator of relative importance of minimizing each term in the objective function [293].  $N_c$  is the control horizon while  $P_g(j|k)$  and  $u_{is}(j|k)$  are the optimized

control actions, grid power and status of the instantaneous shower's switch, respectively, at the  $j^{\text{th}}$  sampling interval based on the most recent measurement at time  $k$ . Normally, MPC optimization problems include both control,  $N_c$ , and predicting,  $N_p$ , horizons. However, only  $N_c$  is considered in this chapter as no state variables are included in the objective function [225]. Hence,  $N_c$  is obtained as,

$$N_c = N - k + 1. \quad (8.26)$$

The first term in objective function (8.25) aims at minimizing the cost of grid power used to power the hot water devices while the second term minimizes the use of the instantaneous shower. To achieve this, the following technical and operational constraints are considered to affect the system,

$$P_w(j) + p_g(j|k) + P_{pv}(j) = P_{hp}u_{hp}(j|k) + P_{is}u_{is}(j|k) + P_l(j), \quad (8.27)$$

$$T_{hp}^{\min} \leq T_{hp}(j|k) \leq T^{\max}, \quad (8.28)$$

$$T_{is}^{\min} \leq T_{is}(j|k) \leq T^{\max}, \quad (8.29)$$

$$-\infty \leq P_g(j|k) \leq \infty, \quad (8.30)$$

$$u_{hp}(j|k) \in \{0, 1\}, \quad (8.31)$$

$$u_{is}(j|k) \in \{0, 1\}, \quad (8.32)$$

where  $T_{hp}^{\min}$  and  $T_{is}^{\min}$  are the minimum allowable temperature ( $^{\circ}\text{C}$ ) of HPWH and instantaneous shower respectively, while  $T^{\max}$  is the maximum allowable temperature ( $^{\circ}\text{C}$ ) for both devices. Equality constraint (8.27) represents power balance in the system, whereby, power from renewable energy sources and grid are used to meet the demand from hot water devices together with the domestic load during every sampling interval,  $j$ . Constraints (8.28) and (8.29) show the state variables, that is, temperature of water from HPWH and instantaneous shower respectively, must be between set minimum and maximum acceptable temperature in every sampling interval,  $j$ . Constraints (8.30)-(8.32) show how the control variables are bound. Grid power,  $P_g$ , is bound such that it not only provides but also accepts power back from the renewable sources. Finally, the status of the HPWH and instantaneous shower switches can take the values of either 0 or 1 representing off and on status respectively.

### 8.2.7.1 MPC Algorithm

Closed-loop MPC control strategy is designed for multi-variable control problems over a finite control horizon. The control action is obtained by solving the open-loop optimal control problem starting

at the current state during each sampling interval [311]. The algorithm for the open-loop optimal control problem has been developed in [201]. In comparison to other control approaches, optimal control and MPC strategies enable the inclusion of constraints in the control problem, by making feasibility of the optimization problem a condition in the decision-making process. Although MPC strategy can be computationally expensive, it provides the only real framework for addressing control of systems involving the state in the presence of constraints. In practice, the predictive aspect of MPC is superior due to its ability to account for the risk of future constraint violation during the current control decision [312].

Just like in the open-loop algorithm [201], control vector  $X$  contains all the control variables such that,

$$X = \begin{bmatrix} u_{hp}(k|k) \dots u_{hp}(k+N_c-1|k) u_{is}(k|k) \dots u_{is}(k+N_c-1|k) \\ P_g(k|k) \dots P_g(k+N_c-1|k) \end{bmatrix}_{3N \times 1}^T \quad (8.33)$$

The controller determines the time,  $k$ , and first works out the control horizon,  $N_c$ . It thereafter solves an open-loop optimization problem with the operating horizon as  $N_c$  and obtains an optimal solution, where only the first element of control variables  $u_{hp}$ ,  $u_{is}$  and  $P_g$  are implemented in the plant. Subsequently, the temperature of water in both devices is measured and fed back to the controller. In the next time step,  $k+1$ , this temperature is used by the controller as the initial temperature for the new  $N_c$ . Other control inputs are updated and the process is repeated until the predetermined end of the operating cycle. In summary, the MPC algorithm solves the optimization problem on-line as follows [197];

1. For time,  $k$ , find the control horizon ( $N_c(k)$ ) using equation (8.26).

2. *Optimization:* Find the optimal solution within the control horizon;

minimize objective function (8.25),

subject to constraints (8.27)-(8.32).

3. From the optimal solution, implement  $[u_{hp}(k|k), u_{is}(k|k), P_g(k|k)]^T$  to the plant.

4. *Feed back:* Measure state variables (temperature)  $T_{hp}(k+1)$  and  $T_{is}(k+1)$ .

5. Set  $k = k + 1$  and update system states and inputs and outputs.
6. Repeat steps 1-5 until  $k$  reaches a predefined value.

This optimization problem is solved using the SCIP solver in optimization interface (OPTI) toolbox, which is a Matlab toolbox for solving optimization problems.

### 8.3 GENERAL DATA

#### 8.3.1 Case study

The farm house in Port Elizabeth, South Africa, studied in [201] is the case used in this chapter. The existing thermostatically controlled HPWH, used to provide hot water in the house, has the parameters shown in Table 8.1. In order to satisfy hot water demand, the thermostat is set at  $50\text{ }^{\circ}\text{C}$ . Further,

**Table 8.1.** Parameters of the HPWH

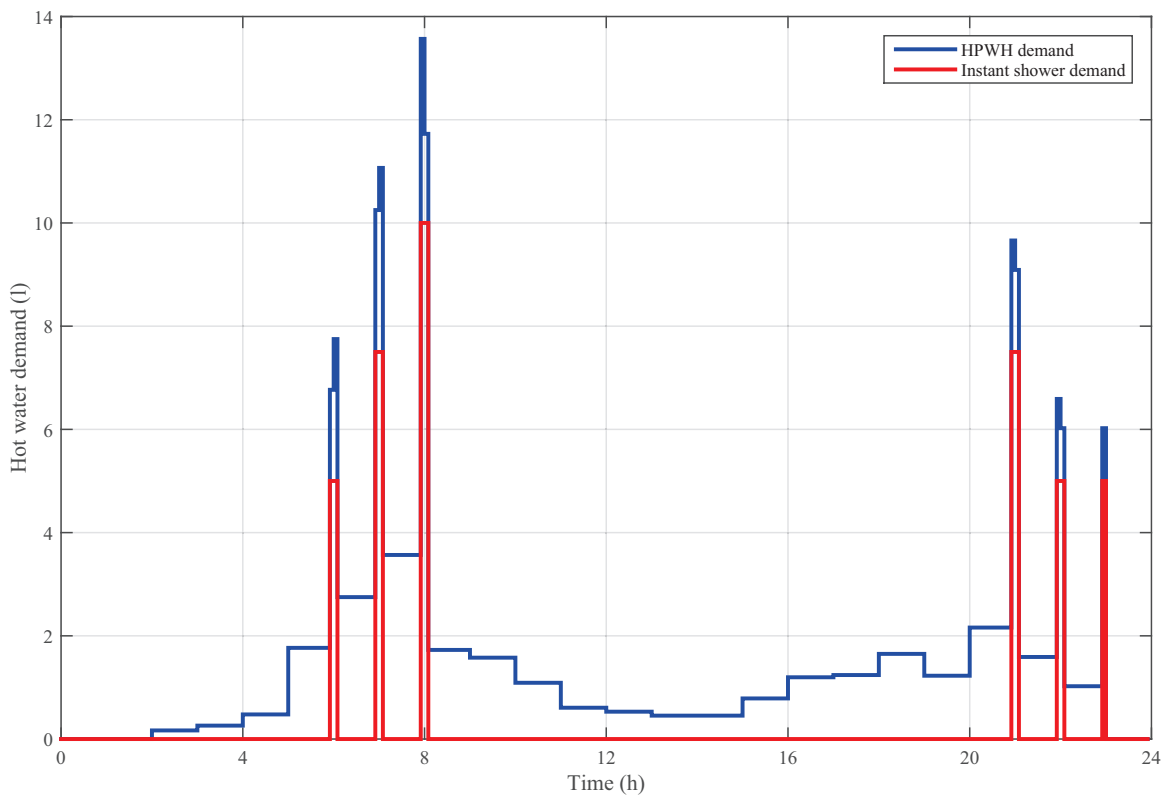
Power (kW)	COP	Volume (l)	$\Delta x$ (m)	k (W/mK)	h (W/m <sup>2</sup> K)
6	3.8	260	0.035	0.055	6.3

plumbing pipes used to transport hot water in the house are  $1/2$  inch in diameter, with the length from the location of the HPWH to the shower being about  $25\text{ m}$ . In this study, Stiebel Eltron IS 60E<sup>1</sup> instantaneous shower, rated at  $8.5\text{ kW}$  is used. Since the shower is the end use requiring hottest water from the HPWH, instantaneous shower is set to maintain the temperature of hot water at  $47 \leq T_{is} \leq 50$  while the HPWH is set at  $45 \leq T_{hp} \leq 50$ .

The hourly hot water demand for the house was measured, with Figure 8.2 showing the demand pattern for a period of one day. Generally, the demand curves for hot water from the HPWH and the shower have two peaks in a day, that is, in the morning and evening, when occupants mostly use hot water for their household chores.

<sup>1</sup><http://www.stiebel-eltron.co.za/is60e.html>





**Figure 8.2.** Overall and shower hot water demand pattern.

Further, the ambient temperature for this region and hourly temperature variation, are obtained from the Southern African Universities Radiometric Network<sup>2</sup>.

### 8.3.2 Renewable energy

Wind and photovoltaic solar power are inputs into this control system. The input parameters of the wind generator and the photovoltaic solar power are given in [66] and [201]. In addition, the average hourly wind speed pattern for a typical day in Port Elizabeth is obtained from the Southern African Universities Radiometric Network<sup>2</sup>.

<sup>2</sup><http://www.sauran.net/>

### 8.3.3 Uncertainty analysis of the measured data

An uncertainty or error analysis is important in order to ascertain the confidence level of the measured hot water demand for both HPWH and instantaneous shower. The uncertainty analysis of the experimental data is carried out using the approach taken by Sichilalu and Xia [67]. Random and instrument's error are considered to affect the measurements of hot water demand in this study. Random errors are generated in MATLAB software with a distribution mean and standard deviation of 0 and 1 respectively while the instrument's absolute uncertainty of  $\pm 0.01$  is provided by the manufacturer. The measured value,  $S_{meas}$ , is therefore given as,

$$S_{meas} = S_{actual} + (Err_{random} \times Err_{inst}), \quad (8.34)$$

where  $Err_{random}$  and  $Err_{inst}$  are the random and instrument errors respectively, while  $S_{actual}$  is the true value. The relative error,  $Err_{relative}$ , is then obtained as,

$$Err_{relative} = \frac{Err_{eff}}{S_{meas}} \%. \quad (8.35)$$

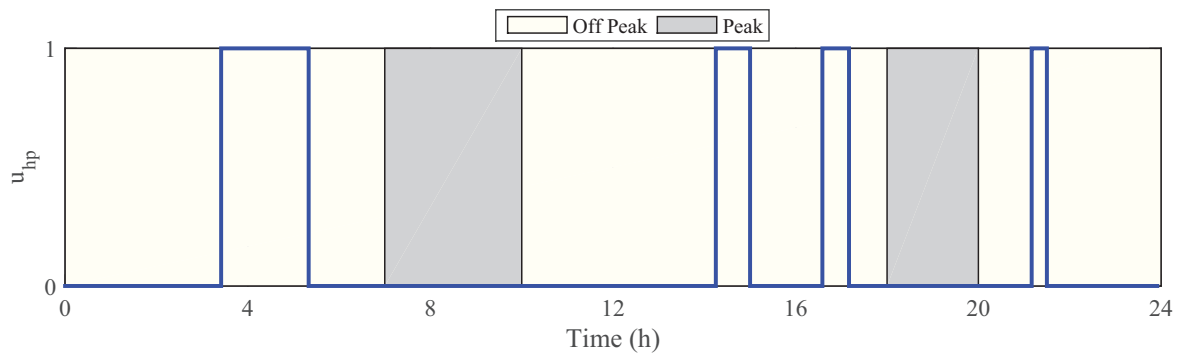
Using the rule of the weakest link, the measurement with the largest relative error is used to determine the final absolute error of the performance index [226], which is the cost of energy in this case.

## 8.4 MPC RESULTS AND DISCUSSION

This section includes simulation results of using MPC to optimally operate the hot water devices, optimal consumption of grid power and the effect on the temperature of hot water from the hot water devices. The legend showing peak and off-peak periods of the TOU tariff is the same throughout this chapter. Thereafter, benefits of the MPC strategy controlling the hot water devices having integrated renewable energy sources are discussed.

### 8.4.1 Optimal operation of HPWH

The MPC strategy optimally operates the HPWH as shown in Figure 8.3 in meeting the overall hot water demand in the house. The HPWH is only operated during the cheaper off-peak TOU period effectively minimizing cost of energy. The controller switches on the HPWH from 03:25-05:15, and predicts that this hot water will be sufficient to meet the high hot water demand in the morning thereby avoiding operating it during the expensive morning TOU peak period. The controller later switches

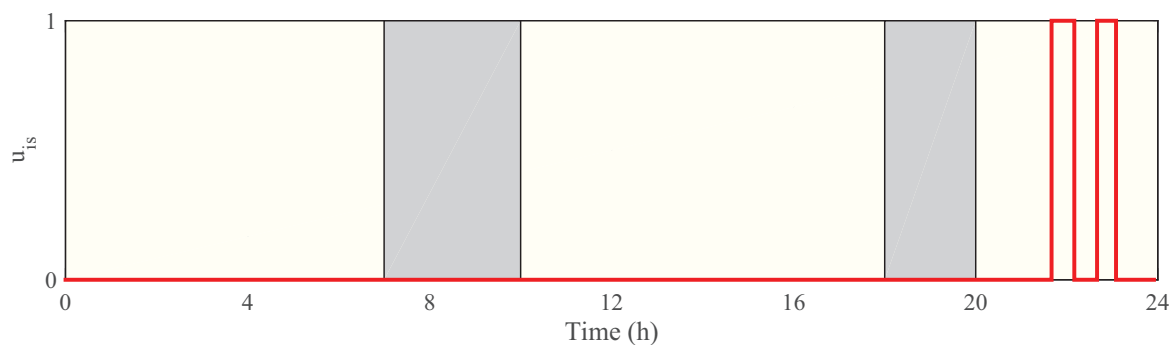


**Figure 8.3.** Optimal operation HPWH to meet overall hot water demand.

on the HPWH again at 14:15-14:55 and 16:35-17:05 upon predicting an increase in water demand in the evening coinciding with the evening TOU peak period. Just like during the morning TOU peak period, it predicts the need to avoid the evening TOU peak, thereby switching the HPWH off until 21:10-21:25 where it switches on to enable meeting the demand for the remaining period of the 24-h operating cycle.

#### 8.4.2 Optimal operation of instantaneous shower

Figure 8.4 shows the optimal operation of instantaneous shower. Just like in optimal operation of



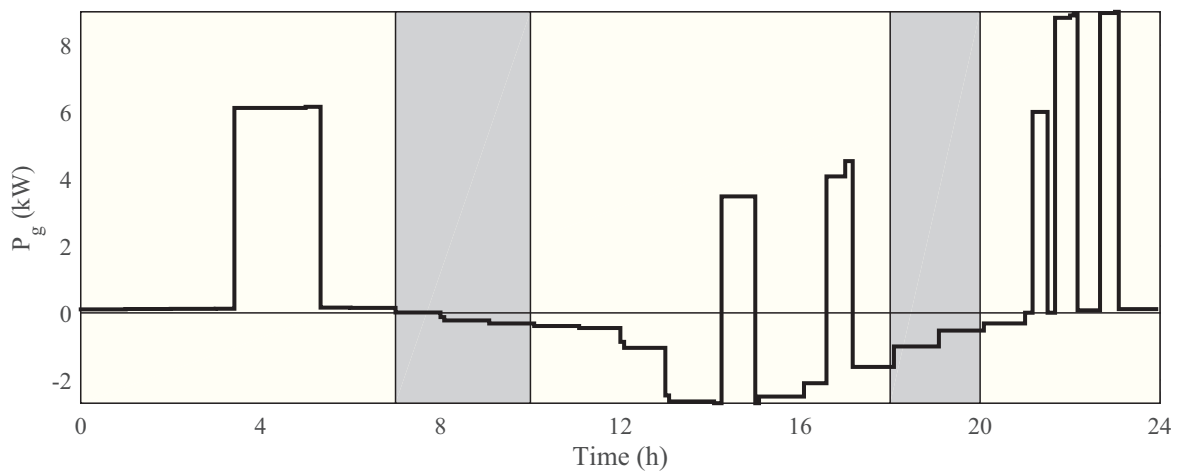
**Figure 8.4.** Optimal operation of instantaneous shower.

