A NUMERICAL STUDY OF NATURAL CONVECTIVE HEAT TRANSFER FROM A HORIZONTAL ISOTHERMAL SQUARE ELEMENT WITH AN UNHEATED ADIABATIC INNER SECTION

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ABSTRACT

A numerical study of natural convective heat transfer from a horizontal isothermal square heated element with a square inner unheated adiabatic section imbedded in a large flat adiabatic surface has been undertaken. The element is at a higher temperature than the surrounding fluid and the cases where it is facing upward and where it is facing downward have been considered. Steady flow has been assumed and fluid properties have been assumed constant except for the density change with temperature which gives rise to the buoyancy forces, this having been treated using the Boussinesq approach. The solution has been obtained by numerically solving the governing equations subject to the boundary conditions using the commercial CFD solver ANSYS FLUENT©. The *k*-epsilon turbulence model was used with full account being taken of buoyancy force effects. The heat transfer rate from the heated element expressed in terms of a Nusselt number is dependent on the Rayleigh number, the size of the inner adiabatic element section relative to the overall size of the element, and the Prandtl number. Results have been obtained only for a Prandtl number of 0.74, i.e., effectively the value for air. Conditions under which laminar, transitional, and turbulent flow exist have been considered. The variation of the Nusselt number with Rayleigh number has been explored in detail for various inner-to-overall element size ratios. The results have been used to determine whether correlation equations that apply for all inner- to-overall element size ratios can be derived.

INTRODUCTION

Natural convective heat transfer from heated upward facing plane horizontal elements of various shapes has been quite extensively studied [1-16]. It has been shown [17-18] that a correlation equation applying to elements of various shapes can be derived providing the Nusselt and Rayleigh numbers are expressed in terms of a suitable mean length scale. However, these studies have essentially only considered cases where the elements cover a single continuous area, examples being square, rectangular, and circular elements. Now, in some non-computer electrical and electronic component air cooling problems, because of the grouping of the components, the heat transfer is essentially from an element that has an outer heated section and an inner unheated section and it is not clear that

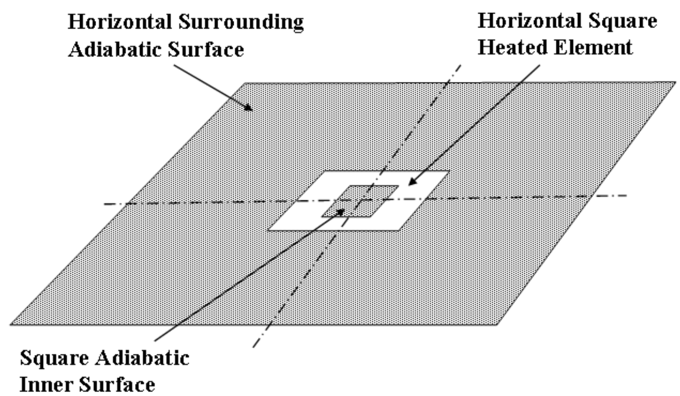

Figure 1 Flow situation considered in present study

correlation equations derived for an element with a continuous area will apply in such situations. Attention therefore has here been given to steady natural convective heat transfer from an isothermal square heated element with an inner unheated adiabatic section imbedded in a large flat adiabatic surface. This situation is shown in Fig. 1 and has here been numerically studied.

Attention has mainly been given to the case where the inner unheated section is square, this being the situation shown in Fig. 2.

