MODULAR PREDICTION OF HEAT TRANSFER UNDER FREE-JETS: SINGLE JET, JET ARRAY, AND THE INFLUENCE OF GRAVITY

Haustein H.D.
School of Mechanical Engineering,
Faculty of Engineering
Tel Aviv University,
Tel Aviv, 69978,
Israel,
E-mail: hermanh@post.tau.ac.il

ABSTRACT

The present study develops the ground work for modular prediction of free-surface jet arrays. Jet arrays generate one of the highest single-phase heat transfer rates, while covering reasonably large areas with good thermal uniformity, relevant to electronics cooling. However, due to liquid evacuation problems, free-jet arrays suffer from flooding, cross-flow and jet interaction, together with the large amount of influencing geometrical parameters, this them very difficult to predict.

For the modular prediction approach to be applied, key issues are here addressed: experiments were conducted employing de-ionized water in both single and basic multiple-jet array (2x2, with local liquid extraction in the jet interaction zones) configurations. Modular conditions, wherein all jets are similar to each other, were created experimentally in a consistent fashion, by use of liquid extraction in the jet-interaction zones. Based on present and previous experimental data the influencing parameters on the pre-jump depth were identified. This description was then used to predict the location of the hydraulic jump (as dependant on the measured post-jump depth). The model combines elements of two previous approaches the shallow-water vs. jump conservation model, and obtains good agreement with available data. In addition conditions were shown for maximizing the distance at which the hydraulic jump occurs - to the point that the supercritical flows of adjacent jets touch (standing fountain type jump). This not only permits prediction of the supercritical flow heat transfer distribution over almost the entire array area, but also reduces the low heat transfer post-jump regions to a minimum. Finally, a more universal single-jet heat transfer model was developed incorporating inherent self-similarities recently identified by the authors and considering all relevant parameters: jet velocity profiles, nozzle-plate spacing, and inclination relative to gravity, to predict stagnation heat transfer as well as its radial decay. It is further identified that the influence of inclination is also of vital importance to free-surface jets (breakage of symmetry) and must be examined in future studies.

By addressing these three key issues the foundation for a modular prediction of heat transfer under a free jet array is laid.

INTRODUCTION

The use of micro-scale jet arrays has become increasingly common for the cooling of electronics equipment, as power densities increase. This is due to it being shown to have several advantages with regards to micro-channels and sprays [1 - 3]. While a large number of studies have demonstrated their successful performance and use [4], their physics remains unclear and their heat transfer prediction varies widely (see Tables in [5, 6]. This is partially due to the large amount of defining parameters of these types of flows, even when only flat-plate of the cooled surfaces are considered. Important parameters of the problem are: jet surroundings (free-surface, submerged, confined, etc.), type of spent liquid extraction (local or distant), and additional geometrical parameters. To simplify the problem and improve understanding the free-surface jet impingement is often preferable as optical access (from above) is improved, boundary conditions are simplified (free-surface) and flow features can be more clearly visible (such as the hydraulic jump). Therein the problem is still rather complex, being described by multiple geometrical (nozzle shape – exit velocity profile, nozzle diameter, nozzle to plate spacing, nozzle to nozzle spacing, heater size – total jets used), flow and liquid parameters (Reynolds number, Prandtl number, Weber number – free jets). Previous parametric studies of multiple free jet impingement [7 - 12] typically found only a weak influence of the nozzle to plate spacing, with a stronger dependency on flow rate (Re) and jet-to-jet spacing (*S*/*d*). Unfortunately, these studies often employed systems with a large numbers of orifice-type jets (drilled in a plate of the scale of jet diameter). This meant that developed flow conditions were not present (i.e. the criterion of L/d•Re should be larger than 0.09), and the exit velocity profile could not be accurately known to allow comparison between studies. Recent numerical studies by the authors have shown the importance of the nozzle-exit velocity profile on the heat transfer distribution in both submerged and free-surface single-jet impingement [13,14]. Consequently, varying trends have been found in the literature:

1. While most studies have found no real dependence of average heat transfer coefficient on nozzle-to-plate spacing, a few have found a strong dependence [15]. In addition, previous studies usually observed flooding of the free-jet configuration in the case of nozzle-plate spacings below 5 diameters (see [7] & [16]). So application-relevant close jet-wall spacing was not examined. This raises the additional consideration of the quality of the drainage provided (and influence of cross-flow) for these jet-arrays systems – which will be addressed in the following section.

2. Conversely, most studies have found that jet-to-jet spacing is of significant importance, though their recommendations differ (20 diameters in [3,5] vs. around 6 diameters in [17]). Whereas, ref. [15] found no significant influence of this parameter.
3. Recent work with larger heaters (ref. [5]), has shown that this parameter is also of importance. Their values varied greatly from those obtained by another recent study [17] employing a heater with an area of 1 mm^2. Concerns regarding the accuracy of results obtained on very-small heaters must be closely examined.
4. These findings clearly raise the question if at these small-scales the performance is dependent on other, edge-effects (flow development, quality of nozzle, etc.). Indeed, an early study on single free-jets [18] found strong nozzle-size dependence below a diameter of 1mm, which they attributed to jet-widening occurring from liquid-nozzle edge interaction at the exit. Similarly, it was found [16] that jet-to-jet interaction may occur before impingement at close jet spacing (S/d<4), which can be explained by jet widening.

More importantly, these findings raise the question of which parameters are truly influential to this problem, and which dimensionless parameters should be the basis for comparison of different experimental studies.

MODULAR PREDICTION APPROACH

In the study presented here, a method of modular prediction of heat transfer for single and multiple free-surface jets is presented, as depicted schematically in Figure 1. This method is based on three pillars: i) Generation of an array where each jet is similar to the next (and each can be modeled by the same building element – outlined by the black dashed line in Fig. 1a) ; ii) as this element contains the hydraulic jump - successful prediction of the location of the hydraulic jump (also under array conditions); and iii) As the hydraulic jump does not allow information to travel upstream (Shock) a suitable single-jet heat transfer distribution model can be used up to it. In other words, the entire influence of an adjacent jet is to shift the location of the hydraulic jump, but not to influence the (supercritical) flow upstream of it. As a final step, there is the need to "stitch" together the prediction of the heat transfer under each individual jet in order to obtain the entire field – a simple mathematical procedure not dealt with here.

Researchers have studied the hydraulic jump ever since the early work of Rayleigh [19], though previous attempts at predicting its location have had limited in success [20, 21]. As a flow-instability (relaxation from supercritical flow) it is very susceptible to influence of initial and system perturbation – leading to a reasonably wide spread of data. The hydraulic jump has been modeled as a shock (wave), a river bore, a standing wave and as a black hole [19, 22, 23, ,24], employing viscous-inviscid flow, as well as shallow water, gas and boundary layer theory [25]. We shall take a closer look at some of these predictions (Bohr et al.[26], Liu & Lienhard [27]) as a foundation to the modelling undertaken here.

