UI FORMULATION FOR CABLE STATE OF EXISTING CABLE-STAYED BRIDGE

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ABSTRACT

As inclined cables are primary load-bearing members of cable-stayed bridges, the accuracy of the method of analysing cable state is a key issue in keeping existing bridges safe. Typical cable characteristics are that they are susceptible to corrosion and tend to sag during their long service life, so it is essential to take these characteristics into account in the structural analysis to determine the actual behaviour of a cable in service. However, most of the recent methods of cable structure analysis are done on the material in a perfect state. The deterioration characteristics, such as cable corrosion and initial sag caused by the cable weight, are disregarded, which makes it difficult to apply the current methods when dealing with cable structures that are in service. By solving the boundary problem of inclined cable using the governing differential equation for the UL (Updated Lagrangian) formulation, this paper introduces a convergence iterative solution method for the analysis of cable structures of existing cable-stayed bridges. When using the iterative solution, it is convenient to determine the relationship between the co-ordinate difference of cable-end position, cable tension and cable weight. With the approach described in this paper, the effect of cable sag can be included without any approximations. Moreover, cable corrosion described by the method leads to good accuracy of results. The method meets the engineering requirements for the analysis of existing long-span cable structures. The results obtained from the method show that it is efficient and reliable. It can be conveniently applied in the analysis of large-displacement cable structures that are in service, which provides a new approach to structural health monitoring of long-span cable-stayed bridges.

1. INTRODUCTION

Inclined cables are primary loading-bearing members of cable-stayed bridges. The accuracy of the method to determine cable state is a key issue in keeping existing bridges safe. In view of the fact that extreme flexibility and susceptibility to corrosion are typical characteristics of steel cable, this leads to high stress levels during its service life. The geometric non-linearity and corrosion effects should therefore be taken into account to describe the actual behaviour of cables.

The geometric non-linearity of cable structures has been studied extensively, and a considerable number of analytical techniques and computer models have been developed and are available. Ernst (1965), Kwan (1998), Mitsugi (1994), and Leonard (1988) modelled cable as a series of straight linear elements. A large number of elements and specific formulations have been proposed to improve the performance of the straight elements taking into account the non-linear behaviour of the cable. Ahmadi-Kashani (1983), Jayaraman and Knudson (1981), and Gambhir and Batchelor (1986) used curved elements to describe the highly non-linear nature of the cable problem. However, most available

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methods are given with the material in a perfect state, and disregard cable deterioration characteristics, such as cable corrosion and initial sag causing by cable weight, which makes the application of the current methods difficult when dealing with cable structures that are in service.

By solving the boundary problem of inclined cable with the governing differential equation of the Updated Lagrangian formulation, a convergence iterative solution method is introduced for the analysis of cable state of existing cable-stayed bridges. Using the approach described in this paper, the effect of cable sag can be included without any approximations. Moreover, cable corrosion characteristic are described, and the method presented makes it possible to solve the divergence problem of stress relaxation of cables that occur when using other techniques, which leads to good accuracy of results. The method meets the engineering requirements of the analysis of existing long-span cable structures.

2. BEHAVIOUR OF INCLINED CABLE SEGMENT

Basic assumptions

As inclined cables of cable-stayed bridges are highly flexible, they undergo large displacements and should therefore be analysed taking into account geometric non-linearity. In order to better describe the behaviour of inclined cables, some assumptions should be made as follows:

1. Inclined cables have no bending and buckling resistance and can sustain only tensile forces.
2. Strains will be small, although large displacements take place when inclined cable is in service.
3. Cable weight loading is uniformly distributed along the length of the cable and cable tension is directed along the tangent to the cable segment.

