# COMBINED EFFECTS OF THERMAL-DIFFUSION AND THERMAL RADIATION ON TRANSIENT MHD NATURAL CONVECTION FLOW IN A VERTICAL CHANNEL

Basant K. Jha[a], Isah B. Y[b, *], and Uwanta I. J[b]
[*]Author for correspondence
[a] Department of Mathematics, Ahmadu Bello University, Zaria, Nigeria
[b] Department of Mathematics, Usmanu Danfodiyo University, Sokoto, Nigeria
E-mail: isah.bala@udusok.edu.ng

## ABSTRACT

This study investigates the transient MHD natural convection flow of viscous incompressible electrically conducting fluid in a vertical channel formed by two infinite vertical parallel plates in the presence of thermal-diffusion and thermal radiation. Analytical solutions for energy and momentum equation are derived using perturbation method for steady state operating condition for small value of radiation parameter. The time dependent energy and momentum equations under relevant initial and boundary conditions are solved using implicit finite difference method. The effects of the various involved parameters on the skin friction and Nusselt number at the channel surfaces are discussed. **A series of numerical experiments shows that the time required to reach steady state velocity, temperature is directly proportional to the Prandtl number of the working fluid for fixed values of other controlling parameters.**

Keywords: Thermal-diffusion, thermal-radiation, MHD, Transient natural convection

## INTRODUCTION

**In nature, many flows are caused not only by the temperature differences but also the concentration differences. The rate of heat transfer is affected by these mass transfer differences especially in industries. The transport process exists in which heat and mass transfer simultaneously take place that results the combine buoyancy effect the thermal diffusion, the phenomenon of heat and mass transfer frequently occurs in chemically processed industries, distribution of temperature and moisture over agricultural fields, dispersion of fog and environment pollution and polymer production Raju *et al.* [1].** However, the impact of magnetic field and thermal radiation **in heat and mass transfer** could be very practicable both naturally and in many branches of science and engineering applications. They play a vital role in many industrial tasks for instance in polymer technology, metallurgy, chemical industry, power and cooling industry for drying, cooling of nuclear reactors and magnetohydrodynamic (MHD) power generators. With span in technology in many directions, the subject of MHD has developed in the use of magnetic fields and the range of fluid and thermal processes [2]. During the production and working life of microelectronic heat transfer devices; an electrically conducting fluid is subject to a magnetic field [3]. In such cases the fluid experiences a Lorentz force which changes the flow velocities. This in turn affects the rate of heat and mass transfer. By knowing these compound and complicated effects, new or improved designs in the manufacturing process can be developed [4]. It is interesting to mention that, the interaction of buoyancy with thermal radiation has increased due to its importance in many practical applications. Radiation effect is important under many isothermal and non-isothermal situations. If the entire system involving the polymer extrusion process is placed in a thermally controlled environment, then radiation could become important. The knowledge of radiation heat transfer in the system can, perhaps, lead to a desired product with a sought characteristic. Also, radiation effects on the convective flow are important in context of space technology and processes involving high temperature and very little is known about the effects of radiation on the boundary layer flow of radiating fluid past body [5].
`

Hence several models with Soret and Dufour effect in different heat and mass transfer problems has been studied. An excellent work on the Soret effect (thermal diffusion effect) on the onset of convective instability has also been investigated [6]. The use of pseudo-spectral Chebyshev collocation method, to analyze the influence of vibration on Soret-driven convection in porous media was carried out [2]. Bourich *et al.* [7] presented an analytical and numerical study on combined magnetic field and an external shear stress subjected to Soret convective fluid. Lin *et al.* [8] accounted for the Soret effect on the rapid heat and mass transfer problem in a slab, employing the Laplace transform method. Recently, Jha *et al.* [9] consider Dufour effect on the free-convection and mass transfer flow in a vertical channel when the boundaries are subjected to symmetric concentration and thermal input. Kumar [10] presented a theoretical treatment of unsteady hydromagnetic flow and heat and mass transfer of an incompressible electrically-conducting and radiating fluid in a vertical channel filled with porous medium taking into account the Soret effects. Kesavaraya *et al.* [11] considered the effect of thermo-diffusion on MHD mixed convective heat and mass transfer flow of a viscous fluid through a porous medium with radiation, heat generation and chemical reaction. Sundhakar *et al.* [12] explained the behaviours of chemically reacted unsteady MHD free convection flow in the presence of thermal diffusion and diffusion thermo effects. Sharma *et al.* [13] analyzed Soret and Dufour effects on unsteady MHD mixed convection flow past a radiative vertical porous plate embedded in a porous medium with chemical reaction. Sarada and Shaner [14] discussed the effect of Soret and Dufour number on an unsteady magnetohydrodynamic free convective fluid flow past a vertical porous plate in the presence of suction or injection. More recently, Ahmed *et al.* [15] carried out a parametric study on radiation, Soret and Dufour effects in MHD channel flow bounded by a long wavy wall and a uniformly moving parallel flat wall.

**The objective of the present work is to examine the** combined effects of thermal-diffusion and thermal radiation on transient MHD natural convection and mass transfer flow in a vertical channel.

## MATHEMATICAL ANALYSIS

Consider transient combined free convective and mass transfer flow of a viscous, incompressible and electrically conduction fluid between two infinite vertical parallel plates. A uniform transverse magnetic field of magnitude $B_0$ is applied in the presence of an incident radiation flux of intensity $q_r$, which absorbed by the plate and transferred to the fluid. At time $t' \le 0$ both the fluid and plates are assumed to be at rest at constant temperature $T_0$ and constant concentration $C_0$ respectively. At time $t' > 0$ the temperature and concentration of the plate situated at $y' = 0$ rise to $T_w$ and $C_w$ while the other plate at a distance $H$ from it, is fixed and maintained at temperature $T_0$ and concentration $C_0$. The stream wise coordinate is denoted by $x'$ taken vertically upward direction and that normal to it is denoted by $y'$. The flow is assumed laminar and fully developed means that the axial ($x'$- direction) velocity depends only on transverse coordinate, $y'$. Since the plates are of infinite length, the velocity, temperature and concentration are function of $y'$ and $t'$ alone. Using the Boussinesq's approximation, the governing equations for the present physical situation in presence of thermal diffusion and thermal radiation in the dimensional form are:

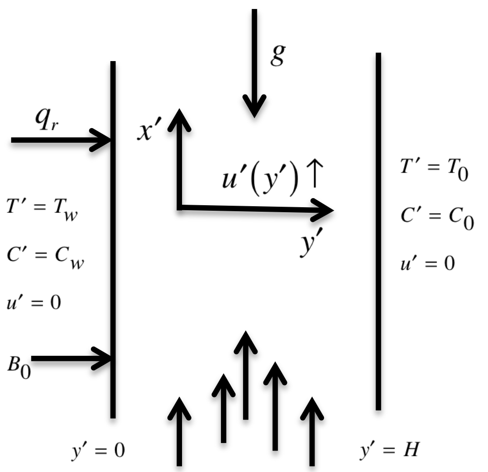


Figure 1: Schematic diagram of the problem

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T' - T_0\right) + g\beta^*\left(C' - C_0\right) - \frac{\sigma_1 B_0^2 u'}{\rho} \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \alpha\left[\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{K}\frac{\partial q_r}{\partial y'}\right] \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D_m\left[\frac{\partial^2 C'}{\partial y'^2}\right] + D_t\left[\frac{\partial^2 T'}{\partial y'^2}\right] \tag{3}$$

$$t' \le 0 : u' = 0, T' = T_0, C' = C_0,\ 0 \le y' \le H$$

$$t' \ge 0 : \begin{cases} u' = 0, T' = T_w, C' = C_w & at\ y' = 0 \\ u' = 0, T' = T_0, C' = C_0 & at\ y' = H \end{cases} \tag{4}$$

where $T'$, is the dimensional temperature of the fluid, $C'$ is the dimensional concentration of the fluid, $\alpha$ is the thermal diffusivity, $K$ is the thermal conductivity, $\rho$ is the density of the fluid, $\beta$ is the coefficient of the thermal expansion, $\sigma_1$ is the fluid electrical conductivity, $D_m$ and $D_t$ are the mass diffusivity and dimensional co-efficient of thermal-diffusion effect respectively, $g$ is the gravitational acceleration and $B_0$ is the strength of applied magnetic field.

The quantity $q_r$ appearing on the right hand side of equation (2) represents the radiative heat flux in the $y'$-direction. Where the radiative heat flux in the $x'$-direction is considered insignificant in comparison with that in the $y'$-direction. The radiative heat flux term in the problem is simplified by using the Rosseland diffusion approximation for an optically thick fluid according to [16].

$$q_r = -\frac{4\sigma}{3\kappa^*}\frac{\partial T'^4}{\partial y'} \tag{5}$$

where $\sigma$ is Stefan-Boltzmann constant and $\kappa^*$ is the mean absorption coefficient. This approximation is valid for intensive absorption, that is, for an optically thick boundary layer. Despite these shortcomings, the Rosseland approximation has been used with success in a variety of problems ranging from the transport of radiation through gases at low density to the study of the effects of radiation on blast waves by nuclear explosion [15]

To obtain the non-dimensional form of the above equations, the following dimensionless variables are introduced.

$$t = \frac{t'\nu}{H^2}, y = \frac{y'}{H}, Pr = \frac{\nu}{\alpha}, M^2 = \frac{\sigma_1 B_0^2 H^2}{\nu\rho}, \phi = \frac{C' - C_0}{C_w - C_0}$$

$$u = u'\nu\left[g\beta H^2\left(T_w - T_0\right)\right]^{-1}, Sc = \frac{\nu}{D_m}, \theta = \frac{T' - T_0}{T_w - T_0},$$

$$R = \frac{4\sigma\left(T_w - T_0\right)^3}{\kappa^* K}, C_T = \frac{T_0}{T_w - T_0}, N = \frac{\beta^*\left(C_w - C_0\right)}{\beta\left(T_w - T_0\right)},$$

$$S_t = \frac{D_t\left(T_w - T_0\right)}{D_m\left(C_w - C_0\right)} \tag{6}$$

Substituting Equations (5) and (6) in Equations (1)–(3), we obtain the following dimensionless equations for velocity, temperature and concentration respectively.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \left[\theta + N\phi\right] - M^2 u \tag{7}$$

$$Pr\frac{\partial\theta}{\partial t}=\left[1+\frac{4}{3}R\left(C_T+\theta\right)^3\right]\left(\frac{\partial^2\theta}{\partial y^2}\right)+4R\left[C_T+\theta\right]^2\left(\frac{\partial\theta}{\partial y}\right)^2 \quad (8)$$

$$Sc\frac{\partial\phi}{\partial t}=\frac{\partial^2\phi}{\partial y^2}+S_t\frac{\partial^2\theta}{\partial y^2} \quad (9)$$

where $N$ is the sustention parameter, $M$ is the magnetic interaction parameter, $Pr$ is the Prandtl number, $R$ is the radiation parameter, $C_T$ is the temperature difference parameter, $Sc$ is the Schmidt number and $S_t$ is the Soret or thermal-diffusion parameter. Initial and boundary conditions in the dimensionless form are

$$t\le 0: u=0,\theta=0,\phi=0, 0\le y\le 1$$

$$t>0:\begin{cases}u=0,\theta=1,\phi=1, \text{at } y=0\\ u=0,\theta=0,\phi=0, \text{at } y=1\end{cases} \quad (10)$$

## ANALYTICAL SOLUTION

The governing equations presented in the previous section are highly nonlinear and exhibited no exact solutions. In general such solution can be very useful in validating computer routines of complicated time dependent two or three-dimensional free convective and radiating conducting fluid and comparison with experimental data. It is therefore of interest to reduce the governing equations of the present problem to the form that can be solved analytically. A special case of the present problem that exhibit analytical solution is the problem of steady state MHD Natural convection and mass transfer flow in a vertical channel with combined thermal-diffusion and thermal radiation effects. The resulting steady state equations and boundary conditions for this special case can be written as

$$\frac{d^2u}{dy^2}-M^2u=-\left[\theta+N\phi\right] \quad (11)$$

$$\left[1+\frac{4R}{3}\left(C_T+\theta\right)^3\right]\frac{d^2\theta}{dy^2}+4R\left[C_T+\theta\right]^2\left(\frac{d\theta}{dy}\right)^2=0 \quad (12)$$

$$\frac{d^2\phi}{dy^2}+S_t\frac{d^2\theta}{dy^2}=0 \quad (13)$$

the boundary conditions are

$$\begin{aligned}&u=0,\theta=1,\phi=1 \quad at \quad y=0\\ &u=0,\theta=0,\phi=0 \quad at \quad y=0\end{aligned} \quad (14)$$

**In order to construct an approximate solution of equations (11) to (13) subject to equation (14), we employ a regular perturbation method by taking a power series expansion in the radiation parameter R such as**

$$\begin{aligned}u(y)&=u_0(y)+Ru_1(y)+O\left(R^2\right)\\ \theta(y)&=\theta_0(y)+R\theta_1(y)+O\left(R^2\right)\\ \phi(y)&=\phi_0(y)+R\phi_1(y)+O\left(R^2\right)\end{aligned} \quad (15)$$

**where $R$ is the radiation parameter $(R\ll 1)$. The second and high order terms of $R$ give correction to $u_0,\theta_0$ and $\phi_0$ that account for thermal radiation effect.** Substituting equations (15) into equations (12) - (14) and equating the like powers of $R$, the required **boundary value problems are:**

$$\frac{d^2u_0}{dy^2}-M^2u_0=-\left[\theta_0+N\phi_0\right] \quad (16)$$

$$\frac{d^2u_1}{dy^2}-M^2u_1=-\left[\theta_1+N\phi_1\right] \quad (17)$$

$$\frac{d^2\theta_0}{dy^2}=0 \quad (18)$$

$$\frac{d^2\theta_1}{dy^2}+\frac{4}{3}\left[C_T+\theta_0\right]^3\frac{d^2\theta_1}{dy^2}+4\left[C_T+\theta_0\right]^2\left(\frac{d\theta_0}{dy}\right)^2=0 \quad (19)$$

$$\frac{d^2\phi_0}{dy^2}+S_t\frac{d^2\theta_0}{dy^2}=0 \quad (20)$$

$$\frac{d^2\phi_1}{dy^2}+S_t\frac{d^2\theta_1}{dy^2}=0 \quad (21)$$

**The relevant boundary conditions to be satisfied are:**

$$\begin{cases}u_0=u_1=\theta_1=\phi_1=0,\theta_0=\phi_0=1 \quad at \quad y=0\\ u_0=u_1=\theta_0=\theta_1=\phi_0=\phi_1=0 \quad at \quad y=1\end{cases} \quad (22)$$