While several previous correlations exist for a single free-jet, both analytically based [28] and empirically based [29,30], recent work by the authors has shown additional self-similarity in free-surface jet impingement [14]. This self-similarity is here used to develop an improved model – suitable for various jet velocity profiles.

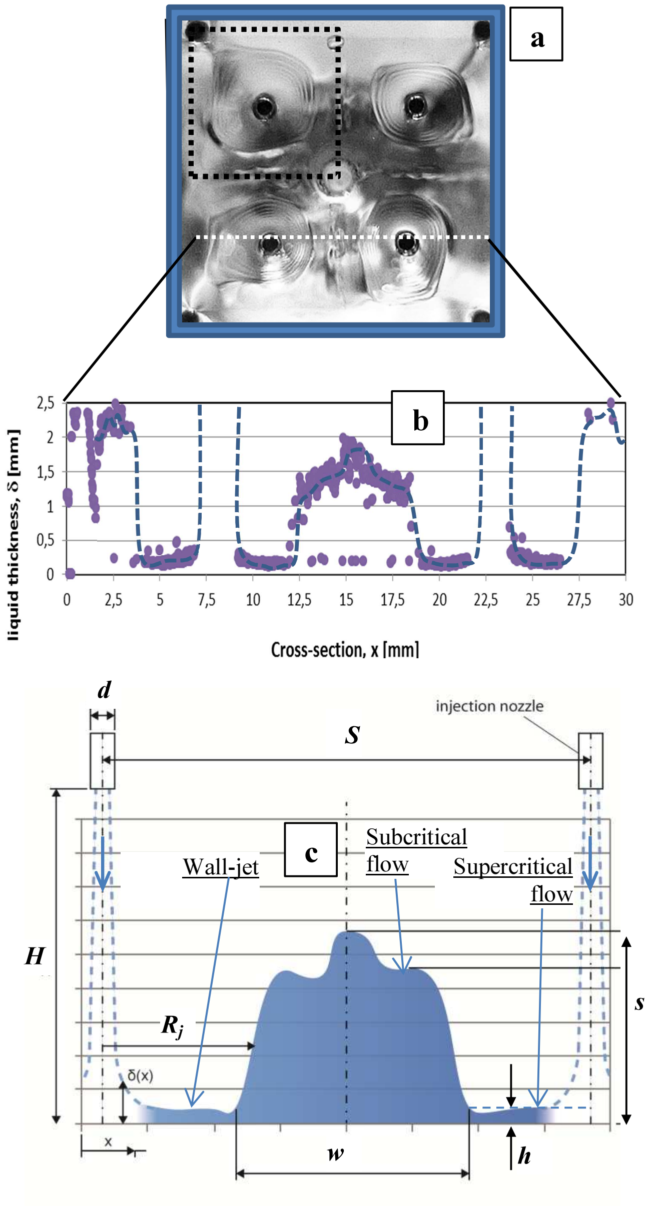

Figure 1 Typical results from the basic 2x2 jet array employed in the present experiments: (a) high speed photography – black dashed line shows modular array element, white dashed line shows location of: (b) liquid thickness measurement; (c) schematic showing jet interaction problem parameters

EXPERIMENTAL SYSTEM

The present study is focused on establishing a clearer view of this complex problem by simplifying and reducing the unknowns. It is identified that underlying the heat transfer problem, the hydrodynamics of single jet and jet interaction are of vital importance to modular prediction. Therein, a simple configuration of 2x2 pipe-type jets running de-ionized was employed, similar to that previously used by the authors [31].To avoid the aforementioned drainage (cross-flow) and flooding problems, a method of liquid extraction was employed. Whereby, suction pipes are located at the jet-meeting points. The basic concept of this type of liquid extraction has been around for more than a decade, with recent application-driven studies showing that it leads to significant enhancement of heat transfer [32, 33]. This form of liquid extraction also increases the inherent symmetry of the problem, their similarity to much larger arrays (modularity) and allows for easier simulations to be performed, which are planned for future study. These type of jets are characterized by a developed parabolic velocity profile, meeting the condition $L/d \cdot Re > 0.08$, for all except the jets with d=1.6mm - for which conditions for generating a 7th power profile are prevalent. These type of small arrays, especially with the local liquid extraction employed here, have been shown to be representative of larger arrays [7,31].

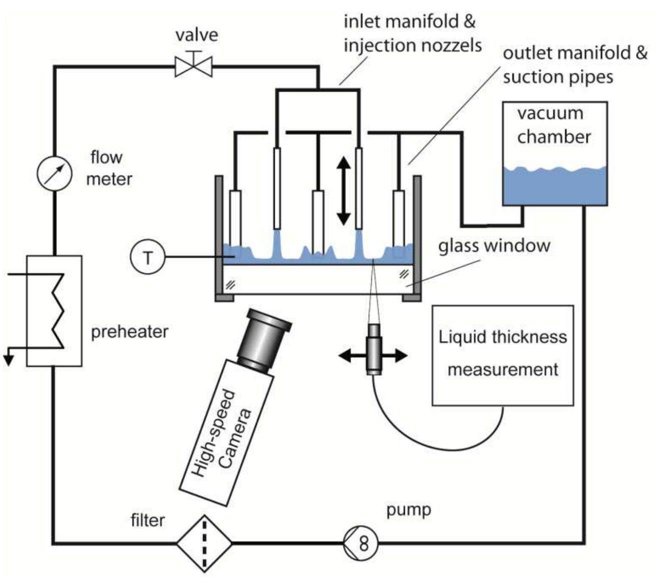

Figure 2 Schematic of experimental free-surface jet array setup

In the present study the influence of nozzle-jet spacing, jet-to-jet spacing and three different jet diameters have been examined. This parametric variation has assisted in identifying their influence on the key issue of the hydraulic jump location. In order to better characterize the resulting free-surface interface a quantative liquid film thickness measurement (optically - through the transparent impingement plate) and high speed photography was employed. Recently, a comprehensive study of the hydrodynamics of the interaction of two adjacent jets in an unbounded configuration has been published [34]. Present results compare well with the findings there, though include the interaction between 4 jets (an array module), and contain quantative liquid thickness measurement, and inter-jet liquid extraction. Previous studies employing quantative liquid thickness measurements have been limited to single jet impingement [35,36] (employing capacitance probes and LIF, accordingly). Thereby the present study builds on the insight provided in these studies, but extends to modelling application relevant conditions (multiple jets with spent liquid extraction).