HPWH, the MPC controller operates the instantaneous shower during the cheaper TOU off-peak periods, effectively minimizing the cost of energy. The controller determines that hot water from HPWH to the instantaneous shower is sufficiently hot to meet the morning shower's water demand. There is therefore no need to switch it on during this period. On the other hand, as the demand from the shower is predicted to rise in the evening, the controller switches on the instantaneous shower on

between 21:40-22:05 and 22:40-23:00 to meet the demand for the shower in this period as the water from HPWH was not sufficiently hot.

#### 8.4.2.1 Optimal power flow through the grid

When the MPC controller optimally operates the hot water devices, the consequent grid power flow is shown in Figure 8.5. The power sold to the grid appears irregular as the control system assumes

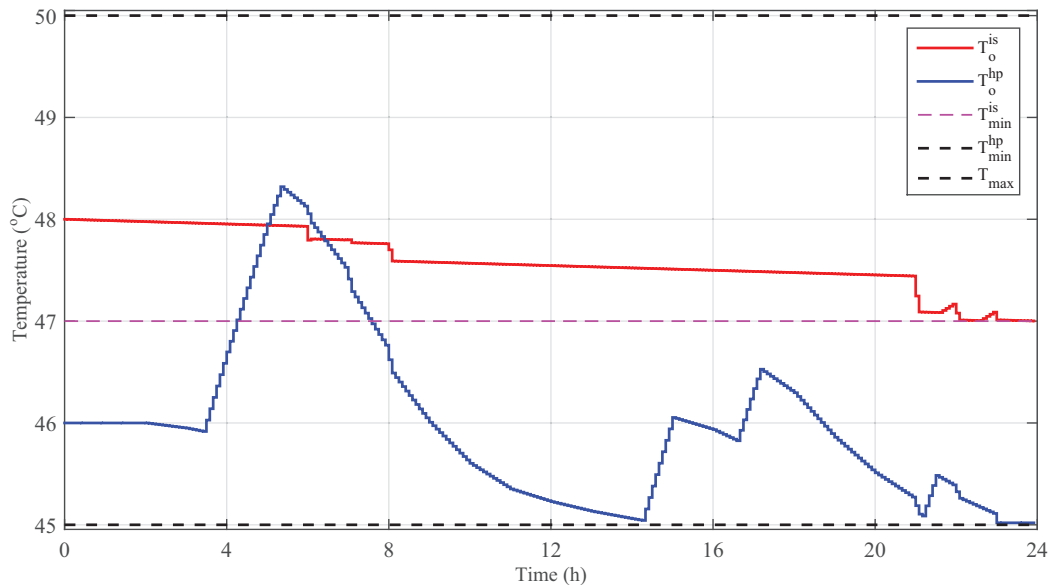


**Figure 8.5.** Optimal grid power flow.

no energy storage takes place in the house. The grid provides minimal power of 0.12 kW from 00:00-03:25 to cater for the domestic load, in the absence of renewable energy. Thereafter, the operation of HPWH in the absence of renewable power from 03:25-05:15 leads to an increase in grid power consumption. During the morning TOU peak, both devices are off and the presence of solar power leads to excess power being sold back to the grid. This is actually important in providing power to other households that need power during the peak TOU period. The sale of power back to the grid increases during the day as the amount of renewable energy increases. The grid power consumption again rises to supplement renewable energy in powering HPWH at 14:15-14:55 and 16:35-17:05. Thereafter, excess power being sold to the grid starts to decline as evening approaches, and by the time the hot water devices are switched on at 21:10-21:25, 21:40-22:05 and 22:40-23:00, only grid energy can be used.

### 8.4.2.2 Hot water temperature variation

Using the MPC strategy for the 24-h operating cycle, the temperature of hot water in both devices varies as shown in Figure 8.6. It can be noted that the MPC controller meets all the set constraints



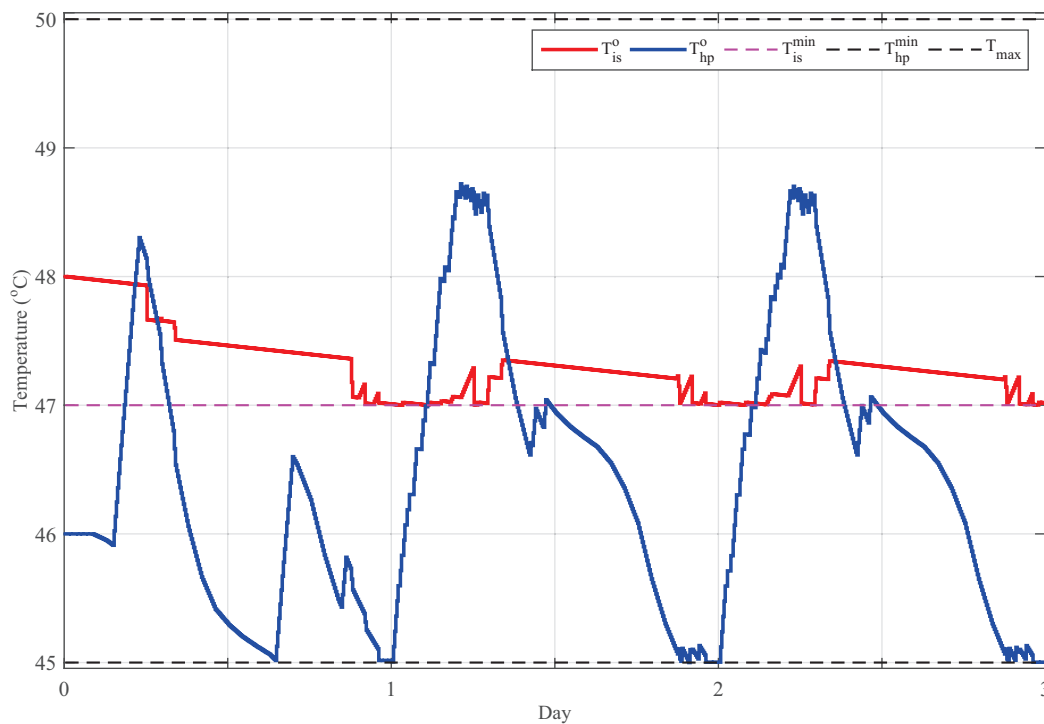
**Figure 8.6.** Variation of temperature of hot water in both devices

in operating both hot water devices. Before switching them on, the temperature of hot water falls gradually due to either standby losses or losses due to drawing of hot water. The temperature of hot water stored in HPWH rises from  $45.92 - 48.32^{\circ}\text{C}$  between 03:25-05:15 when the HPWH is in operation. Thereafter, the temperature starts to fall as the demand for hot water increases up to  $45.04^{\circ}\text{C}$  at 14:15. The risk of violating the minimum temperature constraint necessitates the controller to switch on the HPWH at 14:15-14:55 raising the temperature to  $46.05^{\circ}\text{C}$ . The controller predicts the rising evening water demand and the TOU peak tariff and further switches on the HPWH at 16:35-17:05 raising the temperature up to  $46.53^{\circ}\text{C}$ . This water meets the demand during this peak period until it falls to  $45.09^{\circ}\text{C}$  at 21:10. Again, to avoid violating the minimum temperature constraint, the controller switches on the HPWH up to 21:25 heating the water to  $45.49^{\circ}\text{C}$ . As the controller predicted, this water is sufficient to meet the demand for the remaining period.

In idle state, that is, when the instantaneous shower is not in use, the temperature of water keeps dropping due to standby losses. Such is the case at 00:00-05:45 and 08:05-20:50. However, in its active state, that is, whenever there is demand of hot water in the shower, the temperature variation is

caused either by switching on the instantaneous shower or sufficiently hot water is flowing from the HPWH. In the morning, between 05:45-08:00, the MPC controller detects that water from the HPWH is sufficiently hot for shower use. There is therefore no need of switching on the instantaneous shower as the demand leads to a drop in this temperature to  $47.59^{\circ}\text{C}$ . On the contrary, in the evening shower demand, that is, from 20:55-23:00, the controller operates the instantaneous shower twice as it detects that shower's minimum temperature constraint could be violated as the water becomes cold. Eventually, the controller maintains the water within the required temperature range in the shower.

It is further evident that the temperature of hot water in both hot water devices at the end of the 24-h operating horizon is different from the initial temperatures, implying that initial temperature for the following day will be different from previous day's. This problem is called the turnpike phenomenon and is normally observed in optimization problems with or without terminal constraints [162]. Although open-loop optimal controllers do not guarantee proper operation in the presence of turnpike phenomenon, MPC controllers have been shown to automatically correct it over several days as it uses the previous state of the plant rather than the initial condition [161, 225]. To ascertain proper operation of the MPC controller, simulations are carried out for several days with temperature variation in both devices shown in Figure 8.7. This is carried out with the assumption that conditions remain

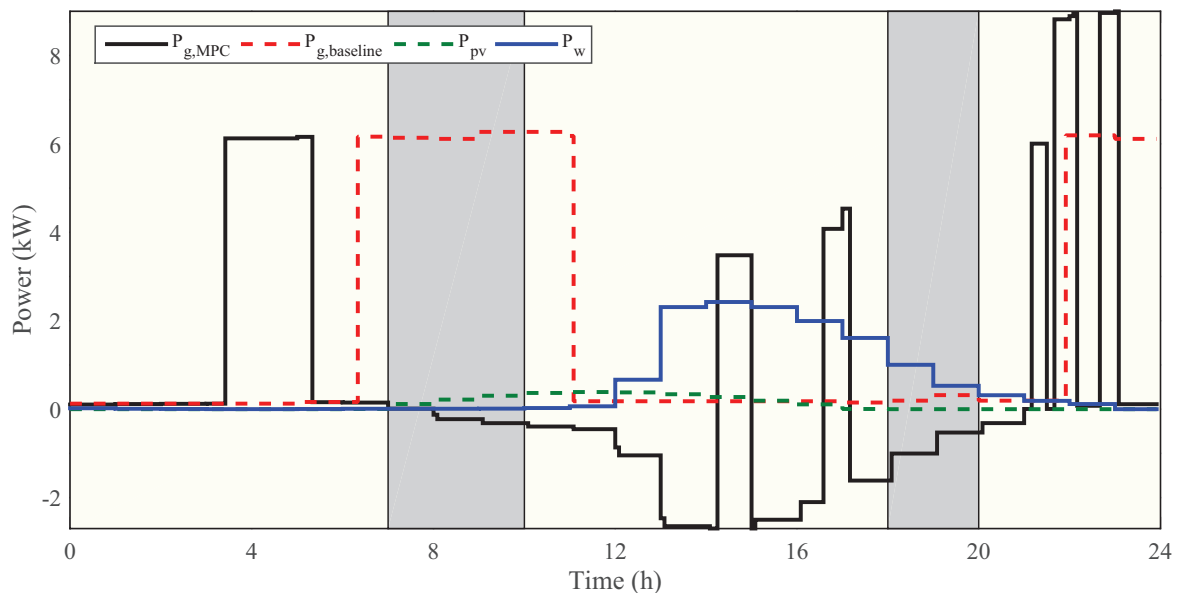


**Figure 8.7.** Temperature variation for three days

similar over these days. Even though initial temperature is different from the final temperature in the first day, the MPC controller adjusts itself such that if subsequent days have same conditions, the temperature of water in both devices would be the same at the end of each day. This subsequently means that initial temperature would also be the same at the start of each new day. In line with the controller's objective to minimize cost of energy and use of instantaneous shower, the temperature of water in both devices at the end of each day is almost the minimum allowable temperature.

### 8.4.3 Discussion

Figure 8.8 shows the flow of power in the system throughout the operating cycle. In the legend,  $P_{g,MPC}$ ,



**Figure 8.8.** Power flow throughout the 24-h operating cycle

is the optimal flow of power in the grid resulting from the use of closed-loop MPC strategy,  $P_{g,baseline}$  is the grid power consumption when the HPWH is controlled by a thermostat and solely powered by grid in the case study.  $P_{pv}$  and  $P_w$  represent the renewable energy generated through PV and wind sources respectively. It can be seen that renewable sources are available during the day, from 07:00 and 12:00 up to 17:00 and 21:00 for PV and wind respectively. As the amount of renewable energy increases during day time, more of it is sold back to the grid (appearing as negative values of  $P_{g,MPC}$ ) whenever the MPC controller switches off both devices. The consumption of power from the grid reaches maximum in the evening when the instantaneous shower is switched on. This is because the

renewable energy being generated in this period is too little making the grid to meet the power demand. The baseline power flow,  $P_{g,baseline}$ , is operated throughout the morning TOU peak to maintain water at the set temperature. This not only leads to higher energy costs being incurred by the end user but it is also against the desire of the power utility to shift load to off-peak periods in order improve the quality of the grid.

The comparison of the day's energy consumption and associated cost in the house is shown in Table 8.2. The baseline in the table is the current scenario in the case study, where the thermostatically

**Table 8.2.** Energy consumption, sales, savings and associated costs.

Baseline		MPC		Savings		Feed-in	
Energy (kWh/day)	Cost (R/day)	Energy (kWh/day)	Cost (R/day)	Energy (%)	Cost (%)	Energy (kWh/day)	Revenue (R/day)
41.00	50.33	27.78	17.46	32.24	65.30	18.76	11.79

controlled HPWH is powered by grid power. Moreover, it is used to meet all the hot water demand in the house necessitating its temperature to be set at a range acceptable in the shower, despite other hot water demands requiring less hot water. This means that the baseline cost of energy is the cost incurred by the thermostatically controlled HPWH under the TOU electricity tariff. The MPC energy is the grid energy consumed upon using the MPC controller integrated with renewable energy sources, while the cost of energy is obtained using the TOU electricity tariff. Savings are obtained from the difference between the baseline and optimal energy consumption and cost respectively. Finally, feed-in energy is the energy from the renewable energy sources, that is, wind and solar PV, that is not used by the two hot water devices as well as domestic load and is therefore sold back to the grid. Since no energy storage is considered, the excess renewable energy is fed back to the grid through an appropriate feed-in tariff to be used by other users within the network. The MPC strategy for heating water using a HPWH and an instantaneous shower powered using integrated renewable energy sources has the potential to save 32.24% grid energy in a day when compared to the baseline. These energy savings result from operation of the HPWH at a lower temperature, meaning, only the required water is heated to higher temperature by the instantaneous shower, and also the use of available renewable energy whenever the devices are switched on. Further, the strategy can lead to a cost savings of up to 65.30% in a day when compared to the thermostat controlled baseline arising from the cost of the energy saved as well as shifting the load to the cheaper off-peak period of the TOU tariff. Incorporation of renewable energy



sources capable of selling excess power back to the grid through an feed-in tariff can lead to the sale of up to  $18.76 \text{ kWh}$  in a day with a revenue of  $R 11.79$  in that day. This revenue is obtained using the off peak price of the TOU tariff as municipalities in South Africa are allowed to buy power back from domestic consumers at the same rate as they sell it. The energy and cost savings obtained in this study compare well with a study conducted on open-loop optimal control of HPWH powered by integrated renewable energy systems [66]. The cost savings in that study are higher (70.74%) as the study only has HPWH as the only energy consuming device, and consequently sells more renewable energy back to the grid.

A further benefit of the strategy involving heating the water for showering at the end use, is the elimination of water wastage through conveyance as the end user waits for hot water to arrive to the shower. For the house in this case, up to  $19 \text{ l}$  of water is saved from being wasted in a day, as this amount is heated by the instantaneous shower whenever there is demand for the shower. Saving this potable water is very important for a dry and developing nation like South Africa, which has high water insecurity.

All measurements have errors or uncertainties present and in this regard, the maximum relative error  $Err_{relative} = 1.99\%$  is obtained in this study, whose effect on the performance index is shown in Table 8.3. The performance index when actual values are used in the controller is 2.29% lower than with

**Table 8.3.** Uncertainty of the performance index

	Optimal energy	Cost	Savings (%)	
	(kWh/day)	(R/day)	Energy	Cost
Baseline	41.00	50.33	-	-
Measured	27.78	17.46	32.24	65.30
Actual	27.14	17.06	33.80	66.10

measured values, leading to 66.10% cost savings compared to the baseline. Therefore, the final absolute error of the control strategy is  $(50.33 - 17.06) \times 1.99\% = 0.66$ , such that the final cost saving of the system is  $R(17.06 \pm 0.66)$  in a day. This performance compares well with Sichilalu and Xia [67], with the difference in final absolute error arising due to the different sampling intervals considered.

