HEFAT2010
7th International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics
19-21 July 2010
Antalya, Turkey

DATA ASSESSMENT OF TWO ERCOFTAC TRANSITION TEST CASES FOR CFD VALIDATION

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ABSTRACT

Turbulence model developers rely on established databases to benchmark advances in turbulence closure methods for Computational Fluid Dynamics (CFD). The European Research Community on Flow, Turbulence and Combustion (ERCOFTAC) provides databases to benchmark different flow regimes. Sub-group 5 of the Special Interest group of Ercoftac (SIG10) on Transition Modelling manages experimental real-flow data to validate the predictive capabilities of transition codes. With experimental data, there is a legitimate expectation for the database user to apply some judgment to overcome acceptable accuracy and consistency issues in the dataset. This work aims to document some of the precautions that are required to interpret and use the Ercoftac dataset to calibrate CFD codes. This paper aims to demonstrate the use of the Ercoftac database to validate a two-dimensional CFD transitional solver, giving more details of the approach adopted, and how to use the Ercoftac experimental data to generate the inflow condition, and in doing so to inform the community on the consistency of the information in this dataset by testing it for the known trends in the mean velocity components and their statistical fluctuations. The objectives are to expose some limitations of the dataset and to evaluate the performance of the RANS transition model implemented in the in-house code Cosmic at the University of Leicester. Velocity measurements from two boundary layer test cases under zero pressure gradient are considered.

INTRODUCTION

Transition is a complex phenomenon, defined as the whole process of change from laminar to turbulent flow. The origin of turbulence and the accompanying transition from laminar to turbulent regime, as often happens, for example, on aircraft wings or past turbine blades, is of fundamental importance for the whole science of fluid mechanics. The European Research Community on Flow, Turbulence and Combustion (ERCOFTAC) is a scientific association of research, education and industry groups in the technology of flow, turbulence and combustion. The Special Interest group of Ercoftac (SIG10) on Transition Modelling is organized into five sub-groups, focussing on different aspects of transition modelling. In particular Sub-group 5 focuses on experimental real-flow data in order to provide a sufficiently experimental database to validate the predictive capabilities of transition codes.

NOMENCLATURE

C_f	[-]	Skin-friction coefficient ($C_f=\tau_w/0.5u^2_\infty\rho_\infty$)
H	[-]	Shape factor
P	[Pa]	Pressure
Pr	[-]	Prandtl number
R	[$Jkg^{-1}K^{-1}$]	Ideal gas constant
Re_θ	[m]	Momentum thickness Reynolds number
S_{ij}	[s^{-1}]	strain-rate-tensor components
T	[K]	Temperature
Tu	[%]	Turbulence intensity $Tu = 100\sqrt{2k/3u^2_\infty}$
U,V	[ms^{-1}]	Velocity components
u',v',w'	[ms^{-1}]	Velocity fluctuations
x	[m]	Cartesian axis direction
y	[m]	Cartesian axis direction
y^+	[-]	$(yu_\tau)/\nu$
δ	[m]	Boundary layer thickness
δ^*	[m]	Displacement thickness
γ	[-]	Intermittency factor, specific heat ratio
θ	[m]	Momentum thickness
ν	[m^2s^{-1}]	Kinematic viscosity

Subscripts

w	Wall conditions
∞	Free stream conditions

PHYSICS OF TRANSITION

Transition affects strongly the evolution of losses and other factors of practical significance, such as the distributions of wall shear stress and surface heat transfer. It is accompanied by many changes in flow characteristics, such as an abrupt change in the law of resistance, and both the skin friction and heat transfer may increase considerably. The most important feature of the phenomenon of transition is the increased diffusivity in the flow. The main reason for its complexity is that, besides the simultaneous presence of laminar and turbulent flow, there is also the interaction between the two regimes. Furthermore, transition in boundary layers is affected by several parameters, such as the streamwise pressure gradient, noise, the Reynolds number, the surface roughness, the wake unsteadiness and the turbulence intensity. These make its prediction very difficult.

