ANNULAR DUCTS: NUSSELT NUMBER CORRELATIONS FOR LAMINAR FLOWS OF LIQUIDS WITH TEMPERATURE DEPENDENT PROPERTIES

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ABSTRACT

In this work, new correlations are proposed for the mean Nusselt number in the entrance region of concentric annular ducts with uniform heat flux boundary conditions specified at the inner wall. These correlations are obtained on the basis of the numerical results of a previous parametric investigation on the effects of temperature dependent viscosity and thermal conductivity in simultaneously developing laminar flows of liquids in straight ducts of constant cross-sections. A superposition method is applied for the estimation of the Nusselt number by considering separately the effects of temperature dependent viscosity and thermal conductivity.

INTRODUCTION

Quite often in laminar flows, the total length of the duct is comparable with that of the entrance region. In such cases, the entrance effects on fluid flow and forced convection heat transfer must be taken into account. Moreover, temperature dependence of fluid properties can also play an important role in the development of the velocity and temperature fields. If the fluid is a liquid, the relative variations of viscosity with temperature are the most relevant, while those of thermal conductivity are, in general, much smaller, and those of density and specific heat capacity are almost negligible [1,2]. As far as velocity distribution and pressure drop are concerned, the main effects of temperature dependent fluid properties can be retained even if only viscosity is allowed to vary with temperature, while the other properties are assumed constant [1,3]. Instead, heat transfer characteristics, namely the Nusselt number, are also influenced by the variations of thermal conductivity [2], which can both increase or decrease with increasing temperature, depending on the nature of the fluid considered.

In the past we carried out systematic numerical analyses of the effects of temperature dependent viscosity and thermal conductivity on forced convection in simultaneously developing laminar flows of liquids in straight ducts with uniform wall heat flux boundary conditions [4,5]. Then we used the numerical data to obtain correlations, suitable for engineering applications, for the mean Nusselt number in the entrance region of circular tubes and square ducts with uniform heat flux boundary conditions specified at the walls [6]. A superposition method has been proved to be applicable in order to estimate the Nusselt number by considering separately the effects of temperature dependent viscosity and thermal conductivity.

In this work, we extend our previous research and we propose correlations for the longitudinally averaged Nusselt number on the inner wall of concentric annular ducts with an adiabatic outer wall and a ratio of the inner to the outer radius equal to 0.75. In our model, we assume that viscosity decreases with increasing temperature according to an exponential relation while thermal conductivity varies linearly with temperature. Suitable dimensionless parameters, namely, viscosity and thermal conductivity Pearson numbers, are used to quantitatively express temperature dependence of corresponding properties [5,6].

STATEMENT OF THE PROBLEM

This study concerns the laminar forced convection in the entrance region of straight annular ducts of constant cross-sections. The fluid is a liquid and heating is assumed to begin at the duct inlet. Uniform values of the axial velocity component u and of the temperature t (i.e., $t = t_e$, $u = u_e = \overline{u}$ and $v = 0$) are specified as the appropriate inlet conditions, being $\overline{u}$ the average axial velocity and v the radial velocity component. A uniform heat flux (Neumann) boundary condition $q'' = k\,\partial t/\partial n = q''_w > 0$ (fluid heating) is applied at the inner wall, while the outer wall is adiabatic. In the previous expression, k is the thermal conductivity and n is the outward normal to the boundary.

As the fluids considered here are liquids, the dynamic viscosity μ is assumed to decrease with increasing temperature according to the widely used exponential formula [3]

$$\mu = \mu_e \exp[-\beta\,(t - t_e)] \tag{1}$$

where μ_e is the value of μ at t_e and $\beta = -(\mathrm{d}\mu/\mathrm{d}t)/\mu = \mathrm{const}$ is positive. The thermal conductivity k can be assumed to both increase or decrease with increasing temperature, depending on the fluid considered, according to the linear relation

$$k = k_e\left[1 + \alpha\left(t - t_e\right)\right] \tag{2}$$

where k_e is the value of k at t_e and $\alpha = (\mathrm{d}k/\mathrm{d}t)/k_e = \mathrm{const}$ can be both positive or negative. By means of simple manipulations, equations (1) and (2) can be cast in the following dimensionless forms [5,6]

$$\frac{\mu}{\mu_e} = \exp(-\mathrm{Pn}_\mu T) \tag{3}$$

$$\frac{k}{k_e} = 1 + \mathrm{Pn}_k T \tag{4}$$

where $T = (t - t_e)\, k_e/(q''_w\, D_h)$ is the dimensionless temperature, D_h is the hydraulic diameter, $\mathrm{Pn}_\mu = \beta\, q''_w\, D_h/k_e$ is the viscosity Pearson number (representing the ratio of the characteristic process temperature difference $q''_w\, D_h/k_e$ to the characteristic temperature difference $1/\beta$ that can produce appreciable viscosity variations) and $\mathrm{Pn}_k = \alpha\, q''_w\, D_h/k_e$ is the thermal conductivity Pearson number (representing the ratio of the characteristic process temperature difference $q''_w\, D_h/k_e$ to the characteristic temperature difference $1/\alpha$ that can produce appreciable thermal conductivity variations).

