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RHEOLOGICAL BEHAVIOUR AND CONCENTRATION DISTRIBUTION OF PARAFFIN SLURRY IN HORIZONTAL RECTANGULAR CHANNEL

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ABSTRACT
The mathematical modeling of liquid-solid flows is very complex because the solid particles distribution in the carrier liquid is heterogeneous or even when moving beds occurs. In this paper, a mathematical modelling and a numerical simulation of the flow behaviour of the paraffin slurry in a horizontal rectangular channel are presented. The pressure drop was presented and analyzed. The diffusion equation was solved, and the concentrations of suspended solid particles in the mixture have been given for some values of the mixture's average velocity.

INTRODUCTION
Thermofluid systems involving liquid- solid flows and phase change arise in many industrial and engineering applications. In spite of the recent developments in experimental and computational techniques for two-phase flows [1], it is still difficult to give reliable predictions of flow characteristics in these systems, because the problem is heavily dependent on the system geometry and the experimental parameters such as flow and particle Reynolds numbers, particle size, and solids loading. Most of the existing correlations are entirely empirical, yielding poor accuracies when applied to systems other than those for which they were obtained [2].

In addition to technologies involving solidification and melting, solid- liquid flows arise in applications dealing with the transport of solid particles by a liquid phase. These applications require an understanding of both the mixture flow and relative flow between the liquid and the solid phases.

However, the mathematical modelling of solid-liquid two phase flow remains complex. In fact the distribution of suspended solid particles is not homogenous, which make changing the mixture thermal and rheological properties.

NOMENCLATURE

A	m^2	surface area
S	m^2	cross section
D_h	m	hydraulic diameter
d	m	diameter
C	(%)	local mass fraction or concentration
x	m	horizontal coordinate
y	m	vertical coordinate
L	m	channel's length
a	m	channel's width
b	m	channel's gap
T	K	temperature
u	m s^{-1}	velocity
V_m	m s^{-1}	mean velocity
μ	Pa .s	viscosity
ρ	kg m^{-3}	density
λ		friction factor
τ	Pa	shear stress
P	Pa	Pressure

Dimensionless numbers

Nu	Nusselt number
Pr	Prandtl number
Re	Reynolds number
He	Hedström number

Subscripts

l	liquid
h	hydraulic
m	mixture, mean
w	wall, water
p	particle
B	Bingham
C	Casson

RHEOLOGICAL MODEL

The paraffin slurries studied in the present work, are made of millimetric particles of paraffin, stabilized in an organic porous polymeric matrix, in suspension in low viscosity liquid (water) (**Figure 1**).

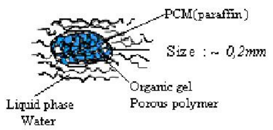

Figure 1 Paraffin particle suspended in the liquid phase

The paraffin used in this work is a commercial product manufactured by EXXON it's called NORPAR 15.

A spectrometric analysis shows that this product is a mixture of five paraffins: tetradecane (33%), pentadecane (43%), hexadecane (17%), heptadecane (5%) and octadecane (2%).

In this paper, the rheoligical behaviour and the pressure drop of the stabilized paraffin slurry flowing in horizontal rectangular channel of 1-m(Length)x0,08-m(Width)x0,006-m(Gap) have been analyzed.

The slurry transportation is divided into three flow patterns as it is shown in **Figure 2**: homogenous flow, heterogeneous flow and stationary bed flow [3]. For the homogenous flow the slurry has high average velocity values and the distribution of solid particles is nearly uniform across the channel cross-section. But when the slurry flow rate decreases, the heterogeneous flow occurs because of the gradient distribution of suspended solid particles in the perpendicular direction to the channel axis with more particles in the upper part. For a reduced slurry flow rate, the moving bed layer at the top occurs with an heterogeneous flow in the bottom, for very low flow rate, the stationary bed at the upper part of the channel is observed [4].

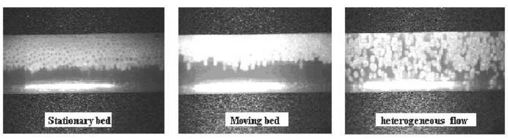

Figure 2 Flow pattern of the paraffin slurry for 6% particles mass fraction

Many different models have been used for the suspension viscosity determination. Most of them essentially extend the work of Einstein on spheres and his equation for viscosity [5] for C<1 %

$$\mu_m = \mu_l\left(1+2,5C\right) \qquad (1)$$

C denotes the concentration of suspended solid particles in the carrier liquid.

