Effect of Axial Wall Conduction and Ambient Heat-in-Leak on the Performance of a Two-Fluid Counter-Flow Cryogenic Heat Exchanger, Using Finite Element Method

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ABSTRACT

A two-fluid, single-pass, counter-flow, cryogenic heat exchanger is investigated for the effect of ambient heat-in-leak to both the fluids and axial conduction in the wall separating the two fluids. In most cryogenics applications, the performance of heat exchangers deteriorates significantly due to heat-in-leak from the ambient and axial wall conduction. Here a small, counter-flow heat exchanger, of the type used in Joule-Thomson refrigerators, experiencing heat-in-leak to both hot & cold fluids, and axial wall conduction, is analyzed adopting the finite element method. Five non-dimensional parameters, including those to account for ambient heat-in-leak and axial wall conduction are defined to present the result. The effect of these parameters on the heat exchanger performance has been analyzed.

The set of non-dimensional governing equations for hot fluid, cold fluid and the wall are obtained by energy balance. These governing equations are solved by FEM using Galerkin's method. Validation is carried out by comparing the results obtained using the present methodology with those published in the literature for limited parameters. The excellent match between the two validates the solution methodology used.

The effect of ambient heat-in-leak and axial wall conduction are studied for their effect on the fluid temperature profiles. Ambient heat-in-leak and axial wall conduction are found to increase the hot fluid exit temperature. The effect of axial wall conduction is found to be more concentrated towards the heat exchanger ends. Further, the effect of ambient heat-in-leak and axial wall conduction and varying non-dimensional parameters are studied for their effect on the hot fluid effectiveness and performance degradation.

INTRODUCTION

Counter-flow heat exchangers are commonly used due to their high effectiveness for cryogenic applications in Joule-Thomson refrigerators, systems that deal with ammonia gas synthesis, purification and liquefaction of hydrogen and helium and others. The performance of these heat exchangers is affected by various irreversibilities such as axial wall conduction, heat-in-leak from the surroundings, flow mal-distribution, property variations in fluids etc; which are often ignored while analyzing heat exchangers working in the normal temperature range.

The effect of axial wall conduction on the performance of high effectiveness heat exchangers has been studied by Kroeger [1]. He has obtained the governing differential equations describing heat exchange between fluids and wall for counter-flow heat exchanger in non-dimensional form. He has considered the basic assumption of constant material properties, and no thermal interaction between heat exchanger and ambient and has solved the equations for both balanced and imbalanced flow. His results show that the axial wall conduction has a pronounced effect on heat exchanger ineffectiveness. The effect of ambient heat-in-leak to both fluid streams separately for a counter-flow heat exchanger has been interpreted by Barron [2]. He has obtained the mathematical relationship to describe the non-dimensional temperature profile and heat transfer rate. His results show that, the performance of a counter-flow heat exchanger is affected significantly due to heat-in-leak from the ambient. The effect of external heat transfer to a double pipe heat exchanger has been presented by Chowdary and Sarangi [3]. They have focused more on the design aspect of the heat exchanger under external heating condition and have presented a graphical relationship between design NTU (number of transfer units) and effective NTU. The two NTU values are found to be equal when there is no heat-in-leak. The performance of a counter-flow heat exchanger with heat loss through the wall at the cold end has been studied by Venkataratnam and Narayanan [4]. They have presented a model which aims towards deriving an expression relating the parasitic heat conducted at the cold end of the heat exchanger, NTU and axial wall conduction. Their results show that the hot fluid exit temperature approaches zero as the value of NTU approaches infinity, irrespective of heat capacity ratio, while the outlet temperature of cold fluid and heat-in-leak at the cold end of the wall remain finite. The performance of counter-flow heat exchangers considering the effect of axial wall conduction and heat-in-leak has been investigated by Gupta and Atrey [5]. Their model considers heat-in-leak only to the cold fluid, and the governing equations have been developed and solved using finite difference method. They have concluded that the degradation is maximum for the balanced flow condition and there is higher degradation due to heat-in-leak at higher values of NTU. A heat exchanger model that includes the effect of axial conduction, parasitic heat loads and property variations has been studied by Nellis [6]. In his paper, he has presented a numerical model of the heat exchanger in which these effects are explicitly modeled. He has derived the governing finite difference equations, made dimensionless, and solved. They have used grid distribution concept for better results. A paper on the effect of ambient heat-in-leak on the performance of a three fluid heat exchanger with three thermal communication, using finite element method have been presented by Krishna et al [7]. They have developed the stiffness and loading matrices for obtaining the solution using FEM (finite element method). In another paper [8], they have adopted a similar approach to study the effect of axial wall conduction on the performance of a three fluid heat exchanger. A mathematical model on the performance of a counter-current heat exchanger in which both fluids are subjected to external heating has been studied by Ameel and HewaVitharana [9]. They describe a drastic decrease in the effectiveness of heat exchanger due to external heating and conclude that an increase in the ambient condition reduces the effectiveness of heat exchanger to produce significant temperature drop of hot fluid. They have concluded that the environmental temperature and thermal conductance ratios are important design parameters under conditions of adverse heat transfer. For effective heat exchanger operation, it is desired to have small conductance ratios, high values of NTU and low values of ambient parameter.