In aerodynamic flows, transition is typically the result of flow instability (Tollmien-Schlichting waves or cross-flow instability). Linear instability theory describes at which streamwise distance small disturbances in the boundary layer become unstable and thus amplify. Where the resulting exponential growth of two-dimensional waves results in finite amplitude waves, the linear theory ceases to be valid. During the growth of the waves, spanwise distortions and three-dimensional non-linear interactions become relevant. Finally, areas of turbulence, denoted as turbulent spots, start to develop in the streamwise direction. These spots grow in the streamwise and spanwise directions until the flow is completely turbulent and thus transition is completed. This instability becomes three-dimensional and non-linear by the formation of vortex loops and eventually results in a non-linear break-down to turbulence. This is often referred to as natural transition. It usually occurs at a low Free-Stream-Turbulence (FST) level < 0.8%. This process of transition gradually appears after a critical value of Reynolds number (Re) is exceeded. In a low-disturbance environment, such as the one found in free flight and in some marine applications, quasi-two-dimensional Tollmien-Schlichting (TS) waves precede transition. Various receptivity processes are responsible for generating these instabilities, which are probably initially three-dimensional and randomly distributed. However, quasi-two-dimensional TS waves are preferentially amplified in the boundary layer. Further downstream, the amplitude of the most amplified waves becomes sufficiently large for nonlinear effects to become important.

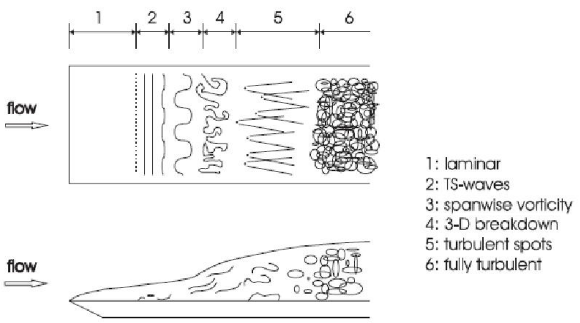

Figure 1 The process of natural transition.

In turbomachinery applications, the main transition mechanism is bypass transition imposed on the boundary layer by high levels of turbulence in the freestream. In this case, low-frequency oscillations in the streamwise velocity appear in the boundary layer. These oscillations are due to streamwise streaks of alternating high and low-velocity and flow visualization studies have shown that the streaks meander slowly sideways and thereby give rise to the observed low-frequency variations. If the energy of the streamwise velocity fluctuations is measured in the boundary layer, it is found to have its maximum in the centre of the boundary layer and exhibit an initial linear amplification with downstream distance, in contrast to amplified TS waves which grow exponentially. The idea behind bypass transition is that the disturbances in the flow cause laminar fluctuations in the boundary layer that initiate spots, or that disturbances are strong enough to enter the boundary layer and initiate turbulent spots immediately. For transition at high freestream turbulence levels (>1%), the first and possibly the second and third stages of the natural transition process may be bypassed, such that turbulent spots are directly produced within the boundary layer by the influence of freestream disturbances. As the spot is born, it grows in the streamwise direction, while the leading edge velocity is larger than trailing edge. Also enlargement in lateral direction takes place. Due to this growth, spots start to overlap each other and thus merge until a complete turbulent boundary layer is obtained. The occurrence of Tollmien-Schlichting waves, spanwise vorticity and three dimensional breakdown is "bypassed", which explains the name. In bypass transition, the turbulent spots are generated more towards the leading edge of a plate (or turbine blade) compared to natural transition.

NUMERICAL METHOD

The intermittency approach to predict transition, which is favoured by the turbomachinery community, is to use the concept of intermittency, to blend together laminar and turbulent flow regimes, based on empirical correlations. Many detailed investigations of the process of transition reveal that, over a certain range of Reynolds numbers around the critical value, the flow becomes intermittent, which means that it alternates in time between being laminar and turbulent. The physical nature of this flow can be properly described with the aid of the intermittency factor γ, which is defined as the fraction of time during which the flow at a given position remains turbulent. By letting the intermittency grow from zero to unity, the start and the evolution of transition can be imposed. Mostly, this is done by multiplying the eddy viscosity in a two-equation turbulence model by the intermittency factor. In other words, once γ is determined, it is multiplied by the eddy viscosity in the mean-flow equations. In the pre-transitional regime, γ is set to zero and γ assumes a positive value only where the model is required to initiate transition. Suzen & Huang [1] developed an intermittency transport model that can produce both the experimentally observed streamwise variation of intermittency and a realistic profile in the cross-stream direction. The model combines the transport equation models of Steelant & Dick [2] and Cho & Chung [3]. Specifically, the transport of intermittency, γ, is given:

$$\frac{\partial(\rho\gamma)}{\partial t}+\frac{\partial(\rho u_j\gamma)}{\partial x_j}=D_\gamma+S_\gamma \qquad 1$$

