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NUMERICAL OPTIMISATION OF CONJUGATE HEXAGONAL COOLING CHANNELS WITH INTERNAL HEAT GENERATION USING AL₂O₃-WATER NANOFLUID

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ABSTRACT

This paper presents a three-dimensional geometric optimisation of conjugate cooling channels in forced convection with internal heat generation within the solid. Hexagonal cooling channels configuration are considered and Al₂O₃-water nanofluid is used as cooling fluid because of its promising superior thermal properties over traditional cooling fluids

The main objective is to optimise the configuration in such a way that the peak temperature is minimised subject to the constraint of fixed global volume of solid material. The cooling fluid is driven through the channels by the pressure difference across the channel. The elemental volume of the structure and hydraulic diameter of the cooling channel were considered as design variables. Also, different values of volume fraction of Al₂O₃ nanoparticles added into water that enhanced the cooling effect are considered. The shape of the channel is allowed to morph to determine the best configuration that gives the lowest peak temperature. A gradient-based mathematic optimisation algorithm (Dynamic-Q) is applied in order to search for the best optimal geometric configuration that improves thermal performance by minimising peak temperature for a wide range of pressure difference. Results obtained show that there are unique optimal geometry for a given pressure difference. Also, the results show that the effects of pressure difference, volume fraction of Al₂O₃ nanoparticles on minimum peak temperature and hydraulic diameter of the channel are consistent with those obtained in the open literature

NOMENCLATURE

A_c	Cross-sectional area of the channel, m ²
A_s	Cross-sectional area of the structure, m ²
Be	Bejan number
C_p	Specific heat at constant pressure, J/kg K

d_h	Hydraulic diameter, m
H	Structure height, m
H	Elemental height, m
I	Mesh iteration index
K	Thermal conductivity, W/Mk
L	Axial length, m
N	Number of channels
N	Normal
P	Pressure, Pa
Pr	Prandtl number
\dot{q}''	Heat flux, W/m ²
q_s'''	Internal heat generation density, W/m ³
\dot{q}	Heat transfer rate, W
R	Thermal resistance
T	Temperature, °C
\vec{u}	Velocity vector, m/s
V	Global structure volume, m ³
v_c	Channel volume, m ³
v_{el}	Elemental volume, m ³
W	Structure width, m
W	Elemental width, m
x, y, z	Cartesian coordinates, m
<i>Greek symbols</i>	
α	Thermal diffusivity, m ² /s
μ	Viscosity, kg/m.s
ν	Kinematics viscosity, m ² /s
ρ	Density, kg/m ³
∂	Differential
∞	Far extreme end, free stream
ϕ	Porosity

Δ	Difference
∇	Differential operator
γ	Convergence criterion
Subscripts	

F	Fluid
In	Inlet
Max	Maximum, peak
Min	Minimum
Opt	Optimum
S	Solid
W	Wall

INTRODUCTION

Nanofluid has emerged as a new class cooling fluid in thermal management and heat transfer analysis due to its promising superior thermal properties over traditional cooling fluids such as water, ethylene glycol, engine oil, glycerol because of its ability to provide and deliver an efficient cooling in heat generating devices with high heat transfer density [1-5]. Nanofluid is a fluid that contains solid particles at a nano scale level which are dispersed into conventional fluids.

The common nanoparticles used to produce nanofluids are : Al_2O_3 , $CaCO_3$, SiO_2 , TiO_2 , MgO , NiO_2 , CuO , SiC , Cu , Ag , Ni , Al , Au . These solid particles like most metals have thermal conductivities greater than that of liquids by the order of magnitude 1 to 3. Therefore thermal cooling is greatly enhanced due to the increase in thermal conductivity of the nanofluid. [1-5]

This kind of research work started with Maxwell when he first introduced colloidal suspension into traditional cooling fluid. The dispersion of micrometer and millimetre sized particles were used then. Later, the work was continued and extended by Choi [1] who first championed and explored the area of nanofluids. Nanofluids are ideally suited in practical applications because their usages have little or no effect on pressure drop due to the fact that the nanoparticles are ultrafine and they appear to behave more like a single-phase fluid than a solid-liquid mixture. A vast range of theoretical and experimental studies have been carried out by scientists on the effective thermal conductivity of nanofluids during this past decade [1-5].