However, the performance of a two fluid counter-flow heat exchanger experiencing ambient heat-in-leak to both fluid streams as well as axial wall conduction has not been studied earlier. This has been studied in the present paper using finite element methodology.

NOMENCLATURE

x	Axial co-ordinate (m)
L	Heat exchanger length (m)
X	Non-dimensional axial co-ordinate as defined in Eq.(4)
$\dot{m}$	Mass flow rate (kg/s)
$\dot{Q}$	Heat transfer rate (W)
C_p	Specific heat at constant pressure (J/kg-K)
T	Temperature (K)
C	Heat capacity rate of the fluids define by the product of $\dot{m}$ and C_p
P	Wetted perimeter for any contact area(m)
A	Surface area for heat transfer as defined by the product of P and L (m^2)
U	Overall heat transfer coefficient (W/m^2-K)
K	Thermal conductivity of the pipe material (W/m-K)
R	Ratio of heat capacity rates as defined in Eq. (4)
NTU	Overall number of transfer units as defined in Eq. (5)
n	Local NTU for each fluid in contact with a specified wall as defined in Eq. (6)
N1 & N2	Shape functions as defined in Eq. (10)

Special Characters

α	Heat-in-leak parameter as defined in Eq. (5)
θ	Dimensionless temperature as defined in Eq. (4)
λ	Longitudinal wall conduction parameter as defined in Eq (6)
ε	Hot fluid thermal effectiveness as defined in Eq. (14 & 15)
τ	Degradation factor as defined in Eq. (16)

Subscript

c	Cold fluid
h	Hot fluid
w	Wall between a pair of fluids as illustrated in Fig. (1)
in	Inlet
out	Outlet

Model formulation

A two-fluid, single-pass and counter-flow heat exchanger has been considered. The system configuration of the heat exchanger is shown in Fig.1. Each of the fluids interacts with each other as well as with the ambient. The effect of the ambient will be a heat leak into the fluids, as the heat exchanger is considered for a cryogenic application.

Fig. 1 shows the schematic diagram of two fluid counter-flow heat exchanger when both the fluids are subjected to external heating along with longitudinal heat conduction across the wall.

The following assumptions have been made for the analysis: (i) The heat exchanger is in a steady state. (ii) All properties are constant with time and space. (iii) Within a stream the temperature distribution is uniform in the transverse direction and equal to the average temperature of the fluid. (iv) There are no phase changes in the fluid streams. (v) There is no heat source or sink in any of the fluids. (vi) The heat transfer area is constant along the length of the heat exchanger. (vii) The temperatures of fluid streams vary only in the longitudinal direction. (viii) Work done by the flow is negligible. (ix) Ambient temperature and external conductance from both the fluids are constant.