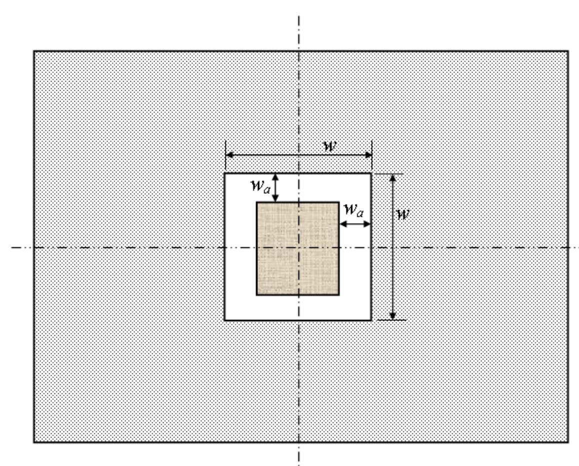

Figure 2 Heated square element with square inner unheated section

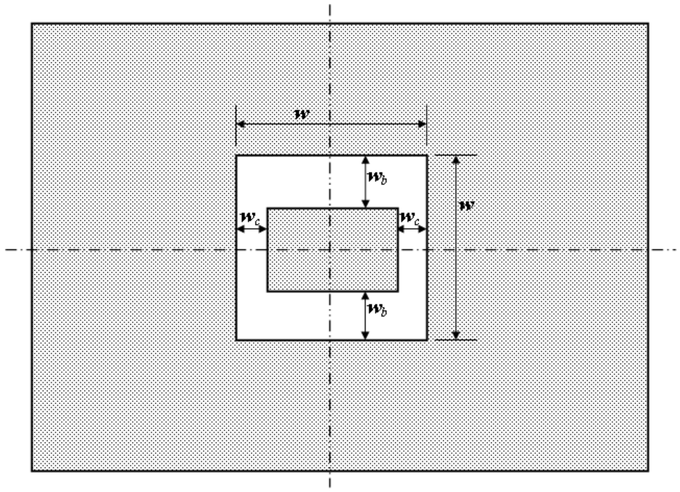

Figure 3 Heated square element with rectangular inner unheated section

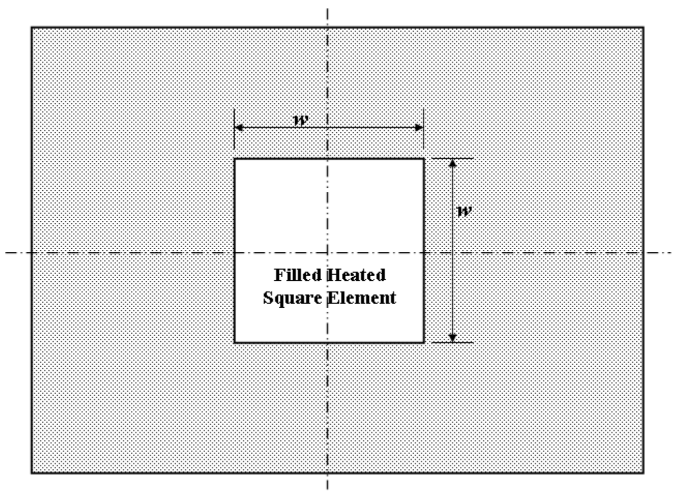

Figure 4 Filled heated square element which has no inner unheated section

Some results have also been obtained for the case where the inner unheated section is rectangular in shape, this situation being shown in Fig. 3. For comparison, results will also be given for the case of a filled square element, i.e., a square element with no inner unheated section. This case is shown in Fig. 4.

The element is at a higher temperature than the surrounding fluid and the case where the heated element is facing upward and the case where the heated element is facing downward have been considered. These two cases are illustrated in Fig. 5. The range of conditions considered is such that laminar, transitional, and turbulent flows occur.

Steady flow has been assumed and fluid properties have been assumed constant except for the density change with temperature which gives rise to the buoyancy forces, this having been treated using the Boussinesq approach. The solution has been obtained by numerically solving the governing equations subject to the boundary conditions using the commercial CFD solver ANSYS FLUENT©. The *k*-epsilon turbulence model was used with full account being taken of buoyancy force effects. The heat transfer rate from the heated element expressed in terms of a Nusselt number is dependent on the Rayleigh number, the size of the inner adiabatic element section relative to the overall size of the element, and the Prandtl number. Results have been obtained only for a Prandtl number of 0.74, i.e., effectively the value for air. Rayleigh numbers between approximately 10^5 and 10^{13} have been considered. The variation of the Nusselt number with Rayleigh number has been explored in detail for various inner-to-overall element size ratios. The results have been used to determine whether correlation equations that apply for all inner- to-overall element size ratios can be derived.