The multiple jet experimental system was not setup to conduct heat transfer experiments, and rather previous single jet experimental and numerical data by the authors [14,37] and data from the literature [18, 28] are relied upon for the heat transfer model development.

NOMENCLATURE

C^*	[-]	Constant in Eq. (1)
C^{**}	[-]	Constant in Eq. (2)
d	[m]	Nozzle/jet-exit diameter
$f(\mathrm{Pr})$	[-]	Function for Prandtl number dependence in stagnation flows
Fr_h	[-]	Froude number based on pre-jump liquid depth, Eq. (3)
g	[m/s^2]	Gravitational acceleration
h	[m]	Pre-jump (supercritical flow) liquid film thickness
h_f	[W/m^{2}K]	Convection coefficient
H	[m]	Nozzle – plate distance
k	[W/mK]	Thermal conductivity
Nu	[-]	Nusselt number, $\mathrm{Nu}= h_f d/k$
Pr	[-]	Prandtl number, $\mathrm{Pr}=\alpha/v$
r	[m]	Radial coordinate
R_j	[m]	Radius of the hydraulic jump
Re	[-]	Jet Reynolds number , $\mathrm{Re}=u_{av}d/v$
s	[m]	Post-jump liquid depth
S	[m]	Inter-jet spacing in an array
T	[K]	Temperature
u	[m/s]	velocity
U	[-]	Ratio of jet centreline to average velocity
x	[-]	Cartesian axis direction – parallel to plate
z	[m]	Cartesian axis direction – normal to plate

Special characters

α	[m^2/s]	Thermal diffusivity
δ	[m]	Local liquid film thickness
θ	[°]	Jet inclination angle
v	[m^2/s]	Viscous diffusivity

Subscripts/Superscripts

av	Average
c	Centerline
f	Flow/convection
j	Jump
m	Stitching power, Eqs. (15) & (17)
n	Radial decay power, Eqs. (13) & (14)
o	Stagnation point

ANALYTICAL-EMPIRICAL MODELLING

In order to employ a modular prediction method, i.e. using single jet heat transfer theory and understanding for predicting multiple jets, the key issues that need to be resolved are predicting the location of the hydraulic jump and finding a reliable universal single-jet heat transfer model. Both these issues are crucial for successful prediction: i) The location of the hydraulic jump will be affected by the spacing between adjacent jets, as their flow interacts and single jet theory starts to breakdown, ii) Existing single jet prediction methods are not universal – they do not consider all relevant parameters: jet velocity profiles, nozzle-plate spacing, inclination relative to gravity or the nature of radial heat transfer decay. These issues are addressed in the following.

Location of Hydraulic Jump

Taking a closer look at previous work on predicting the hydraulic jump (also in review by the authors [38]), it is seen inviscid theory seems to underestimate the hydraulic jump radius and predict a strong dependency on incoming jet diameter, which has not been observed experimentally [39]. Besides that two notable approaches can be seen. On the one hand, the approach presented by Bohr et al. [26] based on shallow-water analysis including viscosity, predicts the jump location as:

$$\frac{R_j}{d} = C^* \,\mathrm{Re}^{5/8}\left(\frac{\nu^2}{gd^3}\right)^{1/8} \tag{1}$$

where R_j is the radius at which the hydraulic jump occurs, d is the jet nozzle diameter, C^* is a constant of order one, Re is the Rayleigh number, ν is the dynamic viscosity and g is the gravitational acceleration. By comparison of prediction by this equation to present experimental data, as extensive data from the literature, it is seen to grossly over-predict. Even if a significantly reducing constant is chosen, C^*=⅛, as Figure 3 shows, the over-prediction persists, and agreement with experiments is not good.

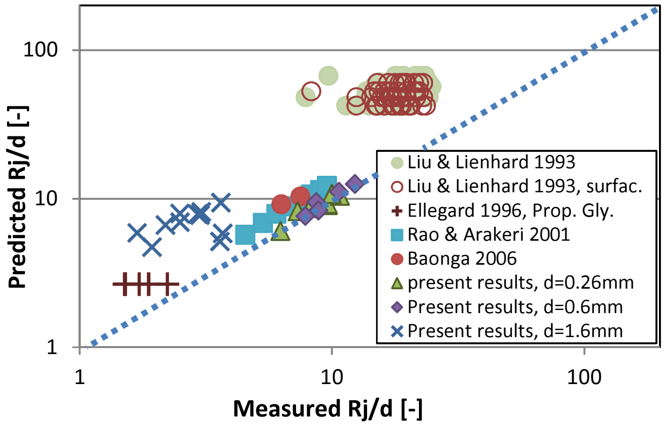

Figure 3 Evaluation of shallow-water model prediction, Eq. (1)

Later work by the same authors [21] found that experimentally the dependence on Reynolds number had a higher power, around ¾, which could be closely obtained by a complex piece-wise theoretical study (matching of upstream and downstream analysis). This modification to Eq. (2) is simply taken as:

$$\frac{R_j}{d} \approx C^{**} \,\mathrm{Re}^{3/4}\left(\frac{\nu^2}{gd^3}\right)^{1/8} \tag{2}$$

Although several studies have found agreement with the prediction of above equations [40, 35], they also noted that as an instability even surface roughness and plate edge chamfer affect R_j, while downstream obstructions or confinement significantly reduce it. When examining the literature as to the cause this significant reduction, one sees that the liquid depth downstream of the jump (s), is inversely proportional to the jump location, and is known to dictate the type of jump formed (single vortex, double vortex, double step, loss of jump [23]). The two previous equations do not contain this important parameter (s), and as Figure 3 shows, can only be considered an upper limit for the jump location – suitable for cases of minimal confinement and backpressure ($s \to h$).

Conversely, in turbulent flow experiments, jump radii and a Reynolds number dependence differ from the above model ($R_j/d \propto \mathrm{Re}^{0.82}$)[27,30]. Following the analysis of Rayleigh [53], in assuming mass and momentum conservation across the jump (or "shock") leads to the relation:

$$\frac{s}{h} = \frac{1}{2}\left(-1+\sqrt{1+8Fr_h^2}\right) \qquad Fr_h = \frac{u_{av}d^2}{8R_j\sqrt{gh^3}} \tag{3}$$

Here Fr_h is the upstream Froude number, h is the upstream depth (pre-jump), s is the downstream depth (post-jump), and u_{av} the average jet velocity. As noted by Liu & Lienhard [27], most expressions for circular hydraulic jumps found in literature (from Watson[39] to Watanabe[21]), employ a variation of this equation. Their comparison with experiments also demonstrated that Eq. (3) mildly over-predicted the jump location. This alternative approach, leading to Eq. (3) is limited by another key factor – as a single equation with 3 unknowns (s, h, R_j) it is unsolvable.