Since the density ρ and the specific heat c are constant, the local Reynolds number $\mathrm{Re} = \rho\, \overline{u}\, D_h/\mu$, the local Prandtl number $\mathrm{Pr} = \mu c/k$ and the local Péclet number $\mathrm{Pe} = \mathrm{Re}\,\mathrm{Pr} = \rho c \overline{u}\, D_h/k$ all vary with temperature because of the variations of μ, of the ratio μ/k and of k, respectively. Therefore we have $\mu/\mu_e = \mathrm{Re}_e/\mathrm{Re}$ and $k/k_e = \mathrm{Pe}_e/\mathrm{Pe}$, where Re_e and Pe_e are the Reynolds and Péclet numbers evaluated at t_e. Moreover, since the viscosity of liquids decreases with increasing temperature ($\beta > 0$), in the case of fluid heating we have $\mathrm{Pn}_\mu > 0$ and $\mathrm{Re}_e/\mathrm{Re} < 1$, while $\mathrm{Pn}_\mu = 0$ and $\mathrm{Re}_e/\mathrm{Re} = 1$ refer to constant viscosity fluids ($\beta = 0$). Instead, since the thermal conductivity can either increase ($\alpha > 0$) or decrease ($\alpha < 0$) with increasing temperature, we have $\mathrm{Pn}_k > 0$ and $\mathrm{Pe}_e/\mathrm{Pe} > 1$ in the first case and $\mathrm{Pn}_k < 0$ and $\mathrm{Pe}_e/\mathrm{Pe} < 1$ in the second one, while $\mathrm{Pn}_k = 0$ and $\mathrm{Pe}_e/\mathrm{Pe} = 1$ refer to constant thermal conductivity fluids ($\alpha = 0$).

As the fluid temperature rises along the duct, the viscosity decreases while Re increases. Therefore, to ensure laminar flow conditions, the local Reynolds number Re_b evaluated at the bulk temperature is only allowed to reach the maximum value $(\mathrm{Re}_b)_{max} = 2,000$, corresponding to the maximum value x_{max} of the axial coordinate x, whereupon computations are stopped. Therefore, taking into account equation (3) and the appropriate heat balance for the duct, the following expression for the maximum value X^*_{max} of the dimensionless axial coordinate $X^* = x/(D_h\, \mathrm{Pe}_e)$ can be obtained [4,5,7]

$$X^*_{max} = \frac{C}{4\,\mathrm{Pn}_\mu\, C_q} \ln\left[\frac{(\mathrm{Re}_b)_{max}}{\mathrm{Re}_e}\right] \tag{5}$$

where C and C_q are the perimeter of the cross section and the heated perimeter, respectively.

Computed results of interest for the present study are represented by the axial distributions of the local Nusselt number $\mathrm{Nu} = hD_h/k_e$, where h is the peripherally averaged local convective heat transfer coefficient defined as

$$h = \frac{q''_w}{t_w - t_b} = \frac{k_e}{D_h\,(T_w - T_b)} \tag{6}$$

In equation (6), t_w and t_b are the wall temperature and the mean bulk temperature, respectively, and T_w and T_b are their dimensionless forms. According to equation (6), the local Nusselt number can be expressed as

$$\mathrm{Nu} = \frac{1}{T_w - T_b} \tag{7}$$

Axial distributions of the longitudinally averaged Nusselt number, defined as [1]

$$\overline{\mathrm{Nu}} = \frac{1}{x}\int_0^x \mathrm{Nu}\,\mathrm{d}x = \frac{1}{X^*}\int_0^{X^*} \mathrm{Nu}\,\mathrm{d}X^* \tag{8}$$

can be obtained by means of an appropriate numerical integration rule.

NUMERICAL PROCEDURE

In laminar duct flows, the effects of axial diffusion can be neglected when $\mathrm{Re}_e > 50$ and $\mathrm{Pe}_e > 50$ [1]. If there is also no recirculation in the longitudinal direction, steady-state flow and heat transfer in straight ducts of constant cross-sections are governed by the continuity and the parabolized Navier-Stokes and energy equations [8,9]. These equations are not reported here for lack of space, but they are reported elsewhere together with the boundary conditions specified on the boundaries of the computational domain [5,7].