An intensive study has been revised lately on nanofluids viscosity by Mahbubul et al [6]. His research showed that viscosity increases with volume fraction and that a nanofluid behaves as a Newtonian fluid at lower particle volume fraction. Roslan et al [7] also discovered that the effect of the viscosity of nanofluid is more prevalent than its thermal conductivity. He concluded that the benefits of using nanofluids in a thermal system cannot be restrained to only its superior thermal conductivity enhancement.

Recently, researchers have investigated the application of nanofluid to heat transfer equipment for heat dissipation. Palm et al [8] presented that nanofluid of Al_2O_3 -water with a volume fraction of 4% could improve the average wall heat transfer coefficient by 25% compared with water base fluid. Nguyen et al [9] used Al_2O_3 nanofluid in an electronic liquid cooling

system, at a nano particle volume concentration of 6.8%, the convective heat transfer coefficient was increased by 40%. This results showed that the heat transfer coefficient of the nanofluid with nano particle. The result was able to show that nanofluids help in improving the heat transfer operation for heat exchangers.

Different researchers have shown through their studies that nanofluids have a higher thermal conductivity than pure base fluids, consequently, they have an outstanding prospect for heat transfer enhancement. Therefore, it is suitable to create a micro channel heat sink containing nanofluids for likely and possible applications in many electronic appliances to be able to enhance the heat transfer performance. Chein and Chuang [10] employed CuO /water nanofluid to a microchannel heat sink (MCHS).

Although, the heat removal technique that uses forced convective heat sink was first carried out by Tuckerman and Pease [11] when microchannel heat sink with water as cooling fluid was used in cooling electronics and proved to have a very high cooling performance. Over the years, different techniques have been adopted in solving the micro-channel heat sinks problem. And because of its great ability for cooling electronic devices, micro channel heat sink has received much attention.

For example, Knight *et al.* [12] presented the governing equations for fluid dynamic and heat transfer in a generalised, dimensionless form along with geometrical relationship that can be used to determine the dimensions of rectangular microchannel heat sink that result in the minimisation of thermal resistance. Bejan and Sciubba [13] applied constructal theory and design [14,15] that have emerged as an evolutionary design philosophy for developing flow architectures that offer greater flow access and system performance of internal heat exchangers. These researchers obtained the design rule for spacing an array of parallel plates to channels so that the heat transfer density of a volume filled with heat-generating components was maximum. The spacing was determined by using the method of asymptotes.

In this paper, our focus constructal theory for heat transfer and fluid flow analysis based on shape and geometry optimisation [16 -19]. The advantage of constructal law in the engineering field is that the flow architecture is not assumed in advance, but is the consequence of allowing the structure to morph [20]. The applications of this theory have been reviewed by Bejan and Lorente [21].

Also, Bello-Ochende *et al.* [22] conducted a three-dimensional optimisation of heat sinks and cooling channels with heat flux using scale analysis and the intersection of asymptotes method based on constructal theory to investigate and predict the design and optimisation of the geometric configurations of the cooling channels. Rocha *et al.* [23] and Biserni *et al.* [24] applied the theory to optimise the geometry of C- and H-shaped cavities respectively that intrude into a solid conducting wall in order to minimise the thermal resistance between the solid and the cavities. Muzychka [25] used this theory and Bejan's intersection of asymptotes method to present an analytical optimisation of circular and non-circular cooling channel geometries. Reis *et al.* [26] optimised the internal configurations of parallel plates and cylindrical

channels using constructal theory to understand the morphology of particle agglomeration and the design of air-cleaning devices. Olakoyejo *et al.* [27] applied constructal theory to optimise cylindrical and square configurations with internal heat generation, where they showed that the minimised peak temperature is a function of the geometry and shape.