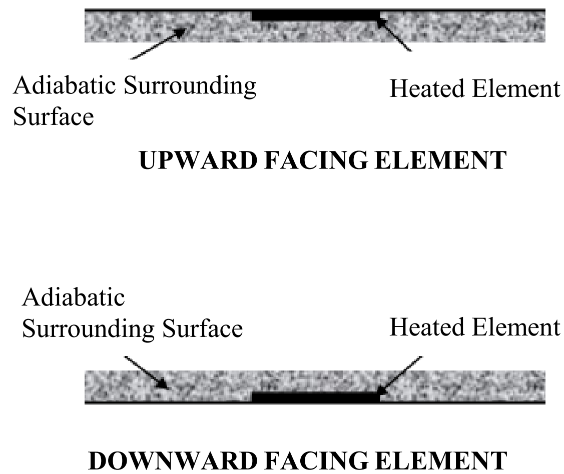

Figure 5 Upward and downward facing heated elements

Natural convective heat transfer from upward facing heated horizontal surfaces with relatively simple shapes has been extensively studied [1-16]. However, most of these studies have considered only situations that involve laminar flow. Natural convective heat transfer from downward facing heated horizontal surfaces has also been quite extensively studied [19-24]. The present study differs from these available studies in that elements with complex shapes have been considered here and results have been obtained for a wide range of Rayleigh numbers covering conditions in which laminar, transitional, and turbulent flow occur. This study is one of a series of investigations of natural convective heat transfer rates from various horizontal heated element arrangements, e.g. see [25-29].

NOMENCLATURE

A	[m^2]	Surface area of heated element
g	[m/s^2]	Gravitational acceleration
k	[W/mK]	Thermal conductivity
m	[m]	Mean size of element
M	[-]	Dimensionless mean size of element, m/w
Nu	[-]	Nusselt number based on w
Nu_m	[-]	Nusselt number based on the mean heated element size m
P	[m]	Total perimeter of the heated element
Pr	[-]	Prandtl number

Q'	[W]	Mean heat transfer rate from element
Ra	[-]	Rayleigh number
Ra_m	[-]	Rayleigh number based on the mean heated element size m
T_f	[K]	Undisturbed fluid temperature
T_w	[K]	Heated element wall temperature
w	[m]	Side length of square heated element
w_a	[m]	Side dimension of the heated element with an unheated square inner section
w_b	[m]	One side dimension of the heated element with a rectangular inner section
w_c	[m]	Other side dimension of the heated element with a rectangular inner section
W_a	[m]	Dimensionless side dimension of the heated element with a square inner section, w_a/w
W_b	[m]	One dimensionless side dimension of the heated element with a rectangular inner section, w_b/w
W_c	[m]	Other dimensionless side dimension of the heated element with a rectangular inner section, w_c/w

Greek Symbols

α	[m^2/s]	Thermal diffusivity
β	[1/K]	Bulk coefficient of thermal expansion
v	[m^2/s]	Kinematic viscosity

SOLUTION PROCEDURE

Steady flow has been assumed. Fluid properties have been assumed constant except for the density change with temperature that gives rise to the buoyancy forces which was treated by means of the Boussinesq type approximation. Radiant heat transfer effects were neglected. Allowance has been made for the possibility that turbulent flow can occur in the system. To deal with this the basic *k*-epsilon turbulence model has been used with standard wall functions and with full account being taken of buoyancy force effects. The k-epsilon model is applied in all calculations and determines when transition commences and when fully turbulent flow is reached. This approach has been demonstrated to provide quite good predictions of the conditions under which turbulence develops, e.g., [30-36].

The commercial CFD solver ANSYS FLUENT© was used to numerically solve the three-dimensional governing equations subject to the boundary conditions. The flow was assumed to be symmetric about the longitudinal center-lines shown in Figs. 2, 3, and 4.

Extensive grid independence and convergence-criteria independence testing was undertaken. With the grids used in obtaining the results presented here this testing indicated that the heat transfer results are grid- and convergence criteria independent to within approximately one per cent.