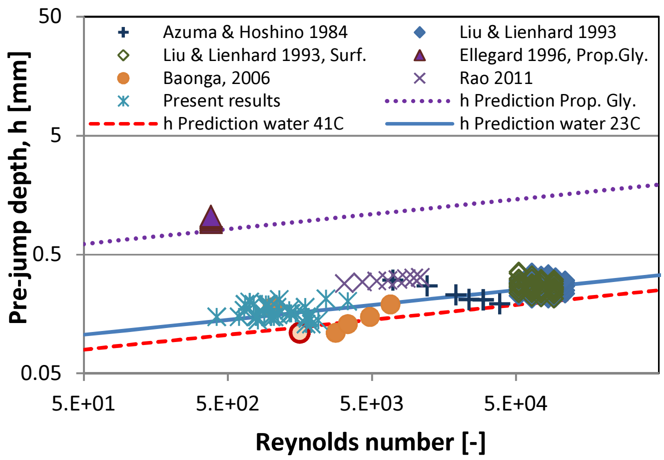

Figure 4 Prediction of pre-jump liquid film thickness by Eq. (4), showing good agreement (R^2=0.958)

The present study employs elements from both these models (Eq. (2) & (3)) in an attempt to reconcile them, into a new model which better captures the phenomena. In order to do this, first the amount of unknowns in Eq. (3) needs to be reduced. By examining extensive data from experimental studies in which liquid profiles were measured under single jets, it is found that the values of h do not vary significantly with jet diameter or even with flow rate. For instance, for water h varies from around 0.1mm up to 0.3mm for an increase of almost three orders of magnitude in Reynolds number. Furthermore, as the pre-jump liquid film is very thin, it bares resemblance to falling liquid films, which is well-described by Nusselt's thin film theory. Considering the characteristic liquid film thickness from that theory, which is related to the viscous length $(v^2/g)^{1/3}$ and examination of the available data trends leads to an expression for pre-jump liquid film thickness:

$$\frac{h}{d} = \frac{\pi}{2}\mathrm{Re}^{1/9}\left(\frac{\nu^2}{gd^3}\right)^{1/3} \tag{4}$$

Note that this equation suggests that pre-jump depth is independent of nozzle diameter, but weakly dependent on Reynolds number and viscous length as defined above. Present values of h together with those found in the literature are plotted against the Reynolds number in Figure 4. The data covers water, water with a surfactant and propylene glycol, nozzle diameters ranging $0.26<d<15$mm and over two orders of magnitude in Reynolds number. Comparing the hollow diamond symbols to the full ones (experimental data from Ref. [27]), shows that surface tension does not have a significant effect on the value of h. On the other hand, viscosity has a significant effect as can be seen from comparison of the water data to that of propylene glycol (from [41]). Even data for water heated to 40°C (from [36]), with viscosity reduced by 35%, stands out as the lowest data point on the graph (hollow circle). as predicted by the dashed line. In the figure, prediction by Eq. (4) is shown to capture this viscosity dependence quite well (3 different values), as well as the general trend.

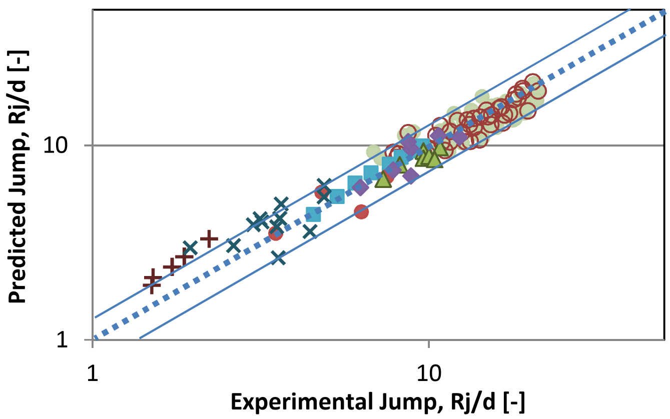

Figure 5 Prediction of hydraulic jump location by Eq. (8), showing good agreement (R^2=0.968); solid lines indicate 25% error, symbols are per Fig. 3

Introducing the upstream liquid film thickness, as described by Eq. (4), into Eq. (3) reduces the amount of unknowns in that equation to two (s, R_j), allowing it to be reordered to the form:

$$Fr_h = \frac{1}{(2\pi)^{3/2}}\mathrm{Re}^{15/18}\frac{d}{R_j} \tag{5}$$

As typical values of Fr_h are high it is possible to approximate Eq. (3) as:

$$\frac{s}{h} \approx \sqrt{2}Fr_h \tag{6}$$

Using this approximation together with the relations given in Eqs. (3) & (4) leads to:

$$\frac{R_j}{d} = \frac{1}{4\sqrt{\pi}}\mathrm{Re}^{17/18}\left(\frac{\nu^2}{gs^3}\right)^{1/3} \tag{7}$$

As we saw the previous model did not contain a term with s which is known to be important, on the other hand it contains a term with d, which does not appear here. Attempting to reconcile Eq. (7) with the previous model, we seek to incorporate the unique terms from both equations and average their Reynolds number dependence, leading to:

$$\frac{R_j}{d} = \frac{4}{3}\mathrm{Re}^{5/6}\left(\frac{\nu^2}{gs^3}\right)^{1/3}\left(\frac{\nu^2}{gd^3}\right)^{1/8} \tag{8}$$

Here the value of the constant was found by comparison to experiments. It can be seen that this equation contains the same influences of both dimensionless length scales (terms in brackets) as suggested by the previous two models, and with an average value of the power of Reynolds number. This value ≈0.833, is slightly higher than previous studies, but quite close to the empirical result of Ref. [30], 0.82. This equation does still contain the post jump depth, s, which must be controlled and preferably measured for prediction of the jump radius, R_j. Therefore the model is evaluated only against data that contains this information, as shown in Figure 5. As the figure shows reasonably good agreement is found using Eq. (8) even under strongly confined conditions, such as in the array experiments conducted here. These confined conditions are typically expressed by an increase in s leading to a decrease in R_j.