In this work, three-dimensional laminar forced convection cooling in a solid structure is investigated. It examines the optimisation of a fixed and finite global volume of solid materials with an array of hexagonal cooling channels, experiencing a uniform internal heat generation, which will result in a minimal global thermal resistance. The influence of Al_2O_3 –water on heat transfer enhancement or minimisation of peak temperature inside a circular cooling channel is studied. We considered volume fractions ranging from 1% to 4% in this analysis. This can be achieved by forcing a coolant to the heated spot in a fast and efficient way to reduce the peak temperature at any point inside the volume that needs cooling.

The objective of this research was done by numerical analysis. The numerical method involves the modelling and discretisation of the computational domain, the solving of conjugate heat transfer by using necessary governing equations, and boundary conditions and the processing of results. A commercial Computational fluid dynamic was employed to search the optimal design variables that minimised thermal resistance. The optimisation process is carried out numerically under total fixed volume and manufacturing constraints.

COMPUTATIONAL MODEL

The schematic, Figure 1, represents the physical configuration of the solid body with heat generation. The system consists of parallel hexagonal cooling channels in a solid structure of fixed global volume V with an internal heat generation q_s''' . The body is cooled by forcing cooling fluid (Al_2O_3 –water) from the left side into the parallel cooling channels by a fixed pressure difference ΔP over the channel length L . The fluid is assumed to be single phase, steady and Newtonian with constant properties.

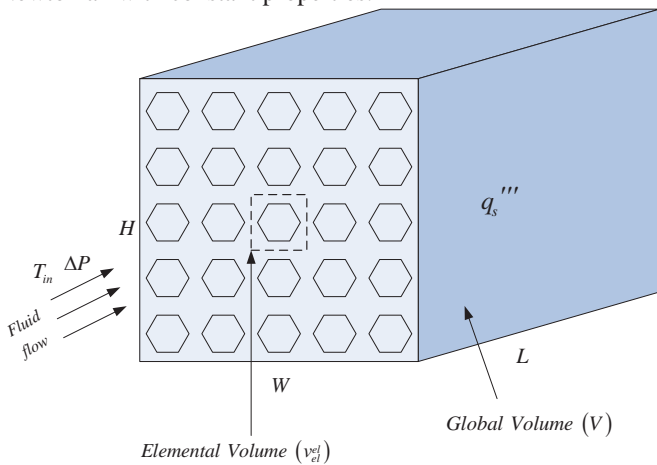


Figure 1. Three-dimensional parallel hexagonal cooling channels across a slab with internal heat generation

An elemental volume is shown in Figure 2 consisting of a hexagonal cooling channel and the surrounding solid used for analysis because of the assumption of the symmetrical heat distribution inside the structure. The heat transfer in the elemental volume is a conjugate problem, which combines heat conduction in the solid and the convection in the flowing fluid.

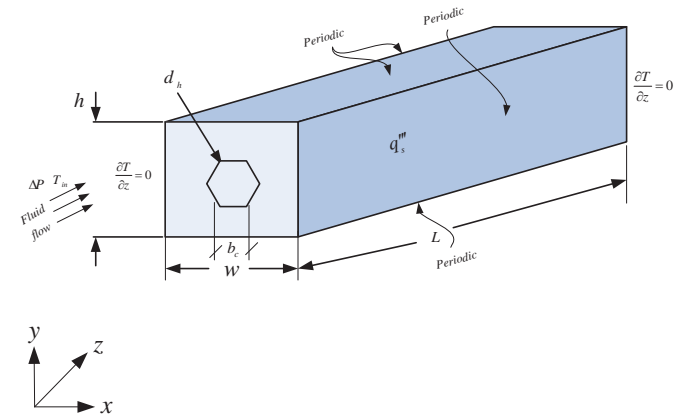


Figure 2. The boundary conditions of the three-dimensional computational domain of the elemental volume

In Fig. 2, an elemental volume, v_{el} , constraint is considered to be composed of an elemental cooling channel of hydraulic diameter, d_h , and the surrounding solid wall defined as:

$$w = h, \quad v_{el} = w^2 L \tag{1}$$

The volume of the unit Hexagonal channel is:

$$v_c = 1.5b_c^2 L \cot\left(\frac{\pi}{6}\right) \tag{2}$$

where, b_c = side dimension and the circumference of an hexagonal duct is:

$$P_c = 6b_c \tag{3}$$

Therefore, the hydraulic diameter of hexagonal channel is:

$$d_h = 4 \frac{A_c}{P_c} \tag{4}$$

where $A_c = 1.5b_c^2 \cot(\pi/6)$ is the cross-sectional area of the channel.