Equation (1) did not take into account the particles size and the particles position, the Einstein theory neglects the interaction between particles and the liquid.

However, Thomas equation takes the interactions between particles into account; it is given as follow [6]:

$$\mu_m = \mu_l\left(1+2,5C+10,05C^2+0,00273\exp(16,6C)\right) \qquad (2)$$

The model is valid for the concentrations up to C=0.625 and particle size ranging from 0.099 to 435mm. It considers that the flow is homogenous.

However, the suspensions viscosity depends also on particles size and shape.

There were several other models developed for the mixture viscosity determination, as shown in **Table 1**.

Table 1

Some expressions of suspension viscosity [16]

Einstein (1906)	$\mu_m = \mu_l\left(1+2,5C\right)$	C<1 % d_p< 2µm
Kunitz (1926)	$\mu_m = \mu_l\dfrac{\left(1+2,5C\right)}{\left(1-C\right)^4}$	10 % ≤ C ≤ 40 %
Guth and Simha (1936)	$\mu_m = \mu_l\dfrac{\left(1+0,5C-0,5C^2\right)}{\left(1-2C-9,6C^2\right)}$	$\mu_m \to \infty$ for $C=23,4\%$
Simha	$\mu_m = \mu_l\left(1+1,5C\left(1+\dfrac{25C}{4f^3}\right)\right)$	1 < f < 2 Dilute suspensions
	$\mu_m = \mu_l\left(1+\dfrac{54}{5f^3}\left(\dfrac{C}{1-\left(C/C_{max}\right)^3}\right)\right)$	C→ C_{max} Very concentrated suspensions
Vand (1948)	$\mu_m = \mu_l\exp\left(\dfrac{2,5C}{1-0,609C}\right)$	No intrerparticule forces
	$\mu_m = \mu_l\exp\left(\dfrac{2,5C+2,7C^2}{1-0,609C}\right)$	
Thomas (1965)	$\mu_m = \mu_l\left(\begin{array}{l}1+2,5C+10,5C^2+\\0,00273\exp\left(16,6C\right)\end{array}\right)$	$\rho_l \cong \rho_p$ 0 < C < 62,5 % 0,1 µm < d_p < 435 µm
Frankel and Acrivos	$\mu_m = \mu_l C\left(1-\left(\dfrac{C}{C_{max}}\right)^{\frac{1}{3}}\right)^{-1}$	Only concentrated suspensions
Mooney	$\mu_m = \mu_l\exp\left(\dfrac{2,5C}{1-KC}\right)$	0,75 < K< 1,5 K depends upon system
Jeffrey	$\mu_m = \mu_l\left(1+AC\right)$	2,5 < A < 10 Ellipsoïdal particles

Several works have been done to establish the rheological behaviour of slurries. Sasaki [7]had used the dilating fluid model (shear thickening)for the ice slurry flow, however Egolf [8] has used the Bingham model which was taken by many authors such as Hansen [9], Frei [10] and Sari [11].The rheological behaviour of the mixture is described by a constitutive relation between the shear stress and the strain rate (velocity gradient) as follows:

$$\tau = \mu \frac{du}{dy} \tag{3}$$

Where u denotes the axial mixture velocity in the channel and μ the viscosity of the slurry.For non Newtonian flows , the viscosity is a function of the shear velocity. The **Table 2** presents some rheological models which can be used to describe the flow behavior of the paraffin slurry.

Table 2
Some rheological models for suspension flow [16]

Bingham	$\tau = \tau_B + \mu_B \frac{du}{dy}$
Power law	$\tau = K(\frac{du}{dy})^n$
Casson (1959)	$\tau^{0,5} = \tau_C^{0,5} + (\mu_C \frac{du}{dy})^{0,5}$
Herschel – Bulkley (1987)	$\tau^n = \tau_0^{n_1} + K_1(\frac{du}{dy})^{n_2}$

In this paper, we have used the Papanastasiou rheological model [12] which can describe the flow behaviour for different shear rate values by the following equation:

$$\tau = \tau_0 (1 - \exp(-m \frac{du}{dy})) + \mu_0 \frac{du}{dy} \tag{4}$$

m denotes the constant time. When m is equal to zero, the Newtonian shear rate is obtained, but for m values approaching infinity the Bingham fluid occurs. The Newtonian behaviour occurs also when τ_0 is zero, τ_0 denotes the yield stress.
As the constant time m is related to the particles concentration C in the carrier fluid, we can say according to the **Figure 3**, that the paraffin slurry behaves as Newtonian fluid for low concentration values(pure water (0%) and 3% slurries), however,the Binghamian behaviour is observed for higher values(6% and 12% paraffin sluries).