where D_γ and S_γ are the diffusion and source term respectively:

$$D_\gamma=\frac{\partial}{\partial x_j}\left\{\left[(1-\gamma)\sigma_{\gamma l}\mu+(1-\gamma)\sigma_{\gamma t}\mu_t\right]\frac{\partial\gamma}{\partial x_j}\right\} \qquad 2$$

$$S_\gamma=(1-\gamma)[(1-F)T_0+F(T_1-T_2)]+T_3 \qquad 3$$

and F is a blending function. The first term, T_0, derives from the model of Steelant and Dick [2],

$$T_0 = C_0\rho\sqrt{u_k u_k}\beta(x) \qquad 4$$

where β(x) = 2f(x)f'(x) and the function f(x) is the following polynomial interpolation function for $\hat{n}\sigma(Re_x - Re_{xt})^2$ around the point x_t of transition onset:

$$f(x) = \frac{ax'^4 + bx'^3 + cx'^2 + dx' + e}{h_1 x' + h_2} \qquad 5$$

with x= x'- x_t, and

$$a = 50\sqrt{\frac{n\sigma}{u}}\,, b = -0.4906\,, c = 0.204\left(\frac{n\sigma}{u}\right)^{-0.5}, \qquad 6$$
$$d = 0\,, e = 0.04444\left(\frac{n\sigma}{u}\right)^{-1.5}, h_1 = 50, h_2 = 10e$$

The T1, T2 and T3 terms are derived from the model of Cho and Chung [3] and are given respectively as:

$$T_1 = \frac{C_1\gamma}{k}\tau_{ij}\frac{\partial u_i}{\partial x_j} \qquad 7$$

$$T_2 = C_2\gamma\rho\frac{k^{1/2}}{\beta^*\omega}\frac{u_i}{\sqrt{u_k u_k}}\frac{\partial u_i}{\partial x_j}\frac{\partial \gamma}{\partial x_j} \qquad 8$$

$$T_3 = C_3\rho\frac{k}{\beta^*\omega}\frac{\partial \gamma}{\partial x_j}\frac{\partial \gamma}{\partial x_j} \qquad 9$$

The blending function, F, provides a smooth passage between the two models in the transition region. It is evaluated as a function of the ratio $k/(S\nu)$, where $S = \sqrt{2S_{ij}S_{ij}}$:

$$F = tanh^4\left[\frac{k/(S\nu)}{200(1-\gamma^{0.1})^{0.3}}\right] \qquad 10$$

The above equation is based on the correlation due to Klebanoff for the distribution of γ in the normal direction to the wall. It renders active the model of Steelant and Dick close to the wall, whereas it applies the model of Cho and Chung in the outer region. The values of the coefficients employed for the present model are:

$$\begin{cases} \sigma_{\gamma l} = 1, \sigma_{\gamma t} = 1 \\ C_0 = 1, C_1 = 1.6, C_2 = 0.16, C_3 = 0.15 \end{cases} \qquad 11$$

By letting the intermittency grow from zero to unity, the start and the evolution of transition can be imposed. Mostly, this is done by multiplying the eddy viscosity in a two-equation turbulence model by the intermittency factor. In other words, once γ is determined, it is multiplied by the eddy viscosity in the mean-flow equations. Turbulent quantities are predicted by using the Wilcox two-equation $k-\omega$ turbulence model [4]. In the pre-transitional regime, γ is set to zero and γ assumes a positive value only where the model is required to initiate transition.

This approach neglects the interaction between the turbulent and non-turbulent parts of the flow during transition. In order to capture this interaction, a conditional averaging technique leading to a set of turbulent and a set of non-turbulent equation for mass, momentum and energy is necessary, as used by Steelant & Dick. The conditional averaging is usually seen as too computationally expensive for engineering applications, as the number of equations doubles. Therefore, the intermittency concept is typically used in combination with globally averaged Navier-Stokes equations and the loss of some physical information is accepted. While the intermittency transport equation defines the intermittency distribution for transitional flows in the simulation, the onset of transition is defined by correlations. The onset of attached flow transition is determined as a function of the turbulence intensity, Tu, for zero pressure gradient. Specifically, for by-pass transition, the Abu-Ghannam & Shaw [9] correlation is used:

$$Re_{\theta t} = 163 + exp(6.91 - Tu) \qquad 12$$

This relation implies the transition start always occurs at a momentum thickness Reynolds number that is 163 or larger. This lower limit stems from the Tollmien-Schlichting stability limit. The length of the transition region is obtained from the Suzen correlation, Eq. 13, based on extensive experimental analysis of key turbulent parameters.