The model equations are solved using a finite element procedure for the analysis of the forced convection of fluids with temperature dependent properties in the entrance region of straight ducts [7,10,11]. The adopted procedure is based on a segregated approach which implies the sequential solution of the momentum and energy equations on a two-dimensional domain in the case of three-dimensional geometries and on a one-dimensional domain in axisymmetric problems, corresponding to the cross-section of the duct. A marching method is then used to move forward in the axial direction. The pressure-velocity coupling is dealt with using an improved projection algorithm already employed by one of the authors (C.N.) for the solution of the Navier-Stokes equations in their elliptic forms [12]. The procedure has already been validated, with reference to both constant and temperature dependent property fluids, by comparing heat transfer and pressure drop results with existing literature data for simultaneously developing laminar flows in straight ducts [10,11,13,14].

SIMULATION PARAMETERS

The cross-sectional geometry considered in this study is that of a concentric annular duct with a ratio of the inner to the outer radius $r_i/r_o = 0.75$. Since the Navier-Stokes equations are solved in their parabolized form, the computational domain corresponding to the annular cross-section, defined taking into account the axial symmetry, is one-dimensional and is discretized using 80 three-node Lagrangian elements and 161 nodes [7]. The axial step is gradually increased from the starting value $\Delta x = 0.0001D_h$ to the maximum value $\Delta x = 0.05D_h$.

In all the computations, the same value is assumed for the entrance Reynolds number ($\mathrm{Re}_e = 100$). Instead, the values $\mathrm{Pr}_e = 5, 20$ and 100 of the Prandtl number at t_e are selected to take into account the behaviours of different liquids. The corresponding values of the Péclet number at t_e are $\mathrm{Pe}_e = 500, 2000$ and 10 000. The values of the dynamic viscosity Pearson number $\mathrm{Pn}_\mu = 0, 1, 2$ and 4 are considered to account for reasonable viscosity temperature dependences. Thus, for the assumed Re_e, the maximum values of the dimensionless axial coordinate given by equation (5) are $X^*_{max} = 1.7474$, 0.8738 and 0.4369 for $\mathrm{Pn}_\mu = 1, 2$ and 4, respectively. For each nonzero value of Pn_μ eight values of Pn_k are selected (four positive and four negative), giving the corresponding value ratios $\mathrm{Pn}_\mu/\mathrm{Pn}_k = \beta/\alpha = \pm 10, \pm 20, \pm 40$ and ± 80. Thus, on the whole, the values $\mathrm{Pn}_k = 0, \pm 0.0125, \pm 0.025, \pm 0.05, \pm 0.1, \pm 0.2$ and ± 0.4 are considered.

SUPERPOSITION METHOD

As demonstrated in [4,5], the effects of temperature dependent properties (viscosity and thermal conductivity) on heat transfer can be illustrated by comparing the local Nusselt number $\mathrm{Nu}_{\mu k}$, obtained for given nonzero values of Pn_μ and Pn_k, with the corresponding local Nusselt number Nu_c, computed for simultaneously developing constant property flows ($\mathrm{Pn}_\mu = \mathrm{Pn}_k = 0$). Therefore, we can assume the value of the ratio $(\mathrm{Nu}_{\mu k} - \mathrm{Nu}_c)/\mathrm{Nu}_c = (\mathrm{Nu}_{\mu k}/\mathrm{Nu}_c) - 1$ as a measure of the combined effects of both temperature dependent viscosity and thermal conductivity. Axial distributions of the ratio $\mathrm{Nu}_{\mu k}/\mathrm{Nu}_c$ for concentric annular ducts and the same values of dimensionless input parameters considered in this paper have been obtained by the authors in the past using the numerical procedure described above and they are reported in [5]. The conclusion reached there is that both temperature dependent viscosity and thermal conductivity can have comparable effects on the Nusselt number, according to the values of Pn_μ and Pn_k. Moreover, it has been demonstrated that if the effects of temperature dependent viscosity and thermal conductivity are considered separately to compute Nu_μ (under the assumptions of temperature dependent viscosity and constant thermal conductivity, i.e., $\mathrm{Pn}_\mu > 0$ and $\mathrm{Pn}_k = 0$) and Nu_k (under the assumptions of temperature dependent thermal conductivity and constant viscosity, i.e., $\mathrm{Pn}_k \neq 0$ and $\mathrm{Pn}_\mu = 0$), a superposition method is applicable in order to obtain approximate values of the Nusselt number $\mathrm{Nu}_{\mu k}$, according to the relation

$$\mathrm{Nu}_{\mu k} - \mathrm{Nu}_c \cong (\mathrm{Nu}_\mu - \mathrm{Nu}_c) + (\mathrm{Nu}_k - \mathrm{Nu}_c) \qquad (9)$$