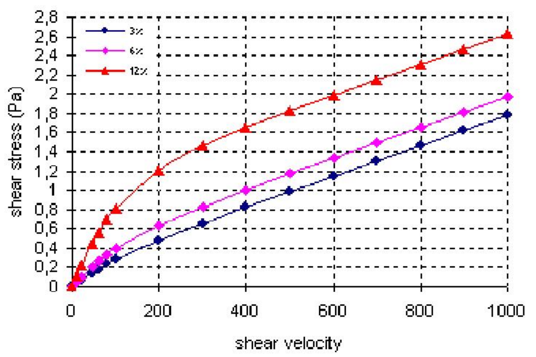

Figure 3 Shear stress versus shear velocity for a paraffin slurry for 3%, 6% and 12% particles concentrations

In an effort of interpretation of the origin of the yield stress,many authors said that it can be interpreted as the minimum stress which overcomes the interaction forces between solid particles and breaks the inter-particles links. The total amount of energy dissipated in the fluid includes both viscous energy dissipation and the energy necessary for particles separation. This implies that τ_0 is directly linked to the rate of formation of particle–particle contacts.

PRESSURE DROP

In the present work, the Binghamian model has been used for describing the rheoligical behaviour of the paraffin slurry. For an homogenous and steady isothermal slurry flow the Darcy-Weissbach equation gives the pressure drop as follows:

$$\Delta p = \lambda \frac{L}{D_h} \frac{\rho v^2}{2} \tag{5}$$

The coefficient λ denotes the friction factor; it can be evaluated using the empirical equation for the Binghamian fluid in laminar flow [13]:

$$\lambda = \frac{64}{\text{Re}}[1 + \frac{He}{6\,\text{Re}} - \frac{He^4}{3\lambda^3\,\text{Re}^4}] \tag{6}$$

The Hedström number He is given as:

$$He = \frac{\rho D_h \tau_0}{\mu_0^2} \tag{7}$$

For the turbulent flow, the friction factor is calculated using the following equation published by Govier and Aziz [14]:

$$\frac{1}{\sqrt{\lambda}} = 2.265 \log_{10}[(1 - \frac{\tau_0}{\tau_w}) \text{Re} \sqrt{\lambda}] \tag{8}$$

The pressure drop of laminar Binghamian slurry flows can also be estimated based on the following approximation of the Buckingham equation (cited by Wasp,1977):

$$\frac{\Delta p}{\Delta L} = \frac{16}{3} \frac{\tau_0}{D_h} + \mu_0 \frac{32 V_m}{D_h^2} \tag{9}$$

The first validation test was done for pure water, in **figure 4** a comparison between numerical and experimental data shows that they are in good agreement and this validate the used model.

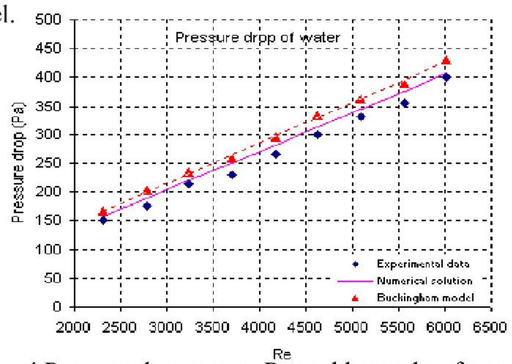

Figure 4 Pressure drop versus Reynolds number for water (Experimental and numerical results for validation)

The **Figure 5** gives the calculated pressure drop compared to the experimental results for different values of concentration 0%, 6% and 12%.

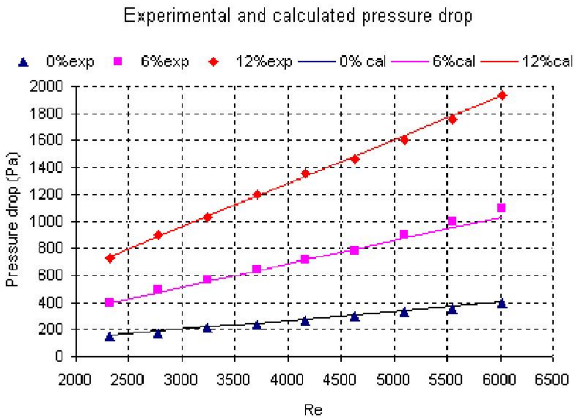

Figure 5 Pressure drop versus Reynolds number (Experimental and numerical results for water, 6% and 12% slurries)

As it is shown in the **Figure 6**, when the concentration decreases and the flow rate increases the pressure drop increases also, this means that the optimisation of the pumping power must be taken into account by choosing the best values of average concentration and flow rate in the channel for minimisation of the pressure drop, if we choose 6% for example, we can say that the pressure drop increases of about 2 times and half when the flow rate goes from 500Kg/h to 1500Kg/h.

