OPTIMAL CONFIGURATION AND THERMAL PERFORMANCE OF HEATED RECTANGULAR BLOCKS UNDER FORCED CONVECTION WITH VOLUME CONSTRAINTS

Mujanayi Katumba J-M[*,§], Bello-Ochende T.[**] and Meyer J.P.[*]
[*] Department of Mechanical and Aeronautical Engineering,
University of Pretoria,
Pretoria, 0002,
South Africa,
[**]Department of Mechanical Engineering,
University of Cape Town
C ape Town, 7701
South Africa
[§]Author for correspondence
E-mail: jmarckatumba@gmail.com

ABSTRACT

This paper presents thermal management of heat transfer density rate from heated blocks mounted on a horizontal wall of a rectangular enclosure and subject to forced convection. The governing equations for mass, momentum and energy for laminar flow and convective heat transfer are solved in three-dimensions using a commercial Computational Fluids Dynamics (CFD) code. First numerical results validated with available experimental data showed that the rate of heat transfer increase with the Reynolds number. Thereafter a numerical optimization procedure is carried out in order to obtain the optimal blocks configuration that maximizes the heat transfer density rate and minimizes the peak temperature in the enclosure by selecting the sides of the blocks as design variables whilst the total volume is maintained constant. In term of the thermal performance of the heat transfer mechanism described by the dimensionless global conductance as well as by the overall Nusselt number, the results showed that optimal configurations were such that none of the blocks aspect ratio was equal to one. However thermal performance was much better when either the height-to-length ratio (*B/G*) or the height-to-width (*B/C*) ratio tends to its maximum. These optimal results obtained numerically are found to be fairly reliable.
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INTRODUCTION

Forced convection mechanism heat transfer has been widely used in both investigation and simulation of the cooling of electronic equipments or entities flush mounted or protruding on a wall as well as in a channel. In order to study the phenomenon of heat transfer from electronics, several works on cooling of heated elements by mixed convection in the laminar regime have been carried out by many authors using experimental procedure [1-7] or numerical analysis [8-14]. From these studies results general characteristics of the cooling mechanism of heated elements were found in to be similar regarding the global thermal performance in accordance with the geometrical configuration of the heated elements. Numerical investigations were carried out in [15, 16] for forced convection in channel. The numerical results which agreed with experimental data proved that the slender-tall blocks enhanced the heat transfer compare with the flatter-short ones for low Reynolds numbers, less than 2000.

Gradually as the years pass electronic entities are being condensed, miniaturized and concentrated on integrated circuits in electronics equipments to improve their functional performance (speed, memories etc)[17]. The miniaturization of heating element causes a considerable increase the heat flux rate within the element as the surface decrease while the generated heat increases causing the temperature of the heated element to raise drastically[18], therefore presents a high risk of overheating which definitely leads to the operational failure of the equipment as the main electronics components are designed to operate under a peak temperature value.

On a thermal management point of view which aims for the maximization of heat removal in order to keep the element under an allowable temperature for functional reasons, forced convection was unable to reach proper cooling performance because of the excess heat generated. However, for reasons of power consumption, reliability, reasonable cost, air forced convection was found to be more advantageous from cooling methods for electronics equipment[18]. In order to prevent or to avoid limitations shown by air convection method, the need of optimization of some pertinent geometric parameters was inevitable to reach better cooling performance. That is how some researchers have begun to focus more on optimization studies of geometric parameters to improve the cooling mechanism in electronics equipments using different method.

Optimization study of spacing between simulated electronics entities in channels was focused in many works carried out by different authors [19-24] to improve thermal performances. Results proved that for equi-distant aligned heated elements arrangement, the cooling performance did not show any improvement. However when distances between consecutive elements followed the geometric series with a spacing ratio greater than 1.2, better global cooling performance was obtained.

Nowadays, numerical optimization tools are added-on to numerical methods process in order to performed automated design (in minimizing or maximizing objectives functions). This presents important advantages such as multidisciplinary design optimization, varieties of constraints, simple and easy to use, fast procedure compare to experimental and analytical methods as illustrated in [25, 26] where the computational results showed very good agreement with experimental data with reliable optimal results.

Bello-Ochende *et al* in [28] used constructal theory to conduct numerical procedure to determine the optimal configuration of a two rows of pin-fins that maximizes the total heat transfer rate by laminar forced convection. The optimal configuration obtained numerically was found to be in good agreement with scales analysis predictions since conduction along the fins and convection transversal to the fins were well balanced.

This paper focuses on a numerical optimization that maximizes the removal of heat from heated blocks mounted on the horizontal wall of a rectangular enclosure cooled by laminar forced convection. This will also reduce the peak temperature in the enclosure.