The governing equations for hot fluid, cold fluid and the wall, obtained by the energy balance, are as follows:

Hot fluid:

$$\frac{d\theta_h}{dX} + R\, n_h (\theta_h - \theta_w) - NTU\, C\, \alpha_h (\theta_o - \theta_h) = 0 \quad (1)$$

Cold fluid:

$$\frac{d\theta_c}{dX} + n_c (\theta_w - \theta_c) + NTU\, \alpha_c (\theta_o - \theta_c) = 0 \quad (2)$$

Wall:

$$R\, \lambda \frac{d^2\theta_w}{dX^2} + n_h (\theta_h - \theta_w) - n_c (\theta_w - \theta_c) = 0 \quad (3)$$

In the above expression θ_h, θ_w and θ_c represent the dimensionless temperature of hot fluid, wall and cold fluid respectively. The following dimensionless parameters have been defined for analysis:

$$\theta = \frac{T - T_{c,in}}{T_{h,in} - T_{c,in}} \qquad X = \frac{x}{L} \qquad R = \frac{C_{min}}{C_{max}} = \frac{C_c}{C_h} \quad (4)$$

$$NTU = \frac{UA}{C_{min}} \qquad \alpha_c = \frac{U_c A_c}{UA} \qquad \alpha_h = \frac{U_h A_h}{UA} \quad (5)$$

$$n_h = \left(\frac{hA}{C}\right)_h \qquad n_c = \left(\frac{hA}{C}\right)_c \qquad \lambda = \frac{K A_c}{C_{min} L} \quad (6)$$

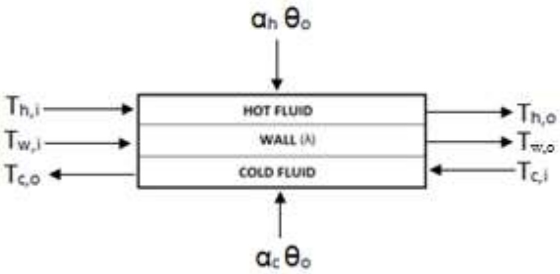

Fig. 1. Schematic diagram of counter-flow two-fluid heat exchanger chosen for analysis with axial wall conduction and ambient heat-in-leak to both the fluids

Finite element method

The heat exchanger is divided into number of small elements. A linear variation of temperature is assumed for hot and cold fluids and wall in a single element. The non-dimensional temperatures (θ_h, θ_c & θ_w) at any section for counter-flow are given by:

$$\theta_h=N_1\theta_{h,in}+N_2\theta_{h,out} \quad (7)$$
$$\theta_c=N_2\theta_{c,in}+N_1\theta_{c,out} \quad (8)$$
$$\theta_w=N_1\theta_{w,in}+N_2\theta_{w,out} \quad (9)$$

Where, N_1 and N_2 are the shape functions and are given by:

$$N_1=1-X, \text{ and } N_2=X \quad (10)$$

Using Galerkin's method of minimizing the weighted residual [6], the governing equations are reduced to a set of algebraic linear equations. These discretized governing equations are written in matrix form for each element as:

$$[K]\ \{\theta\} = \{F\} \quad (11)$$

Where, [K] is known as stiffness matrix and is a (6x6) matrix for each element; {θ} is a non-dimensional temperature vector, and {F} is the load vector.
The stiffness matrix is assembled for all the elements to get the globalized stiffness matrix, and then the boundary conditions are applied. The systems of equation are solved by MATLAB, to get the non-dimensional temperatures along the length of the HX.