RESULTS

The solution depends on whether a square or rectangular inner section is considered (see Figs. 2 and 3). The solution has the following parameters:

- The Rayleigh number, Ra, based on the side length, w, of the square element (see Fig. 1) and the difference between the surface temperature of the heated section, T_w, and the temperature of the undisturbed fluid well away from the system, T_f, i.e.:

$$Ra = \frac{\beta g w^3 (T_w - T_f)}{v\alpha} \tag{1}$$

- Whether the inner section has a square or rectangular shape. If the inner section has a square shape the solution will depend on the dimensionless size of the heated element sections, i.e., on $Wa = wa / w$ (see Fig. 2) while if the inner section has a rectangular shape the solution will depend on the dimensionless sizes of the two heated element sections, i.e., on both $W_b = w_b / w$ and $W_c = w_c / w$ (see Fig. 3).

- The Prandtl number, Pr.

- The orientation of the heated element, i.e., whether it is facing upward or facing downward (see Fig. 5).

Because of the applications that were the motivation for this study, results have been obtained for only a Prandtl number of 0.74, i.e., effectively the value for air. For elements with square inner sections results have been obtained for W_a values between 0.125 and 0.375 while for elements with rectangular inner sections results have only been obtained for the case where $W_b = 0.375$ and $W_c = 0.125$.

For comparison, results have also been obtained for the case where there is no inner section, i.e., for the case of a complete square element. This case is shown in Fig. 5. Rayleigh numbers of between approximately 10^4 and 10^{12} have been considered. Results have been obtained for both upward and downward facing heated element orientations.

The mean heat transfer rate from the heated surface, $\overline{Q}'$ has been expressed in terms of a mean Nusselt number based on the side length, w, of the square heated element and on the difference between the heated section surface temperature, T_w, and the temperature of the undisturbed fluid well away from the system, T_f. That is:

$$Nu = \frac{\bar{Q}' w}{kA(T_w - T_f)} \tag{2}$$

where A is the surface area of the heated surface, i.e., $4\, w_a (w - w_a)$ in the case of an element with a square inner section and $2\, w\, (w_b + w_c) - 4w_c^2$ in the case of an element with a rectangular inner section. Because a fixed value of Pr is being considered Nu is a function of Ra, the element geometry, and the element orientation, i.e., upward or downward facing.

Typical variations of the mean Nusselt number with Rayleigh number for heated elements with a square unheated inner sections for the upward facing and the downward facing element cases are shown in Figs. 6, 7, and 8. These figures give results for different values of the size of the square unheated inner section. From these figures it will be seen that in all cases the range of Rayleigh numbers considered covers those in which laminar, transitional, and turbulent flows exist.

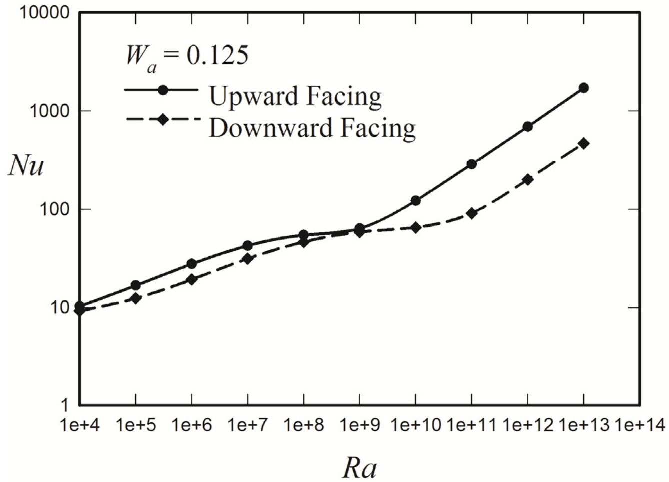

Figure 6 Variation of Nusselt number based on *w* with Rayleigh number also based on *w* for a square heated element having a square inner unheated section for the case where W_a (see Fig. 2) = 0.125