This new model for the jump location has directly led to the definition of a new dimensionless spacing between adjacent impinging jets in the form of:

$$S^* = \frac{R_j}{d}\frac{d}{S} = \frac{4}{3}\mathrm{Re}^{5/6}\left(\frac{\nu^2}{gs^3}\right)^{1/3}\left(\frac{\nu^2}{gd^3}\right)^{1/8}\left(\frac{d}{S}\right) \tag{9}$$

Evaluating the terms in brackets for the experimental data obtained here it is found that a good approximation is given by:

$$S^* = 0.091\,\mathrm{Re}^{5/6}\frac{d}{S} \tag{10}$$

Although the value of the constant, agrees with that found by introducing characteristic values into Eq. (9) for d=1.6mm, it seems suitable for the d=0.6mm data as well. Plotting the width of the inter-jet post hydraulic jump zone (here termed the inter-jet hump) against the dimensionless scaling given in Eq. (10), as shown in Figure 6, reveals a clear transition in jet interaction. At around a value of S^*=1/2, by definition, the adjacent jet's supercritical flows begin to meet up, this leads to a rapid narrowing of the hump to around one jet diameter and the occurrence of a standing fountain. This phenomenon has recently been extensively studied and described for two adjacent jets [34], and is expected to prevent the heat transfer degradation typically associated with the post hydraulic jump area – the negative jet interaction. From a modelling point of view, the Nusselt number prediction of each jet, developed for the supercritical flow zone, covers the majority of the heated surface. This leaves only very small ("corner") zones to be interpolated for in order to calculate the entire heat transfer distribution or average transfer coefficient.

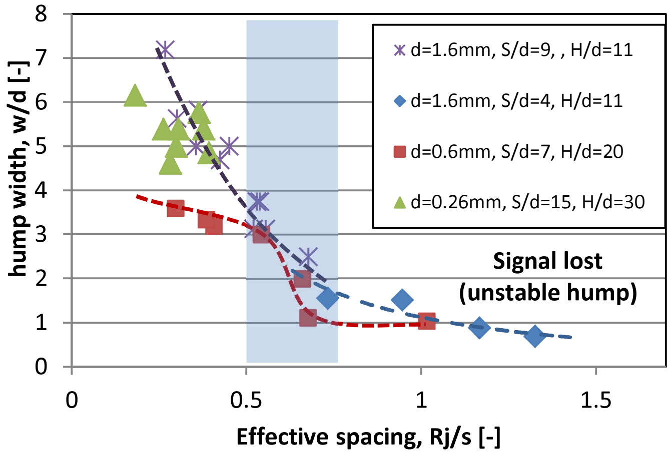

Figure 6 Dimensionless inter-jet hump width as dependent on self-similar jet spacing Eq. (10), showing transition to standing fountain between $\frac{1}{2}<R_j/s<\frac{3}{4}$

Heat Transfer Distribution

Regarding the second key issue for modular prediction, the model for the Nusselt number, several options exist. In the literature several models can be found: the analytical model of Liu et al. [28] is based on boundary layer theory applied in a piece-wise fashion to describe the different zones (stagnation, boundary layer emergence, thermal boundary layer emergence, etc.). However, this piece-wise model it not very convenient for use, and as it is based on a uniform velocity profile is not applicable to all jets. The empirical model of Ma et al. [29] has a similar form to the previous model, though with slightly different constants. While the empirical model of Stevens and Webb [30] was developed for turbulent water flows and employs only two zones (stagnation and wall-jet) with a stitching function between them. Recent simulations by the authors have shown that for laminar jets (and as will be shown here for weakly turbulent ones as well) additional self-similarity exists in the various zones, which can be used to simplify modelling [14]. This previous study was limited to the evolution of a parabolic velocity profile, emerging from the nozzle. It is therefore extended, following the same methodology, in order to develop an improved more useable model – suitable for various jet velocity profiles.

Previous studies have found that the impinging velocity profile is of vital importance to the form of the heat transfer distribution. In a previous study by the authors a parabolic velocity profile and its relaxation during its pre-impingement flight, was studied by numerical simulations [14]. That study identified the importance of the term of the *local* centerline velocity, normalized by the average *exit* velocity for the heat transfer. The present study similarly characterizes any intermediate velocity profile (between parabolic and uniform) by its *exit* centerline velocity to average *exit* velocity. Assuming, as typically is the case, that Reynolds number is quite high or nozzle-plate distance is small, such that $H/(d{\cdot}\mathrm{Re})\rightarrow 0$, means that this is similar to the velocity profile which impinges.

If the velocity profile, exiting the nozzle, is known, or can be measured or reasonably estimated, then the above described velocity ratio can be defined. This, ratio can then be used to form a new equation for predicting the stagnation Nusselt number, applicable for almost any monotonous velocity profile – from parabolic to uniform, under the mentioned assumptions:

$$\mathrm{Nu}_o = \frac{3}{4}\mathrm{Re}^{1/2}\left(\mathrm{Pr}^*\right)^{1/3} f(\mathrm{Pr})\left(\frac{u_c}{u_{av}}\right)_o \qquad (11)$$

Here Nu_o is the stagnation Nusselt number, Re is the Reynolds number based on jet exit conditions, Pr is the Prandtl number, u is velocity and subscripts c and av are centerline and average, accordingly. The constant of ¾ is based on the value suggested by theory [28] for a uniform velocity profile ($u_c/u_{av}=1$), and Pr^* is an average value of the Prandtl number needed for converting the type of dependence in that model to the form in the present, given by $f(\mathrm{Pr})$. This is the complex function of Prandtl number suitable for stagnation flow – validated for $0.07<\mathrm{Pr}<1300$ [14], and approximated according to [42] as:

$$f(\mathrm{Pr}) = \begin{cases} \dfrac{\sqrt{2\,\mathrm{Pr}/\pi}}{1+0.804552\sqrt{2\,\mathrm{Pr}/\pi}} & \mathrm{Pr}\le 0.15 \\ 0.53898\,\mathrm{Pr}^{0.4} & 0.15\le \mathrm{Pr}\le 3 \\ 0.60105\,\mathrm{Pr}^{1/3}-0.050848 & \mathrm{Pr}\ge 3 \end{cases} \qquad (12)$$

By comparison to the above mentioned literature it is found that the most suitable value for the unknown constant in Eq. (11), is around the value of water, $\mathrm{Pr}^*=6.5$. With this value the prediction by Eq. (11) agrees quite well with the asymptotic values suggested by Liu et al. [28] at *both* ends of the curve (parabolic profile and uniform profile), as shown in Figure 7. This stagnation heat transfer model, Eq. (11), innovates by incorporating the jet velocity profile and a Prandtl number dependence suitable to a wider range of liquids.