Boundary conditions

Assuming that the two ends of the walls are adiabatic, the following boundary conditions are applied to the globalized stiffness matrix:

$$X = 0: \quad \theta_h = 1, \quad \frac{\partial\theta_w}{\partial X} = 0 \quad (12)$$

$$X = 1: \quad \theta_c = 0, \quad \frac{\partial\theta_w}{\partial X}=0 \quad (13)$$

Effectiveness

The effectiveness of any heat exchanger is the ratio of actual heat transfer to the maximum possible heat transfer. Due to various losses such as heat-in-leak from ambient, axial wall conduction, etc; the cold fluid does not absorb all heat transferred by the hot fluid. Hence effectiveness of hot fluid is different from cold fluid. In this paper, effectiveness is based on the hot fluid and is expressed mathematically as:

$$\varepsilon_h = \frac{q_{hot}}{q_{max}} = \frac{C_h(T_{h,in}-T_{h,out})}{C_{min}(T_{h,in}-T_{c,in})} \quad (14)$$

The above expression can be expressed in terms of dimensionless parameter as:

$$\varepsilon_h = \frac{(1-\theta_{h,out})}{R} \quad (15)$$

Degradation factor

Degradation factor, τ, is defined to evaluate the extent of deterioration in the performance of the heat exchanger due to ambient heat-in-leak and axial wall conduction. As stated by Gupta and Atrey [5], the degradation factor for the hot fluid is defined as the ratio of the loss in thermal effectiveness due to ambient heat-in-leak and axial wall conduction under no loss conditions and is given by :

$$\tau_h = \frac{\Delta\varepsilon}{\varepsilon} = \frac{\varepsilon_{NC,NHL} - \varepsilon_{WC,WHL}}{\varepsilon_{NC,NHL}} \quad (16)$$

Where, NC-no conduction, WC-with conduction, NHL-no heat leak, WHL-with heat leak.

Validation of the present methodology

In the present paper, a counter-flow heat exchanger experiencing axial wall conduction and heat-in-leak to both hot & cold fluids is analyzed, adopting the finite element method. Five non-dimensional parameters, including those to account for ambient heat-in-leak and axial wall conduction have been identified. The effect of these parameters on the heat exchanger performance has been analyzed.

The methodology has been validated by comparison with a similar model on a counter-flow heat exchanger reported previously by Ameel and Hewavitharana [9]. They have studied a counter-flow heat exchanger in which both the fluids are subjected to external heating. However they have not considered the effect of axial wall conduction. The present model has been compared with their model by neglecting the effect of axial wall conduction. The comparison has been shown in Fig.2. From the two figures it is observed that the results obtained from the present model in this paper, have compared very well with those presented by Ameel and Hewavitharana[9].

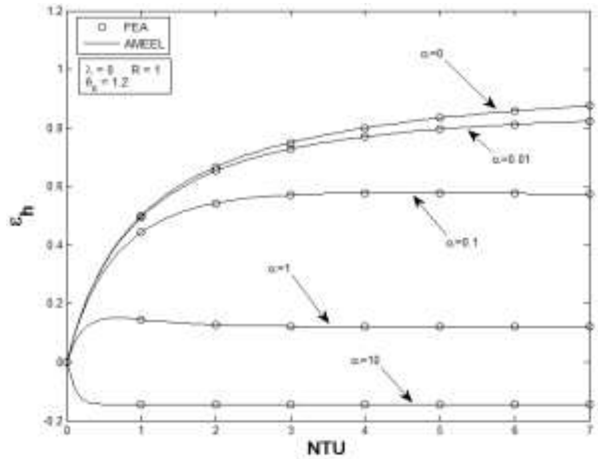

Fig.2 variation of hot fluid effectiveness with NTU. Comparison of present values (FEM values) with Ameel and Hewavitharina values [9]. Values of other non-dimensional parameters: $\lambda = 0$, θ_o=1.2, NTU = 2

Result and Discussion

Temperatures profiles

The effect of ambient heat-in-leak and axial wall conduction on the temperature distributions of the hot and cold fluids are shown in Figs.3a - 3c, for the balanced flow (R=1) condition. The effect of ambient heat-in-leak alone is shown in Fig. 3a, while the effect of axial wall conduction is shown in Fig. 3b. The combined effect is shown in Fig. 3c.