with an accuracy which can be considered satisfactory in most situations. With reference to the ratios $\mathrm{Nu}_{\mu k}/\mathrm{Nu}_c$, $\mathrm{Nu}_\mu/\mathrm{Nu}_c$ and $\mathrm{Nu}_k/\mathrm{Nu}_c$, equation (9) can be recast in the form [4,5]

$$\frac{\mathrm{Nu}_{\mu k}}{\mathrm{Nu}_c} \cong \left(\frac{\mathrm{Nu}_{\mu k}}{\mathrm{Nu}_c}\right)' = \frac{\mathrm{Nu}_\mu}{\mathrm{Nu}_c} + \frac{\mathrm{Nu}_k}{\mathrm{Nu}_c} - 1 \qquad (10)$$

where $(\mathrm{Nu}_{\mu k}/\mathrm{Nu}_c)'$ is the approximate value of $\mathrm{Nu}_{\mu k}/\mathrm{Nu}_c$ given by the superposition method. Therefore, the values of the differences $(\mathrm{Nu}_\mu/\mathrm{Nu}_c) - 1$ and $(\mathrm{Nu}_k/\mathrm{Nu}_c) - 1$ approximately measure the separate effects of temperature dependent viscosity and thermal conductivity, respectively.

In this work, previous numerical results concerning axial distributions of the ratios $\mathrm{Nu}_{\mu k}/\mathrm{Nu}_c$, $\mathrm{Nu}_\mu/\mathrm{Nu}_c$ and $\mathrm{Nu}_k/\mathrm{Nu}_c$ [5] have been used to obtain, by means of a suitable numerical integration rule according to equation (8), the corresponding distributions of the ratios $\overline{\mathrm{Nu}}_{\mu k}/\overline{\mathrm{Nu}}_c$, $\overline{\mathrm{Nu}}_\mu/\overline{\mathrm{Nu}}_c$ and $\overline{\mathrm{Nu}}_k/\overline{\mathrm{Nu}}_c$. Then, it has been verified that the following relation, obtained by combining equations (8) and (9), still holds true with an accuracy which can be considered satisfactory in most situations

$$\frac{\overline{\mathrm{Nu}}_{\mu k}}{\overline{\mathrm{Nu}}_c} \cong \left(\frac{\overline{\mathrm{Nu}}_{\mu k}}{\overline{\mathrm{Nu}}_c}\right)' = \frac{\overline{\mathrm{Nu}}_\mu}{\overline{\mathrm{Nu}}_c} + \frac{\overline{\mathrm{Nu}}_k}{\overline{\mathrm{Nu}}_c} - 1 \qquad (11)$$

In equation (11), $(\overline{\mathrm{Nu}}_{\mu k}/\overline{\mathrm{Nu}}_c)'$ represents the approximate value of $\overline{\mathrm{Nu}}_{\mu k}/\overline{\mathrm{Nu}}_c$ given by the superposition method. As an example, axial distributions of the ratio $(\overline{\mathrm{Nu}}_{\mu k}/\overline{\mathrm{Nu}}_c)'$ for $\mathrm{Pr}_e = 20$, the three values of Pn_μ considered and different values of Pn_k are reported in Figure 1 and compared with the corresponding distributions of $\overline{\mathrm{Nu}}_{\mu k}/\overline{\mathrm{Nu}}_c$. As can be seen, even for the highest values of Pn_μ and the highest/lowest values of Pn_k the dashed curves, representing axial distributions of the ratio $(\overline{\mathrm{Nu}}_{\mu k}/\overline{\mathrm{Nu}}_c)'$, are very close to the solid ones giving the numerical solutions for $\overline{\mathrm{Nu}}_{\mu k}/\overline{\mathrm{Nu}}_c$, thus confirming the validity of equation (11). Similar comparisons could be carried out for $\mathrm{Pr}_e = 5$ and 100, but are not reported here for the sake of brevity. Anyway, the maximum positive and negative relative errors in the approximation of $\overline{\mathrm{Nu}}_{\mu k}/\overline{\mathrm{Nu}}_c$ by means of $(\overline{\mathrm{Nu}}_{\mu k}/\overline{\mathrm{Nu}}_c)'$ are $\varepsilon^+_{max} = 0.13\%$ and $\varepsilon^-_{max} = -0.31\%$ in the range $10^{-4} \leq X^* \leq X^*_{max}$ with $\mathrm{Pr}_e = 5$, 20 and 100 and different values of Pn_μ and Pn_k.