NOMENCLATURE

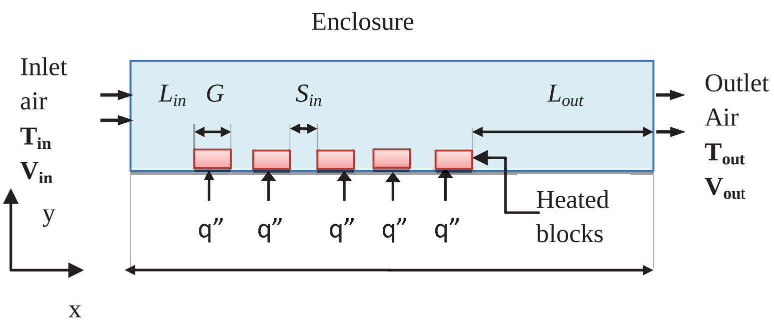
A	[m^2]	Surface area
B	[m]	Heated block
C	[m]	Heated block span-wise length
C_p	[-]	Pressure coefficient
D_h	[m]	Enclosure hydraulic diameter
G	[m]	Heated block stream-wise length
H	[m]	Enclosure height
L_x	[m]	Enclosure length
Nu	[-]	Nusselt number
P	[atm]	Pressure
Ᵽ	[-]	Dimensionless Pressure difference
Pr	[-]	Prandtl number
Re	[-]	Reynolds number
S	[m]	Side-to-side distance consecutive blocks
T	[°C]	Temperature
V	[m^3]	Volume
W	[m]	Enclosure width
c_p	[J/Kg.K]	Specific heat
g	[m/s^2]	Standard acceleration gravity
h	[W/m.K]	Heat transfer coefficient
k	[W/m.K]	Thermal conductivity
n	[-]	Normal direction
q''	W/m^2	Heat flux
u, v, w	[-]	Velocity components in x. y. z directions
x, y, z	[-]	System coordinate directions
x^+	[-]	Dimensionless distance along the wall
Special characters		
α	[m2/s^2]	Thermal diffusivity
μ	[Kg/m.s]	Dynamic viscosity
υ	[m^2/s]	Kinematic viscosity
θ	[-]	Dimensionless temperature
ρ	[Kg/m^3]	Fluid density
γ	[-]	Standard convergence criterion
Subscripts		
c		Characteristic
ch		Channel
ext		External: Towards external heat sink
dev		Fully developed
f		Fluid
in		Inlet
max		Maximum
min		Minimum

out	Outlet
ref	Reference
s	Solid
w	walls

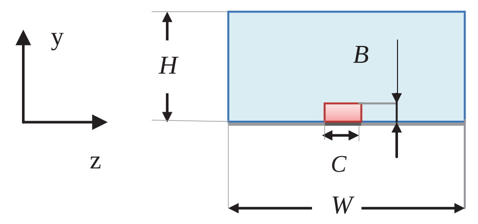
PHYSICAL AND MATHEMATICAL MODEL

Physical Model

Cooling of aligned rectangular blocks mounted on the bottom wall of an enclosure by forced convection is being analyzed in three-dimensions. The physical model and computational domain of the studied problem is showed on Figure 1 (a) and (b). The enclosure has the streamwise length L_x, the height H and the width W that are set in accordance with the theoretical considerations regarding laminar forced convection within a rectangular enclosure provided in the literature [8-10, 13, 29-33] to avoid thermal influence of the boundaries conditions on the model.



(a)



(b)

Figure 1 Geometric representation and physical model of the domain: in the x-y plane (a) and in the y-z plane (b)

The identical heated blocks have the width *C*, the lengths G and the height B. The volumes of each block as well as their total volume are maintained constant while the three dimensions are set as variables in range as considered in previous works [1, 14, 16, 34]. The blocks are being heating from their bottom face with a constant heat flux. The inlet air temperature is 20°C, and the velocity is maintained in order to ensure a laminar flow regime with the Reynolds varying from 100 to 1000. The enclosure faces are adiabatic.

The constraints are described from Eq.(1) to (10) and set such as the upstream face of the first flow is located a L_{in} more than 3 *G*, while the downstream distance behind the last block L_{out}, have to be greater than 10 *G*, and long enough to ensure the recirculation flow remains within the computational domain, and each lateral distance is set to be equal to 5 *G*. We set *G* = 0.05 m for the initial geometry.

$$\frac{C}{G} = 1 \tag{1}$$

$$\frac{B}{G} = 0.5 \tag{2}$$

$$\frac{W}{G} = 11 \tag{3}$$

$$\frac{H}{G} = 3 \tag{4}$$

$$\frac{L_x}{G} = 30 \tag{5}$$

$$\frac{L_{in}}{G} = 5 \tag{6}$$

$$\frac{L_{out}}{G} = 28 \tag{7}$$

$$GBC = V_i \tag{8}$$

$$\sum_1^n V_i = V_{tot} \tag{9}$$

$$L_{in} + \sum_1^n G + \sum_1^{n-1} S_i + L_{out} = L_x \tag{10}$$

Where V_i , V_{tot} and L_x are constants.

n is the total number of the heated blocks which is 5.

Governing Equations

We assumed that the air as Newtonian fluid. The flow is three-dimensional and is assumed to be steady, incompressible and laminar. All the thermo-physical properties are also assumed constant. The buoyancy induced effects as well as the viscous heat dissipation with the radiative effect when compared to convection is negligible.

The governing differential equations of mass, momentum and energy conservation using for fluid flow and heat transfer in the computational domain in their nondimensional form using the hydraulic diameter D_h as the characteristic length are expressed as follow:

$$\nabla.\rho\vec{v} = 0 \quad (11)$$

$$\rho(\vec{v}.\nabla\vec{v}) = -\nabla P + \mu\nabla^2\vec{v} \quad (12)$$

$$\rho C_p(\vec{v}.\nabla T) = k_f\nabla^2 T \quad (13)$$

Where:

$$\nabla^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2} \quad (14)$$

The energy equation for the solid is:

$$k_s\nabla^2 T = 0 \quad (15)$$

The continuity of the flux at the heated block-air interface is:

$$k_s \left.\frac{\partial T}{\partial n}\right|_w = k_f \left.\frac{\partial T}{\partial n}\right|_w \quad (16)$$

The continuity of the temperature at the heated block-air interface is:

$$T_s = T_f \quad (17)$$

The boundaries conditions for the thermal configuration of the fluid are described as follow:

No-slip and adiabatic conditions at $y = z = 0$

$$\vec{v} = 0 \quad (18)$$

$$\frac{\partial T}{\partial x} = 0 \quad (19)$$

And at the inlet (x = 0)

$$v = w = 0 \quad (20)$$

$$T = T_{in} \quad (21)$$

$$P = P_{in} \quad (22)$$

Fully developed inlet flow

$$u = \vec{v}_{dev} \quad (23)$$

At the outlet (x = L_x), the flow assumed to be fully developed

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = \frac{\partial w_z}{\partial z} = 0 \quad (24)$$

$$\frac{\partial T}{\partial x} = 0 \quad (25)$$

$$P_{out} = P_{atm} \quad (26)$$

For different Reynolds number *Re* and based on the the hydraulic diameter D_h of the enclosure as the characteristic length some dimensionless will be used number to measure the thermal performance of the cooling process which are:

- the dimensionless temperature θ given by:

$$\theta = \frac{T - T_{in}}{(q'' D_h / k_f)} \quad (27)$$

- the overall or global dimensionless conductance is defined as:

$$\overline{C} = \frac{Q'' D_h}{k_f(T_{ave} - T_{in})} \quad (28)$$

- the Nusselt number that are evaluated using.

$$Nu_x = \frac{hD_h}{k_f} = \frac{-1}{\theta_s}\frac{\partial\theta_f}{\partial\theta_s} \quad (29)$$

$$\overline{N}u_i = \frac{\int_{A_i} Nu_x dx}{A_i} and \quad (30)$$

$$\overline{N}u_{ave} = \frac{\sum \overline{N}u_i A_i}{\sum A_i} \quad (31)$$

- the dimensionless pressure difference

$$\overline{P} = \frac{\Delta P D_h^2}{(\mu\alpha_f)} \quad (32)$$

$$\Delta P = P - P_{ref} = C_p\left(\frac{1}{2}\rho_{ref} v_{ref}^2\right) \quad (33)$$

- The Reynolds number is defined as:

$$\mathrm{Re}_{D_h} = \frac{U_{in} D_h}{\nu} \quad (34)$$

$$D_h = \frac{4A_{ch}}{P} \quad (35)$$

Where A_{ch} is the channel cross section.

The enclosure walls are maintained adiabatic and the blocks are heated from the bottom face with a constant heat flux of 500 W/m^2.K.

We assume constant thermal properties of air as the convective medium, that are taken at the inlet temperature, T_{in}= 20°C as given in Table 2. The heated blocks are identical, homogenous materials with isotropic properties and consist of silicon wafer, metal frame and packaging materials having a mean thermal conductivity of k_s = 2.63 as assumed in [19, 21, 38].

Nu_i is the mean Nusselt number over a single surface, Nu_{ave} is the overall Nusselt number and C_p is the Pressure coefficient [35]. P_{ref} is the reference pressure which is the atmospheric pressure, ρ_{ref} and v_{ref} are the air reference density and velocity taken at the inlet condition.

NUMERICAL ANALYSIS AND CODE VALIDATION

Code Validation

A computational fluid dynamics (CFD) code, Star-CCM 8.02 [35] is used to conduct all the simulations in three dimensions for the laminar forced convection process within the enclosure. In Eqs. (11) to (13), the pressure-velocity coupling is solved using the Semi-Implicit Method for Pressure-Linked Equations (SIMPLE) well explained by Patankar and Spalding [36]. The domain is discretised using a polyhedral meshing of the core volume obtained using a finite-volume method for segregated flow where the second order upwind scheme is used for the diffusive terms and the convective terms in the momentum and energy equations. This coincided when the normalized simulation residuals fall below 10^{-7} for momentum and the energy equations and 10^{-6} the continuity equation. The convergence criterion at each step for the overall temperature as the quantity monitored was:

$$\gamma = \frac{\left| T_{ave_i} T_{ave\,i-1} \right|}{T_{ave\,i}} \leq 10^{-2} \tag{36}$$

Where i is the mesh index according to refinement obtain form varying the base size. We refined the mesh by decrease the base size to satisfy the convergence criteria (27) and to validate the results accuracy.

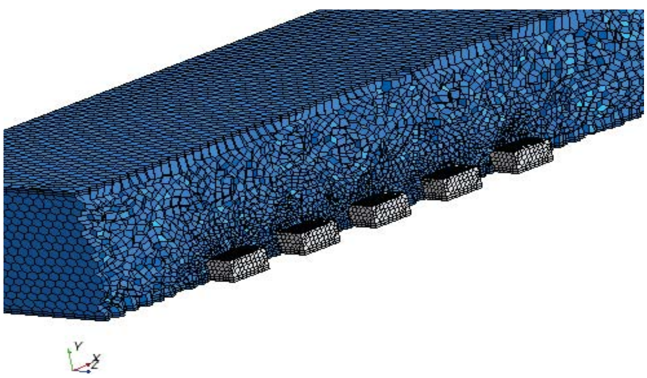


Figure 2 The discretised three-dimensional computational domain

Table 1 Grid independent study for the air forced convection mechanism for Re = 500

Base size	Cells	T_{max}	T_{ave}	$\gamma = \frac{\left| T_{ave\,i} - T_{ave\,i-1} \right|}{T_{avex\,i}}$
0.08	40370	132.66	112.45	
0.07	49172	130.63	111.07	0.0124246
0.06	91731	134.53	113.40	0.02054674
0.05	13815	133.79	113.13	0.00229804
0.04	18624	133.62	113.14	0.00008838