The solutions of equations (16) to (21) subject to boundary conditions (17) are

$$\theta(y)=1-y+R\left[2B^2\left(y-y^2\right)-\frac{4R}{3}\left(y-y^3\right)+\frac{1}{3}\left(y-y^4\right)\right] \quad (23)$$

$$\phi(y)=1-y+R\left[2B^2S_t\left(y^2-y\right)-\frac{4BS_t}{3}\left(y^3-y\right)+\frac{S_t}{3}\left(y^4-y\right)\right] \quad (24)$$

$$\begin{aligned}u(y)=&\frac{1+N}{M^2 sinh(M)}\left[(1-y)sinh(M)+sinh(My-M)\right]\\ &+R\left[\frac{\chi_0}{M^2 sinh(M)}sinh(My-M)+\left(1-\frac{sinh(My)}{sinh(M)}\right)\chi_0+\left(y-\frac{sinh(My)}{sinh(M)}\right)\lambda\right.\\ &+\left(y^2-\frac{sinh(My)}{sinh(M)}\right)\chi_2+\left(y^3-\frac{sinh(My)}{sinh(M)}\right)\chi_3\\ &\left.+\left(y^4-\frac{sinh(My)}{sinh(M)}\right)\chi_4\right]\end{aligned} \quad (25)$$

Using (23), we write the steady state rate of heat transfer (Nusselt number) on the boundary:

$$Nu = -\left.\frac{d\theta}{dy}\right|_{y=0} = \left[1 - R\left(2B^2 - \frac{4B}{3} + \frac{1}{3}\right)\right]\left[1 + \left(1 + C_T\right)^3\right] \quad (26)$$

Also, using (18), we write the skin friction coefficient as

$$\tau = -\left.\frac{du}{dy}\right|_{y=0} = \frac{1+N}{M^2 sinh(M)}\left[Mcosh(M) - sinh(M)\right]$$
$$+R\left[2\chi_1 + 2\chi_2 + 3\chi_3 + 4\chi_4 - \frac{Mcosh(M)}{sinh(M)}\left(\chi_1 + \chi_2 + \chi_3 + \chi_4\right)\right] \quad (27)$$

where $B = 1 + C_T$ while the constant $\chi_{i's}$ are not given in order to reduce the size of the work.

## NUMERICAL PROCEDURE

The nonlinear partial differential equations (7) - (9) are solved numerically using semi-implicit finite difference scheme. We used forward difference formulas for all time derivatives and approximate both the second and first spatial derivatives with second order central differences. The semi-implicit finite difference equation corresponding to equation (7) - (9) is as follows

$$\frac{u_i^{(K+1)} - u_i^{(K)}}{\delta t} = \frac{u_i^{(K+1)} - 2u_i^{(K+1)} + u_i^{(K+1)}}{(\delta y)^2}$$
$$-Mu_i^{(K)} + \left[\theta_i^{(K)} + N\phi_i^{(K)}\right] \quad (28)$$

$$Pr = \frac{\theta_i^{(K+1)} - \theta_i^{(K)}}{\delta t} =$$
$$\left[1 + \frac{4}{3}R\left(C_T + \theta_i^{(K)}\right)^3\right]\left(\frac{\theta_{i-1}^{(K+1)} - 2\theta_i^{(K+1)} + \theta_{i+1}^{(K+1)}}{(\delta y)^2}\right)$$
$$+4R\left[C_T + \theta_i^{(K)}\right]^2\left(\frac{\theta_{i+1}^{(K)} - \theta_{i-1}^{(K)}}{2(\delta y)}\right)^2 \quad (29)$$

$$Sc\frac{\phi_i^{(K+1)} - \phi_i^{(K)}}{\delta t} = \frac{\phi_{i-1}^{(K+1)} - 2\phi_i^{(K+1)} + \phi_{i+1}^{(K+1)}}{(\delta y)^2}$$
$$+S_t\frac{\theta_{i-i}^{(K)} - 2\theta_i^{(K)} + \theta_{i+1}^{(K)}}{(\delta y)^2} \quad (30)$$

with the following initial and boundary conditions:

$$\left.\begin{array}{l} u_{i,0} = 0, \theta_{i,0} = 0, \phi_{i,0} = 0, \ for\ \ all\ i = 0 \\ u_{0,j} = 1, \ \theta_{0,j} = 1, \phi_{0,j} = 1 \\ u_{M,j} = 0, \theta_{M,j} = 0\phi_{0,j} = 1 \end{array}\right\} \quad (31)$$

Using the known values of $\theta$ and $\phi$ at grid point $t = 0$ and reducing the solution to tri-diagonal matrix we obtained temperature and concentration fields at time $t_{i+1} = t_i + \delta t$ using the known values of the previous time $t = t_i$ for all $i = 1, 2, \ldots M - 1$. Then the velocity field is evaluated using the already known value of temperature and concentration fields obtained at $t_{i+1} = t_i + \delta t$. These processes are repeated till the required solution of $\theta$, $\phi$ and $u$ are gained at convergence criteria.

$$abs\left|\left(u,\theta,\phi\right)_{exact} - \left(u,\theta,\phi\right)_{num}\right| < 10^{-3} \quad (32)$$

The iterative system does not restrict time step and the technique is always convergent and unconditionally stable

## RESULT AND DISCUSSION

In the present work, numerical calculation on transient MHD natural convection and mass transfer flow of viscous incompressible electrically conducting fluid in a vertical channel formed by two infinite vertical parallel plates in the presence of thermal-diffusion and thermal radiation was carried out. The radiation parameter $(R)$ in the present work, is in the range of $0 \le R \le 1.2$ because large value of $R$, lead to finite time temperature blow up since the terms associated with $R$ are strong heat sources [17], the temperature difference $(C_T)$, thermal-diffusion parameter $(S_t)$, Schmidt number $(Sc)$, the sustention parameter $(N)$ appearing in eqns. 7 and 12 measures the relative importance of mass and thermal–diffusion in the buoyancy–driven flow is taken positive for thermally assisting flows to enhance the fluid velocity and Prandtl number $(Pr)$ chosen as 0.71 and 7.0 that physically represent two fluids air and water, respectively. Moreover, time is chosen between $0.2 \le t \le 8.5$ so as to capture the transient behavior of both velocity and temperature. Besides, all other parameters are taken arbitrary. To clearly give an account of the flow governing parameters on velocity, temperature, skin friction, and Nusselt number, line graphs are depicted in Figures 2 through 18. The numerical scheme was validated using the steady state (perturbation) solution obtained from equation (16) and (17); and our results are found in good agreement between numerical and perturbation solution in a steady state situation as depicted in table (1).