CORRELATIONS FOR THE MEAN NUSSELT NUMBER

Taking advantage of equation (11), a correlation for $\overline{\mathrm{Nu}}_{\mu k}$ has been obtained by assembling the different correlations for $\overline{\mathrm{Nu}}_c$, $\overline{\mathrm{Nu}}_\mu/\overline{\mathrm{Nu}}_c$ and $\overline{\mathrm{Nu}}_k/\overline{\mathrm{Nu}}_c$ obtained from computed results for flows of liquids with constant property ($\mathrm{Pn}_k = \mathrm{Pn}_\mu = 0$), temperature dependent viscosity ($\mathrm{Pn}_k = 0$ and $\mathrm{Pn}_\mu > 0$) and temperature dependent thermal conductivity ($\mathrm{Pn}_\mu = 0$ and $\mathrm{Pn}_k \neq 0$), respectively. All the correlations reported in this paper have been developed using the nonlinear least-square Levenberg-Marquardt algorithm implemented in the Gnuplot 4.0 software package and are only valid for $r_i/r_o = 0.75$.

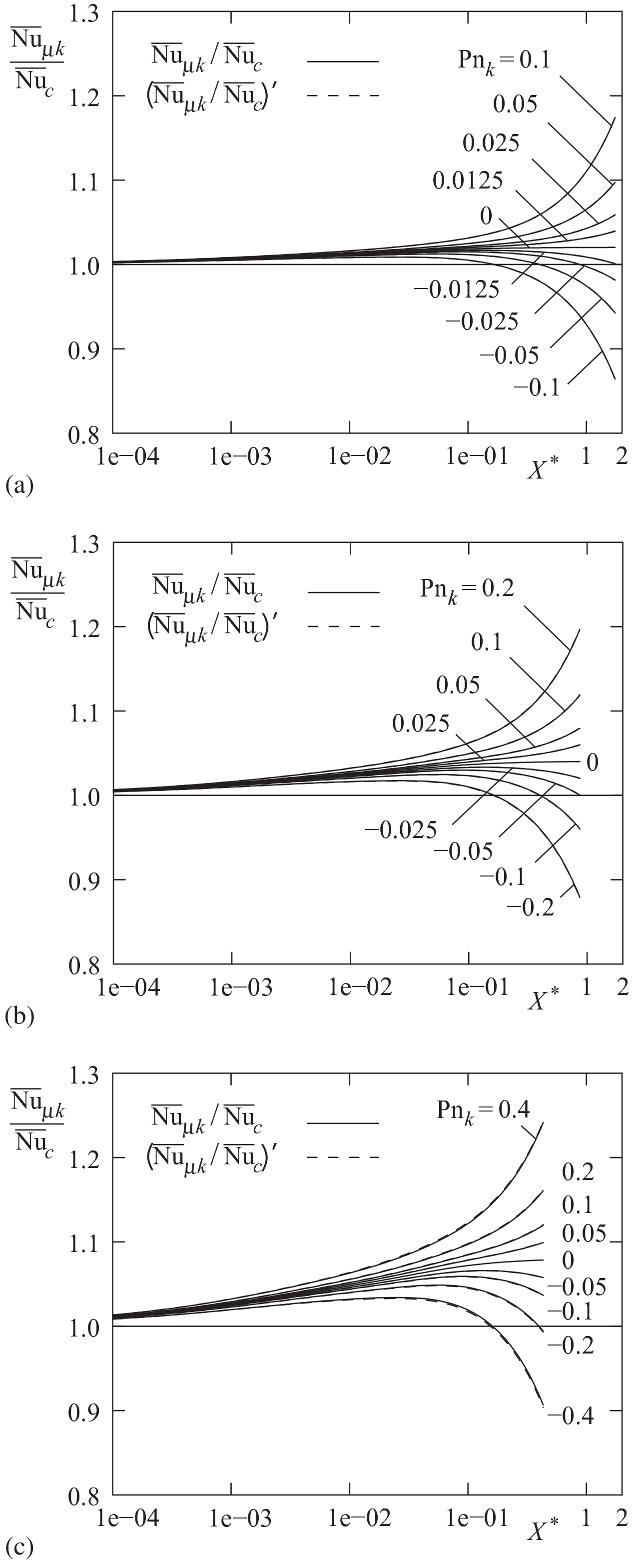

Figure 1 Comparison of axial distributions of the ratios $\overline{\mathrm{Nu}}_{\mu,k}/\overline{\mathrm{Nu}}_c$ and $\left(\overline{\mathrm{Nu}}_{\mu,k}/\overline{\mathrm{Nu}}_c\right)'$ for simultaneously developing laminar flows in annular ducts with $\mathrm{Pr}_e = 20$, different values of Pn_k and: (a) $\mathrm{Pn}_\mu = 1$, (b) $\mathrm{Pn}_\mu = 2$ and (c) $\mathrm{Pn}_\mu = 4$.