Table 1 illustrates how grid independence was achieved. The mesh refinement was made by reducing the base side with a value 0.01 after each simulation which increased the grid density. Figure 3 shows how the Reynolds number based on the hydraulic diameter as the characteristic length varies with the inlet air velocity. Using Re_{Dh} = 500 as mean value, the convergence criterion was reached for a base size of 0.05 m, this model generated 138150 cells. It was found that a further increase in the grid density led to insignificant change in both the maximum and the average temperature. We validated our numerical procedure by comparing the resulting average Nusselt number in the channel obtained in this work with those calculated by the Wirtz and Dykshoorn correlation given by Eq. (35) based on their experimentally data for in-line arrays of heated blocks in a rectangular cross section channel [37].

$$Nu_L = 0.6\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{0.33} \tag{37}$$

Within the laminar regime, this equation gives the average Nusselt number of the heated blocks in the channel as a function of the Reynolds number based on the same characteristic length expressed by the subscript L. Pr is the Prandtl number taken at the inlet temperature of the air. Figure 4 depicts the similarity between the Nusselt numbers obtained for various Reynolds numbers with both methods.

Numerical Procedure

Investigations were first carried out to perform numerical simulations for air forced cooling of five aligned heated blocks in the enclosure. Keeping geometric dimensions unchanged in all simulations (as given in equation (1) – (3)), the results showed how the global thermal performance is strongly influence by the Reynolds number that varied from 100 to 1000 to keep the regime laminar in the enclosure.

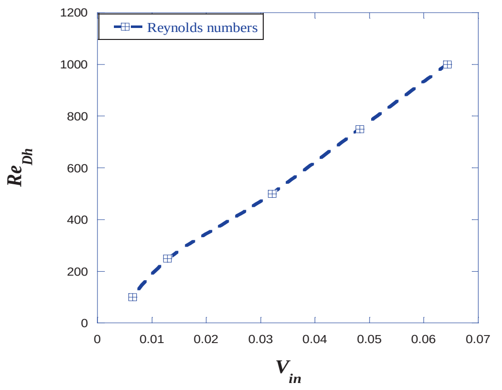


Figure 3 Variation of Reynolds number with the inlet cooling air velocity.

Table 2 Thermo-physical properties of air at T_{in} = 20 °C, 1 atm

Density ρ (kg/m^3)	Thermal diffusivity α_f (m^2/s^2)	Thermal conductivity k (W/m.K)	Dynamic viscosity μ (Kg/m.s)	Kinematic viscosity ν (m^2/s)	Specific Heat c_p (J/kg.K)
1.204	2.074 x10^{-5}	0.02514	1.825 x10^{-5}	1.516 x10^{-5}	1007

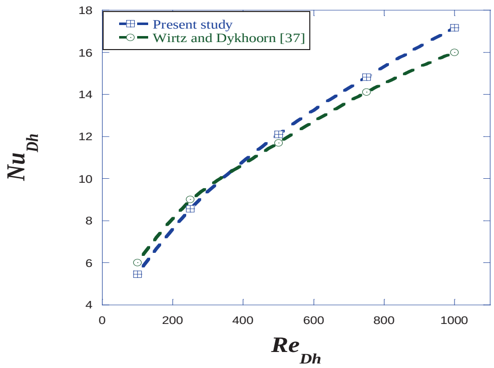


Figure 4 Validation of code for computed average Nusselt number

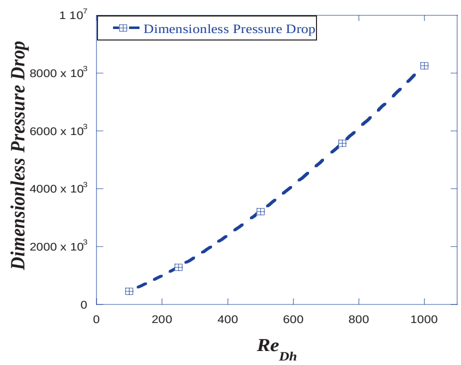


Figure 5 Variation of the dimensionless pressure drop with the Reynolds number

Figures 6 and 7 show how the increase in the Reynolds number increases the mixing within the fluid flow which improves the heat transfer density rate. Therefore the overall temperature as well as the maximum temperature decreases in the channel. These results are in good agreement with those obtained experimentally and numerically in [14, 15, 34, 39].

NUMERICAL OPTIMIZATION

The optimization procedure is conducted to obtain the optimal geometric configuration of the heated blocks that maximizes the heat transfer density rate to the cooling fluid with various inlet velocities. As seen in the literature, the global thermal performance for forced convection heat transfer from

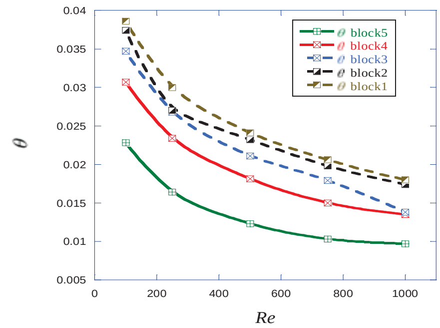


Figure 6 Influence of Reynolds number variation on different heated blocks temperature for the initial configuration

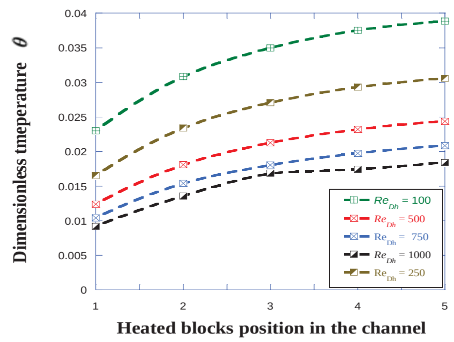


Figure 7 Reynolds numbers effect on the on the aligned heated blocks temperature in the enclosure in the streamwise direction

heated sources depends not only on the thermal properties but also on the geometric characteristics of the model such as enclosure dimensions, heated elements dimensions as well as the distance between elements [14, 19, 21, 34]. Each optimization process is also conducted using the CFD code Star-CCM+. However, it is important to point out the optimization problem within Star-CCM+ environment is solve using HEEDS-Optimate+ code.