The pressure drop is an important quantity in the design of pumping systems for slurries and liquid-solid flows. In addition to this pressure drop, settling of particles is an important concern for economical transport of liquid-solid flows because settling particles generally need additional pumping power, flow blockage through restricted sections, and other difficulties. In the present work, we have used non settling paraffin particles so there is no need to more supplied power to make the mixture in a motion state.

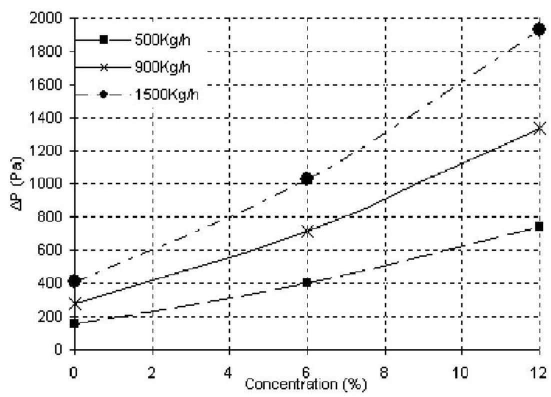

Figure 6 Pressure drop versus concentration for different flow rate values

CONCENTRATION DISTRIBUTION OF PARTICLES

Paraffin particles in water may be transported in various ways, the most significant of which are as follows: Convection, Diffusion and Sedimentation.

Convection is simply the movement of the particles as a result of flow, whereas diffusion and sedimentation cause particles to depart from flow lines. These types of motion are hindered by fluid drag. The particles are supposed rigid spheres of homogeneous diameter d_p (mean diameter) that is sufficiently small compared with the characteristic dimension of flow. Whenever a particle moves in a fluid, a drag force, F_D, is experienced. This is most conveniently presented as a product of two terms: a force term and a dimensionless drag coefficient, C_D. The force term is just the dynamic pressure multiplied by an effective area of the particle, *S*. Thus

$$F = \frac{1}{2}\rho_l S u^2 C_D \qquad (10)$$

The drag coefficient C_D is a function of the Reynolds number, *Re,* given by the following formula:

$$\mathrm{Re} = \frac{\rho_l d_p u}{\mu_l} \qquad (11)$$

Care has to be taken in choosing an appropriate area, S, because the drag coefficient depends greatly on how S is defined, especially for highly asymmetric shapes, such as fibers. The simplest procedure, and the one adopted here, is to define S as the projected area of the particle on a plane normal to the direction of flow. So, for a sphere, with diameter d_p,

$$S = \frac{\pi d_p^2}{4} \qquad (12)$$

The dependence of drag coefficient on the Reynolds number is of paramount importance.

At low values of Reynolds number*,* viscous effects dominate over inertial effects and flows are orderly and laminar. As Re increases (by increasing fluid velocity) inertial forces become more important, leading to vortices and, eventually, to turbulence.

For very low Reynolds numbers (Re<0.1), the so-called "creeping flow" conditions
apply and the drag coefficient for a sphere takes a simple form, first derived by Stokes:

$$C_D = \frac{24}{\mathrm{Re}} \qquad (13)$$

This expression is good enough for most particles in natural waters, but in those cases where the Reynolds number is too large for the Stokes expression to apply, the drag coefficient has to be obtained in a different manner. There is no fundamental theory that gives the drag coefficient under non-creeping flow conditions. Instead, we have to rely on experimentally determined values and empirical equations. For spheres, and Re values in the range of 1–100, the following expression gives a good approximation to the drag coefficient:

$$C_D = \frac{24}{\mathrm{Re}} + \frac{6}{1+\sqrt{\mathrm{Re}}} + 0,4 \qquad (14)$$