**Figure 2(a & b) depicts the influence of perturbation parameter $R$. During the course of the work it is observed that for $R \le 0.1$ the perturbation solution coincides with the numerical solution see Figure 2(a). For $R \ge 0.1$, the perturbation solution breaks down. And its increasing effect becomes more magnified as $R \ge 0.5$ as shown in Figure 2(b).**

The effect of thermal–diffusion parameter $S_t$ is highlighted in Figure 3. From these two Figures (a & b) it is observed that the value of the steady–state velocity is qualitatively same irrespective of values of $S_t$ and $Pr$ of the two fluids air $(Pr = 0.71)$ or water $(Pr = 7.0)$ yet the result shows buoyant influence of dimensional time $t$ and thermal-diffusion parameter during transient state as depicted

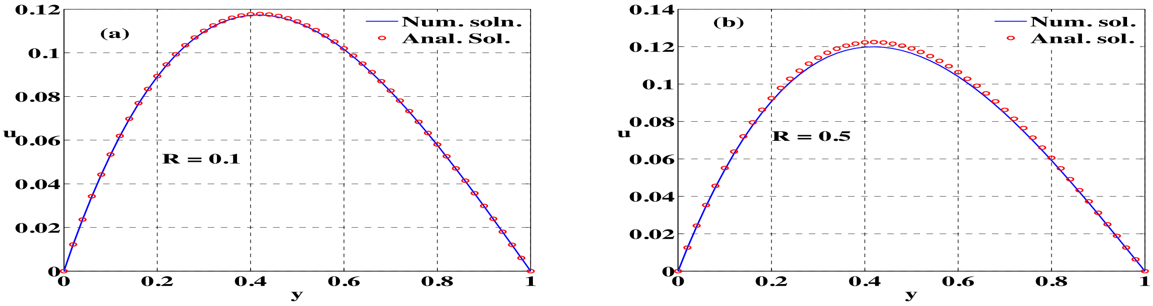


**Figure 2.** Effect of perturbation parameter on velocity for fixed values of $C_T = 0.01, N = 1, M = 1, Sc = 2, S_t = 0.2$ .

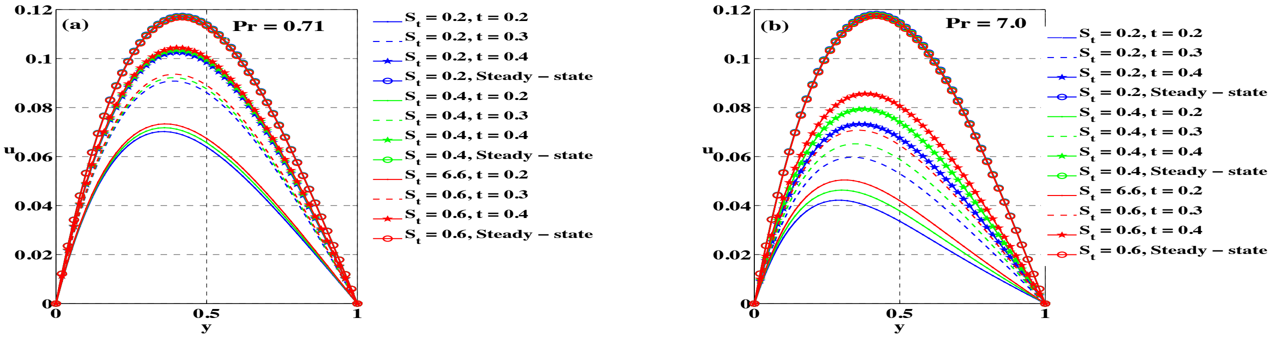


**Figure 3.** Effect of thermal-diffusion parameter on velocity for fixed values of $C_T = 0.01, N = 1, M = 1, Sc = 2,$

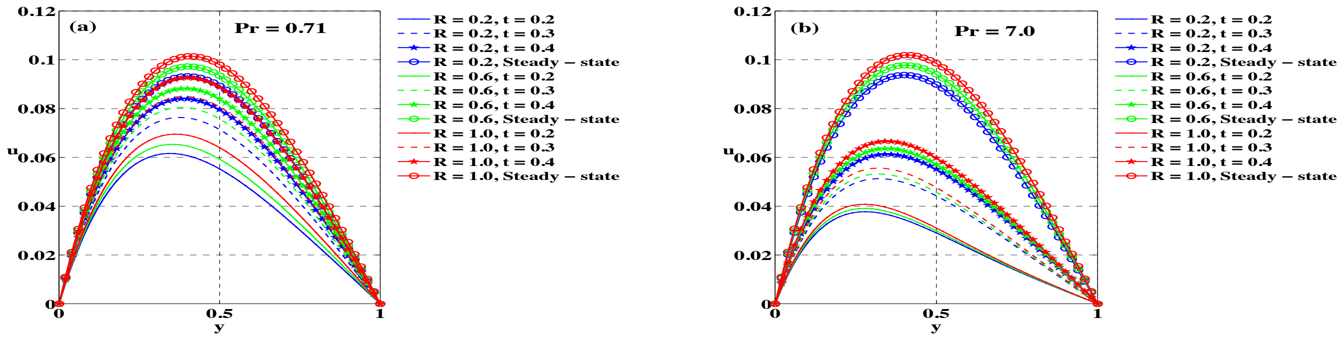


**Figure 4**. Effect of radiation parameter on velocity for fixed values of $C_T = 0.01, N = 1, M = 1, Sc = 2, S_t = 0.2$ .

Figure 3a in comparison with 3b.

Figure 4(a & b) shows the influence of the thermal radiation parameter $R$ on transient and steady–state velocity. It is observed from these figures that the velocity increases with increase in nondimensional time. There is significant influence of $R$ and nondimensional time on the transient **state velocity**. In addition, the steady–state values of the two fluids are independent on $Pr$ .

It is observe from Figure 5(a &b) that the velocity decreases with increase in the value of Sc. The Figures also delineate that the velocity increases with nondimensional time and ultimately reaches its steady–state value.

According to Figure 6(a & b), the velocity increases with increase in sustention (buoyancy ratio) parameter $N$ and nondimensional time t during transient and steady–state. It is also noted that the transient state values are higher in Figure 6a $(Pr = 0.71)$ in comparison with Figure 6b $(Pr = 7.0)$ while the inverse is the case during steady–state due to the influence of Prandtl number $Pr$ .

Figure 7 depict the temperature profiles for different values of thermal radiation $R$ and time t for fixed value of $C_T = 0.01$ . From this Figure we observed that temperature increases with increase in thermal radiation $R$ and nondimensional time t until steady–state value reach. It is recorded in Figure 7b that temperature increases more significantly as $R$ and t increases reference to value of Prandtl number $(Pr = 7.0)$ in comparison with 7a $(Pr = 0.71)$. However, during transient state the temperature is high incased of air in comparison with water see Figure 7(a & b). This is due to the physical fact that, as the Prandtl number increases; the thermal diffusivity of the fluid reduces which results in a corresponding decrease in the fluid temperature.