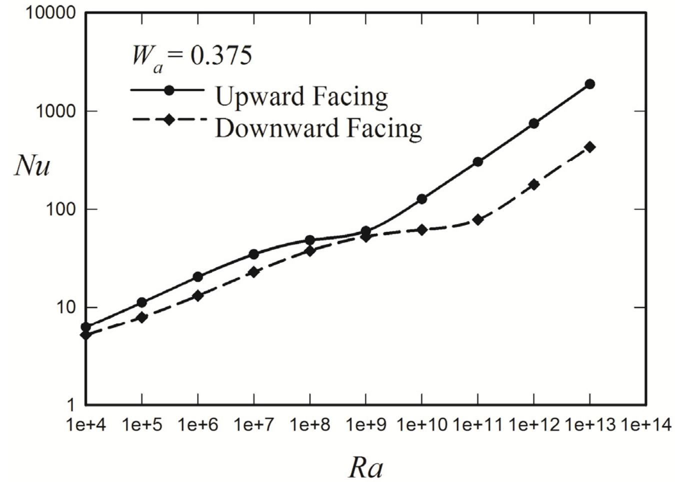

Figure 8 Variation of Nusselt number based on *w* with Rayleigh number also based on *w* for a square heated element having a square inner unheated section for the case where W_a (see Fig. 2) = 0.375

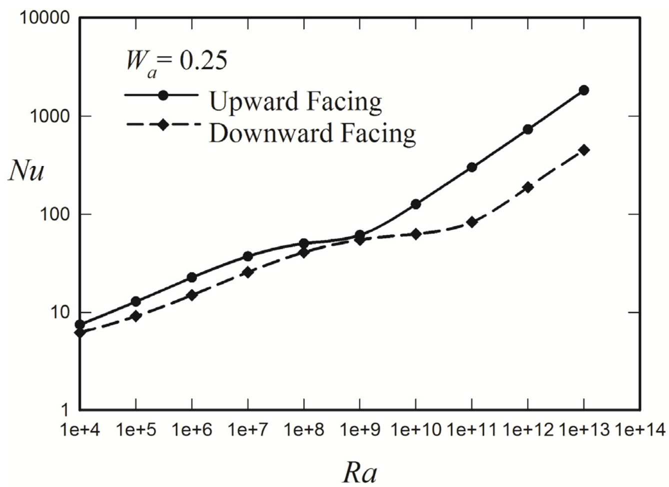

Figure 7 Variation of Nusselt number based on *w* with Rayleigh number also based on *w* for a square heated element having a square inner unheated section for the case where W_a (see Fig. 2) = 0.25

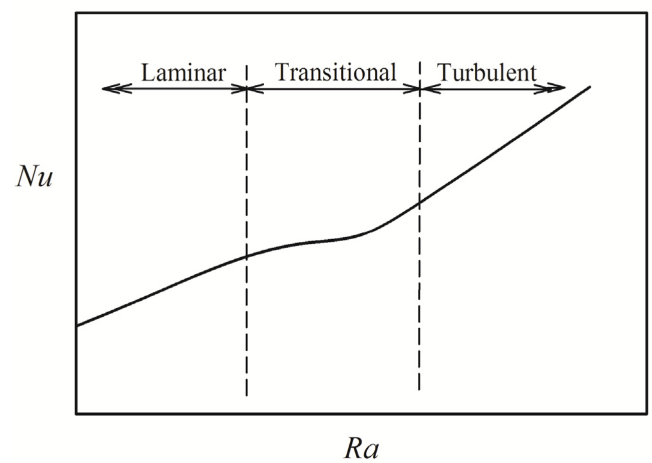

Figure 9 General form of the variation of Nusselt number with Rayleigh number showing the effects of the changes from laminar flow to transitional flow to turbulent flow on the form of this variation

The forms of the Nusselt number variation that exist with these three types of flow are shown schematically in Fig. 9. It will also be seen from Figs. 6, 7, and 8 that, as is to be expected, the Nusselt number at a given Rayleigh number is higher for the upward facing element case than it is for the downward facing element case. It will also be seen from Figs. 6, 7, and 8 that in all cases transition from laminar to turbulent flow commences at a lower Rayleigh number in the upward facing element case than it does in the downward facing element case. The transition in the upward facing case starts at a Rayleigh number of roughly 10^7 and in the downward facing case at a Rayleigh number of roughly 10^8.

Typical variations of the mean Nusselt number with Rayleigh number for a square heated element with a rectangular unheated inner section for the upward facing and the downward facing element cases are shown in Fig. 10. It will be seen that these variations have the same basic form as those that exist with square elements with square unheated inner sections.

For comparison, variations of the mean Nusselt number with Rayleigh number for a filled square heated element, i.e., a square heated element with no inner unheated section, for the upward facing and the downward facing element cases are shown in Fig. 11. It will be seen that transition from laminar to turbulent flow commences at higher Rayleigh numbers in the filled square element case than it does in the square element with an unheated inner section case.