$$\hat{n}\sigma = \left(\frac{n\nu^2}{U^3}\right)\sigma = 1.8 * 10^{-11}Tu^{7/4} \qquad 13$$

DESCRIPTION OF THE TEST CASES

The test cases chosen to provide initial profiles and to validate the results of the transitional flow solver are ERCOFTAC Transitional Flow Benchmark cases T3A- and T3A of Roach & Brierley [5], featuring hot-wire traverses of a 1.7m long flat plate transitional two dimensional isothermal boundary layer flow with zero pressure gradient. The case T3A- has a freestream velocity of 19.8 m/s and a FST level of 0.9% and test case T3A has a freestream velocity of 5.1 m/s and a higher FST level of 3.3%. T3A- and T3A are classified as the low and moderate FST intensity cases respectively.

The computational domain is made up of 260352 rectangular computational cells, with 1356 cells in the x-direction and 192 cells in the y-direction. The total length of the test section is 1.4m. For accuracy, grid points are clustered in regions of large gradients, but spread out elsewhere for economy. The stretching factor has been chosen in order to have a value of $y+ < 0.2$ for the first point away from the wall, and such that the viscous region of the boundary layer ($y+ < 30$) contains more than 40 cells (see Figure 2).

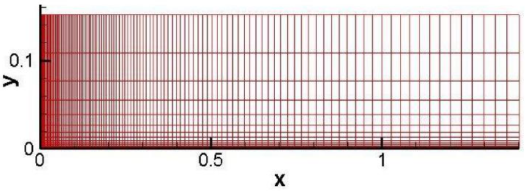

Figure 2 Sketch of computational mesh: For clarity, one point every 12 in both the x and y directions is shown, and the x-axis to y-axis ratio is 0.4.

At the streamwise distance of x=495mm and x=95mm from the leading edge, for T3A- and T3A respectively, a laminar profile (γ=0) is imposed as the inflow of the computational domain. As far as the inflow boundaries in the computation, these are derived from a numerical solution of the Blasius equation that is re-scaled to match the experimental laminar boundary layer parameters, see Table 1. Inflow profiles of specific turbulent kinetic energy and specific turbulent kinetic energy dissipation are imposed at the inflow because transition is strongly affected by the decay of turbulence in the free-stream. Therefore, it is very important for a correct analysis of the results to assign the appropriate inlet boundary condition. It is not possible to start from experimental initial conditions from k and ω derived directly from tabulated hot-wire probe traverse results in this Ercoftac database, both for the uncertainty of the experimental data and because it is not easy to estimate the dissipation profile starting from velocity data. The inlet turbulent kinetic energy is fixed according to the freestream turbulence level. Initially γ is set to zero throughout the flow field. On solid wall boundaries and at the freestream, a zero gradient of γ is assumed. On outflow boundaries γ is extrapolated from inside the domain to the outer boundaries.

		C_f	δ^* [mm]	θ [mm]	Re_θ	H
T3A	Experimental	0.003723	0.882	0.344	117.4	2.562
	Numerical	0.003731	0.918	0.354	118.1	2.593
T3A-	Experimental	0.00078	1.083	0.416	593.20	2.606
	Numerical	0.00080	1.091	0.425	594.30	2.611

Table1 Boundary layer parameters at inflow, 95mm, for the T3A case and at 495mm for the T3A- case.

The temperature distribution across the boundary layer was determined using Crocco's law. The adiabatic wall temperature due to frictional heating is given by:

$$T_w = T_\infty \left[1 + \sqrt{P_r}\frac{\gamma - 1}{2}\left(\frac{\overline{U}}{\sqrt{\gamma RT}}\right)^2\right] \qquad 14$$

The walls were modelled as adiabatic by the use of the temperature recover factor, the square root of Prandlt number. The static pressure was imposed as constant across the boundary layer.