HEEDS (Hierarchical Evolutionary Engineering Design System) is an add-on optimization tool to the Star-CCM+ code that uses the SHERPA (Simultaneous Hybrid Exploration that is Robust, Progressive and Adaptive) algorithm to perform design exploration, optimization, and design of experiment and robustness studies. The optimization process is carried out without leaving the Star-CCM+ environment and the method used in Optimate+ or HEEDS-Multidisciplinary Design Optimization (MDO) is very easy to use and it doesn't require any expertise in optimization algorithms and applications, this has many advantages as its scheme works with the principle of simultaneous multiple search method instead of proceeding by sequential steps. Furthermore the SHERPA is and does not require tunable parameters but adjust them automatically because of its hybrid adaptation. This makes it a robust and an efficient method in term of time saving during the process. During the optimization process, the computational domain is automatically remeshed at each run according to the geometry.

The objective of this optimization problem being to minimize the peak temperature and to maximize the heat flux rate to the cooling fluid, the optimization algorithms will therefore use the results from numerical analyses and simulations, to guide the search for an optimal configuration [40, 41] by specifying the objective functions and setting the design variables as well as the constraints in the process.

We conducted the Pareto Multi Objective as the design optimization option in order to perform the minimization of T_{ave} together with the maximization of Nu_{ave} which are chosen as objective functions. For each value of the Reynolds number, we conducted the numerical optimization by selecting the sides of the blocks as the design variables in the range of 0.025 to 0.05. The total volume of each block is maintained constant as expressed in Eqs. (8) and (9). Therefore the spacing between blocks might also vary according to optimal blocks shape but the total streamwise length of the channel is constraint by Eq. (10).

RESULTS

The optimization process is applied in the computational domain to constant computational volume of 0.0003125 mm^3 for each block for different pressure drop varying with the Reynolds numbers range from 100 to 1000. Optimal blocks dimensions that maximize the rate of heat transfer and minimize peak temperature in the channel are listed in tables 3 and 4. The temperature contours of optimized geometries for Re_{Dh} = 100, 500 and 1000 are shown in figures 8, 9 and 10 respectively.

Results show that optimal blocks' geometric are taller and slender compared to the initial configuration. These optimum blocks' geometric ratios show that taller-slender blocks are better than shorter-flatter ones in term of heat transfer rate. The optimal blocks' geometry also showed that the total block's area in contact with the coolant is larger compare to the initial geometry which had a larger bottom surface. This has the consequences that the overall Nusselt number increases with the surfaces area. With

Table 3 Optimal blocks dimensions and optimal spacing for different Reynolds number

Re_{Dh}	B_{opt}	G_{opt}	C_{opt}	$S_{i,opt}$
100	0.05	0.05	0.025	0.5
250	0.0482	0.04523	0.02864	0.05477
500	0.041332	0.0421	0.0359	0.0579
750	0.05	0.025	0.05	0.075
1000	0.05	0.025	0.05	0.075

Table 4 Resulting ratio for optimal blocks geometry

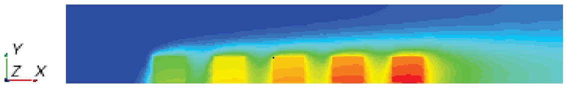
Re_{Dh}	B_{opt}/G_{opt}	C_{opt}/G_{opt}	$S_{i,opt}/G_{opt}$
100	1	0.5	1
250	1.06566	0.633	1.211
500	0.98	0.853	1.375
750	2	2	3
1000	2	2	3

this specific configuration the blocks arrangement is such that the spacing between consecutive blocks is greater compare to initial configuration, therefore improves the heat transfer rate. This decreases both the average and the peak temperature in the enclosure

This effect is more significant for the first block that benefits from the inlet flow impact which improves the rate of heat transfer of the first element as its upstream face is larger as is illustrated in figure 11. Heat transfer rate from subsequent blocks downstream is enhanced by the fact that in the laminar regime, when the blocks height increases, the clearance height above to block is reduced, this rises the flow velocity over each block and recirculation zone is being moved from the top of the blocks into the downstream spaces which are larger so that more flow can penetrate the cavities in between blocks.

Therefore the flow increases the recirculation above the block as a result it decreases the thermal boundary thickness of vertical faces while mixing with the vortex and then reduces the isolating level of the recirculation flow. This has an effect of removing further the heat form blocks and reducing the surface temperature. Heat flux rate at the last block is further enhanced by the flow rate as the thermal transport due the downstream recirculation increases. Similar results were also observed in two-dimensional experimental and numerical investigations in [15, 16].

Results listed in tables 3 and 4 illustrate that optimal blocks' ratio are very similar for low inlet air velocity ($100 \leq ReDh \leq 250$) and we increase inlet flow velocity r, which also increases the Reynolds number ($750 \leq ReDh \leq 1000$), we also observed very closed similarities in optimal geometry of the blocks.