The presence of buoyancy force due to the gravitational field can further complicate the transport phenomena in the flow of PCM suspensions. There have been very few studies concerning buoyancy-driven convection of PCM suspensions. The diffusion phenomenon and the buoyancy make particles moving upward, a gradient of particles concentration in the y ascendant direction appears.
Based on the mass transport of the dispersed particles in the continuous fluid, we can express the conservation equation for the dispersed particles as follows:

$$\frac{\partial(\rho C)}{\partial t}+\nabla(\rho C\vec{v}_m)=-\nabla(\rho D_c\nabla C)+\nabla(\rho C(1-C)\vec{V}_r) \qquad (15)$$

Where Dc is the local diffusion coefficient associated with Brownian diffusion of the suspension particles, which is usually much smaller than the thermal diffusivity. The second term on the right-hand side of the equation (15) accounts for the mass flux due to the gravitational lift of the paraffin particles in the liquid, it is related to the relative velocity v_r of the particles which is expressed as:

$$\vec{v_r}=\frac{\rho_p-\rho_m}{18\mu_l f_{drag}}d_p^2(\vec{g}-\frac{\partial\vec{v_m}}{\partial t}-\vec{v_m}\nabla\vec{v_m}) \qquad (16)$$

And the drag function f_{drag} is calculated by using Schiller and Naumann equations [15]:

$$f_{drag}=1+0.15\,\mathrm{Re}_p^{0.687} \quad \text{if} \quad \mathrm{Re}_p\le 1000 \qquad (17)$$

And $$f_{drag}=0.0183\,\mathrm{Re}_p \quad \text{if} \quad \mathrm{Re}_p\succ 1000 \qquad (18)$$

The inlet velocity profile (for x=0) is supposed to be parabolic for laminar flow [16].

For low Reynolds numbers values, the relative velocity term is not so important and the second term on the right-hand side of the equation (15) could be neglected, then this equation projected on the y axis for an isothermal steady state, becomes:

$$D_c\frac{d^2C(y)}{dy^2}+C(y)\frac{dv(y)}{dy}+v(y)\frac{dC(y)}{dy}=0 \qquad (19)$$

The equation (19) can be similar to that one given by Schmidt and Rouse and cited by Nasr-El-Din and Shook [17] when v(y) approaches the local hindered terminal velocity w:

$$D_c\frac{d^2C(y)}{dy^2}+w\frac{dC(y)}{dy}=0 \qquad (20)$$

The diffusion coefficient of a spherical particle with diameter d_p, is related to temperature and the liquid's viscosity; it can be given by the Stokes-Einstein equation:

$$D_c=\frac{k_BT}{3\pi d_p\mu} \qquad (21)$$

Where k_B is the Boltzmann constant and *T* is the absolute temperature.Doron has given the diffusion coefficient D_c by the following equation [18]:

$$D_c=0,052\sqrt{\frac{f_i}{8}}D_h u \qquad (22)$$

Here f_i denotes the friction factor suggested by Doron[17] and evaluated from the Colebrook formula which is expressed as:

$$\frac{1}{\sqrt{2f_i}}=-0,86Ln(\frac{d_p}{3,7D_h}+\frac{2,51}{\mathrm{Re}\sqrt{2f_i}}) \qquad (23)$$

Numerical resolution of equation (19) gives the particle's concentration distribution in paraffin slurries with average mass fraction of 6% (**Figure 7**) and 12% (**Figure 8**) for some mean velocity values.