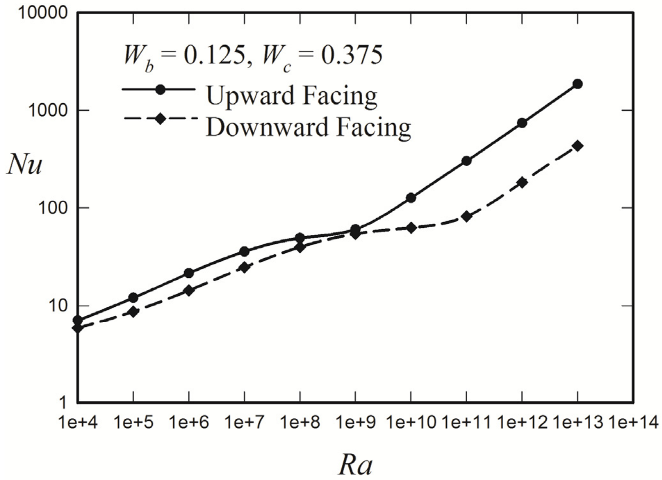

Figure 10 Typical variation of Nusselt number based on *w* with Rayleigh number also based on *w f*or a square heated element having a rectangular inner unheated section for the case where W_b (see Fig. 3) = 0.125 and W_c (see Fig. 3) = 0.375

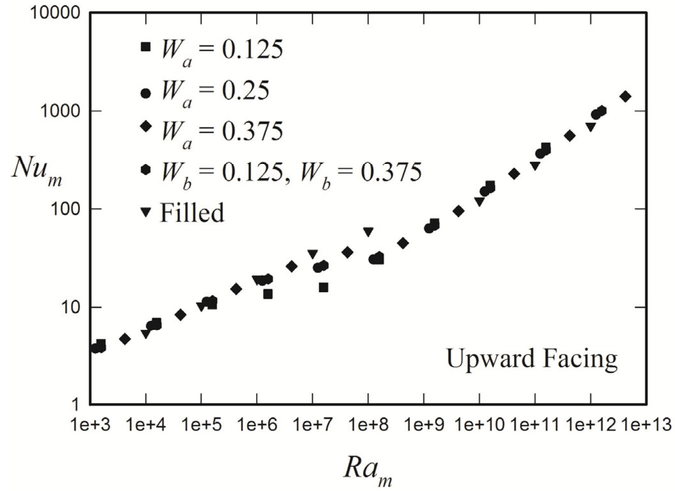

Figure 12 Variations of Nusselt number based on *m* with Rayleigh number also based on *m* for all situations considered for the case where the elements face upward

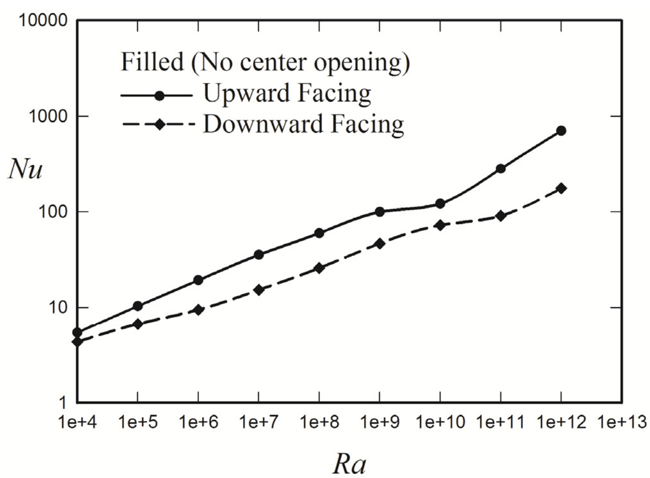

Figure 11 Variation of Nusselt number based on *w* with Rayleigh number also based on *w* for a square heated element having no inner unheated section, i.e., for a filled square element