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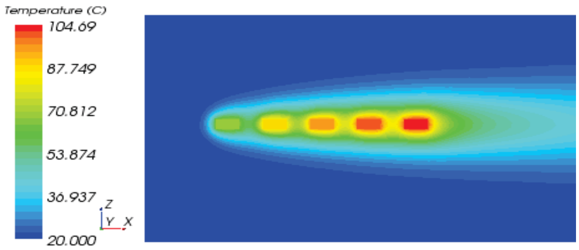
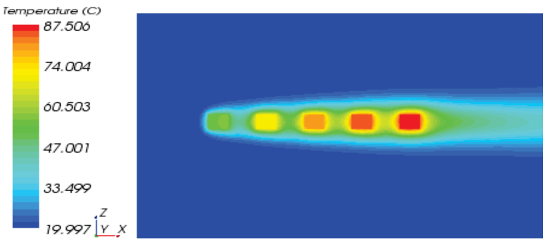


(b)

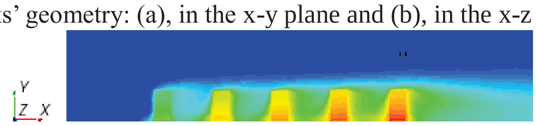
Figure 8 Temperature contour for Re = 100 for the optimal blocks' geometry: (a), in the x-y plane and (b), in the x-z plane

(a)

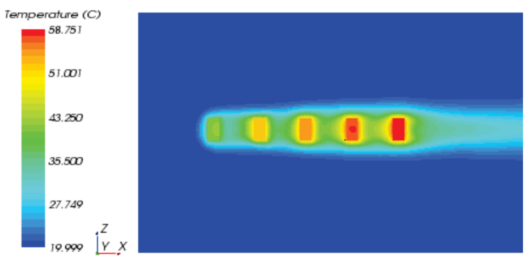


(b)

Figure 9 Temperature contour for Re = 500 for the optimal blocks' geometry: (a), in the x-y plane and (b), in the x-z plane



(a)



(b)

Figure 10 Temperature contour for Re = 1000 for the optimal blocks' geometry: (a), in the x-y plane and (b), in the x-z plane

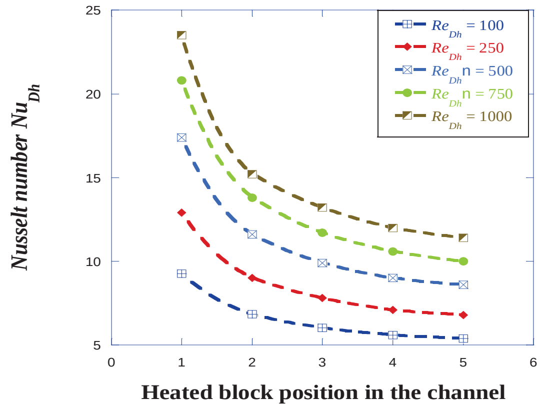


Figure 11 Variation of the Nusselt number of individual heated block in the enclosure with the Reynolds number

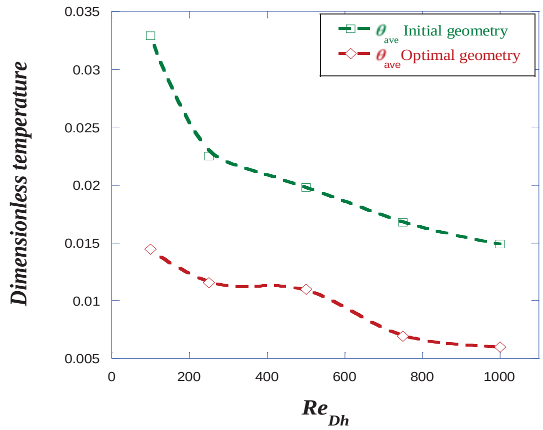


Figure 12 Comparison of the dimensionless overall temperature between the optimal blocks' geometry and the initial geometry

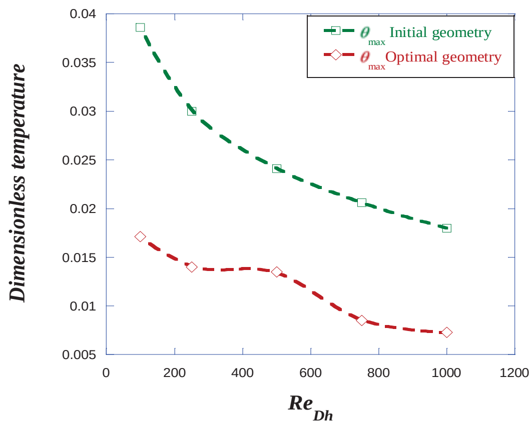


Figure 13 Comparison of the dimensionless maximum temperature between the optimal blocks' geometry and the initial geometry

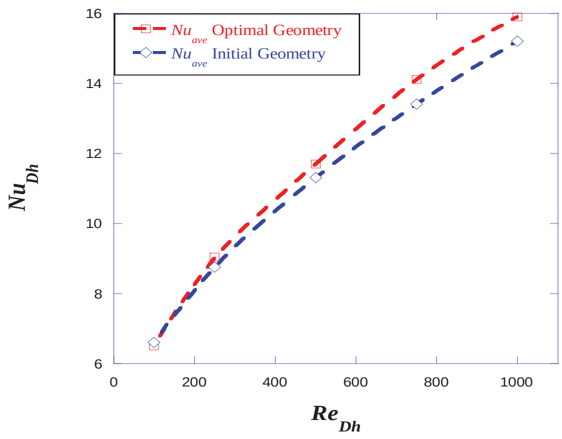


Figure 14 Comparison of the overall Nusselt number between the optimal block's geometry and the initial geometry

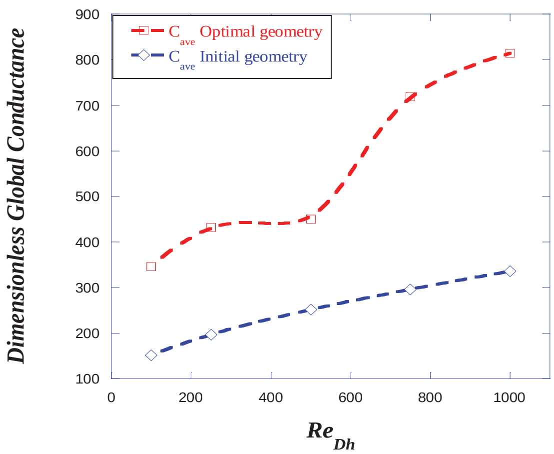


Figure 14 Comparison of the dimensionless global conductance between the optimal blocks' geometry and the initial geometry

When we increase inlet flow velocity number, which also increases the Reynolds number ($750 \leq Re_{Dh} \leq 1000$), we also observed very closed similarities in optimal configuration of the blocks.