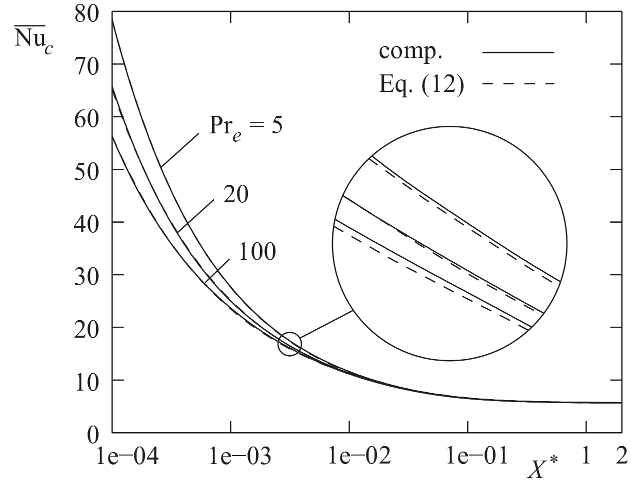

Figure 2 Comparison of computed and predicted axial distributions of the mean Nusselt number $\overline{\mathrm{Nu}}_c$ on the heated inner wall of annular ducts with the considered values of Pr_e (zoom: 10-time magnification).

Liquids with constant properties

The results obtained under the assumption of constant property flow ($\mathrm{Pn}_k = \mathrm{Pn}_\mu = 0$) have been used to obtain a correlation, valid for $X^* \geq 10^{-4}$, for the mean Nusselt number $\overline{\mathrm{Nu}}_c$ on the heated inner wall of concentric annular ducts with $r_i/r_o = 0.75$

$$\overline{\mathrm{Nu}}_c = 5.6443 + \frac{0.070\,(X^*)^{-1.35}}{1 + 0.119\,\mathrm{Pr}_e^{-0.08}\,(X^*)^{-n}} \qquad (12)$$

where 5.6443 is the asymptotic value of Nu_c for fully developed conditions and the exponent n depends on the Prandtl number Pr_e according to the relation

$$n = 0.801\,\mathrm{Pr}_e^{0.0304} - 0.000155\,\mathrm{Pr}_e \qquad (13)$$

Comparison are shown in Figure 2 between the computed axial distribution of the mean Nusselt number $\overline{\mathrm{Nu}}_c$ on the heated inner wall of annular ducts and the corresponding predictions of the correlation (12). As can be seen, the agreement is quite good for all the values of Pr_e considered. The maximum positive and negative relative errors in the approximation of $\overline{\mathrm{Nu}}_c$ by means of equation (12) are $\varepsilon^+_{max} = 1.14\%$ and $\varepsilon^-_{max} = -1.52\%$, while the maximum residual standard deviation is $s_{max} = 0.53\%$.

Liquids with temperature dependent viscosity

The results obtained for annular ducts for $\mathrm{Pn}_k = 0$ and $\mathrm{Pn}_\mu > 0$ have first been used to obtain a correlation for asymptotic value of the ratio $\overline{\mathrm{Nu}}_\mu/\overline{\mathrm{Nu}}_c$

$$\left(\frac{\mathrm{Nu}_\mu}{\mathrm{Nu}_c}\right)_{fd} = 1 + 0.0204\,\mathrm{Pn}_\mu + 0.000051\,\mathrm{Pn}_\mu^2 \qquad (14)$$

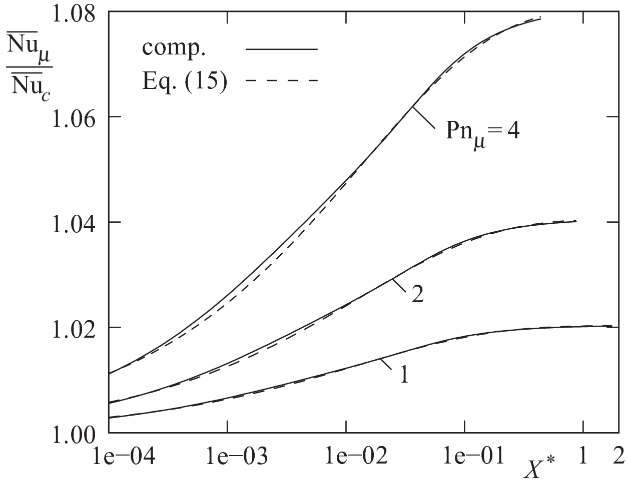

Figure 3 Comparison of computed and predicted axial distributions of the ratios $\overline{\mathrm{Nu}}_\mu/\overline{\mathrm{Nu}}_c$ for simultaneously developing laminar flows in annular ducts with $\mathrm{Pr}_e = 20$ and $\mathrm{Pn}_k = 0$.