The combination of HPWH and instantaneous shower powered using renewable energy sources can operate efficiently saving both energy, water and their associated cost if proper control strategies are employed. In reality, there are unpredictable disturbances present in any system. Previous research has shown that open-loop optimal control normally deals with uncertainties that do not significantly change the demand pattern and are more predictable while closed-loop MPC strategy robustly deals with disturbances that aren't easily predictable and significantly change the pattern of water demand [157, 225]. This benefit comes at more complexity and a higher computation and implementation cost than open-loop optimal control strategy [225].

### 8.5 ECONOMIC ANALYSIS

It is important to evaluate the cost effectiveness of implementing the renewable energy and control strategies based on comprehensive consideration of various cost and revenue components. One effective method is the present worth method that discounts back all future elements of the financial analysis of a project to their present worth, apart from capital costs that are already given in present terms. Thereafter, positive and negative elements of the cash flow are summed and if the net present value (NPV) is positive, then the investment is financially attractive [254]. When such an analysis is carried out throughout the life of the project, it is called life cycle cost (LCC) analysis. Costs included in analysis of LCC include cost of acquisition, operation, maintenance and disposal [255]. Therefore,

$$LCC = C_c + C_o + C_s, \quad (8.36)$$

where  $C_c$  is the capital cost,  $C_o$  is the operation cost and  $C_s$  is the salvage cost at the end of life of the system. Capital cost includes total cost of labour, acquiring and installing the system. In the operation stage, operation cost mainly includes energy and maintenance cost incurred during the service life. Finally, salvage cost is the cost incurred at the end of system's life including the salvage value of the system, cost of removal and disposal. Equation (8.36) can be written in terms of the discounting factor, that is, the factor by which future cash flows must be multiplied with to get the present worth, as,

$$LCC = C_c + \sum_{n=1}^m \frac{C_o(n)}{(1+r)^n} + C_s, \quad (8.37)$$

where  $n$  and  $m$  are the number of years and project lifetime respectively, while  $r$  is the discounting factor. In this study, the real discount rate is used as it represents a more realistic purchasing power. The real discount rate is normally given by the difference between the average interest and inflation rate. The interest rate represents the opportunity value of time, that is, the compensation that should be

paid to defer additional expenditure in the current year until a later year. The price of the equipment, useful life and cost of labour are based on the South African market rates<sup>3</sup> as well as estimates found in relevant literature [313]. Some assumptions are made while carrying out LCC analysis of the system; The real discount rate and maintenance costs are assumed to be constant throughout the life of the system. The annual maintenance cost is assumed to be 1% of the total cost of the system. Further, cost of electricity has been increasing annually at an average rate of 9.6% over the last ten years<sup>4</sup>. With this increase expected to continue in future, and the introduction of carbon tax set to begin, the operation cost and control benefits are assumed to increase proportionately. The annualized cost and revenue are average values from daily values obtained when simulations are carried out over the four seasons having varying hot water consumption behaviour as well as weather parameters. It is important to note that although the HPWH's COP varies with variation of ambient temperature [314], the average temperature variation in the case studied is not large, and therefore, the COP was assumed to be constant throughout all the seasons. Table B.1 shows the LCC analysis of the MPC control of both hot water devices powered by integrated renewable energy systems. It is taken that all capital investment is done in the beginning of the project, and all components of the system will be operational for the 20 year duration. In the analysis, the expenses are indicated using negative values (brackets) while revenue is indicated as positive values. Interest<sup>5</sup> and inflation<sup>6</sup> rates of 6.95% and 6.3%, respectively, which are the average rates in South Africa for the last 5 years are used to obtain the time value of money. The discounted cash flows continuously increase the cumulative cash flows in each year, and the year which the cumulative cash flows becomes zero indicates the break even point or the payback period. Further, the renewable energy benefit is obtained using the REFIT tariff in eThekweni Municipality. In this study, the strategy has a payback period of about 9 years and 4 months. Although this payback compares well with a similar study carried out in the same region by Sichilalu [22], which had a payback period of 8 years and 8 months, the methodology is slightly different. This study is more realistic, despite having an extra energy consuming device, since it uses the real discount rate while Sichilalu used the inflation rate. Further, the expected increase in operation cost and resulting effect on the benefits have been included in this study. This high payback period could even go lower if the low REFIT rates in South Africa are enhanced to make investing in domestic renewable energy more attractive. It is therefore recommended that policy makers should look at ways of making the REFIT rates in the country more attractive. Finally, the instantaneous shower is an energy consuming device

---

<sup>3</sup>[www.sustainable.co.za](http://www.sustainable.co.za)

<sup>4</sup>[www.eskom.co.za/CustomerCare/TariffsAndCharges/Pages/Tariff\\_History.aspx](http://www.eskom.co.za/CustomerCare/TariffsAndCharges/Pages/Tariff_History.aspx)

<sup>5</sup>[www.tradingeconomics.com/south-africa/interest-rate](http://www.tradingeconomics.com/south-africa/interest-rate)

<sup>6</sup>[www.tradingeconomics.com/south-africa/inflation-cpi](http://www.tradingeconomics.com/south-africa/inflation-cpi)

that has a significant merit of increasing water efficiency and convenience in the shower at a minimal investment.

## 8.6 SYNOPSIS

In this chapter, a closed-loop MPC strategy for optimally operating both HPWH and instantaneous shower is designed. This strategy ensures efficient use of both energy and water while meeting hot water demand in the house. The MPC controller operates the devices during the cheaper TOU off-peak period, hence leading to savings on cost of energy. Additionally, maximizing the use of renewable energy not only saves energy-not-delivered from the grid, but also brings in revenue by selling excess renewable energy back to the grid through an appropriate feed-in tariff. The closed-loop MPC strategy is superior than open-loop optimal control strategy due to its robustness in dealing with unpredictable disturbances present in the system. This however, comes at a higher computational and implementation cost, though the choice of the controller depends on the specific application. The use of instantaneous shower placed at the shower end use has double benefits. First, it allows the HPWH to be operated at a slightly lower temperature that is allowable for most other end uses leading to lower energy consumption. Secondly, it helps in saving water that cools in the pipes, which is normally drained as end users wait for hot water to arrive. This control strategy can potentially save 32.24% and 19 l of energy and water in a day respectively. This robust closed-loop control strategy operating both water heating devices powered mainly using renewable energy sources marks a step closer to zero-energy buildings. A realistic LCC analysis is carried out to determine the economic feasibility of investing in this strategy. The analysis shows that the system would have payback period of 9 years and 4 months, which compares well with similar studies carried out in South Africa.

This system is suitable for home owners who intend to increase the efficiency of use of both energy and water, and simultaneously lower the cost of both resources. It is suitable for owners keen on lowering the green gas house emission by integrating renewable energy to supply energy-efficient equipment. The MPC strategy is a cost-effective solution to overcome the weaknesses of thermostatic control. To implement the control strategy in a real word scenario, the model must be translated to C and then compiled for a specific hardware while verifying in each step<sup>7</sup>. An open-loop optimal controller solves a given optimization problem for the entire operating cycle and then sends optimal

---

<sup>7</sup>[www.mathworks.com/company/newsletters/articles/from-matlab-to-embedded-c.html](http://www.mathworks.com/company/newsletters/articles/from-matlab-to-embedded-c.html)

commands to power the devices either using grid or renewable energy. The MPC strategy requires temperature sensors to feed back the temperature of water in both devices to the controller during every time step. The controller then uses the current temperature to predict the future behaviour of the devices. This research can be advanced by incorporating other renewable energy sources such as biogas as well as considering energy storage systems like batteries and fuel cells. The effect of using energy storage systems on LCC analysis for countries whose REFIT rates are not very attractive should be investigated further.

# CHAPTER 9 COMPARISON OF LIFE CYCLE COST ANALYSIS

## 9.1 INTRODUCTION

This chapter looks at the comparison of LCC analysis of the control systems in both cold and hot water categories. The comparison is important for investors while making holistic decisions on the appropriate control strategy based on performance benefits and economic analysis. The method discussed in Section 8.5 is used to carry out the LCC analysis, which includes the acquisition, operation, maintenance, disposal costs and revenues accrued as a result of implementing the optimal control strategies. In this chapter, the maintenance cost, inflation and operation cost are assumed to be constant over the entire 20 year period under consideration. Further, the acquisition cost is taken to only take place at the onset of the project, and all components are operational for the entire period. Any cost incurred by the investor is represented as negative cash flows and represented in brackets<sup>1</sup>.

## 9.2 LCC COMPARISON IN COLD WATER CATEGORY

This thesis presents open-loop optimal and closed-loop MPC strategies as suitable for optimally operating alternative water supply options. Chapter 3 discusses a pump storage scheme using both open-loop and closed-loop MPC strategies, for buildings with unreliable water supply. To enhance water conservation, this controller is advanced by incorporating rooftop rain water harvesting in Chapter 4, grey water recycling in Chapter 5 and finally the integrated control strategies, both open-loop optimal and MPC, are discussed in Chapter 6. Subsequently, the LCC analysis of the MPC strategy is developed

---

<sup>1</sup>[www.accaglobal.com/ng/en/member/professional-standards/members-standards/international-standards.html](http://www.accaglobal.com/ng/en/member/professional-standards/members-standards/international-standards.html)

and discussed in Section 6.5.4 with the results shown in Table A.1. Using the same procedure, the LCC analysis of the open-loop optimal control strategy in integrated water recycling system is carried out and the results are shown in Table A.2. The results show that the open-loop control strategy is slightly cheaper than MPC strategy due to extra components required to implement the MPC strategy, despite its superiority. Both cases, however, do not payback within their lifetime mainly due to low price of water from municipalities. This case is not unique to South Africa [229], though, as studies from other regions in the world have also shown that water recycling strategies at the moment do no payback within their lifetime [230–232]. However, with climate change, population increase and depletion of few sources of fresh water, it is expected that the cost of potable water will increase and these alternatives will start making economic sense in the near future. The two strategies are however extremely useful in instances where potable water supply from the municipality is unreliable, forcing home owners to seek alternative supply options. The initiatives therefore not only lead to energy and water conservation but also increases the reliability and security of water supply in homes.

### 9.3 LCC COMPARISON IN HOT WATER CATEGORY

Similar to previous section, open-loop optimal and closed-loop MPC strategies are designed and developed to optimally control hot water devices while incorporating renewable energy sources. Chapter 7 discusses the performance of an open-loop optimal control strategy while Chapter 8 advances the controller by introducing feedback in the system so as to make it more robust. The LCC analysis of the two control strategies are shown in Tables B.1 and B.2. Using the open-loop control strategy, the investor can break even (payback period) after about 8 years and 10 months while the MPC strategy pays back after 9 years and 4 months. As alluded to in previous chapters, the MPC strategy is more expensive and complex to implement due to extra components required, such as sensors and more wiring, leading to a higher initial investment and hence longer time to pay back. The analysis shows that besides the performance advantages that the two control strategies possess, they also make economic sense to adopt them. Further benefits such as reduction of CO<sub>2</sub> emission can increase the benefits leading to a shorter payback period. It is important to note that this LCC analysis agrees with similar studies conducted in the region [22].

## 9.4 SYNOPSIS

The comparison of LCC analysis of both open-loop and MPC strategies in cold and hot water categories has given clear economic analysis that is important for decision and policy makers who are keen to conserve both energy and water. The strategies in the hot water category that incorporate renewable energy have shown that they can pay back the investment within their lifetime, while the strategies in cold water category do not. Each of the strategies discussed in this thesis have their importance in resource conservation and it is important for policy makers, investors and home owners in different countries to carefully weigh the implications of these strategies based on their geographical location.



## CHAPTER 10 CONCLUSIONS

This research designs optimal controllers for energy-water nexus management in residential buildings. The work is divided into two broad categories: cold water and hot water systems. In each category, the most resource intensive end uses and alternative and practical supply of energy and water are identified and the relevant optimal controllers developed. Open-loop optimal control and closed-loop model predictive control (MPC) systems are validated using a case study, and then life cycle cost (LCC) analysis done to determine economic feasibility of the proposed interventions. To begin with, energy and water systems in each category are mathematically modelled as functions of control variables. Thereafter, open-loop optimal and closed-loop MPC controllers are developed with the aim of minimizing energy cost subject to the TOU electricity tariff, amount of energy and potable consumed and maximizing use of alternative sources of these resources.

The heat pump water heater (HPWH) and instantaneous shower are used as the DSM mitigation devices for hot water use in the house. On the other hand, lawn irrigation scheduling is used as the demand side management (DSM) intervention for cold water use. Further, two most abundant renewable energy sources in South Africa, solar and wind, are modelled to power the hot water devices, with grid used both as a back up and storage of excess renewable energy as it accepts it through a feed-in tariff. This strategy not only reduces the demand for grid power, but also greatly reduces greenhouse gas emissions in the country and brings in revenue to the home owner. In similar veins, pumping-storage scheme, rooftop rain water harvesting and grey water recycling offer alternatives in supply of water to the house, increasing reliability of water supply while reducing demand for potable water and waste water services.

This work therefore endeavours to integrate various supply alternatives with resource intensive end uses in a seamless, convenient, economical and practical way.

## 10.1 SUMMARY

To achieve energy-water nexus DSM for cold water category, pump-storage scheme, rooftop rain water harvesting and grey water recycling are identified as possible water supply alternatives. Lawn irrigation is the most water intensive end use in South African homes, mainly due to low rainfall and high evaporation rates. Increased demand for municipal water has led to unreliability of supply necessitating some home owners to pump and store water in overhead or rooftop tanks. The two optimal controllers are developed for such home owners, where the cost of pumping energy subject to the TOU electricity tariff and maintenance are minimised. Both controllers meet all the constraints but adapt differently to disturbances and uncertainties present in the system. Therefore, adoption of either controllers is subject to the nature of each application and present uncertainties as recommended in Chapter 3. The second alternative water supply, rooftop rain water harvesting, is modelled to provide water for lawn irrigation for two practical scenarios discussed in Chapter 4. Direct scheduling using open-loop optimal control is done to improve the efficiency of potable water by preventing over-irrigation. Thereafter, rooftop water harvesting is subsumed, where rain water collected from the roof is stored in a tank. The open-loop controller operates a booster pump, subject to the TOU tariff, that takes water from the tank to the lawn at the required pressure and whenever required. This intervention not only ensures efficiency of use of water and pumping energy is achieved but also water conservation takes place by using stored rain water to irrigate while municipal water acts as back up. Recycling grey water from suitable end uses such as shower and washing machine is useful in providing water for non-potable end uses such as lawn irrigation and toilet flushing. Therefore, open-loop and closed-loop controllers are designed in Chapter 5 to safely and reliably operate a typical grey water recycling system, essentially enhancing energy efficiency and water conservation. This requires the use of two water pumps if municipal water is unreliable and one if reliable. Energy efficiency of the pumps is achieved through minimising the cost of pumping energy subject to the TOU tariff by the controllers. Finally, an integrated alternative water supply system incorporating pumping-storage scheme, grey and rooftop water recycling is proposed in Chapter 6. Similar to previous chapters, both control strategies are designed to operate the system safely and reliably with the aim of achieving resource efficiency and conservation. Life cycle cost (LCC) analysis is then carried out for the integrated system, and it shows that despite the practicability and the need of these cold water alternatives, they would not pay back their investment within their life time. This is attributed to low cost of potable water in the country such that end users with reliable water supply do not see the need to conserve the two resources. However, those with unreliable water supply are adopting the alternative systems as a way of ensuring reliability of the limited supply of

water available. The proposed controllers would go a long way in ensuring the systems operate safely, conveniently and reliably.

Heat pumps water heaters (HPWHs) are very energy efficient in providing hot water due to their high coefficient of performance. However, their central location in a house means that water that cools in the pipes is lost in remote end uses such as the shower, as end users normally allow it to run down the drain while waiting for the arrival of hot water. In energy-water nexus DSM, an instantaneous shower is important to prevent wastage of the cold water. Further, energy conservation takes place by powering both HPWH and instantaneous shower using integrated renewable systems (solar photovoltaic and wind) tied to the grid. An open-loop optimal control strategy is proposed in Chapter 7, which shows that it is possible to integrate and operate renewable energy devices and efficient devices in the house. The controller minimizes the use of grid energy and instantaneous shower while maximizing the use of renewable energy. It further coordinates the sale of excess renewable energy back to the grid. Thereafter, this control system is advanced by introducing feedback to the system, hence developing the closed-loop MPC controller in Chapter 8. In addition to resource and cost savings achieved by the two control systems, LCC analysis is carried out on the intervention. Unlike the cold water management system, the hot water system with integrated renewable energy systems is actually economically feasible as it paybacks within 10 years of operation.