CONCLUSION

In this study a three-dimensional geometric optimization has been numerical carried out to maximize the heat transfer density rate from rectangular heated blocks mounted on the enclosure bottom wall. A constant heat flux is applied on the bottom of the heated blocks under forced convection. During the optimization procedure the objective functions are the overall temperature as well as the overall Nusselt number in the enclosure, the geometric blocks dimensions are set as design variables and the total volume of the blocks is fixed as a constraint.

Constant thermo-physical properties of the cooling air at the inlet temperature were considered. For specific Reynolds numbers range from 100 to 1000, the validation numerical result was done by comparing the similarity in the trends of the blocks' Nusselt number curves with experimental data. Optimized results were presented in terms of overall Nusselt number, the dimensionless temperature and the global dimensionless conductance in the channel.

Besides the Reynolds number influence on the heat transfer density rate, the blocks shape strongly affects thermal performance of the convection mechanism. The heat transfer is improved when the blocks' geometric ratios are not equal to 1.

Optimal results compared to the initial geometry gave a average improvement in term of the Nusselt number of 5% and an average decrease of about 50 % for peak temperature.

REFERENCES

[1] A. Bar-Cohen and A. D. Kraus, *Advances in thermal modeling of electronic components and systems*, 1988.

[2] A. B. McEntire and B. W. Webb, "Local forced convective heat transfer from protruding and flush-mounted two-dimensional discrete heat sources," *International Journal of Heat and Mass Transfer*, vol. 33, pp. 1521-1533, 1990.

[3] B. A. Jubran, S. A. Swiety, and M. A. Hamdan, "Convective heat transfer and pressure drop characteristics of various array configurations to simulate the cooling of electronic modules," *International Journal of Heat and Mass Transfer*, vol. 39, pp. 3519-3529, 1996.

[4] S. Baskaya, U. Erturhan, and M. Sivrioglu, "An experimental study on convection heat transfer from an array of discrete heat sources," *International Communications in Heat and Mass Transfer*, vol. 32, pp. 248-257, 2005.

[5] A. Dogan, M. Sivrioglu, and S. Baskaya, "Experimental investigation of mixed convection heat transfer in a rectangular channel with discrete heat sources at the top and at the bottom," *International Communications in Heat and Mass Transfer*, vol. 32, pp. 1244-1252, 2005.

[6] A. Dogan, M. Sivrioglu, and S. Baskaya, "Investigation of mixed convection heat transfer in a horizontal channel with discrete heat sources at the top and at the bottom," *International Journal of Heat and Mass Transfer*, vol. 49, pp. 2652-2662, 2006.

[7] T. Pirasaci and M. Sivrioglu, "Experimental investigation of laminar mixed convection heat transfer from arrays of protruded heat sources," *Energy Conversion and Management*, vol. 52, pp. 2056-2063.

[8] J. Davalath and Y. Bayazitoglu, "Forced Convection Cooling Across Rectangular Blocks," *Journal of Heat Transfer*, vol. 109, pp. 321-328, 1987.

[9] S. H. Kim and N. K. Anand, "Laminar developing flow and heat transfer between a series of parallel plates with surface mounted discrete heat sources," *International Journal of Heat and Mass Transfer*, vol. 37, pp. 2231-2244, 1994.

[10] T. J. Young and K. Vafai, "Convective cooling of a heated obstacle in a channel," *International Journal of Heat and Mass Transfer,* vol. 41, pp. 3131-3148, 1998.

[11] F. C. Hung TC, "Conjugate heat transfer anakysis for the passive enhancement of electronic cooling through geometric modification in a mixed convective domain," *Numerical Heat Transfer, Part A: Applications,* vol. 35, pp. 519-535, 2014/09/01 1999.

[12] Y. P. Cheng, T. S. Lee, and H. T. Low, "Numerical analysis of mixed convection in three-dimensional rectangular channel with flush-mounted heat sources based on field synergy principle," *International Journal for Numerical Methods in Fluids,* vol. 52, pp. 987-1003, 2006.

[13] B. Premachandran and C. Balaji, "Conjugate mixed convection with surface radiation from a horizontal channel with protruding heat sources," *International Journal of Heat and Mass Transfer,* vol. 49, pp. 3568-3582, 2006.

[14] T. J. Young and K. Vafai, "Experimental and Numerical Investigation of Forced Convective Characteristics of Arrays of Channel Mounted Obstacles," *Journal of Heat Transfer,* vol. 121, pp. 34-42, 1999.

[15] M. El Nakla, "Forced Convection Heat Transfer in Two-Dimensional Ribbed Channels with Varying Heat Flux Profiles Using ANSYSâ€™ Software and Modeling," *Arabian Journal for Science and Engineering,* vol. 39, pp. 2157-2164.