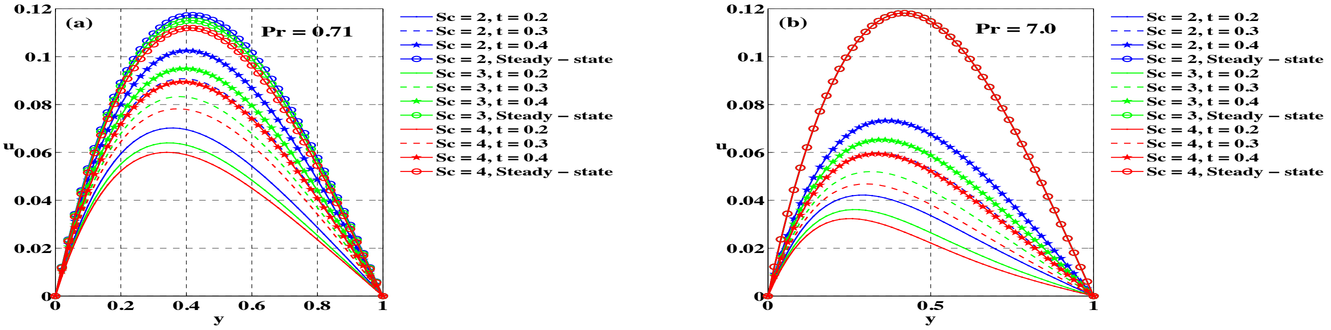


**Figure 5.** Effect of Schmidt number on velocity for fixed values of $C_T = 0.01, N = 1, M = 1, S_t = 0.2$ .

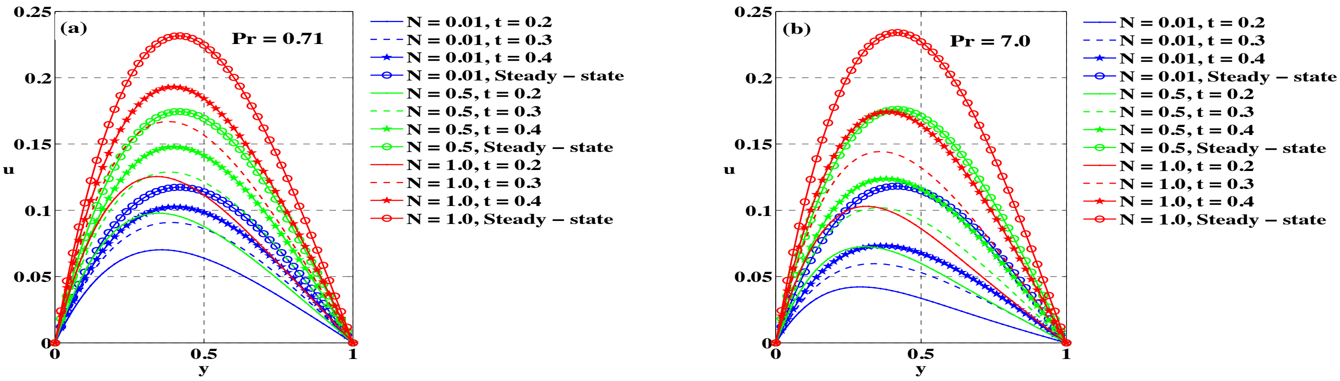


**Figure 6.** Effect of sustention parameter on velocity for fixed values of $C_T = 0.01, N = 1, M = 1, Sc = 2, S_t = 0.2$ .