However, the void fraction or porosity of the unit structure can be defined as:

$$\phi = \frac{v_c}{v_{el}} \tag{5}$$

For a fixed length of the channel, we have

$$A_s = HW \tag{6}$$

Therefore, the total number of channels in the structure arrangement for the two configurations can be defined as:

$$N = \frac{HW}{hw} \quad (7)$$

The temperature distribution in the model was determined by solving the equation for the conservation of mass, momentum and energy numerically. The discretised three-dimensional computational domain of the configuration is shown in Fig. 3.

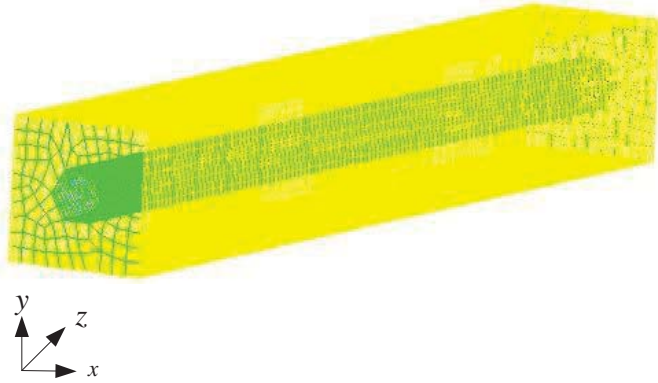


Figure 3. The discretised 3-D computational domain for a Hexagonal cooling channel

The governing differential equations used for the fluid flow and heat transfer analysis in the unit volume of the structure are:

$$\nabla \cdot \vec{u} = 0 \quad (8)$$

$$\rho_{nf} (\vec{u} \cdot \nabla \vec{u}) = -\nabla P + \mu_{nf} \nabla^2 \vec{u} \quad (9)$$

$$(\rho c_p)_{nf} (\vec{u} \cdot \nabla T) = k_{nf} \nabla^2 T \quad (10)$$

Energy equation for a solid given as:

$$k_s \nabla^2 T + q_s'' = 0 \quad (11)$$

The continuity of the heat flux at the interface between the solid and the liquid is given as:

$$k_s \left. \frac{\partial T}{\partial n} \right|_s = k_{nf} \left. \frac{\partial T}{\partial n} \right|_{nf} \quad (12)$$

A no-slip boundary condition is specified at the wall of the channel, $\vec{u} = 0$, at the inlet ($x = 0$), $u_x = u_y = 0$, $T = T_{in}$ and

$$P = P_{in} \quad (13)$$

At the outlet ($x = L$), zero normal stress, $P_{out} = 1 \text{ atm}$

At the solid boundaries,

$$\nabla T = 0 \quad (14)$$

We examined Al_2O_3 –water nanofluid mathematically; The thermophysical properties of water and Al_2O_3 nanoparticles taken at 300 K were used during the calculations [28]. Three different nanoparticle volume fractions were used (0.0%, 2% and 4%). The density, viscosity, specific heat capacity and thermal conductivity of Al_2O_3 –water nanofluids are determined by employing the following equations[29-33]:

$$\rho_{nf} = (1 - \phi) \rho_{bf} + \phi \rho_p \quad (15)$$

$$(\rho c_p)_{nf} = (1 - \phi) (\rho c_p)_{bf} + \phi (\rho c_p)_p \quad (16)$$

$$\mu_{nf} = \frac{1}{(1 - \phi)^{2.5}} \mu_{bf} \quad (17)$$

$$k_{nf} = \left(\frac{k_p + (n-1)k_f - (n-1)\phi(k_f - k_p)}{k_p + (n-1)k_f + \phi(k_f - k_p)} \right) \times k_f \quad (18)$$

where ϕ is the particle volume fraction and the subscripts p, f, nf correspond to particle, fluid and nanofluid. The spherical particles are assumed for the nanoparticles with $n=3$.