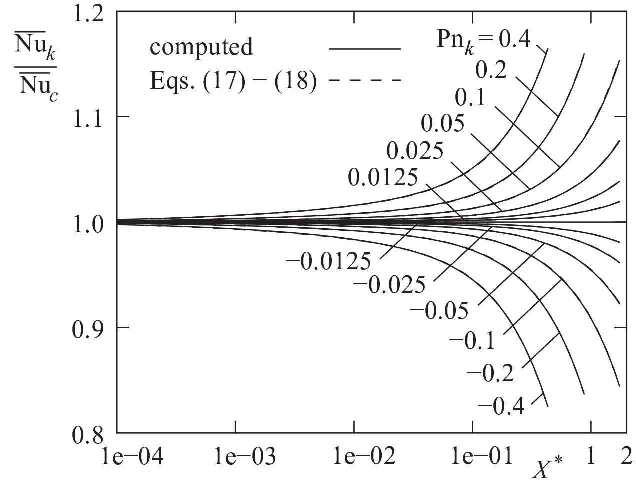

Figure 4 Comparison of computed and predicted axial distributions of the ratios $\overline{\mathrm{Nu}}_k/\overline{\mathrm{Nu}}_c$ for simultaneously developing laminar flows in annular ducts with $\mathrm{Pr}_e = 20$, $\mathrm{Pn}_\mu = 0$.

and then one for the ratio $\overline{\mathrm{Nu}}_\mu/\overline{\mathrm{Nu}}_c$ as a function of X^*

$$\begin{aligned}\frac{\overline{\mathrm{Nu}}_\mu}{\overline{\mathrm{Nu}}_c} = 1 &+ \left(0.0204\,\mathrm{Pn}_\mu - 0.000051\,\mathrm{Pn}_\mu^2\right) \\ &\cdot \{1 - \exp\left[-\left(1.43 - 0.44\,\mathrm{Pn}_\mu\right)(X^*)^m \right. \\ &\left. -4.0\,(X^*)^{0.4}\right]\}\end{aligned} \tag{15}$$

In equation (15), the exponent m depends on the Prandtl number Pr_e and can be expressed as

$$m = 0.55\,\mathrm{Pr}_e^{-0.15} \tag{16}$$

The range of validity of the above correlation is $5 \leq \mathrm{Pr}_e \leq 100$, $1 \leq \mathrm{Pn}_\mu \leq 4$ and $10^{-4} \leq X^* \leq X^*_{max}$. As an example, computed axial distributions of the ratios $\overline{\mathrm{Nu}}_\mu/\overline{\mathrm{Nu}}_c$ for simultaneously developing laminar flows in annular ducts with $\mathrm{Pr}_e = 20$ and $\mathrm{Pn}_k = 0$ are successfully compared in Figure 3 with those calculated using equation (15). For $\mathrm{Pr}_e = 5$, 20 and 100, the maximum positive and negative relative errors in the approximation of $\overline{\mathrm{Nu}}_\mu/\overline{\mathrm{Nu}}_c$ by means of equation (15) are $\varepsilon^+_{max} = 0.16\%$ and $\varepsilon^-_{max} = -0.17\%$, while the maximum residual standard deviation is $s_{max} = 0.068\%$. Therefore, we can conclude that the agreement between computed and approximate results is very good.

Liquids with temperature dependent thermal conductivity

The results obtained for $\mathrm{Pn}_\mu = 0$ and $\mathrm{Pn}_k \neq 0$ have been used to obtain two correlations for the ratio $\overline{\mathrm{Nu}}_k/\overline{\mathrm{Nu}}_c$ as a function of X^*, one for $\mathrm{Pn}_k > 0$

$$\begin{aligned}\frac{\overline{\mathrm{Nu}}_k}{\overline{\mathrm{Nu}}_c} &= 1 + 0.0425\,\mathrm{Pn}_k\,\{1 - \exp\left[-11(X^*)^n\right]\} \\ &+ \left(0.871\,\mathrm{Pn}_k - 0.042\,\mathrm{Pn}_k^2\right) X^*\end{aligned} \tag{17}$$

and one for $\mathrm{Pn}_k < 0$

$$\begin{aligned}\frac{\overline{\mathrm{Nu}}_k}{\overline{\mathrm{Nu}}_c} &= 1 + 0.0455\,\mathrm{Pn}_k\,\{1 - \exp\left[-11(X^*)^n\right]\} \\ &+ \left(0.850\,\mathrm{Pn}_k - 0.150\,\mathrm{Pn}_k^2\right) X^*\end{aligned} \tag{18}$$

In both cases the exponent n depends on the Prandtl number Pr_e and can be expressed as

$$n = 0.50\,\mathrm{Pr}_e^{-0.022} \tag{19}$$

The above correlations are valid for $5 \leq \mathrm{Pr}_e \leq 100$, $0.0125 \leq |\mathrm{Pn}_k| \leq 0.4$ and $10^{-4} \leq X^* \leq X^*_{max}$.