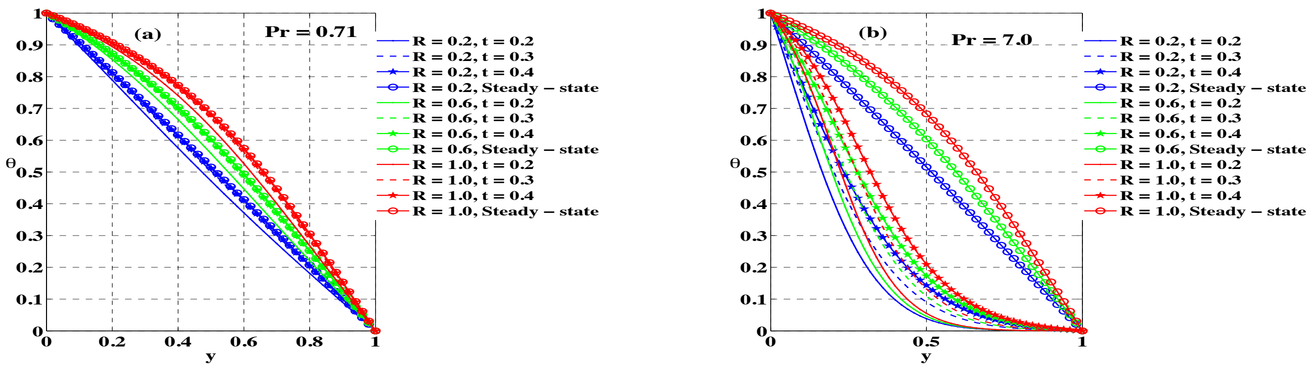


**Figure 7.** Effect of radiation parameter on temperature for fixed value of $C_T = 0.01$ .

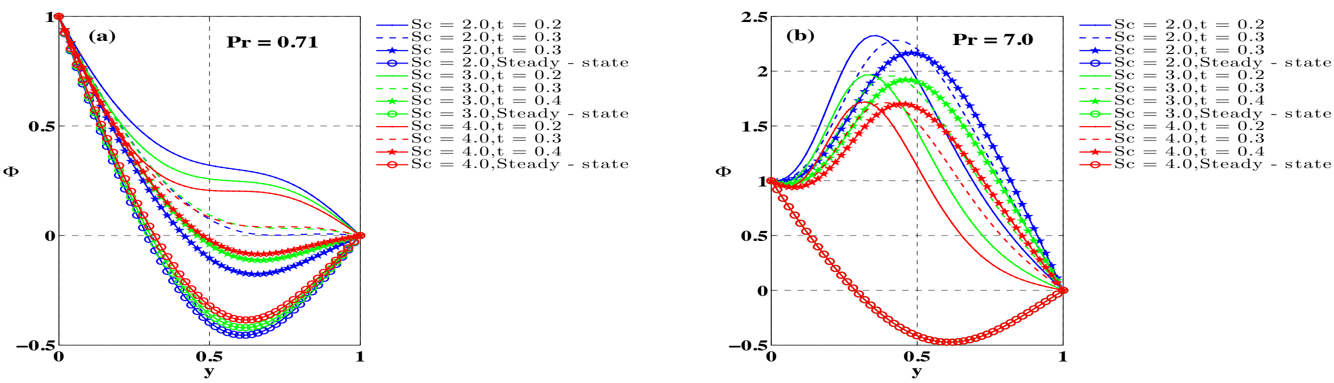


**Figure 8.** Effect of Schmidt number on concentration for fixed values of $C_T = 0.01, R = 1.0, S_t = 5.0$ .

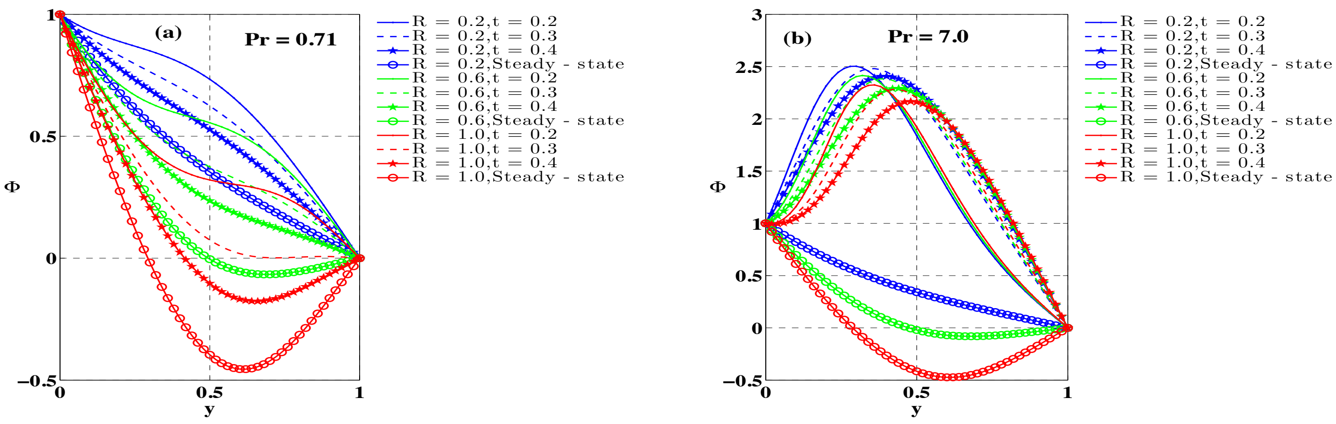


**Figure 9.** Effect of radiation parameter on concentration for fixed values of $C_T = 0.01$, $Sc = 2.0$, $S_t = 5.0$ .

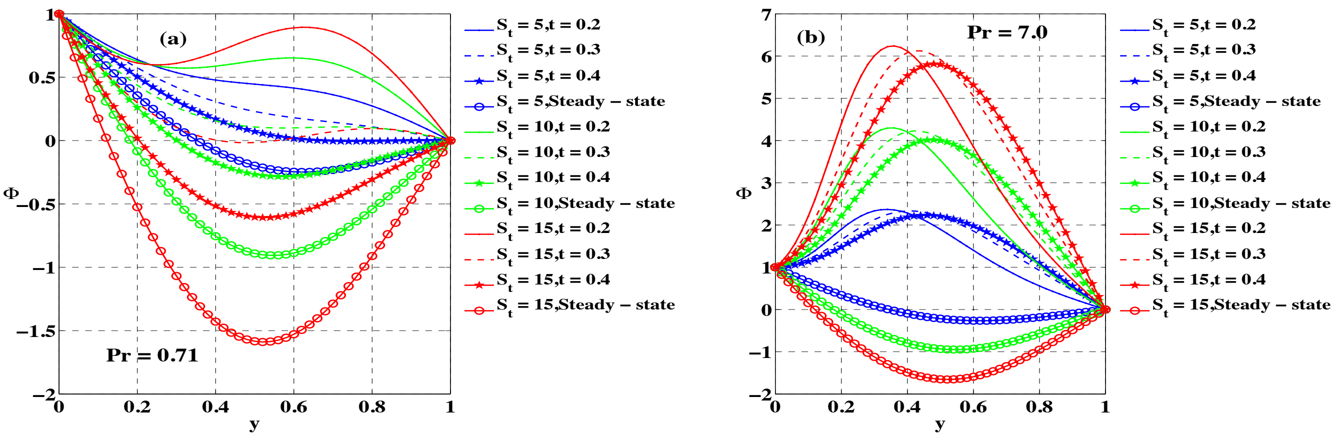


**Figure 10.** Effect of thermal-diffusion parameter on concentration for fixed values of $C_T = 0.01$, $R = 0.8$, $Sc = 2.0$

**Figures 8 – 10 explain the effects of increasing the Schmidt number $Sc$, thermal radiation $R$ and thermal–diffusion $S_t$. With all other parameters constant, increasing $Sc$ and $R$ decreases the concentration see Figures 8 and 9 respectively. On the other hand increasing $S_t$ enhances the concentration as illustrated in Figure 10. These Figures (8, 9 & 10) further shows that as time increases concentration decreases and finally attains its steady state values. Also in Figure 8 the numerical values of steady state concentration increases with increase of $Sc$ in case of air while the steady state concentration value is independent of $Sc$ when $Pr = 7.0$ see Figures 8a and b respectively. It is observed from Figures 9 and 10 that the steady state concentration decreases and strictly depend on the values of $R$ and $S_t$. It is therefore important to mention that for large Radiation parameter $(R)$ or thermal diffusion $(S_t)$ or both result to negative values of concentration at large time. This is due to the fact that at large time the value of $\partial^2\theta/\partial y^2$ appearing at the right hand side of equation (9) is negative and is multiplied with $S_t$.**

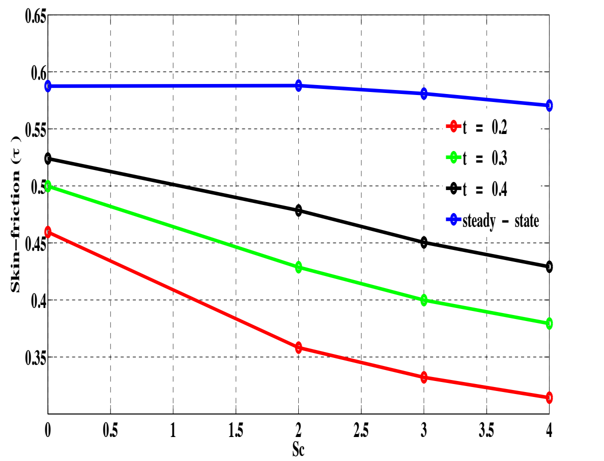


**Figure 11.** Skin friction against $Sc$ at y = 0 for fixed values of $C_T = 0.01$, $M = 1$, $N = 1$, $Pr = 0.71$, $R = 0.1$, $S_t = 0.2$ .

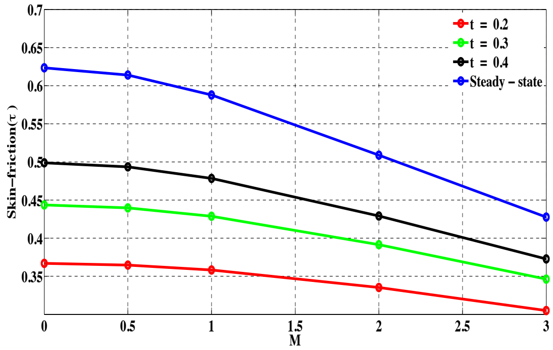


**Figure 12.** Skin friction against $M$ at y = 0 for fixed values of $C_T = 0.01$, $N = 1$, $Pr = 0.71$, $R = 0.1$, $Sc = 2.0$, $S_t = 0.2$ .