It is noticed from Fig. 3a that the exit temperatures of both the fluids are significantly increased due to ambient heat-in-leak. The hot fluid exit temperature is enhanced by 11.64%, while the cold fluid exit temperature is enhanced by 1.73% for α = 0.005. It is observed that the temperature enhancement is more at the mid-span of the heat exchanger than at the exit. As the value of α increase the exit temperatures of both hot and cold fluids increase.

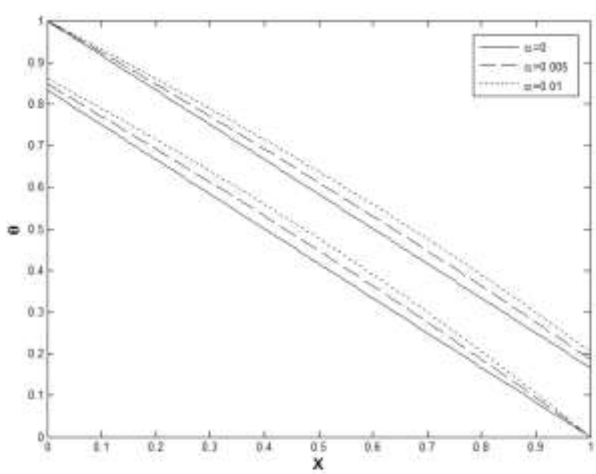

Fig.3a Hot and cold fluid non-dimensional temperature distribution within the heat exchanger for NTU = 5, θ_0 = 1.2, λ = 0 and R = 1

Fig.3b shows the effect of axial wall conduction independently on the temperature distribution within the heat exchanger. The hot fluid exit temperature is enhanced by 14.58%, while the cold fluid exit temperature is reduced by 1.73% for λ = 0.04. It is observed that the gradient of the temperature profiles are altered due to axial wall conduction. From the Fig. 3b it is seen that the hot fluid temperature distribution suffers a reduction in temperature due to axial conduction at the inlet end and an increase at the exit end, with a point near the mid-span of the heat exchanger where there is a cross over. At this point it is observed that the effect of axial wall conduction is zero. The reason for this is that increased wall conduction results in more amount of heat being transferred from the hot fluid at the inlet and hence the temperature gradient reduces. But towards the downstream side, the heat transferred from the hot fluid goes on decreasing and the temperature of the hot fluid gets enhanced. The cold fluid suffers the exact opposite effect. An increase in axial wall conduction enhances its temperature at the inlet due to increased heat received from the hot fluid and a reduction in its temperature towards the exit.

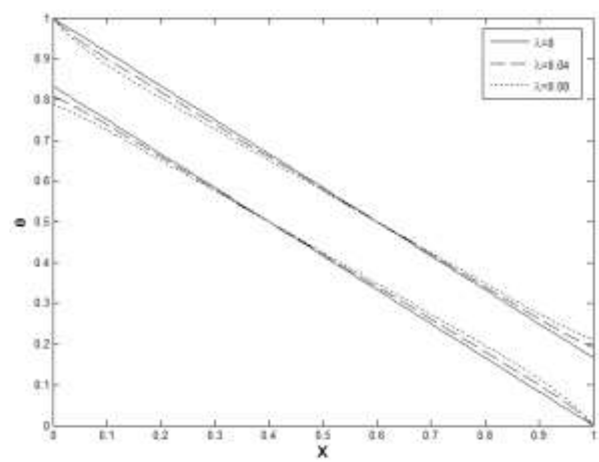

Fig.3b Hot and cold fluid non-dimensional temperature distribution within the heat exchanger for NTU = 5, θ_0 = 1.2, α = 0 and R = 1

Fig.3c shows the combined effect of axial wall conduction and ambient heat-in-leak on the temperature distribution within the heat exchanger. As the values of λ and α increase the exit temperature of hot fluid increases significantly, where as cold fluid exit temperature decreases. The hot fluid exit temperature is enhanced by 11.78%, while the cold fluid exit temperature is reduced by 2.75% for λ = 0.04 and α=0.01. The temperature profile is almost linear for λ = 0.