The mitigations introduced in this research have huge effect on the security of the two resources if widely adopted. Benefits include lower bills incurred by the end users, reliability in energy and water supply, reduced demand for both resources, reduced greenhouse gas emissions, revenue from the sale of renewable energy, reduced demand for waste water services and eventually reduced pressure to increase the capacity of existing infrastructure.

## 10.2 RECOMMENDATIONS AND FUTURE WORK

The optimal control strategies proposed in this research have shown their ability in ensuring resource efficiency and conservation takes place in residential buildings. However, various uncertainties and disturbances affect the performances of the controllers differently. The open-loop controller is suitable in instances where uncertainties do not change the demand pattern during the operating cycle. However, in cases where the uncertainties and disturbances majorly affect the demand pattern, the closed-loop

MPC strategy is suitable. From the outputs of this research, it is evident that the price of municipal water really affects the economic feasibility of rain water harvesting and grey water recycling systems. Policy makers should therefore critically look at the pricing of water to include all expenses incurred in acquiring, treating and distributing to end users such as energy, environmental and social costs. Further, due to scarcity of water in South Africa, the stress will only get worse. The government should therefore encourage the uptake of resource conservation methods through incentives, rebates and public education on the importance of these interventions.

Although the objectives set for this research have been met, there is room for further improvement of the control systems. Potential opportunities include,

1. The research is carried out in two broad categories. However, one control strategy incorporating both categories can be developed to operate the resource efficient devices. Thereafter, an economic analysis should be conducted to evaluate whether the complete intervention is economically sound.
2. Incorporation of other renewable energy sources such as biomass and combined heat and power (CHP) systems could further lead more benefits such as increased grid reserves through lowered demand.
3. The interventions proposed in this work seem to have high capital cost. Other benefits such as environmental, social and carbon emission reduction should be incorporated in the design and economic analysis so as to capture all the benefits arising from the interventions.
4. This work involved domestic residential buildings. Therefore, the research can be advanced by considering the interventions for commercial buildings. It is actually expected that the cold water interventions would be economically feasible in commercial buildings. Therefore, benefits achieved would even be greater.
5. The two control systems can be implemented in a typical building and experiments carried out on their stability in performance.

## REFERENCES

- [1] Y. Sokona and F. Denton, “Climate change impacts: can Africa cope with the challenges?” *Climate Policy*, vol. 1, no. 1, pp. 117–123, 2001.
- [2] A. S. Stillwell, C. W. King, M. E. Webber, I. J. Duncan, and A. Hardberger, “The energy-water nexus in Texas,” *Ecology and Society*, pp. 1–20, 2010.
- [3] M. W. Rosegrant, X. Cai, and S. A. Cline, “Global water outlook to 2025: averting an impending crisis,” International Food Policy Research Institute, Washington, D.C., U.S.A., Tech. Rep., September 2002.
- [4] J. Conti, “International energy outlook,” U.S. Energy Information Administration (EIA), Washington, D.C., U.S.A., Tech. Rep., May 2016.
- [5] M. Pablo-Romero, R. Pozo-Barajas, and R. Yñiguez, “Global changes in residential energy consumption,” *Energy Policy*, vol. 101, pp. 342–352, 2017.
- [6] A. Fry, R. Martin, E. Haden, and M. Martin, “Water facts and trends,” WBCSD, Geneva, Switzerland, Tech. Rep., August 2005. [Online]. Available: [www.wbcsd.org](http://www.wbcsd.org)
- [7] E. M. Maleka, L. Mashimbye, and P. Goyns, “South African energy synopsis,” Department of Energy, Pretoria, South Africa, Tech. Rep., 2010.
- [8] K. D. Ilesanmi and D. D. Tewari, “Sectoral energy consumption in South Africa and its implication for economic growth,” *Evolution*, vol. 9, pp. 3523 – 3530, 2015.

## REFERENCES

---

- [9] C. du Plessis, D. Irurah, and R. Scholes, "The built environment and climate change in South Africa," *Building Research & Information*, vol. 31, no. 3-4, pp. 240–256, 2003.
- [10] D. B. Brooks, "An operational definition of water demand management," *International Journal of Water Resources Development*, vol. 22, no. 4, pp. 521–528, 2006.
- [11] P. Warren, "A review of demand-side management policy in the UK," *Renewable and Sustainable Energy Reviews*, vol. 29, pp. 941–951, 2014.
- [12] Organisation for Economic Co-operation and Development, *Environmental Performance Reviews: South Africa*. OECD Publishing, Paris, France, 2013.
- [13] F. Im, S. Mahajan, A. Denis, I. Gillson, G.-M. Lange, A. Leiman, T. Losse-Mueller, P. Ngwenya, S. Hussain, M. Brahmbhatt, and M. Toman, "South Africa economic update - focus on green growth," World Bank, Washington, D.C., U.S.A., Tech. Rep., 2011.
- [14] Pew, *Who's Winning the Clean Energy Race?* Pew Charitable Trusts, Washington D.C., U.S.A., 2014.
- [15] J. Adewumi, A. Ilemobade, and J. Van Zyl, "Treated wastewater reuse in South Africa: Overview, potential and challenges," *Resources, Conservation and Recycling*, vol. 55, no. 2, pp. 221–231, 2010.
- [16] J. Mutamba, "Water service delivery challenges-what role for water demand management?: municipal water services," *Water Wheel*, vol. 13, no. 3, pp. 30–33, 2014.
- [17] Department of Water Affairs and Forestry, "Water conservation and demand management - national strategy framework," DWAF, Pretoria, South Africa, Tech. Rep., 1999.
- [18] Y. A. Sha'aban, B. Lennox, and D. Laurí, "PID versus MPC performance for siso dead-time dominant processes," *IFAC Proceedings Volumes*, vol. 46, no. 32, pp. 241–246, 2013.

## REFERENCES

---

- [19] D. E. Kirk, *Optimal Control Theory: An Introduction*. Dover Publications Inc., New York, U.S.A., 2012.
- [20] D. Q. Mayne, J. B. Rawlings, C. V. Rao, and P. O. Scokaert, “Constrained model predictive control: Stability and optimality,” *Automatica*, vol. 36, no. 6, pp. 789–814, 2000.
- [21] D. Setlhaolo, “Optimal management of household load under demand response,” Ph.D. dissertation, University of Pretoria, Pretoria, South Africa, 2015.
- [22] S. M. Sichilalu, “Optimal control of renewable energy/grid hybrid systems with heat pump load,” Ph.D. dissertation, University of Pretoria, Pretoria, South Africa, 2016.
- [23] H. Tazvinga, B. Zhu, and X. Xia, “Energy dispatch strategy for a photovoltaic–wind–diesel–battery hybrid power system,” *Solar Energy*, vol. 108, pp. 412 – 420, 2014.
- [24] B. Wang and X. Xia, “Optimal maintenance planning for building energy efficiency retrofitting from optimization and control system perspectives,” *Energy and Buildings*, vol. 96, pp. 299 – 308, 2015.
- [25] Y. Fan and X. Xia, “A multi-objective optimization model for energy-efficiency building envelope retrofitting plan with rooftop pv system installation and maintenance,” *Applied Energy*, vol. 189, pp. 327–335, 2017.
- [26] Y. Guo and B. W. Baetz, “Sizing of rainwater storage units for green building applications,” *Journal of Hydrologic Engineering*, vol. 12, no. 2, pp. 197–205, 2007.
- [27] M.-D. Su, C.-H. Lin, L.-F. Chang, J.-L. Kang, and M.-C. Lin, “A probabilistic approach to rainwater harvesting systems design and evaluation,” *Resources, Conservation and Recycling*, vol. 53, no. 7, pp. 393–399, 2009.
- [28] D. Zhang, R. M. Gersberg, C. Wilhelm, and M. Voigt, “Decentralized water management: rainwater harvesting and greywater reuse in an urban area of Beijing, China,” *Urban Water Journal*, vol. 6, no. 5, pp. 375–385, 2009.

## REFERENCES

---

- [29] N.-B. Chang, B. J. Rivera, and M. P. Wanielista, "Optimal design for water conservation and energy savings using green roofs in a green building under mixed uncertainties," *Journal of Cleaner Production*, vol. 19, no. 11, pp. 1180–1188, 2011.
- [30] P. Rousseau and G. Greyvenstein, "Enhancing the impact of heat pump water heaters in the South African commercial sector," *Energy*, vol. 25, no. 1, pp. 51–70, 2000.
- [31] A. C. Christian, A. Rahman, and G. J. Mwangi, "Economic analysis and feasibility of rainwater harvesting systems in urban and peri-urban environments: A review of the global situation with a special focus on Australia and Kenya," *Water*, vol. 8, no. 4, pp. 1–21, 2016.
- [32] J. L. du Plessis, "Estimating domestic outdoor water demand for residential estates," Master's thesis, Stellenbosch University, Stellenbosch, South Africa, April 2014.
- [33] H. Jacobs, "Residential water information management," *South African Journal of Information Management*, vol. 3, pp. 1 – 12, 2008.
- [34] A. Hepbasli and Y. Kalinci, "A review of heat pump water heating systems," *Renewable and Sustainable Energy Reviews*, vol. 13, no. 6, pp. 1211–1229, 2009.
- [35] J. Meyer and M. Tshimankinda, "Domestic hot water consumption in South African houses for developed and developing communities," *International Journal of Energy Research*, vol. 21, no. 7, pp. 667–673, 1997.
- [36] X. Xia and L. Zhang, "Industrial energy systems in view of energy efficiency and operation control," *Annual Reviews in Control*, vol. 42, pp. 299–308, 2016.
- [37] S. Arlosoroff, "Water demand management," in *International Symposium on Efficient Water Use in Urban Areas, Kobe, Japan*, 1999.
- [38] G. Hondroyiannis, S. Lolos, and E. Papapetrou, "Energy consumption and economic growth: assessing the evidence from Greece," *Energy Economics*, vol. 24, no. 4, pp. 319–336, 2002.



## REFERENCES

---

- [39] A. Pegels, “Renewable energy in South Africa: Potentials, barriers and options for support,” *Energy policy*, vol. 38, no. 9, pp. 4945–4954, 2010.
- [40] N. M. Odhiambo, “Electricity consumption and economic growth in South Africa: A trivariate causality test,” *Energy Economics*, vol. 31, no. 5, pp. 635–640, 2009.
- [41] K. Menyah and Y. Wolde-Rufael, “Energy consumption, pollutant emissions and economic growth in South Africa,” *Energy Economics*, vol. 32, no. 6, pp. 1374–1382, 2010.
- [42] X. Xia, J. Zhang, and W. Cass, “Energy management of commercial buildings—a case study from a POET perspective of energy efficiency,” *Journal of Energy in Southern Africa*, vol. 23, no. 1, pp. 23–31, 2012.
- [43] X. Xia and J. Zhang, “Energy audit from a POET perspective,” in *Int. Conf. on Appl. Energy, Singapore*, 2010, pp. 1200–1209.
- [44] X. Xia and J. Zhang, “Energy efficiency and control systems—from a POET perspective,” *IFAC Proceedings Volumes*, vol. 43, no. 1, pp. 255–260, 2010.
- [45] A. Middelberg, J. Zhang, and X. Xia, “An optimal control model for load shifting—with application in the energy management of a colliery,” *Applied Energy*, vol. 86, no. 7, pp. 1266–1273, 2009.
- [46] Statistics South Africa, “South Africa energy statistics,” Statistics South Africa, Pretoria, South Africa, Tech. Rep., 2008.
- [47] H. Zhang, X. Xia, and J. Zhang, “Optimal sizing and operation of pumping systems to achieve energy efficiency and load shifting,” *Electric Power Systems Research*, vol. 86, pp. 41–50, 2012.
- [48] L. Zhang, X. Xia, and J. Zhang, “Improving energy efficiency of cyclone circuits in coal beneficiation plants by pump-storage systems,” *Applied Energy*, vol. 119, pp. 306 – 313, 2014.

## REFERENCES

---

- [49] L. Zhang and X. Xia, "A model predictive control for coal beneficiation dense medium cyclones," *IFAC Proceedings Volumes*, vol. 47, no. 3, pp. 9810–9815, 2014.
- [50] E. J. Meyer and I. Craig, "The development of dynamic models for a dense medium separation circuit in coal beneficiation," *Minerals Engineering*, vol. 23, no. 10, pp. 791–805, 2010.
- [51] X. Zhuan and X. Xia, "Development of efficient model predictive control strategy for cost-optimal operation of a water pumping station," *IEEE Transactions on Control Systems Technology*, vol. 21, no. 4, pp. 1449–1454, 2013.
- [52] T. Mathaba and X. Xia, "A parametric energy model for energy management of long belt conveyors," *Energies*, vol. 8, no. 12, pp. 13 590–13 608, 2015.
- [53] Y. Shen and X. Xia, "Adaptive parameter estimation for an energy model of belt conveyor with dc motor," *Asian Journal of Control*, vol. 16, no. 4, pp. 1122–1132, 2014.
- [54] S. Zhang and X. Xia, "Optimal control of operation efficiency of belt conveyor systems," *Applied Energy*, vol. 87, no. 6, pp. 1929 – 1937, 2010.
- [55] B. Numbi and X. Xia, "Optimal energy control of a crushing process based on vertical shaft impactor," *Applied Energy*, vol. 162, pp. 1653–1661, 2016.
- [56] B. Numbi, J. Zhang, and X. Xia, "Optimal energy management for a jaw crushing process in deep mines," *Energy*, vol. 68, pp. 337 – 348, 2014.
- [57] B. Numbi and X. Xia, "Systems optimization model for energy management of a parallel HPGR crushing process," *Applied Energy*, vol. 149, pp. 133 – 147, 2015.
- [58] W. Badenhorst, J. Zhang, and X. Xia, "Optimal hoist scheduling of a deep level mine twin rock winder system for demand side management," *Electric Power Systems Research*, vol. 81, no. 5, pp. 1088–1095, 2011.

## REFERENCES

---

- [59] A. Chatterjee, L. Zhang, and X. Xia, "Optimization of mine ventilation fan speeds according to ventilation on demand and time of use tariff," *Applied Energy*, vol. 146, pp. 65–73, 2015.
- [60] M. Chou, X. Xia, and C. Kayser, "Modelling and model validation of heavy-haul trains equipped with electronically controlled pneumatic brake systems," *Control Engineering Practice*, vol. 15, no. 4, pp. 501–509, 2007.
- [61] X. Xia and J. Zhang, "Modeling and control of heavy-haul trains," *IEEE Control Systems Magazine*, vol. 31, no. 4, pp. 18–31, 2011.
- [62] X. Zhuan and X. Xia, "Optimal scheduling and control of heavy haul trains equipped with electronically controlled pneumatic braking systems," *IEEE Transactions on Control Systems Technology*, vol. 15, no. 6, pp. 1159–1166, 2007.
- [63] X. Zhuan and X. Xia, "Fault-tolerant control of heavy-haul trains," *Vehicle System Dynamics*, vol. 48, no. 6, pp. 705–735, 2010.
- [64] S. Ntsaluba, B. Zhu, and X. Xia, "Optimal flow control of a forced circulation solar water heating system with energy storage units and connecting pipes," *Renewable Energy*, vol. 89, pp. 108–124, 2016.
- [65] S. Sichilalu, X. Xia, and J. Zhang, "Optimal scheduling strategy for a grid-connected photovoltaic system for heat pump water heaters," *Energy Procedia*, vol. 61, pp. 1511–1514, 2014.
- [66] S. Sichilalu, T. Mathaba, and X. Xia, "Optimal control of a wind-PV-hybrid powered heat pump water heater," *Applied Energy*, vol. 185, pp. 1173–1184, 2017.
- [67] S. M. Sichilalu and X. Xia, "Optimal power dispatch of a grid tied-battery-photovoltaic system supplying heat pump water heaters," *Energy Conversion and Management*, vol. 102, pp. 81–91, 2015.
- [68] S. Sichilalu and X. Xia, "Optimal energy control of grid tied pv–diesel–battery hybrid system powering heat pump water heater," *Solar Energy*, vol. 115, pp. 243–254, 2015.