The in-house multi-block finite volume time-resolved RANS Cosmic scheme is used to compute the seven conservative variables (density, specific momentum, specific stagnation energy, specific turbulent kinetic energy, specific turbulent dissipation rate, and intermittency) that define the flow state in the discretized Navier-Stokes equations. The code uses a two-dimensional finite-volume four-point stencil approximate Riemann solver with a Monotone Upstream-centred Scheme for Conservation Laws (MUSCL) interpolation by Van Leer et al. [5] to compute the convective fluxes. This gives up to a third order accurate reconstruction of the spatial gradients in regions of smooth flow. To prevent numerical oscillations in region of rapidly changing flow, the spatial gradients are limited by the introduction of a Total Variation Diminishing Scheme (TVD) by Sweby [6]. The scheme is explicit and a standard multi-stage second-order Runge-Kutta (RK) integration is used to time-march the flow. The turbulent flux vector is estimated using a second-order accurate gradient reconstruction method, based on the Gauss divergence theorem.

DATABASE INTEGRITY

The database is a valuable tool for the transitional flow community. Still, in both datasets, some discrepancies were found from what is stated in the description of the experiment and in the database information, specifically with respect to the normal velocity component, the turbulence isotropy, and the pressure gradient.

In the Ercoftac database, case T3A, as well as in the case T3A-, the sign of the cross-stream velocity u_2 is negative, as shown in Figure 3(a) and 3(b) respectively. However, u_2 should be positive definite, as the displacement thickness grows monotonically along the flat plate, giving a negative $\partial u_1/\partial$ x and a positive $\partial u_2/\partial y$ so that the incompressible continuity equation (2) is satisfied.

$$\frac{\partial u_i}{\partial x_i} = 0, \qquad i = 1, 2 \qquad 15$$

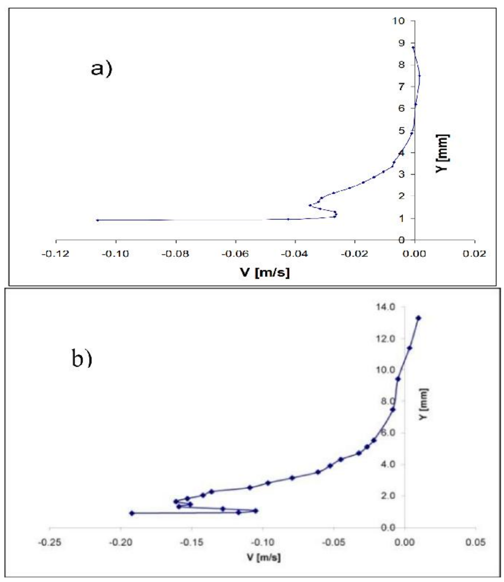

Figure 3 Cross-stream velocity measurements for T3A (a) and T3A- (b).

Moreover, the first data points closest to the wall in Figure 3(a) and 3(b) are suspect; the velocity magnitude above these points decreases monotonically, whereas at these points the trend reverses; it is only possible to assume that the odd behaviour of u_2 near the wall is associated with measurements problems with the hot-wire when this was very close to the flat plate surface. Therefore, they should not be inserted in the dataset. The wall-normal velocity data was re-interpreted invoking the principle of conservation of mass and using the streamwise velocity profile. It was concluded that, by reversing the sign of the wall-normal velocity and removing suspect measurements close to the wall, this field can be made to be physically consistent with the measured streamwise velocity profile.

As far as the free-stream turbulence is concerned, it is possible to verify that the turbulence is not isotropic, contrary to what is stated in the experimental details of the database. The database reports that the generated turbulence is extremely homogeneous and isotropic, with a streamwise-to-normal fluctuating velocity ratio of about 1.005 by means of turbulence generating grids. But looking at table 2 that reports the data taken for the database away from the wall, it is noticeable that the ratio of streamwise-to-normal fluctuating velocity is quite far away the ratio of 1.005, considering the Cross-Wire (CW) anemometry data.

It is shown that the inflow Reynolds stresses have an important statistical anisotropy that may lead to poor agreement with predictions from turbulence models that use the Boussinesq approximation and may show improvements by using an explicit Reynolds stress model.

	X. [mm]	δ [mm]	Y [mm]	u' [m/s]	v' [m/s]	w' [m/s]	u'/v' [m/s]	u'/w' [m/s]
T3A	495	3.33	13	0.130	0.1	0.117	1.300	1.111
	695	3.97	16	0.108	0.089	0.098	1.212	1.102
T3AM	95	2.71	8.78	0.124	0.101	0.118	1.227	1.051
	195	4.17	13.30	0.112	0.096	0.105	1.170	1.066

Table2 Fluctuating velocity and their ratio at two different boundary layer thickness for the two test cases.