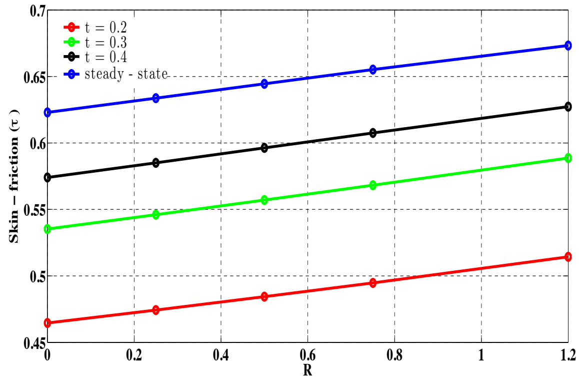


**Figure 13.** Skin friction against $R$ at y = 0 for fixed values of $C_T = 0.01$, $M = 1$, $N = 1$, $Pr = 0.71$, $Sc = 2.0$, $S_t = 0.2$ .

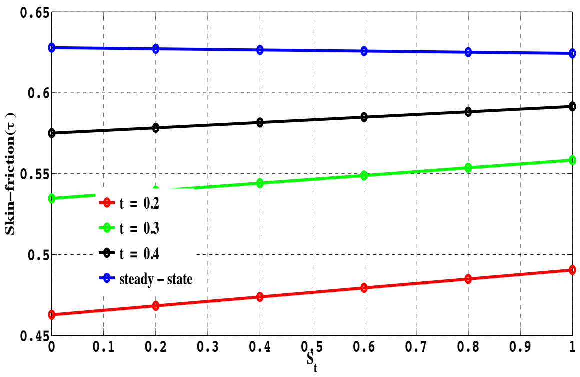


**Figure 14.** Skin friction against $S_t$ at y = 0 for fixed values of $C_T = 0.01$, $M = 1$, $N = 1$, $Pr = 0.71$, $R = 0.1$, $Sc = 2.0$ .

In Figures 11 through 16, the variation of skin friction for air $(Pr = 0.71)$ and water $(Pr = 7.0)$ at the plate $y = 0$ are narrated. Figure 11 and 12 depict the variation of skin friction with respect to $Sc, M$ and time. From the set out of these Figures it reflect that the skin friction increases with time and ultimately reaches its steady state value. It is observed from these two figures that as $Sc$ and $M$ increases the skin friction decreases.

Figures 13, 14, 15 and 16 illustrate in particular the increase in the values of skin friction with increasing $R, S_t, C_T, N$ and time t, when all other parameter values are fixed. See Figures 13 through 16 respectively.

Figures 17 and 18 reveal that as dimensional time increases Nusselt number decreases and finally achieved its steady–state value. Furthermore Nusselt number **increases with increase of $C_T$ (temperature difference) while the reverse is true for $R$.**

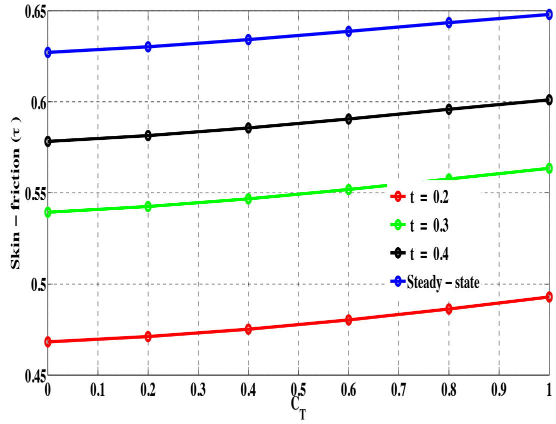


**Figure 15.** Skin friction against $C_T$ at y = 0 for fixed values of $M = 1$, $N = 1$, $Pr = 0.71$, $R = 0.1$, $Sc = 2.0$, $S_t$ .

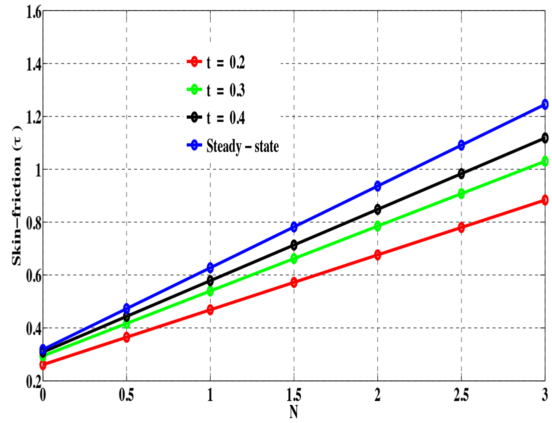


**Figure 16.** Skin friction against $N$ at y = 0 for fixed values of $C_T = 0.01$, $M = 1$, $Pr = 0.71$, $R = 0.1$, $Sc = 2.0$, $S_t = 0.2$ .

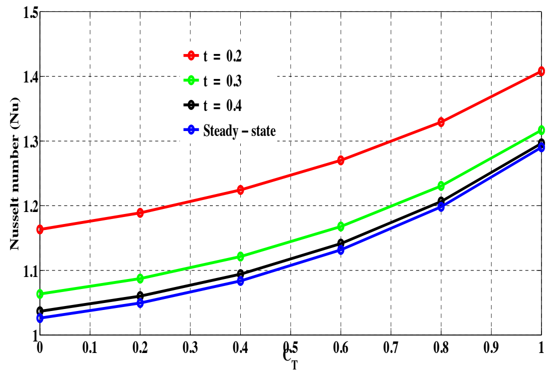


**Figure 17** Nusselt number against $C_T$ at y = 0 for fixed value of $R = 0.1$ .

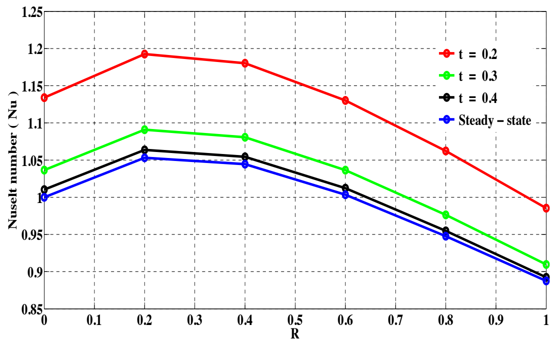


**Figure 18**. Nusselt number against R at y = 0 for fixed value of $C_T = 0.01$.

## RESULT VALIDATION

In order to verify the accuracy of our results, we have considered the analytical solutions (steady–state) obtained from equation (25) and that of semi-implicit finite difference equation (29). These computed results are tabulated in Table 1. It is interesting to observe from this Table 1 that the transient and steady state solutions results (under some limiting conditions) are in very good agreement at large value of time $(t)$, which clearly shows the correctness of our numerical (computed) scheme.