## REFERENCES

---

- [69] S. Sichilalu, H. Tazvinga, and X. Xia, “Optimal control of a fuel cell/wind/pv/grid hybrid system with thermal heat pump load,” *Solar Energy*, vol. 135, pp. 59–69, 2016.
- [70] D. Setlhaolo and X. Xia, “Optimal scheduling of household appliances with a battery storage system and coordination,” *Energy and Buildings*, vol. 94, pp. 61–70, 2015.
- [71] D. Setlhaolo and X. Xia, “Combined residential demand side management strategies with coordination and economic analysis,” *International Journal of Electrical Power & Energy Systems*, vol. 79, pp. 150–160, 2016.
- [72] X. Ye and X. Xia, “Optimal metering plan for measurement and verification on a lighting case study,” *Energy*, vol. 95, pp. 580–592, 2016.
- [73] X. Ye, X. Xia, and J. Zhang, “Optimal sampling plan for clean development mechanism energy efficiency lighting projects,” *Applied energy*, vol. 112, pp. 1006–1015, 2013.
- [74] H. Carstens, X. Xia, and X. Ye, “Improvements to longitudinal clean development mechanism sampling designs for lighting retrofit projects,” *Applied Energy*, vol. 126, pp. 256–265, 2014.
- [75] N. Wang, J. Zhang, and X. Xia, “Energy consumption of air conditioners at different temperature set points,” *Energy and Buildings*, vol. 65, pp. 412–418, 2013.
- [76] J. Mei, B. Zhu, and X. Xia, “Model predictive control for optimizing indoor air temperature and humidity in a direct expansion air conditioning system,” in *27th Control and Decision Conference (CCDC), Qingdao, China*. IEEE, 2015, pp. 2491–2496.
- [77] B. Wang, X. Xia, and J. Zhang, “A multi-objective optimization model for the life-cycle cost analysis and retrofitting planning of buildings,” *Energy and Buildings*, vol. 77, pp. 227–235, 2014.
- [78] M. Michael, L. Zhang, and X. Xia, “An optimal model for a building retrofit with LEED standard as reference protocol,” *Energy and Buildings*, vol. 139, pp. 22 – 30, 2017.

## REFERENCES

---

- [79] Z. Wu and X. Xia, "Optimal switching renewable energy system for demand side management," *Solar Energy*, vol. 114, pp. 278–288, 2015.
- [80] H. Tazvinga, X. Xia, and J. Zhang, "Minimum cost solution of photovoltaic–diesel–battery hybrid power systems for remote consumers," *Solar Energy*, vol. 96, pp. 292–299, 2013.
- [81] Z. Wu, H. Tazvinga, and X. Xia, "Demand side management of photovoltaic-battery hybrid system," *Applied Energy*, vol. 148, pp. 294 – 304, 2015.
- [82] N. I. Nwulu and X. Xia, "Optimal dispatch for a microgrid incorporating renewables and demand response," *Renewable Energy*, vol. 101, pp. 16–28, 2017.
- [83] M. Berrittella, A. Y. Hoekstra, K. Rehdanz, R. Roson, and R. S. Tol, "The economic impact of restricted water supply: A computable general equilibrium analysis," *Water Research*, vol. 41, no. 8, pp. 1799–1813, 2007.
- [84] S. S. Roy and M. Rouault, "Spatial patterns of seasonal scale trends in extreme hourly precipitation in South Africa," *Applied Geography*, vol. 39, pp. 151–157, 2013.
- [85] A. Letsoalo, J. Blignaut, T. De Wet, M. De Wit, S. Hess, R. S. Tol, and J. Van Heerden, "Triple dividends of water consumption charges in South Africa," *Water Resources Research*, vol. 43, no. 5, 2007.
- [86] F. Otieno and G. Ochieng, "Water management tools as a means of averting a possible water scarcity in South Africa by the year 2025," *Water SA*, vol. 30, no. 5, pp. 120–124, 2004.
- [87] Department of Water Affairs and Forestry, "Water conservation and demand management strategy for the water services sectors," DWAF, Pretoria, South Africa, Tech. Rep., August 2004.
- [88] G.-M. Lange, "An approach to sustainable water management in Southern Africa using natural resource accounts: the experience in Namibia," *Ecological Economics*, vol. 26, no. 3, pp. 299–311, 1998.

## REFERENCES

---

- [89] Organisation for Economic Co-operation and Development, *Benefits of Investing in Water and Sanitation: An OECD Perspective*. OECD Publishing, Paris, France, 2011.
- [90] K. Vairavamoorthy, S. D. Gorantiwar, and A. Pathirana, “Managing urban water supplies in developing countries—climate change and water scarcity scenarios,” *Physics and Chemistry of the Earth, Parts A/B/C*, vol. 33, no. 5, pp. 330–339, 2008.
- [91] A. Siddiqi and L. D. Anadon, “The water-energy nexus in Middle East and North Africa,” *Energy Policy*, vol. 39, no. 8, pp. 4529–4540, 2011.
- [92] D. Bauer, M. Philbrick, and B. Vallario, “The water-energy nexus: Challenges and opportunities,” U.S. Department of Energy, Washington, D.C., U.S.A., Tech. Rep., 2014.
- [93] P. H. Gleick, “Water and energy,” *Annual Review of Energy and the Environment*, vol. 19, no. 1, pp. 267–299, 1994.
- [94] L. Hardy, A. Garrido, and L. Juana, “Evaluation of Spain’s water-energy nexus,” *International Journal of Water Resources Development*, vol. 28, no. 1, pp. 151–170, 2012.
- [95] J. Pittock, K. Hussey, and S. McGlennon, “Australian climate, energy and water policies: conflicts and synergies,” *Australian Geographer*, vol. 44, no. 1, pp. 3–22, 2013.
- [96] A. Gu, F. Teng, and Y. Wang, “China energy-water nexus: Assessing the water-saving synergy effects of energy-saving policies during the eleventh five-year plan,” *Energy Conversion and Management*, vol. 85, pp. 630–637, 2014.
- [97] F. Kahrl and D. Roland-Holst, “China’s water–energy nexus,” *Water Policy*, vol. 10, no. S1, pp. 51–65, 2008.
- [98] A. S. Vieira and E. Ghisi, “Water-energy nexus in low-income houses in Brazil: the influence of integrated on-site water and sewage management strategies on the energy consumption of water and sewerage services,” *Journal of Cleaner Production*, vol. 133, pp. 145–162, 2016.

## REFERENCES

---

- [99] R. Malik, “Water-energy nexus in resource-poor economies: the Indian experience,” *International Journal of Water Resources Development*, vol. 18, no. 1, pp. 47–58, 2002.
- [100] L. Landu and A. C. Brent, “Environmental life cycle assessment of water supply in South Africa: The Rosslyn industrial area as a case study,” *Water SA*, vol. 32, no. 2, pp. 249–256, 2006.
- [101] E. Friedrich, S. Pillay, and C. Buckley, “Environmental life cycle assessments for water treatment processes a South African case study of an urban water cycle,” *Water SA*, vol. 35, no. 1, pp. 73–84, 2009.
- [102] S. Nair, B. George, H. M. Malano, M. Arora, and B. Nawarathna, “Water–energy–greenhouse gas nexus of urban water systems: Review of concepts, state-of-art and methods,” *Resources, Conservation and Recycling*, vol. 89, pp. 1–10, 2014.
- [103] L. Handia, J. M. Tembo, and C. Mwiindwa, “Potential of rainwater harvesting in urban Zambia,” *Physics and Chemistry of the Earth, Parts A/B/C*, vol. 28, no. 20, pp. 893–896, 2003.
- [104] O. R. Al-Jayyousi, “Greywater reuse: towards sustainable water management,” *Desalination*, vol. 156, no. 1-3, pp. 181–192, 2003.
- [105] E. Wanjiru and X. Xia, “Optimal energy-water management in urban residential buildings through grey water recycling,” *Sustainable Cities and Society*, vol. 32, pp. 654–668, 2017.
- [106] A. Dixon, D. Butler, and A. Fewkes, “Water saving potential of domestic water reuse systems using greywater and rainwater in combination,” *Water Science and Technology*, vol. 39, no. 5, pp. 25–32, 1999.
- [107] S. Bennett, “A brief history of automatic control,” *IEEE Control Systems Magazine*, vol. 16, no. 3, pp. 17–25, 1996.
- [108] A. MacFarlane, “The development of frequency-response methods in automatic control,” *IEEE Transactions on Automatic Control*, vol. 24, no. 2, pp. 250–265, 1979.

## REFERENCES

---

- [109] S.-Z. He, S. Tan, F.-L. Xu, and P.-Z. Wang, “Fuzzy self-tuning of pid controllers,” *Fuzzy Sets and Systems*, vol. 56, no. 1, pp. 37–46, 1993.
- [110] P. Hajela and C.-Y. Lin, “Genetic search strategies in multicriterion optimal design,” *Structural Optimization*, vol. 4, no. 2, pp. 99–107, 1992.
- [111] Š. Kozák, “From PID to MPC: Control engineering methods development and applications,” in *Cybernetics Informatics (K&I)*, Levoča, Slovakia, 2016, pp. 1–7.
- [112] A. Bemporad, F. Borrelli, and M. Morari, “Model predictive control based on linear programming - the explicit solution,” *IEEE Transactions on Automatic Control*, vol. 47, no. 12, pp. 1974–1985, 2002.
- [113] B. Cirak, “Comparison of MPC and PID controls of Sirnak water supply network system,” *International Journal of Scientific Research in Science & Technology*, vol. 2, no. 1, pp. 28 – 36, 2015.
- [114] R. Banos, F. Manzano-Agugliaro, F. Montoya, C. Gil, A. Alcayde, and J. Gómez, “Optimization methods applied to renewable and sustainable energy: A review,” *Renewable and Sustainable Energy Reviews*, vol. 15, no. 4, pp. 1753–1766, 2011.
- [115] S. C. Bengea and R. A. DeCarlo, “Optimal control of switching systems,” *Automatica*, vol. 41, no. 1, pp. 11–27, 2005.
- [116] H. Tazvinga, B. Zhu, and X. Xia, “Optimal power flow management for distributed energy resources with batteries,” *Energy Conversion and Management*, vol. 102, pp. 104–110, 2015.
- [117] P. F. Boulos, Z. Wu, C. H. Orr, M. Moore, P. Hsiung, and D. Thomas, “Optimal pump operation of water distribution systems using genetic algorithms,” in *American Water Works Association Distribution System Symposium, Denver, Colorado, U.S.A.*, 2001, pp. 1–10.
- [118] J. Reca, A. Garcia-Manzano, and J. Martinez, “Optimal pumping scheduling for complex irrigation water distribution systems,” *Journal of Water Resources Planning and Management*,



## REFERENCES

---

- vol. 140, no. 5, pp. 630–637, 2014.
- [119] J. Reça, A. Garcia-Manzano, and J. Martinez, “Optimal pumping scheduling model considering reservoir evaporation,” *Agricultural Water Management*, vol. 148, pp. 250 – 257, 2015.
- [120] A. Santhosh, A. M. Farid, and K. Youcef-Toumi, “Real-time economic dispatch for the supply side of the energy-water nexus,” *Applied Energy*, vol. 122, pp. 42–52, 2014.
- [121] A. Santhosh, A. M. Farid, and K. Youcef-Toumi, “Optimal network flow for the supply side of the energy-water nexus,” in *International Workshop on Intelligent Energy Systems*. IEEE, 2013, pp. 155 – 160.
- [122] S. H. Jeong, R. Gulbinas, R. K. Jain, and J. E. Taylor, “The impact of combined water and energy consumption eco-feedback on conservation,” *Energy and Buildings*, vol. 80, pp. 114–119, 2014.
- [123] G. Boccaletti, M. Grobbel, and M. Stuchtey, “The business opportunity in water conservation,” McKinsey & Company, Inc., London, U.K., Tech. Rep., 2010.
- [124] G. Bonvin, A. Samperio, C. L. Pape, V. Mazauric, S. Demassej, and N. Maïzi, “A heuristic approach to the water networks pumping scheduling issue,” *Energy Procedia*, vol. 75, pp. 2846 – 2851, 2015.
- [125] S. Pachauri, “An analysis of cross-sectional variations in total household energy requirements in India using micro survey data,” *Energy Policy*, vol. 32, no. 15, pp. 1723–1735, 2004.
- [126] E. M. Malatji, J. Zhang, and X. Xia, “A multiple objective optimisation model for building energy efficiency investment decision,” *Energy and Buildings*, vol. 61, pp. 81–87, 2013.
- [127] Z. Wu, X. Xia, and B. Wang, “Improving building energy efficiency by multiobjective neighborhood field optimization,” *Energy and Buildings*, vol. 87, pp. 45–56, 2015.

## REFERENCES

---

- [128] S. Chen and B. Chen, “Urban energy–water nexus: A network perspective,” *Applied Energy*, 2016.
- [129] M. Talebpour, O. Sahin, R. Siems, and R. Stewart, “Water and energy nexus of residential rainwater tanks at an end uselevel: Case of Australia,” *Energy and Buildings*, vol. 80, pp. 195–207, 2014.
- [130] C. Cheng, “Study of the inter-relationship between water use and energy conservation for a building,” *Energy and Buildings*, vol. 34, no. 3, pp. 261 – 266, 2002.
- [131] C. Cheung, K. Mui, and L. Wong, “Energy efficiency of elevated water supply tanks for high-rise buildings,” *Applied Energy*, vol. 103, pp. 685–691, 2013.
- [132] B. Baisa, L. W. Davis, S. W. Salant, and W. Wilcox, “The welfare costs of unreliable water service,” *Journal of Development Economics*, vol. 92, no. 1, pp. 1–12, 2010.
- [133] R. Porter, *The economics of water and waste in three African capitals*. Ashgate, Farnham, U.K., 1997.
- [134] A. du Plessis, T. Harmse, and F. Ahmed, “Quantifying and predicting the water quality associated with land cover change: a case study of the Blesbokspruit catchment, South Africa,” *Water*, vol. 6, no. 10, pp. 2946–2968, 2014.
- [135] P. J. Ashton, D. Hardwick, and C. Breen, “Changes in water availability and demand within South Africa’s shared river basins as determinants of regional social-ecological resilience,” unpublished.
- [136] H. Jacobs, L. Geustyn, and B. Loubser, “Water–how is it used at home?” in *Proceedings of the Water Institute of South Africa Biennial Conference, Kruger, South Africa*, 2006, pp. 1–12.
- [137] H. Jacobs, L. Geustyn, B. Loubser, and B. Van Der Merwe, “Estimating residential water demand in Southern Africa: technical paper,” *Journal of the South African Institution of Civil Engineering*, vol. 46, no. 4, pp. p–2, 2004.

## REFERENCES

---

- [138] P. W. Mayer and W. DeOreo, “Residential end uses of water,” Water Research Foundation, New York, U.S.A., Tech. Rep., 1999.
- [139] M. Heinrich, *Water End Use and Efficiency Project (WEEP)*. Branz, Porirua, New Zealand, 2007.
- [140] M. Loh and P. Coghlan, *Domestic water use study in Perth, Western Australia, 1998-2001*. Water Corporation, Perth, Australia, 2003.
- [141] P. Roberts, “Yarra valley water: 2004 residential end use measurement study,” Yarra Valley Water Melbourne, Australia, Tech. Rep., 2005.
- [142] M. A. Jiménez-Bello, A. Royuela, J. Manzano, A. G. Prats, and F. Martínez-Alzamora, “Methodology to improve water and energy use by proper irrigation scheduling in pressurised networks,” *Agricultural Water Management*, vol. 149, pp. 91–101, 2015.
- [143] B. Barán, C. von Lüken, and A. Sotelo, “Multi-objective pump scheduling optimisation using evolutionary strategies,” *Advances in Engineering Software*, vol. 36, no. 1, pp. 39–47, 2005.
- [144] R. Qi, H. Mei, C. Chen, and F. Qian, “A fast MPC algorithm for reducing computation burden of MIMO,” *Chinese Journal of Chemical Engineering*, vol. 23, no. 12, pp. 2087–2091, 2015.
- [145] S. Abirami, Z. Hussain, S. Muthu, and A. Kumar, “Performance comparison of different controllers for a level process,” *Int. Journal of Engineering Research and Applications*, vol. 4, no. 3, pp. 341–344, 2014.
- [146] A. Li, “Comparison between Model Predictive Control and PID control for water-level maintenance in a two-tank system,” Master’s thesis, University of Pittsburgh, Pittsburgh, Pennsylvania, U.S.A., 2010.
- [147] E. M. Wanjiru and X. Xia, “Energy-water optimization model incorporating rooftop water harvesting for lawn irrigation,” *Applied Energy*, vol. 160, pp. 521–531, 2015.