In order to obtain a full description of the heat transfer under an impinging free-jet, we now look beyond the stagnation point, into the wall-jet region (up to the hydraulic jump). By fitting curves to several measured distributions, for various velocity profiles taken from previous works by the authors [14,37] and some unpublished experimental data, it was possible to establish the decay of the heat transfer in the wall-jet region, as follows:

$$\mathrm{Nu}_r = \mathrm{Re}^{1/2} f(\mathrm{Pr})\left(\mathrm{Nu}_o^*\right)^{1/2}\left(\frac{r}{d}\right)^n \quad (13)$$

This equation contains a dependence on the value of the stagnation Nusselt number – a higher starting point for the decay with radius, *r*. Regarding the value of the rate of decay, *n*, previous works have suggested a value of ½ [28] - equivalent to a constant height pre-jump liquid film, while recent simulations suggest somewhat higher values – which would be equivalent to the thickening of the wall-jet liquid film found for these profiles [39, 14]. As higher stagnation heat transfer decays more rapidly downstream, this power should depend on stagnation Nusselt number (which contains this dependence):

$$n = \frac{1}{2}\left(\frac{2}{3} + \frac{1}{4}\frac{\mathrm{Nu}_o}{\mathrm{Re}^{1/2} f(\mathrm{Pr})}\right) \quad (14)$$

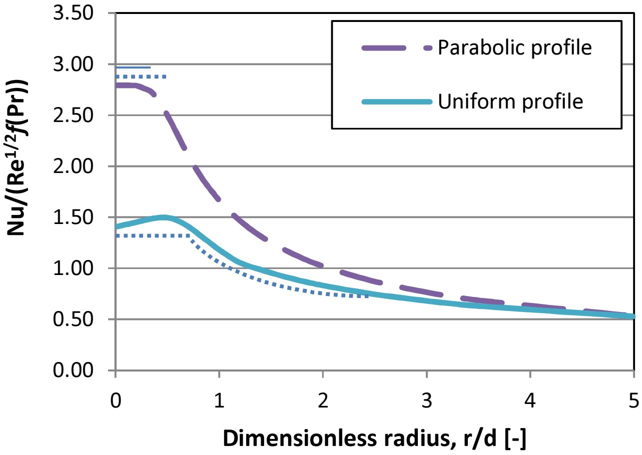

Figure 7 Nusselt number predicted by Eqs. (15) & (17), for the two limiting velocity profiles vs. dashed line - theory [29], solid line – simulation [14]

The stagnation Nusselt number can now be "stitched" to the radial one (given in Eqs. (13) & (14)) in a similar form to that used in Ref. [30]:

$$\mathrm{Nu} = \left(\left(\mathrm{Nu}_o\right)^{-m} + \left(\mathrm{Nu}_r\right)^{-m}\right)^{-\frac{1}{m}} \quad (15)$$

Here a value of m=7 was found to give smooth curves, similar to experimental data. This combined equation gives a radial heat transfer decay which fulfils several important observed physical trends: i) Velocity profiles with a higher centreline value (i.e. steeper gradients as in a parabolic profile) generate a higher stagnation Nusselt number; ii) For higher values of stagnation Nusselt number (e.g. parabolic profile) the radial decay is faster; iii) Beyond a certain distance downstream the heat transfer becomes independent of the incoming velocity profile (in the range of 3-5d); iv) As found in previous simulations and experiments, when the velocity profile is near to uniformity, an off center peak emerges. In order to model the latter, the criterion for the off-center peak is adapted from previous work [14] and converted into terms of velocity:

$$U = \frac{u_c}{u_{av}} < 1.2 \quad or \quad \mathrm{Nu}_o < \frac{5}{3}\mathrm{Re}^{1/2} f(\mathrm{Pr}) \quad (16)$$

If this condition is met then a slight modification to Eq. (15) should be used, up to the radius where both equations agree – around r/d=1:

$$\frac{r}{d} < 1: \quad \mathrm{Nu}\big|_{U<1.2} = \left(\left(1-\frac{r}{d}\right)\left(\mathrm{Nu}_o\right)^{-m} + \frac{r}{d}\left(\mathrm{Nu}_r\right)^{-m}\right)^{-\frac{1}{m}} \quad (17)$$

In Figure 7 these heat transfer curves are plotted and compared to previous theory and simulations.

Unfortunately, the exit velocity profile of the jet is not often characterized in heat transfer experiments. However, it is possible to estimate the form of the profile in the limiting cases: High Reynolds number, short/orifice type nozzle – *uniform velocity profile*; Low Reynolds number, pipe-type nozzle – *parabolic velocity profile*. As an example of the former, Figure 8 shows a comparison of experimentally measured heat transfer distribution (taken together with liquid film thickness measurement, in [37]) to that predicted by several models [28,30], including the present one. For the case of uniform velocity profile, all models predict the stagnation heat transfer well for Re=2900. Conversely, the full profile measured at Re=3550 is under-predicted at stagnation by both the other models. The theoretical model [28] under-estimates the heat transfer, probably because it was developed for purely laminar flow, and has been shown to over-predict the liquid film thickness (under-predict the velocity) [14]. The purely empirical model [30], developed for turbulent conditions (Re>4000) better captures the wall-jet part of the heat transfer. While the present, analytical-empirical model seems to capture the entire curve better, right up to the location of the hydraulic jump at r/d=3.6.

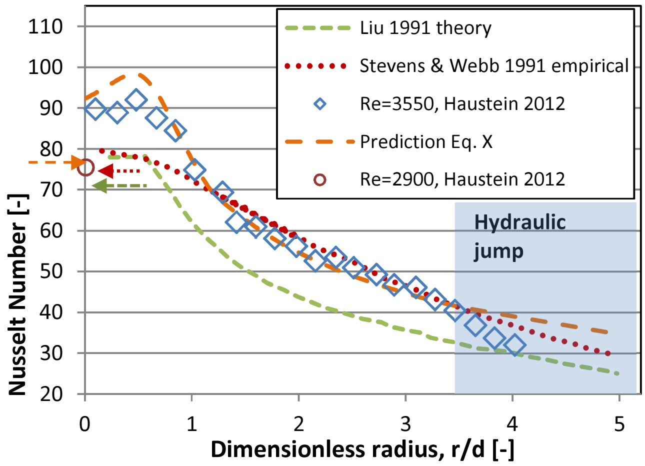

Figure 8 Prediction of heat transfer distribution under a uniform velocity profile free jet (Re=2900, 3550; H/d=6) by three different models

Influence of Gravity

For a more complete description of impinging free-jets, inclination (gravity) is of importance and especially relevant to many applications (cooling for machining/drilling, metal sheet quenching and other manufacturing processes). For examining the influence of inclination relative to gravity's direction, highly resolved thermal measurements were conducted of a *horizontal* free jet impinging on a transparent heater. These measurements were compared to simulations in an attempt to better understand the results, as shown in Fig. 9. Agreement in form between simulation and experiment was very good though the simulations overestimated the heat transfer in the stagnation zone by around 50 percent. Both methods showed that already in the stagnation zone asymmetry exists due to the influence of gravity. The details of the mechanism causing this asymmetry remain to be clarified – as it may be due to either weak inclination of the jet (around 3 degrees) or redistribution of the incoming velocity profile prior to impingement.