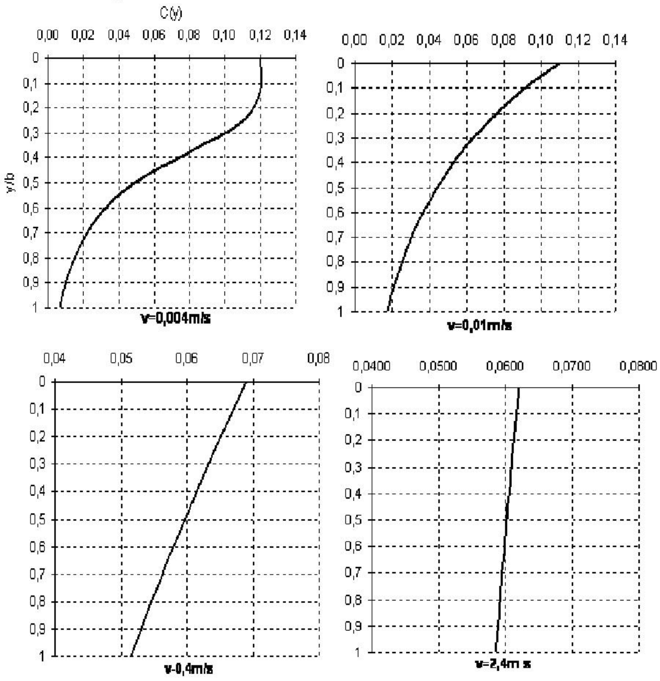

Figure 7 : Concentration distributions for 6% paraffin slurry

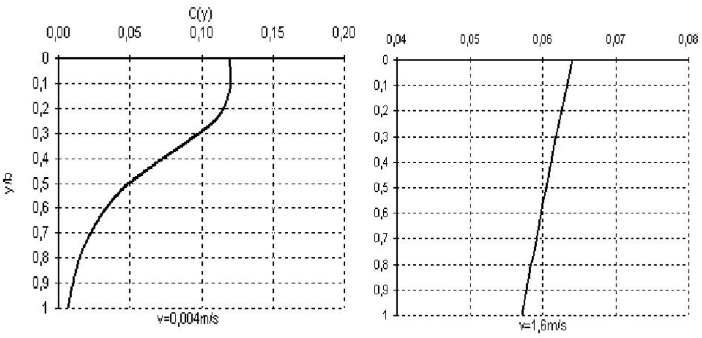

Figure 8 : Concentration distributions for 12% paraffin slurry

For an average velocity of about 0,004ms^{-1} ,a moving bed appears both in an average concentration C_m of 6 and 12 %.However, when the velocity increases the heterogeneous flow occurs , it becomes nearly homogenous for high values of about 2,4m/s.

PARAFFIN SLURRY'S EFFECTIVE VISCOSITY

So many different models can be used to describe the average slurry's viscosity such us Ball –Richmond's equation and Krieger-Dougherty's theory [5]. However, none of them

considers the slurry as heterogeneous mixture or even a flow with a moving or a stationary bed.
In this paper, a model based on Thomas equation of viscosity was used to evaluate the average viscosity of paraffin slurry flowing between two horizontal flat plates with a rectangular cross section S.The mixture's viscosity can be expressed using the equation of Thomas (equation (2)) as a function of C(y):

$$\mu_m(y) = \mu_l \left(1 + 2,5C(y) + 10,05C^2(y) + 0,00273\exp(16,6C(y))\right)$$

The average viscosity of the slurry is calculated by integration through the cross section S using the following expression:

$$\overline{\mu}_m = \frac{1}{S}\int_S \mu_m(y)dS = \frac{1}{b}\int_0^b \mu_m(y)dy \qquad (24)$$

The **Figure 9** shows the effective viscosity variations depending on velocities. It is clear that for low concentration values the viscosity is almost independent of the average velocities and when the concentration increases and the velocity decreases the viscosity increases rapidly.

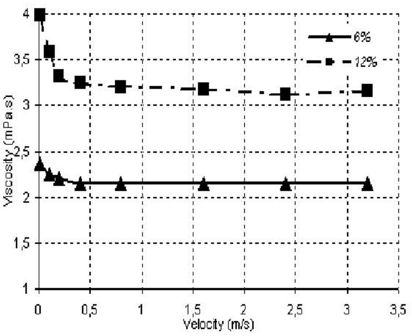

Figure 9 Paraffin slurry viscosity as function of velocity

Concerning the effective viscosity variations depending on concentrations for some temperature values, the **Figure 10** shows that at the same temperature, the viscosity increases when the mass fraction of particles increases, and for increasing temperature the viscosity decreases, this explains that when paraffin particles are frozen (for temperatures less than the melting temperature: about 7^0C) the mixture becomes more viscous and the pumping supplied power should increase for such cases.

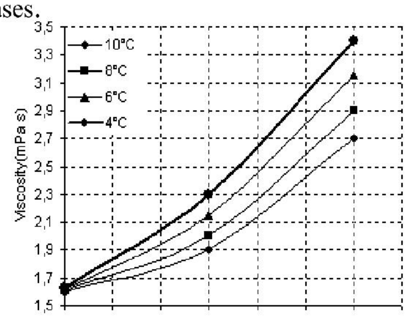

Figure 10 Paraffin slurry viscosity as function of mass fraction

CONCLUSION

In this study, the rheological behaviour of the paraffin and water mixture have been analyzed and the Binghamian model was validated by comparison of the experimental and the calculated results of pressure drop for water and paraffin slurries with 6% and 12 % average mass fraction.
The gradient of concentration caused by the buoyancy effects has been studied and the paraffin concentrations presented. For low concentration's values where stationary and moving beds are involved, or even for heterogeneous flows, so many repercussions would affect the heat transfer in the heat exchanger.

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