[16] Y. Zeng and K. Vafai, "An Investigation of Convective Cooling of an Array of Channel-Mounted Obstacles," *Numerical Heat Transfer, Part A: Applications,* vol. 55, pp. 967-982, 2014/11/11 2009.

[17] G. E. Moore, " Cramming More Components onto Integrated Circuits,", *Electron* vol. 38, pp. 114–117, 1965.

[18] R. Schmidt, "Challenges in Electronic Coolingâ€"Opportunities for Enhanced Thermal Management Techniquesâ€"Microprocessor Liquid Cooled Minichannel Heat Sink," *Heat Transfer Engineering,* vol. 25, pp. 3-12, 2014/08/31 2004.

[19] Y. Liu, S. Chen, and B. W. Shiu, "An optimum spacing problem for four chips on a horizontal substrate â€“ mixed convection," *Computational Mechanics,* vol. 26, pp. 470-477, 2000-11-01 2000.

[20] S. Chen, Y. Liu, S. F. Chan, C. W. Leung, and T. L. Chan, "Experimental study of optimum spacing problem in the cooling of simulated electronic package," *Heat and Mass Transfer,* vol. 37, pp. 251-257, 2001-04-01 2001.

[21] S. Chen and Y. Liu, "An optimum spacing problem for three-by-three heated elements mounted on a substrate," *Heat and Mass Transfer,* vol. 39, pp. 3-9, 2002-11-01 2002.

[22] A. K. da Silva, S. Lorente, and A. Bejan, "Optimal distribution of discrete heat sources on a plate with laminar forced convection," *International Journal of Heat and Mass Transfer,* vol. 47, pp. 2139-2148, 2004.

[23] T. Dias Jr and L. F. Milanez, "Optimal location of heat sources on a vertical wall with natural convection through genetic algorithms," *International Journal of Heat and Mass Transfer,* vol. 49, pp. 2090-2096, 2006.

[24] R. R. Madadi and C. Balaji, "Optimization of the location of multiple discrete heat sources in a ventilated cavity using artificial neural networks and micro genetic algorithm," *International Journal of Heat and Mass Transfer,* vol. 51, pp. 2299-2312, 2008.

[25] K.-Y. Kim and S.-S. Kim, "Shape optimization of rib-roughened surface to enhance turbulent heat transfer," *International Journal of Heat and Mass Transfer,* vol. 45, pp. 2719-2727, 2002.

[26] H.-M. Kim and K.-Y. Kim, "Shape optimization of three-dimensional channel roughened by angled ribs with RANS analysis of turbulent heat transfer," *International Journal of Heat and Mass Transfer,* vol. 49, pp. 4013-4022, 2006.

[27] A. Bejan and S. Lorente, "Constructal theory of generation of configuration in nature and engineering," *Journal of Applied Physics,* vol. 100, pp. -, 2006.

[28] T. Bello-Ochende, J. P. Meyer, and A. Bejan, "Constructal multi-scale pinâ€“fins," *International Journal of Heat and Mass Transfer,* vol. 53, pp. 2773-2779.

[29] G. I. Sultan, "Enhancing forced convection heat transfer from multiple protruding heat sources simulating electronic components in a horizontal channel by passing cooling," *Microelectronics Journal* vol. 31, pp. 773-779., 2000.

[30] S. V. Garimella and P. A. Eibeck, "Heat transfer characteristics of an array of protruding elements in single phase forced convection," *International Journal of Heat and Mass Transfer,* vol. 33, pp. 2659-2669, 1990.

[31] A. Korichi and L. Oufer, "Heat transfer enhancement in oscillatory flow in channel with periodically upper and lower walls mounted obstacles," *International Journal of Heat and Fluid Flow,* vol. 28, pp. 1003-1012, 2007.

[32] B. R. Amirouche Y, "Numerical simulation of laminar mixed convection air-cooling from an array of heated electronic components mounted in a vertical channel," in *International Renewable Energy Congress* Sousse, Tunisia, 2010

[33] A. Korichi and L. Oufer, "Unsteady Heat Transfer and Pressure Drop in Channels with Obtacles Mounted on the Upper and Lower Walls," *Numerical Heat Transfer, Part A: Applications,* vol. 48, pp. 711-729, 2014/12/23 2005.

[34] T. J. Young and K. Vafai, "Convective flow and heat transfer in a channel containing multiple heated obstacles," *International Journal of Heat and Mass Transfer,* vol. 41, pp. 3279-3298, 1998.

[35] STAR-CCM+, " USER GUIDE ", 8.02 ed: CD-adapco, 2013.

[36] S. V. Patankar and D. B. Spalding, "A calculation procedure for heat, mass and momentum transfer in three-dimensional parabolic flows," *International Journal of Heat and Mass Transfer,* vol. 15, pp. 1787-1806, 1972.

[37] S.J.Kim and S. W. Lee, "Air Cooling Technology for Electronic Equipment," p.cm, Ed. Florida: CRC Press LLC, 1996.

[38] Y. Liu, N. Phan-Thien, C. W. Leung, and T. L. Chan, "An optimum spacing problem for five chips on a horizontal substrate in a vertically insulated enclosure," *Computational Mechanics,* vol. 24, pp. 310-318, 1999.

[39] S. R. G Mebarki, A Hamza, "Heat Transfer Enhancement by Flow Control in a Rectangular Horizontal Channel," *International Journal of Materials, Mechanics and Manufacturing,* vol. 1, pp. 171-176, May 2013.

[40] "An Efficient and Robust Optimization/Search Algorithm," in *red cedar TECHNOLOGY.* vol. WP-1023 Rev.05.08.

[41] HEEDS, "- mdo Getting Started Guide," Red Cedar Technology, 2011.