From the Fig. 3c it is seen that the hot fluid temperature distribution suffers a reduction at the inlet end and an increase at the exit end where as cold fluid temperature distribution suffers an increase at the inlet end and reduces at the exit end.

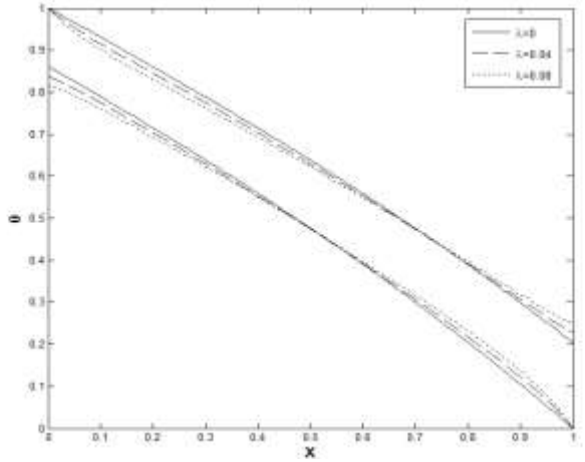

Fig.3c Hot and cold fluid non-dimensional temperature distribution within the heat exchanger for NTU = 5, θ_0 = 1.2, α = 0.01 and R = 1

Effect of NTU

The effect of NTU on degradation factor τ_h is shown in fig. 4a. It shows the contribution of λ and θ_o individually as well as in a combined manner on the degradation factor (τ_h). It is observed that, τ_h increases with NTU, for all values of θ_o and λ. The τ_h is increased by 305% when θ_o increased to 4.67 from 1 at NTU = 12 and λ = 0.04. For given value of θ_o (say θ_o = 1) the τ_h is increased by 88.67 % for λ = 0.04 and NTU = 12. Generally the percentage increase in τ_h decreases as the value of NTU increases as shown in Table 4a. For the given value of θ_o percentage increase in the τ_h increases as the value of λ increases. This shows that the performance of heat exchanger degrades more at higher values of θ_o , λ and NTU.