The next item that was examined is the pressure gradient, which should be dp/dx = 0 for the two test cases. In the wind tunnel where the measurements were taken, the bottom wall can be inclined to produce a zero pressure gradient on the test surface, which is hung from the ceiling of the working section. Figure 4 shows the trend of the pressure along the flat plate using the experimental values of density and temperature and applying simply the equation of state for the ideal gas $P = \rho RT$.

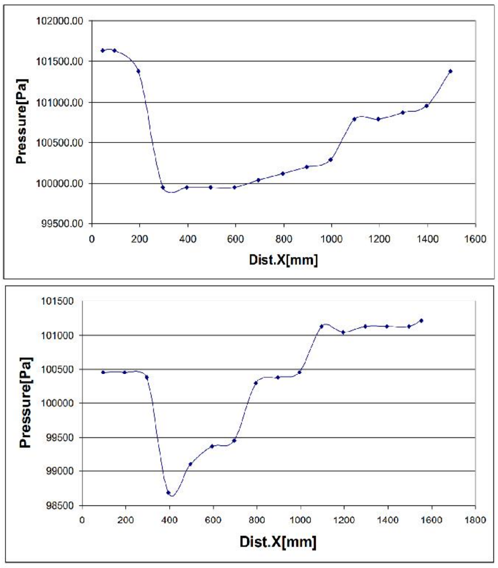

Figure 4 Pressure vs. distance in X for T3A case (top) and T3AM case (bottom)

In fact, the experimental data does not show a constant value of the free-stream velocity along the flat plate, as shown in Figure 5.

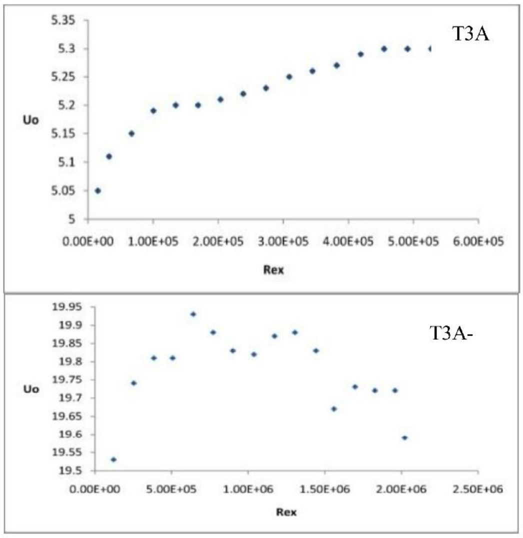

Figure 5 Free-stream velocity along the flat plate for both test cases.

The streamwise pressure gradient was also examined over the predominantly laminar portion of the boundary layer, using the integral method of Thwaites. This is an empirical relationship, based on the observation that a laminar boundary layer obeys the following relationship:

$$\frac{u_e}{\nu}\frac{\partial\theta^2}{\partial x} = 0.45 - 6\frac{\theta^2}{\nu}\frac{\partial u_e}{\partial x} \qquad 16$$

The above equation may be analytically integrated to find the momentum thickness θ. After θ is found, the following relations are used to compute the shape factor $H=\delta^*/\theta$

$$H = 2.61 - 3.75\lambda + 5.24\lambda^2 \quad for\ 0 \le \lambda \le 0.1$$

$$H = 2.472 + \frac{0.0147}{0.107 + \lambda} \quad for - 0.1 \le \lambda \le 0$$

where, $\lambda = (\theta^2/\nu)(\partial U/\partial x)$ is the dimensionless pressure gradient. This regression displayed a departure from the design test conditions of a zero streamwise pressure gradient. This departure may be due to the flat plate leading edge effect propagating some distance downstream.

NUMERICAL RESULTS

The computational result for the T3A case is shown in Figure 6. The predicted skin friction coefficients and shape factor variation have been compared with the experimental data. The shape factor indicates the region where the boundary layer tends to be turbulent. A reduction in H indicates transition is about to occurs. In a flow with zero pressure gradient, the shape factor before transition should be constant and equal to 2.61. The experimental data showed a premature decline of H from the laminar value, on the other hand, the computation showed a pure laminar flow before the transition onset. This can be explained by a non-zero streamwise pressure gradient in the flow field. Also, for the skin friction, the computational results are in good agreement with the experimental ones. The start of transition is taken as the location where the skin friction coefficient starts to deviate from its analytical laminar value. This is shown to be at $Re_x \approx 1.3*10^5$.