Computed axial distributions of the ratios $\overline{\mathrm{Nu}}_k/\overline{\mathrm{Nu}}_c$ for simultaneously developing laminar flows in annular ducts with $\mathrm{Pr}_e = 20$ and $\mathrm{Pn}_\mu = 0$ are reported in Figure 4 together with those yielded by equations (17) and (18). Also in this case the agreement is quite good, with maximum positive and negative relative errors in the approximation of $\overline{\mathrm{Nu}}_k/\overline{\mathrm{Nu}}_c$ by means of equations (17) and (18) that, for $\mathrm{Pr}_e = 5$, 20 and 100, are $\varepsilon^+_{max} = 0.26\%$ and $\varepsilon^-_{max} = -0.12\%$, while the maximum residual standard deviation is $s_{max} = 0.067\%$.

Liquids with temperature dependent viscosity and thermal conductivity

The correlations proposed in the previous sections can be used to predict the value of $\overline{\mathrm{Nu}}_{\mu k}$. In fact, according to equation (11), we can write

$$\begin{aligned}\overline{\mathrm{Nu}}_{\mu k} = \overline{\mathrm{Nu}}_c\,\frac{\overline{\mathrm{Nu}}_{\mu k}}{\overline{\mathrm{Nu}}_c} &\cong \overline{\mathrm{Nu}}_c \left(\frac{\overline{\mathrm{Nu}}_{\mu k}}{\overline{\mathrm{Nu}}_c}\right)' \\ &= \overline{\mathrm{Nu}}_c \left(\frac{\overline{\mathrm{Nu}}_\mu}{\overline{\mathrm{Nu}}_c} + \frac{\overline{\mathrm{Nu}}_k}{\overline{\mathrm{Nu}}_c} - 1\right)\end{aligned} \tag{20}$$

Table 1 Maximum absolute values $|\varepsilon|_{max}$ (%) of the relative error in the approximation of $\overline{\mathrm{Nu}}_{\mu k}$ by means of equation (20).

$\mathrm{Pn}_\mu/\mathrm{Pn}_k$	Pn_μ		
	1	2	4
−10	1.43	1.38	1.80
−20	1.43	1.36	1.78
−40	1.43	1.34	1.75
−80	1.43	1.34	1.75
80	1.43	1.34	1.74
40	1.42	1.34	1.73
20	1.42	1.32	1.71
10	1.41	1.34	1.67

where $\overline{\mathrm{Nu}}_c$, $\overline{\mathrm{Nu}}_\mu/\overline{\mathrm{Nu}}_c$ and $\overline{\mathrm{Nu}}_k/\overline{\mathrm{Nu}}_c$ are given by equations (12), (15), (17), and (18), respectively. The maximum absolute values $|\varepsilon|_{max}$ of the relative error in the approximation of $\overline{\mathrm{Nu}}_{\mu k}$ by means of equation (20) for $\mathrm{Pr}_e = 5$, 20 and 100 and different values of Pn_μ and $\mathrm{Pn}_\mu/\mathrm{Pn}_k$ are reported in Table 1 for the considered annular cross-sectional geometry. The good agreement between computed and approximate results confirms the validity of the proposed approach even for ducts with not simply connected cross-sections.

CONCLUSIONS

In this work, new correlations, suitable for engineering applications, for the mean Nusselt number in the entrance region of concentric annular ducts with uniform heat flux boundary conditions specified at the inner wall have been proposed. These correlations have been obtained on the basis of the results of a previous parametric investigation on the effects of temperature dependent viscosity and thermal conductivity in simultaneously developing laminar flows of liquids in straight ducts of constant cross-sections. In these studies, a finite element procedure has been employed for the numerical solution of the parabolized momentum and energy equations. Viscosity and thermal conductivity have been assumed to vary with temperature according to an exponential and to a linear relation, respectively, while the other fluid properties are held constant. The temperature dependences of viscosity and thermal conductivity have been quantitatively expressed by the corresponding Pearson numbers. Axial distributions of the mean Nusselt number, obtained by numerical integration from those of the local Nusselt number, have been used as input data in the derivation of the proposed correlations.

A superposition method has been proved to be applicable in order to estimate the Nusselt number by considering separately the effects of temperature dependent viscosity and thermal conductivity. Therefore, two distinct correlations have been proposed, one for flows of liquids with temperature dependent viscosity and one for flows of liquids with temperature dependent thermal conductivity, in addition to that obtained for flows of constant property fluids.

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