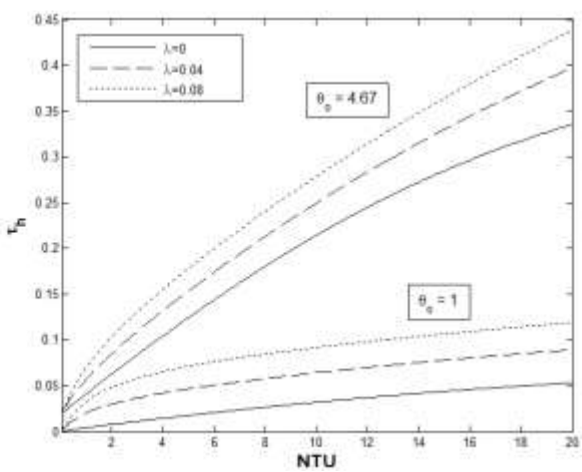

Fig.4a Effect of NTU on the degradation factor for α = 0.005 and R = 1

The variation of hot fluid effectiveness ε_h with NTU is shown in Fig. 4b. It is observed that for θ_o = 1 and for lower values of NTU (upto NTU = 6) hot fluid effectiveness increases with NTU. For higher values of NTU the effectiveness almost remains constant. For higher values of θ_o (say θ_o = 4.67) effectiveness increases initially with NTU (upto NTU = 5) but for higher values of NTU the effectiveness decreases with NTU. Table 4b shows the percentage decrease in effectiveness due to increases in θ_o and λ. The hot fluid effectiveness decreases with increase in the axial wall conduction parameter λ. As increase in λ leads to decrease in thermal interaction between the hot and the cold fluids due to which the outlet temperature of the hot fluid increases. Hence the effectiveness decreases. For higher values of θ_o the thermal interaction between the cold fluid and ambient is more. Consequently the heat transfer from hot fluid to cold fluid is reduced leading to increase in the hot fluid exit temperature. Thus the hot fluid effectiveness decreases.

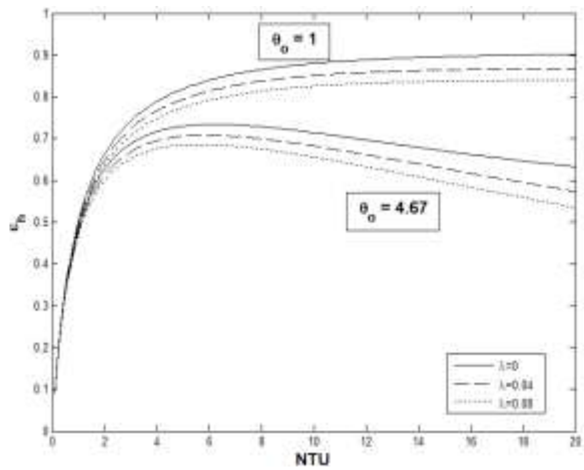

Fig.4b Effect of NTU on the hot fluid effectiveness for α = 0.005 and R = 1

CONCLUSION

A two-fluid counter-flow cryogenic heat exchanger is investigated for the effect of axial wall conduction and ambient heat-in-leak to both the fluids, using finite element method.

The following conclusion can be drawn based on the results presented above.

(i) Various factors such as heat-in-leak and longitudinal conduction influences the performance of the heat exchanger apart from the operational parameters.

(ii) The effect of ambient heat-in-leak and longitudinal wall conduction is to increase the hot fluid exit temperature. Hence the hot fluid effectiveness decreases and thereby increasing the degradation factor, with increases in λ, α_h and α_c.

(iii) The temperature enhancement is more at the mid-span of the heat exchanger than at the exit due to ambient heat-in-leak.

(iv) It is observed from the graphs that the gradient of the temperature profiles are altered due to axial wall conduction. The hot fluid temperature distribution suffers a reduction in temperature due to axial conduction at the inlet end and an increase at the exit end, with a point near the mid-span of the heat exchanger where there is a cross over. Such cross over occurs because of the fact that increased wall conduction results in more amount of heat is transferred from hot fluid at the inlet, but at outlet heat transfer from hot fluid get reduced and hence the temperature of hot fluid gets enhanced.

(v) The combined effect of axial wall conduction and ambient heat-in-leak significantly increases the hot fluid exit temperature, whereas cold fluid exit temperature decreases.
(vi) For higher values of θ_o the thermal interaction between the cold fluid and ambient is more. Consequently the heat transfer from hot fluid to cold fluid is reduced leading to increase in the hot fluid exit temperature. Thus the hot fluid effectiveness decreases.
(vii) For lower values of NTU the effectiveness of hot fluid increases, but at higher values it becomes constant.
The validation of the finite element methodology by comparing with the previous published results shows the versatility of this methodology.