**Table 1.** Comparison of the numerical values of the transient velocity obtained using finite difference method and the steady state velocity obtained by perturbation method.

$C_T = 0.01, M = 1, N = 1, Sc = 2, S_t = 0.2, Pr = 7.0, R \ll 1$

| Transient state | Steady state | $\lvert error \rvert$ |
|---|---|---|
| 0 | 0 | 0 |
| **0.012124** | **0.012125** | $\mathbf{0.049347 \times 10^{-5}}$ |
| **0.023470** | **0.023471** | $\mathbf{0.140197 \times 10^{-5}}$ |
| **0.043905** | **0.043906** | $\mathbf{0.181774 \times 10^{-5}}$ |

## CONCLUSION

**The thermal radiation effects on the time dependent fully developed MHD free convective flow in a vertical channel in the presence of thermal diffusion effect has been** examined **in this treatise. The radiative heat flux term in the energy equation is simplified by using Rosseland approximation.** The time dependent **mathematical model relevant to the present physical situation under appropriate initial and boundary conditions** are solved numerically by employing a well known implicit finite difference method. The **steady state** version of **the physical situation has been solved** using perturbation method. **The influence of the various involved parameters** on the velocity, temperature and concentration fields are **shown and discussed. The** skin friction and Nusselt number are obtained and illustrated graphically. **The main findings are summarised as follows:**

i. An increase in $Sc$ and $M$ decreases the velocity field while an increase in $N, R$ and $S_t$ increases the velocity field.

ii. An increase in $R$ qualitatively enhances the temperature field.

iii. Velocity and temperature fields increases with dimensionless time parameter $t$.

iv. Concentration found to decrease with increase in $Sc, R$ and dimensionless time $t$ while $S_t$ increases the concentration during unsteady state and decreasing characteristic at steady state.

**ACKNOLEDGEMENT:** One of the authors Isah Bala Yabo is thankful to Usmanu Danfodiyo University, Sokoto for financial support.

## REFERENCES

[1] Raju, K, V. S., Sudhakar Reddy, T., Raju, M., C., Venkataramana, S. and Varma, S. V. K., Heat and mass transfer effects on unsteady free convection boundary layer flow past an impulsively started vertical surface with Newtonian heating, *International Journal of Scientific Research,* Vol. 2, No. 2, pp. 263 – 265.

[2] Mojtabi, M.C.C., Razi, Y.P., Maliwan, K. and Mojtabi, A., Influence of vibration on Soret–driven convection in porous media, *Numerical Heat Transfer. Part A: Applications*. Vol. 46, No. 10, 2004, pp. 981–993.

[3] Anjali Devi S. P. and Uma Devi R., Soret and Dufour effects on MHD slip flow with thermal radiation over a porous rotating infinite disk. *Communication Nonlinear Science and Numerical Simulation*. Vol. 16, 2011, pp. 1917–1930.

[4] Mahmud S., Tasnim S. H., Mamun M. A. H., Thermodynamic analysis of mixed convection in a channel with transverse hydromagnetic effect. *International Journal of Thermal Science*. Vol. 42, 2003, pp. 731–740.

[5] Abd El-Aziz, M., Thermal-diffusion and diffusion-thermo effects on combined heat and mass transfer by hydromagnetic three-dimensional free convection over a permeable stretching surface with radiation. *Physics Letters A*. Vol. 372, 2008, 263– 272.

[6] Joly, F., Vasseur, P. and Labrosse, G., Soret instability in a vertical Brinkman porous enclosure, *Numerical Heat Transfer. Part A: Applications*. Vol. 39, No. 4, 2001, pp. 339–359.

[7] Bourich, M., Hasnaoui, M., Amahmid, A., Er-Raki, M.,and Mamou, M., Analytical and numerical study of combined effects of a magnetic field and an external shear stress on Soret convection in a horizontal porous enclosure, *Numerical Heat Transfer, Part A: Applications*. Vol. 54 No.11, 2008, pp. 1042–1060.

[8] Lin, H.C., Char, M.I. and Chang, W.J., Soret effects on non-Fourier heat and non-Fickian mass diffusion transfer in a slab, *Numerical Heat Transfer. Part A: Applications*. Vol.

55, No. 12, 2009, pp.1096–1115.

[9] Jha, B.K. Samaila,A.K.and Ajibade, A.O., Diffusion-thermo effects on free convective heat and mass transfer flow in a vertical channel with symmetric boundary conditions, *Journal of Heat Transfer,* Vol. 133, No. 052502 2011, pp. 1 – 8.

[10] Kumar, R., Thermo diffusion and radiative heat effects on unsteady oscillatory mixed convection MHD flow through a planer channel filled with porous medium,*International Journal of Physical and Mathematical Sciences,* Vol. 3, No.1, 2012,13pages.

[11] Kesavaraya, D. C., Satyanarayana, P.V., and

Venkataramana S.V., Radiation and thermo-diffusion effects on mixed convective heat and mass transfer flow of a viscous dissipated fluid over a vertical surface in the presence of chemical reaction with heat source, *International Journal of Scientific Engineering and Technology,*Vol. 2 No. 2, 2013, pp. 56 –72.

[12] Sudhakar, K., Raju, R.S. and Rangamma, M., Chemical reaction effect on an unsteady MHD free convection flow past an infinite vertical accelerated plate with constant heat flux, thermal diffusion and diffusion thermo, *International Journal of Modern and Engineering Research,* Vol. 2, No. 5, 2012, pp. 3329 – 3339.

[13] Sharma B. K., Yadav, K., Mishra, N. K. and Chaudhary, R.C., Soret and Dufour effects on unsteady MHD mixed convectionflow past a radiative vertical porous plate embedded in a porous medium with chemical reaction. *Applied Mathematics*, Vol. 3 2012, pp.717 – 723.

[14] Sarada, K. and Shankar, B., The effects of Soret and Dufour on an unsteady MHD free convection flow past a vertical porous plate in the presence of suction or injection, *Research Inventy: International Journal of Engineering and Science,* Vol. 2, No. 7, 2013 pp. 13 – 25.

[15] Ahmed, N., Bhaattacharya, D.J. and Barua, D.P., Radiation, Soret and Dufour effects in MHD channel flow bounded by a long wavy wall and a uniformly moving parallel flat wall, *Journal of Applied Mathematics and Fluid Mechanics,* Vol. 6 No.1, 2014, pp. 1 – 19.

[16] Ali Agha, H. Bouaziz, M. N. and Hanini, S., Free convection boundary layer flow from a vertical flat plate embedded in a Darcy medium filled with a nanofluid: Effects of magnetic field and thermal radiation, *Arab Journal of Science Engineering* Vol. 39, 2014, 8331–8340.

[17] Makinde, O. D. and Chinyoka, T (2010). Numerical investigation of transient heat transfer to hydromagnetic channel flow with radiative heat and convective cooling, *Communication in Nonlinear Science and Numerical Simulation*, Vol.15, 2010, pp. 3919 – 3930.