## REFERENCES

---

- [148] O. Pozos, C. A. Gonzalez, J. Giesecke, W. Marx, and E. A. Rodal, "Air entrapped in gravity pipeline systems," *Journal of Hydraulic Research*, vol. 48, no. 3, pp. 338–347, 2010.
- [149] M. Genius, E. Hatzaki, E. Kouromichelaki, G. Kouvakis, S. Nikiforaki, and K. P. Tsagarakis, "Evaluating consumers willingness to pay for improved potable water quality and quantity," *Water Resources Management*, vol. 22, no. 12, pp. 1825–1834, 2008.
- [150] A. M. Bagirov, A. Barton, H. Mala-Jetmarova, A. Al Nuaimat, S. Ahmed, N. Sultanova, and J. Yearwood, "An algorithm for minimization of pumping costs in water distribution systems using a novel approach to pump scheduling," *Mathematical and Computer Modelling*, vol. 57, no. 3, pp. 873–886, 2013.
- [151] J. Naoum-Sawaya, B. Ghaddar, E. Arandia, and B. Eck, "Simulation-optimization approaches for water pump scheduling and pipe replacement problems," *European Journal of Operational Research*, 2015.
- [152] M. Behandish and Z. Wu, "Concurrent pump scheduling and storage level optimization using meta-models and evolutionary algorithms," *Procedia Engineering*, vol. 70, pp. 103–112, 2014.
- [153] T. Mathaba, X. Xia, and J. Zhang, "Analysing the economic benefit of electricity price forecast in industrial load scheduling," *Electric Power Systems Research*, vol. 116, no. 0, pp. 158 – 165, 2014.
- [154] D. Setlhaolo, X. Xia, and J. Zhang, "Optimal scheduling of household appliances for demand response," *Electric Power Systems Research*, vol. 116, pp. 24 – 28, 2014.
- [155] G. Gamrath, T. Fischer, T. Gally, A. M. Gleixner, G. Hendel, T. Koch, S. J. Maher, M. Miltenberger, B. Müller, M. E. Pfetsch, C. Puchert, D. Rehfeldt, S. Schenker, R. Schwarz, F. Serrano, Y. Shinano, S. Vigerske, D. Weninger, M. Winkler, J. T. Witt, and J. Witzig, "The SCIP optimization suite 3.2: ZIB-Report 15-60," Zuse Institute Berlin, Germany, Tech. Rep., 2016.
- [156] S. J. Qin and T. A. Badgwell, "A survey of industrial model predictive control technology," *Control Engineering Practice*, vol. 11, no. 7, pp. 733–764, 2003.

## REFERENCES

---

- [157] A. J. Van Staden, J. Zhang, and X. Xia, “A model predictive control strategy for load shifting in a water pumping scheme with maximum demand charges,” *Applied Energy*, vol. 88, no. 12, pp. 4785–4794, 2011.
- [158] L. Wang, *Model Predictive Control System Design and Implementation Using MATLAB®*. Springer Science & Business Media, New York, U.S.A., 2009.
- [159] R. Willis, R. A. Stewart, P. Williams, C. Hacker, S. Emmonds, and G. Capati, “Residential potable and recycled water end uses in a dual reticulated supply system,” *Desalination*, vol. 272, no. 1, pp. 201–211, 2011.
- [160] X. Zhuan and X. Xia, “Optimal operation scheduling of a pumping station with multiple pumps,” *Applied Energy*, vol. 104, pp. 250 – 257, 2013.
- [161] M. Gugat, E. Trélat, and E. Zuazua, “Optimal Neumann control for the 1D wave equation: finite horizon, infinite horizon, boundary tracking terms and the turnpike property,” *Systems & Control Letters*, vol. 90, pp. 61–70, 2016.
- [162] T. Faulwasser and D. Bonvin, “On the design of economic NMPC based on an exact turnpike property,” *IFAC-PapersOnLine*, vol. 48, no. 8, pp. 525–530, 2015.
- [163] J. Zhang and X. Xia, “A model predictive control approach to the periodic implementation of the solutions of the optimal dynamic resource allocation problem,” *Automatica*, vol. 47, no. 2, pp. 358–362, 2011.
- [164] M. Hafeez, J. Bundschuh, and S. Mushtaq, “Exploring synergies and tradeoffs: Energy, water, and economic implications of water reuse in rice-based irrigation systems,” *Applied Energy*, vol. 114, pp. 889 – 900, 2014.
- [165] A. Plappally and J. Lienhard, “Energy requirements for water production, treatment, enduse, reclamation, and disposal,” *Renewable and Sustainable Energy Reviews*, vol. 16, pp. 4818–4848, 2012.

## REFERENCES

---

- [166] A. Fidar, F. Memon, and D. Butler, “Environmental implications of water efficient microcomponents in residential buildings,” *Science of the Total Environment*, vol. 408, pp. 5828–5835, 2010.
- [167] S. Muthukumarana, K. Baskarana, and N. Sextonb, “Quantification of potable water savings by residential water conservation and reuse - a case study,” *Resources, Conservation and Recycling*, vol. 55, pp. 945–952, 2011.
- [168] World Water Assessment Programme, “The United Nations world water development report 4: Managing water under uncertainty and risk, facts and figures,” UNESCO, Paris, France, Tech. Rep., 2012.
- [169] UNESCO, WMO, and IAEA, *The state of the Resource*. UNESCO Publishing and Berghahn Books, New York, U.S.A., 2006, vol. 2.
- [170] M. Gulati, I. Jacobs, A. Jooste, D. Naidoo, and S. Fakir, “The water-energy-food security nexus: Challenges and opportunities for food security in South Africa,” *Aquatic Procedia*, vol. 1, pp. 150 – 164, 2013.
- [171] A. Dubreuil, E. Assoumou, S. Bouckaert, S. Selosse, and N. Maïzi, “Water modeling in an energy optimization framework– the water-scarce Middle East context,” *Applied Energy*, vol. 101, pp. 268 – 279, 2013.
- [172] Y. Chiu, C. Liaw, and L. Chen, “Optimizing rainwater harvesting systems as an innovative approach to saving energy in hilly communities,” *Renewable Energy*, vol. 34, no. 3, pp. 492 – 498, 2009.
- [173] S. J. Kenway, R. Scheidegger, T. A. Larsen, P. Lant, and H.-P. Bader, “Water-related energy in households: A model designed to understand the current state and simulate possible measures,” *Energy and Buildings*, vol. 58, no. 0, pp. 378 – 389, 2013.
- [174] U. E. Ekpenyong, J. Zhang, and X. Xia, “Mathematical modelling for the social impact to energy efficiency savings,” *Energy and Buildings*, vol. 84, pp. 344 – 351, 2014.

## REFERENCES

---

- [175] B. Gumbo, “The status of water demand management in selected cities of Southern Africa,” *Physics and Chemistry of the Earth, Parts A/B/C*, vol. 29, no. 15, pp. 1225 – 1231, 2004.
- [176] H. Léville, H. Sally, and J. Cour, “Testing water demand management scenarios in a water-stressed basin in South Africa: application of the WEAP model,” *Physics and Chemistry of the Earth, Parts A/B/C*, vol. 28, no. 20, pp. 779–786, 2003.
- [177] J. M. Kahinda, A. Taigbenu, and R. Boroto, “Domestic rainwater harvesting as an adaptation measure to climate change in South Africa,” *Physics and Chemistry of the Earth, Parts A/B/C*, vol. 35, no. 13, pp. 742 – 751, 2010.
- [178] J. M. Kahinda and A. Taigbenu, “Rainwater harvesting in South Africa: Challenges and opportunities,” *Physics and Chemistry of the Earth, Parts A/B/C*, vol. 36, no. 14, pp. 968 – 976, 2011.
- [179] D. J. Sample and J. P. Heaney, “Integrated management of irrigation and urban storm-water infiltration,” *Journal of Water Resources Planning and Management*, vol. 132, no. 5, pp. 362–373, 2006.
- [180] S. Davis, M. D. Dukes, and G. Miller, “Landscape irrigation by evapotranspiration-based irrigation controllers under dry conditions in Southwest Florida,” *Agricultural Water Management*, vol. 96, no. 12, pp. 1828–1836, 2009.
- [181] S. Davis and M. Dukes, “Irrigation scheduling performance by evapotranspiration-based controllers,” *Agricultural Water Management*, vol. 98, no. 1, pp. 19 – 28, 2010.
- [182] J. Blonquist Jr., S. Jones and D. Robinson, “Precise irrigation scheduling for turfgrass using a subsurface electromagnetic soil moisture sensor,” *Agricultural Water Management*, vol. 84, no. 1, pp. 153 – 165, 2006.
- [183] A. Bocanegra-Martínez, J. M. Ponce-Ortega, F. Nápoles-Rivera, M. Serna-González, A. J. Castro-Montoya, and M. M. El-Halwagi, “Optimal design of rainwater collecting systems for

## REFERENCES

---

- domestic use into a residential development,” *Resources, Conservation and Recycling*, vol. 84, pp. 44 – 56, 2014.
- [184] D. J. Sample, J. Liu, and S. Wang, “Evaluating the dual benefits of rainwater harvesting systems using reliability analysis,” *Journal of Hydrologic Engineering*, vol. 18, no. 10, pp. 1310–1321, 2012.
- [185] D. J. Sample and J. Liu, “Optimizing rainwater harvesting systems for the dual purposes of water supply and runoff capture,” *Journal of Cleaner Production*, vol. 75, pp. 174–194, 2014.
- [186] S. Ward, D. Butler, and F. A. Memon, “Benchmarking energy consumption and  $CO_2$  emissions from rainwater-harvesting systems: an improved method by proxy,” *Water and Environment Journal*, vol. 26, no. 2, pp. 184–190, 2012.
- [187] A. Dimoudi and M. Nikolopoulou, “Vegetation in the urban environment: microclimatic analysis and benefits,” *Energy and Buildings*, vol. 35, no. 1, pp. 69–76, 2003.
- [188] M. McCready and M. Dukes, “Landscape irrigation scheduling efficiency and adequacy by various control technologies,” *Agricultural Water Management*, vol. 98, no. 4, pp. 697 – 704, 2011.
- [189] R. T. Marler and J. S. Arora, “The weighted sum method for multi-objective optimization: new insights,” *Structural and Multidisciplinary Optimization*, vol. 41, no. 6, pp. 853–862, 2010.
- [190] D. Fang and B. Chen, “Linkage analysis for the water–energy nexus of city,” *Applied Energy*, vol. 189, pp. 770 – 779, 2017.
- [191] N. Tsolakis and L. Anthopoulos, “Eco-cities: An integrated system dynamics framework and a concise research taxonomy,” *Sustainable Cities and Society*, vol. 17, pp. 1–14, 2015.
- [192] K. Yumkella and P. Yillia, “Framing the water-energy nexus for the post-2015 development agenda,” *Aquatic Procedia*, vol. 5, pp. 8–12, 2015.



## REFERENCES

---

- [193] M. Jacobsen, M. Webster, and K. Vairavamoorthy, *The Future of Water in African Cities: Why Waste Water?* World Bank Publications, Washington D.C., U.S.A., 2012.
- [194] C. E. Kontokosta and R. K. Jain, “Modeling the determinants of large-scale building water use: Implications for data-driven urban sustainability policy,” *Sustainable Cities and Society*, vol. 18, pp. 44–55, 2015.
- [195] M. Shatat, M. Worall, and S. Riffat, “Opportunities for solar water desalination worldwide: review,” *Sustainable Cities and Society*, vol. 9, pp. 67–80, 2013.
- [196] N. I. Nwulu and X. Xia, “Implementing a model predictive control strategy on the dynamic economic emission dispatch problem with game theory based demand response programs,” *Energy*, vol. 91, pp. 404–419, 2015.
- [197] B. Zhu, H. Tazvinga, and X. Xia, “Switched model predictive control for energy dispatching of a photovoltaic-diesel-battery hybrid power system,” *IEEE Transactions on Control Systems Technology*, vol. 23, no. 3, pp. 1229–1236, 2015.
- [198] Y. Cai, W. Yue, L. Xu, Z. Yang, and Q. Rong, “Sustainable urban water resources management considering life-cycle environmental impacts of water utilization under uncertainty,” *Resources, Conservation and Recycling*, vol. 108, pp. 21–40, 2016.
- [199] H. H. Savenije and P. Van Der Zaag, “Water as an economic good and demand management paradigms with pitfalls,” *Water International*, vol. 27, no. 1, pp. 98–104, 2002.
- [200] S. Mutambara, M. B. Darkoh, and J. R. Atlhopheng, “A comparative review of water management sustainability challenges in smallholder irrigation schemes in Africa and Asia,” *Agricultural Water Management*, vol. 171, pp. 63–72, 2016.
- [201] E. M. Wanjiru, S. M. Sichilalu, and X. Xia, “Optimal control of heat pump water heater-instantaneous shower using integrated renewable-grid energy systems,” *Applied Energy*, vol. 201, pp. 332–342, 2017.

## REFERENCES

---

- [202] E. M. Wanjiru, S. M. Sichilalu, and X. Xia, “Optimal integrated diesel grid-renewable energy system for hot water devices,” *Energy Procedia*, vol. 103, pp. 117–122, 2016.
- [203] R. E. Engström, M. Howells, G. Destouni, V. Bhatt, M. Bazilian, and H.-H. Rogner, “Connecting the resource nexus to basic urban service provision—with a focus on water-energy interactions in New York City,” *Sustainable Cities and Society*, vol. 31, pp. 83–94, 2017.
- [204] Z. Ren, W. Y. Chan, X. Wang, J. Anticev, S. Cook, and D. Chen, “An integrated approach to modelling end-use energy and water consumption of Australian households,” *Sustainable Cities and Society*, vol. 26, pp. 344–353, 2016.
- [205] H. Kim and J. Haberl, “Development and application of weather-normalized monthly building water use model,” *Energy and Buildings*, vol. 69, pp. 267–277, 2014.
- [206] M. Abdellatif and A. Al-Shammaš, “Review of sustainability in buildings,” *Sustainable Cities and Society*, vol. 14, pp. 171–177, 2015.
- [207] M. Lee, B. Tansel, and M. Balbin, “Goal based water conservation projections based on historical water use data and trends in Miami-Dade County,” *Sustainable Cities and Society*, vol. 1, no. 2, pp. 97–103, 2011.
- [208] C. Pahl-Wostl, D. Tàbara, R. Bouwen, M. Craps, A. Dewulf, E. Mostert, D. Ridder, and T. Taillieu, “The importance of social learning and culture for sustainable water management,” *Ecological Economics*, vol. 64, no. 3, pp. 484–495, 2008.
- [209] P. Díaz, P. Stanek, N. Frantzeskaki, and D. H. Yeh, “Shifting paradigms, changing waters: Transitioning to integrated urban water management in the coastal city of Dunedin, USA,” *Sustainable Cities and Society*, 2016.
- [210] E. Rozos and C. Makropoulos, “Assessing the combined benefits of water recycling technologies by modelling the total urban water cycle,” *Urban Water Journal*, vol. 9, no. 1, pp. 1–10, 2012.