However, as the results emphasis that gravity must be taken into consideration when a detailed description of the heat transfer under a jet is required, especially with regards to the occurrence and location of the hydraulic jump (easily supressed in gravity's direction).

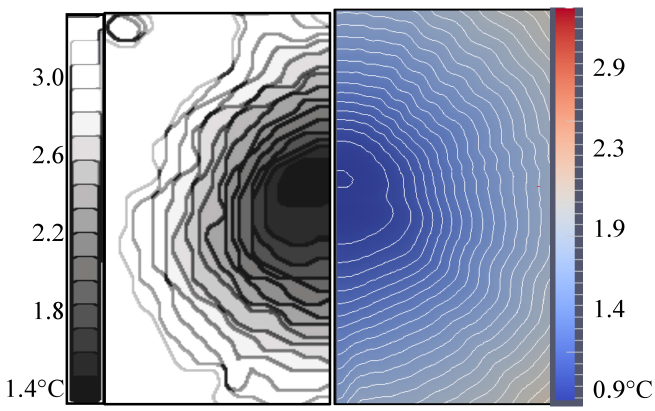

Figure 9 Measured vs. Simulated heat transfer under a horizontal impinging jet (Re=3500, H/d=5.5, $\theta_0 \approx 3°$) showing up-down asymmetry within the stagnation zone

CONCLUSION

The development of a model for modular prediction of heat transfer under an array of free-surface jets has been presented. It was further identified that the key obstacles to this approach, are:

i) **Generating a modular array** (whereby all jets are similar to each other), under which single jet theory can be used to describe the majority of the heat transfer area (only the jet-interaction zones require interpolation between adjacent predictions);
ii) Predicting the **location of the hydraulic jump** under these modular conditions, up to where the single jet theory strictly applies – this also leads to the ability to predict (or generate conditions which reduce) the undesirable post-jump jet interaction zones;
iii) Establishing a **universal single-jet heat transfer model** incorporating all relevant parameters: jet velocity profiles, nozzle-plate spacing, inclination relative to gravity or the nature of radial heat transfer decay.

To address the first issue, experiments were conducted employing de-ionized water in both single and basic multiple-jet array (2x2, with local liquid extraction in the jet interaction zones) configurations. It was found that when local liquid extraction is employed a high level of similarity between jets can be established, to permit modular prediction.

Regarding the second issue, two previous limited theoretical approaches were re-examined (shallow-water vs. jump conservation) and through analytical-empirical development were reconciled into a new model for the jump location. This new model required developing a prediction of the pre-jump depth and still requires measurement or control of the post-jump depth. Under these limitations good agreement was found with the available detailed experimental data.

Finally, the last issue was addressed by reviewing a few existing models for heat transfer distribution under a single-jet – both theoretical (based on the integral boundary approach) and empirical (based on nominally turbulent water flows), and employing inherent similarities found in a previous numerical study by the authors. These were incorporated into a new model which addresses the often neglected jet velocity profile (related to jet-nozzle shape) in a comprehensive way. This model is shown to agree with the previous models under certain conditions and surpass them under others. It is further identified that the influence of inclination is also of vital importance to free-surface jets (breakage of symmetry) and must be examined in future studies.

By addressing these three key issues the foundation for a modular prediction of heat transfer under a free jet array is laid. Future studies are planned in which this prediction will be compared to local and average heat transfer under a larger scale symmetric array (3x3, with liquid extraction at each flow junction).

REFERENCES

[1] Weigand B. and Spring S., Multiple Jet Impingement – a Review, *Heat Transfer Research*, vol. 42, no. 2, 2011, pp. 101-142

[2] Robinson A. J., Thermal–Hydraulic Comparison of Liquid Microchannel and Impinging Liquid Jet Array Heat Sinks for High-Power Electronics Cooling, *IEEE Transactions on Components and Packaging Technologies*, vol. 32, no. 2, 2009

[3] Fabbri M., Jiang S., and Dhir V. K., A Comparative Study of Cooling of High Power Density Electronics Using Sprays and Microjets, *Transactions of the ASME*, vol. 127, 2005

[4] Wang E. N., Zhang L., Jiang L., Koo J. M., Maveety J. G., Sanchez E. A., Goodson K. E. and Kenny T. W., Micromachined Jets for Liquid Impingement Cooling of VLSI Chips, *Journal of Microelectromechanical Systems*, vol. 13, no. 5, 2004

[5] Bhunia, A., and Chen, C. L. On the Scalability of Liquid Microjet Array Impingement Cooling for Large Area Systems. *Journal of Heat Transfer*, 133(6), 064501, 2011.

[6] Fabbri, M. and Dhir, V. K., Optimized heat transfer for high power electronic cooling using arrays of microjets, *Journal of Heat Transfer*, 127(7), 2005, pp. 760–769

[7] Pan Y., Webb B.W., Heat Transfer Characteristics of Arrays of Free Surface Liquid Jets, *Journal of Heat Transfer*, vol. 117, 1995, pp. 878–883

[8] Womac D.J., Incopera F.P., and Ramadhyani S., Correlating equations for impingement cooling of small heat sources with multiple circular liquid jets, *Journal of Heat Transfer*, vol. 116, 1994, pp. 482- 486

[9] Womac D. J., Ramadhyani S. and Incropera F.P., Correlating Equations for Impingement Cooling of Small Heat Sources with Single Circular Liquid Jets, *Journal of Heat Transfer*, vol. 115, 1993, pp. 106-115

[10] Yonehara N. and Ito I., Cooling Characteristics of Impinging Multiple Water Jets on a Horizontal Plane, *Technol. Rep. Kyushu Univ.*, vol. 24, 1998, pp. 267-281

[11] Whelan B. and Robinson A.J., Nozzle Geometry Effects in Liquid Jet Array Impingement,, Applied Thermal Engineering, vol. 29 (11-12), 2009, pp. 2211-2221

[12] Jiang S., Heat Removal Using Microjet Arrays and Microdroplets in Open and Closed Systems for Electronic Cooling, Ph. D. dissertation, University of California, 2002

[13] Rohlfs W., Haustein H. D., Garbrecht O., and Kneer R., Insights into the Local Heat Transfer of a Submerged Impinging Jet: Influence of Local Flow Acceleration and Vortex-Wall interaction, *International Journal of Heat and Mass Transfer*, vol. 55, no. 25-26, 2012, pp. 7728-7736

[14] Rohlfs, W., Ehrenpreis, C., Haustein, H. D., and Kneer, R., Influence of viscous flow relaxation time on self-similarity in free-surface jet impingement. *International Journal of Heat and Mass Transfer*, 78, 2014, pp. 435-446.