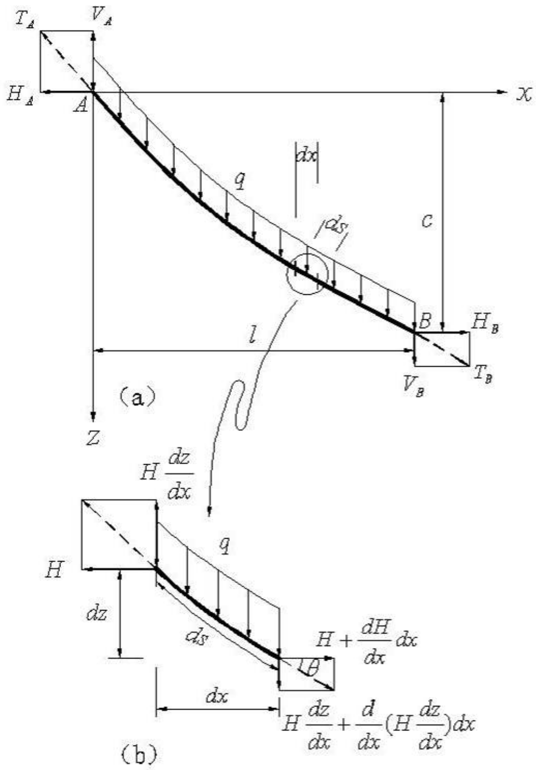

Figure 1. Cable equilibrium

Equilibrium equations of inclined cable

The inclined cable shown in Figure 1 (a) is suspended between two fixed points A and B. Let q denote the intensity of cable weight loading per unit length of the segment. The horizontal coordinate x is the independent variable. The dependent variables will be the components of tension H and V, the vertical deflection z, and the deformed and undeformed lengths of the cable S and S_0. A free-body diagram of a differential length of the cable segment is shown in Figure 1 (b).

The equilibrium of forces on the differential length are given as:

$$\begin{cases} \Sigma X = 0, \dfrac{dH}{dx}dx = 0 \\ \Sigma Z = 0, \dfrac{d}{dx}(H\dfrac{dz}{dx})dx + qds = 0 \end{cases} \tag{1}$$

where

H is the horizontal component of cable tension which is constant throughout the span since no longitudinal loads are acting.

Because the following geometric constraint must be satisfied:

$$\left(\frac{dx}{ds}\right)^2 + \left(\frac{dz}{ds}\right)^2 = 1 \tag{2}$$

we can rewrite equation (1) as:

$$\begin{cases} H = H_0 = constant \\ H_0\dfrac{d^2z}{dx^2} + q\sqrt{1+(\dfrac{dz}{dx})^2} = 0 \end{cases} \tag{3}$$

which is the classic catenary equation for the deflected profile z of the arc.

The boundary conditions at the cable supports A and B are:

$$\begin{cases} x = 0, z = 0 \\ x = l, z = c \end{cases} \tag{4}$$

Applying the boundary conditions and integrating equation (3) twice , we obtain:

$$z = \frac{H_0}{q}\left[\cosh\alpha - \cosh(\frac{2\beta x}{l} - \alpha)\right] \tag{5}$$

$$\alpha = \sinh^{-1}\left[\frac{\beta(c/l)}{\sinh\beta}\right] + \beta \tag{6}$$

$$\beta = \frac{ql}{2H_0} \tag{7}$$

The behaviour of a catenary segment of inclined cable is described by equations (5) to (7) in terms of an as yet unknown horizontal force H_0. If β were specified, the non-linear equation (5) could be solved numerically for β and hence H_0.

3. DESCRIPTION OF CABLE CORROSION

Cables for cable-stayed bridges are susceptible to corrosion during their service life. Because the accuracy in describing cable state is critical in the evaluation of the tensile forces of the cables, which in turn are responsible for the strength of the whole structure, cable corrosion deterioration should therefore be taken into account to model the actual behaviour of cables, although corrosion problems associated with cable-supported structures tend to be unique and complex (Hopwood and Havens, 1984). We consider mechanisms of cable corrosion deterioration that result in the reduction in cable cross-sectional area. We make this assumption here for the reason that simplified solutions can be developed that can satisfy the engineering requirements. Let A_0 denote the cross-sectional area of the inclined cable in a perfect state, and A^* be the impaired area due to cable corrosion. Now we can define the effective area $\tilde{A}$ as:

$$\tilde{A} = A_0 - A^* \tag{8}$$

Then the corresponding corrosion ratio D can be defined in the form:

$$D = \frac{A^*}{A_0} = \frac{A_0 - \tilde{A}}{A_0} \tag{9}$$

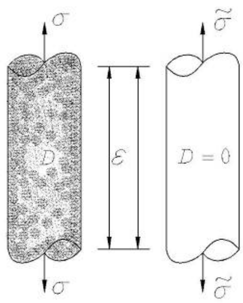

Figure 2. Equivalent-strain for corrosion cable

Since inclined cable can only be loaded by tension, the normal stresses of an inclined cable in a perfect state and in a corrosion state may be defined by Cauchy stress σ and effective stress $\tilde{\sigma}$ respectively. We obtain:

$$\sigma = \frac{T}{A_0} \qquad \tilde{\sigma} = \frac{T}{\tilde{A}} \tag{10}$$

Since $\sigma A_0 = \tilde{\sigma}\tilde{A}$ and according to equation (9) we obtain:

$$\tilde{\sigma} = \frac{\sigma}{1-D} \tag{11}$$

By introducing Lemaitre's (1971) equivalent-strain principle, which is a useful method for evaluating damage in metal structures subjected to tension and dynamic loading, the constitutive relationship of inclined cable in a corrosion state can be determined by the Cauchy stress of cable in a perfect state. Using the principle, it can be proved that corroded cable subjected to Cauchy stress σ is equivalent to perfect cable subjected to effective stress $\tilde{\sigma}$ on condition that the corresponding strains are the same, as shown in Figure 2. The principle can be written as:

$$\varepsilon = \frac{\sigma}{\tilde{E}} = \frac{\tilde{\sigma}}{E} = \frac{\sigma}{(1-D)E} \tag{12}$$

Then from equations (11) and (12), the effective modulus of elasticity $\tilde{E}$ for corroded cable can be arrived at by:

$$\tilde{E} = \frac{\tilde{A}}{A_0} E \tag{13}$$

Based on the above discussion, the corrosion characteristic of inclined cable has been taken into consideration by rational discounting of the modulus of elasticity, and a convenient way to determine the actual cable state of existing cable structures in engineering has been developed.

4. DETERMINATION OF TENSION AND DEFORMATION OF CORROSION CABLE

According to the geometry of the differential length as shown in Figure 1, the stretched length S of the inclined cable is obtained from equation (5) as follows:

$$S = \int ds = \int_0^l \sqrt{1 + (\frac{dz}{dx})^2}\, dx = \frac{l}{\beta}\sqrt{\sinh^2 \beta + (\frac{\beta c}{l})^2} \tag{14}$$

Also, since the tension must be directed along the tangent to the arc:

$$T = \frac{H_0}{\cos\theta} \tag{15}$$

Based on Lemaitre’s equivalent-strain principle, the catenary deformation is calculated as:

$$\varepsilon = \frac{T}{\tilde{E}A_0} = \frac{H_0}{\tilde{E}A_0 \cos\theta} = \frac{ql}{2\tilde{E}A_0 \beta \cos\theta} \tag{16}$$

Thus the increased length of the inclined cable is:

$$S - S_0 = \int_0^S \varepsilon ds = \frac{ql^2}{4\tilde{E}A_0\beta}\left\{\frac{\coth\beta}{\beta}\left[\sinh^2\beta + 2(\frac{\beta c}{l})^2\right] + 1\right\} \tag{17}$$

Substituting equation (14) in equation (17) yields:

$$\frac{l}{\beta}\sqrt{\sinh^2 \beta + (\frac{\beta c}{l})^2} - S_0 = \frac{ql^2}{4\tilde{E}A_0\beta}\left\{\frac{\coth\beta}{\beta}\left[\sinh^2\beta + 2(\frac{\beta c}{l})^2\right] + 1\right\} \tag{18}$$

Rewriting equation (17) as a function of β, we obtain:

$$F(\beta) = \sqrt{c^2 + (\frac{l\sinh\beta}{\beta})^2} - S_0 - \frac{ql^2}{4\tilde{E}A_0\beta}\left\{\frac{\coth\beta}{\beta}\left[\sinh^2\beta + 2(\frac{\beta c}{l})^2\right] + 1\right\} \tag{19}$$