The measure of performance is the minimum global peak temperature, which could be expressed in a dimensionless form as:

$$(T_{max})_{min} \quad (19)$$

And it is a function of the optimised design variables and other parameters.

$$(T_{max})_{min} = f(d_h, w_{el}, \phi, \phi) \quad (20)$$

NUMERICAL PROCEDURE AND GRID ANALYSIS

The simulation work began by fixing the length of the channel, applied pressure difference, porosity, internal heat generation and material properties and we kept varying the values side dimension and hydraulic diameter of the channel in order to identify the best (optimal) internal configuration, which minimised the peak temperature. The numerical solution of the continuity, momentum and energy Equations. (8) - (11) along with the boundary conditions (12) - (14) was obtained by using a three-dimensional commercial package FLUENT™ [34]. FLUENT™ uses a finite volume method to convert the governing partial differential equations into a system of discrete algebraic equations by discretizing the computational domain. The details of the method were explained by Patankar [35]. The FLUENT™ was coupled with a geometry and mesh generation package GAMBIT [36] using MATLAB [37] to allow the automation and running of the simulation process. The computational domain was discretised using hexahedral/wedge elements. A second-order upwind scheme was used to discretise the combined convection and diffusion terms in the momentum and energy equations. The SIMPLE algorithm was then employed to solve the coupled pressure-velocity fields of the transport equations. After the simulation had converged, an output file was obtained containing all the necessary simulation data and results for the post-processing and analysis. The solution was assumed to have converged when the normalised residuals of the mass and momentum equations fall below 10^{-6} and while the residual convergence of energy equation was set to less than 10^{-10} . The number of grid cells used for the simulations varied for different elemental volume and porosities. However, grid independence tests for several mesh

refinements were carried out to ensure the accuracy of the numerical results. The convergence criterion for the overall thermal resistance as the quantity monitored is:

$$\gamma = \frac{|(T_{\max})_i - (T_{\max})_{i+1}|}{|(T_{\max})_i|} \leq 10^{-5} \quad (21)$$

where i is the mesh iteration index. The mesh is more refined as i increases. The $i-1$ mesh is selected as a converged mesh when the criterion (21) is satisfied.

To ensure accurate results, several grid independence tests were conducted until a mesh size with negligible changes in excess heat transfer was obtained. Table 1 shows the grid independence test for hexagonal cooling channel configuration cell densities of 19622, 32796, 41630, and 48510 which were used for the grid independence test. It was observed that almost identical results were predicted when 478483 and 571300 cell densities were used. Therefore, any increase in the cell density beyond 478483 had a negligible effect on the numerical result.

Table 1: Grid independence study for hexagonal cooling channel configuration with

Number of nodes	Number of cells	T_{\max}	$\gamma = \frac{ (T_{\max})_i - (T_{\max})_{i+1} }{ (T_{\max})_i } \leq 10^{-5}$
19622	13520	27.78085	-
32796	13520	27.84381	0.0022611
41630	31750	27.84368	0.0000044
48510	48510	27.78085	0.0000011

NUMERICAL RESULTS

In this section, we present results for the case when the elemental volume was in the range of $0.025 \text{ mm}^3 \leq v_{el} \leq 5 \text{ mm}^3$ and the nanoparticles volume fraction is fixed at $\phi = 0.2$, $\varphi = 0\%$ and 2% with a fixed length of $L = 10 \text{ mm}$, and fixed applied pressure differences of $\Delta P = 50 \text{ kPa}$. The thermal conductivity of the solid structure (silicon) is 148 W/m.K , and the internal heat generation within the solid was taken to be fixed at 100 W/cm^3 . The thermophysical properties of water [38] and that of Al_2O_3 nanoparticles [41] used in this study were based at 300 K and the inlet Al_2O_3 -water nanofluid temperature was fixed at this temperature.