[15] Jiji L.M., Dagan Z., Experimental Investigation of Single-Phase Multijet Impingement Cooling of an Array of Microelectronic Heat Sources, W. Aung (Ed*.), Cooling Technology for Electronic Equipment*, Hemisphere Publishing Corporation, 1988, pp. 333-351

[16] Robinson A.J., Schnitzler E., An Experimental Investigation of Free and Submerged Miniature Liquid Jet Array Impingement Heat Transfer, *Experimental Thermal and Fluid Science*, vol. 32, 2007, pp. 1–13

[17] Michna G. J., Browne E. A., Peles Y. and Jensen M. K., The Effect of Area Ration on Microjet Array Heat Transfer, *International Journal of Heat and Mass Transfer*, vol. 54, 2011, pp. 1782-1790

[18] Ellison E., and Webb B.W., Local heat transfer to impinging liquid jets in the initially laminar, transitional, and turbulent regimes, *International Journal of Heat and Mass Transfer*, vol. 37, 1994, pp. 1207-1216

[19] Rayleigh, L., On the theory of long waves and bores, *Proceedings of the Royal Society Land.*, 3, 1914, pp. 813–816

[20] Ellegaard, C., Hansen, A. E., Haaning, A., Hansen, K., Marcussen, A., Bohr, T., ... , and Watanabe, S. Cover illustration: Polygonal hydraulic jumps. *Nonlinearity*, 12(1), 1999, pp. 1

[21] Watanabe, S., Putkaradze, V., and Bohr, T., Integral methods for shallow free-surface flows with separation. *Journal of Fluid Mechanics*, 480, 2003, pp. 233-265.

[22] Bohr, T., Putkaradze, V., and Watanabe, S., Averaging theory for the structure of hydraulic jumps and separation in laminar free-surface flows, *Physics Review Letters*, 1997 , pp. 1038–1041

[23] Bush, J. W. M., Aristoff, J. M., and Hosoi, A. E., An experimental investigation of the stability of the circular hydraulic jump, *Journal of Fluid Mechanics*, 558, 2006 , pp. 33–52

[24] Ray, A. K. and Bhattacharjee, J. K., Standing and travelling waves in the shallow-water circular hydraulic jump, *Physics Letters A*, 371(3), 2007, pp. 241–248

[25] Whitham, G. B., *Linear and Nonlinear Waves*, vol. 42, JohnWiley and Sons, 2011

[26] Bohr, T., Dimon, P., and Putkaradze, V., Shallow-water approach to the circular hydraulic jump. *Journal of Fluid Mechanics*, 254, 1993, pp. 635-648.

[27] Liu, X., and Lienhard, J.H., V., The hydraulic jump in circular jet impingement and in other thin liquid films, *Experiments in Fluids*, 15(2), 1993, pp. 108–116

[28] Liu, X., Lienhard, J. H., and Lombara, J. S., Convective heat transfer by impingement of circular liquid jets, *Journal of Heat Transfer*, 113(3), 1991, pp. 571–582

[29] Ma, C. F., Gan, Y., Tian, Y., Lei, D., and Gomi, T., Liquid jet impingement heat transfer with or without boiling, *Journal of Thermal Science*, 2, 1993, pp. 32–49

[30] Stevens, J. and Webb, B. W., Local heat transfer coefficients under an axisymmetric, single-phase liquid jet, *Journal of Heat Transfer*, 113(1), 1991, pp. 71–78

[31] Haustein, H. D., Joerg, J., Rohlfs, W., and Kneer, R., Influence of micro-scale aspects and jet-to-jet interaction on free-surface liquid jet impingement for micro-jet array cooling, *Thermal and Thermomechanical Phenomena in Electronics Systems, ITHERM*, 2014

[32] Natarajan G., and Bezama R. J., Microjet Cooler with Distributed Returns, *Heat Transfer Engineering*, vol. 28(8-9), 2007, pp. 779-787

[33] Brunschwiler S., Rothuizen H., Fabbri M., Kloter U., Michel B., Bezama R. J. and Natarajan G., „Direct Liquid Jet-Impingement Cooling with Micron-sized Nozzle Array and Distributed Return Architecture, *10th Thermal and Thermomechanical Phenomena in Electronics Systems, ITHERM*, 2006, pp. 196-203

[34] Kate R. P., Das P. K. and Chakraborty S., An Experimental Investigation on the Interaction of Hydraulic Jumps Formed by Two Normal Impinging Circular Liquid Jets, *Journal of Fluid Mechanics* (Cambridge University Press), vol. 590, 2007, pp. 355–380

[35] Rao A. and Arakeri J.H., Wave Structure in the Radial Film Flow with a Circular Hydraulic Jump, *Experiments in Fluids*, vol. 31, 2001, pp. 542-549

[36] Baonga, J., Louahlia-Gualous, H., and Imbert, M., Experimental study of the hydrodynamic and heat transfer of free liquid jet impinging a flat circular heated disk, *Applied Thermal Engineering*, 26(11-12), 2006, pp. 1125 – 1138

[37] Haustein, H. D., Tebrügge, G., Rohlfs, W., and Kneer, R. Local heat transfer coefficient measurement through a visibly-transparent heater under jet-impingement cooling. *International Journal of Heat and Mass Transfer*, 55(23), 2012, pp. 6410-6424.

[38] Kneer, R., Haustein, H. D., Ehrenpreis, C., and Rohlfs, W. Flow Structures and Heat Transfer in Submerged and Free Laminar Jets, *Keynote at IHTC 2014*, Kyoto, Japan, 2014.

[39] Watson, E., The radial spread of a liquid jet over a horizontal plane, *Journal of Fluid Mechanics*, 20(03), 1964, pp. 481–499

[40] Hansen, S. H., Hørl¨uck, S., Zauner, D., Dimon, P., Ellegaard, C., and Creagh, S. C., Geometric orbits of surface waves from a circular hydraulic jump, *Physics Review E*, 55, 1997, pp. 7048–7062

[41] Ellegaard, C., Hansen, A. E., Haaning, A., and Bohr, T., Experimental results on flow separation and transitions in the circular hydraulic jump. *Physica Scripta*, 1996 (T67), pp. 105.

[42] Liu X., Gabour L.A., Lienhard J.H. V, Stagnation-point heat transfer during impingement of laminar liquid jets: analysis including surface tension, *Journal of Heat Transfer* 119 (3) 1993, pp. 99–105