## REFERENCES

---

- [211] J. Wilcox, F. Nasiri, S. Bell, and M. S. Rahaman, “Urban water reuse: A triple bottom line assessment framework and review,” *Sustainable Cities and Society*, vol. 27, pp. 448–456, 2016.
- [212] A. Stec and S. Kordana, “Analysis of profitability of rainwater harvesting, gray water recycling and drain water heat recovery systems,” *Resources, Conservation and Recycling*, vol. 105, pp. 84–94, 2015.
- [213] World Health Organization, *Guidelines for the Safe Use of Wastewater, Excreta and Greywater: Excreta and Grey Water Use in Agriculture*. WHO, Geneva, Switzerland, 2006, vol. 4.
- [214] P. L. Du Pisani, “Direct reclamation of potable water at Windhoek’s Goreangab reclamation plant,” *Desalination*, vol. 188, no. 1-3, pp. 79–88, 2006.
- [215] T. Asano, M. Maeda, and M. Takaki, “Wastewater reclamation and reuse in japan: overview and implementation examples,” *Water Science and Technology*, vol. 34, no. 11, pp. 219–226, 1996.
- [216] A. Mankad, “Decentralised water systems: Emotional influences on resource decision making,” *Environment International*, vol. 44, pp. 128–140, 2012.
- [217] A. Stoakley, “Alternative water management in Pretoria, South Africa: An investigation into public perceptions of water recycling,” in *People and the Planet Conference Proceedings*, 2013, pp. 1–20.
- [218] C. Santasmasas, M. Rovira, F. Clarens, and C. Valderrama, “Grey water reclamation by decentralized MBR prototype,” *Resources, Conservation and Recycling*, vol. 72, pp. 102–107, 2013.
- [219] K. Thirugnanasambandham, V. Sivakumar, and J. Prakash Maran, “Performance evaluation and optimization of electrocoagulation process to treat grey wastewater,” *Desalination and Water Treatment*, vol. 55, no. 7, pp. 1703–1711, 2015.
- [220] S. Godfrey, P. Labhasetwar, and S. Wate, “Greywater reuse in residential schools in Madhya Pradesh, India- A case study of cost–benefit analysis,” *Resources, Conservation and Recycling*,

## REFERENCES

---

- vol. 53, no. 5, pp. 287–293, 2009.
- [221] M. Hasan, M. Shafiquzzaman, J. Nakajima, A. K. T. Ahmed, and M. S. Azam, “Application of a low cost ceramic filter to a membrane bioreactor for greywater treatment,” *Water Environment Research*, vol. 87, no. 3, pp. 233–241, 2015.
- [222] E. Friedler and M. Hadari, “Economic feasibility of on-site greywater reuse in multi-storey buildings,” *Desalination*, vol. 190, no. 1, pp. 221–234, 2006.
- [223] M. Imteaz and A. Shanableh, “Feasibility of recycling grey-water in multi-storey buildings in Melbourne,” *World Sustainability Forum*, vol. 2, pp. 1–7, 2012.
- [224] A. N. Bdour, M. R. Hamdi, and Z. Tarawneh, “Perspectives on sustainable wastewater treatment technologies and reuse options in the urban areas of the mediterranean region,” *Desalination*, vol. 237, no. 1, pp. 162–174, 2009.
- [225] E. M. Wanjiru, L. Zhang, and X. Xia, “Model predictive control strategy of energy-water management in urban households,” *Applied Energy*, vol. 179, pp. 821–831, 2016.
- [226] F. Wamalwa, S. Sichilalu, and X. Xia, “Optimal control of conventional hydropower plant retrofitted with a cascaded pumpback system powered by an on-site hydrokinetic system,” *Energy Conversion and Management*, vol. 132, pp. 438–451, 2017.
- [227] P. Valdiserri and C. Biserni, “Energy performance of an existing office building in the northern part of Italy: Retrofitting actions and economic assessment,” *Sustainable Cities and Society*, vol. 27, pp. 65–72, 2016.
- [228] N. H. Wong, S. F. Tay, R. Wong, C. L. Ong, and A. Sia, “Life cycle cost analysis of rooftop gardens in Singapore,” *Building and environment*, vol. 38, no. 3, pp. 499–509, 2003.
- [229] A. Ilemobade, O. Olanrewaju, and M. Griffioen, “Greywater reuse for toilet flushing at a university academic and residential building,” *Water SA*, vol. 39, no. 06, pp. 351–360, 2013.

## REFERENCES

---

- [230] Z. Li, F. Boyle, and A. Reynolds, "Rainwater harvesting and greywater treatment systems for domestic application in Ireland," *Desalination*, vol. 260, no. 1, pp. 1–8, 2010.
- [231] M. Fountoulakis, N. Markakis, I. Petousi, and T. Manios, "Single house on-site grey water treatment using a submerged membrane bioreactor for toilet flushing," *Science of the Total Environment*, vol. 551, pp. 706–711, 2016.
- [232] S. Jabornig, "Overview and feasibility of advanced grey water treatment systems for single households," *Urban Water Journal*, vol. 11, no. 5, pp. 361–369, 2014.
- [233] C. K. Makropoulos and D. Butler, "Distributed water infrastructure for sustainable communities," *Water Resources Management*, vol. 24, no. 11, pp. 2795–2816, 2010.
- [234] J. Rice, A. Wutich, D. D. White, and P. Westerhoff, "Comparing actual de facto wastewater reuse and its public acceptability: A three city case study," *Sustainable Cities and Society*, vol. 27, pp. 467–474, 2016.
- [235] B. Verrecht, C. James, E. Germain, R. Birks, A. Barugh, P. Pearce, and S. Judd, "Economical evaluation and operating experiences of a small-scale MBR for nonpotable reuse," *Journal of Environmental Engineering*, vol. 138, no. 5, pp. 594–600, 2011.
- [236] G. Alaerts and N. Dickinson, Eds., *Water for a Changing World - Developing Local Knowledge and Capacity*. CRC Press, Delft, The Netherlands, 2008.
- [237] D. Grey and C. W. Sadoff, "Sink or swim? water security for growth and development," *Water Policy*, vol. 9, no. 6, pp. 545–571, 2007.
- [238] P. B. Cobbinah, M. O. Erdiaw-Kwasie, and P. Amoateng, "Africa's urbanisation: Implications for sustainable development," *Cities*, vol. 47, pp. 62–72, 2015.
- [239] A. Elaiw, X. Xia, and A. Shehata, "Application of model predictive control to optimal dynamic dispatch of generation with emission limitations," *Electric Power Systems Research*, vol. 84, no. 1, pp. 31–44, 2012.

## REFERENCES

---

- [240] M. Chou and X. Xia, "Optimal cruise control of heavy-haul trains equipped with electronically controlled pneumatic brake systems," *Control Engineering Practice*, vol. 15, no. 5, pp. 511–519, 2007.
- [241] P. Mukheibir, "Water resources management strategies for adaptation to climate-induced impacts in South Africa," *Water Resources Management*, vol. 22, no. 9, pp. 1259–1276, 2008.
- [242] L. H. Hoy, "A proactive water supply shortage response plan focusing on the green industry in the Rand Water supply area," Ph.D. dissertation, University of South Africa, Pretoria, South Africa, 2009.
- [243] E. L. Bohensky, "A social-ecological systems perspective on water management in South Africa," Ph.D. dissertation, University of Pretoria, Pretoria, South Africa, 2006.
- [244] D. Christova-Boal, R. E. Eden, and S. McFarlane, "An investigation into greywater reuse for urban residential properties," *Desalination*, vol. 106, no. 1-3, pp. 391–397, 1996.
- [245] E. M. Wanjiru, S. M. Sichilalu, and X. Xia, "Model predictive control of heat pump water heater-instantaneous shower powered with integrated renewable-grid energy systems," *Applied Energy*, vol. <https://doi.org/10.1016/j.apenergy.2017.05.033>, 2017.
- [246] E. Wanjiru, S. Sichilalu, and X. Xia, "Optimal operation of integrated heat pump-instant water heaters with renewable energy," *Energy Procedia*, vol. 105, pp. 2151–2156, 2017.
- [247] R.-H. Kim, S. Lee, J. Jeong, J.-H. Lee, and Y.-K. Kim, "Reuse of greywater and rainwater using fiber filter media and metal membrane," *Desalination*, vol. 202, no. 1-3, pp. 326–332, 2007.
- [248] E. Ghisi and S. M. de Oliveira, "Potential for potable water savings by combining the use of rainwater and greywater in houses in southern Brazil," *Building and Environment*, vol. 42, no. 4, pp. 1731–1742, 2007.
- [249] E. Rozos, C. Makropoulos, and D. Butler, "Design robustness of local water-recycling schemes," *Journal of Water Resources Planning and Management*, vol. 136, no. 5, pp. 531–538, 2009.

## REFERENCES

---

- [250] J. Loux, R. Winer-Skonovd, and E. Gellerman, “Evaluation of combined rainwater and grey-water systems for multiple development types in mediterranean climates,” *Journal of Water Sustainability*, vol. 2, no. 1, pp. 55–77, 2012.
- [251] B. G. Lipták, *Instrument Engineers’ Handbook: Process Control and Optimization*. CRC Press, Florida, U.S.A., 2005, vol. 2.
- [252] X. Xia and J. Zhang, “Operation efficiency optimisation modelling and application of model predictive control,” *IEEE/CAA Journal of Automatica Sinica*, vol. 2, no. 2, pp. 166–172, 2015.
- [253] U. E. Ekpenyong, J. Zhang, and X. Xia, “How information propagation in social networks can improve energy savings based on time of use tariff,” *Sustainable Cities and Society*, vol. 19, pp. 26–33, 2015.
- [254] F. M. Vanek, L. D. Albright, and L. T. Angenent, *Energy Systems Engineering: Evaluation and Implementation*. McGraw-Hill Professional, New York, U.S.A., 2012.
- [255] B. L. Capehart, W. C. Turner, and W. J. Kennedy, *Guide to Energy Management*. The Fairmont Press, Inc., Lilburn, G.A., U.S.A., 2006.
- [256] J. W. Bull, *Life Cycle Costing: For the Analysis, Management and Maintenance of Civil Engineering Infrastructure, Illustrated Edition*. Whittles Publishing, Dunbeath, U.K., 2015.
- [257] S. Lal and A. Raturi, “Techno-economic analysis of a hybrid mini-grid system for Fiji islands,” *International Journal of Energy and Environmental Engineering*, vol. 3, no. 1, pp. 1–10, 2012.
- [258] D. Weisser, “On the economics of electricity consumption in small island developing states: a role for renewable energy technologies?” *Energy Policy*, vol. 32, no. 1, pp. 127 – 140, 2004.
- [259] L. Pérez-Lombard, J. Ortiz, and C. Pout, “A review on buildings energy consumption information,” *Energy and buildings*, vol. 40, no. 3, pp. 394–398, 2008.

## REFERENCES

---

- [260] K. van Alphen, M. P. Hekkert, and W. G. van Sark, “Renewable energy technologies in the Maldives- Realizing the potential,” *Renewable and Sustainable Energy Reviews*, vol. 12, no. 1, pp. 162 – 180, 2008.
- [261] X. Liu, S.-K. Lau, and H. Li, “Optimization and analysis of a multi-functional heat pump system with air source and gray water source in heating mode,” *Energy and Buildings*, vol. 69, pp. 1–13, 2014.
- [262] H. Singh, A. Muetze, and P. Eames, “Factors influencing the uptake of heat pump technology by the UK domestic sector,” *Renewable Energy*, vol. 35, no. 4, pp. 873–878, 2010.
- [263] F. Alam and T. Theos, “A new generation energy efficient residential house in Australia,” in *Proceedings of the 4th BSME-ASME International Conference on Thermal Engineering, Bangladesh, India*, 2008, pp. 27–29.
- [264] E. Ghisi, S. Gosch, and R. Lamberts, “Electricity end-uses in the residential sector of Brazil,” *Energy Policy*, vol. 35, no. 8, pp. 4107–4120, 2007.
- [265] A. S. Vieira, R. A. Stewart, and C. D. Beal, “Air source heat pump water heaters in residential buildings in Australia: Identification of key performance parameters,” *Energy and Buildings*, vol. 91, pp. 148–162, 2015.
- [266] K. Chua, S. Chou, and W. Yang, “Advances in heat pump systems: A review,” *Applied Energy*, vol. 87, no. 12, pp. 3611–3624, 2010.
- [267] J. Ji, G. Pei, T.-t. Chow, W. He, A. Zhang, J. Dong, and H. Yi, “Performance of multi-functional domestic heat-pump system,” *Applied Energy*, vol. 80, no. 3, pp. 307–326, 2005.
- [268] S. Sichilalu and X. Xia, “Optimal power control of grid tied pv-battery-diesel system powering heat pump water heaters,” *Energy Procedia*, vol. 75, pp. 1514–1521, 2015.
- [269] L. Kreuder and C. Spataru, “Assessing demand response with heat pumps for efficient grid operation in smart grids,” *Sustainable Cities and Society*, vol. 19, pp. 136–143, 2015.



## REFERENCES

---

- [270] N. Aste, R. Adhikari, and M. Manfren, “Cost optimal analysis of heat pump technology adoption in residential reference buildings,” *Renewable Energy*, vol. 60, pp. 615–624, 2013.
- [271] B. Alimohammadisagvand, J. Jokisalo, S. Kilpeläinen, M. Ali, and K. Sirén, “Cost-optimal thermal energy storage system for a residential building with heat pump heating and demand response control,” *Applied Energy*, vol. 174, pp. 275–287, 2016.
- [272] D. S. Sowmy and R. T. Prado, “Assessment of energy efficiency in electric storage water heaters,” *Energy and Buildings*, vol. 40, no. 12, pp. 2128–2132, 2008.
- [273] H. M. Sachs, J. Talbot, and N. Kaufman, “Emerging hot water technologies and practices for energy efficiency as of 2011,” American Council for an Energy Efficient Economy, Washington D.C., U.S.A., Tech. Rep. A112, feb 2012.
- [274] S. Luo, “Achieving water conservation and energy efficiency with electric tankless water heaters,” *American Water Works Association*, vol. 102, no. 2, pp. 84–85, 2010.
- [275] S. Ashok, “Optimised model for community-based hybrid energy system,” *Renewable Energy*, vol. 32, no. 7, pp. 1155–1164, 2007.
- [276] H. Li, Z. Chen, and H. Polinder, “Optimization of multibrid permanent-magnet wind generator systems,” *IEEE Transactions on Energy Conversion*, vol. 24, no. 1, pp. 82–92, March 2009.
- [277] A. Belkaid, I. Colak, and O. Isik, “Photovoltaic maximum power point tracking under fast varying of solar radiation,” *Applied Energy*, vol. 179, pp. 523–530, 2016.
- [278] M. Collares-Pereira and A. Rabl, “The average distribution of solar radiation-correlations between diffuse and hemispherical and between daily and hourly insolation values,” *Solar Energy*, vol. 22, no. 2, pp. 155–164, 1979.
- [279] R. Rasmussen, J. MacArthur, E. Grald, and G. Nowakowski, “Performance of engine-driven heat pumps under cycling conditions,” *ASHRAE Trans.:(United States)*, vol. 93, no. 2, pp. –, 1987.

## REFERENCES

---

- [280] M. Kim, M. S. Kim, and J. D. Chung, “Transient thermal behavior of a water heater system driven by a heat pump,” *International Journal of Refrigeration*, vol. 27, no. 4, pp. 415–421, 2004.
- [281] A. A. Hegazy and M. Diab, “Performance of an improved design for storage-type domestic electrical water-heaters,” *Applied Energy*, vol. 71, no. 4, pp. 287–306, 2002.
- [282] H. Mehling, L. F. Cabeza, S. Hippeli, and S. Hiebler, “PCM-module to improve hot water heat stores with stratification,” *Renewable Energy*, vol. 28, no. 5, pp. 699–711, 2003.
- [283] A. U. Atmaca, A. Ereğ, and H. M. Altay, “Comparison of two numerical approaches to the domestic hot water circuit in a combi boiler appliance,” *Energy and Buildings*, vol. 127, pp. 1043–1056, 2016.
- [284] M. Gustafson, J. Baylor, and G. Epstein, “Direct water heater load control-estimating program effectiveness using an engineering model,” *IEEE Transactions on Power Systems*, vol. 8, no. 1, pp. 137–143, 1993.
- [285] A. H. Fanney, R. R. Zarr, and J. Ketay-Paprocki, “Thermal performance of residential electric water heaters using alternative blowing agents,” *Transactions- American Society of Heating Refrigerating and Air Conditioning Engineers*, vol. 106, no. 2, pp. 377–394, 2000.
- [286] P. Kepplinger, G. Huber, and J. Petrasch, “Field testing of demand side management via autonomous optimal control of a domestic hot water heater,” *Energy and Buildings*, vol. 127, pp. 730–735, 2016.
- [287] P. Dolan, M. Nehrir, and V. Gerez, “Development of a Monte Carlo based aggregate model for residential electric water heater loads,” *Electric Power Systems Research*, vol. 36, no. 1, pp. 29–35, 1996.
- [288] R. Milward, “Electric tankless water heating: competitive assessment,” Global Energy Partners, LLC, Lafayette, C.A., U.S.A., Tech. Rep. 1285-5-04, March 2005. [Online]. Available: <http://www.gepllc.com/>

## REFERENCES

---

- [289] G. P. Henze, D. P. Yuill, and A. H. Coward, “Development of a model predictive controller for tankless water heaters,” *HVAC&R Research*, vol. 15, no. 1, pp. 3–23, 2009.
- [290] R. K. Johnson and C. A. Clark, “Field evaluation of two demand electric water heaters.” *ASHRAE Transactions*, vol. 112, no. 1, pp. 426–435, 2006.
- [291] G. Johnson and I. Beausoleil-Morrison, “The calibration and validation of a model for predicting the performance of gas-fired tankless water heaters in domestic hot water applications,” *Applied Energy*, vol. 177, pp. 740–750, 2016.
- [292] D. Kosar and J. Stair, “Short-term performance of gas-fired tankless water heater: Laboratory characterization,” *ASHRAE Transactions*, vol. 119, pp. 48–69, 2013.
- [293] S. S. Rao, *Engineering Optimization: Theory and Practice*. John Wiley & Sons, New Jersey, U.S.A., 2009.
- [294] M. P. Todaro and S. C. Smith, *Economic Development*, 12th ed. Pearson College Division, London, U.K., 2014.
- [295] C. Brandoni and B. Bošnjaković, “HOMER analysis of the water and renewable energy nexus for water-stressed urban areas in Sub-Saharan Africa,” *Journal of Cleaner Production*, vol. <http://dx.doi.org/10.1016/j.jclepro.2016.07.114>, 2016.
- [296] T. Alton, C. Arndt, R. Davies, F. Hartley, K. Makrelov, J. Thurlow, and D. Ubogu, “Introducing carbon taxes in South Africa,” *Applied Energy*, vol. 116, pp. 344–354, 2014.
- [297] M. Eissa, “Demand side management program evaluation based on industrial and commercial field data,” *Energy Policy*, vol. 39, no. 10, pp. 5961–5969, 2011.
- [298] N. I. Nwulu and X. Xia, “Multi-objective dynamic economic emission dispatch of electric power generation integrated with game theory based demand response programs,” *Energy Conversion and Management*, vol. 89, pp. 963–974, 2015.