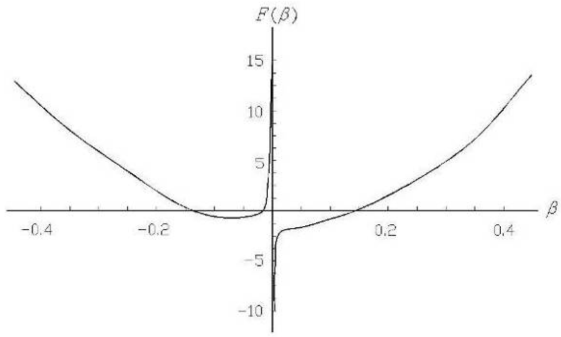

Figure 3. Graph of function $F(\beta)$

Equation (19) could be solved directly for β using numerical procedures. Figure 3 is obtained by applying four equalised increments, and a modified Newton-Raphson iteration was performed after each increment to achieve a convergence tolerance of 0.001. It is easy to verify that $F(0) \to -\infty$ and $F(+\infty) \to +\infty$, and there is just one positive root in $\beta \in (0,+\infty)$ from Descartes' rule of signs, which is the required value of $F(\beta)$. Once we obtain a solution to equation (19), equation (7) can be used to determine H_0. With reference to Figure 1(a), static equilibrium equations will be involved in determining the horizontal and vertical components of cable tension at two supports.

$$H_0 = \frac{ql}{2\beta} \tag{20}$$

$$V_B = -H_0\left\{\frac{\beta c}{l}\coth\beta - \sqrt{\sinh^2\beta + (\frac{\beta c}{l})^2}\right\} \tag{21}$$

$$V_A = H_0\left\{\frac{\beta c}{l}\coth\beta + \sqrt{\sinh^2\beta + (\frac{\beta c}{l})^2}\right\} \tag{22}$$

$$H_B = H_0 \tag{23}$$

$$H_A = -H_0 \tag{24}$$

and therefore the tension of the inclined cable is:

$$T = \frac{H_0}{\sinh\beta}\left\{\sqrt{\sinh^2\beta + (\frac{\beta c}{l})^2}\cosh(\beta - \frac{2\beta x}{l}) + \frac{\beta c}{l}\sinh(\beta - \frac{2\beta x}{l})\right\} \tag{25}$$

When the cable-end position is not changed relative to the vertical and horizontal distance, the cable shape and cable stress can be calculated directly by applying the convergence iterative solution method in this paper. Based on the approach presented, loading techniques associated with the behaviour of large-displacement inclined cable during the application of loads are then obtained. We should first take the unstressed cable state as the initial state, during which there is no cable weight loading and the inclined cable is suspended in a straight line between two fixed supports. If the first step increment of loading is applied, c and l will be obtained by the coordinate difference of the cable-end position. We then obtain iterative solution β, and the deformed length of the cable as well as the cable tension can consequently be determined. To improve the accuracy of the convergence iterative solution method presented in this paper, it is recommended that dead load be applied in small increments. In this way the effect of cable sag caused by cable weight loading can be included without any approximations.

On the other hand, cables would become slack and stress relaxation would occur if the relative cable-end position is changed during service life. The divergence problem will be encountered if Ernst's equivalent modulus is used in this case. Such problems can be avoided by adopting the method presented with the incremental loading scheme. If the cable-end position is taken as the current reference configuration after the previous increment load has been applied, we may search for a displacement increment due to the current added increment load. Therefore the actual cable-end position after the current increment load has been applied is an iterative solution, which can be as theoretically accurate as required.

5. CONCLUSIONS

By introducing Lemaitre's equivalent-strain principle, the corrosion characteristic of inclined cable is taken into consideration by rational discounting of the modulus of elasticity. A convenient engineering method has been developed and explored to determine the actual cable state of existing cable structures.

The scheme presented for the calculation of large cable displacements makes it possible to solve the divergence problem of stress relaxation of cables caused by the use of other techniques, and provides a new approach to structural health monitoring of long-span cable-stayed bridges.

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