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Details of the matrices in Eq. (11) for Galerkin's method :

Appendix

$$[K] = \begin{bmatrix} (-\frac{1}{2}+\frac{Cn_h}{3}+\frac{NTU.C\alpha_h}{3}) & (-\frac{Cn_h}{3}) & 0 & (\frac{1}{2}+\frac{Cn_h}{6}+\frac{NTU.C\alpha_h}{6}) & (-\frac{Cn_h}{6}) & 0 \\ (\frac{n_h}{3}) & (-C\lambda-\frac{n_h}{3}-\frac{n_c}{3}) & (\frac{n_c}{3}) & (\frac{n_h}{6}) & (C\lambda-\frac{n_h}{6}-\frac{n_c}{6}) & (\frac{n_c}{6}) \\ 0 & (\frac{n_c}{3}) & (-\frac{1}{2}-\frac{n_c}{3}-\frac{NTU.\alpha_c}{3}) & 0 & (\frac{n_c}{6}) & (\frac{1}{2}-\frac{n_c}{6}-\frac{NTU.\alpha_c}{6}) \\ (-\frac{1}{2}+\frac{Cn_h}{6}+\frac{NTU.C\alpha_h}{6}) & (-\frac{Cn_h}{6}) & 0 & (\frac{1}{2}+\frac{Cn_h}{3}+\frac{NTU.C\alpha_h}{3}) & (-\frac{Cn_h}{3}) & 0 \\ (\frac{n_h}{6}) & (C\lambda-\frac{n_h}{6}-\frac{n_c}{6}) & (\frac{n_c}{6}) & (\frac{n_h}{3}) & (-C\lambda-\frac{n_h}{3}-\frac{n_c}{3}) & (\frac{n_c}{3}) \\ 0 & (\frac{n_c}{6}) & (-\frac{1}{2}-\frac{n_c}{6}-\frac{NTU.\alpha_c}{6}) & 0 & (\frac{n_c}{3}) & (\frac{1}{2}-\frac{n_c}{3}-\frac{NTU.\alpha_c}{3}) \end{bmatrix}$$

$$\{\theta\} = \begin{bmatrix} \theta_{h,in} \\ \theta_{w,in} \\ \theta_{c,out} \\ \theta_{h,out} \\ \theta_{w,out} \\ \theta_{c,in} \end{bmatrix} \qquad \{F\} = \begin{bmatrix} \frac{NTU.C\alpha_h\theta_o}{2} \\ 0 \\ -\frac{NTU.\alpha_c\theta_o}{2} \\ \frac{NTU.C\alpha_h\theta_o}{2} \\ 0 \\ -\frac{NTU.\alpha_c\theta_o}{2} \end{bmatrix}$$

Table.4A	θ_O =1			θ_O =4.67		
NTU	λ =0	λ =0.04	λ =0.08	λ =0	λ =0.04	λ =0.08
4	0.0144	0.0420	0.0653	0.1040	0.1317	0.1551
8	0.0265	0.0578	0.0845	0.1805	0.2131	0.2406
12	0.0371	0.0700	0.0981	0.2450	0.2835	0.3144
16	0.0459	0.0804	0.1093	0.2964	0.3445	0.3800
20	0.0531	0.0893	0.1191	0.3356	0.3974	0.4386
		% increase			% increase	
		191.67	353.47		26.63	49.13
		121.51	218.86		18.06	33.29
		88.67	164.42		15.71	28.32
		75.16	138.12		16.22	28.20
		68.17	124.29		18.41	30.69

Table.4B	θ_O =1			θ_O =4.67		
NTU	λ =0	λ =0.04	λ =0.08	λ =0	λ =0.04	λ =0.08
4	0.7885	0.7664	0.7478	0.7168	0.6946	0.6759
8	0.8653	0.8375	0.8138	0.7285	0.6995	0.6751
12	0.8889	0.8584	0.8325	0.6969	0.6614	0.6329
16	0.8980	0.8655	0.8383	0.6622	0.6169	0.5836
20	0.9018	0.8673	0.8389	0.6327	0.5739	0.5346
		% decrease			% decrease	
		2.80	5.16		3.09	5.70
		3.21	5.95		3.98	7.33
		3.43	6.34		5.09	9.18
		3.62	6.65		6.84	11.86
		3.82	6.97		9.29	15.50