Figure 4 indicates the maximum peak temperature as a function of the dimensionless hydraulic diameter of the hexagonal channel for a given pressure difference. It shows that there exists an optimum channel hydraulic diameter, which lies in the range $0.002 \leq d_h/L \leq 0.03$ that minimises the peak temperature. Any further increase or decrease in the design variable beyond the optimal values indicates that the working fluid is not properly engaged in the cooling process. This is detrimental to the global performance of the system. The results also show that the optimal arrangement of the elemental volume for the entire structure at this fixed pressure difference should be very small in order to achieve better cooling.

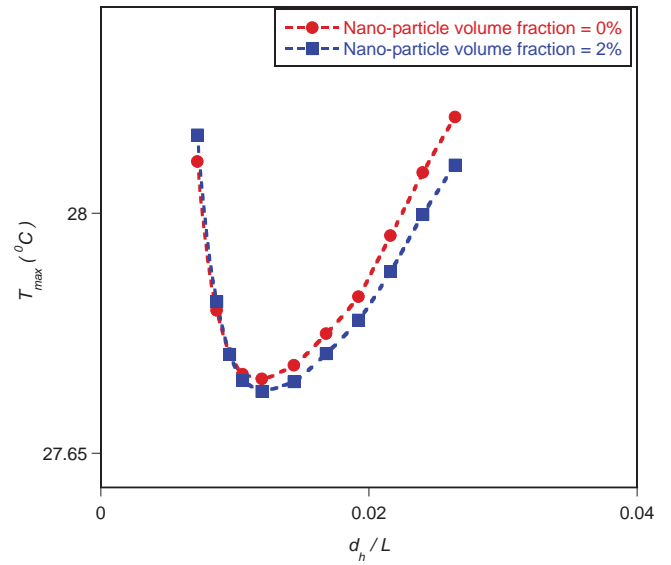


Figure 4 Effect of optimised hydraulic diameter, d_h , on the peak temperature

MATHEMATICAL OPTIMISATION

This section introduces an optimisation algorithm that will search and identify the optimal design variables at which the system will perform best. A numerical algorithm, Dynamic-Q [39], is employed and incorporated into the finite volume solver and grid (geometry and mesh) generation package by using MATLAB for more efficient and better accuracy in determining the optimal performance.

The Dynamic-Q is a multidimensional and robust gradient-based optimisation algorithm, which does not require an explicit line search. The technique involves the application of a dynamic trajectory LFOPC optimisation algorithm to successive quadratic approximations of the actual problem [41]. The details of the Dynamic-Q and applications can be found in open literature [39-42].

OPTIMISATION CONSTRAINT

The constraint ranges for the optimisation are:

$$\begin{aligned} 0.025 \text{ mm}^3 \leq v_{el} \leq 5 \text{ mm}^3, \quad h = w, \quad 0 \leq d_h \leq w, \\ 0.1 \leq \phi \leq 0.2, \quad 0\% \leq \varphi \leq 4\%, \end{aligned} \quad (27)$$

NUMERICAL OPTIMISATION RESULTS AND DISCUSSION

In this section, we discussed the optimization results of cooling channel geometry of forced convection heat transfer using nanofluid and studied the effect of nano-particles volume concentration on the heat transfer enhancement. We present results for the case when the elemental volume was in the range of $0.025 \text{ mm}^3 \leq v_{el} \leq 5 \text{ mm}^3$ and the porosity was fixed $\phi = 0.2$ with a fixed length of $L = 10 \text{ mm}$, Al_2O_3 nanoparticles was used and disperse into conventional water at the

range of 1 to 4% volume fraction for the numerical analysis. The optimisation process was repeated for applied dimensionless pressure differences ranging from $\Delta P = 10$ kPa to $\Delta P = 50$ kPa.

Figure 5 shows the minimised peak temperature as a function of pressure difference at two different porosities of 0.1 and 0.2. The results show that the minimised global peak temperature monotonically decreases as the pressure difference increases for different values of nanoparticles volume fraction and porosities. Also, it shows that as the nanoparticles volume fraction increases, the minimised peak temperature decreases for porosity of 0.1. However, at porosity of 0.2, the minimised peak temperature is fairly the same at nanoparticles volume fraction of 2% and 4% as the peak does not sensitive to increase in nanoparticles volume fraction despite the fact that the effective thermal conductivity increases when dispersed nanoparticles into the conventional fluid (water).