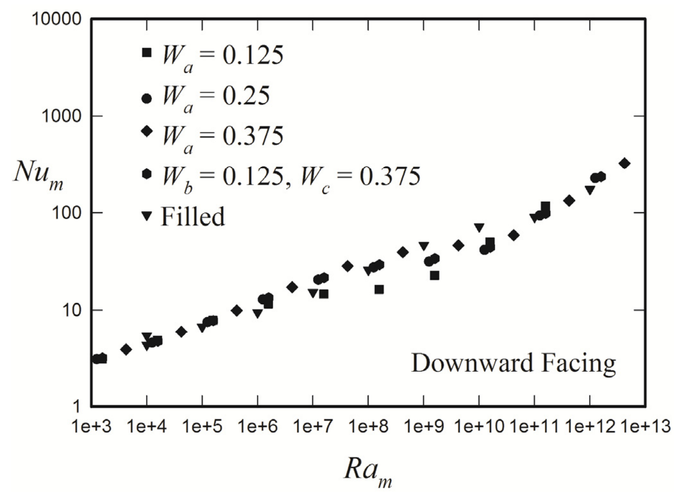

Figure 13 Variations of Nusselt number based on *m* with Rayleigh number also based on *m* for all situations considered for the case where the elements face downward

Now it is often assumed (e.g., see [17-18]) for natural convection from horizontal heated elements with shapes that are less complex than those being considered here that if a mean element size, *m*, defined by:

$$m = 4\frac{A}{P} \tag{3}$$

is introduced and if Nusselt and Rayleigh numbers based on this mean element size are used then the variations of Nusselt number with Rayleigh number will be the same for all element shapes. In the above equation *A* is the surface area of the heated element and *P* is the total perimeter of the heated element. Now for a square element with a square unheated inner section:

$$A = 4w_a(w - w_a) \tag{4}$$

and in the case of a square element with a rectangular unheated inner section:

$$A = 2w(w_b + w_c) - 4w_b w_c \tag{5}$$

Also for a square element with a square unheated inner section:

$$P = 8(w - w_a) \tag{6}$$

and for a square element with a rectangular unheated inner section:

$$P = 8w - 4(w_b + w_c) \tag{7}$$

Therefore for a square element with a square unheated inner section:

$$m = 4\frac{4w_a(w - w_a)}{8(w - w_a)} = 2w_a \text{, i.e., } M = 2W_a \qquad (8)$$

where $M = m/w$ and for a square element with a rectangular unheated inner section:

$$m = 4\,\frac{2w(w_b + w_c) - 4w_b w_c}{8w - 4(w_b + w_c)}\text{, i.e.,}$$

$$M = 2\,\frac{W_b + W_c - 2W_b W_c}{2 - W_b - W_c} \qquad (9)$$

Using these equations the values of M for the elements in all of the situations considered above have been found and the variations of Nu_m with Ra_m for all of these situations have been determined. The results are shown in Figs 12 and 13 which give results for the upward facing case and the downward facing case respectively. It will be seen from these results that in the laminar flow and turbulent flow regions the variations of Nusselt number with Rayleigh number are essentially the same for all element shapes for both the upward facing case and the downward facing case. In the transition region, however, because of the dependence of the Rayleigh number at which transition starts on the geometrical situation considered the results in this region for the different situations considered are not correlated by assuming that Nu_m varies with Ra_m for all of the situations considered.

CONCLUSIONS

The results obtained in the present numerical study show that:

1. In all the geometrical situations considered the Nusselt number at a given Rayleigh number is higher for the upward facing element case than it is for the downward facing element case.
2. In all the geometrical situations considered transition from laminar to turbulent flow commences at a lower Rayleigh number in the upward facing element case than it does in the downward facing element case.
3. If Nusselt and Rayleigh numbers based on the mean element size $m = 4A/P$, where A is the surface area of the heated element and P is the total perimeter of the heated element, then in the laminar flow region and in the turbulent flow region the variations of Nusselt number based on m with Rayleigh number based on m are essentially the same for all element shapes considered both for the upward facing case and for the downward facing case. In the transition region, however, the variations of Nusselt number based on m with Rayleigh number based on m are not the same for all the geometrical situations considered due basically to the dependence of the transition Rayleigh number on the element shape.

ACKNOWLEDGEMENTS

This work was supported by the Natural Sciences and Engineering Research Council of Canada (NSERC) through its Discovery Grant Program.

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