## REFERENCES

---

- [299] S. Zhang and X. Xia, "A new energy calculation model of belt conveyor," in *IEEE AFRICON, 2009, Nairobi, Kenya*. IEEE, 2009, pp. 1–6.
- [300] U. E. Ekpenyong, J. Zhang, and X. Xia, "An improved robust model for generator maintenance scheduling," *Electric Power Systems Research*, vol. 92, pp. 29–36, 2012.
- [301] A. Elaiw, X. Xia, and A. Shehata, "Hybrid DE-SQP and hybrid PSO-SQP methods for solving dynamic economic emission dispatch problem with valve-point effects," *Electric Power Systems Research*, vol. 103, pp. 192–200, 2013.
- [302] X. Xia, J. Zhang, and A. Elaiw, "A model predictive control approach to dynamic economic dispatch problem," in *PowerTech, 2009 IEEE Bucharest*. IEEE, 2009, pp. 1–7.
- [303] X. Xia and J. Zhang, "Mathematical description for the measurement and verification of energy efficiency improvement," *Applied Energy*, vol. 111, pp. 247–256, 2013.
- [304] E. Joubert, S. Hess, and J. Van Niekerk, "Large-scale solar water heating in south africa: Status, barriers and recommendations," *Renewable Energy*, vol. 97, pp. 809–822, 2016.
- [305] C. Dang, X. Wang, X. Wang, Y. Xiao, B. Gong, and Y. Fang, "Electrical model and control method of heat pump water heaters for promoting renewable integration," in *International Conference on Electricity Distribution, Xi'an, China*. IEEE, 2016, pp. 1–5.
- [306] E. Mathioulakis, G. Panaras, and V. Belessiotis, "Artificial neural networks for the performance prediction of heat pump hot water heaters," *International Journal of Sustainable Energy*, pp. 1–20, 2016.
- [307] X. Jin, J. Maguire, and D. Christensen, "Model predictive control of heat pump water heaters for energy efficiency," National Renewable Energy Laboratory (NREL), Golden, C.O., U.S.A., Tech. Rep., 2014.
- [308] R. M. Willis, R. A. Stewart, D. P. Giurco, M. R. Talebpour, and A. Mousavinejad, "End use water consumption in households: impact of socio-demographic factors and efficient devices,"

## REFERENCES

---

- Journal of Cleaner Production*, vol. 60, pp. 107–115, 2013.
- [309] B. Numbi and S. Malinga, “Optimal energy cost and economic analysis of a residential grid-interactive solar pv system-case of eThekweni municipality in South Africa,” *Applied Energy*, vol. 186, pp. 28–45, 2017.
- [310] C. R. Wilkes, A. D. Mason, and S. C. Hern, “Probability distributions for showering and bathing water-use behavior for various us subpopulations,” *Risk Analysis*, vol. 25, no. 2, pp. 317–337, 2005.
- [311] B. Mayer, M. Killian, and M. Kozek, “A branch and bound approach for building cooling supply control with hybrid model predictive control,” *Energy and Buildings*, vol. 128, pp. 553–566, 2016.
- [312] M. Guay, V. Adetola, and D. DeHaan, *Robust and Adaptive Model Predictive Control of Nonlinear Systems*. Institution of Engineering and Technology (IET), Herts, U.K., 2013.
- [313] T. Mahlia and P. Chan, “Life cycle cost analysis of fuel cell based cogeneration system for residential application in Malaysia,” *Renewable and Sustainable Energy Reviews*, vol. 15, no. 1, pp. 416–426, 2011.
- [314] S. Ikeda, W. Choi, and R. Ooka, “Optimization method for multiple heat source operation including ground source heat pump considering dynamic variation in ground temperature,” *Applied Energy*, vol. 193, pp. 466–478, 2017.

## ADDENDUM A LCC ANALYSIS OF COLD WATER INTERVENTIONS

**Table A.1.** LCC analysis of MPC strategy on integrated water recycling system.

Year	Initial investment (R)	Salvage value (R)	Annual cost (R)		Revenue (R)	Total (R)	Discounting factor	Cash flows (R)	
			Operation	Maintenance				Discounted	Cumulative
0	Water tanks	(3 500)	1 800						
	Pumps	(280.00)	50						
	UV purifier and filters	(4 500.00)	1 700						
	Controller	(16 900.00)							
	Accessories	(15 000.00)							
	Installation cost	(14 000.00)							
	Total Capital	(51 630.00)					1.00	(51 630.00)	(51 630.00)
1			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.94	(3 926.58)	(55 556.58)
2			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.88	( 3 684.85)	(59 241.43)
3			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.83	(3 458.01)	(62 699.43)
4			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.78	(3 245.13)	(65 944.56)

continued to next page ...

... continued from previous page

Year	Initial investment (R)	Salvage value (R)	Annual cost (R)		Revenue (R)	Total (R)	Discounting factor	Cash flows (R)	
			Operation	Maintenance				Discounted	Cumulative
5			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.73	(3 045.35)	(68 989.91)
6			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.68	(2 857.87)	(71 847.78)
7			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.64	(2 681.94)	(74 529.72)
8			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.60	(2 516.83)	(77 046.55)
9			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.56	(2 361.89)	(79 408.45)
10			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.53	(2 216.49)	(81 624.94)
11			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.50	(2 080.04)	(83 704.98)
12			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.47	(1 951.99)	(85 656.97)
13			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.44	(1 831.82)	(87 488.79)
14			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.41	(1 719.05)	(89 207.85)
15			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.39	(1 613.23)	(90 821.07)
16			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.36	(1 513.91)	(92 334.99)
17			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.34	(1 420.71)	(93 755.70)
18			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.32	(1 333.25)	(95 088.95)
19			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.30	(1 251.18)	(96 340.13)
20			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.28	(1 174.15)	(97 514.28)

**Table A.2.** LCC of open-loop optimal control strategy on integrated water recycling system.

Year	Initial investment (R)	Salvage value (R)	Annual cost (R)		Revenue (R)	Total (R)	Discounting factor	Cash flows (R)	
			Operation	Maintenance				Discounted	Cumulative
0	Water tanks	(3 500)	1 800						
	Pumps	(280.00)	50						
	UV purifier and filters	(4 500.00)	1 700						
	Controller	(16 900.00)							
	Accessories	(5 000.00)							
	Installation cost	(14 000.00)							
	<b>Total Capital</b>	<b>(41 630.00)</b>					<b>1.00</b>	<b>(41 630.00)</b>	<b>(41 630.00)</b>
1			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.94	(3 926.58)	(45 556.58)
2			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.88	(( 3 684.85)	(49 241.43)
3			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.83	(3 458.01)	(52 699.43)
4			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.78	(3 245.13)	(55 944.56)
5			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.73	(3 045.35)	(58 989.91)
6			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.68	(2 857.87)	(61 847.78)
7			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.64	(2 681.94)	(64 529.72)
8			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.60	(2 516.83)	(67 046.55)
9			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.56	(2 361.89)	(69 408.45)
10			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.53	(2 216.49)	(71 624.94)
11			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.50	(2 080.04)	(73 704.98)
12			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.47	(1 951.99)	(75 656.97)
13			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.44	(1 831.82)	(77 488.79)

continued to next page ...



... continued from previous page

Year	Initial investment (R)	Salvage value (R)	Annual cost (R)		Revenue (R)	Total (R)	Discounting factor	Cash flows (R)	
			Operation	Maintenance				Discounted	Cumulative
14			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.41	(1 719.05)	(79 207.85)
15			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.39	(1 613.23)	(80 821.07)
16			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.36	(1 513.91)	(82 334.99)
17			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.34	(1 420.71)	(83 755.70)
18			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.32	(1 333.25)	(85 088.95)
19			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.30	(1 251.18)	(86 340.13)
20			(4 966.20)	(1 500.00)	2 282.04	(4 184.16)	0.28	(1 174.15)	(87 514.28)

## ADDENDUM B LCC ANALYSIS OF HOT WATER INTERVENTIONS

**Table B.1.** Life cycle cost analysis of the water heating strategy using MPC strategy.

Year	Initial investment (R)	Salvage value (R)	Annual cost (R)		Revenue (R)		Total (R)	Discounting factor $(1+r)^{-n}$	Cash flows (R)	
			Operation	Maintenance	Control benefit	Renewable energy			Discounted	Cumulative
0	Heat pump	(25 775.85)	2300							
	Instantaneous shower	(3 000.00)								
	Solar panels	(27 500.00)								
	Wind generator	(23 500.00)	1200							
	Controller	(22 900.00)								
	Accessories	(15 000.00)								
	Installation cost	(14 000.00)								
	Total Capital	(128 175.87)						1.00	(128 175.87)	(128 175.87)
1			(5 861.90)	(1 141.76)	12 117.09	3 927.30	9 040.73	0.99	8 982.35	(119 193.52)
2			(6 424.64)	(1 141.76)	13 280.33	4 304.32	10 018.25	0.99	9 889.27	(109 304.25)

continued to next page ...

... continued from previous page

Year	Initial investment (R)	Salvage value (R)	Annual cost (R)		Revenue (R)		Total (R)	Discounting factor $(1+r)^{-n}$	Cash flows (R)	
			Operation	Maintenance	Control benefit	Renewable energy			Discounted	Cumulative
3			(7 041.41)	(1 141.76)	14 555.24	4 717.54	11 089.61	0.98	10 876.14	(98 428.11)
4			(7 717.38)	(1 141.76)	15 952.55	5 170.42	12 263.82	0.97	11 950.08	(86 478.03)
5			(8 458.25)	(1 141.76)	17 483.99	5 666.78	13 550.76	0.97	13 118.82	(73 359.21)
6			(9 270.24)	(1 141.76)	19 162.45	6 210.79	14 961.24	0.96	14 390.80	(58 968.41)
7			(10 160.19)	(1 141.76)	21 002.05	6 807.03	16 507.13	0.96	15 775.21	(43 193.20)
8			(11 135.57)	(1 141.76)	23 018.25	7 460.50	18 201.42	0.95	17 282.04	(25 911.16)
9			(12 204.58)	(1 141.76)	25 228.00	8 176.71	20 058.37	0.94	18 922.20	(6 988.97)
10			(13 376.22)	(1 141.76)	27 649.88	8 961.67	22 093.58	0.94	20 707.53	13 718.56
11			(14 660.34)	(1 141.76)	30 304.27	9 821.99	24 324.17	0.93	22 650.95	36 369.52
12			(16 067.73)	(1 141.76)	33 213.48	10 764.90	26 768.90	0.93	24 766.53	61 136.05
13			(17 610.23)	(1 141.76)	36 401.98	11 798.34	29 448.32	0.92	27 069.58	88 205.62
14			(19 300.81)	(1 141.76)	39 896.57	12 930.98	32 384.97	0.91	29 576.76	117 782.38
15			(21 153.69)	(1 141.76)	43 726.64	14 172.35	35 603.54	0.91	32 306.24	150 088.63
16			(23 184.45)	(1 141.76)	47 924.40	15 532.89	39 131.09	0.90	35 277.79	185 366.42
17			(25 410.15)	(1 141.76)	52 525.14	17 024.05	42 997.28	0.90	38 512.94	223 879.37
18			(27 849.53)	(1 141.76)	57 567.55	18 658.36	47 234.63	0.89	42 035.14	265 914.50
19			(30 523.08)	(1 141.76)	63 094.04	20 449.56	51 878.76	0.88	45 869.90	311 784.40
20			(33 453.30)	(1 141.76)	69 151.06	22 412.72	56 968.73	0.88	50 045.03	361 829.43

**Table B.2.** Life cycle cost analysis of the water heating strategy using open-loop strategy.

Year	Initial investment (R)	Salvage value (R)	Annual cost (R)		Revenue (R)		Total (R)	Discounting factor $(1+r)^{-n}$	Cash flows (R)	
			Operation	Maintenance	Control benefit	Renewable energy			Discounted	Cumulative
0	Heat pump	(25 775.87)	2300							
	Instantaneous shower	(3 000.00)								
	Solar panels	(27 500.00)								
	Wind generator	(23 500.00)		1200						
	Controller	(22 900.00)								
	Accessories	(5 000.00)								
	Installation cost	(14 000.00)								
		Total Capital		(118 175.87)					1.00	(118 175.87)
1			(8 906.00)	(2 500.00)	11 757.00	17 792.50	18 143.50	0.94	17 026.56	(101 149.31)
2			(8 906.00)	(2 500.00)	11 757.00	17 792.50	18 143.50	0.88	15 978.38	(85 170.94)
3			(8 906.00)	(2 500.00)	11 757.00	17 792.50	18 143.50	0.83	14 994.72	(70 176.21)
4			(8 906.00)	(2 500.00)	11 757.00	17 792.50	18 143.50	0.78	14 071.62	(56 104.59)
5			(8 906.00)	(2 500.00)	11 757.00	17 792.50	18 143.50	0.73	13 205.35	(42 899.24)
6			(8 906.00)	(2 500.00)	11 757.00	17 792.50	18 143.50	0.68	12 392.41	(30 506.83)
7			(8 906.00)	(2 500.00)	11 757.00	17 792.50	18 143.50	0.64	11 629.51	(18 877.31)
8			(8 906.00)	(2 500.00)	11 757.00	17 792.50	18 143.50	0.60	10 913.58	(7 963.73)
9			(8 906.00)	(2 500.00)	11 757.00	17 792.50	18 143.50	0.56	10 241.73	2 278.00
10			(8 906.00)	(2 500.00)	11 757.00	17 792.50	18 143.50	0.53	9 611.23	11 889.23
11			(8 906.00)	(2 500.00)	11 757.00	27 192.50	27 543.50	0.50	9 019.55	20 908.78

continued to next page ...

... continued from previous page

Year	Initial investment (R)	Salvage value (R)	Annual cost (R)		Revenue (R)		Total (R)	Discounting factor $(1+r)^{-n}$	Cash flows (R)	
			Operation	Maintenance	Control benefit	Renewable energy			Discounted	Cumulative
12			(8 906.00)	(2 500.00)	11 757.00	17 792.50	18 143.50	0.47	8 464.29	29 373.07
13			(8 906.00)	(2 500.00)	11 757.00	17 792.50	18 143.50	0.44	7 943.21	37 316.28
14			(8 906.00)	(2 500.00)	11 757.00	17 792.50	18 143.50	0.41	7 454.22	44 770.50
15			(8 906.00)	(2 500.00)	11 757.00	17 792.50	18 143.50	0.39	6 995.32	51 765.82
16			(8 906.00)	(2 500.00)	11 757.00	17 792.50	18 143.50	0.36	6 564.68	58 330.51
17			(8 906.00)	(2 500.00)	11 757.00	17 792.50	18 143.50	0.34	6 160.55	64 491.05
18			(8 906.00)	(2 500.00)	11 757.00	17 792.50	18 143.50	0.32	5 781.30	70 272.35
19			(8 906.00)	(2 500.00)	11 757.00	17 792.50	18 143.50	0.30	5 425.39	75 697.74
20			(8 906.00)	(2 500.00)	11 757.00	17 792.50	18 143.50	0.28	5 091.40	80 789.14