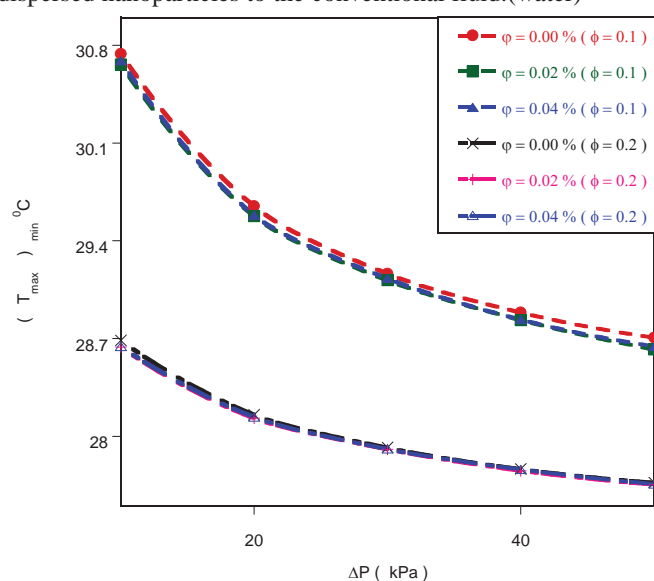


Figure 5. The effect of pressure difference on the minimised peak temperature

Also, Figs. 6 shows the optimal behaviours of the geometry with respect to applied pressure difference for different values of nanoparticles volume fraction. It shows that the optimal hydraulic diameter decreases as the dimensionless pressure differences increase for different values of nanoparticles volume fraction. Also, there exists a unique optimal geometry for each of the applied pressure differences for different values of nanoparticles volume fraction for the configuration.

CONCLUSION

This paper applied constructal theory and design to present the numerical search for geometric structures of conjugate cooling channels in forced convection with internal heat generation within the solid for an array of parallel hexagonal cooling channels. The effects of geometrical parameters and nanofluid were comprehensively studied and Al_2O_3 -water nanofluid was used as cooling fluid because of its

promising superior thermal properties over traditional cooling fluids. The numerical results obtained show that there are unique optimal geometries for a given pressure difference for the configuration considered. Also, the results show that the minimum peak temperature decreases as pressure difference, and porosity increase. Again, it shows that the minimised peak temperature is almost the same at nanoparticles volume fraction of 2% and 4% as the peak does not sensitive to increase in nanoparticles volume fraction within his range despite the fact that the effective thermal conductivity increases when dispersed nanoparticles into the conventional fluid. Future work will consider the case where the nanoparticles volume fraction dispersed into the conventional fluid is beyond 4%.

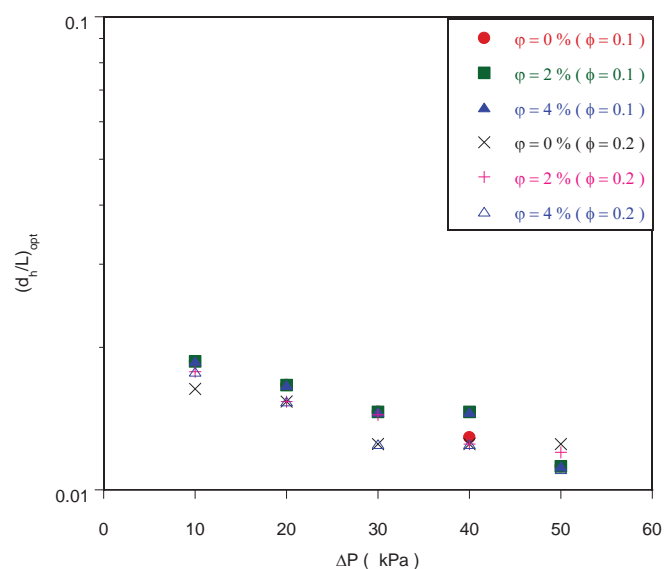


Figure 6. Effect of pressure difference on optimised hydraulic diameter, d_h

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