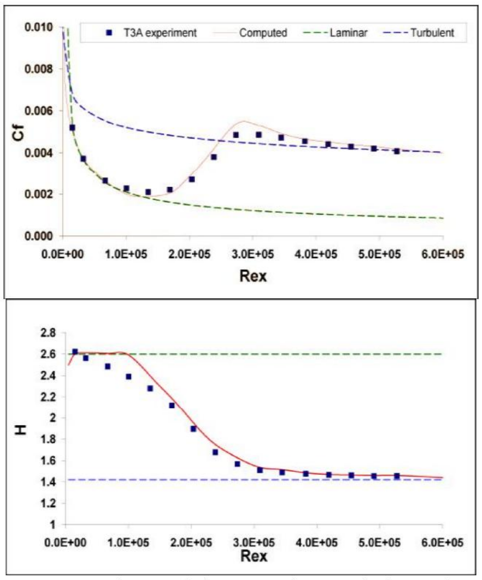

Figure 6 Comparison of the experimental shape factor and skin friction coefficient against computational results for the T3A test case.

As for the T3A- case, the predicted skin friction variation has been compared with the experimental data, in Figure 7. The model responds correctly to change in free-stream Tu; the location of transition moves downstream as Tu decreases. However, in the post-transition region of the T3A- case, the model under-predicts C_f because the low Tu makes the flow undergo natural transition and there is no appearance of a fully turbulent boundary layer in this region.

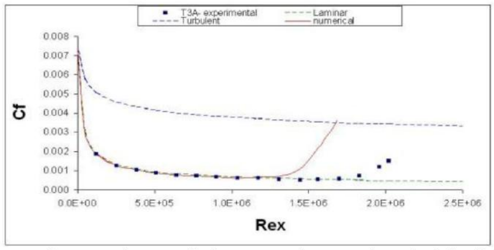

Figure 7 Comparison of the experimental and skin friction coefficient against computational results for the T3A- test case.

CONCLUSION

The Ercoftac database remains an established and very valuable tool for advancing transition model research. The dataset for the two test cases analyzed has to be used with care, so the practitioners should be aware of the issues encountered. Perhaps an extension of the explanatory notes accompanying the database should be done. This investigation helps turbulence model developers in their validation activities by showing how the data can be re-interpreted in a way that is consistent with the boundary layer governing equations and the known physics of these near-wall flows. The tested transition model works well at a moderate FST level and less well at a lower Tu. This different behaviour is probably due to the different physics of the two transition mechanisms in the two test cases. Possibly in the T3A- test case with a FST level of 0.9%, the influence of TS waves might be not be negligible.

ACKNOWLEDGMENTS

This research project has been supported by a Marie Curie Early Stage Research Training Fellowship of the European Community's Sixth Framework Programme under contract number MEST CT 2005 020301. I would like to express my sincere gratitude and appreciation to Prof. J.P. Gostelow for his support and invaluable advice.

REFERENCES

[1] Suzen, Y. B. and Huang, P. G., Model of Flow Transition Using an Intermittency Transport Equation, *Journal of Fluid Engineering*, Vol. 122, 2000, pp. 273–284

[2] Steelant, J. and Dick, E., Modeling of Bypass Transition with Conditioned Navier-Stokes Equations Coupled to an Intermittency Transport Equation, *International Journal for Numerical Methods in Fluids*, Vol. 23, 1996, pp. 193–220

[3] Cho, J. R. and Chung, M. K., A k-ε-γ equation turbulence model, *Journal of Fluid Mechanics*, Vol. 237, 1992, pp. 301–322

[4] Wilcox, D. C., *Turbulence Modeling for CFD*, 1993

[5] Roach, P.E. & Brierley, D.H., The influence of a turbulent free stream on zero pressure gradient transitional boundary layer development. Part 1: testcases T3A and T3B. Cambridge University Press, 1990

[6] Van Leer B., Thomas J., Roe P. and Newsome R., A Comparison of numerical flux formulas for Euler and Navier-Stoke equation, *AIAA 8th Computational Fluid Dynamics Conference*, Ed., No. 87-1104, 1987, pp. 36–41

[7] Sweby P., High resolution schemes using flux limiters for hyperbolic conservative laws, SIAM Journal of Numerical Analysis, vol.21, no.5, October 1984. pp. 995-1011